Secular relaxation of self-gravitating planes and disc

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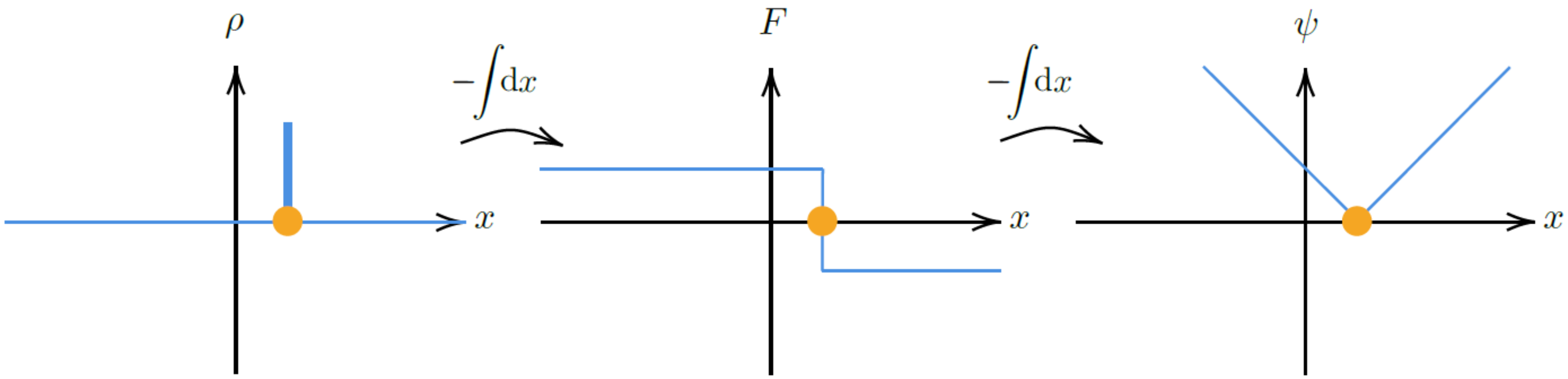
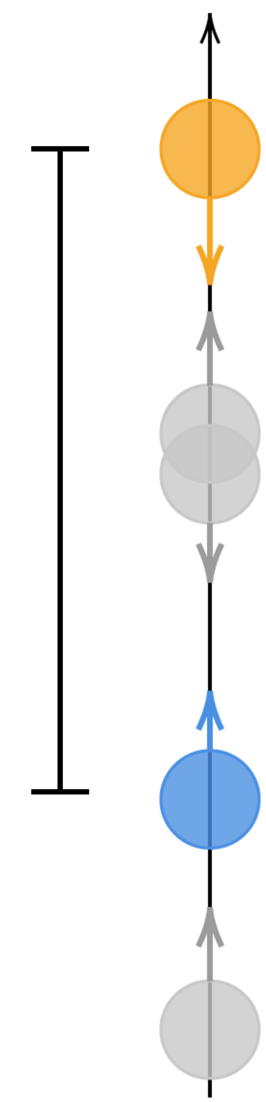
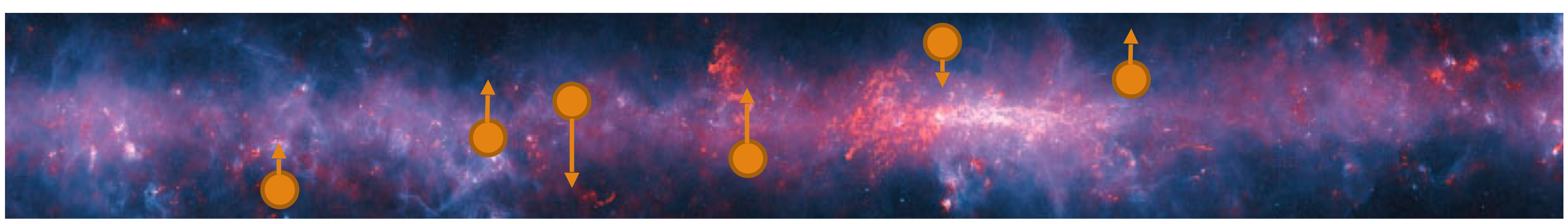
The one-dimensional self-gravitating system (planes)



