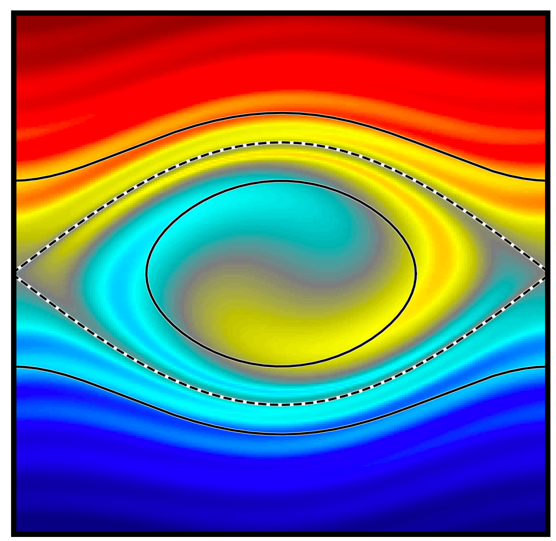
Galactic Bar Resonances with Diffusion



Chris Hamilton
Institute for Advanced Study
@ KITP CosmicWeb23

Hamilton et. al (2022)

Submitted to ApJ

Galactic bar resonances with diffusion:
an analytic model with implications for bar-dark matter halo dynamical friction

Chris Hamilton,[1] Elizabeth Tolman,[1] Lev Arzamasskiy,[1] and Vinícius N. Duarte[2]

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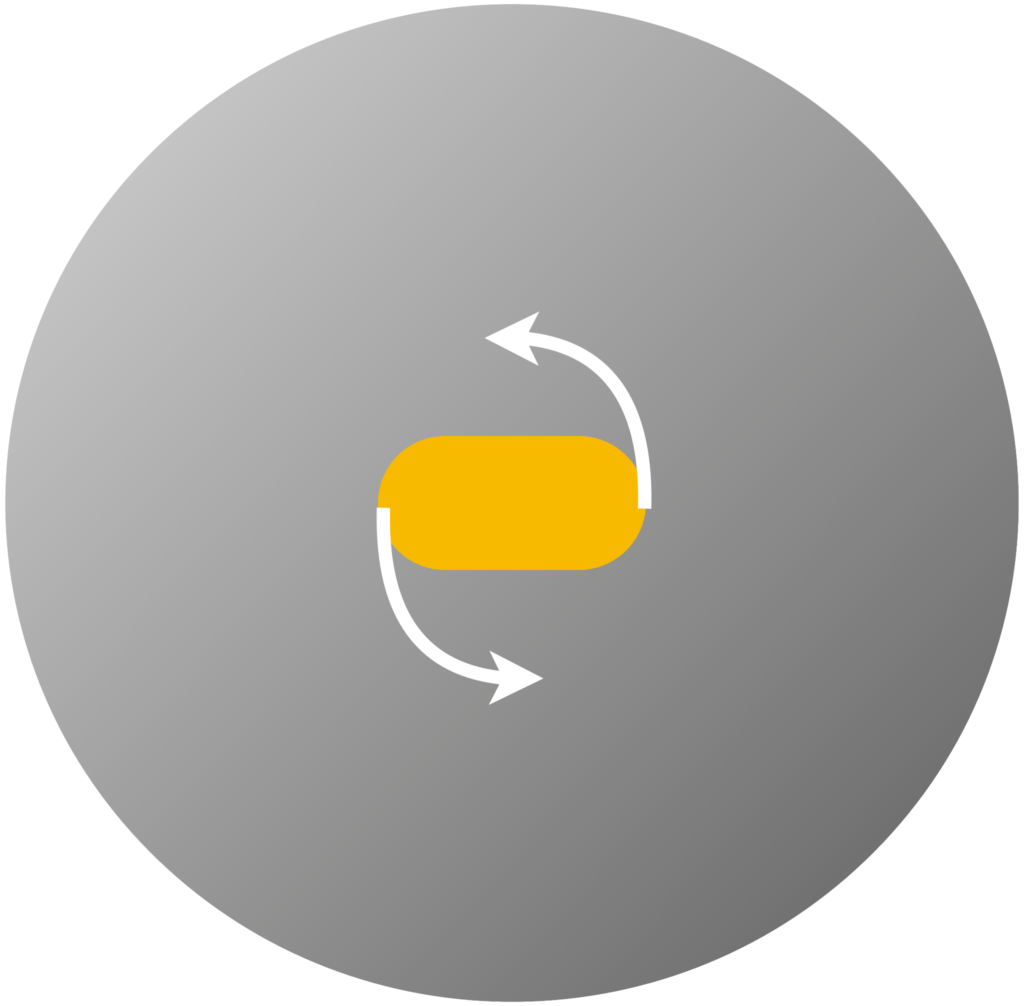
[2] *Princeton Plasma Physics Laboratory, Princeton University, Princeton, NJ 08543*




Bar-Halo Interaction



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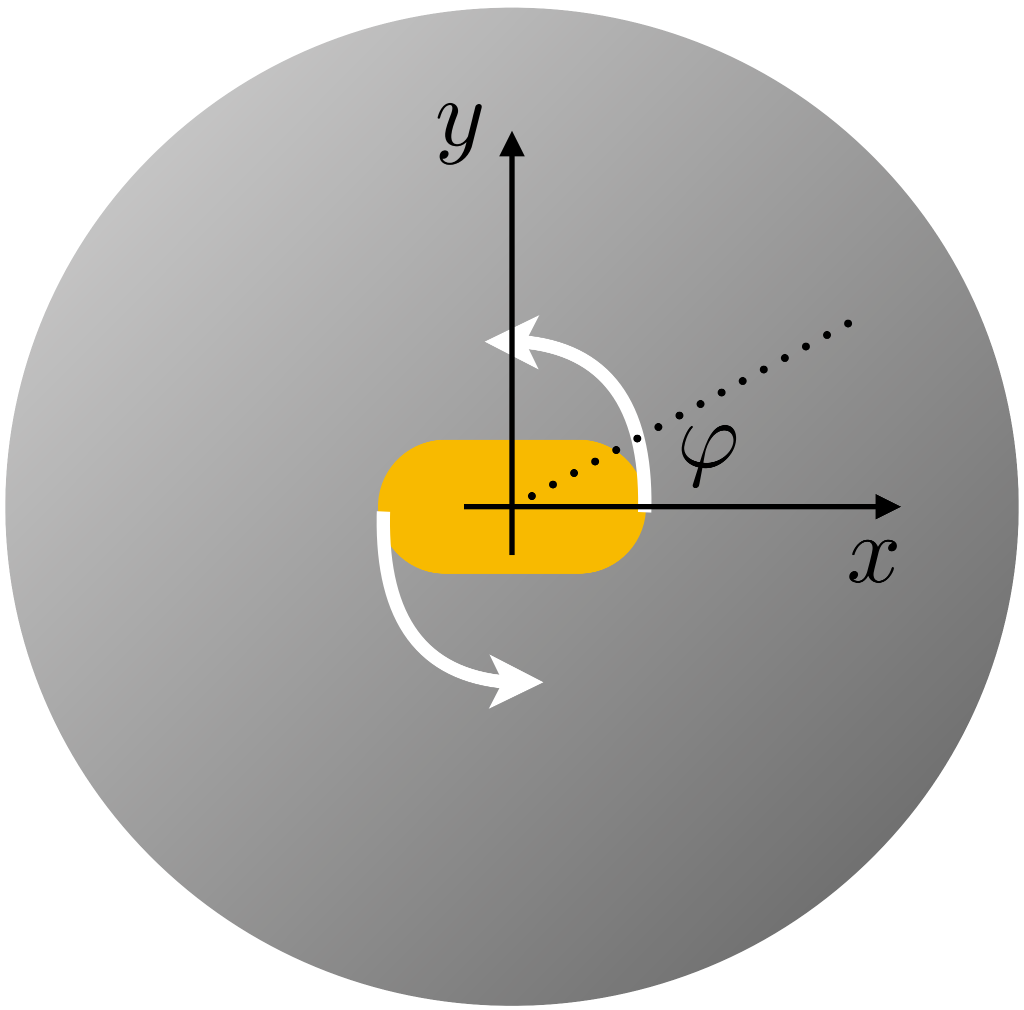
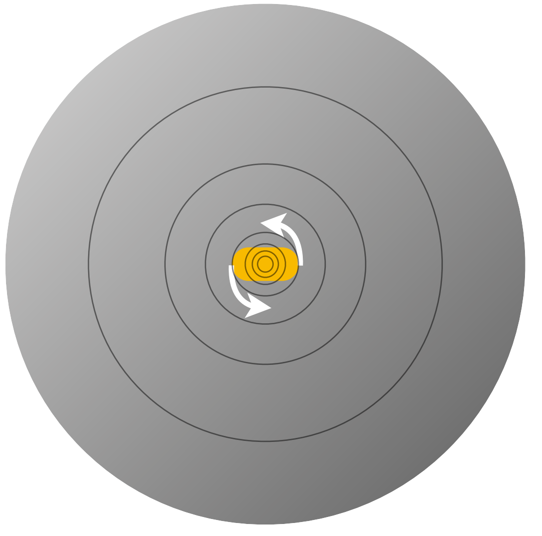


Bar rotates in azimuth with pattern speed Ω_{p}

What happens?

What Happens?

t=0



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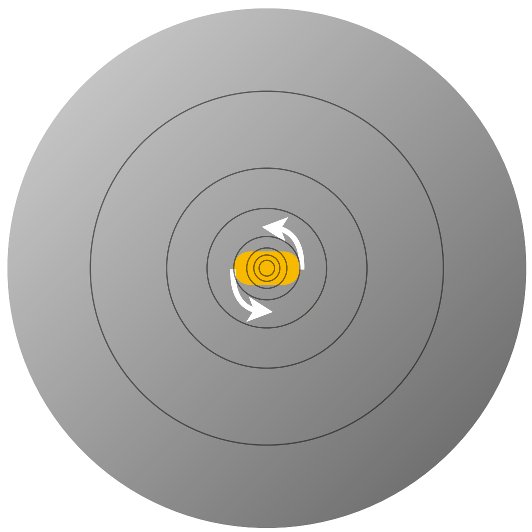
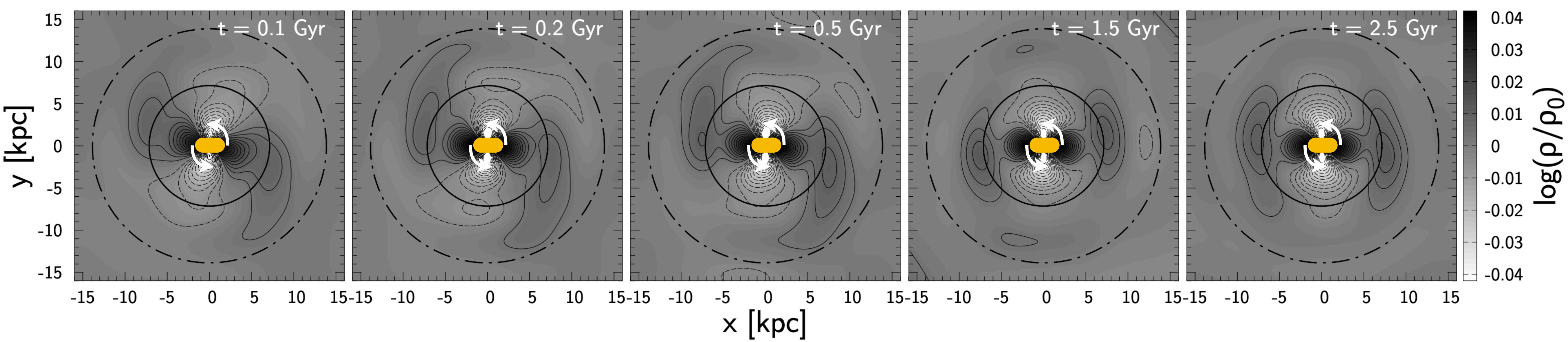


Dark matter density response for constant Ω_p
(Chiba & Schonrich 2022)



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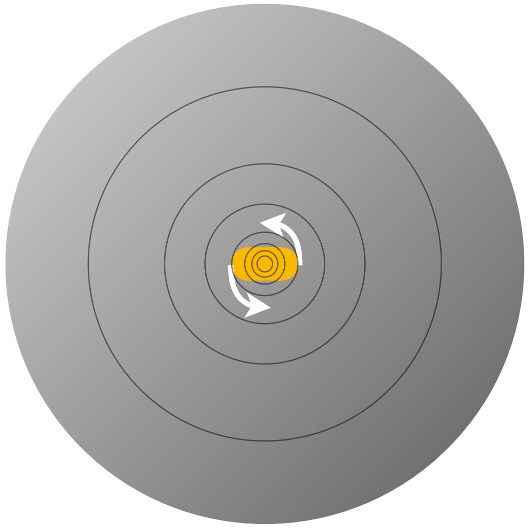

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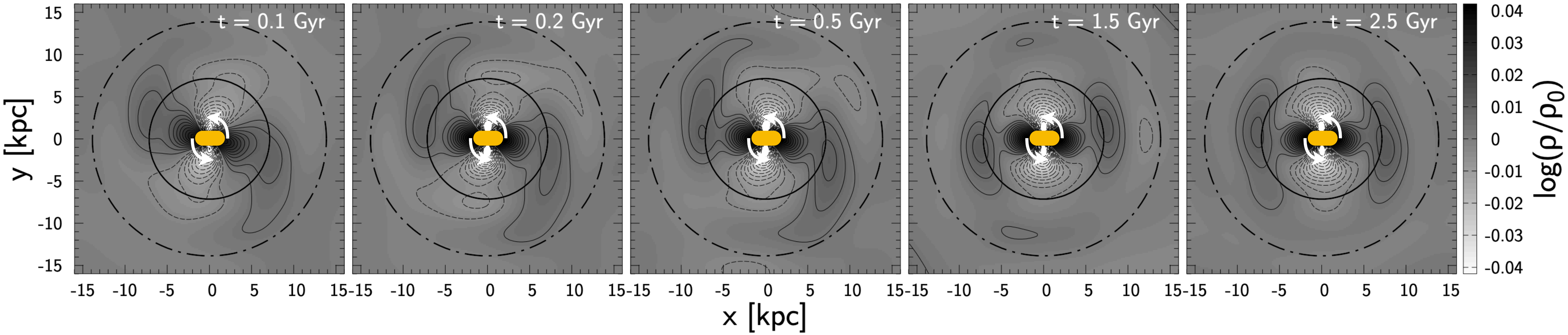
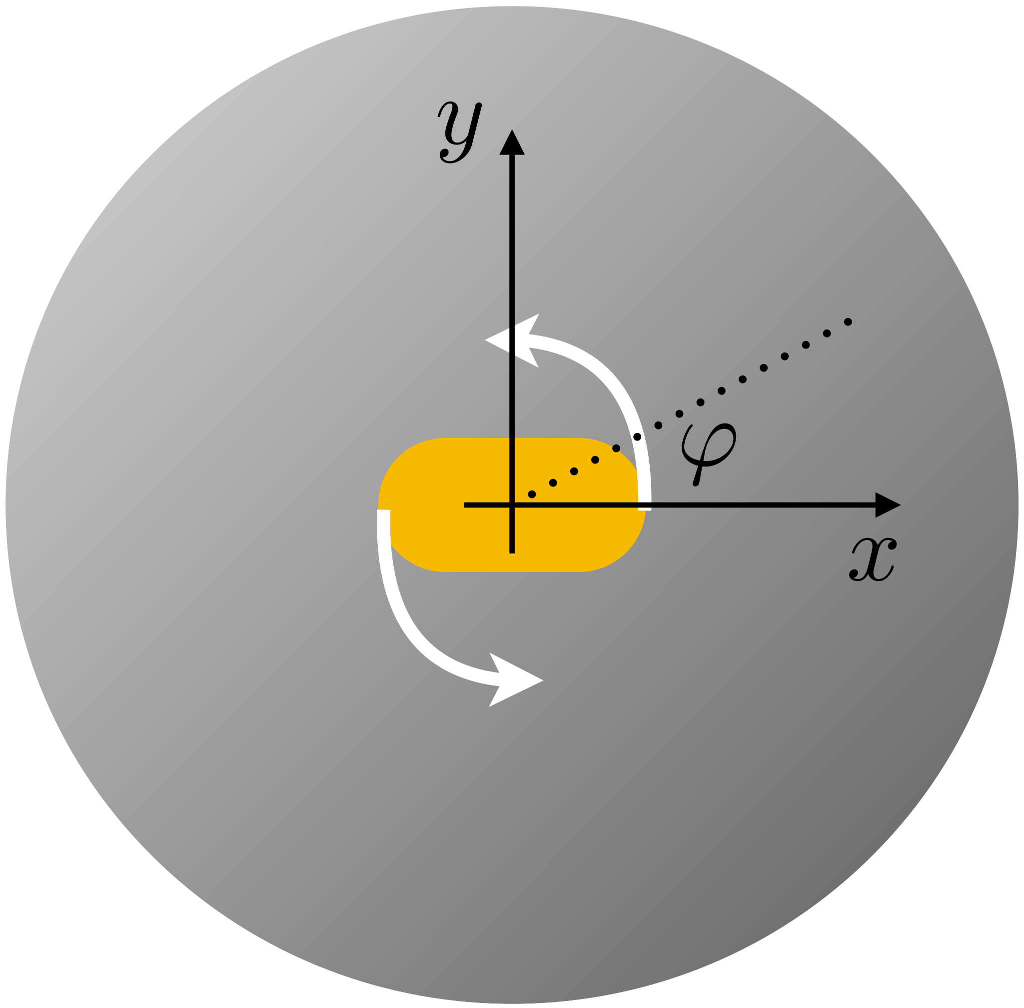


