Analytics of the Cosmic Web
A systematic attempt

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VNiVERSiDAD D SALAMANCA

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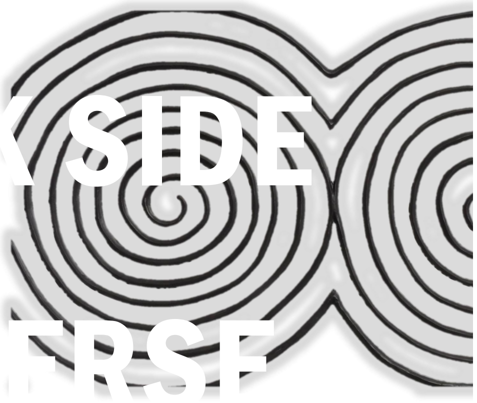

Organizing DSU2023 in Kigali, Rwanda.
Hope to see you there!

https://eaifr.ictp.it/events/dsu-2023/

WEBPAGE: https://eaifr.org/events/dsu-2023/

Organisers:

- Paolo Creminelli, ICTP, Trieste
- Joern Kersten, Uni Bergen
- Roy Maartens, UWC, South Africa
- Shoaib Munir, ICTP-EAIFR
- Marcello Musso, Uni Salamanca
- Riccardo Sturani, ICTP-SAIFR
- Filippo Vernizzi, IPhT, Saclay
- Gabrijela Zaharijas, Uni Nova Gorica

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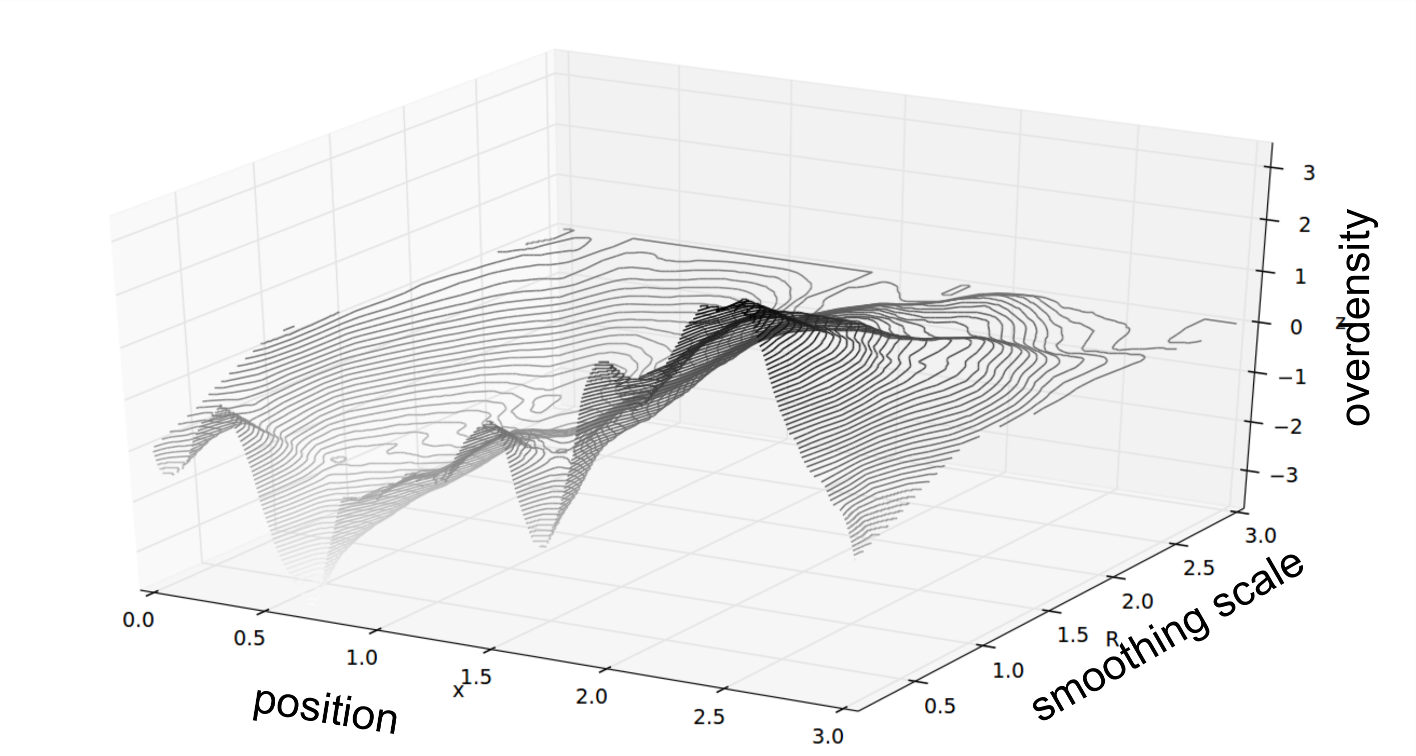
Finding proto-halos

- Find protohalo patches in the IC
- After smoothing, 4-dimensional landscape in $\mathbf{x}$ and R
- Look for peaks of critical height
- Peak constraint fixes position $\mathbf{x}$
- Threshold on the peak height fixes the smoothing scale R
- Which filter?



Press & Schechter 74, BBKS86, BCEK91, ….

1. “My filter is better than yours”

Or: getting the place right

Matter vs energy peaks

- Geometrical radius: $R^3 = 3V/4\pi$
- $$\frac{\dot{R}^2}{2} - \frac{GM}{R} - \frac{\Lambda}{6}R^2 \simeq -\frac{5}{3}\frac{GM}{R_{\rm in}}\delta_{R,{\rm in}}$$
- Mass: $M = \frac{4\pi}{3}\bar{\rho}(1+\delta_R)R^3$

vs

- Inertial radius: $\frac{R_I^2}{5} \equiv \int_V \frac{{\rm d}^3 r}{3M}\rho\,|\mathbf{r}-\mathbf{r}_{\rm cm}|^2$
- $$\frac{\dot{R}_I^2}{2} - \frac{GM_I}{R} - \frac{\Lambda}{6}R_I^2 \simeq -\frac{5}{3}\frac{GM_I}{R_{I,{\rm in}}}\epsilon_{\rm in}$$
- Inertial mass: $M_I = \frac{4\pi}{3}\bar{\rho}(1+\epsilon_R)R_I^3$

- Governed by **matter** overdensity

$$\delta_R \equiv \frac{1}{V}\int_V {\rm d}^3 r\,\delta(\mathbf{r})$$

- Governed by **energy** overdensity

$$\epsilon_R \equiv 5\int_V \frac{{\rm d}^3 r}{MR_I^2}\rho\,\mathbf{r}\cdot(\nabla\phi - \nabla\phi_{\rm cm})$$

Matter vs energy peaks

- Characteristic time ~ $(1/\delta_R)^{3/2}$
- Halos of mass M are peaks of $\delta_R(\mathbf{x})$
- In Fourier space:

$$\delta_R = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \delta(\mathbf{k})\, \frac{3 j_1(kR)}{kR}$$

vs

- Characteristic time ~ $(1/\epsilon_R)^{3/2}$
- Halos of mass M are peaks of $\epsilon(\mathbf{x})$
- In Fourier space:

$$\epsilon_R = \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \delta(\mathbf{k})\, \frac{15 j_2(kR)}{(kR)^2}$$

(extra power of $1/k$)

What is the advantage?

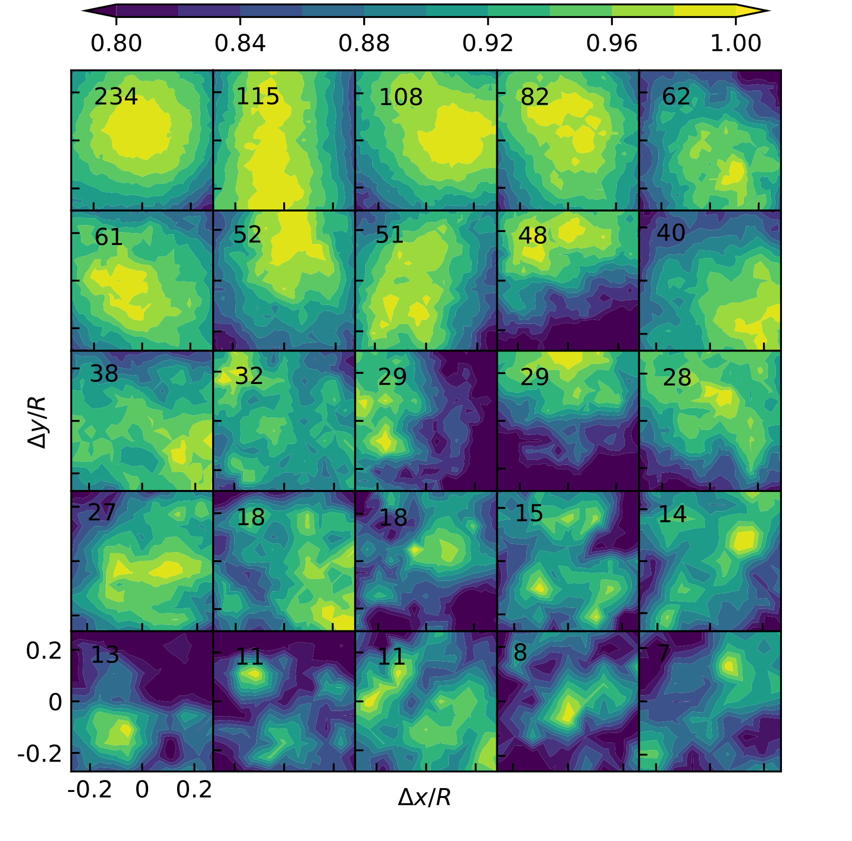
- R is very sensitive to the halo boundary (it actually is...)
- No dynamical meaning in $\nabla \delta_R = 0$
- More small-scale power. $\langle (\nabla^2 \delta_R)^2 \rangle$ diverges in ΛCDM.
- Usually resort to Gaussian filter. Blurred physical interpretation

