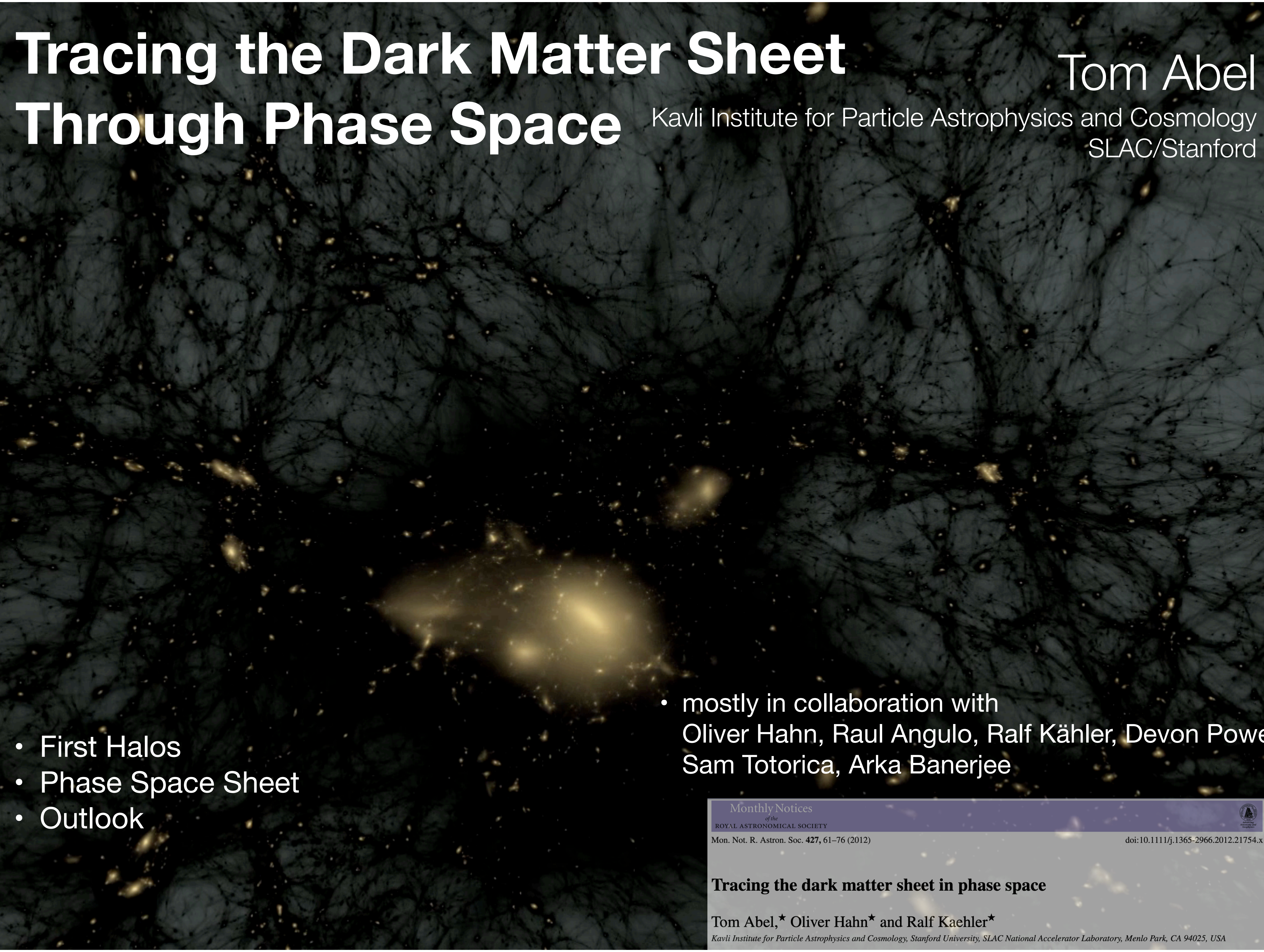
Tracing the Dark Matter Sheet Through Phase Space

Tom Abel
Kavli Institute for Particle Astrophysics and Cosmology
SLAC/Stanford



- First Halos
- Phase Space Sheet
- Outlook

- mostly in collaboration with
  Oliver Hahn, Raul Angulo, Ralf Kähler, Devon Powe
  Sam Totorica, Arka Banerjee

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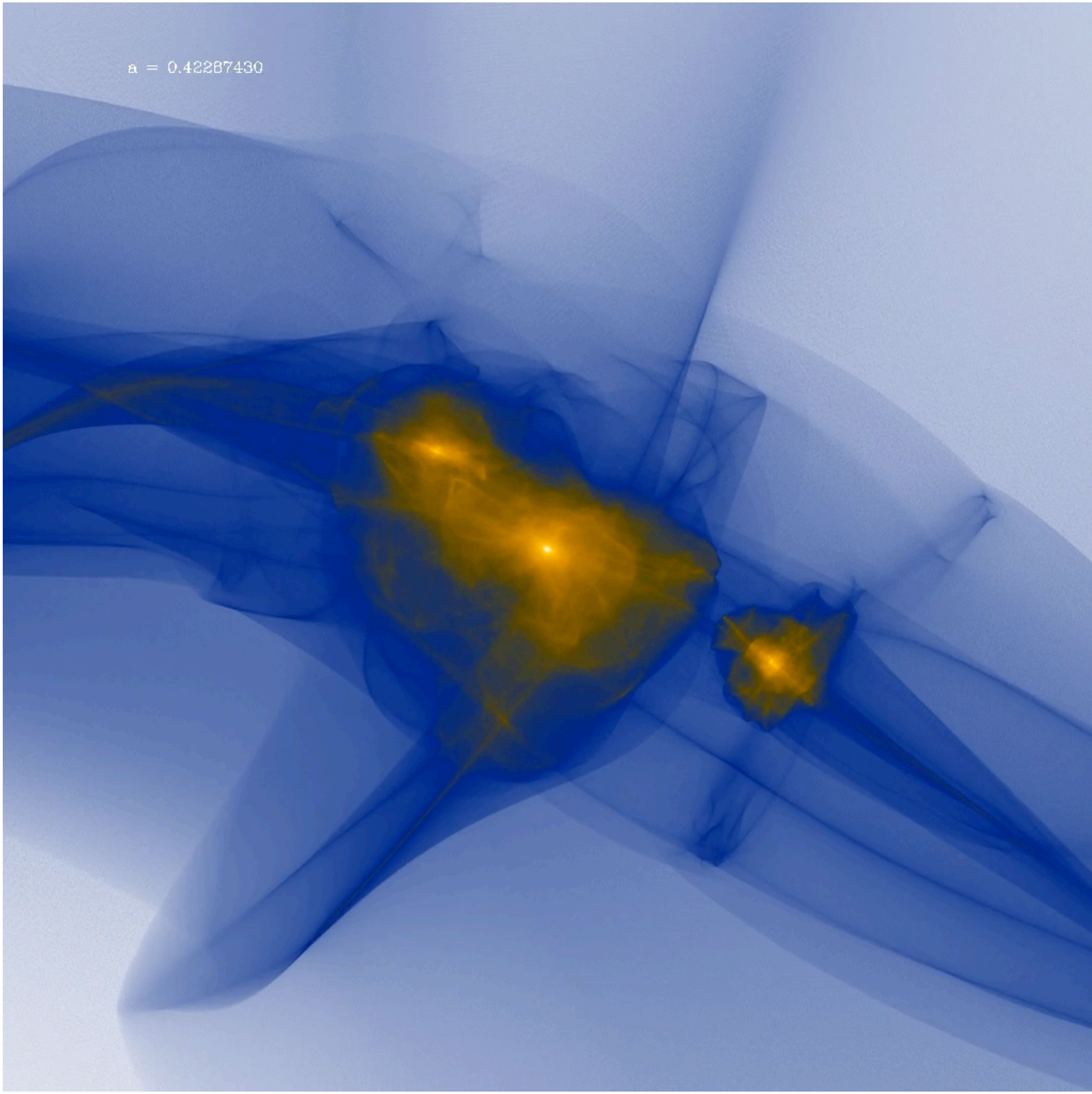
Tracing the dark matter sheet in phase space

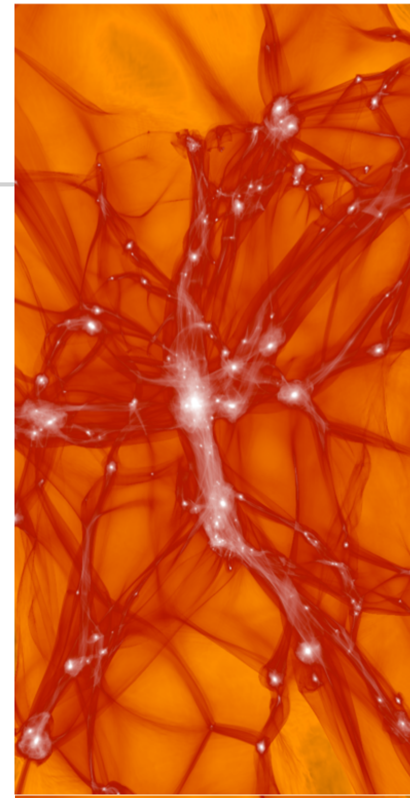
Tom Abel,★ Oliver Hahn★ and Ralf Kaehler★
Kavli Institute for Particle Astrophysics and Cosmology, Stanford University, SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA

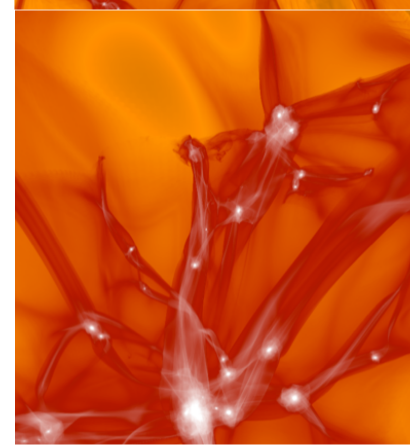
Warm dark matter halo with refinement and quadratic elements