This produces a backreaction torque on bar: *dynamical friction*

Bar-Halo Interaction



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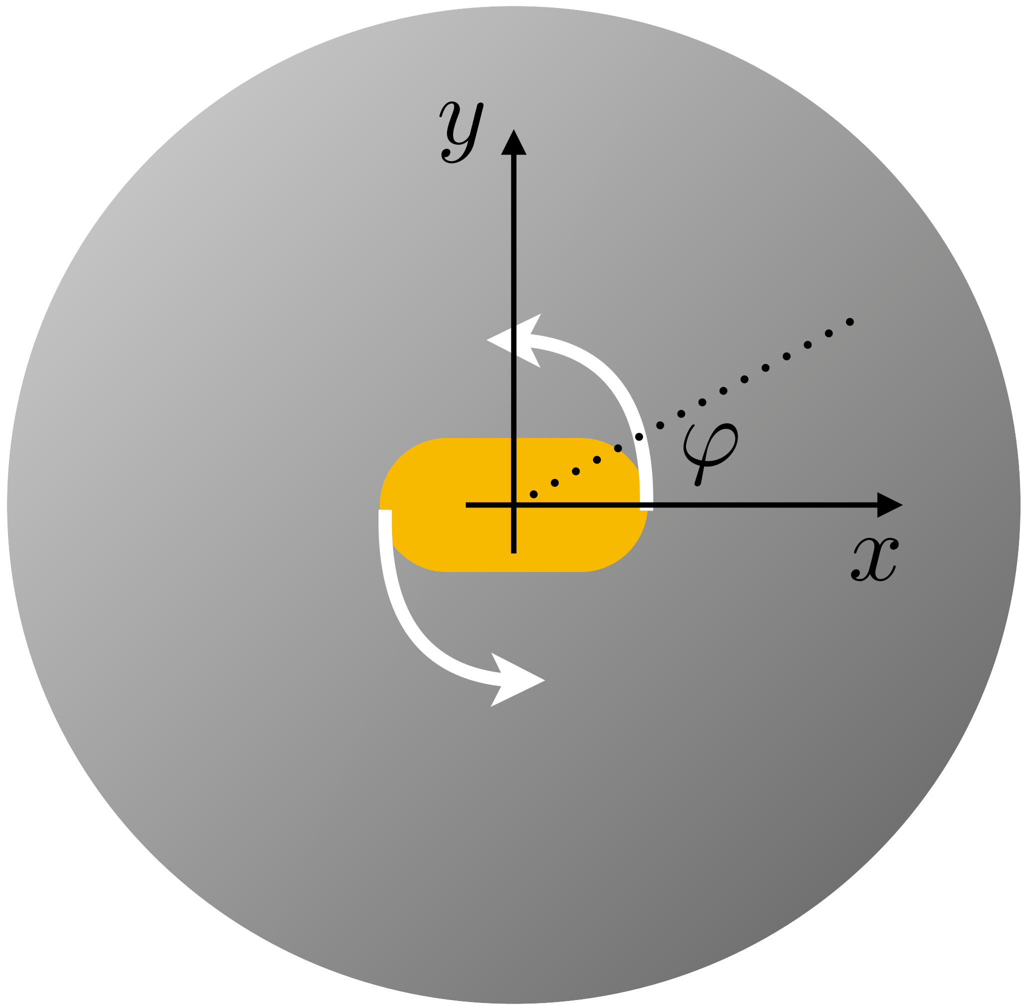
$$H = \frac{\mathbf{v}^2}{2} + \Phi_0(\mathbf{x}) + \delta\Phi(\mathbf{x}, t)$$

BAR-HALO INTERACTION



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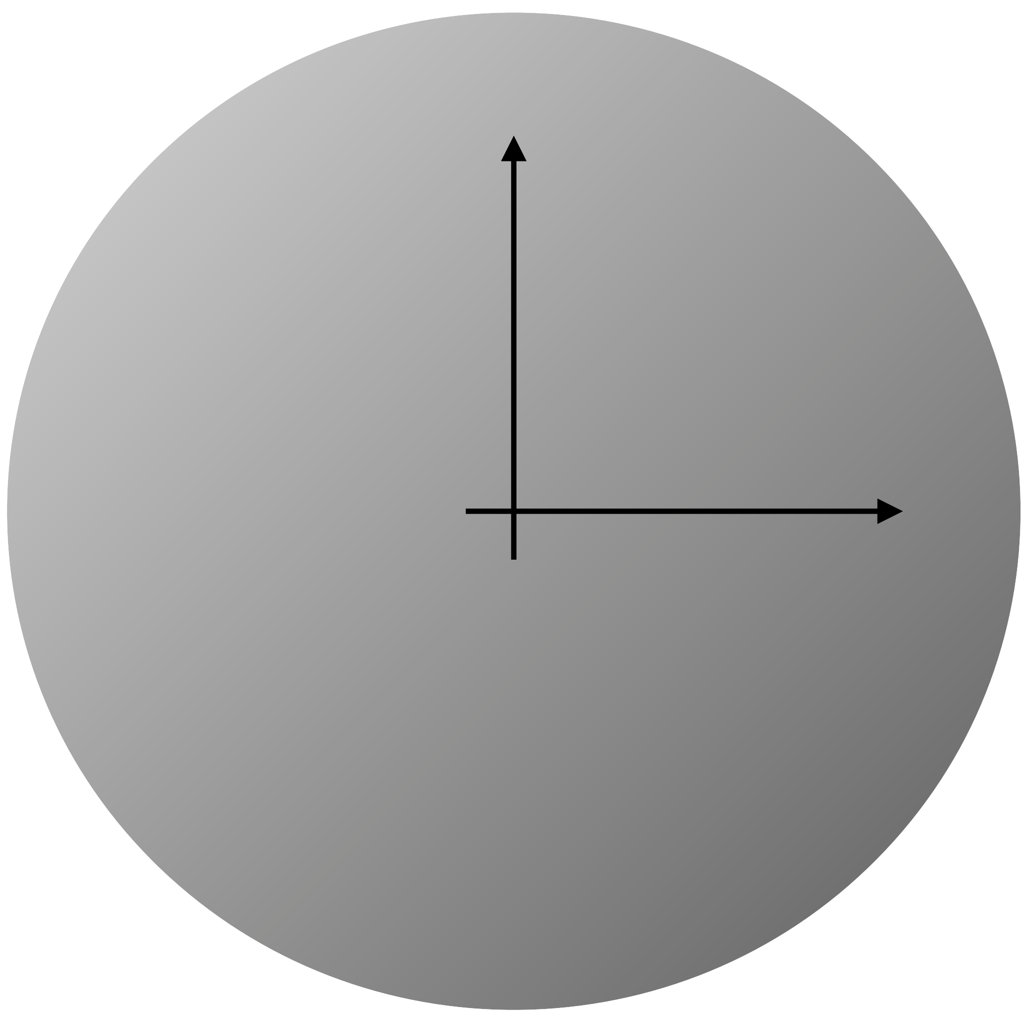
UNPERTURBED DARK MATTER ORBITS



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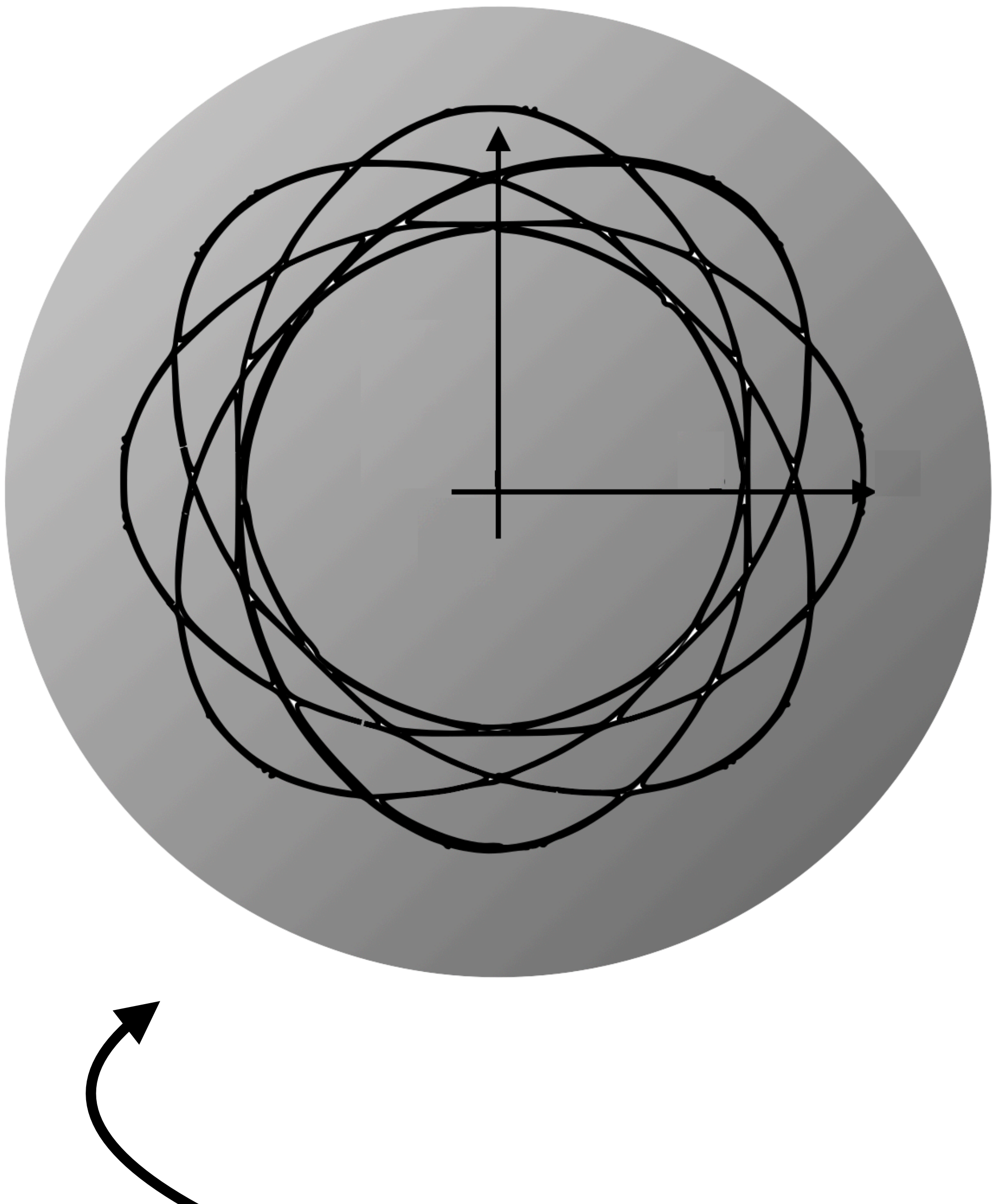

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All orbits in spherical potentials look like this

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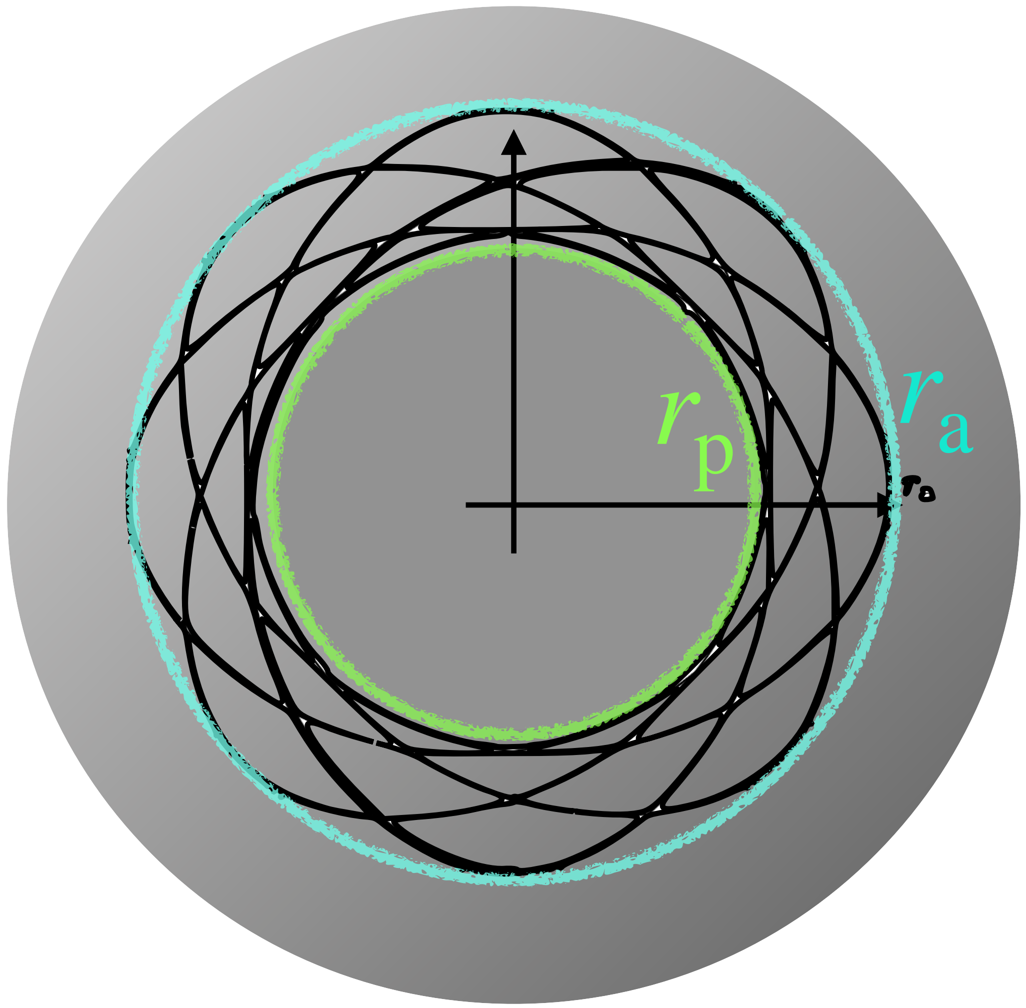

