Connectivity of the Cosmic Web.
Multiplicity of its nodes

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Isle of Skye: 2D peaks and ridges

Image credit: Zoltan Gabor, http//zoltangabor.com

Isle of Skye: 2D peaks and ridges

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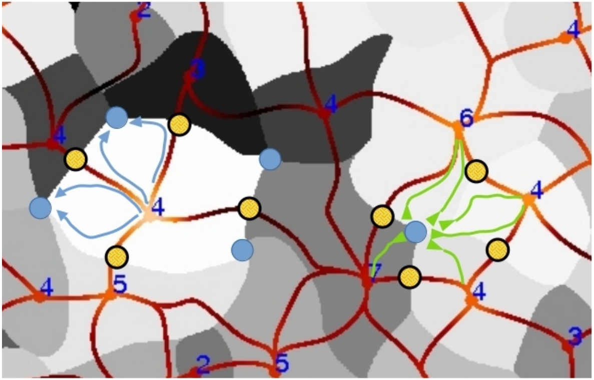
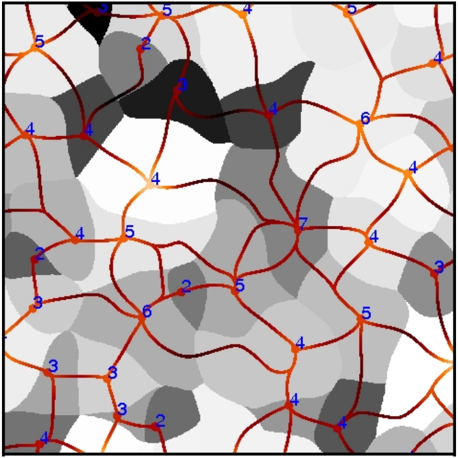
Fully connected Global Skeleton



- Global Skeleton is a boundary of the region that can be reached going everywhere uphill from a given minimum (ascending manifold)
- Global Skeleton is fully connected, no line can terminate anywhere but extrema or saddles.
- The region that you can reach from a given maximum going strictly downhill (descending manifold) is segmented depending in which minimum you end up. Skeleton lines separate these segments. On the boundary of descending manifold there are saddle points, through which skeleton line passes to the next maximum.

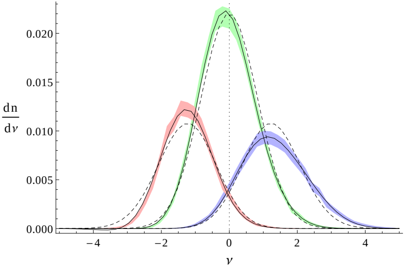
Connectivity of Fully connected Global Skeleton

- skeleton is a set of gradient lines connecting peaks
- Connectivity κ is number of peaks the given peak is connected to
- skeleton line between peaks passes through one saddle point
- κ as the function of peak properties, and/or restrictions put on saddles is the main object
- Properties of saddles are critical for percolation
- but not every segment of the skeleton is a dense filament

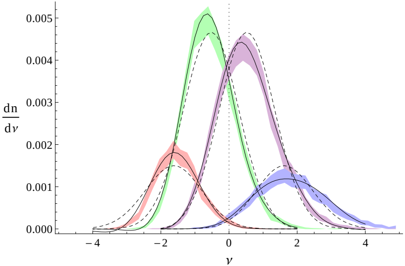


Basic info on Gaussian Peaks and Extrema Counts (BBKS)

2D



3D



$$\langle n_{\rm peak}\rangle = \frac{1}{8\sqrt{3}\pi R_*^2},\quad \langle n_{\rm saddle}\rangle = \frac{1}{4\sqrt{3}\pi R_*^2}$$

$$R_p \approx 3.7 R_* = 3.7\frac{\sigma_1}{\sigma_2}$$

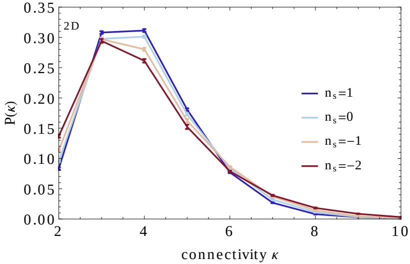
$$\frac{\langle n_{\rm saddle}\rangle}{\langle n_{\rm peak}\rangle} = 2$$