vs

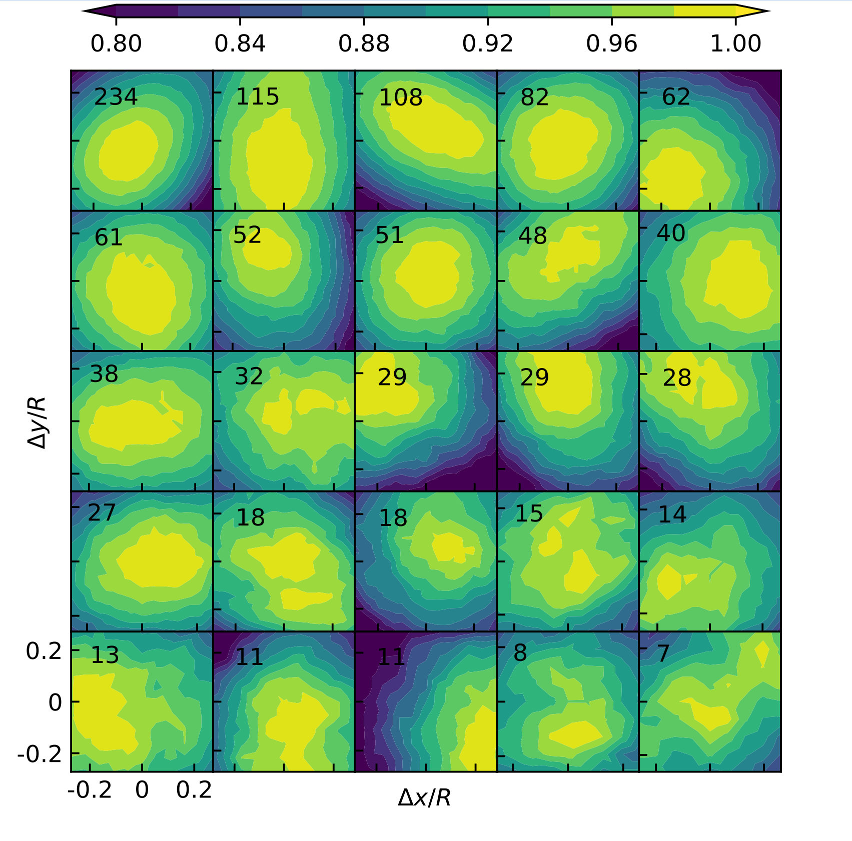
- R_I is density weighted, less sensitive to halo boundary at late times
- $\nabla \epsilon_R$ ~ dipole moment. $\nabla \epsilon_R = 0$ implies radial infall
- Less small-scale power. $\langle (\nabla^2 \epsilon_R)^2 \rangle$ remains finite.
- No need to “tweak” the filter. Clearly rooted in the EoM

Testing the energy peak ansatz

Mean matter overdensity field δ_R



Mean energy overdensity field ϵ_R



- Energy peaks are a better proxy for protohalo centers!

Other filters?

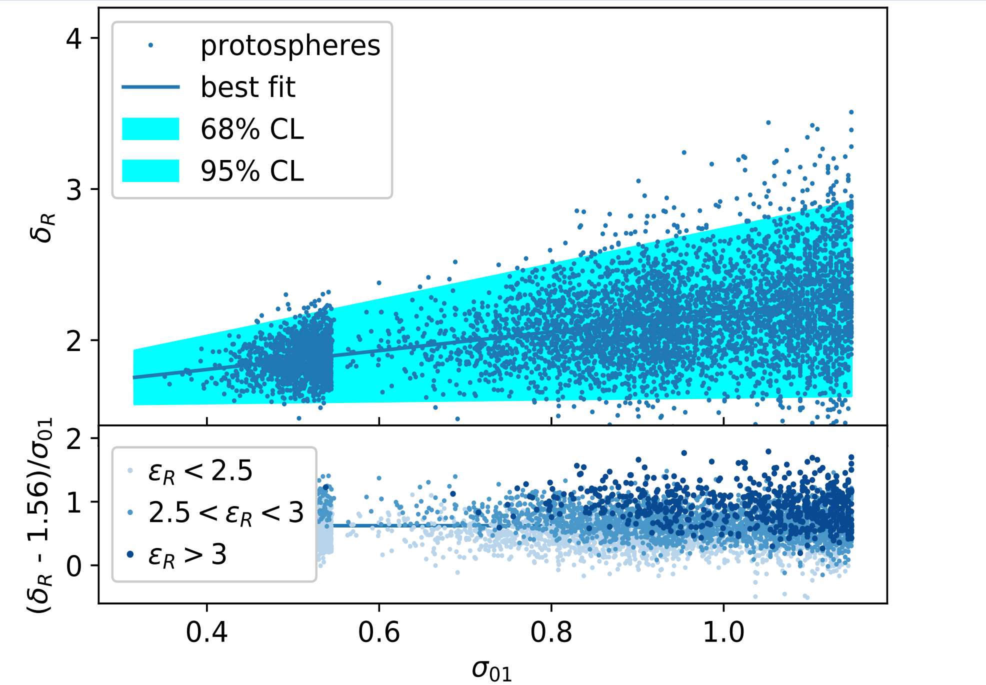
- There are other popular filter choices (Gaussian, sharp-k…)
- They have interesting mathematical properties that can make calculations easier
- However, they are not obviously connected to physical quantities
- They don’t have a clear dynamical meaning (to me...)

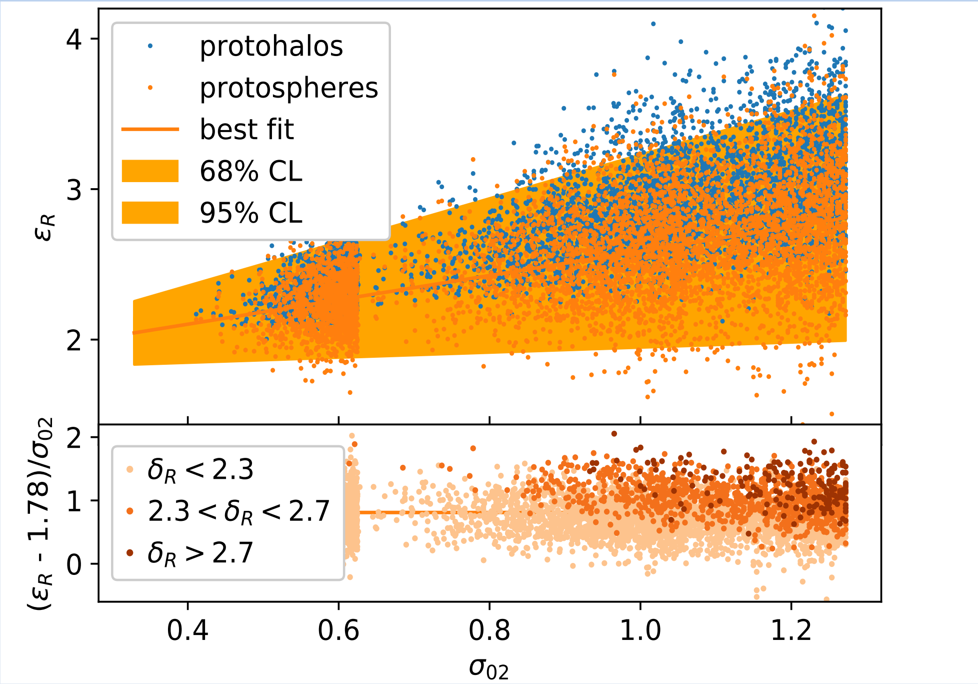
2. The threshold

Or: getting the mass right

Peak height (= threshold)

- To predict a mass, the peak height must cross a threshold. However:





- The measured value of δ_R is not really $\delta_c = 1.686$
- But ϵ_R is not a constant either...