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Azimuthal circulation + radial oscillation between pericentre and apocentre

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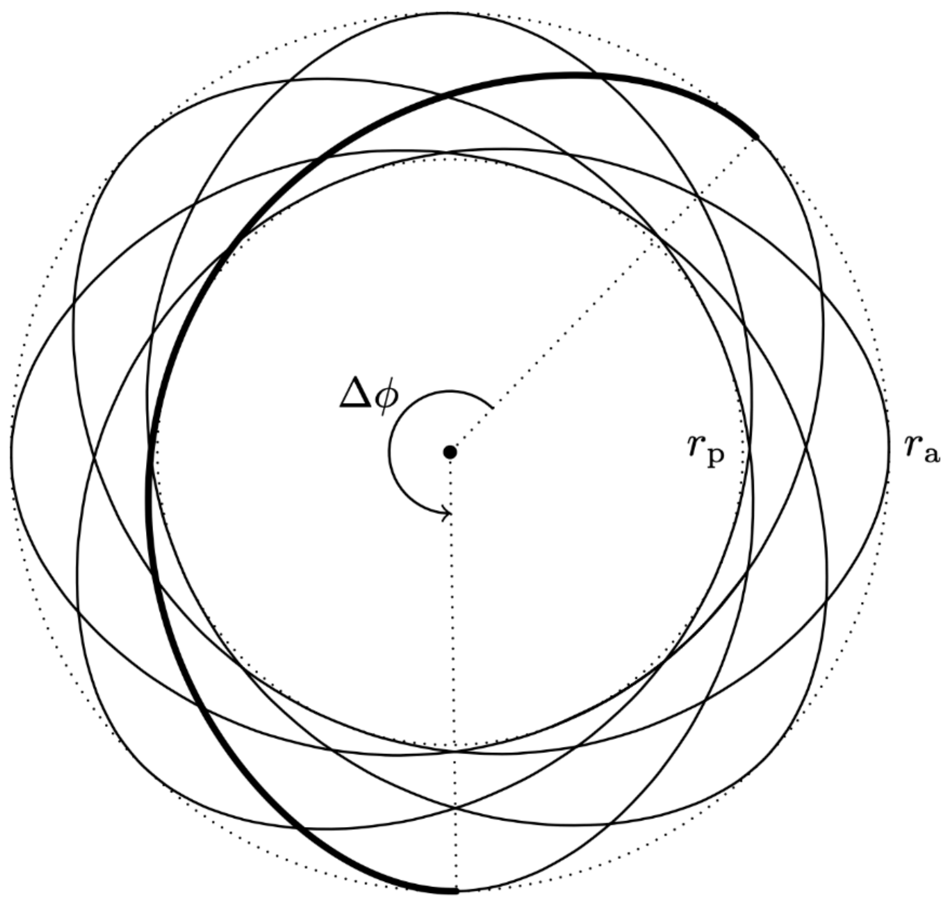

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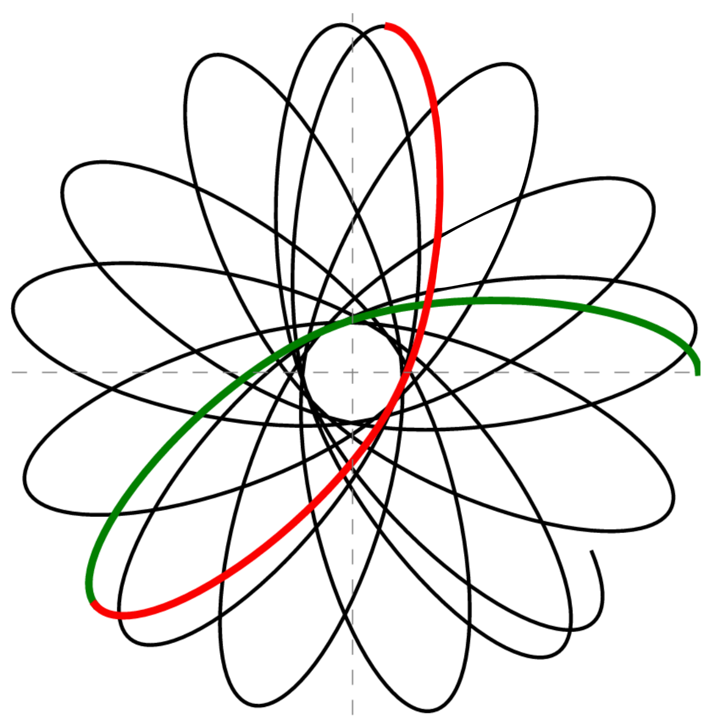

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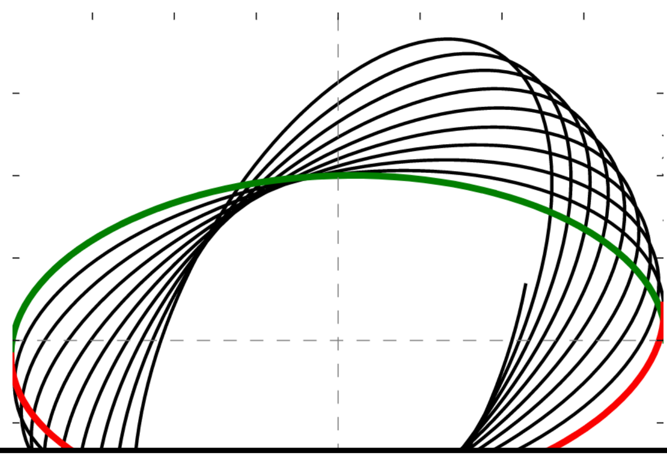
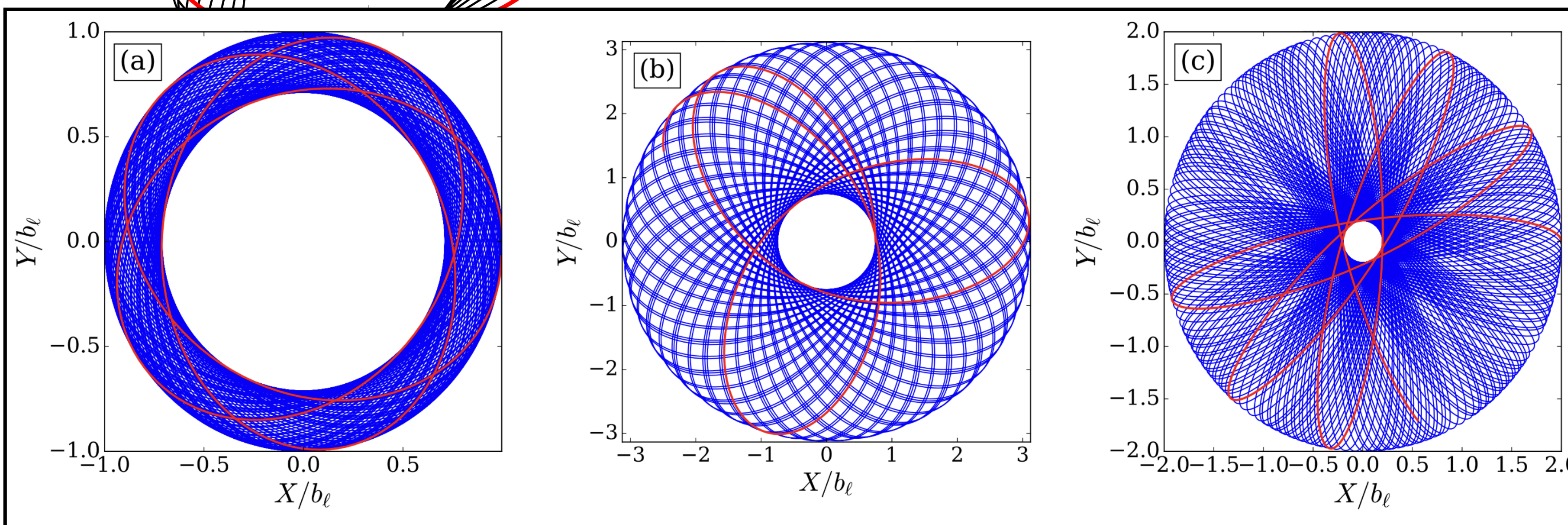




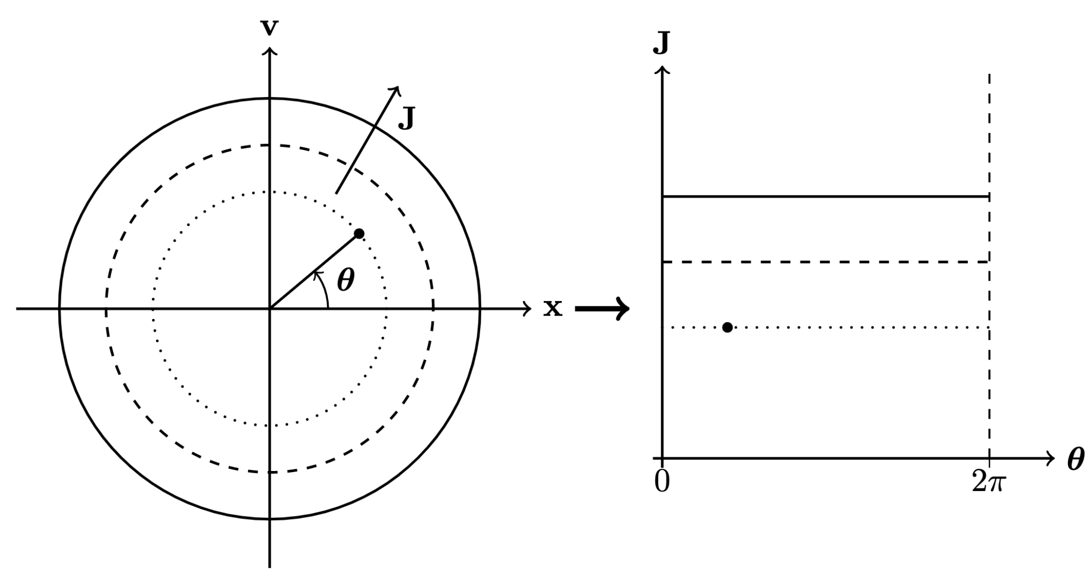
Angle-action Coordinates

Change of variables:

$$(\boldsymbol{x}, \boldsymbol{v}) \to (\boldsymbol{\theta}, \boldsymbol{J})$$

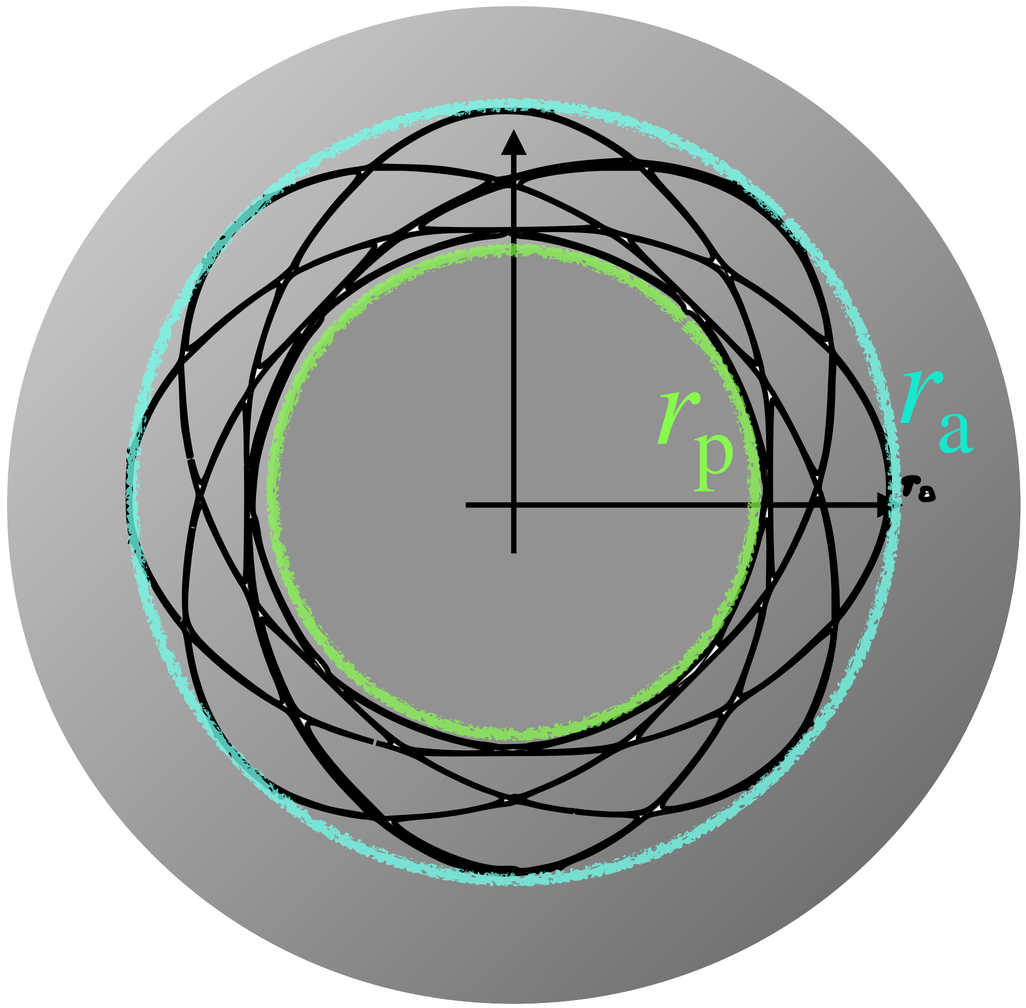
Such that $H = \frac{\mathbf{v}^2}{2} + \Phi_0(\mathbf{x})$ only depends on $\boldsymbol{J}$

$$\frac{\mathrm{d}\boldsymbol{\theta}}{\mathrm{d}t} = \frac{\partial H}{\partial \boldsymbol{J}} = \boldsymbol{\Omega}(\boldsymbol{J}) \qquad \frac{\mathrm{d}\boldsymbol{J}}{\mathrm{d}t} = -\frac{\partial H}{\partial \boldsymbol{\theta}} = 0,$$



$$\boldsymbol{\theta}(t) = \boldsymbol{\theta}_0 + \boldsymbol{\Omega}(\boldsymbol{J})\,t \quad ; \quad \boldsymbol{J}(t) = \text{const.}$$

Angle-action Coordinates



For spherical systems,

$$\boldsymbol{J} = (J_r, L)$$

$$\boldsymbol{\theta} = (\theta_r, \theta_\phi)$$

Angular momentum

$$L = r^2 \frac{\mathrm{d}\phi}{\mathrm{d}t}$$

Radial action

$$J_r = \frac{1}{\pi}\int_{r_{\rm p}}^{r_{\rm a}} \mathrm{d}r \, \sqrt{2(E - \Phi(r)) - L^2/r^2}$$

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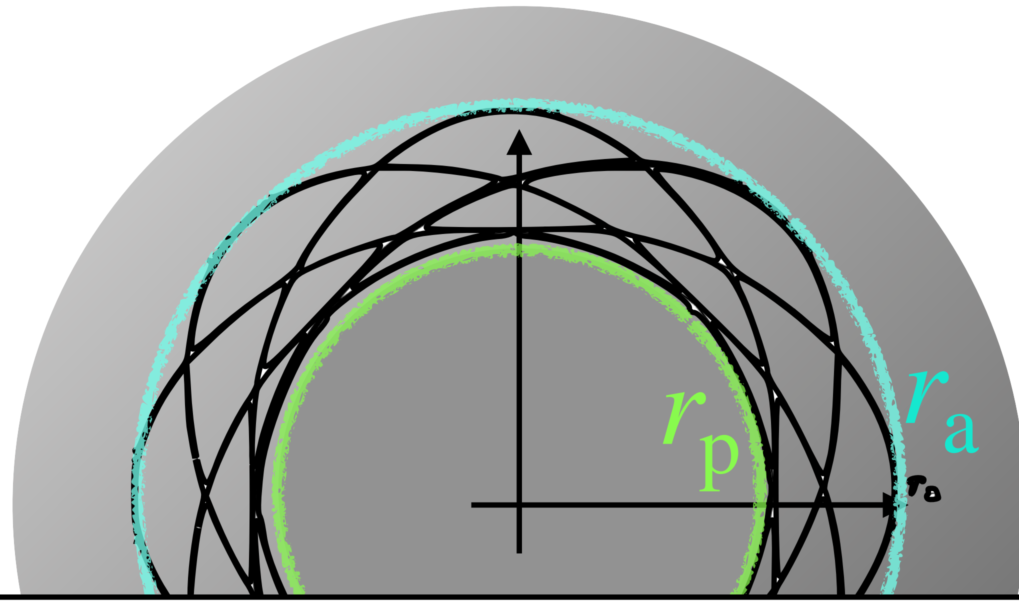

Action J tells you which orbit you are on;

Angle θ tells you where you are on that orbit.