Analogous to first cold dark matter halos



Cosmological N-body simulations

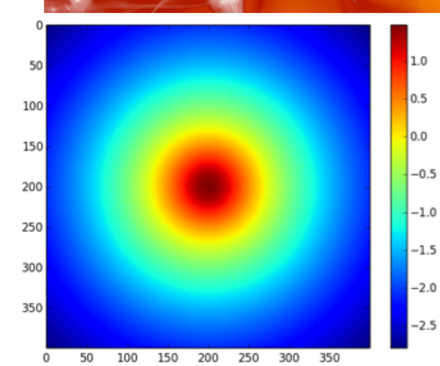


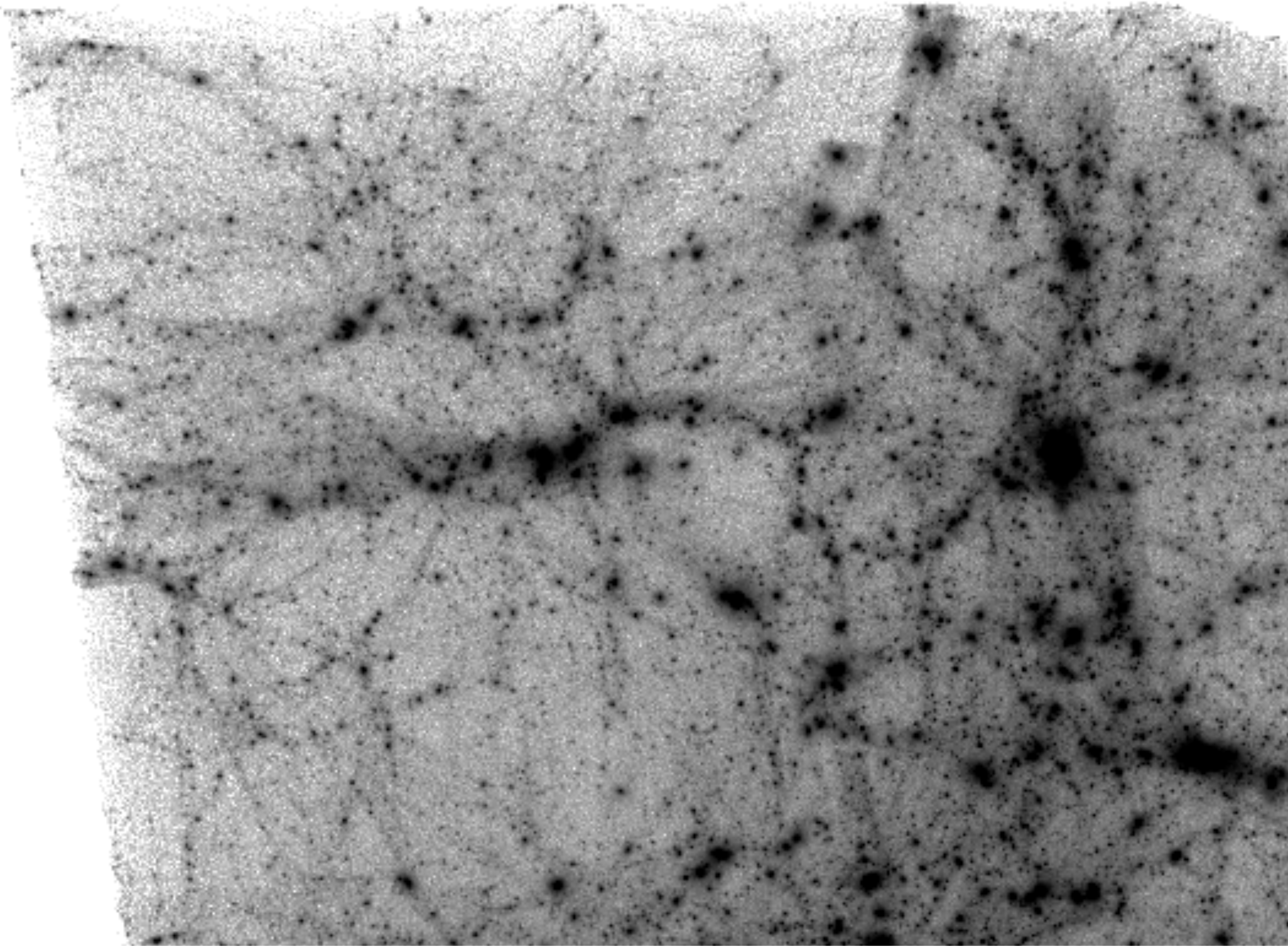


$$\dot{\mathbf{x}} = \mathbf{v}(t) \qquad \dot{\mathbf{v}}_{\mathbf{i}} = -\sum_{i \neq j}^{N} G m_i m_j \frac{(\mathbf{x}_{\mathbf{j}} - \mathbf{x}_{\mathbf{i}})}{|\mathbf{x}_{\mathbf{j}} - \mathbf{x}_{\mathbf{i}}|^3}$$

- All modern cosmological simulation codes only differ in how they accelerate the computation of the sum over all particles to obtain the net force
- End result are simply the positions and velocities of all particles
- Softening of forces (add epsilon^2 in denominator) avoids singularities.
- Limit N goes to infinity must give correct answer, right?
- Plummer softening

$$\dot{\mathbf{v}}_i = -\sum_{i \neq j}^{N} G\, m_i\, m_j \frac{(\mathbf{x}_j - \mathbf{x}_i)}{(|\mathbf{x}_j - \mathbf{x}_i|^2 + \epsilon^2)^{3/2}}$$





GRAVITY:

TOTAL MASS DENSITY

POISSON EQUATION: $\nabla^2 \phi = 4\pi G \rho$

CONTINUUM DESCRIPTION

$$F/m = -\nabla \phi$$

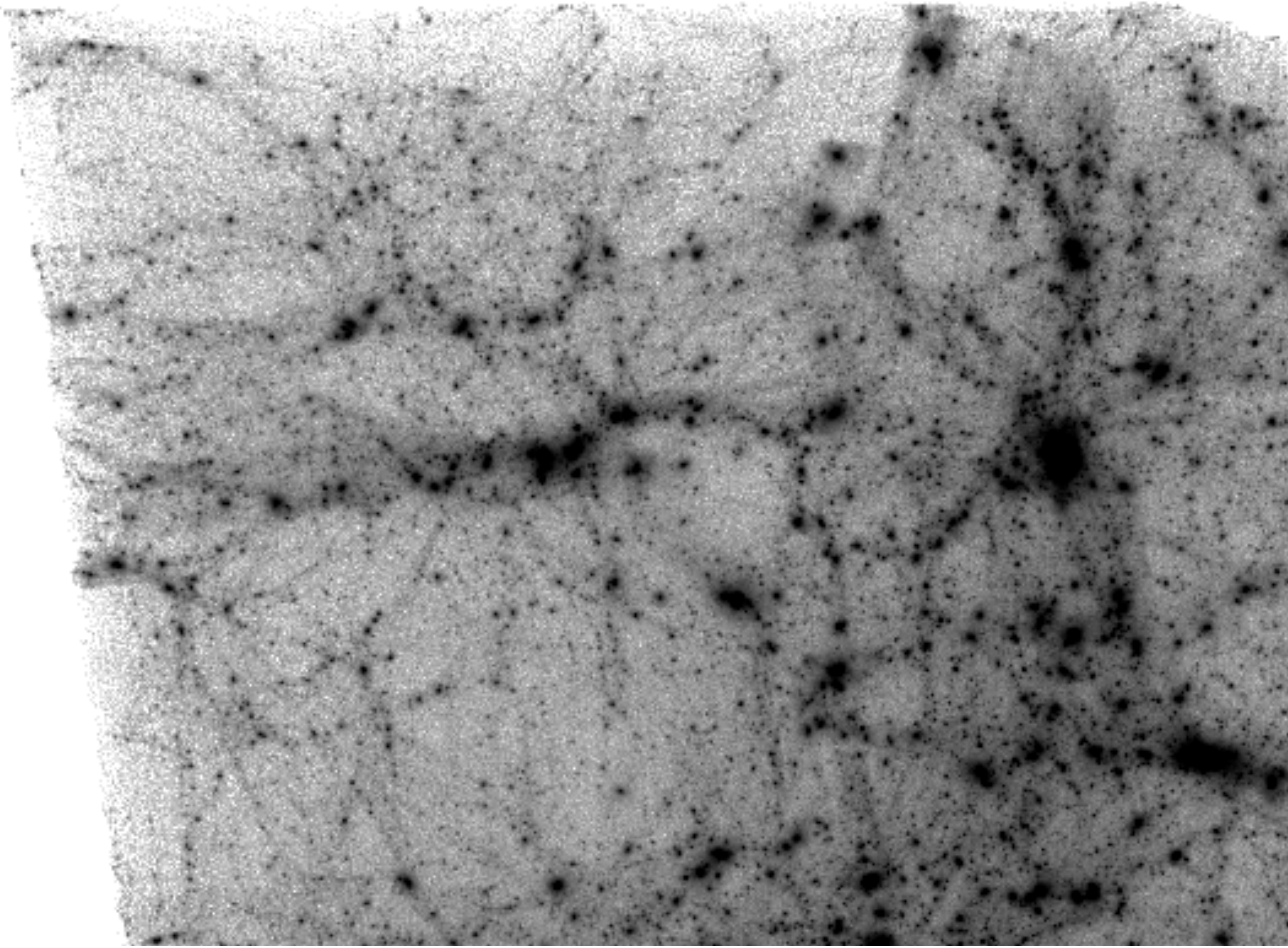
VLASOV EQUATION $\frac{\partial f}{\partial t} + v \nabla_x f - \nabla \phi \nabla_v f = 0$

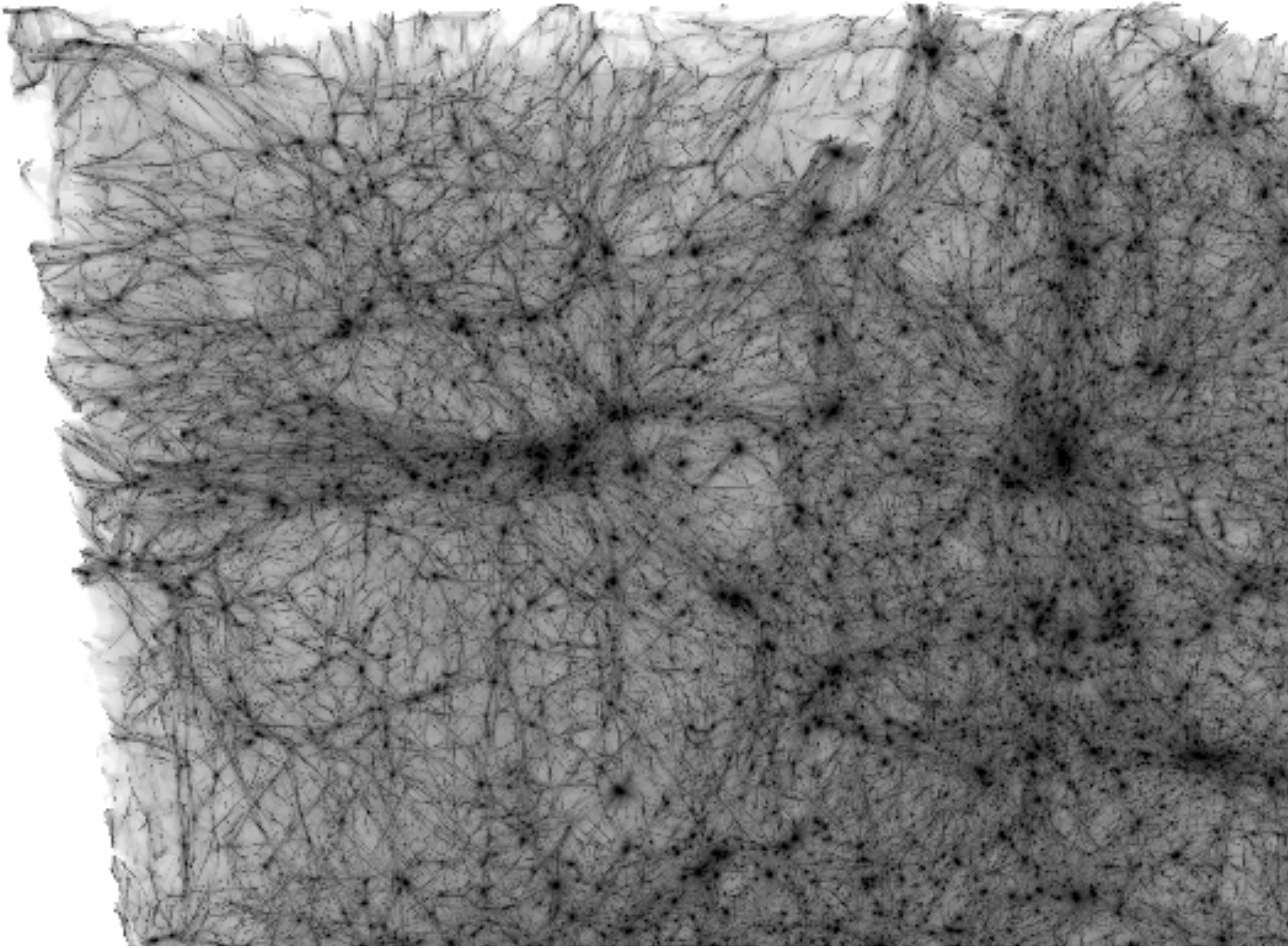
FOR $N \rightarrow \infty$ ↕ IDENTICAL

N POINT MASSES: $\vec{a}_j = -\sum_{i \neq j} \frac{G m_i}{|\vec{x}_j - \vec{x}_i|^2 + \varepsilon^2} \frac{(\vec{x}_j - \vec{x}_i)}{|\vec{x}_j - \vec{x}_i|}$

PARTICLE PICTURE

Particle "advection": $\frac{\partial \vec{x}_i}{\partial t} = \vec{v}$; $\frac{\partial \vec{v}_i}{\partial t} = \vec{a}_j$





The Dark Matter Sheet?