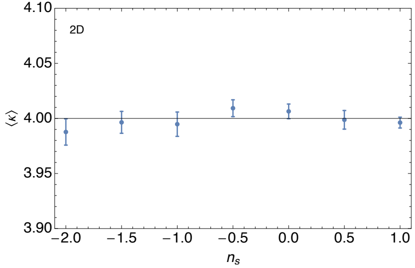
$$\langle n_{\mp--}\rangle = \frac{29\sqrt{15}\mp 18\sqrt{10}}{1800\pi^2 R_*^3}$$

$$R_p \approx 4.2 R_* = 4.2\frac{\sigma_1}{\sigma_2}$$

$$\frac{\langle n_{\rm saddle}\rangle}{\langle n_{\rm peak}\rangle} \approx 3.055$$

Connectivity of the Global Skeleton: 2D

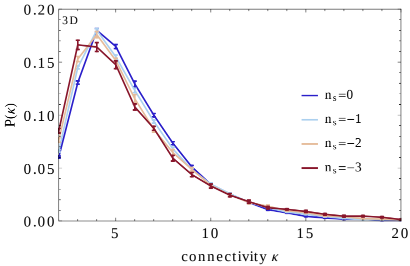


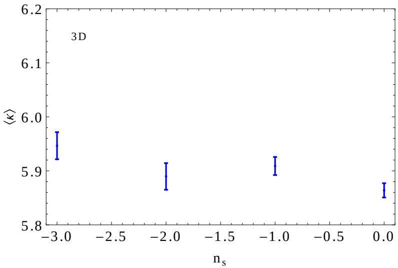


Notable Result:
2D $\langle\kappa\rangle \approx 4$ - average connections of a peak

Matches well with
$2\langle n_{sad}\rangle/\langle n_{max}\rangle = 4$

Connectivity of the Global Skeleton: 3D

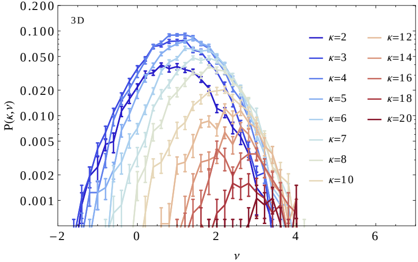


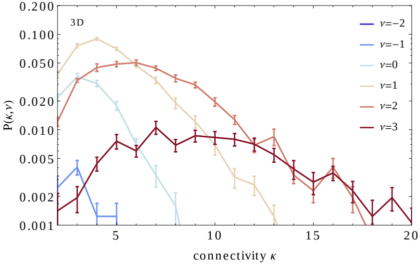


Notable Result:
3D $\langle\kappa\rangle \approx 5.9$ - average connections of a peak

Matches with
$2\langle n_{sad}\rangle/\langle n_{max}\rangle \approx 6.1$

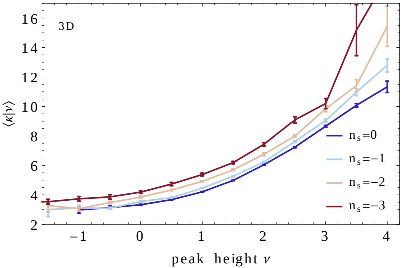
Joint PDF $P(\kappa, \nu)$ in 3D





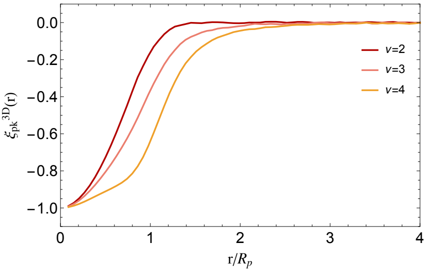
Notable Result:

- Higher the peak more connections it tends to have
- mean $\langle \kappa | \nu \rangle \sim 6.5\ (\nu = 2),\ 10\ (\nu = 3)$
- Peaks with large number of connections are predominantly high