1D Poisson equation : $\Delta\psi = 2G\,\rho$

$U(x, x') \propto |x - x'|$





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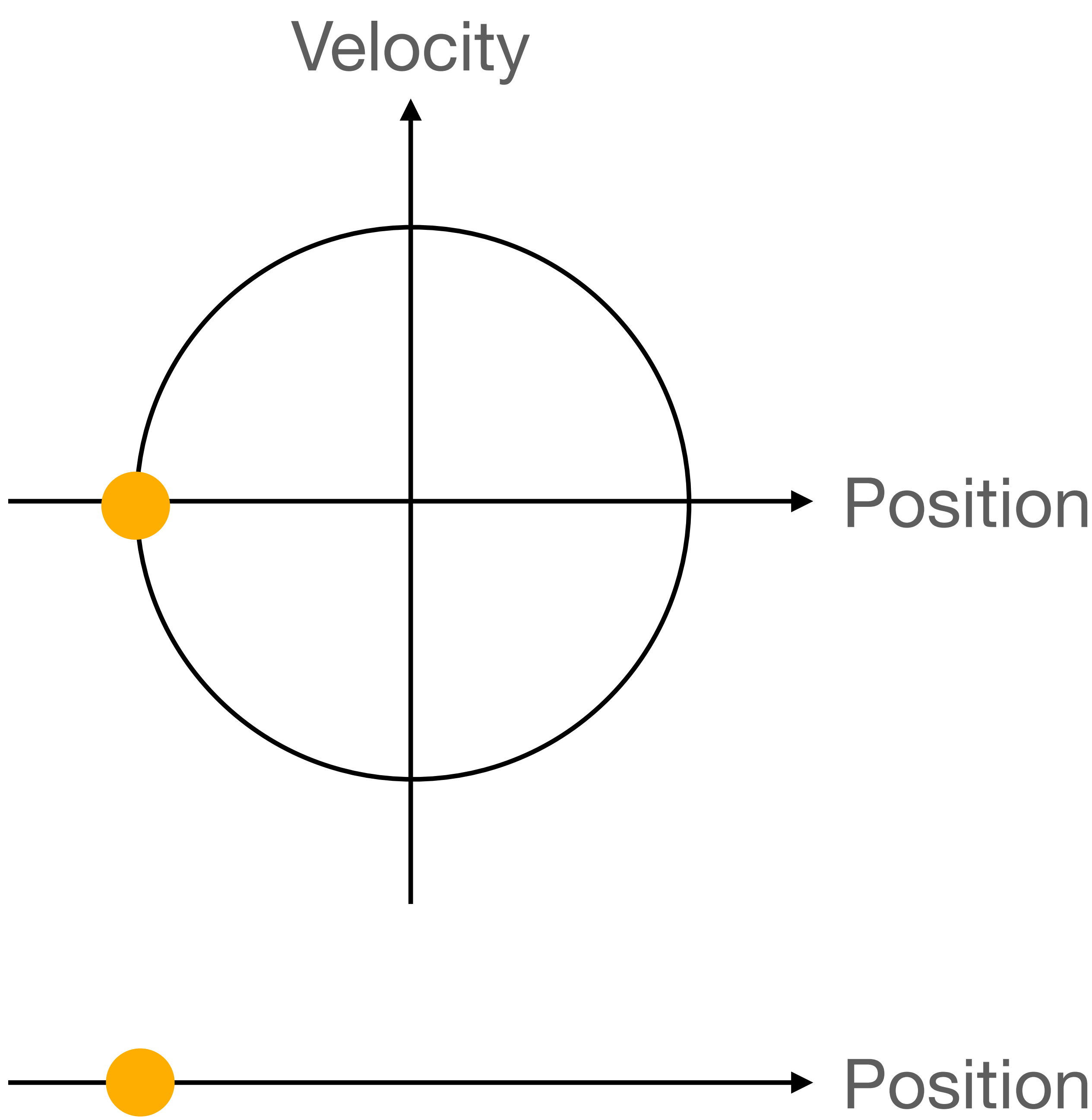
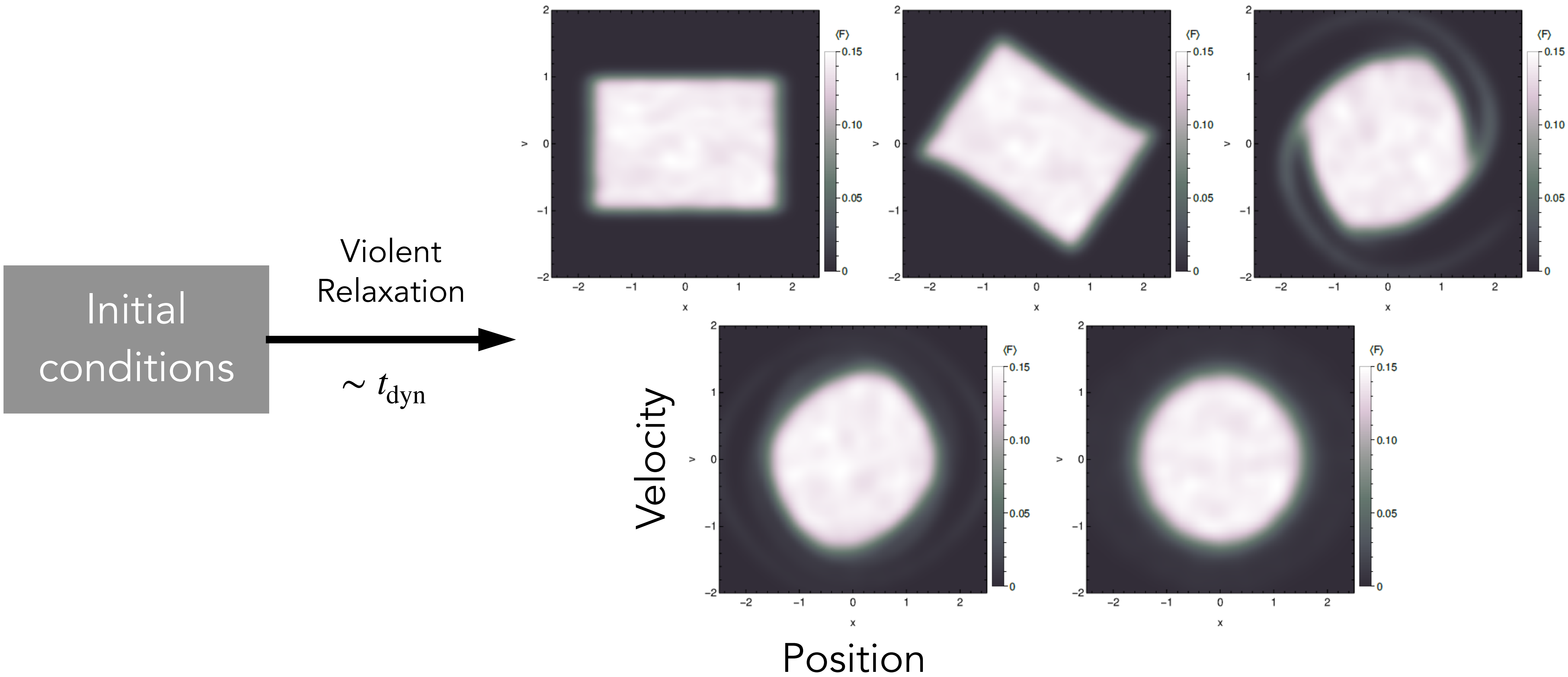
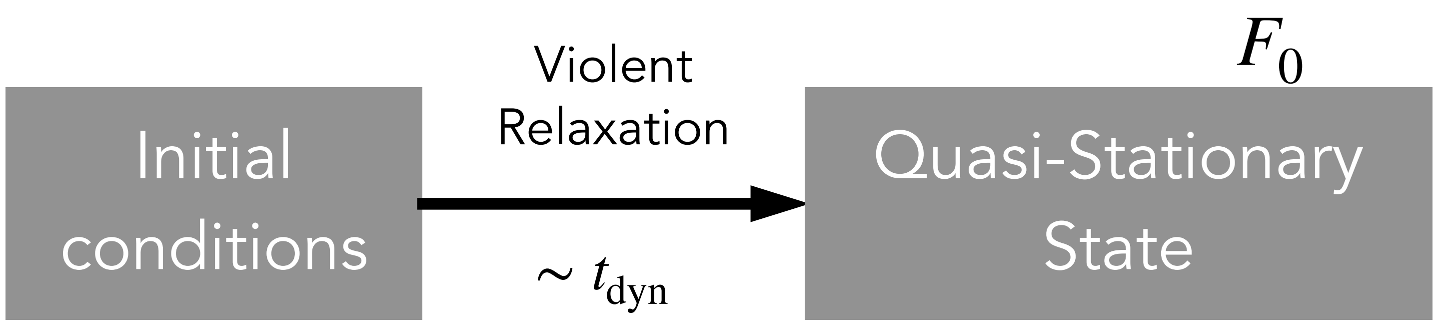


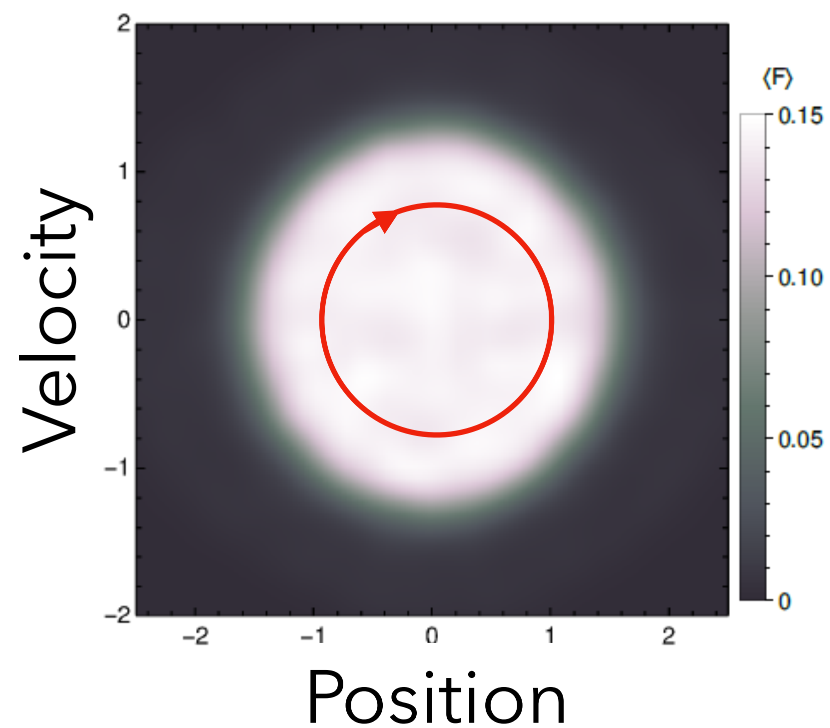
Typical fate of isolated self-gravitating systems