Fluid

OF DARK MATTER PARTICLES IN THE MILKY WAY :

$$N_{DM} \approx 10^{67}\left(\frac{100\,GeV}{m_{DM}}\right) \gg \text{\# OF STARS IN THE UNIVERSE}$$

$$\gg \text{\# OF PARTICLES THAT FIT ON A COMPUTER}$$

USING ALL THE COMPUTERS IN THE WORLD : $\lesssim 10^{17}$ particles

SOLVE VLASOV-POISSON SYSTEM INSTEAD.

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla_x f + \vec{a}\cdot\vec{\nabla}_v f = 0$$

$$\vec{a} = -\vec{\nabla}\phi$$

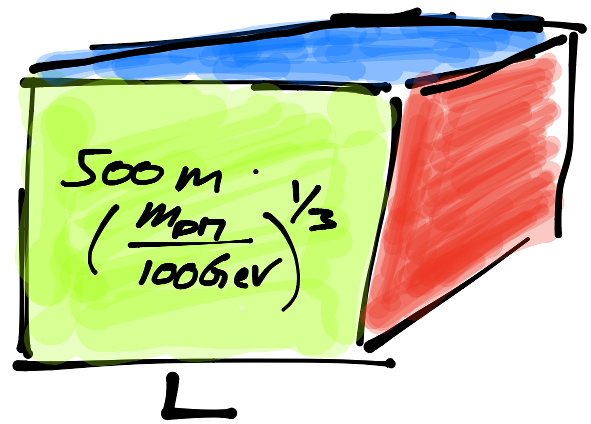
$$\nabla^2\phi = 4\pi G\rho$$

f: distribution function IN PHASE SPACE

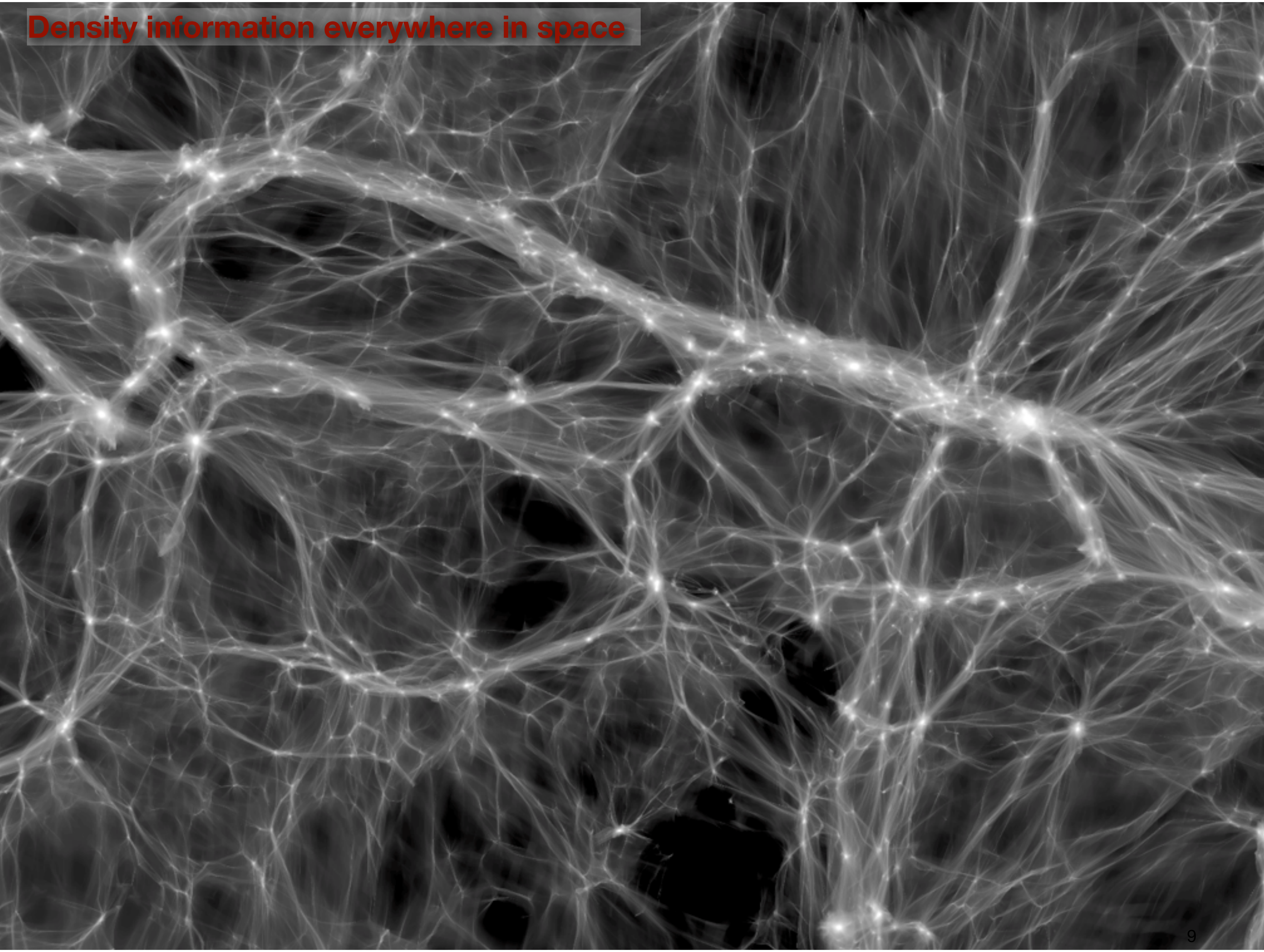
ϕ: potential

FOR PHASE SPACE ELEMENT TO CONTAIN 10^6 PARTICLES @ MEAN DENSITY IT HAS TO BE LARGER THAN

$$L \sim 500\,m\left(\frac{m_{DM}}{100\,GeV}\right)^{1/3}$$



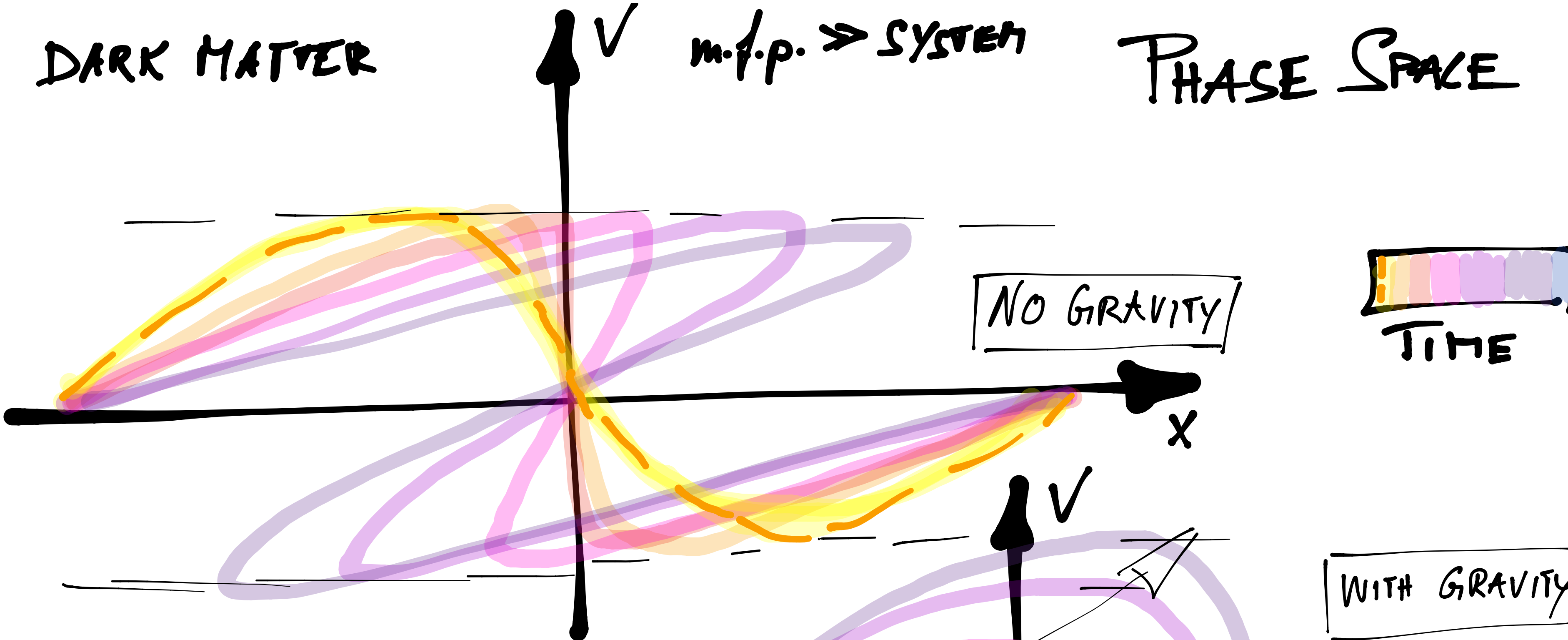