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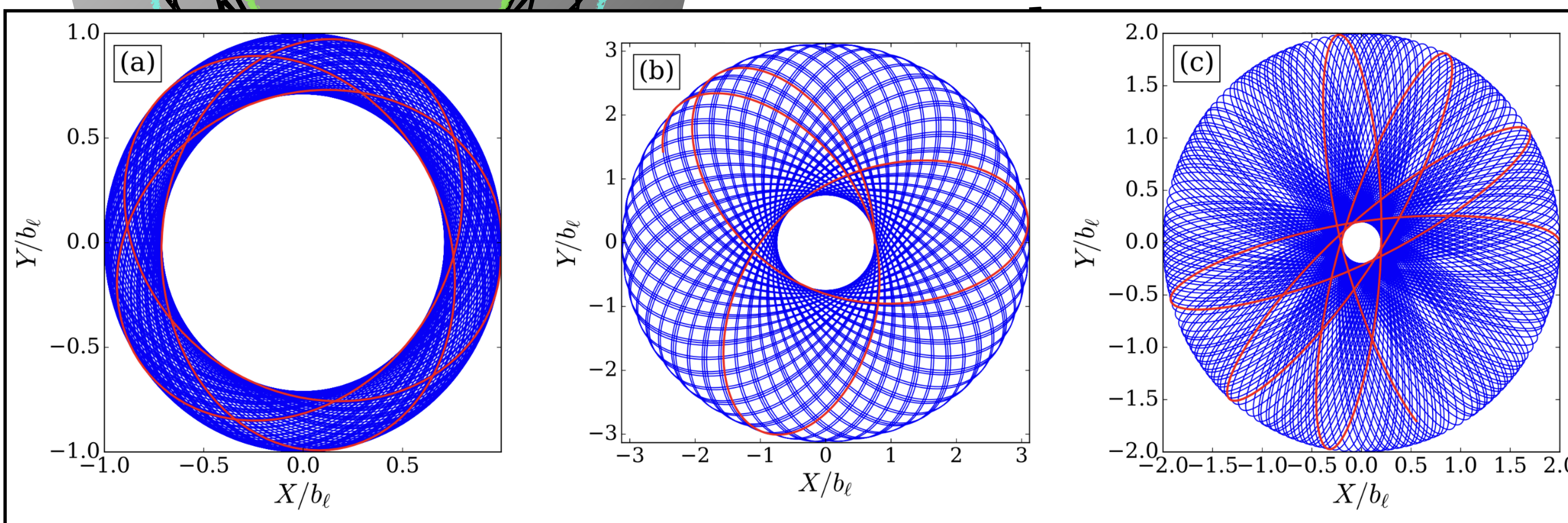
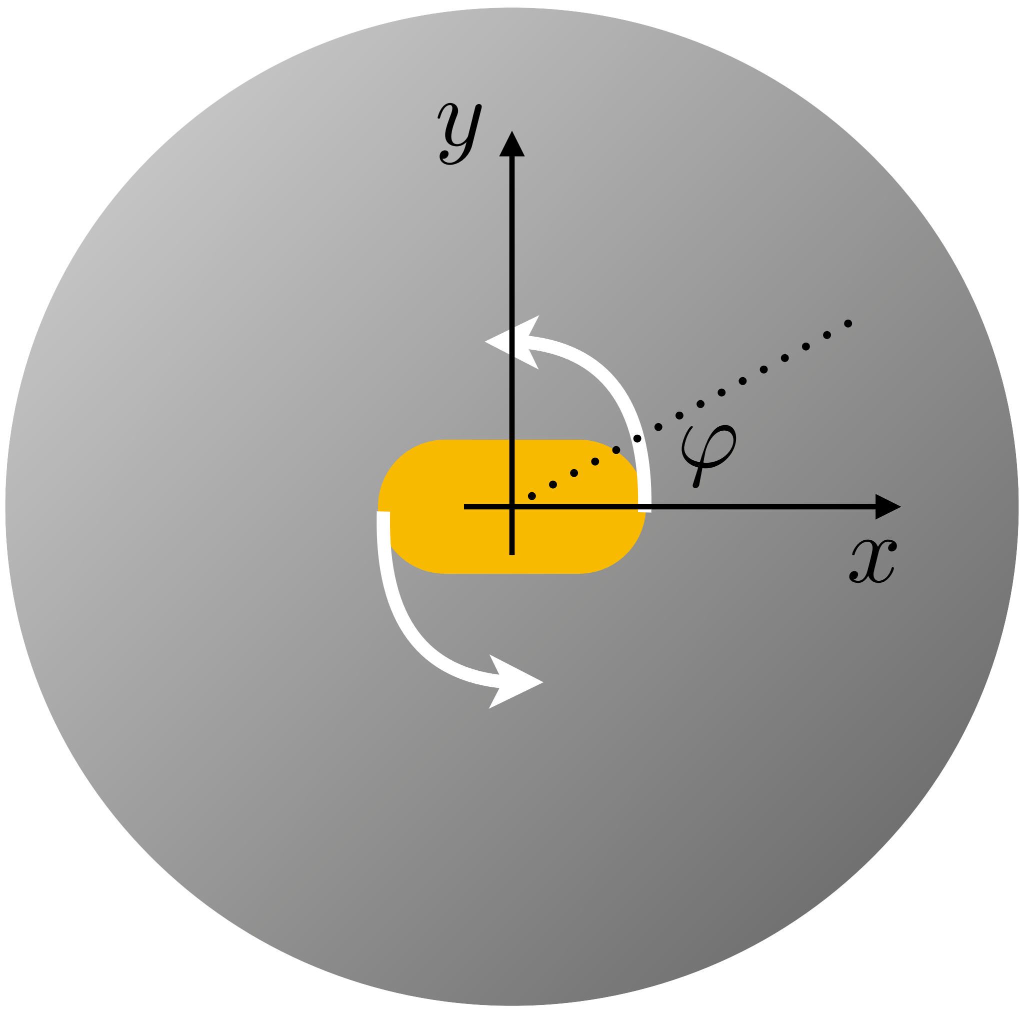


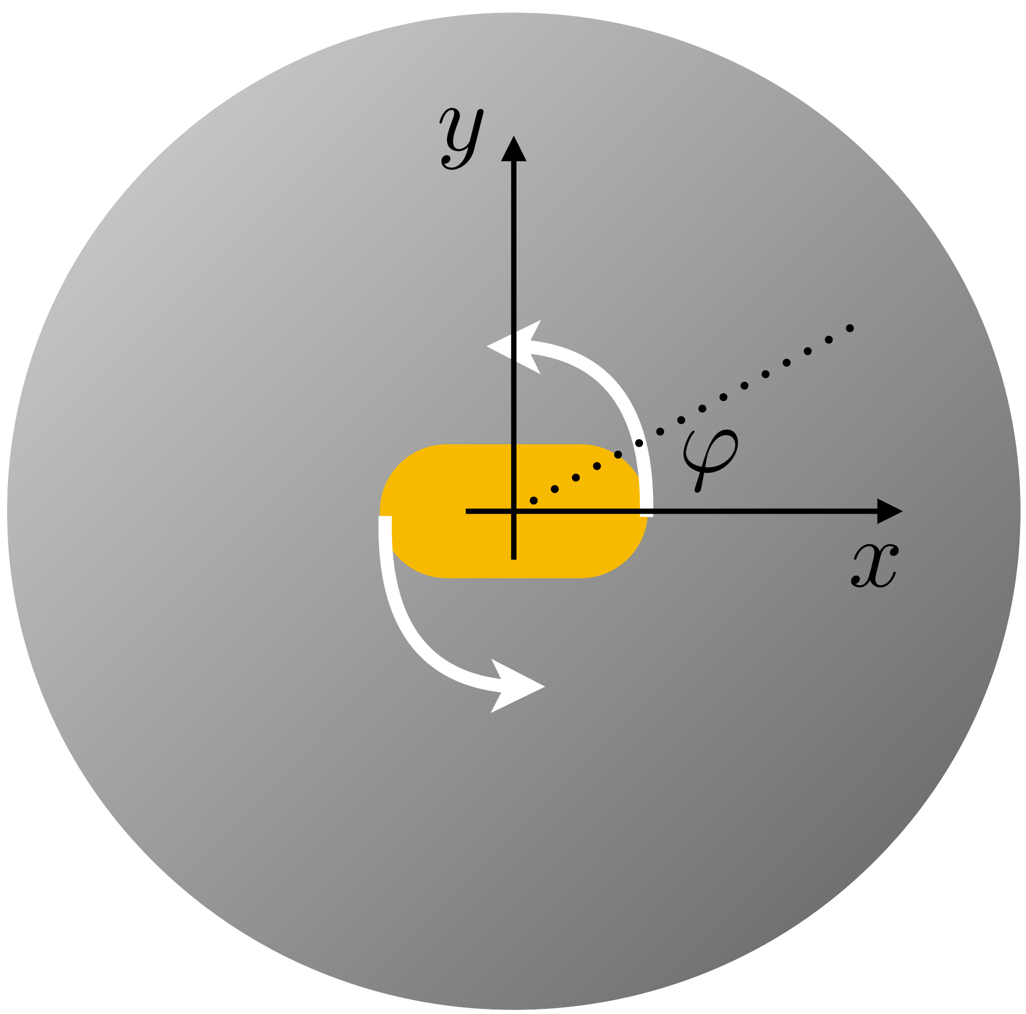
Bar-Halo Interaction



Bar rotates in azimuth with pattern speed $\Omega_{\rm p}$

Dark matter particles have actions $\mathbf{J} = (J_r, L, L_z)$
& corresponding orbital frequencies $\boldsymbol{\Omega} = (\Omega_r, \Omega_\psi, \Omega_\varphi)$

Bar-Halo Interaction



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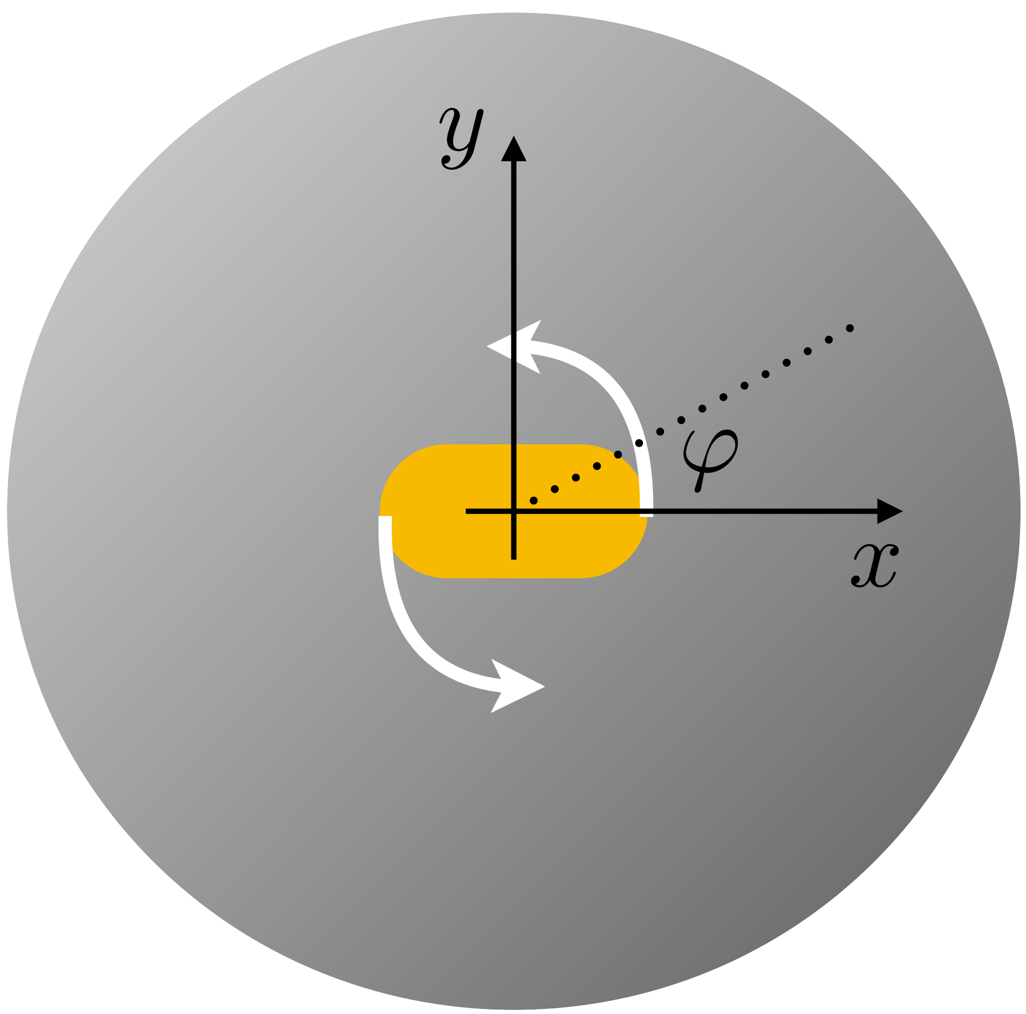


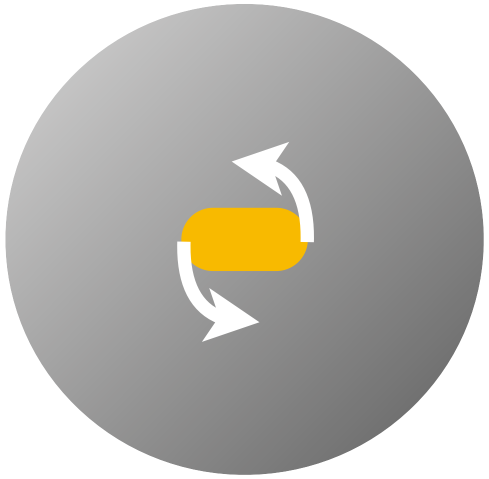
Dark matter particle motion is governed by

$$H = H_0(\boldsymbol{J}) + \delta\Phi(\boldsymbol{\theta}, \boldsymbol{J}, t)$$

Unperturbed halo Hamiltonian

Bar perturbation

$$\left|\frac{\delta\Phi}{\Phi_0}\right| \sim 2\%$$



WHAT IS THE FRICTIONAL TORQUE?