Typical fate of isolated self-gravitating systems

$$\frac{\partial F_0}{\partial t} = [H_0, F_0] = 0$$





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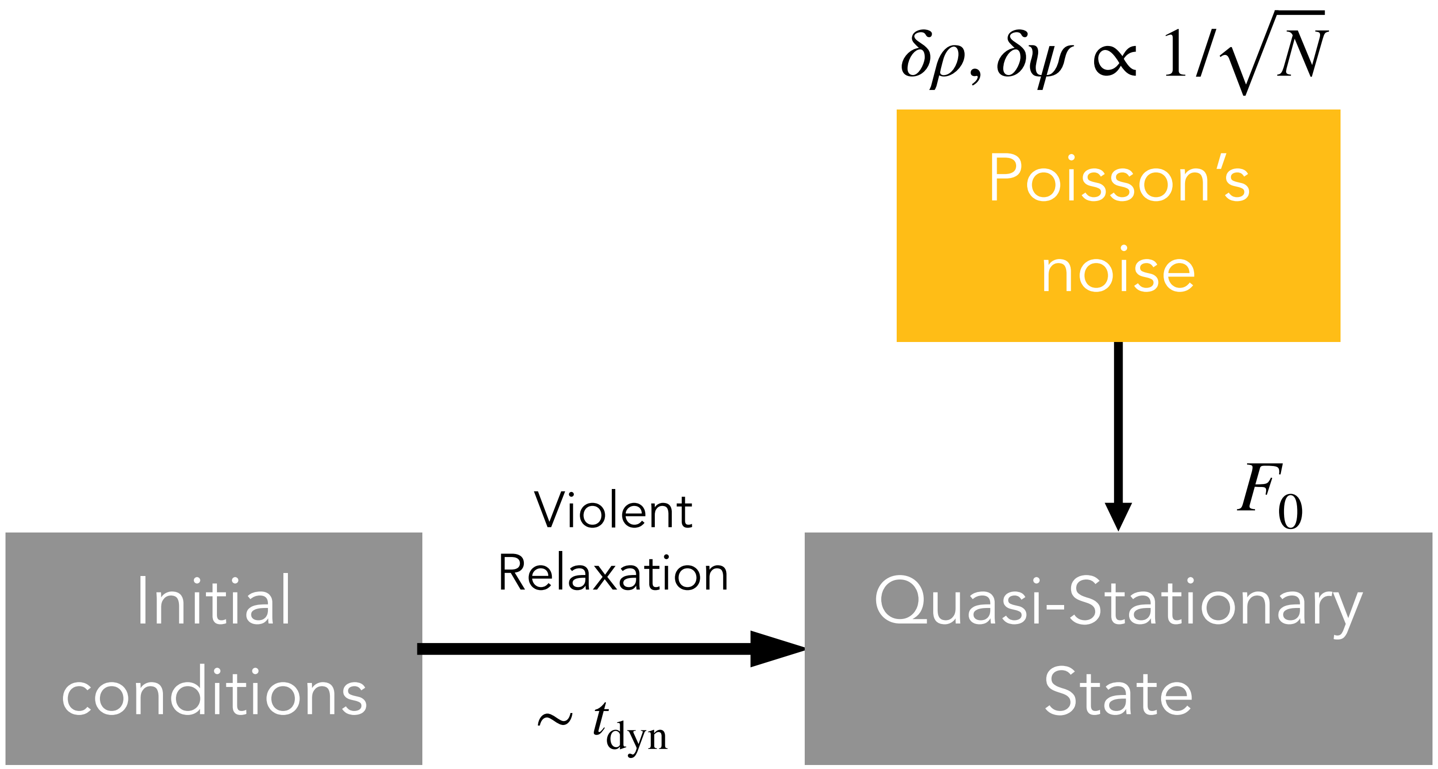

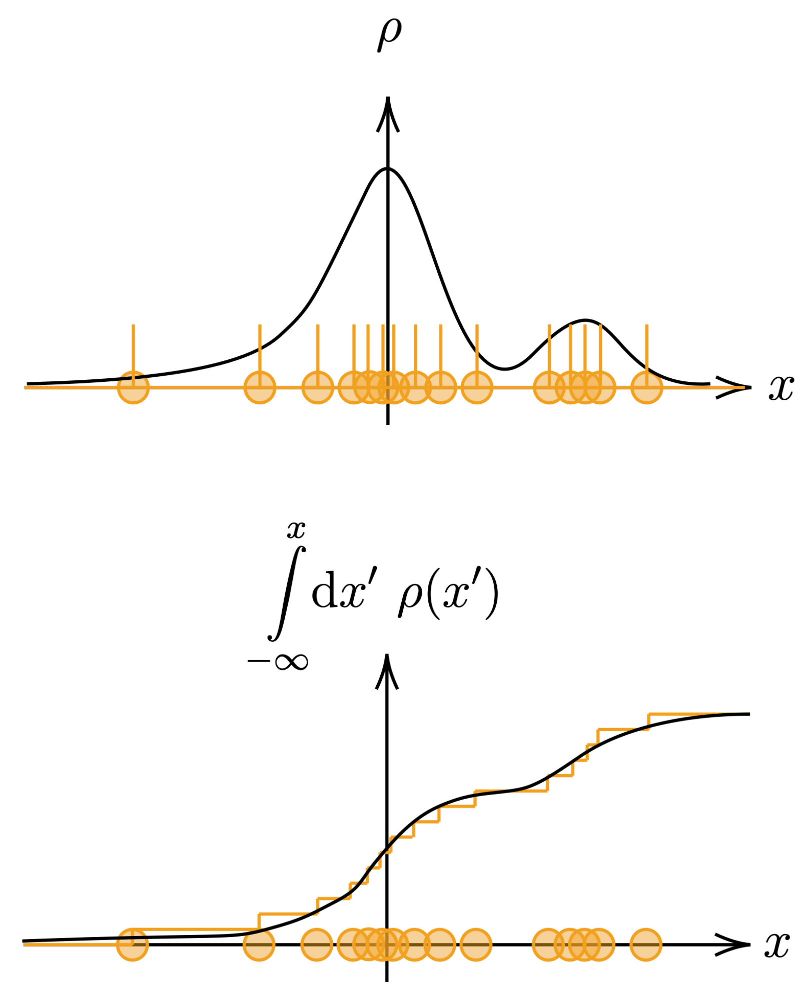

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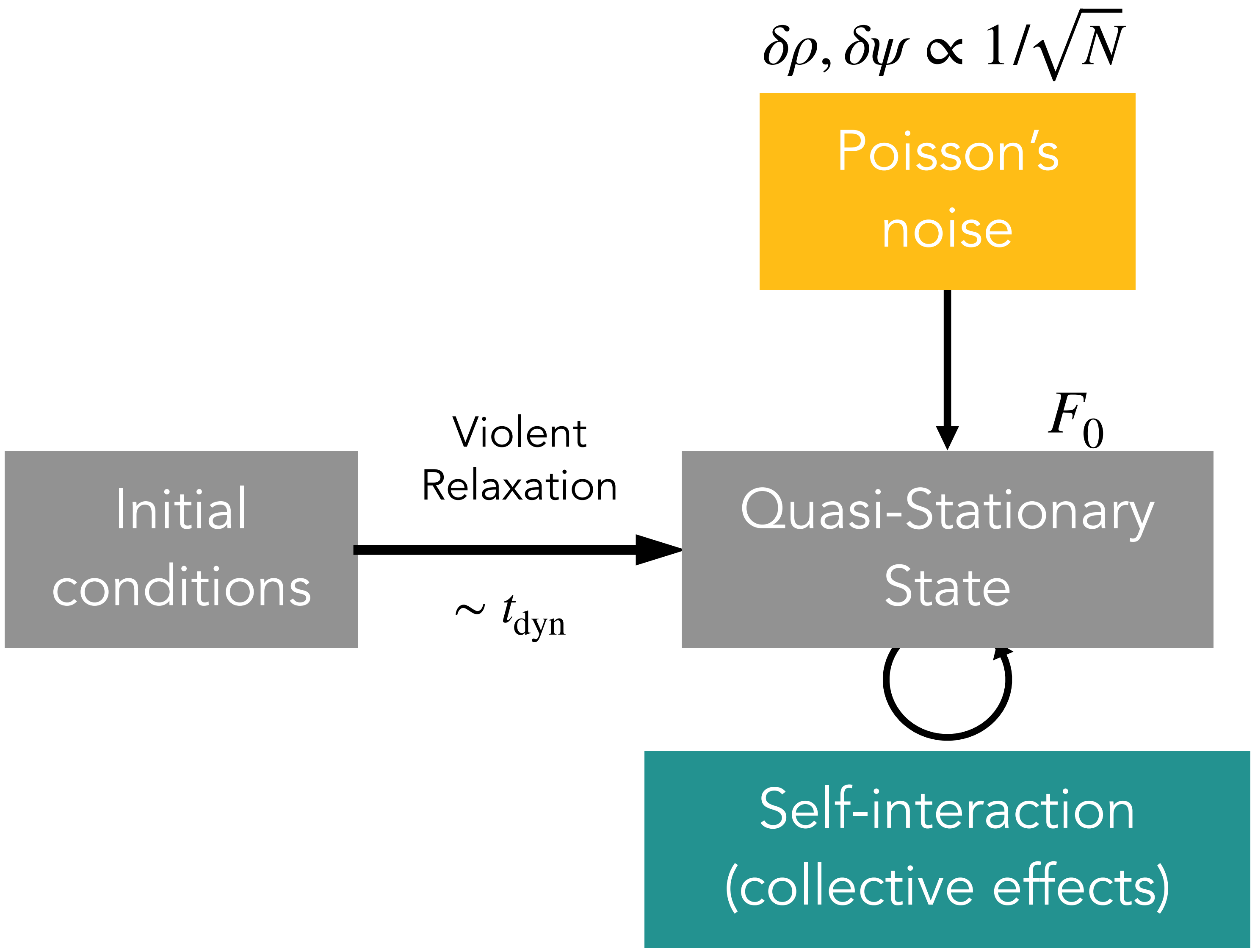