Peak height (= threshold)

At least two effects contribute to the scatter and must be modeled:

1) Anisotropy of the environment.

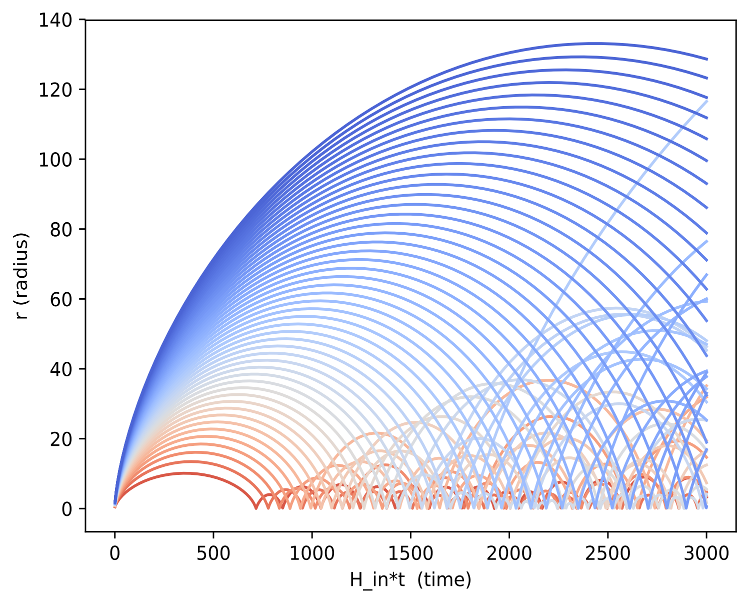
- Tidal shear slows down collapse. Need a higher initial overdensity to counter it.
- Follows from (neglected) anisotropic second order terms in the EoM

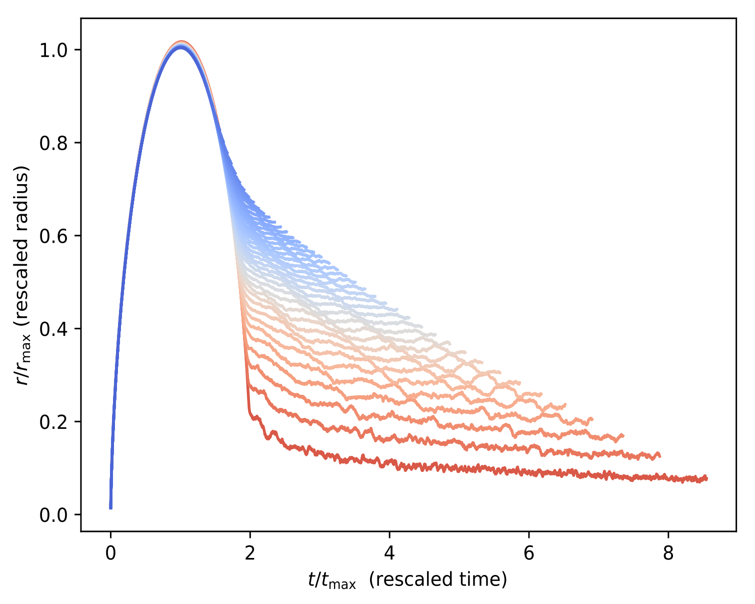
2) Different formation times.

- Early forming haloes come from higher initial overdensities
- But most halo finders are blind to formation times
- Effect also exists in spherical symmetry
- Need to model the multi-stream regime beyond shell crossing

Spherical model of virialization

- Shells cross the center at different times, and then start crossing each other. Mass and energy within each shell are NOT conserved in multi-stream regime
- The radius of mass-conserving spheres freezes (null mean velocity)

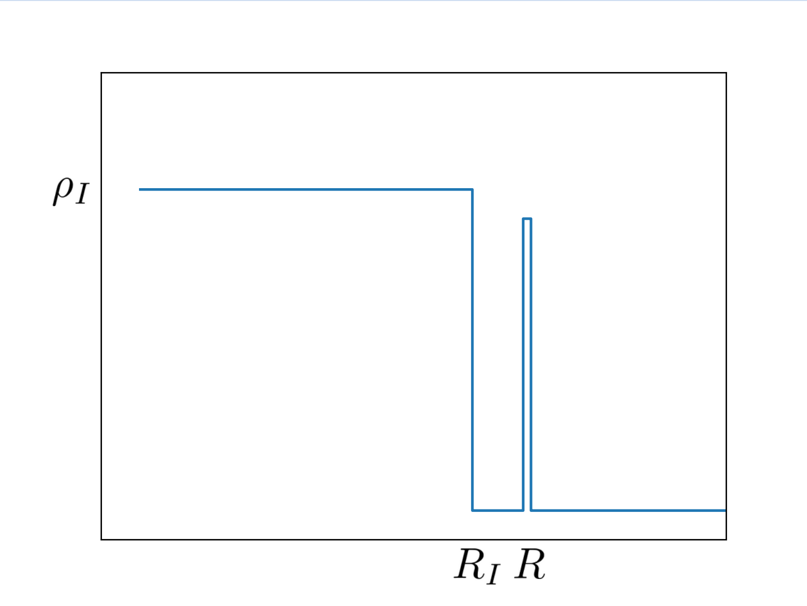


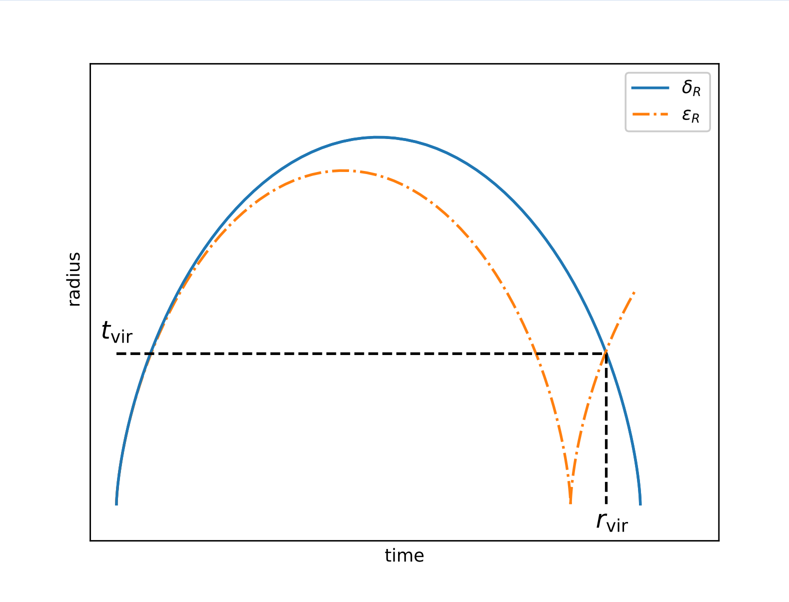


- The virialization radius of each shell is NOT half of the turnaround radius

Spherical model of virialization

- Can we get some analytical insight from an equivalent configuration?
- Two spherical collapse solutions with overdensity δ_R and ϵ_R, intersecting after bounce