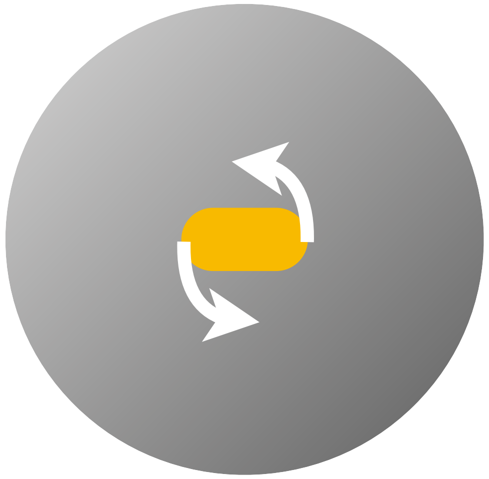
Torque on one halo particle:

$$\frac{\mathrm{d}L_z}{\mathrm{d}t} = -\frac{\partial \delta \Phi}{\partial \theta_\varphi}$$

Total torque on bar:

$$\mathcal{T}(t) = \int \mathrm{d}\boldsymbol{\theta}\mathrm{d}\boldsymbol{J}\, f(\boldsymbol{\theta}, \boldsymbol{J}, t) \frac{\partial \delta \Phi(\boldsymbol{\theta}, \boldsymbol{J}, t)}{\partial \theta_\varphi}$$

Challenge is to compute the perturbed $f(\boldsymbol{\theta}, \boldsymbol{J}, t)$



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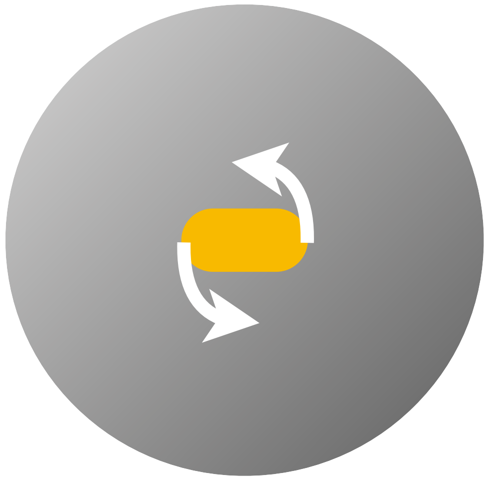


LINEAR THEORY (LBK72)

Compute perturbed DF using linearized Vlasov equation:

$$f_{\boldsymbol{N}}(\boldsymbol{J},t) = i\boldsymbol{N}\cdot\frac{\partial f_0}{\partial \boldsymbol{J}}\int_0^t \mathrm{d}t'\,\delta\Phi_{\boldsymbol{N}}(\boldsymbol{J},t')\mathrm{e}^{-i\boldsymbol{N}\cdot\boldsymbol{\Omega}(t-t')}$$

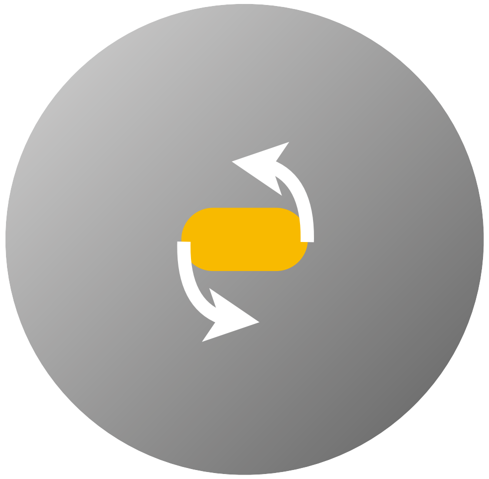
Result:

$$\mathcal{T}_{\boldsymbol{N}}^{\mathrm{lin}}(t) \equiv (2\pi)^3 N_\varphi \int \mathrm{d}\boldsymbol{J}\,|\delta\Phi_{\boldsymbol{N}}|^2\,\boldsymbol{N}\cdot\frac{\partial f_0}{\partial \boldsymbol{J}}\,\frac{\sin[(\boldsymbol{N}\cdot\boldsymbol{\Omega}-N_\varphi\Omega_{\mathrm{p}})t]}{\boldsymbol{N}\cdot\boldsymbol{\Omega}-N_\varphi\Omega_{\mathrm{p}}}$$

Time-asymptotic *LBK torque*:

$$\mathcal{T}_{\boldsymbol{N}}^{\mathrm{LBK}} \equiv (2\pi)^3 N_\varphi \int \mathrm{d}\boldsymbol{J}\,|\delta\Phi_{\boldsymbol{N}}|^2\,\boldsymbol{N}\cdot\frac{\partial f_0}{\partial \boldsymbol{J}}\;\pi\delta\left(\boldsymbol{N}\cdot\boldsymbol{\Omega}-N_\varphi\Omega_{\mathrm{p}}\right)$$

This is (a) finite, (b) negative, and (c) resonant



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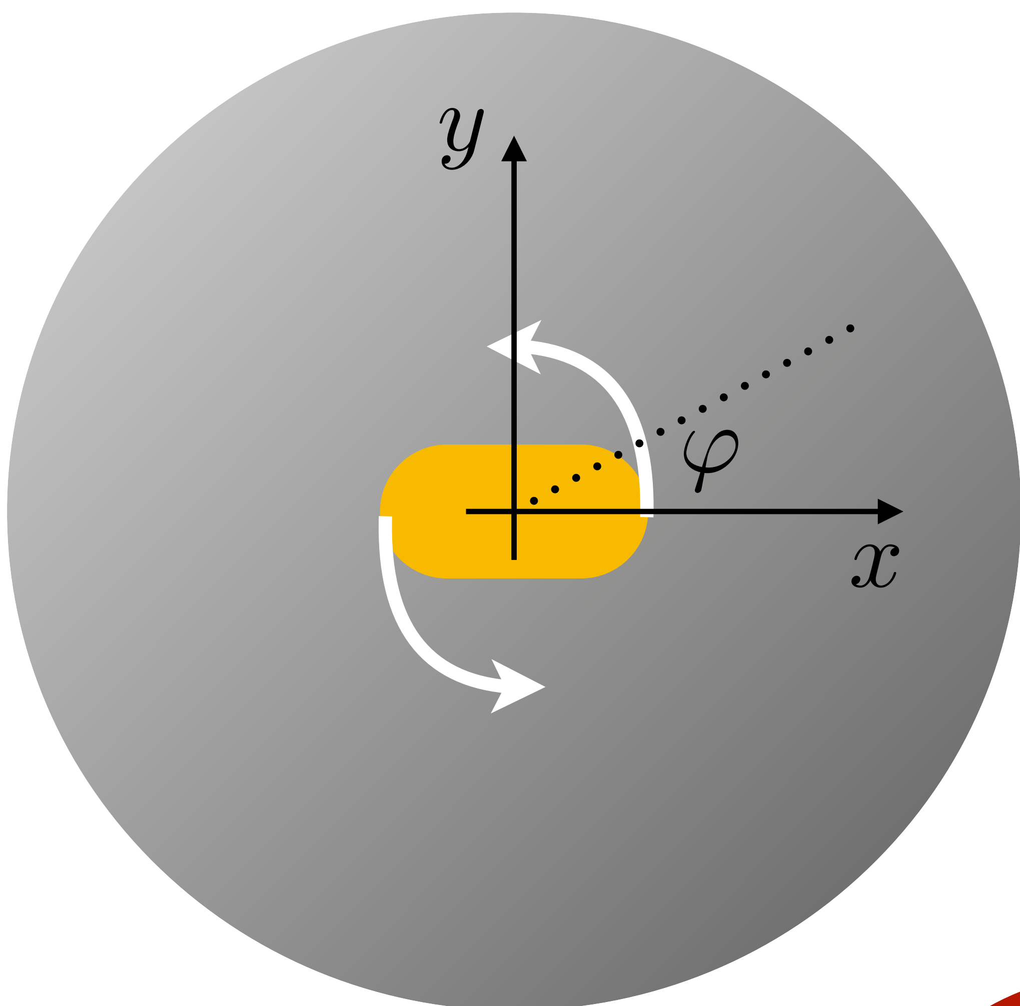
And (d) redolent of Landau damping

Nonlinear Theory (TW84)



Dark matter particle motion is governed by

$$H = \frac{\mathbf{v}^2}{2} + \Phi_0(\mathbf{x}) + \delta\Phi(\mathbf{x}, t)$$

Key point: if you are sufficiently close to resonance, the bar is not a linear perturbation!

$$\mathbf{N} \cdot \boldsymbol{\Omega} = N_\varphi \Omega_\mathrm{p}$$

(so LBK predicts its own demise)

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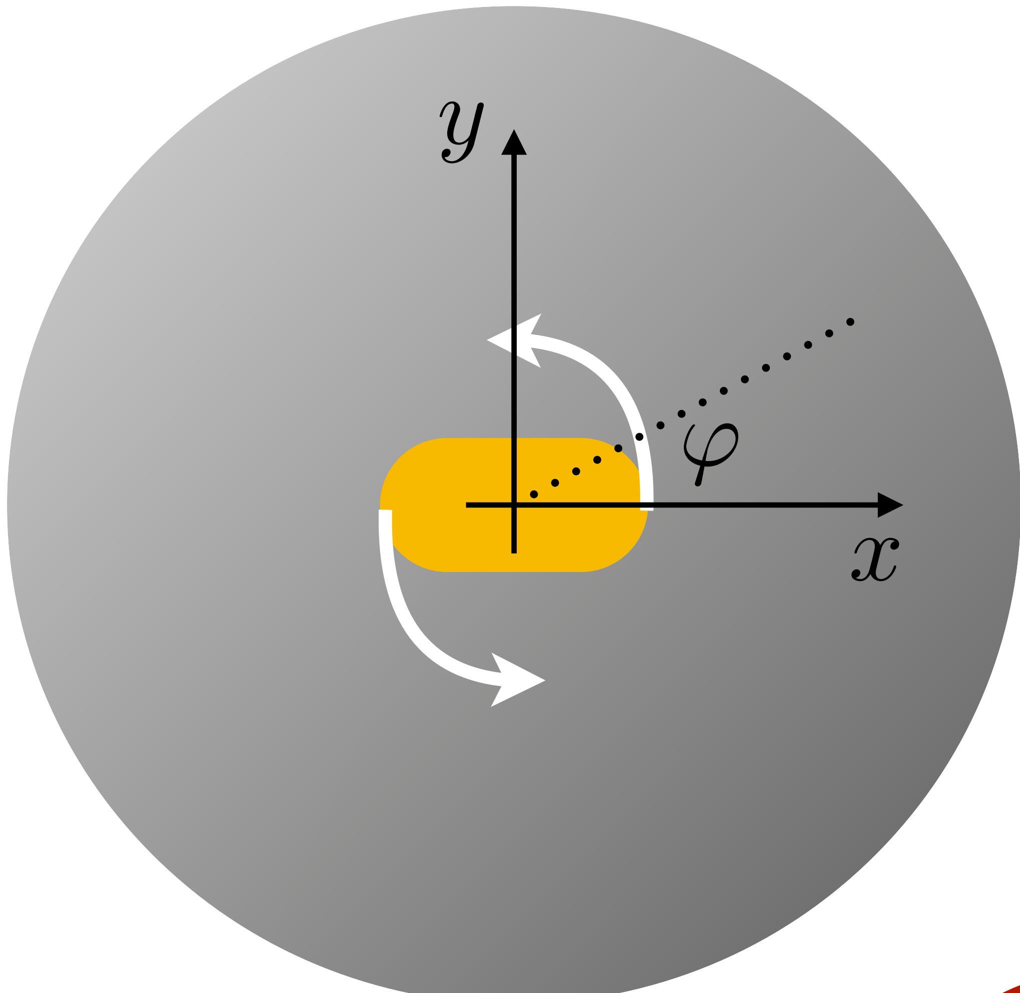

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O'Neill (1965)
Mazitov (1965)

SLOW-FAST VARIABLES

Key idea: in the vicinity of each resonance, replace one of your angle variables with the *slow angle*

$$\theta_s \equiv \boldsymbol{N} \cdot \boldsymbol{\theta} - N_\varphi \Omega_{\mathrm{p}} t$$

The corresponding *slow action is*

$$J_{\mathrm{s}} \equiv L_z / N_\varphi$$

Then particle motion can be described in slow angle-action space using the Hamiltonian

$$\mathcal{H} = H_0(J_{\mathrm{s}}) - N_\varphi \Omega_{\mathrm{p}} J_{\mathrm{s}} + \sum_{k \neq 0} \Psi_k(J_{\mathrm{s}}) \exp(ik\theta_{\mathrm{s}})$$