Density information everywhere in space

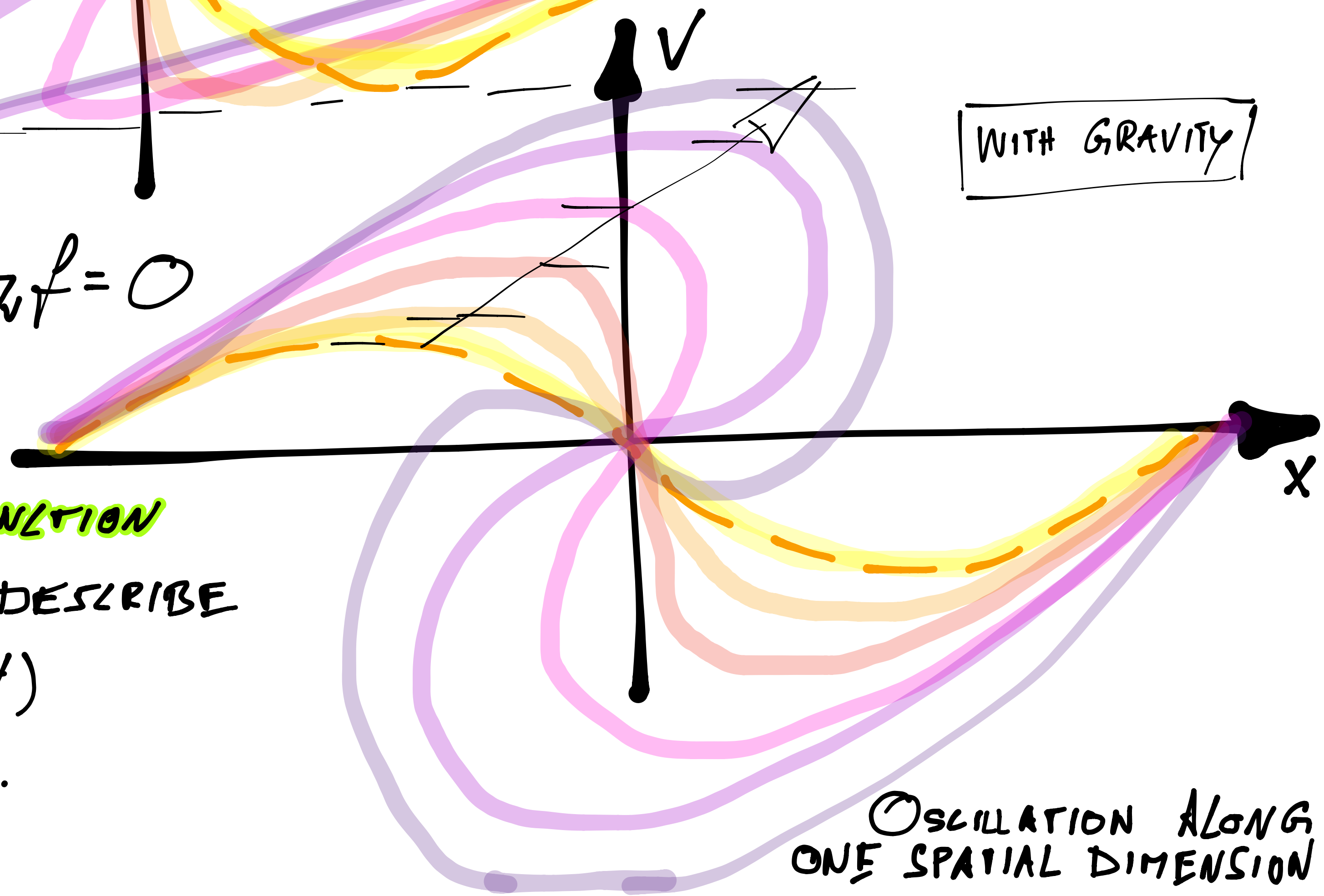


DARK MATTER

m.f.p. ≫ SYSTEM

PHASE SPACE





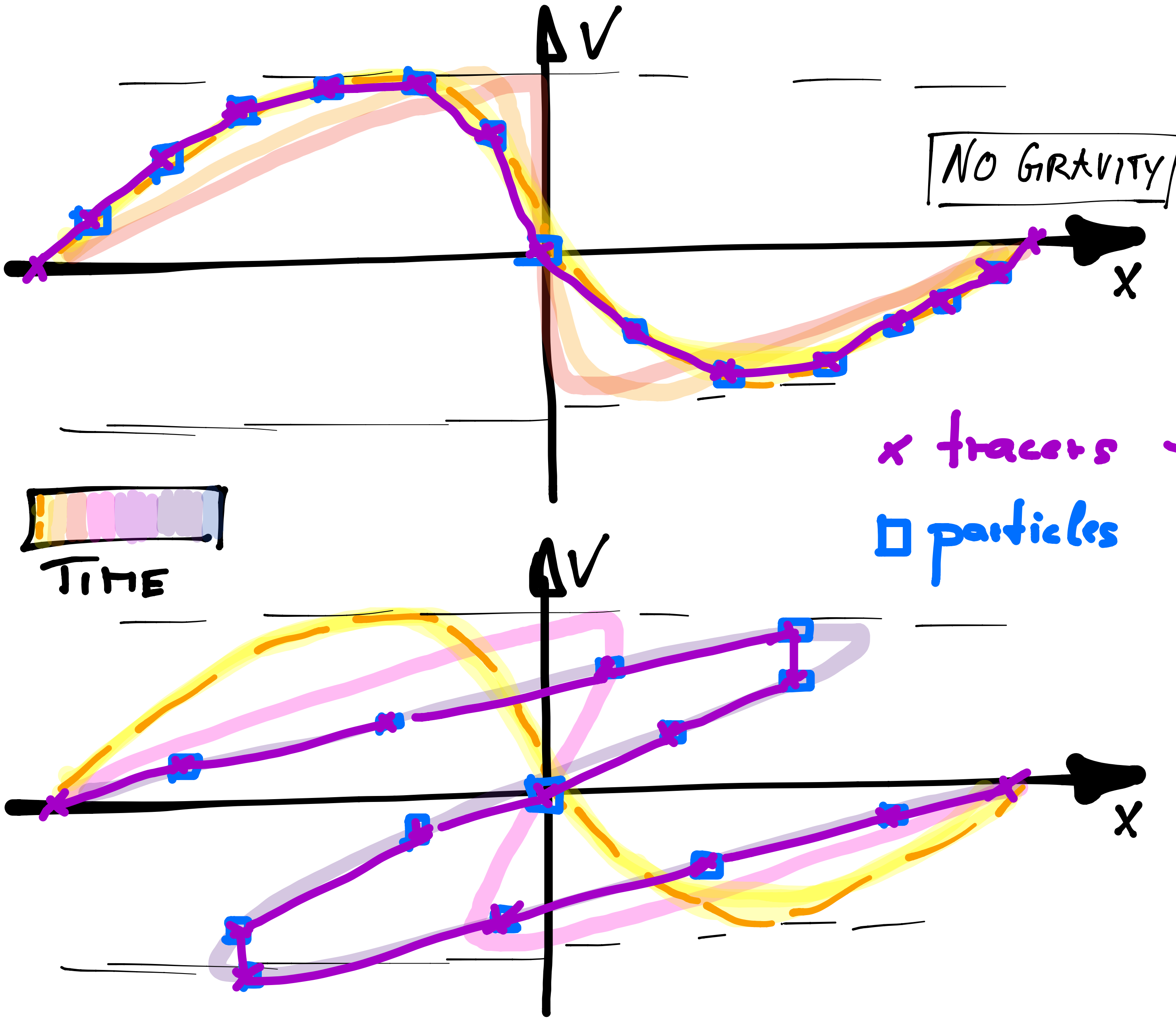
$$\frac{\partial f}{\partial t} + v \nabla_x f - \nabla_x \phi \nabla_v f = 0$$

VLASOV EQU.

USE **DISTRIBUTION FUNCTION** IN PHASE SPACE TO DESCRIBE EVOLUTION: $f(\vec{x}, \vec{v}, t)$

YES. 7 DIMENSIONS...

Discretize Dark Matter Distribution: Mass or Volume?



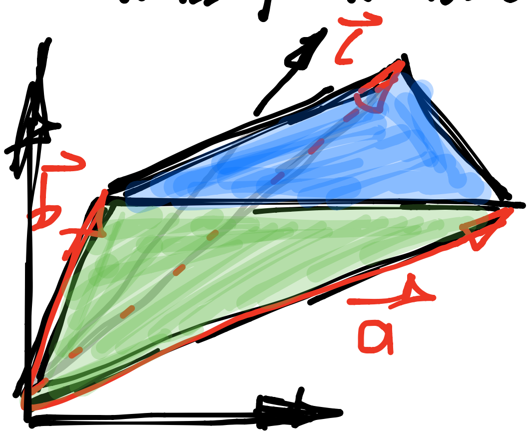
COUNT PARTICLES IN RANDOM VOLUMES GIVES AVERAGE DENSITIES

THINK MASS BETWEEN PARTICLES !

⟹ DENSITY KNOWN EVERYWHERE !

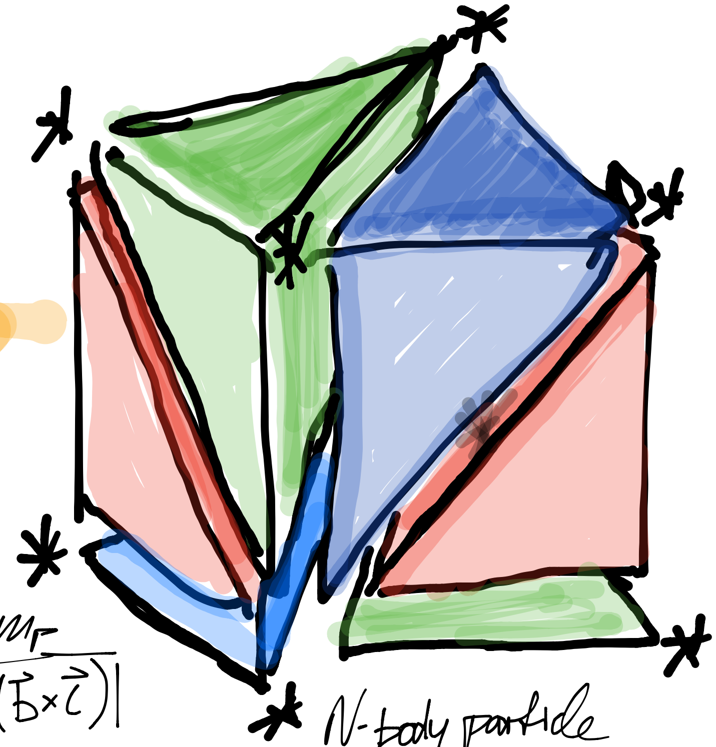
TESSELATE 3D MANIFOLD & TRACK IN 6D PHASE SPACE

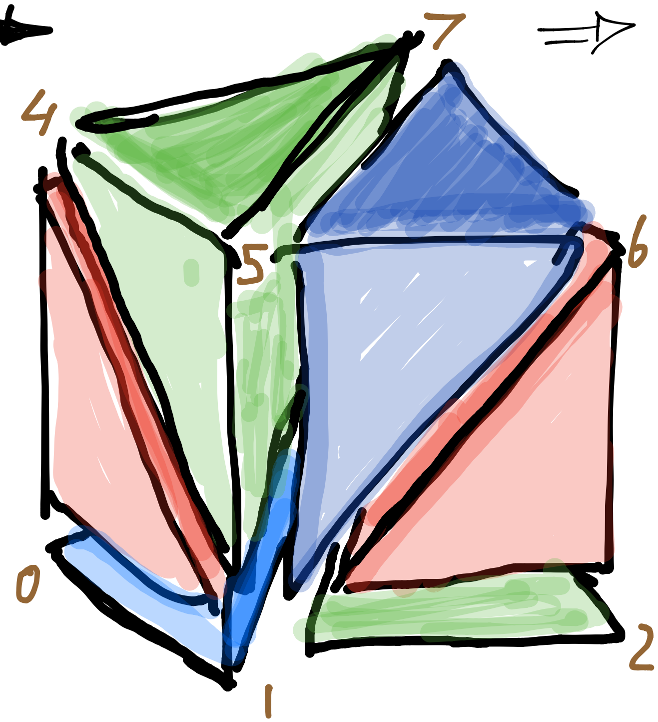
- NATURAL TESSELATION SPLITS CUBE INTO 6 EQUAL SIZED TETRAHEDRA
- mass per tetrahedron = 1/6 of DM particle mass.