Towards connectivity theory

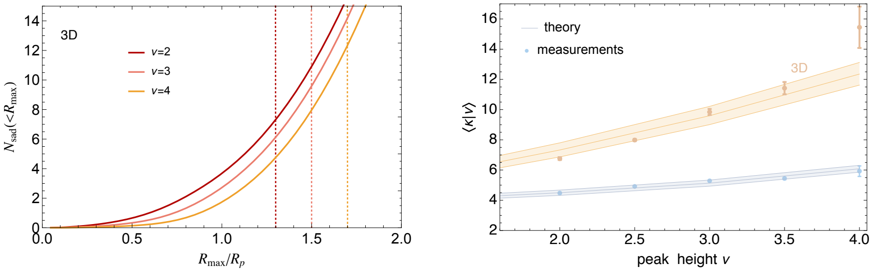
Idea: Count the number of saddles up to $R_{\max}$ . . . , conditional on the properties of the peak. But what is $R_{\max}$? Some characteristic size of a peak-patch around the peak. Let us look (in 3D) where the neighbouring peaks are using peak-peak correlation function



They are at the end of the exclusion zone, which for high central peak $\nu \geq 2$ it increases with ν roughly linearly

$$R_{max} \approx (0.9 + \nu/5) R_p \qquad R_{\max} = 1.2, 1.5, 1.8\ R_p, \quad \nu = 2, 3, 4$$

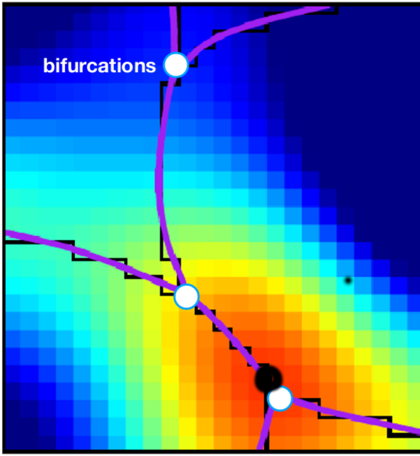
Estimating κ by counting saddles to the next peak



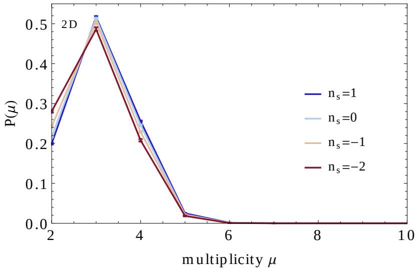
Number of saddles to distance R_{max} conditional on the height ν of the peak translates to peak connectivity $\langle\kappa|\nu\rangle$

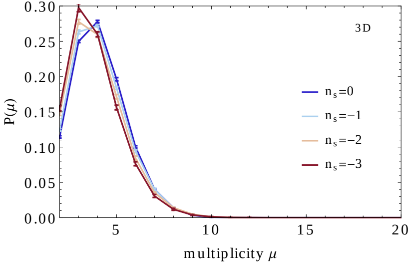
Local Skeleton

- Near the peak real filaments follows ridges of the density and formal skeleton lines at high resolution just duplicate few real filaments and need to be coarse-grained.
- Task is to count peak multiplicity μ – the number of real filaments that leave a peak vicinity (e.g. R_{vir})
- Of course very locally each peak is elliptical and has just two ridges !
- But further away such (coarse grained) skeleton line bifurcate, and $\mu = \kappa - N_{bifur}(R > R_{vir})$.
- Such **local** filament structure reflects merger history and flow pattern around in the peak-patch.



Multiplicity μ of the peaks



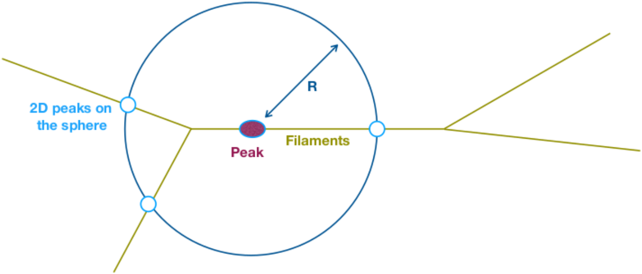


2D: $\langle\mu\rangle = 3$ 3D: $\langle\mu\rangle = 4$

- Notable Results: average number of branches from a peak
- Interesting link: Adhesion model that enforces these values by construction
- $N_{\text{bifurcations}} = \kappa - \mu$
- Issues: not all branches have high density and are physically important