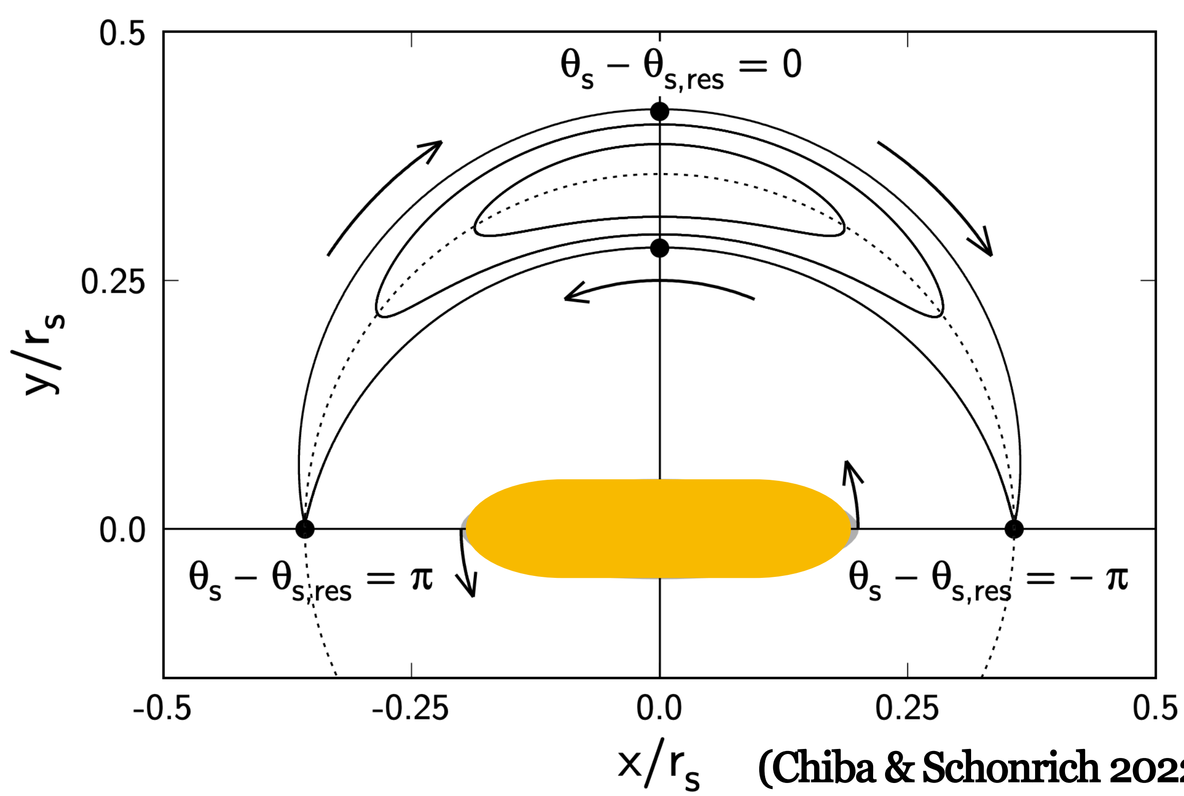
Slow-fast Variables

Example: <u>corotation resonance</u>. *Slow angle* is

$$\theta_{\rm s} = \theta_{\rm s}(0) + 2[\Omega_\varphi - \Omega_{\rm p}]t$$

The corresponding *slow action* is

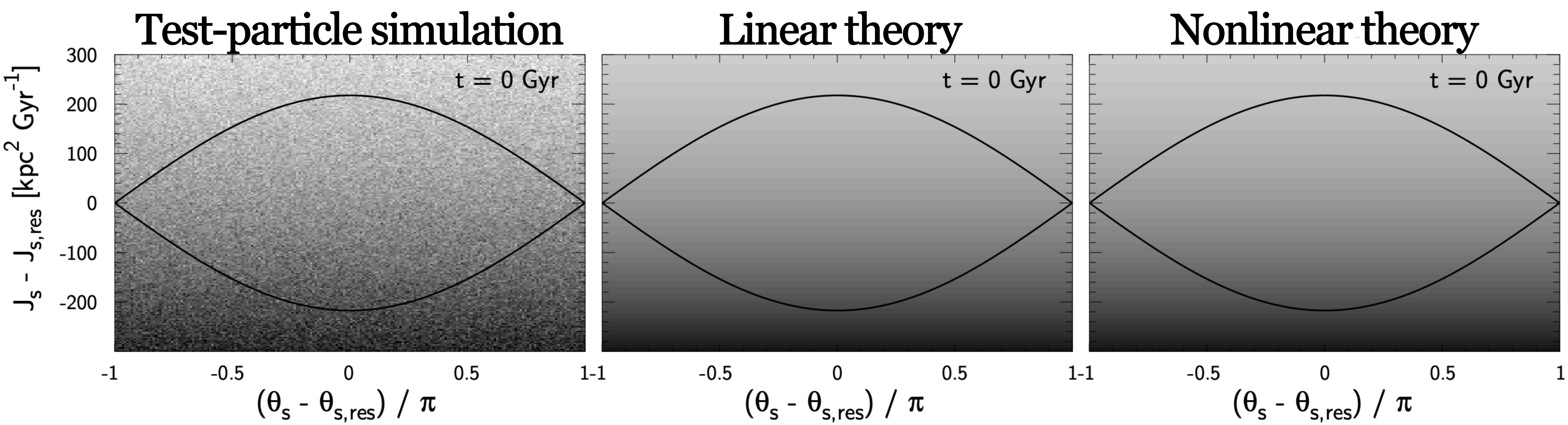
$$J_{\rm s} = L_z/2$$



(Chiba & Schonrich 2022)

Slow-fast Variables

Illustration of linear vs nonlinear theory from Chiba & Schonrich (2022)



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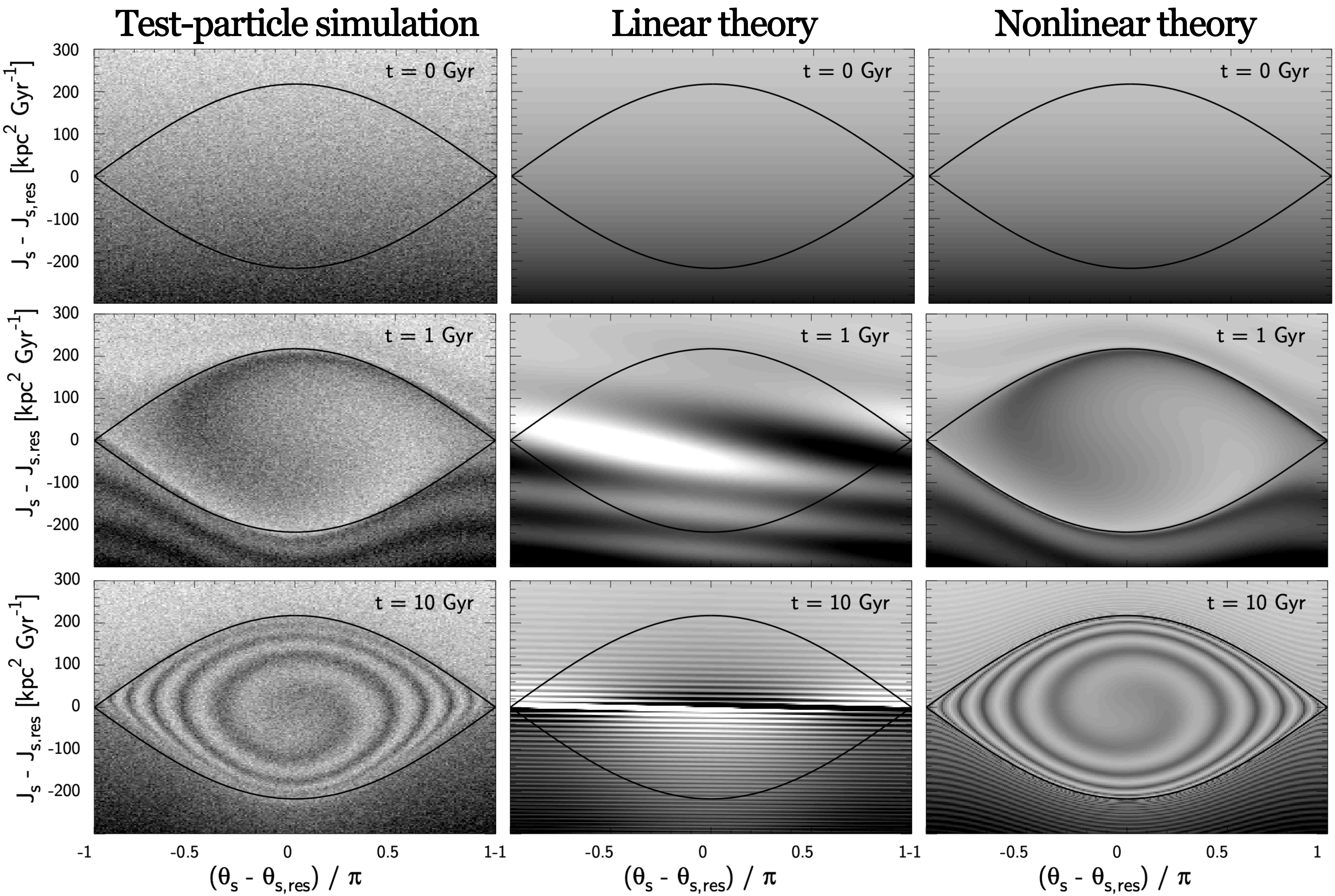
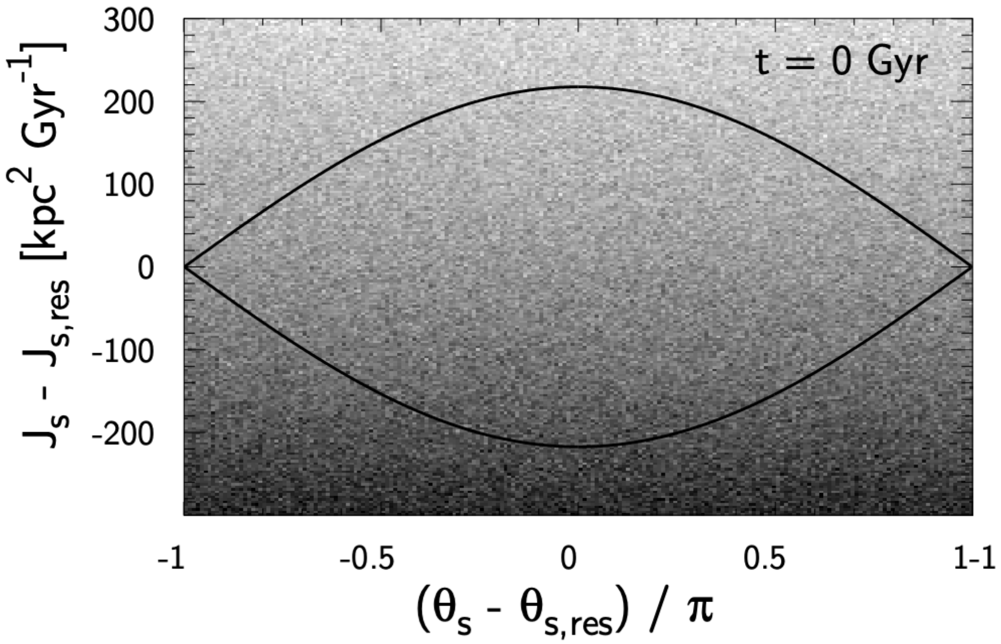
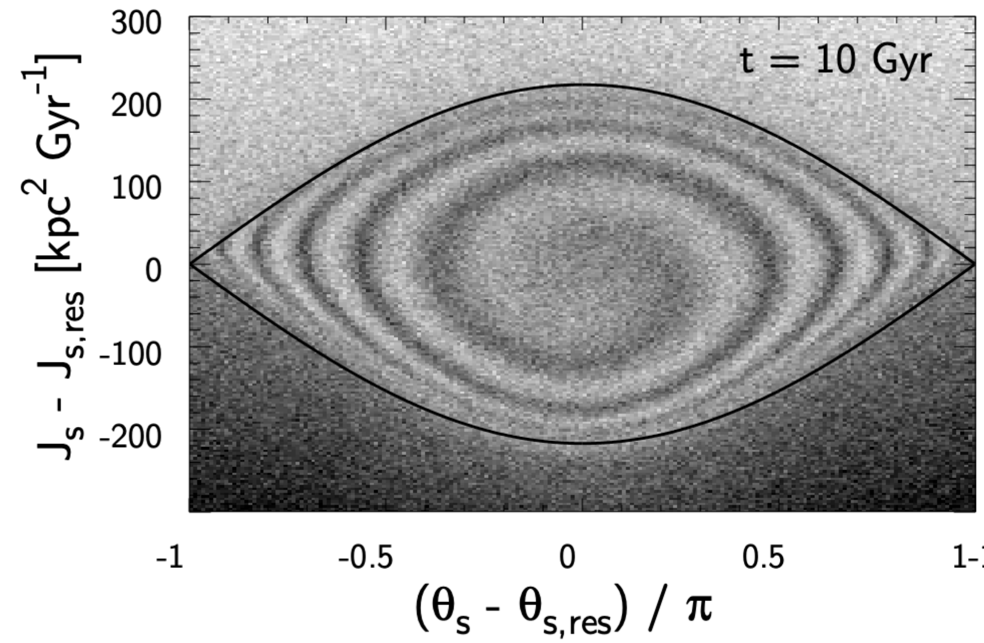


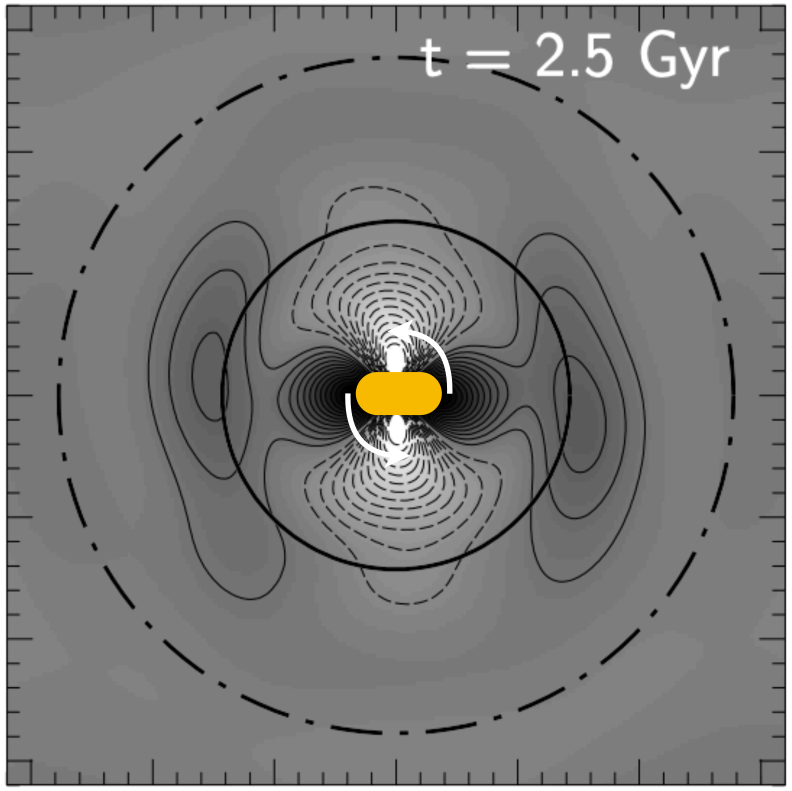
Phase-mixing

in slow angle-action phase space

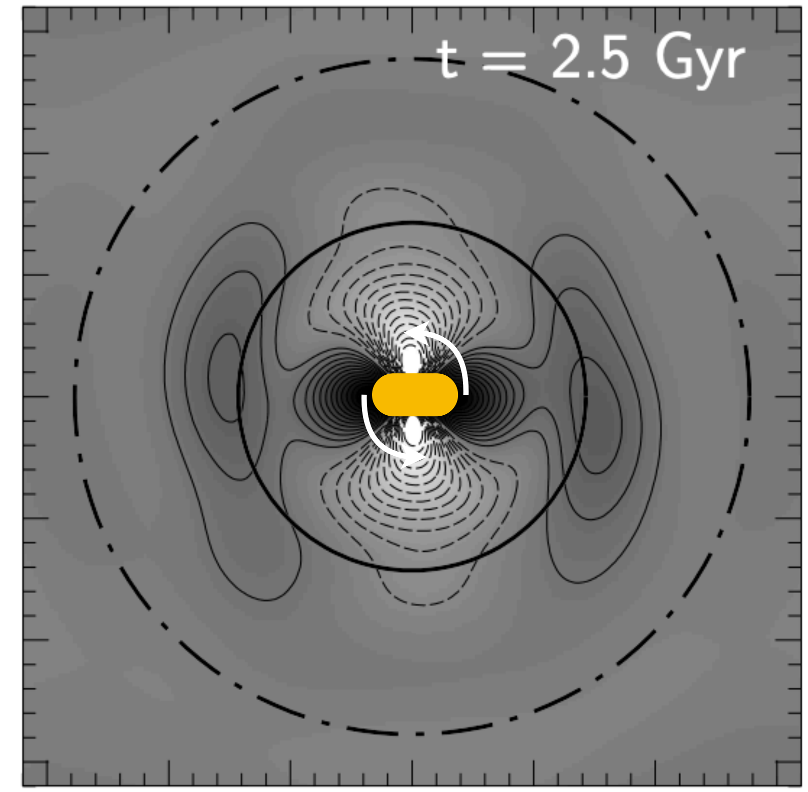




in physical space



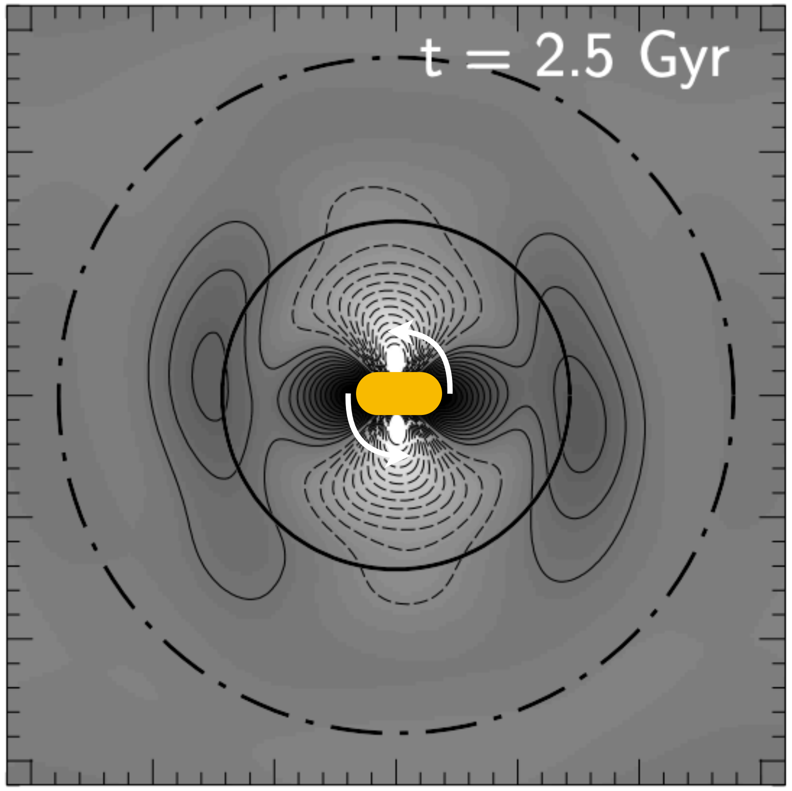
Nonlinear Theory (TW84)



Phase mixing leads to a symmetric density distribution surrounding the bar, and hence no torque

$$\mathcal{T}_{\boldsymbol{N}}^{\mathrm{TW84}} = 0$$

Nonlinear Theory (TW84)



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Diffusion?