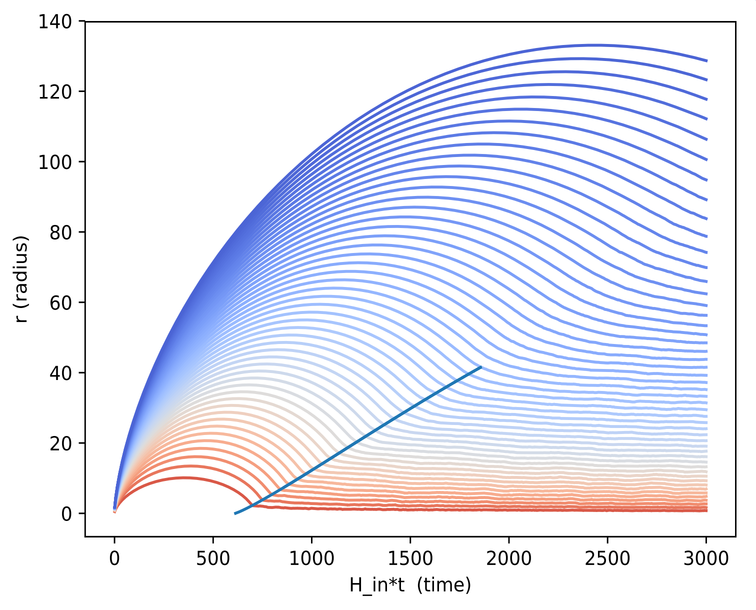


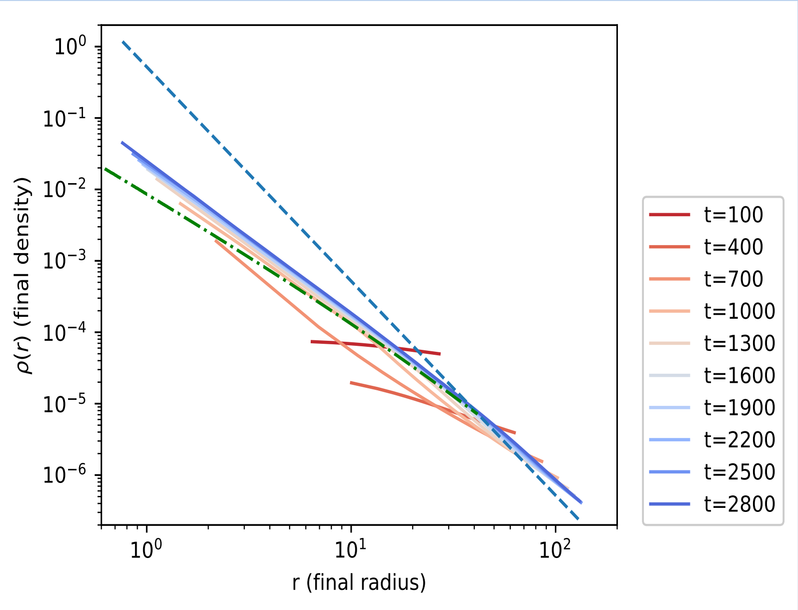


- Solve for r_{vir} and t_{vir} , and repeat for every R...

Spherical model of virialization

- … and it seems to work quite well!





- Can also predict the final profile
- The threshold is actually a relation between δ_R and ϵ_R

3. The Minimum Energy Principle

Or: getting the shape right

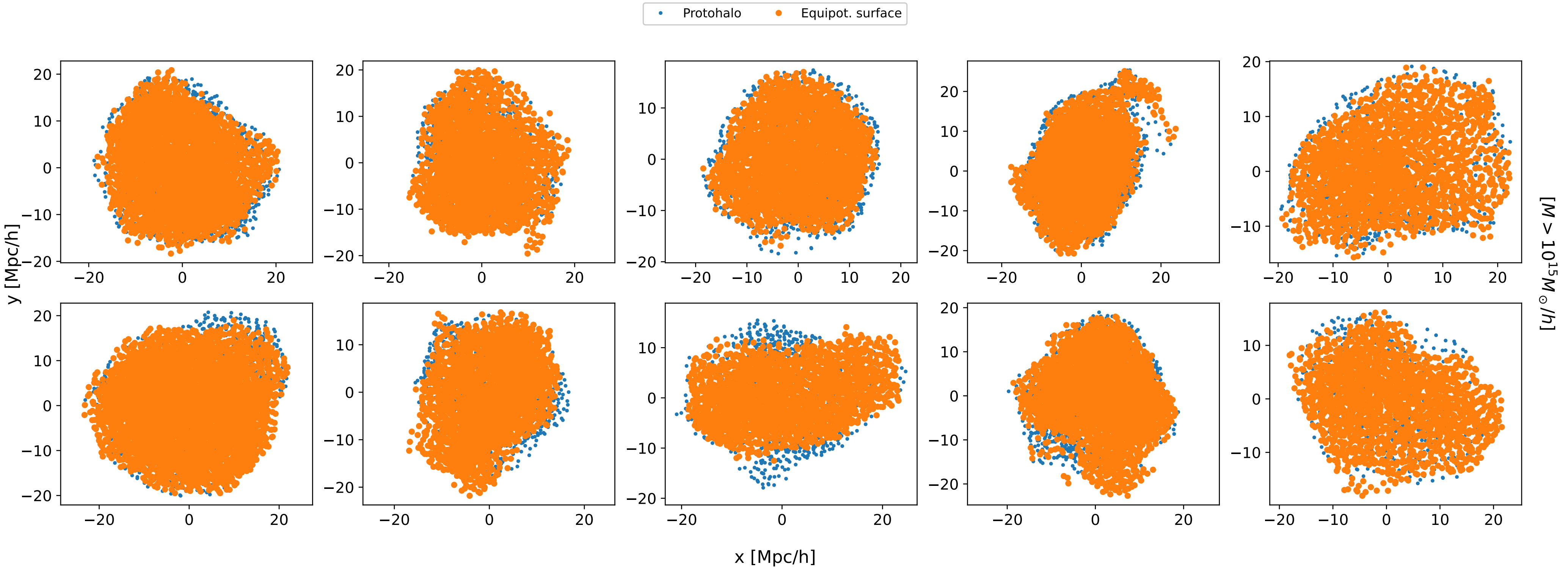
Shape of maximal ϵ

- Once a spherical peak is found, one can further increase ϵ (decrease E) by deforming the sphere at fixed volume.
- The inertial radius R_I of the deformed region collapses even faster
- The boundary of the region of maximal ϵ (minimal E) must be an isosurface of

$$\mathcal{V}(\mathbf{r}) \equiv (\mathbf{r} - \mathbf{r}_{\mathrm{cm}}) \cdot \left[\nabla\phi - \nabla\phi_{\mathrm{cm}} - \frac{\epsilon}{3}(\mathbf{r} - \mathbf{r}_{\mathrm{cm}}) \right]$$

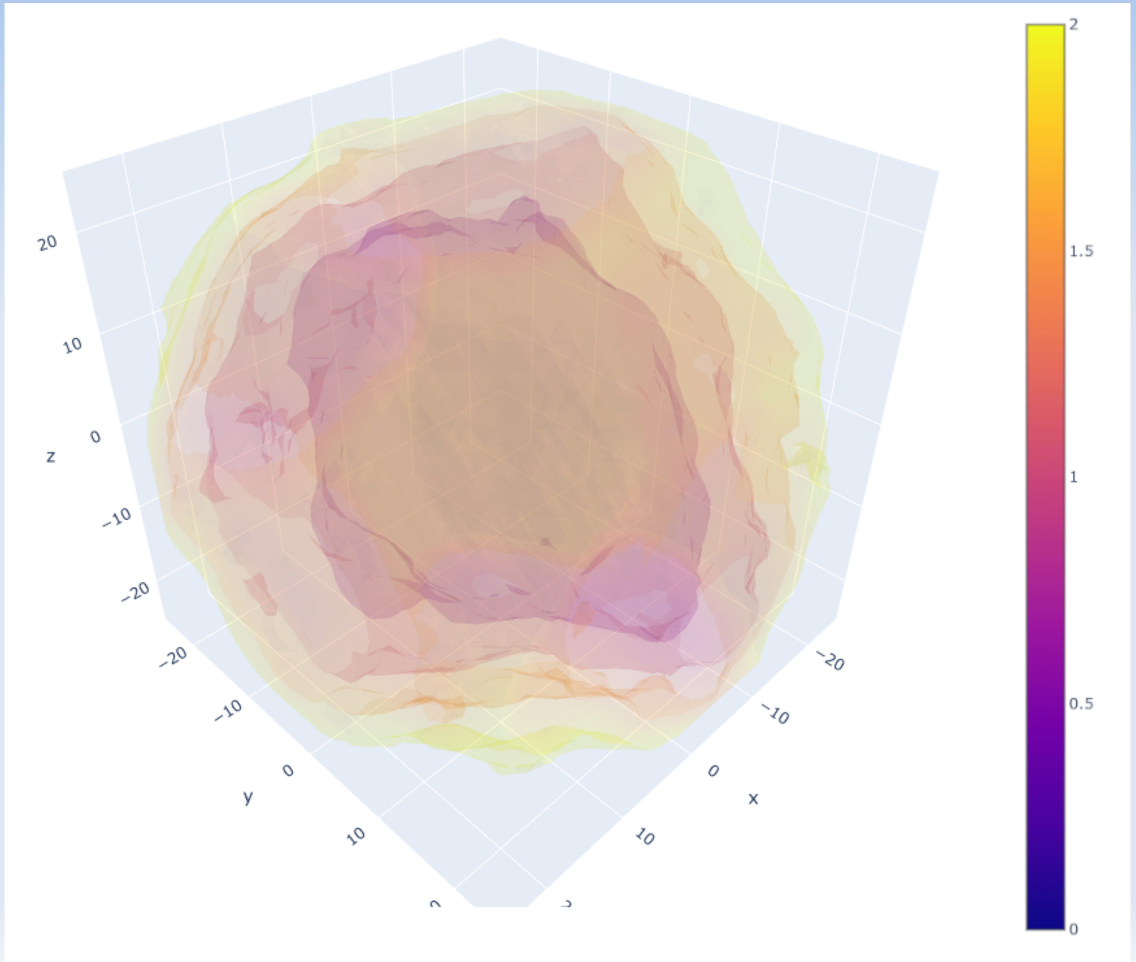
- Proxy for protohalo shape and boundary!
- Longest axis in the direction of maximum compression (orthogonal to the filament)
- Can predict initial torques