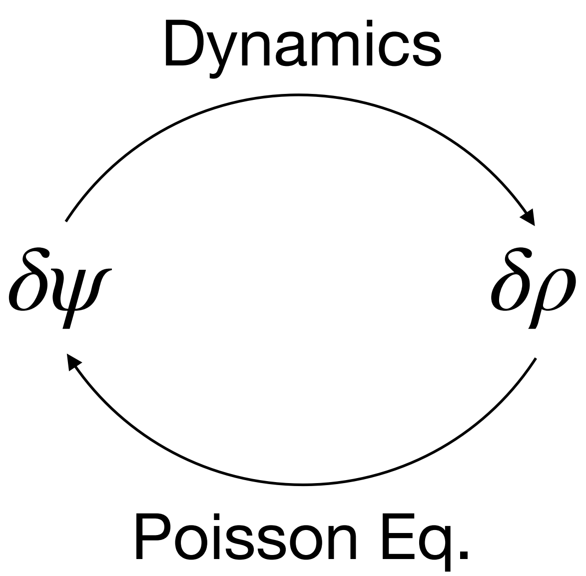

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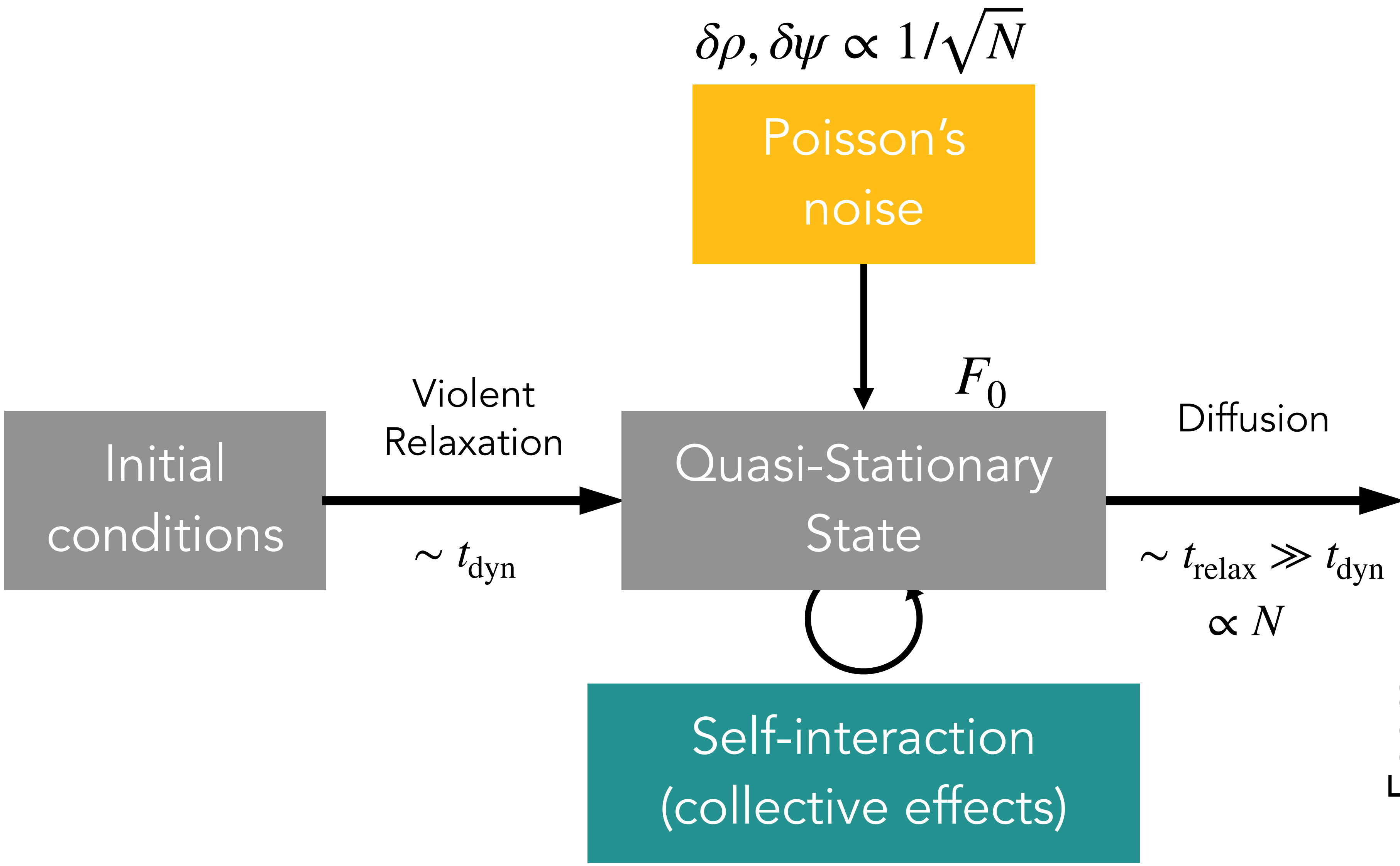