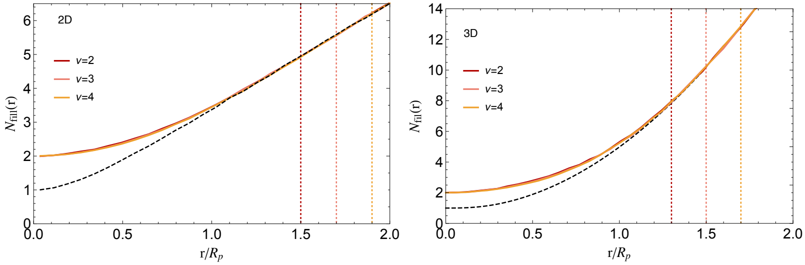
Local multiplicity: towards theoretical predictions

Let us count filaments $N_{fil}(R)$ as maxima on a sphere around the peak



$$\kappa = N_{fil}(R_{max}), \qquad \mu = N_{fil}(R_{vir})$$

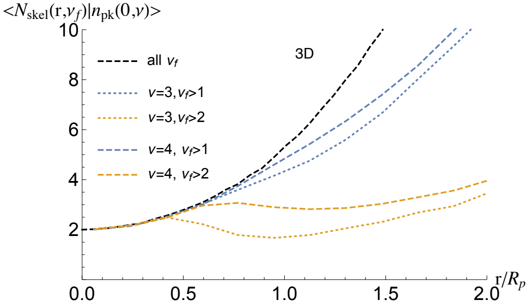
Counting filaments as maxima on a sphere around the peak



Notable points

- At $r \sim 0$ $N_{fil} = 2$ if we have peak at the center. $N_{fil} = 1$ otherwise
- $N_{fil}(r)$ does not depend on peak height, but distance to the next peak does, which explains dependence of connectivity on height

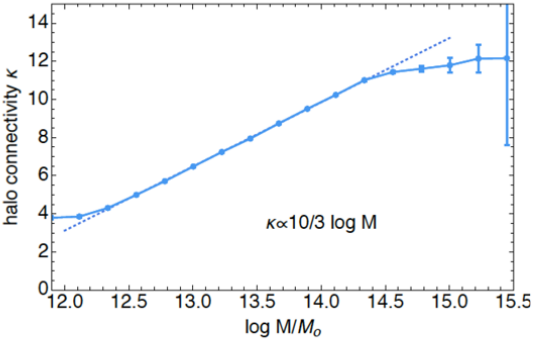
Not all filaments are equally prominent. Counting important ones



- Number of dense $\nu_f > 2$ filamentary bridges is increasing with the height of the central peak
- Not very rare $\nu = 3$ central peak has two (branches of) dense filaments, i.e it sits in one dominant filament on average
- Rare $\nu = 4$ peak is at intersection of three prominent branches.

Application: Halo connectivity as function of mass z=0

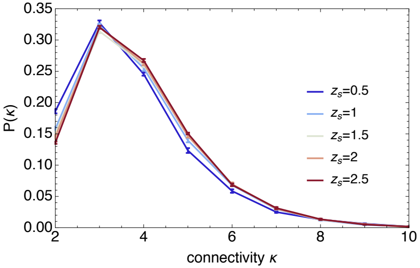
- Measurements in simulations reveal scaling law of connectivity as the function of log mass of halo $\kappa \propto 10/3 \log M$
- Theory: height and scale of the peak can be translated into mass ...



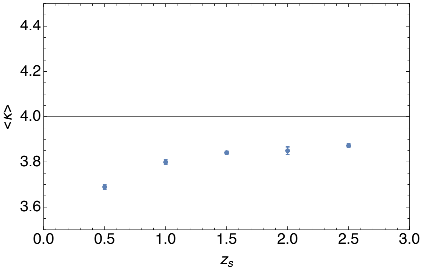
Application: weak lensing convergence maps

- no-massive-neutrinos convergence maps from columbialensing.org
- 12.25 deg^2 2D maps with 1.9 arcmin resolution
- 5 source redshifts, $z_s = 0.5, 1, 1.5, 2, 2.5$.
- non-Gaussian effects reduce connectivity

$P(\kappa)$



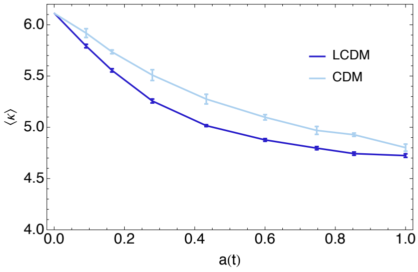
$\langle\kappa\rangle$



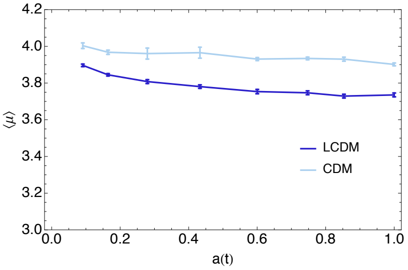
Application: Time evolution of the Cosmic Web connectivity

- In cosmological simulations, as density becomes more non-Gaussian, connectivity of the Cosmic Web decreases
- This leads to model dependent history of the connectivity at different redshifts.