Protohaloes vs equipotential surfaces



Equipotential surfaces

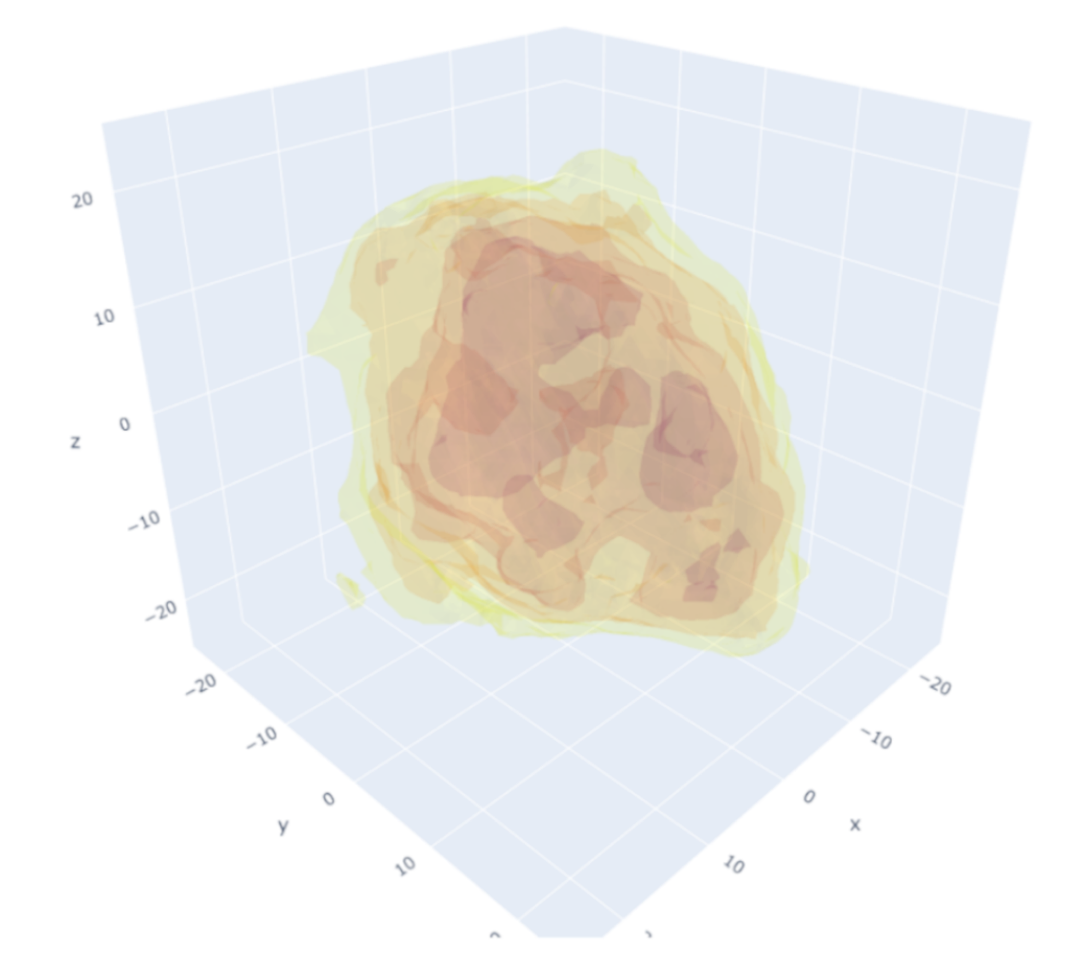
- Nested equipotential surfaces with different overdensity ϵ and volume V describe the mass accretion history
- Excursion sets of peaks of arbitrary shape!



Equipotential surfaces

- Zooming in, the surfaces of constant infall potential $\mathcal{V}$ may fragment
- Natural prediction of a merger event
- The notion of critical event, $\det(\nabla\nabla\epsilon) = 0$, may be replaced by

$$\nabla\mathcal{V} = 0$$



Conclusions

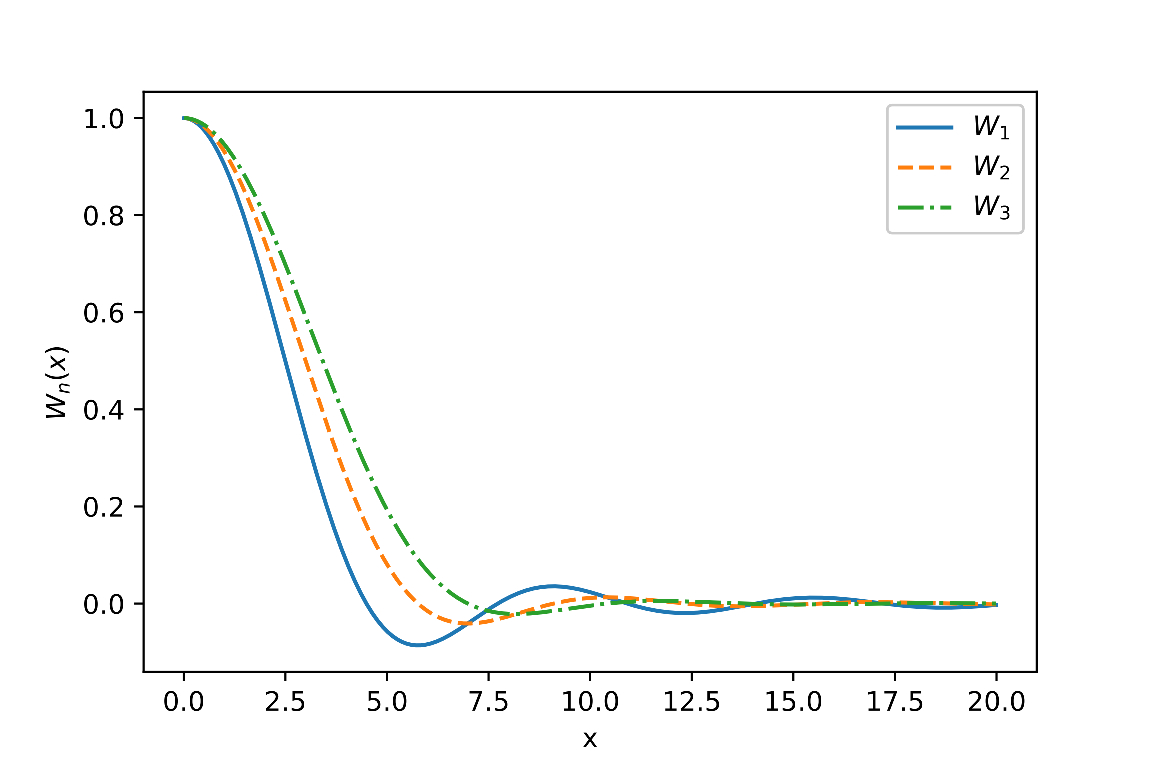
- Protohaloes are peaks of the initial energy overdensity field. Not densest but most energetically bound initial regions, having fastest collapse times.
- Peaks in ϵ_R are convergent matter flows. Initial evolution matches perturbation theory. Final high mean density results dynamically, not put in “by hand”.
- Using ϵ_R instead of δ_R simply means changing the filter (to a more convergent one)
- Energy density peaks are better behaved, and better proxies for protohalo centers
- The threshold contains both anisotropic corrections AND scatter in formation times.
- A model of spherical virialization leeds to a relation between ϵ_R and δ_R
- Protohalo shapes and alignments are well described by equipotential surfaces

Open questions and outlook

- Can we predict critical value ϵ_c ? Must model virialization (in progress)
- Relation with halo finder? Ellipsoidal? FOF? Energy-based?
- Angular momentum? (in progress)
- How to improve even more? Account for non-conservation of energy?
- Final shear/shape alignments?
- (Assembly) bias? Voids? Skeleton/cosmic web?
- Primordial BHs?
- …

Thank you!!