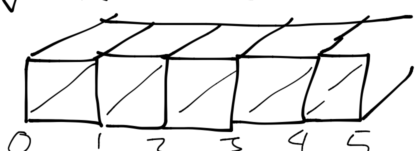
$$V = \frac{|\vec{a}\cdot(\vec{b}\times\vec{c})|}{6}$$

$$\Rightarrow \rho = \frac{m_p}{6V} = \frac{m_p}{|\vec{a}\cdot(\vec{b}\times\vec{c})|}$$





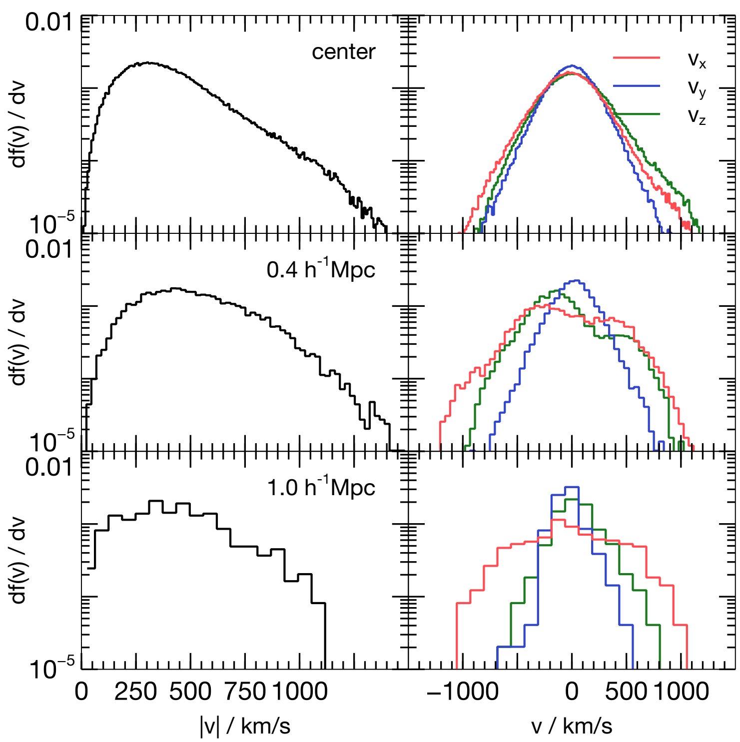
- Number the edges of the cube
- think of lattice
- Looping over

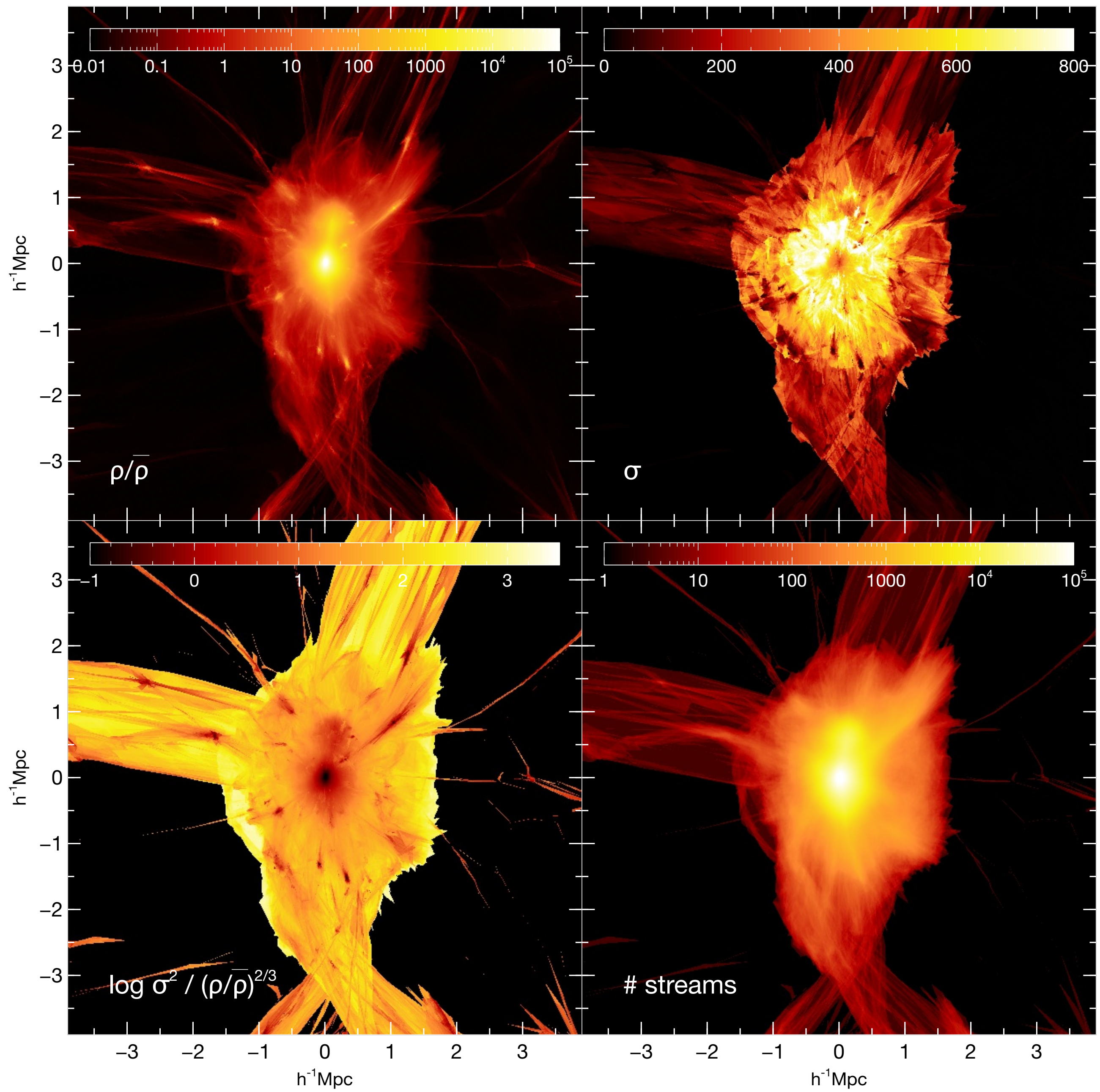


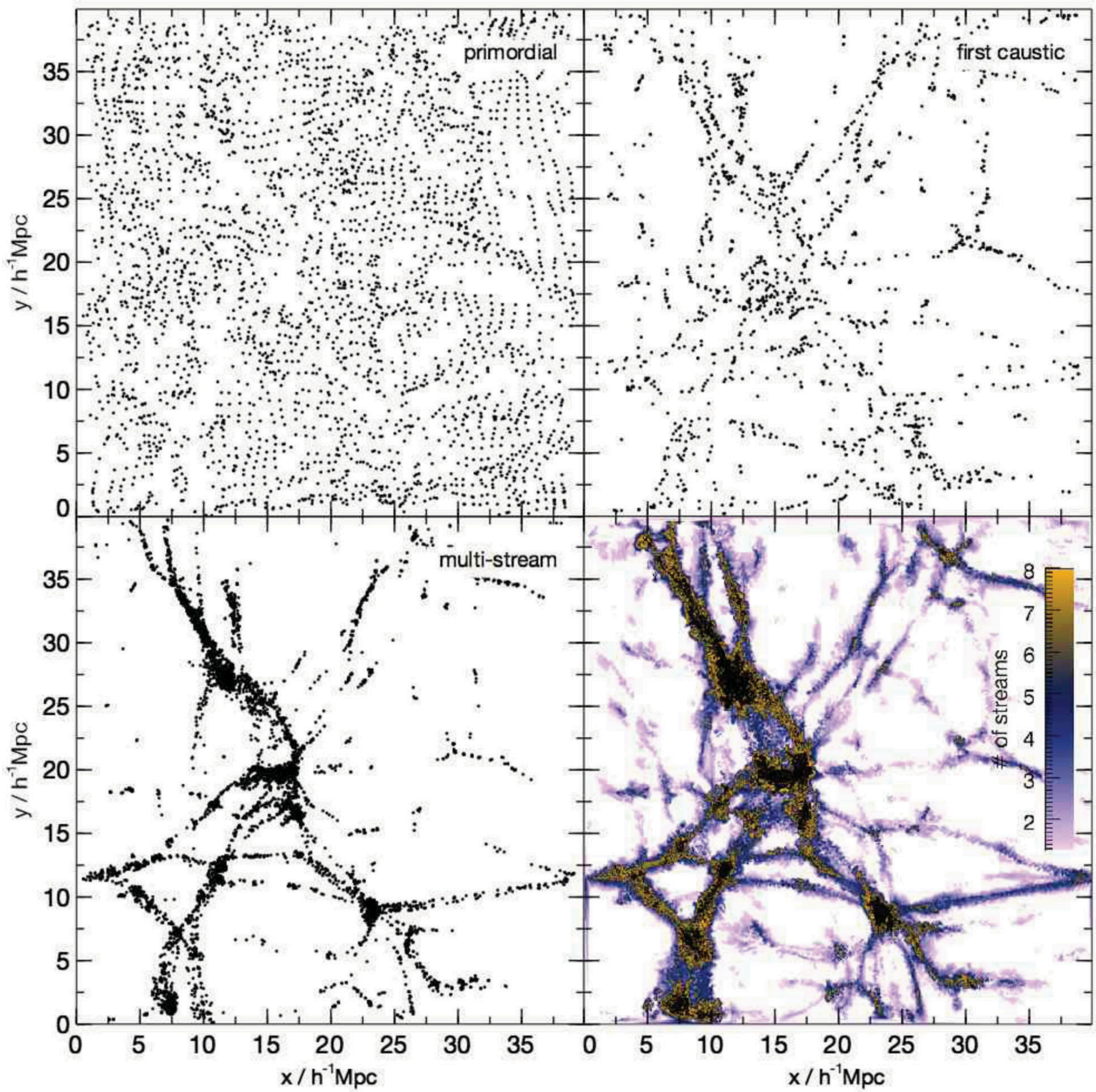
The initial cartesian (LAGRANGIAN lattice generates the 6N tetrahedra.

All microphysical phase space information available

can probe fine-grained phase space structure.







primordial
first caustic
multi-stream
# of streams
y / h⁻¹Mpc
x / h⁻¹Mpc

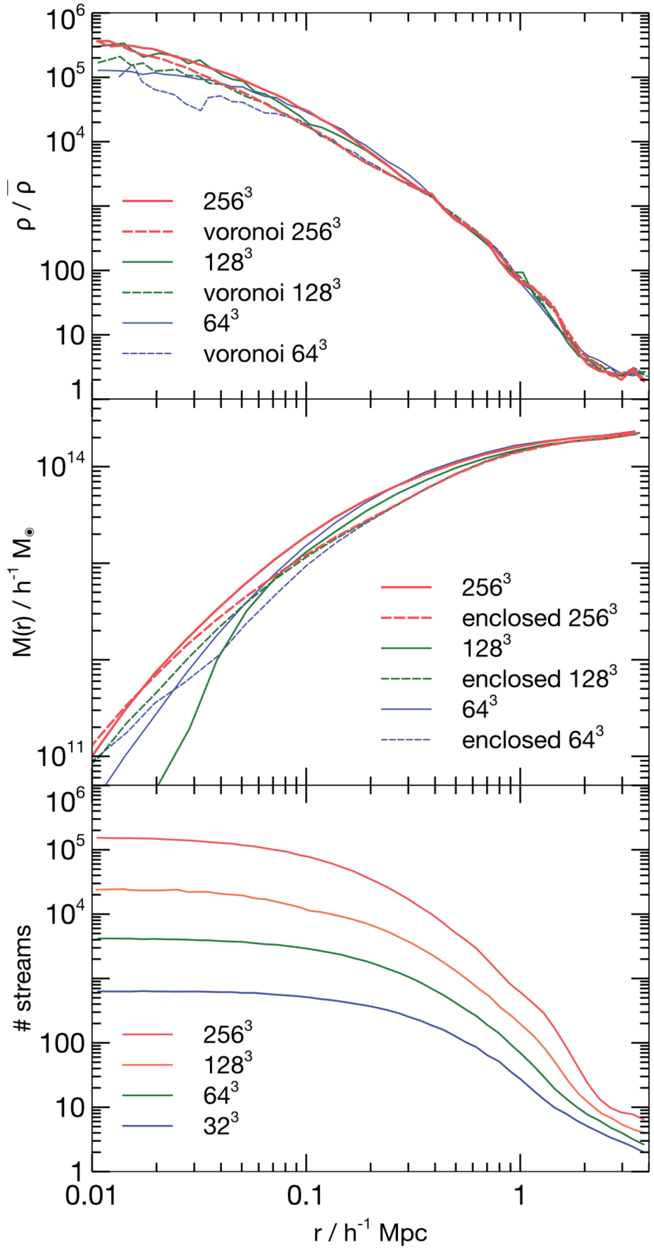


Figure 14. DM density profile in the most massive halo of $2 \times 10^{14}\,\mathrm{M}_\odot$ with $R_{\rm vir} \approx 1\,\mathrm{Mpc}$ at redshift zero. The overdensity in the top, the enclosed DM (middle) as calculated from the density in the top panel and the number of streams (bottom panel) are shown for all the different resolution simulations studied in this paper. The density profile estimated from our method starts to differ from the conventional estimate at scales as large as one-third of the virial radius. As expected in CDM, the number of streams does not converge also in radial profiles as the resolution is increased.