Both LBK and TW84 were solving a *collisionless* problem:

$$\frac{\partial f}{\partial t} + \{f, \mathcal{H}\} = 0$$

In reality, there always exists some collisionality/diffusion (e.g. due to substructure, molecular clouds, ...):

$$\frac{\partial f}{\partial t} + \{f, \mathcal{H}\} = C[f]$$

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Q. How does diffusion affect these delicate resonant phenomena?

DIFFUSION?

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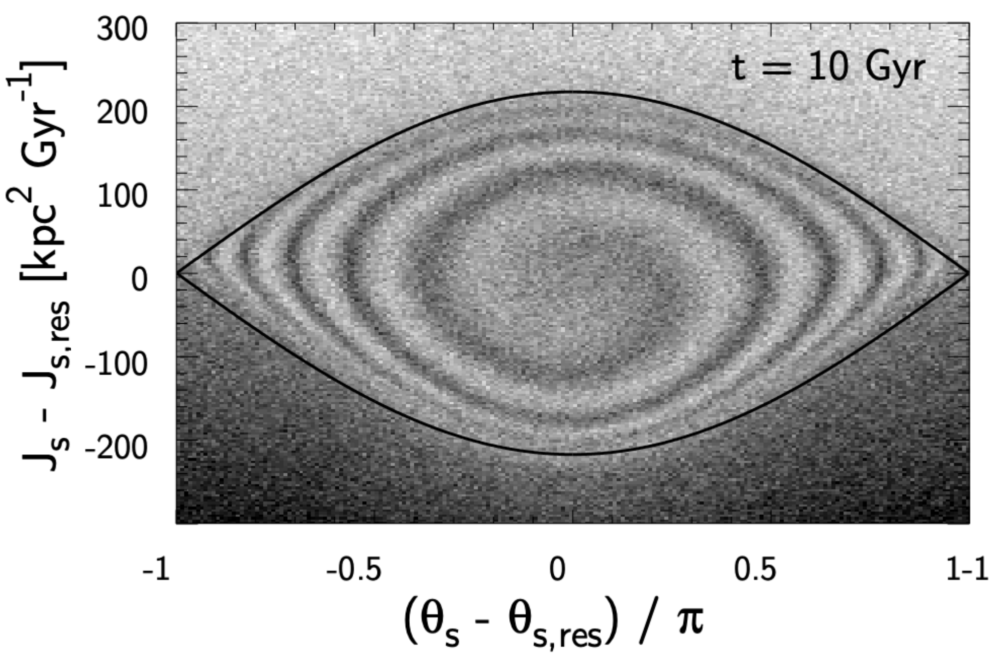
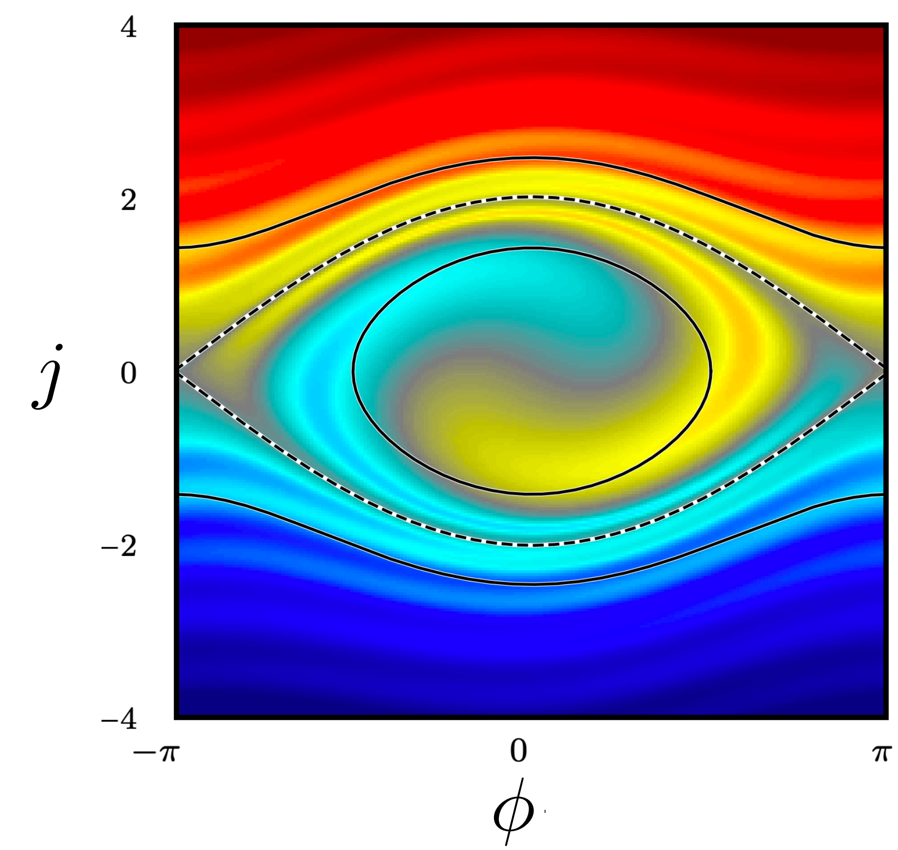
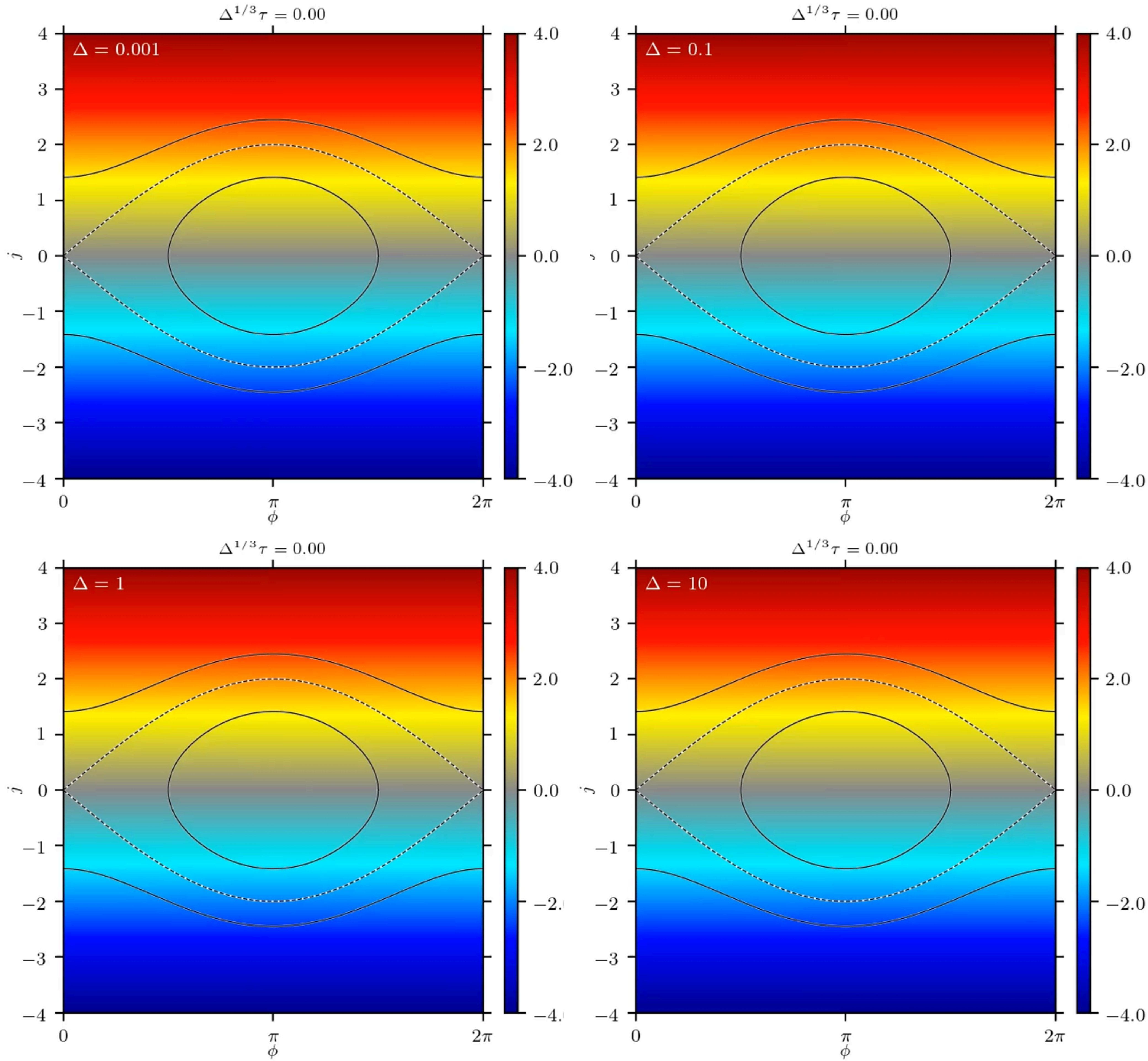


non-dimensionalize the slow angle-action variables



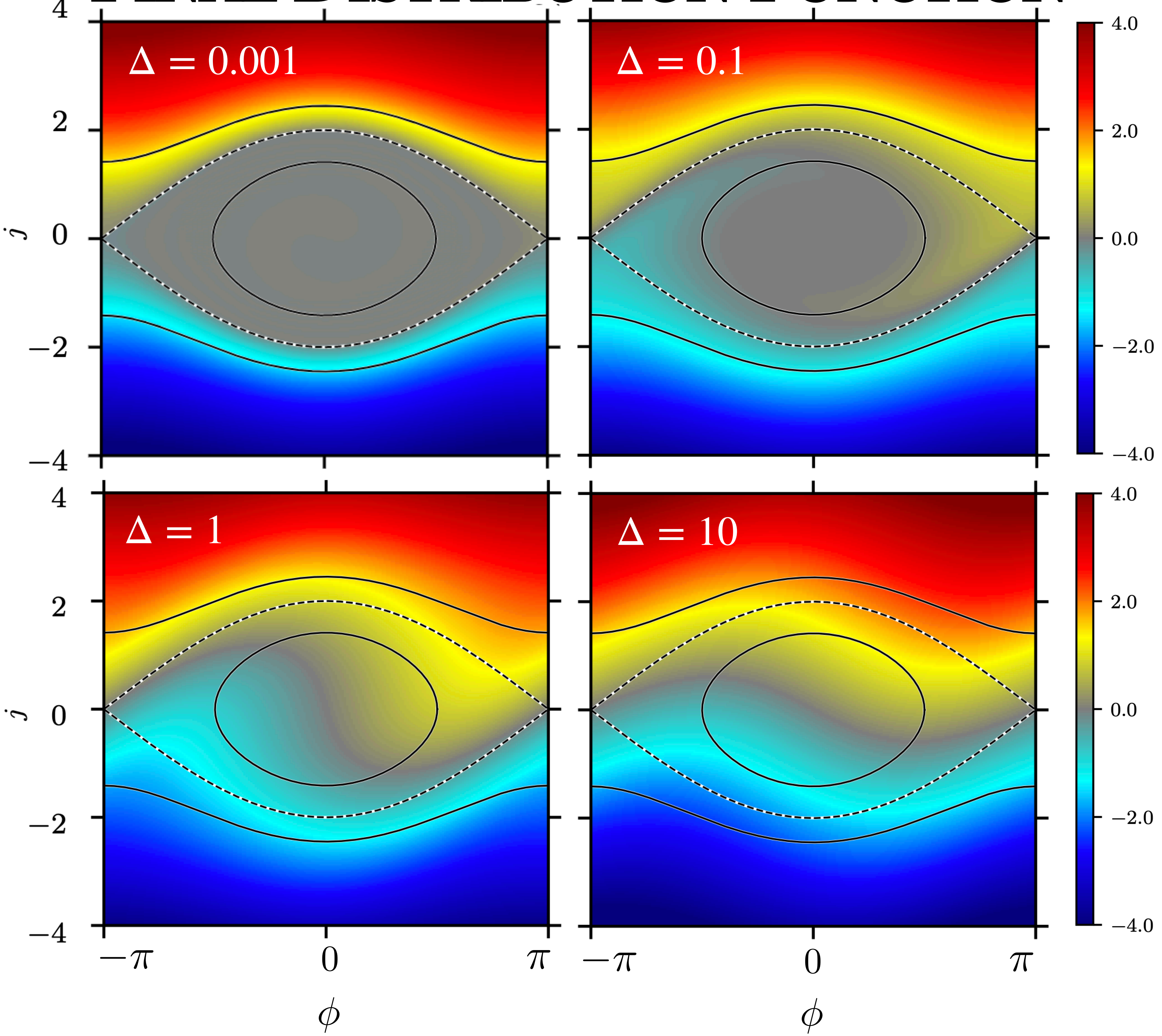
Then,

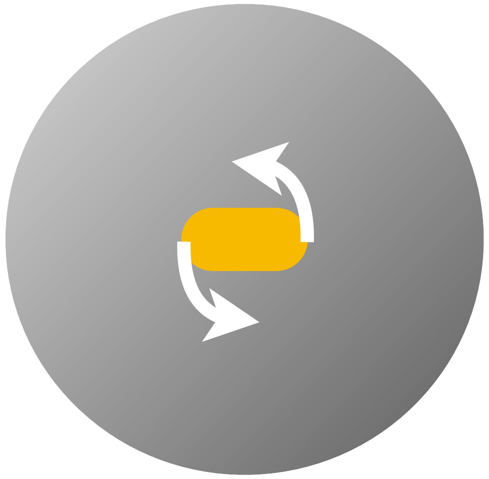
$$\frac{\partial f}{\partial \tau} + j\frac{\partial f}{\partial \phi} - \sin\phi\frac{\partial f}{\partial j} = \Delta\frac{\partial^2 f}{\partial j^2} \quad \text{where} \quad \Delta \equiv \frac{t_{\rm lib}}{t_{\rm diff}}$$



$\Delta^{1/3}\tau = 0.00$
$\Delta = 0.001$
$\Delta = 0.1$
$\Delta = 1$
$\Delta = 10$
j
ϕ
0
π
2π
4
3
2
1
−1
−2
−3
−4
4.0
2.0
0.0
−2.0
−4.0

Final Distribution Function





WHAT IS THE FRICTIONAL TORQUE?