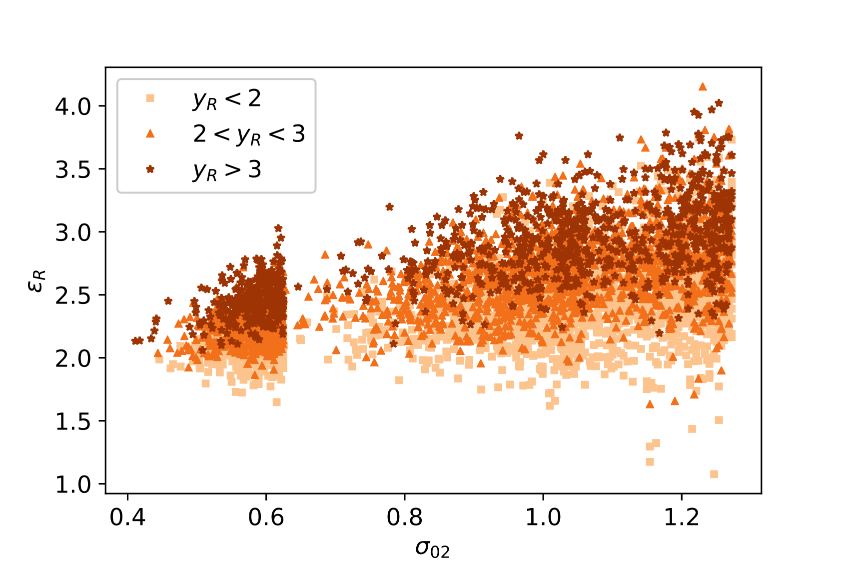
The filters

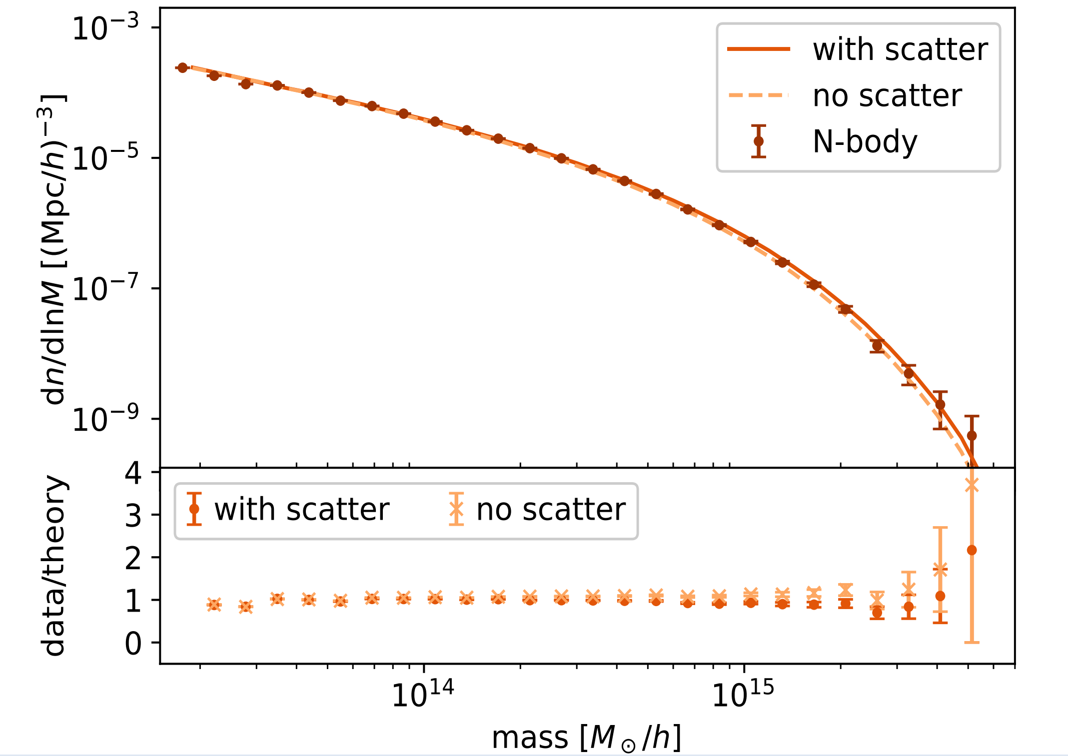


- The W_1 filter converges more slowly and has more pronounced wiggles

Halo mass function

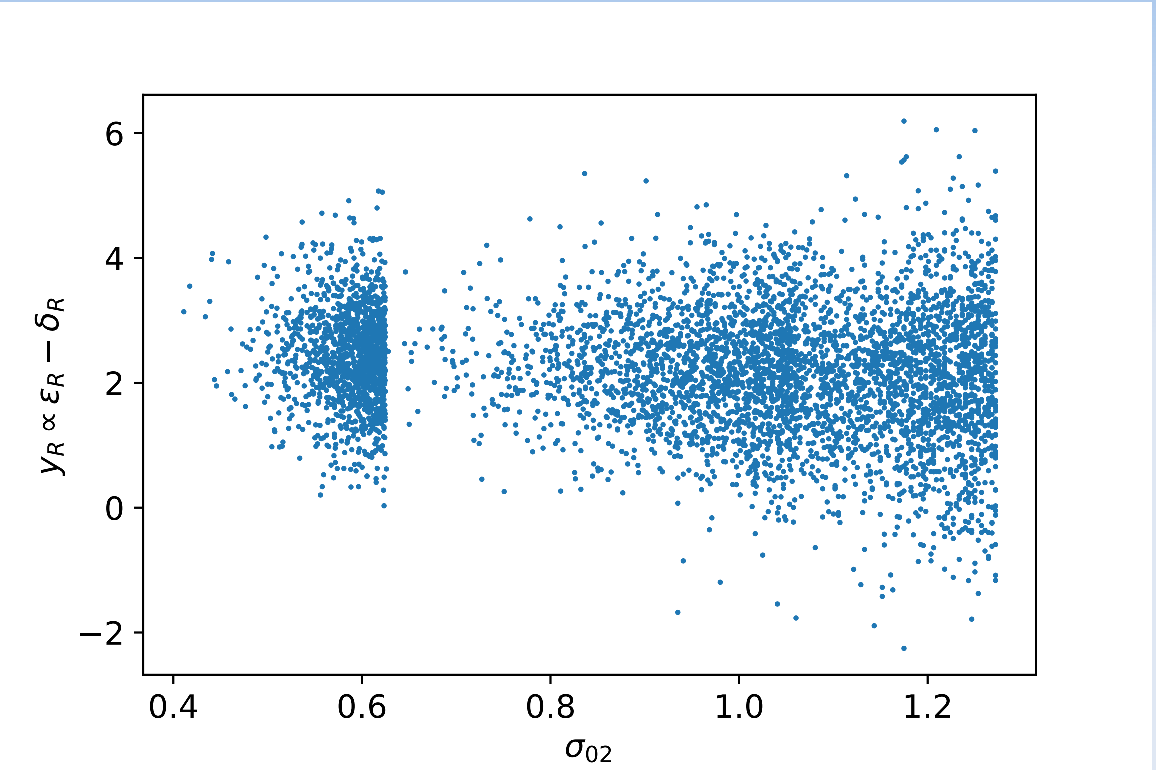
- Predicted, using a fit to ϵ_R





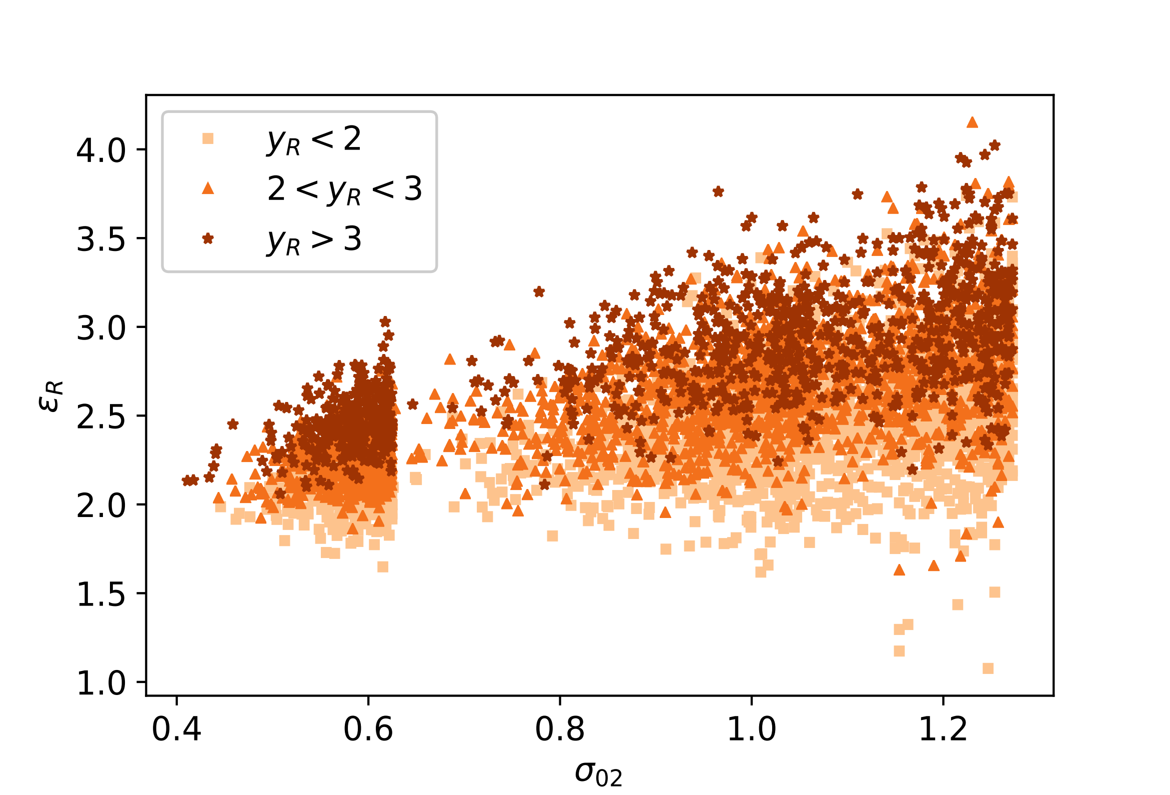
- Scatter can describe assembly bias.