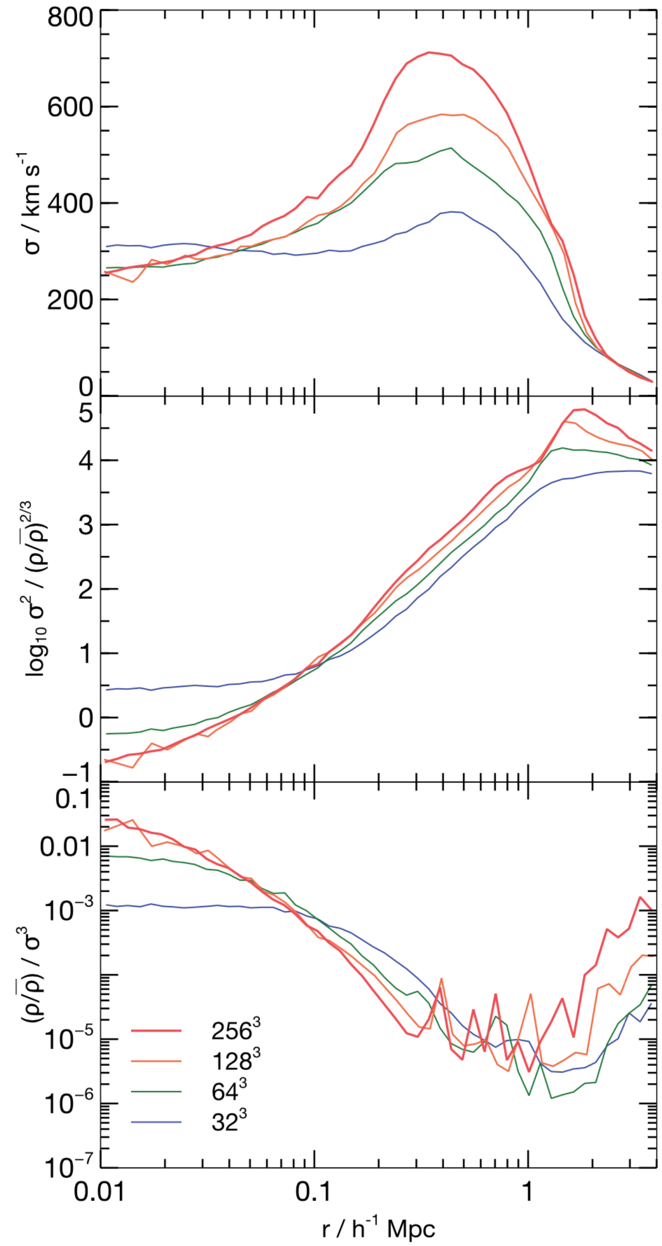
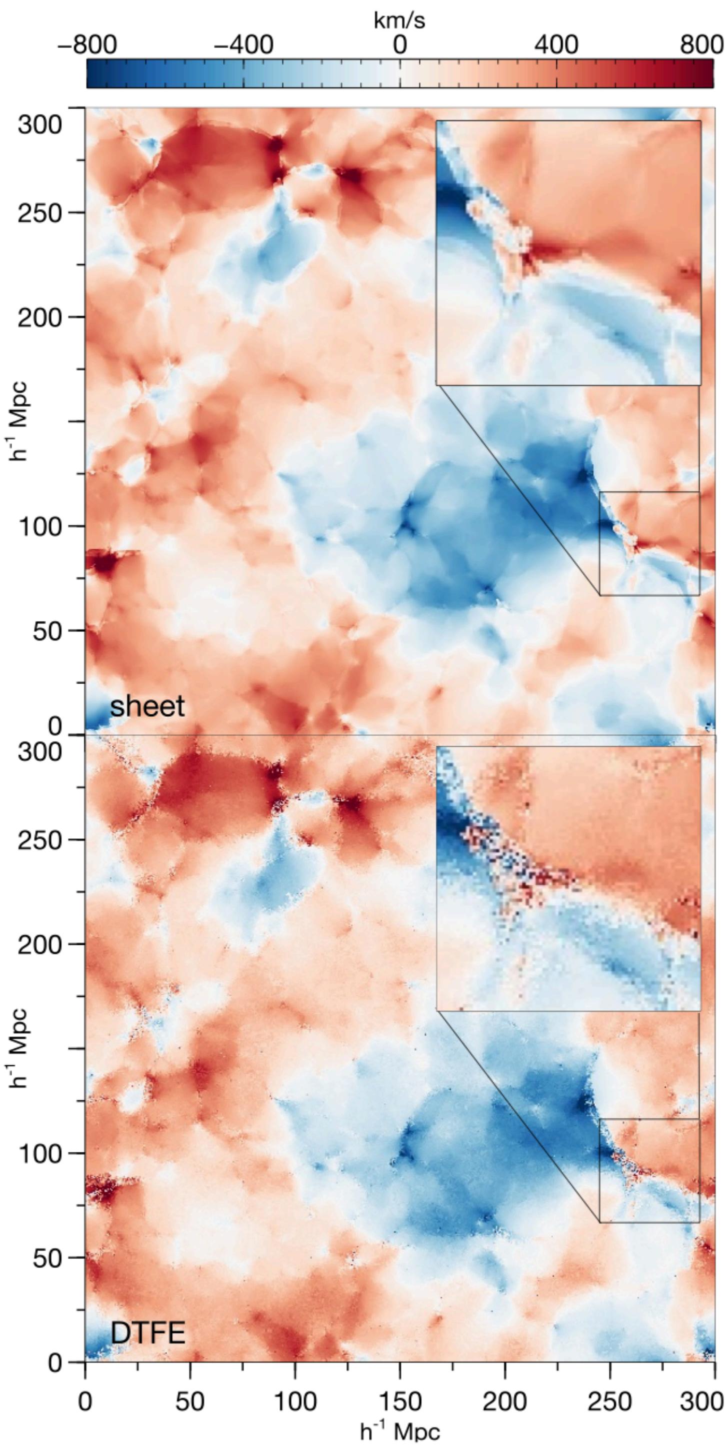
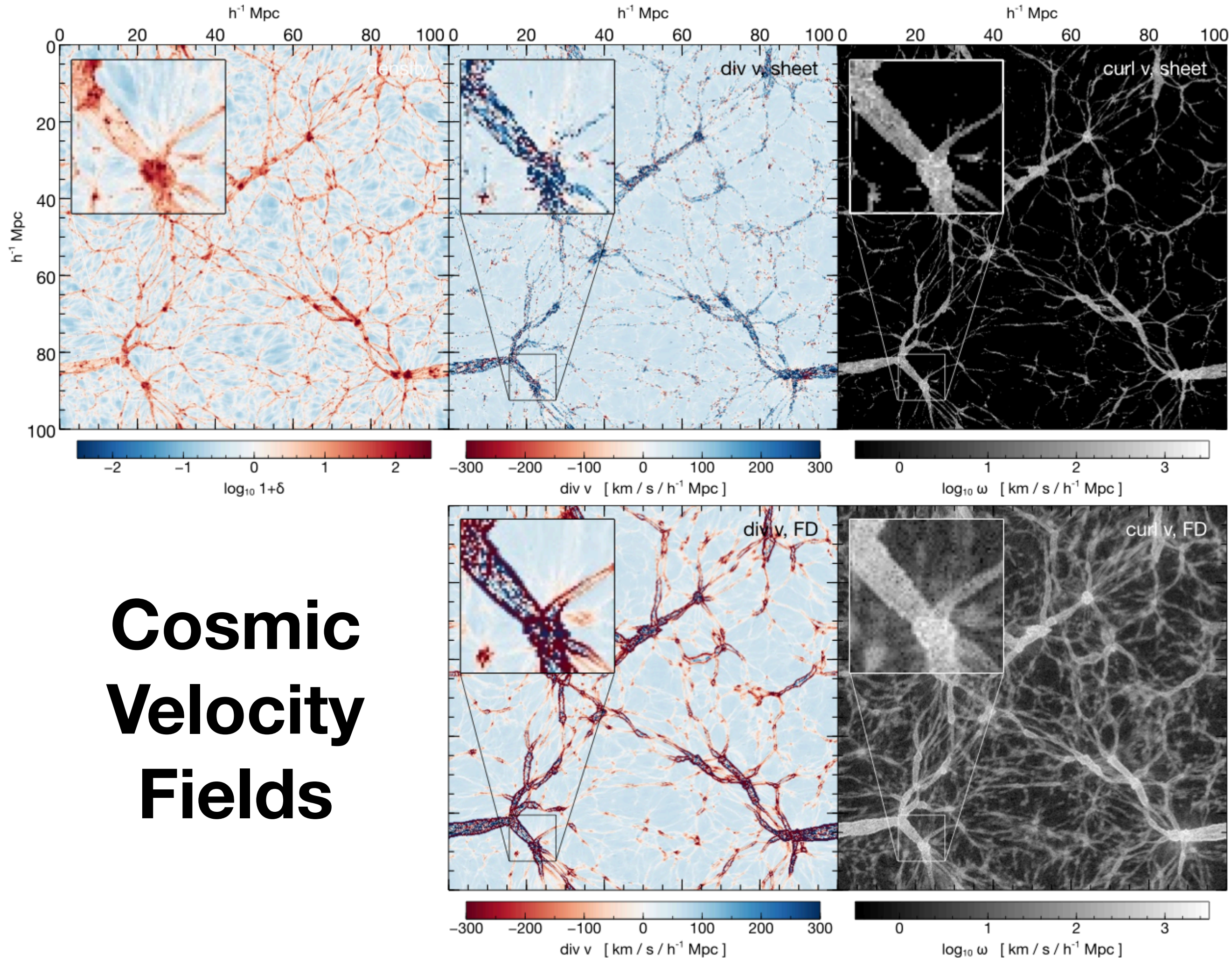
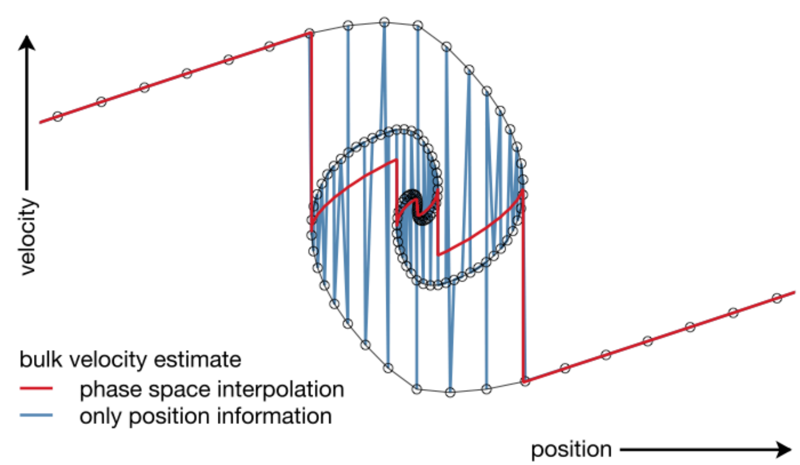


Figure 15. Radial spherically averaged profiles of the velocity dispersion (top), the DM 'entropy' $\sigma^2/(\rho/\bar{\rho})^{2/3}$ (middle) and the pseudo-phase-space density (bottom) for the same halo as in Fig. 14. The velocity dispersion is remarkably flat inside about one-tenth of the virial radius. The DM 'entropy' profile also shows signs of already converging at the modest resolutions employed here. Using the microscopic velocity dispersion of our approach which removes the bulk flows does not give the typical power-law behaviour in the pseudo-phase-space density found when using the total dispersion of particles at those radii.