$\langle \kappa \rangle$

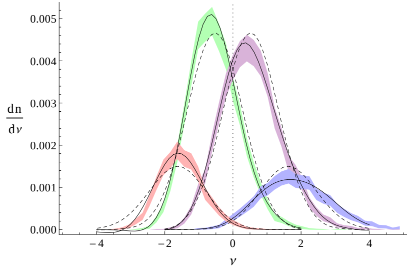


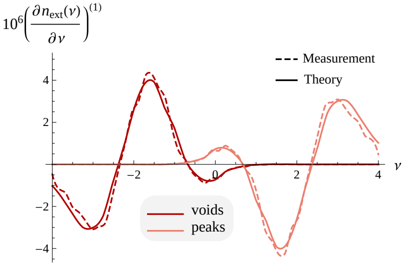
$\langle \mu \rangle$



Non-Gaussian 3D Extrema Counts (Gay et al, 2011)

$$\langle n_{\mp--}\rangle = \frac{29\sqrt{15}\mp 18\sqrt{10}}{1800\pi^2 R_*^3} + \frac{5\sqrt{5}}{24\pi^2\sqrt{6\pi}R_*^3}\left(\langle q^2 J_1\rangle - \frac{8}{21}\langle {J_1}^3\rangle + \frac{10}{21}\langle J_1 J_2\rangle\right)$$





At $\sigma \approx 0.2$

$$\frac{\langle n_{saddle}\rangle}{\langle n_{peak}\rangle} \approx 2.8 \quad \Rightarrow \langle\kappa\rangle \approx 5.6$$

(cubic moments evaluate, with some spectral dependence, to ≈ 0.1, see Gay, Pichon, Pogosyan, 2011)

Summary and what is left behind the scenes

- Description of the filametary Cosmic Web poses many questions that can be formulated in geometrical or topological language.
- This allows for novel computation and powerful analytical technique as well as methods of analysis of simulations and data.
- Skeleton, in different versions, is one such technique that has been proven extremely helpful 'marking' large scale structure for study its effect, for example, on galaxy properties and formation (not in this talk).
- In this talk we focused on connectivity of the skeleton of the Cosmic Web, which allowed as to formulate analytical explanation of how it works.
- We found evidence of increased connectivity for rare peaks – i.e more massive galaxy clusters – and it explained theoretically.
- Evolving cosmic density field becomes non-Gaussian as non-linearity developes. This affects prominence of the filaments and connectivity of the Web. Different cosmological models, as the result, has different history of connectivity which may be observed.
- Formalism of geometrical measures in non-gaussian regime that we developed (not in this talk) can provide the basis for analysing (mildly) non-linear skeleton. Simple example of direct extrema counts support simulation measurements on decrease of connectivity as non-linearity increases.

Cosmic Web in 2D Adhesion Model

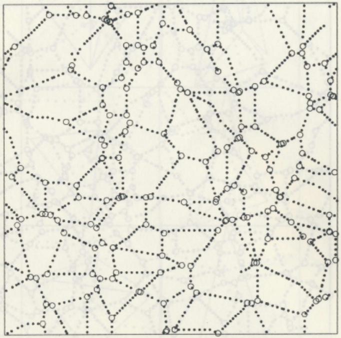


Fig.9b

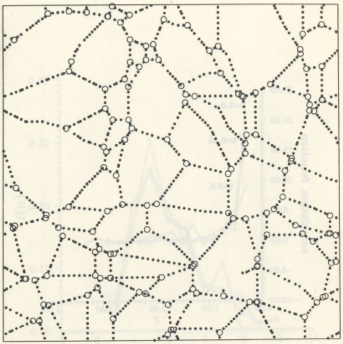


Fig.9c

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