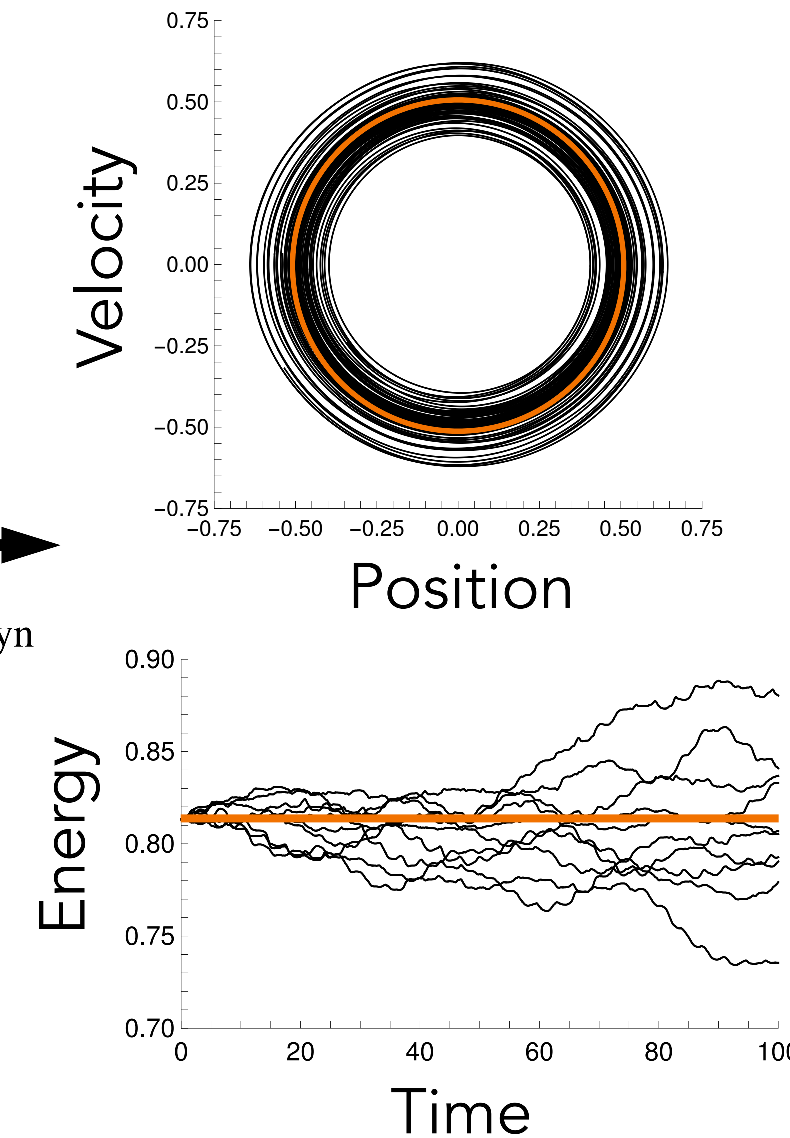

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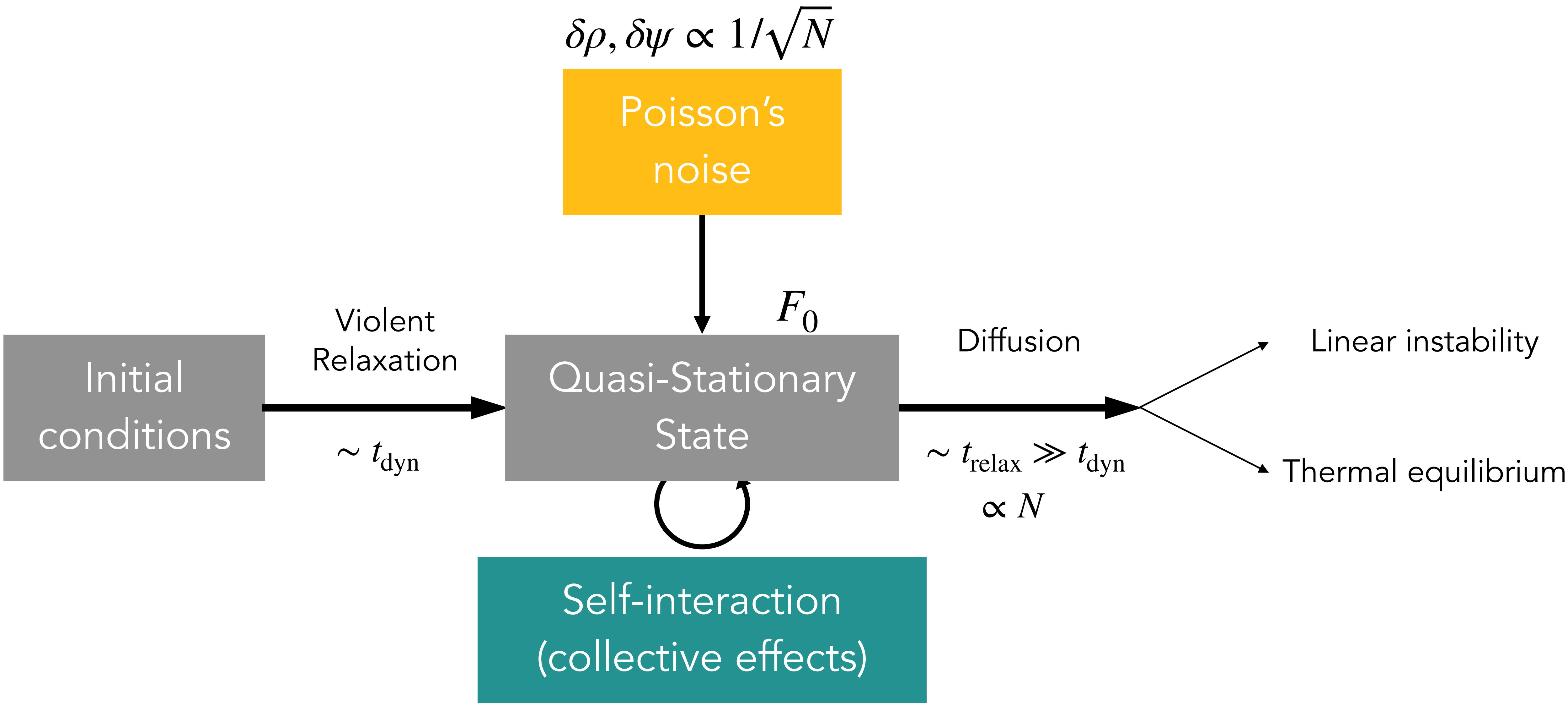
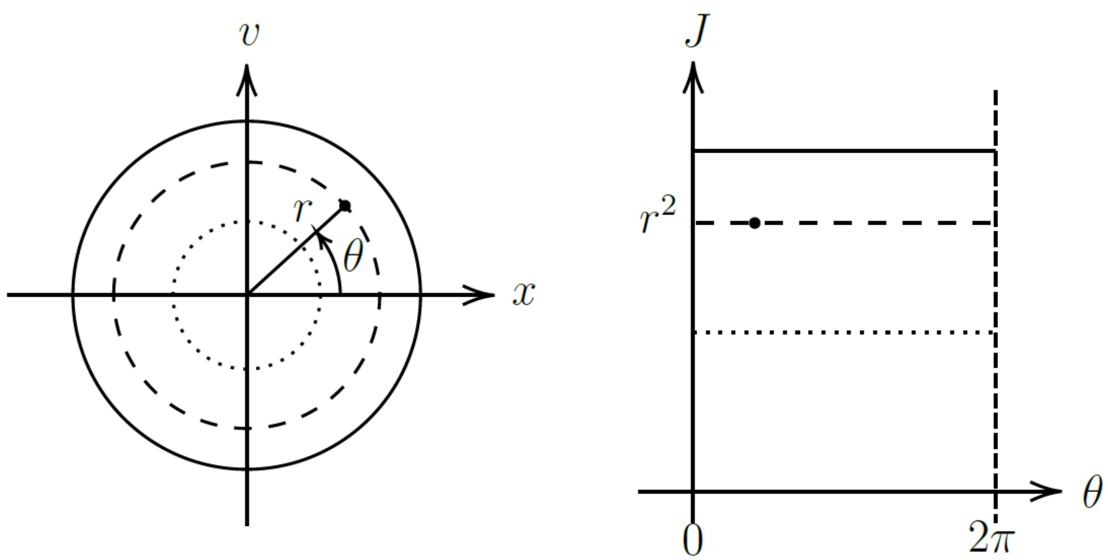


Inhomogeneous Balescu-Lenard equation

Key points:

- Angle-action variables



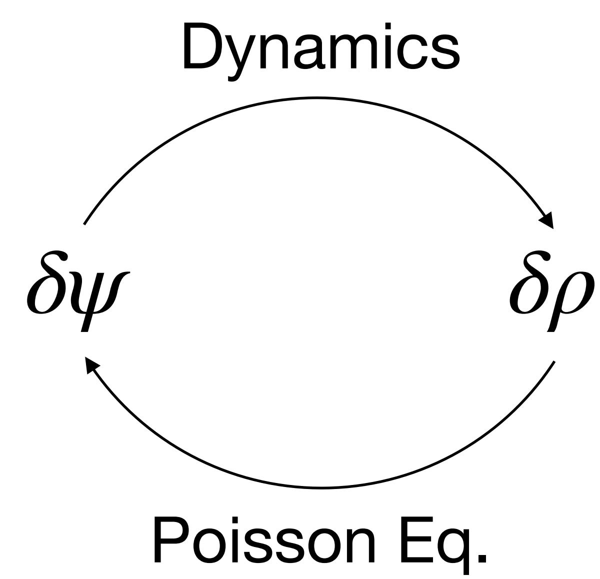
$$J = \mathrm{cst}$$

$$\theta = \Omega(J)\, t + \theta_0$$

- Resonant coupling

$$\delta_{\mathrm{D}}(n\Omega - n'\Omega')$$

- Collective effects



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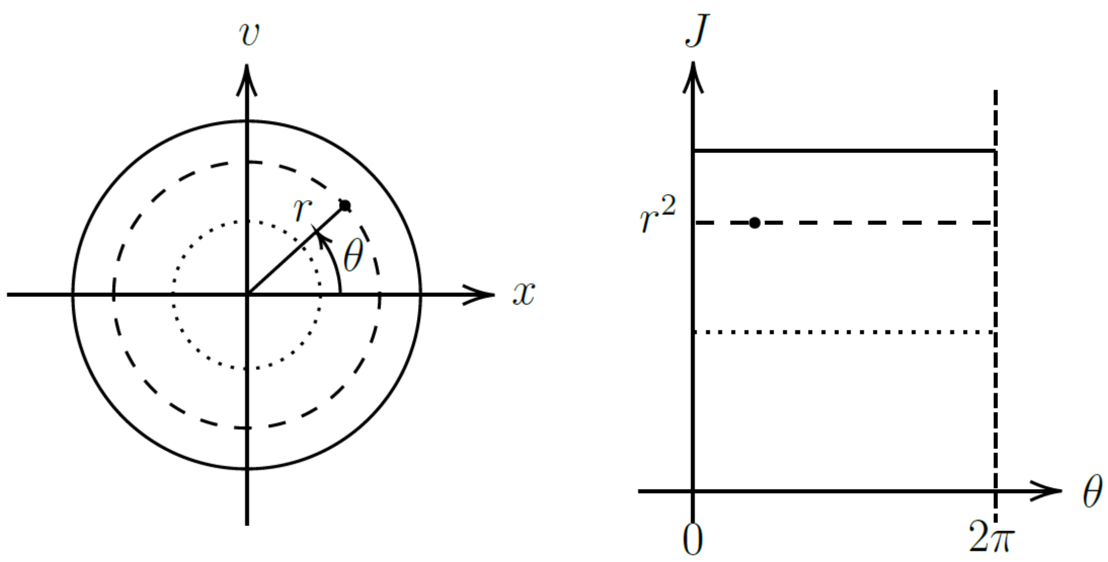