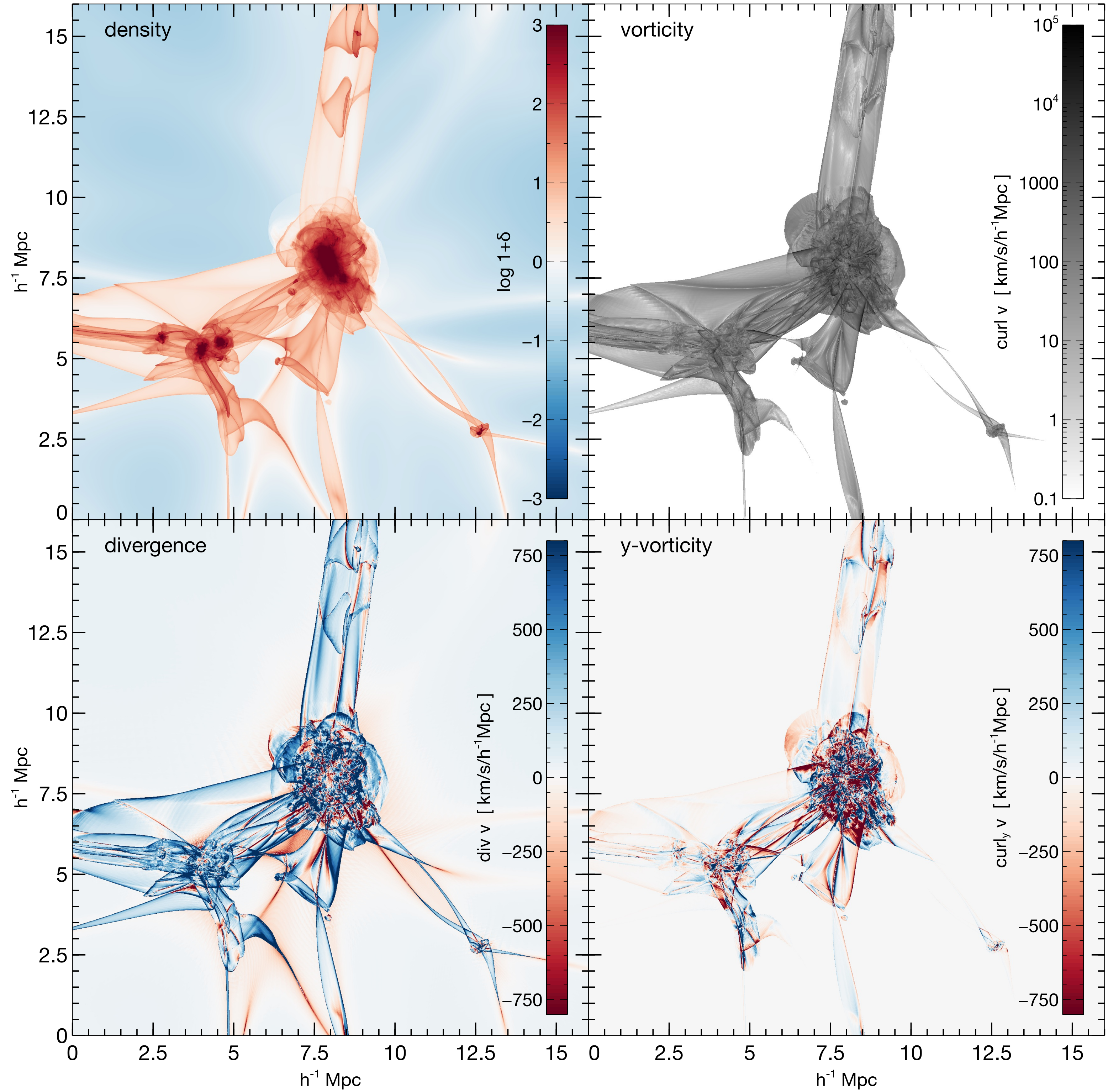


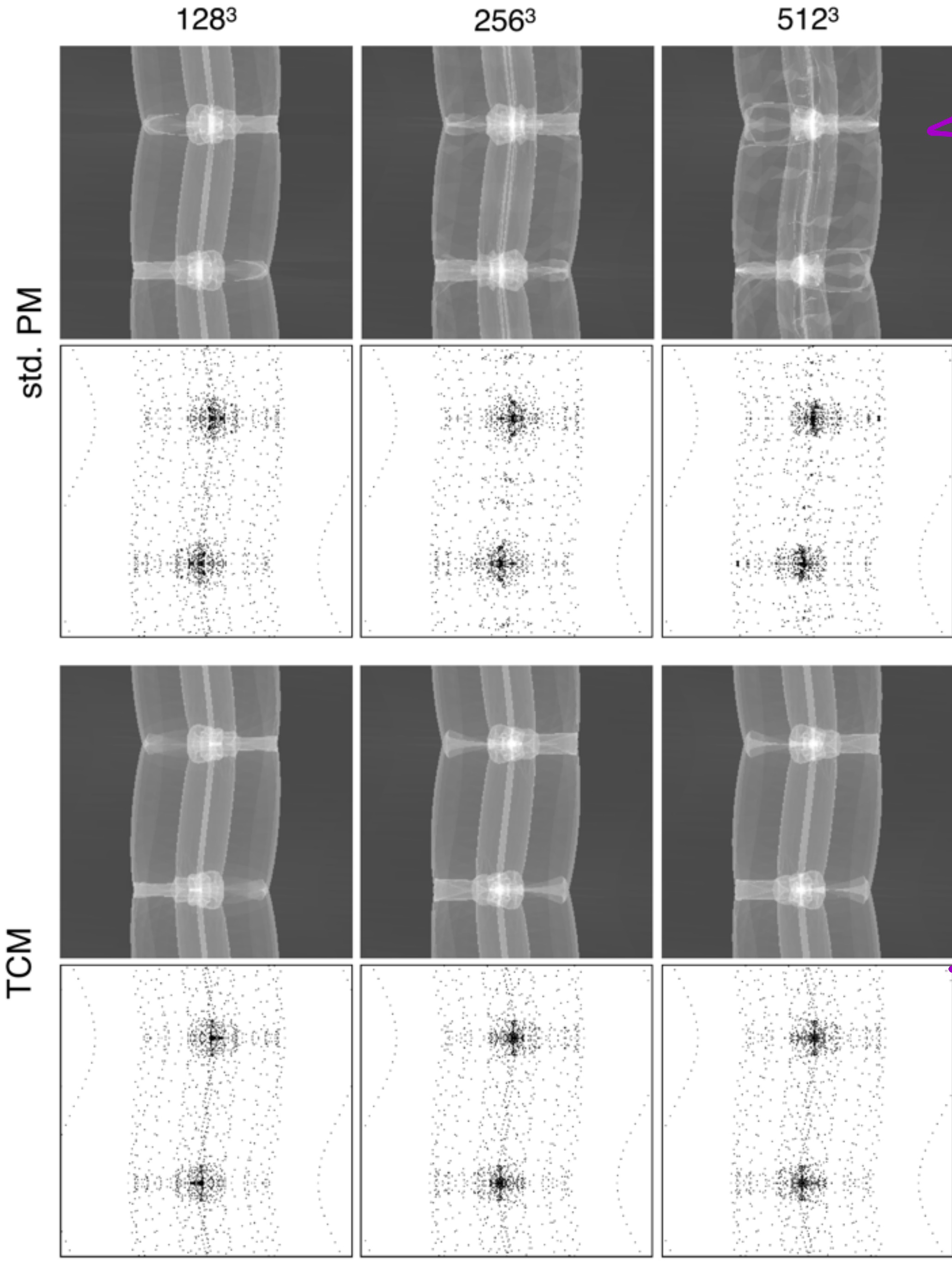
Cosmic Velocity Fields



The properties of cosmic velocity fields

Oliver Hahn,[1★] Raul E. Angulo[2] and Tom Abel[3]


density
vorticity
divergence
y-vorticity
log 1+δ
curl v [km/s/h⁻¹Mpc]
div v [km/s/h⁻¹Mpc]
curl_y v [km/s/h⁻¹Mpc]
h⁻¹ Mpc
0
2.5
5
7.5
10
12.5
15
3
2
1
0
−1
−2
−3
10⁵
10⁴
1000
100
10
1
0.1
750
500
250
0
−250
−500
−750



two body scattering undesirable ...

Note how the new visualization technique helps in spotting errors in the N-body integration

Constant mass resolution
Vary force resolution

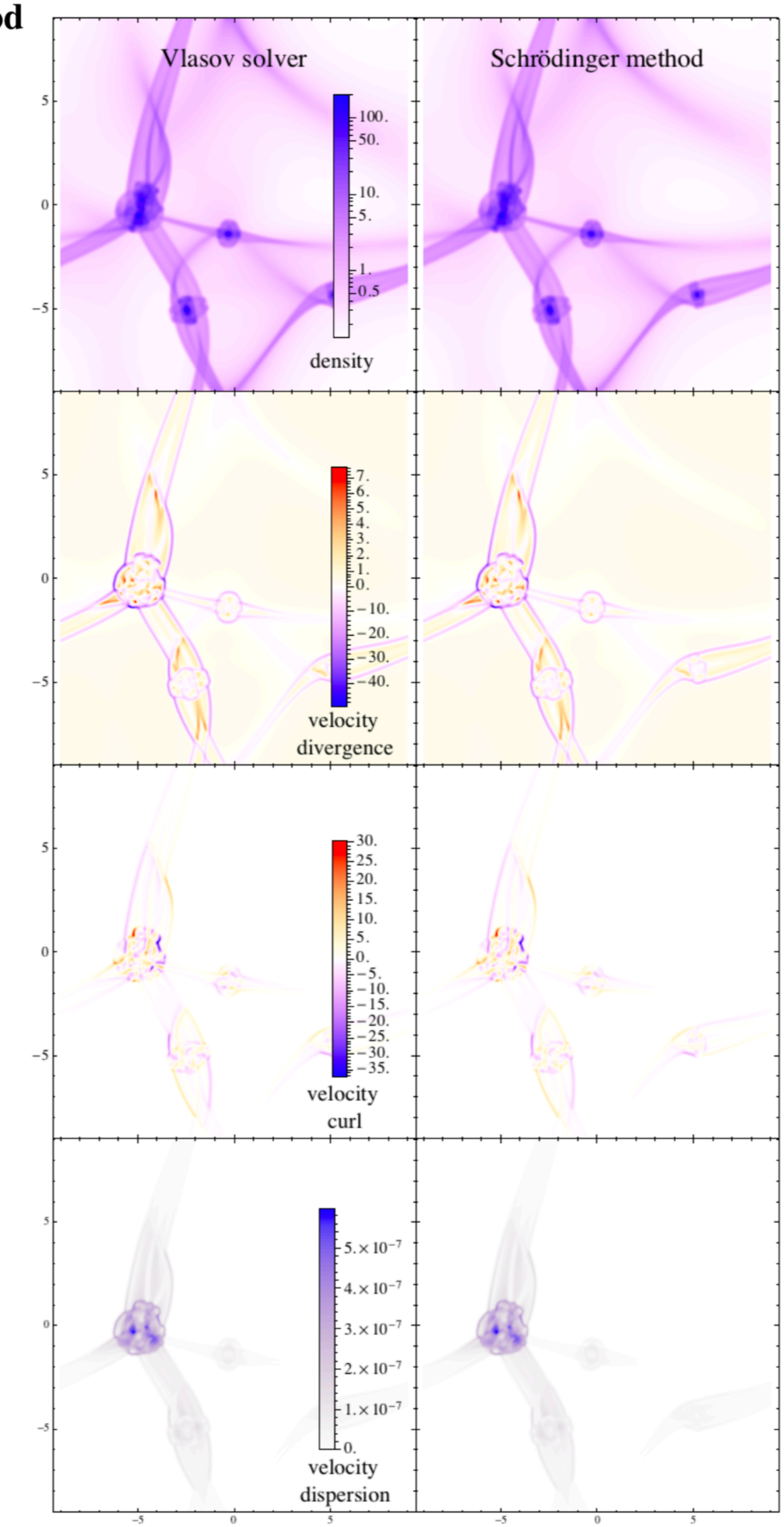
excellent convergence behavior of our new method.