Excursion set slopes



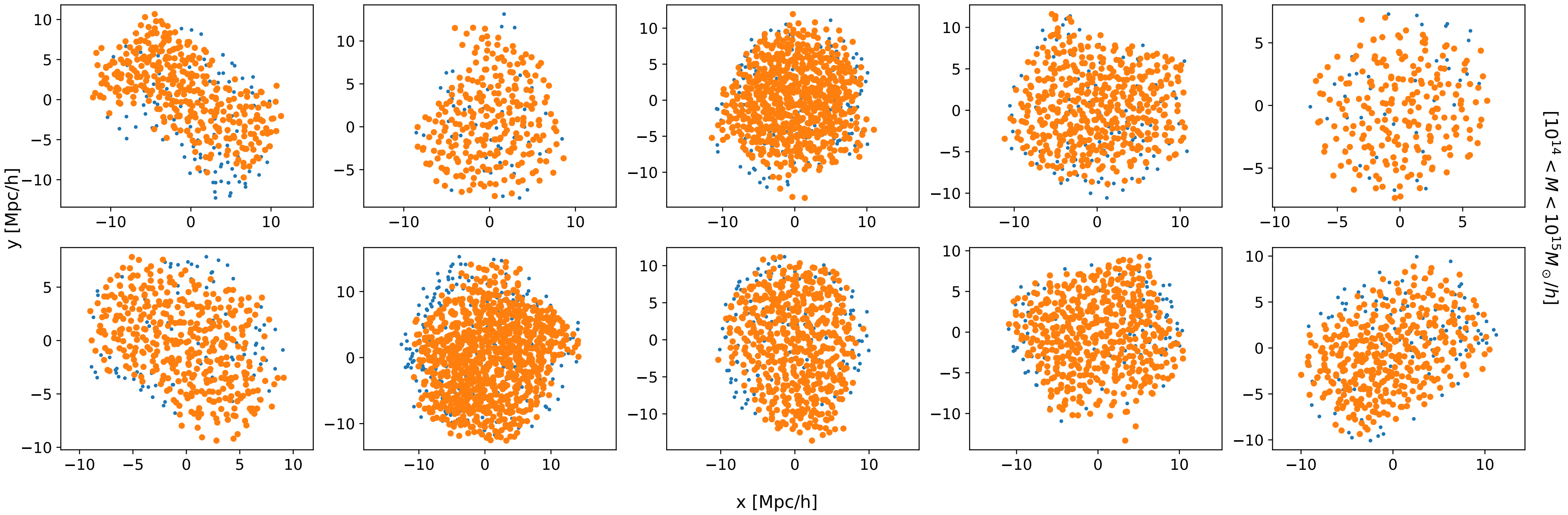
- The "slope of the excursion set" $-\mathrm{d}\epsilon_R/\mathrm{d}R$ at the center is always positive. Consistent with the peak ansatz.

Excursion set slopes

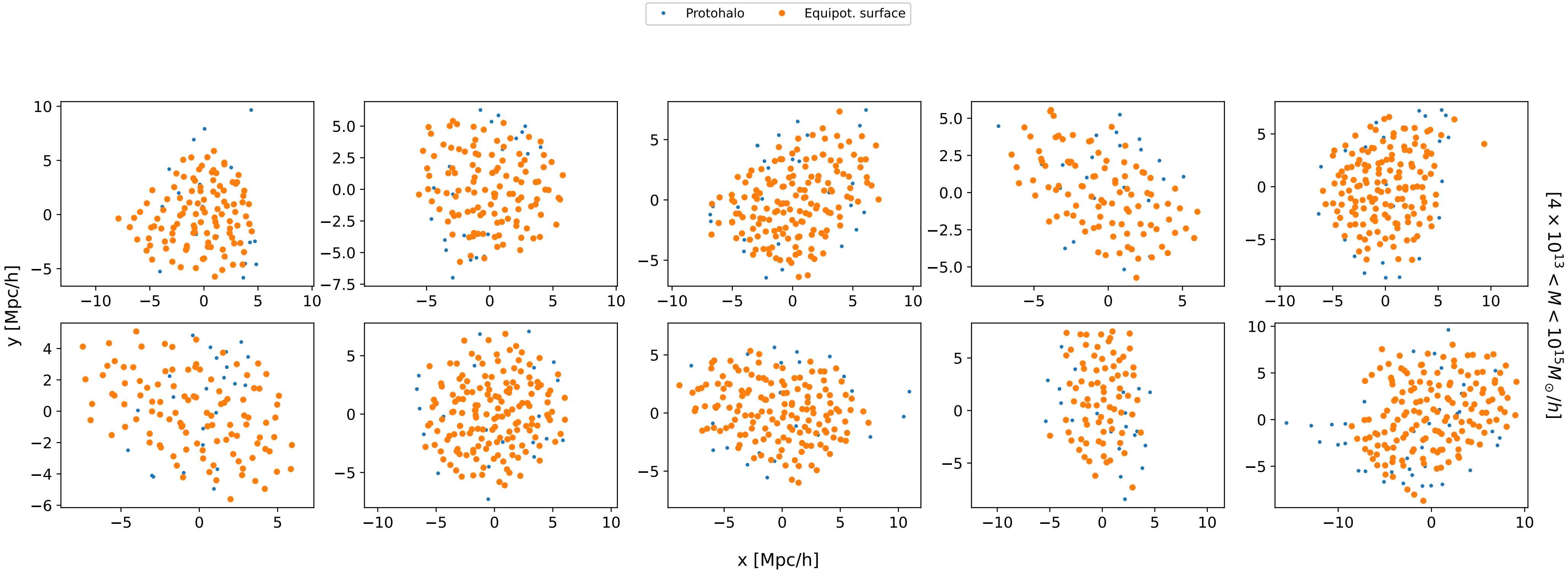


- Peak height and excursion set slope correlate.
- What is the slope for the final halo? Accretion rate?