Torque on one halo particle:

$$\frac{\mathrm{d}L_z}{\mathrm{d}t} = -\frac{\partial \delta \Phi}{\partial \theta_\varphi}$$

Total torque on bar = - total torque on halo:

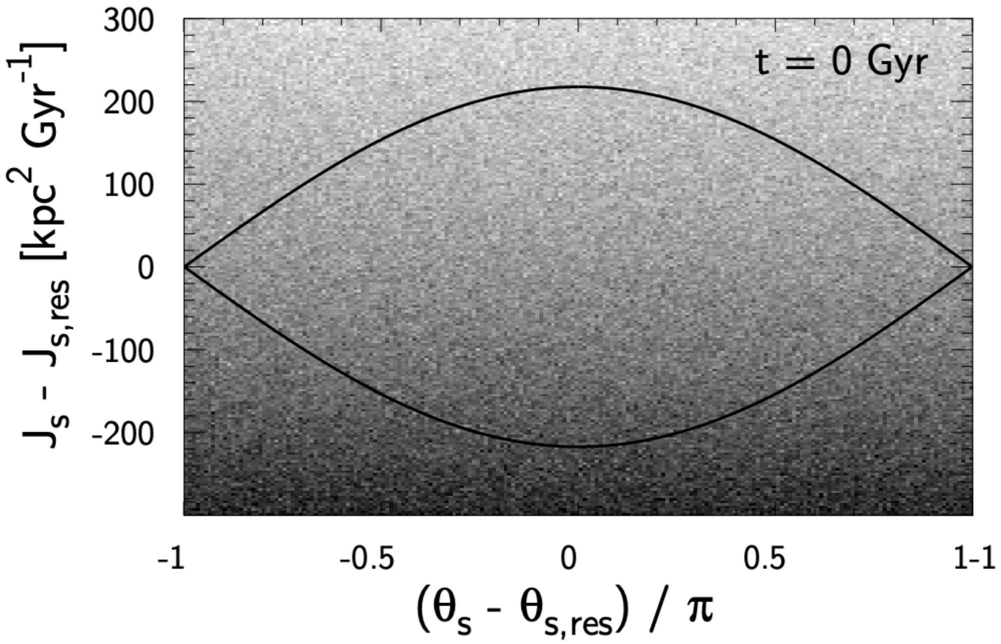
$$\mathcal{T}(t) = \int \mathrm{d}\boldsymbol{\theta}\mathrm{d}\boldsymbol{J}\, f(\boldsymbol{\theta}, \boldsymbol{J}, t) \frac{\partial \delta\Phi(\boldsymbol{\theta}, \boldsymbol{J}, t)}{\partial \theta_\varphi}$$

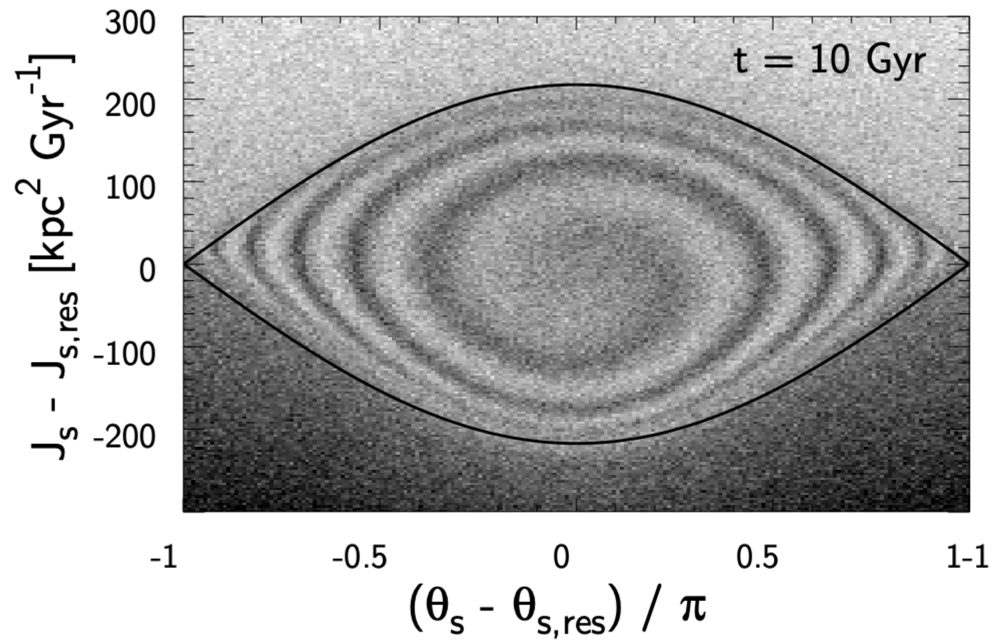
Compute $f(\boldsymbol{\theta}, \boldsymbol{J}, t)$ from

$$\frac{\partial f}{\partial \tau} + j\frac{\partial f}{\partial \phi} - \sin\phi \frac{\partial f}{\partial j} = \Delta \frac{\partial^2 f}{\partial j^2}$$

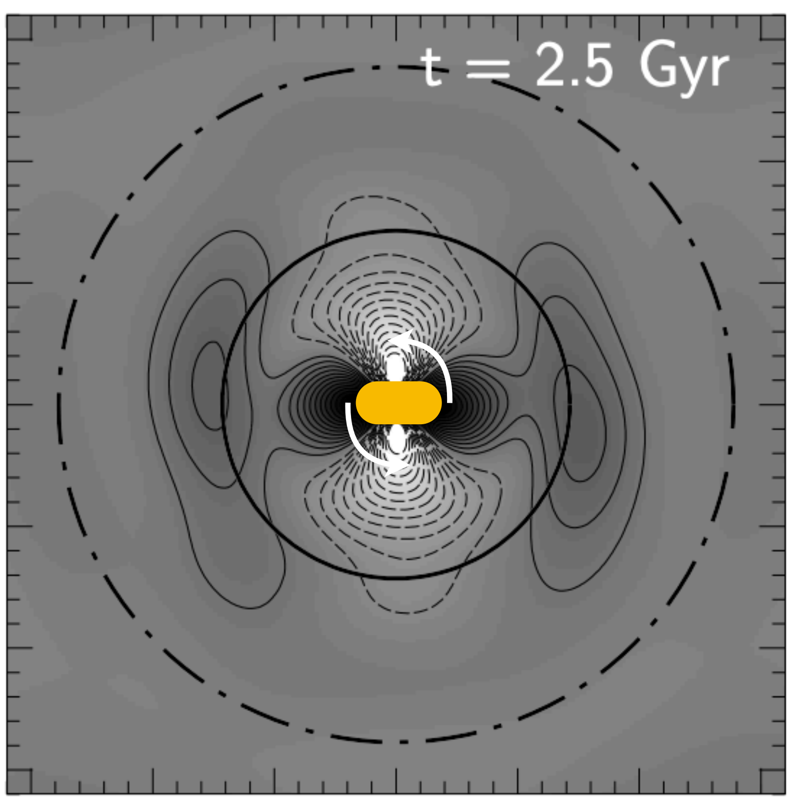
Phase Mixing

(Chiba & Schonrich 2022)

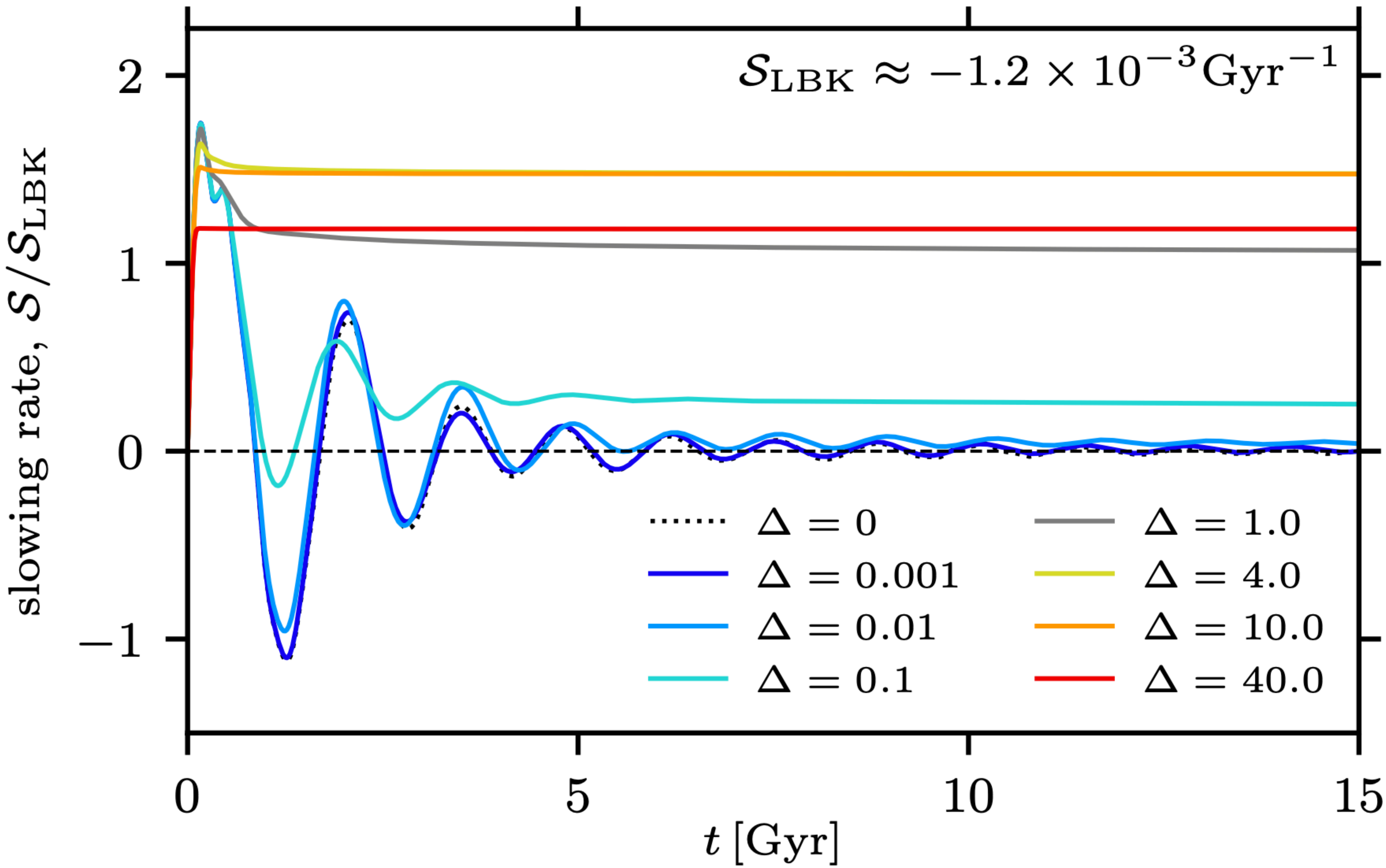




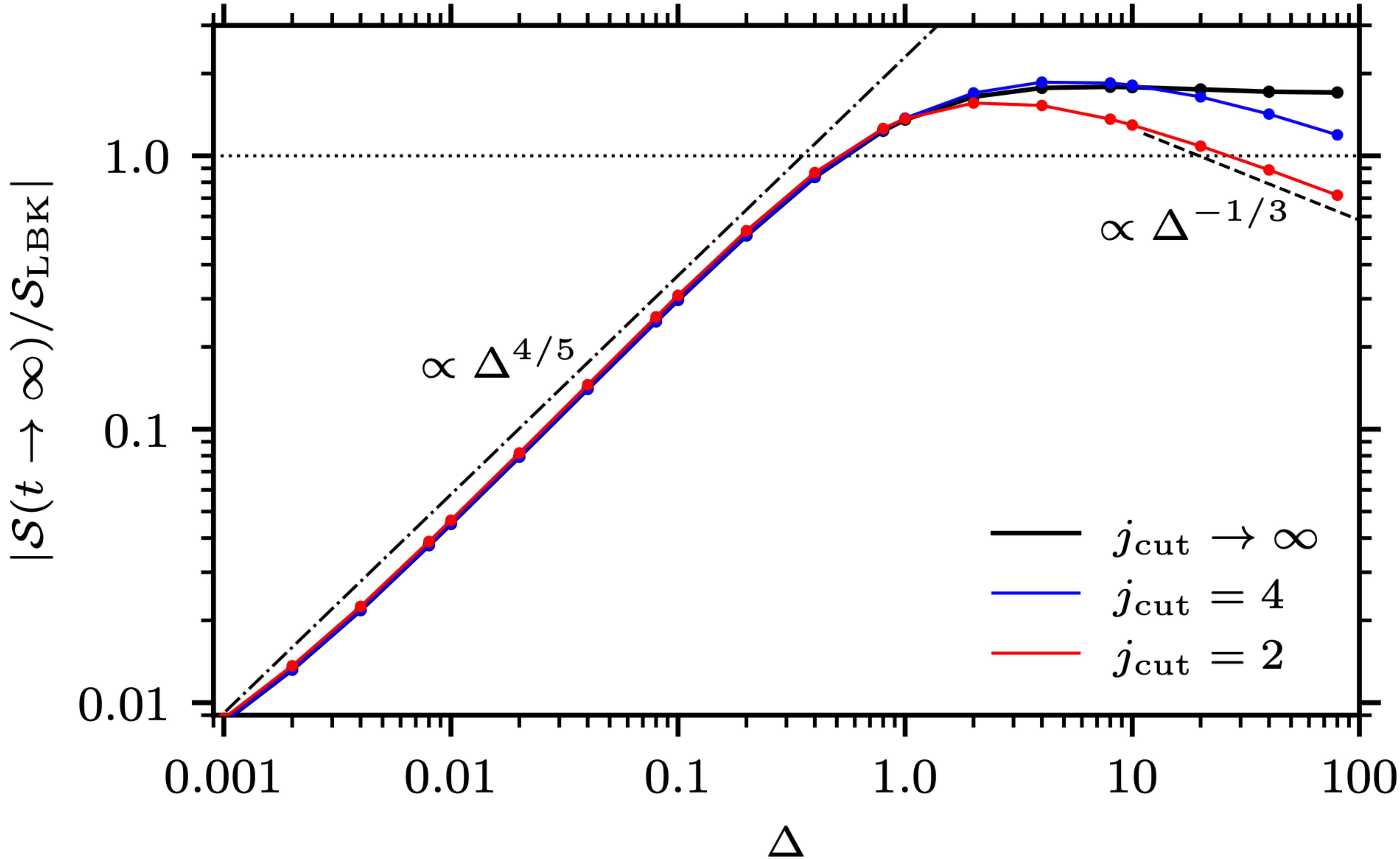
in physical space



How does torque depend on diffusion?



TIME-ASYMPTOTIC TORQUE



Literature on Wave-particle Interactions

Plasma kinetics / **Galactic dynamics**

	Collisionless	Collisional	
Linear theory	Landau (1946) **Lynden-Bell & Kalnajs (1972)** **Weinberg (2004)**	Auerbach (1977) Catto (2020)	
	$\Delta = 0$	$0 < \Delta \ll 1$	$\Delta \gg 1$
Nonlinear theory (w/particle trapping)	O'Neil (1965); Mazitov (1965) **Tremaine & Weinberg (1984)** **Chiba & Schönrich (2022)**	Pao (1988) Petviachvili (1999) <— **this paper (Hamilton et al. 2022)**	Berk et al. (1997) Duarte & Gorelenkov (2019) —>