Solving the Vlasov equation in two spatial dimensions with the Schrödinger method

Michael Kopp,[1,2,*] Kyriakos Vattis,[1,3,†] and Constantinos Skordis[1,2,‡]

Physical Review D, Volume 96, December 2017

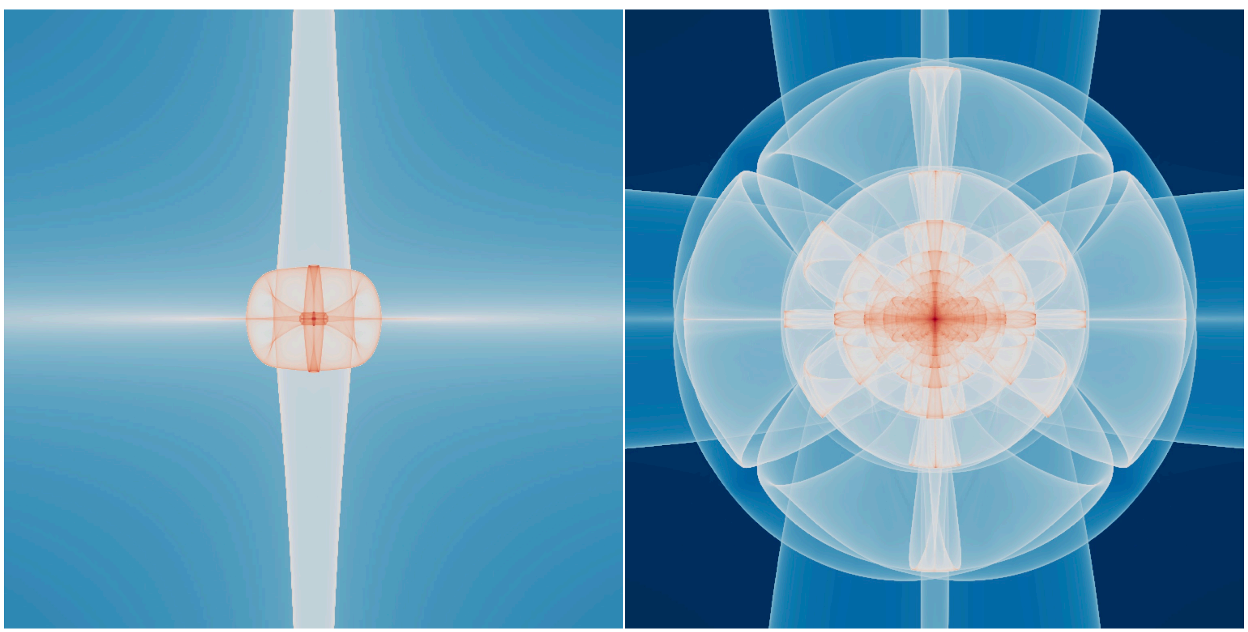
FIG. 1. Time snapshots of a two-dimensional cosmological simulation evolved to the present time $a = 1$. Both codes were started with the same single-stream initial conditions, set up using the Zel'dovich approximation at $a_{\text{ini}} = 1/51$. From top to bottom: density, velocity divergence, velocity curl and trace of the velocity dispersion tensor. The left column shows the smoothed results of the 2D-version of the Vlasov solver `ColDICE` [13]. The result of the Schrödinger method is shown in the right column. The differences are barely visible by eye. A quantitative comparison is presented in Sec. IV.



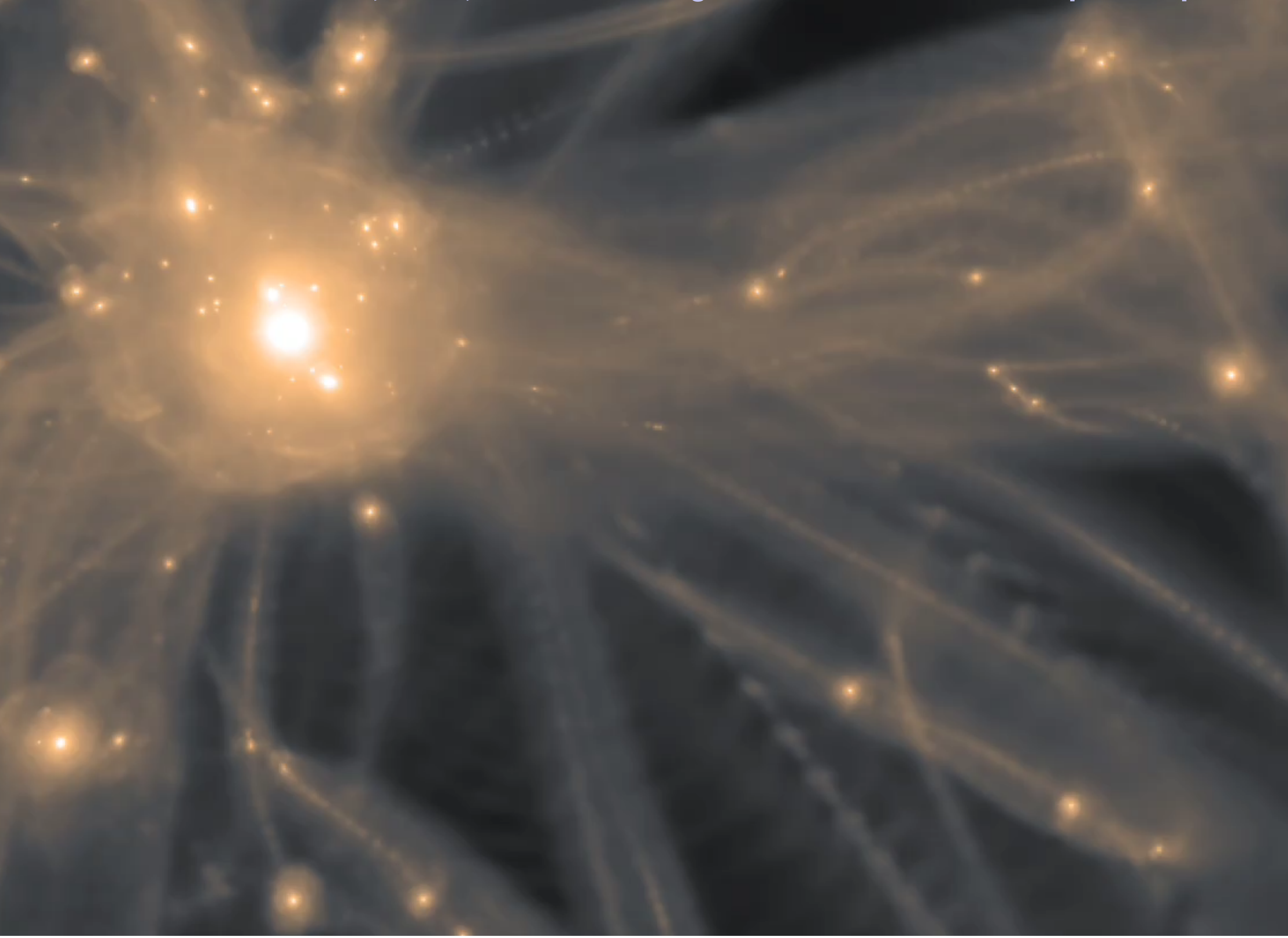
used:

`ColDICE`: a parallel Vlasov-Poisson solver using moving adaptive simplicial tessellation

Thierry Sousbie[a,b,c,*], Stéphane Colombi[a]



2012 MNRAS.427: Abel, Hahn, Kaehler Tracing the dark matter sheet in phase space



N-body

Quadrilaterals + refinement

Quadrilaterals

125=5x5x5 points initially in cube
N-body 2e6 particle reference

Linear tets + refinement

Linear tets

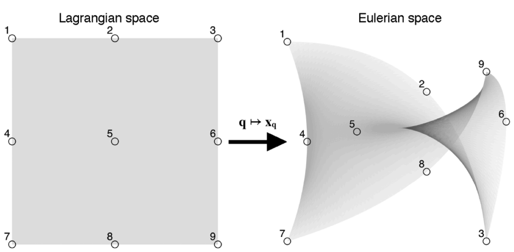


Figure 1. Mapping between Lagrangian space and Eulerian configuration space. The mapping can become multivalued in projection to configuration space and can have singular points and curves. Here, as an example, the mapping of nine points forming a square in Lagrangian space are shown under a biquadratic map to phase–space and then projected into configuration space.

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An adaptively refined phase–space element method for cosmological simulations and collisionless dynamics

Oliver Hahn[1★] and Raul E. Angulo[2]

[1] *Department of Physics, ETH Zurich, CH-8093 Zürich, Switzerland*
[2] *Centro de Estudios de Física del Cosmos de Aragón, Plaza San Juan 1, Planta-2, E-44001 Teruel, Spain*

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Clear accuracy gains with higher order interpolation schemes.
Shown here in the test case of a cube evolving in a static potential.
Second order interpolation is clearly advantageous.

Lagrangian Tessellation: What's it good for? Not complete list:

- Analyzing N-body sims, including web classification, velocity dispersion, profiles, resolution study (Abel, Hahn, Kaehler 2012)
- DM visualization (Kaehler, Hahn, Abel 2012)
- Better Numerical Methods (Hahn, Abel & Kaehler 2013, Hahn, Angulo & Abel 2014, Angulo and Hahn 2016, Sousbie & Colombi 2016)
- Finally reliable WDM mass functions below the cutoff scale Angulo, Hahn, Abel 2013
- Gravitational Lensing predictions, Angulo, Chen, Hilbert & Abel 2014
- Cosmic Velocity fields, Hahn, Angulo, Abel 2014
- The SIC method for Plasma simulations (Vlasov/Poisson) (Kates-Harbeck, Totorica, Zrake & Abel 2016, JCompPhys)
- Exact overlap integrals of Polyhedra (Powell & Abel 2015 JCompPhys)
- Adaptively refined phase-space, Hahn & Angulo 2016
- ColDICE: A parallel Vlasov-Poisson solver using moving adaptive simplicial tessellation, Sousby & Colombi 2016
- Void profiles, Wojtak, Powell, Abel 2016 MNRAS
- Stücker, Busch, & White 2017: Median density of the Universe
- East, Wojtak, Abel compare numerical GR and Newtonian cosmology 2017
- Powell & Abel 2018, Beam Tracing for radiation transport
- Banerjee, et al: Noiseless Cosmological Neutrino simulations, 2018
- Totorica, et. al. Weibel instabilities, shocks, particle acceleration in PIC simulations 2018
- "Nonthermal electron and ion acceleration by magnetic reconnection in large laser-driven plasmas" Totorica, Hoshino, Abel, Fiuza 2020
- Powell et al DM annihilation,
- Totorica & Abel: Vlasov-Maxwell (2023)