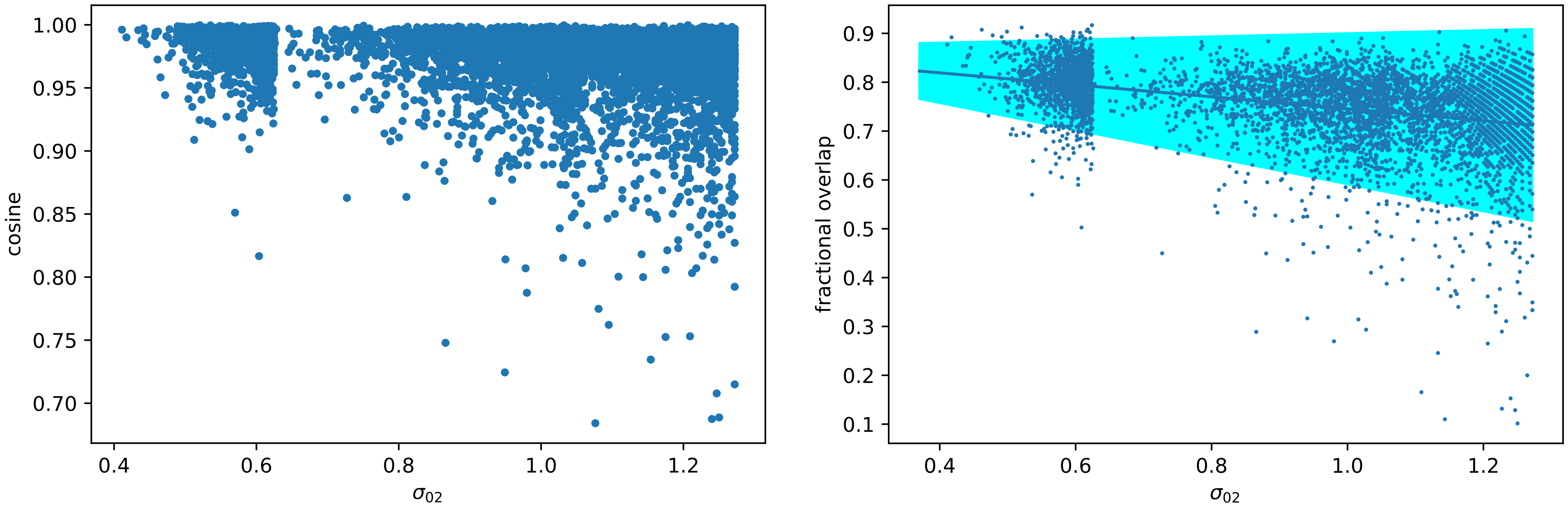
Protohaloes vs equipotential surfaces



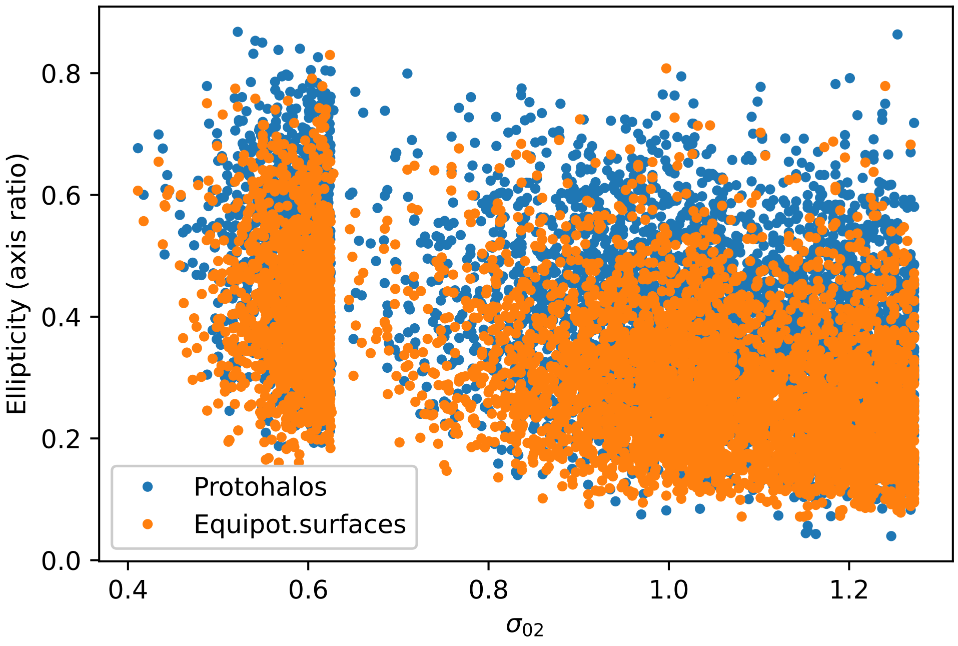
Protohaloes vs equipotential surfaces

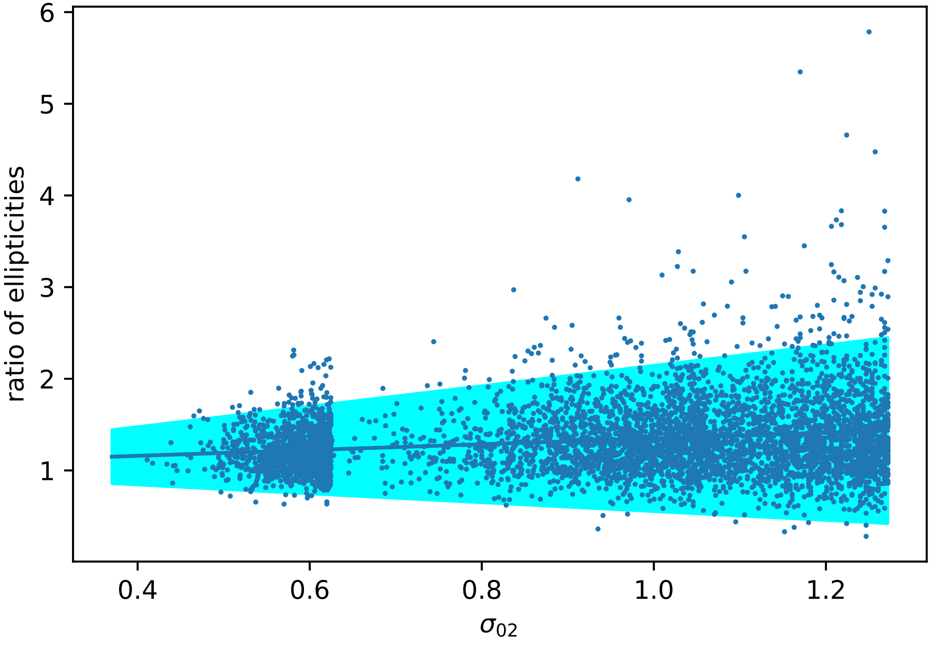


Alignments

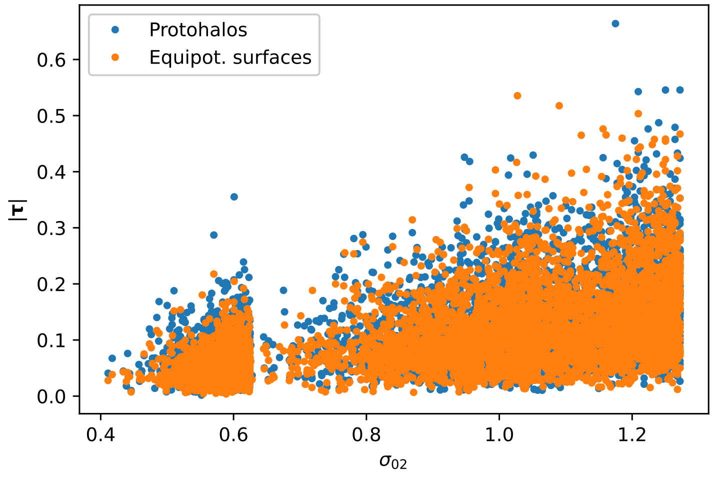

1.00
0.95
0.90
0.85
0.80
0.75
0.70
cosine
0.4
0.6
0.8
1.0
1.2
σ02
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
fractional overlap
0.4
0.6
0.8
1.0
1.2
σ02

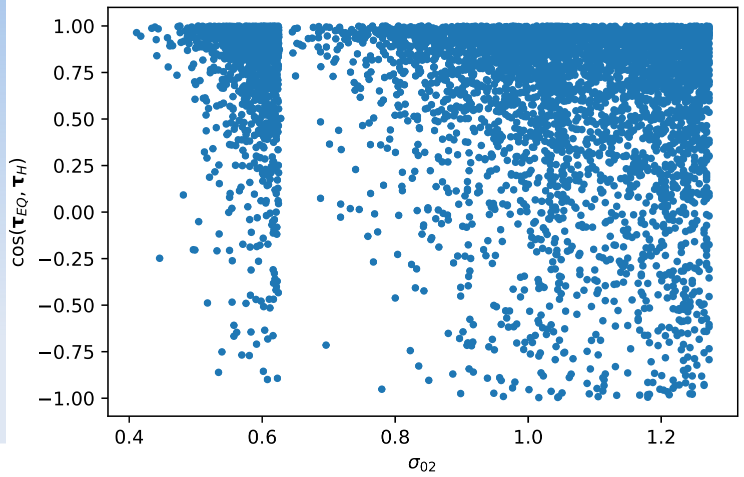
Ellipticities





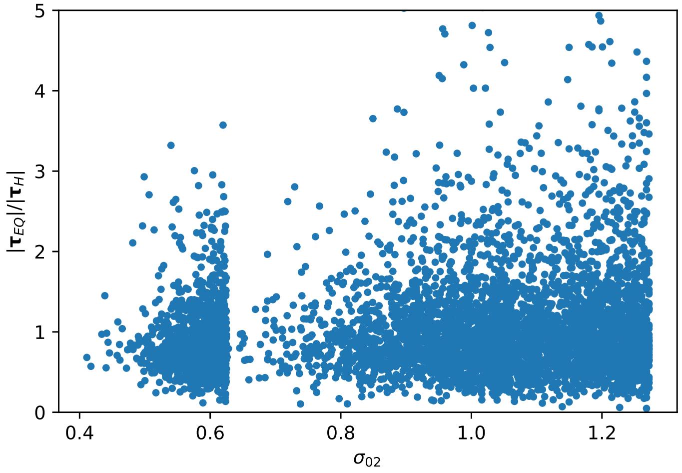
Torques





$$\tau_i = -\frac{MR_I^2}{5}\varepsilon_{ijk}\epsilon_{jk}$$

$\epsilon_{ij} =$ energy ovedensity tensor.
No approx here!



It does not always work...

- At low mass, sometimes the prediction fails
(here, < 40% of protohalo particles identified correctly)