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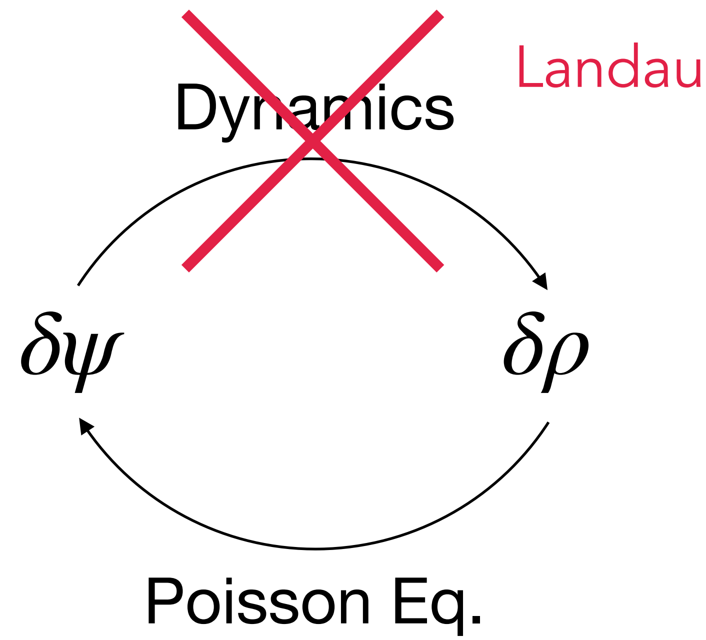

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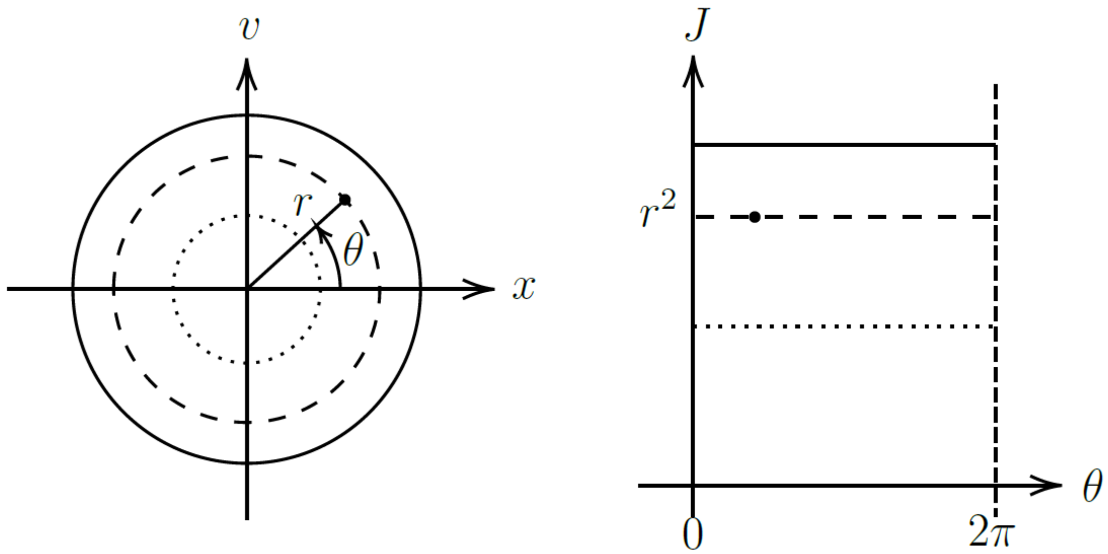

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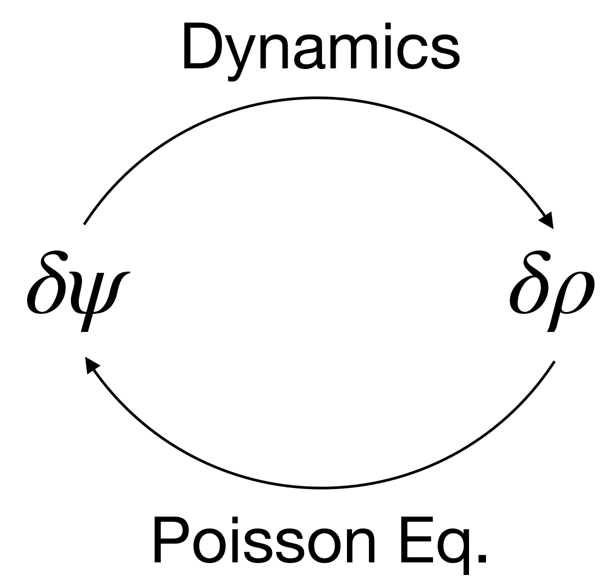


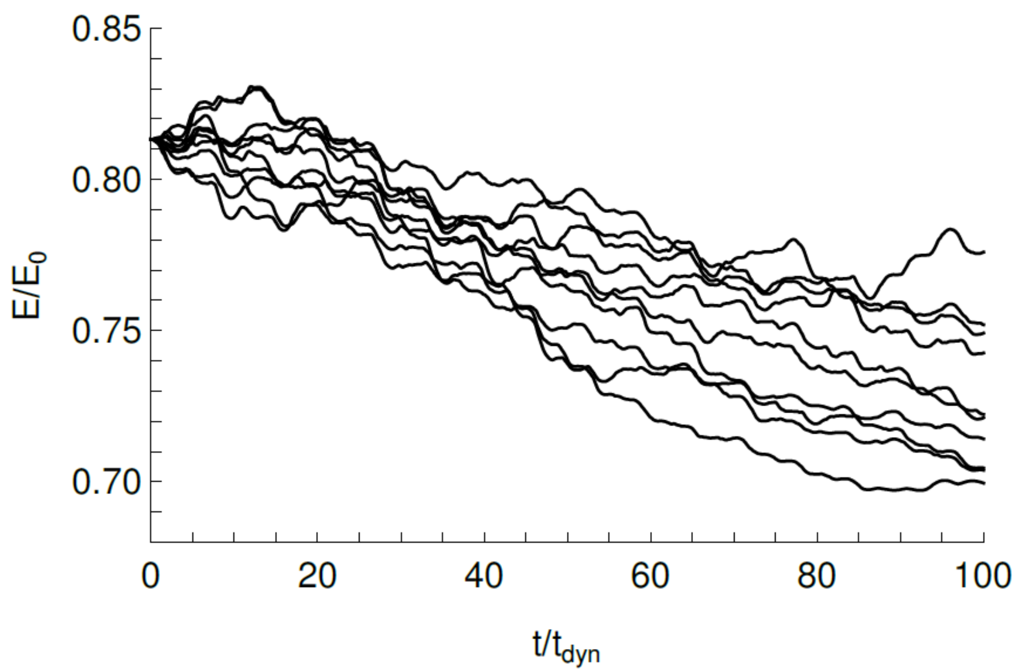
Hypotheses:

- $N \gg 1$
- Integrable system
- Resonances over encounters
- Time-scale separation
- « Sufficiently » stable
- Uncorrelated Poisson noise

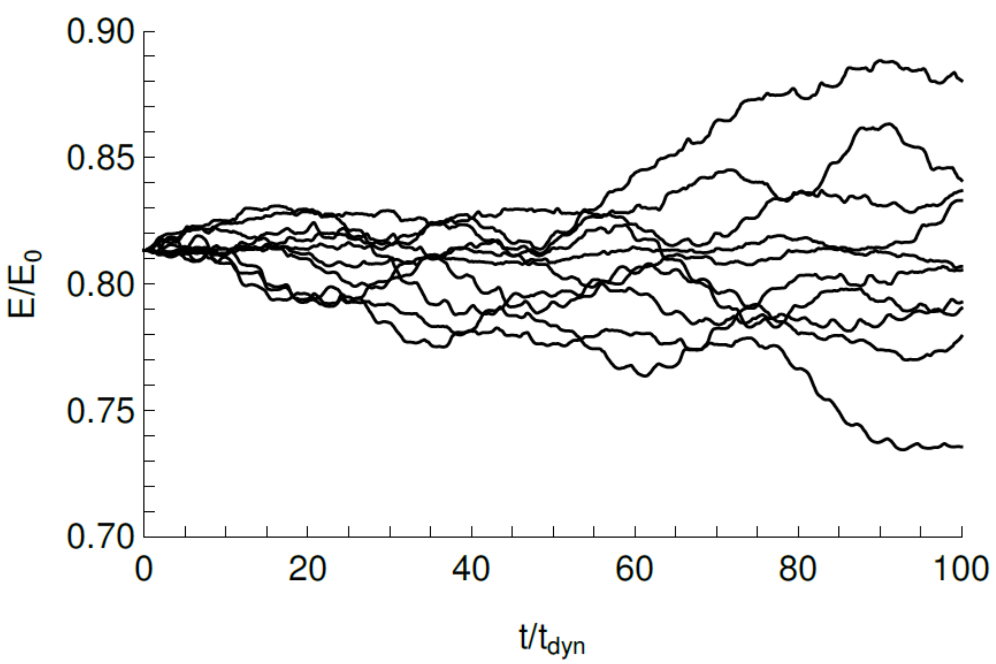
Fokker-Planck form

$$\frac{\partial F_0}{\partial t} = -\frac{\partial \mathscr{F}}{\partial J} = -\frac{\partial}{\partial J}\left[\mathrm{A}(J) F_0(J) - \frac{1}{2}\mathrm{D}(J)\frac{\partial F_0}{\partial J}\right]$$

Dynamical friction



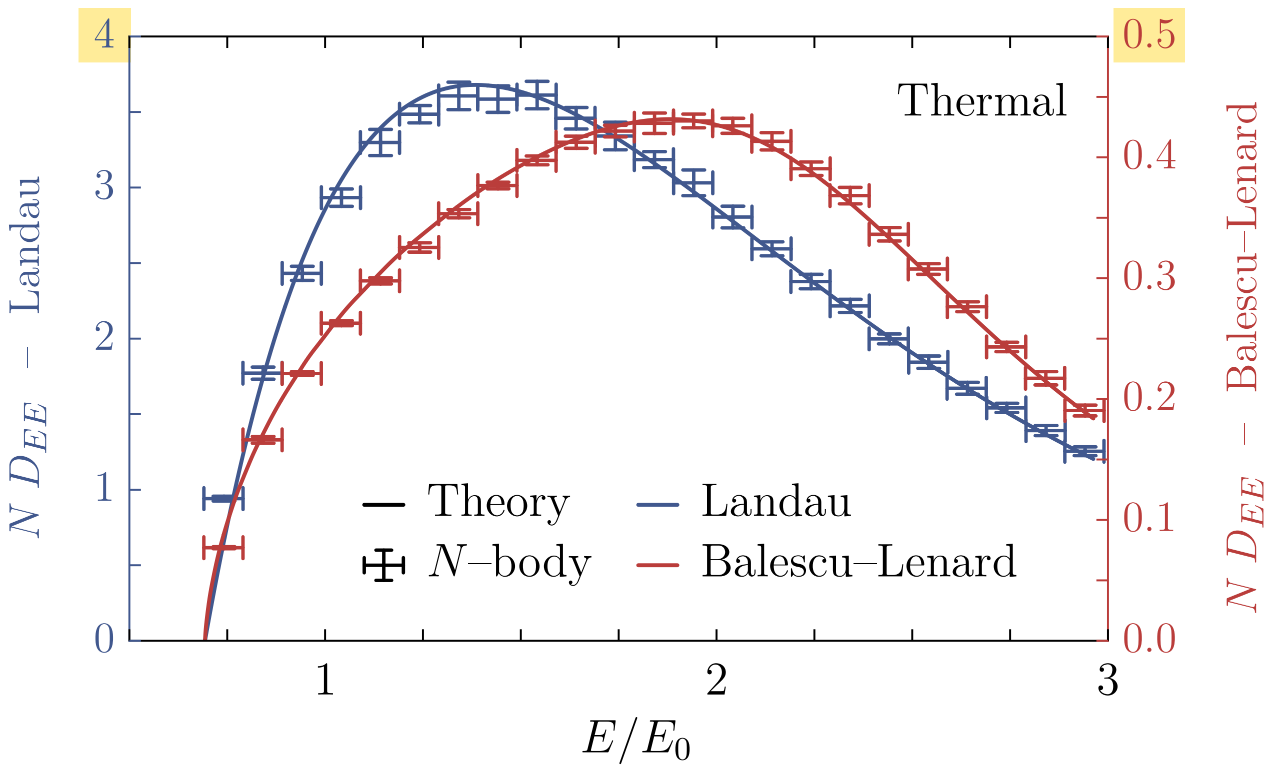
Diffusion

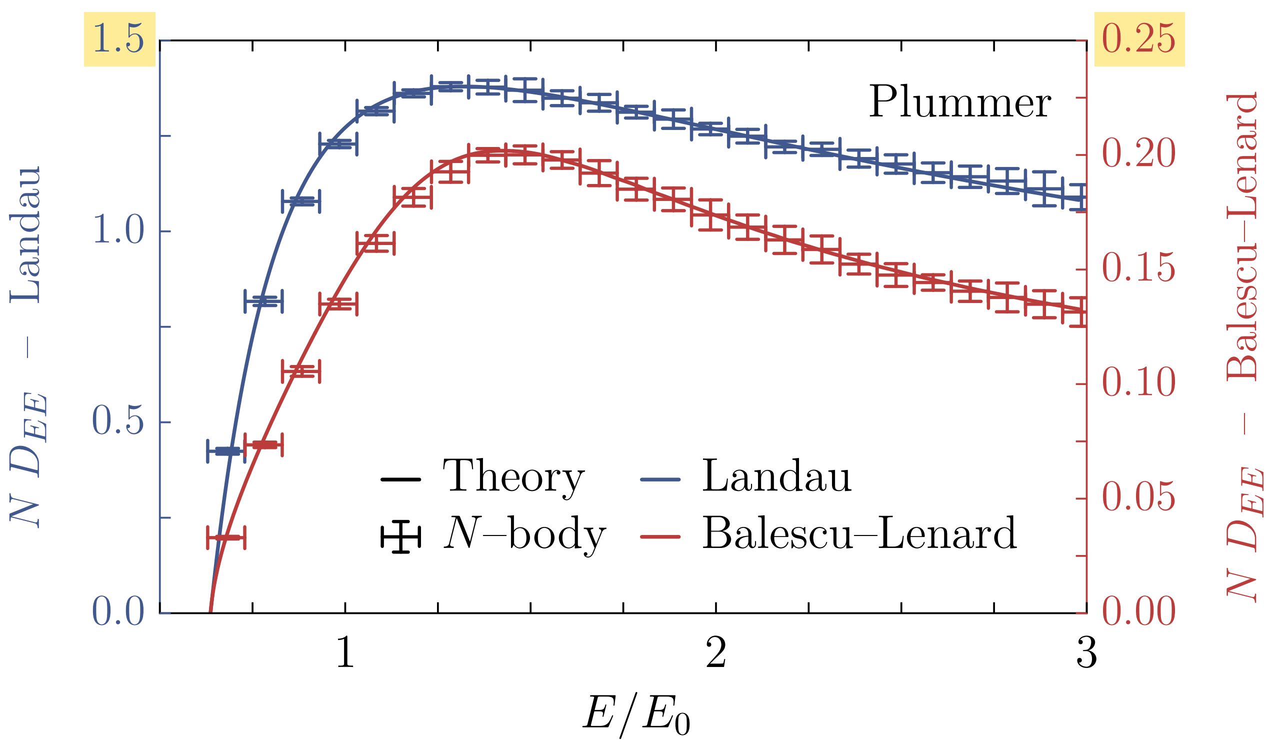


Prediction vs measurements

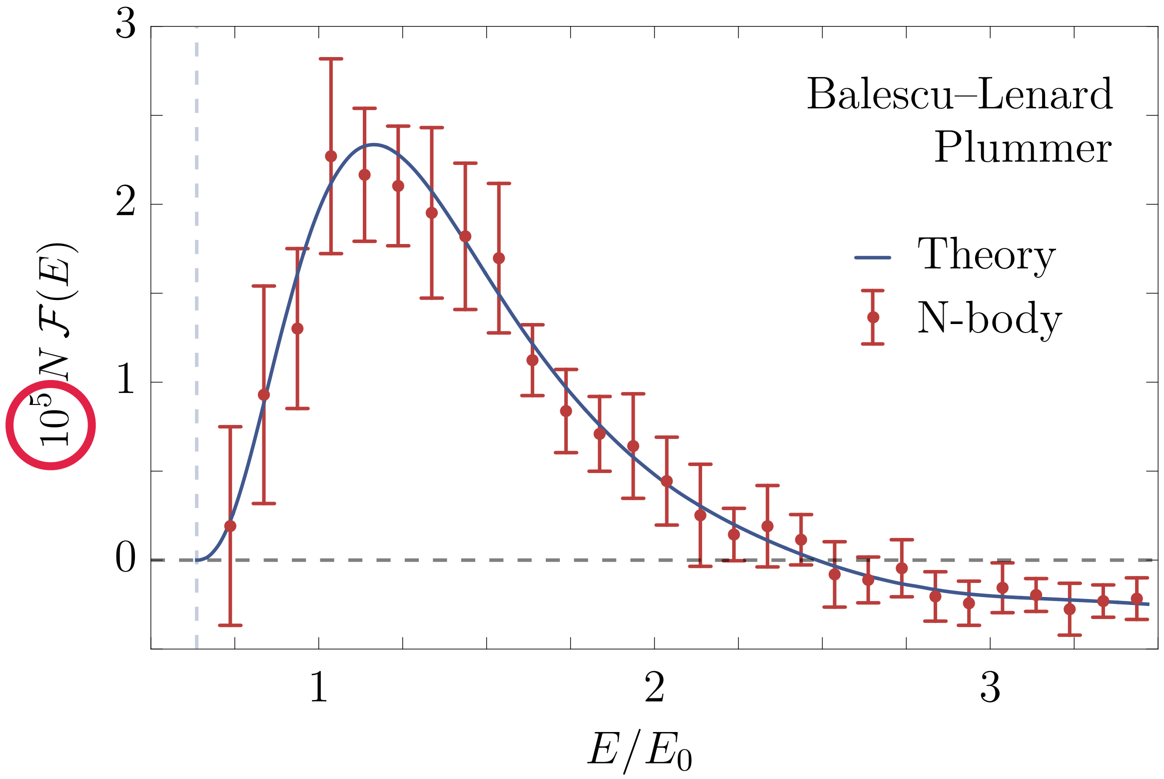
Roule+(2022)

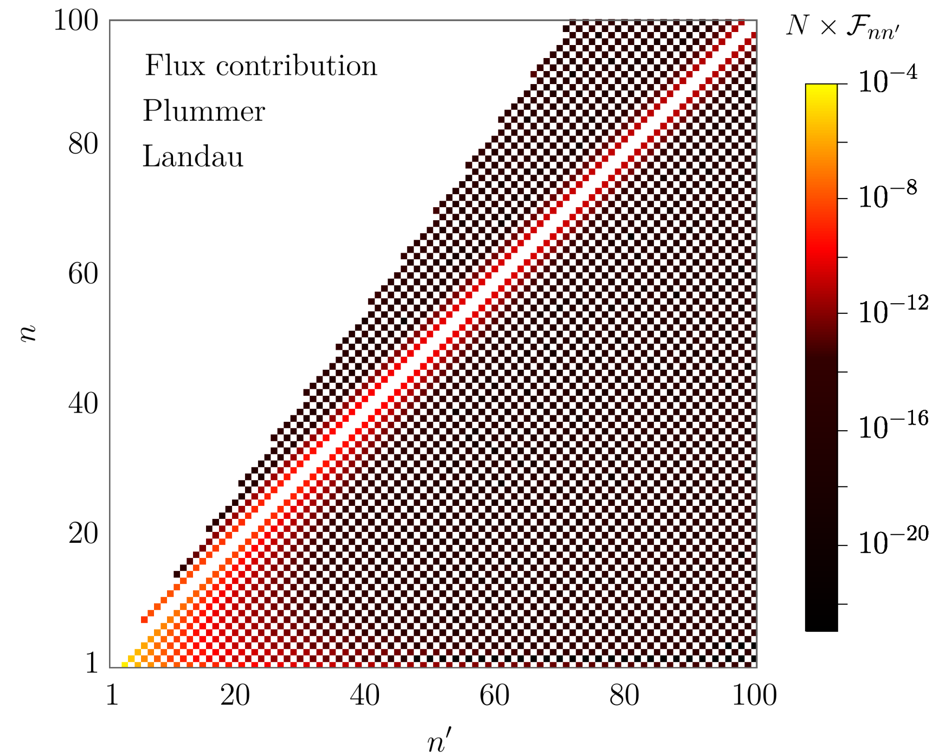
Diffusion





Flux



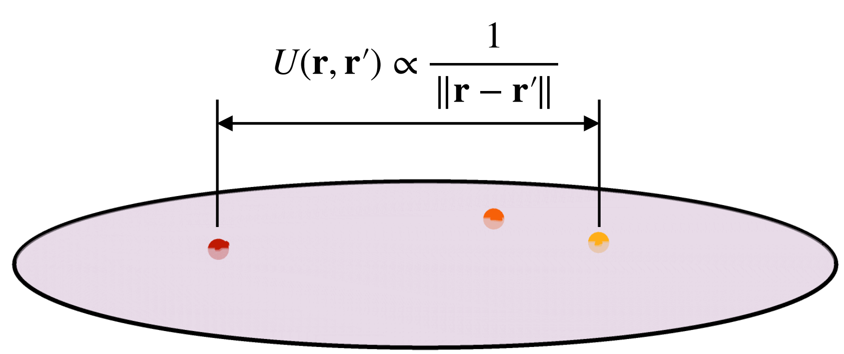


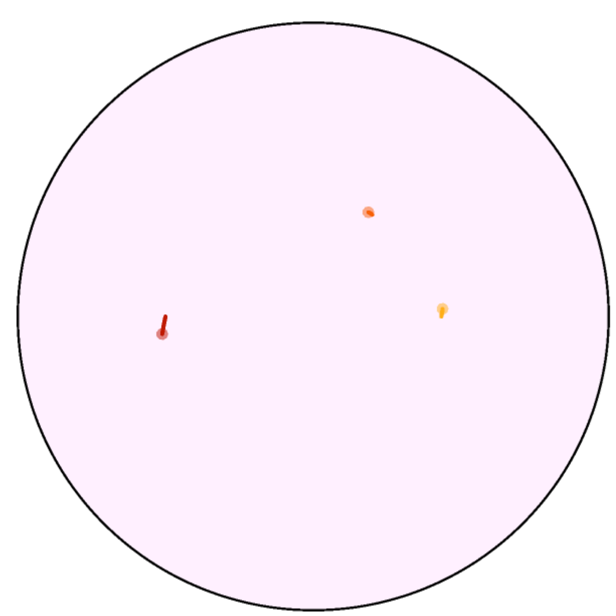
Conclusion on 1D self-gravitating system

- Balescu-Lenard validation
- Quasi kinetic blocking
- Damping collective effects
- Resonances and collective effects matter

Razor-thin disc

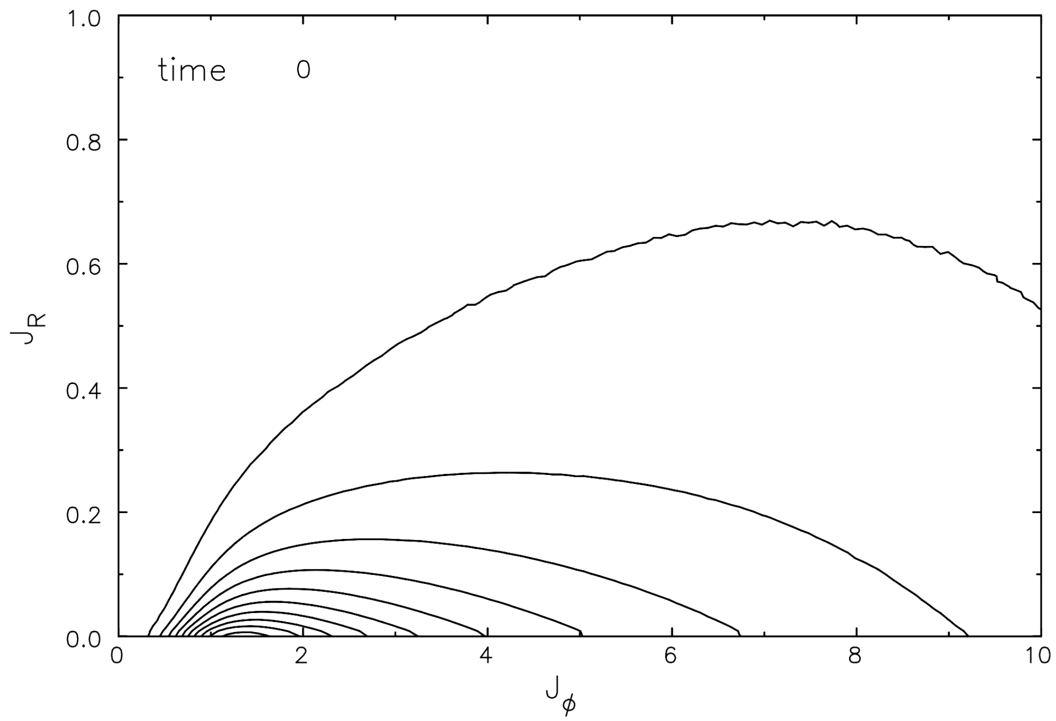
Axisymmetric-potential motion



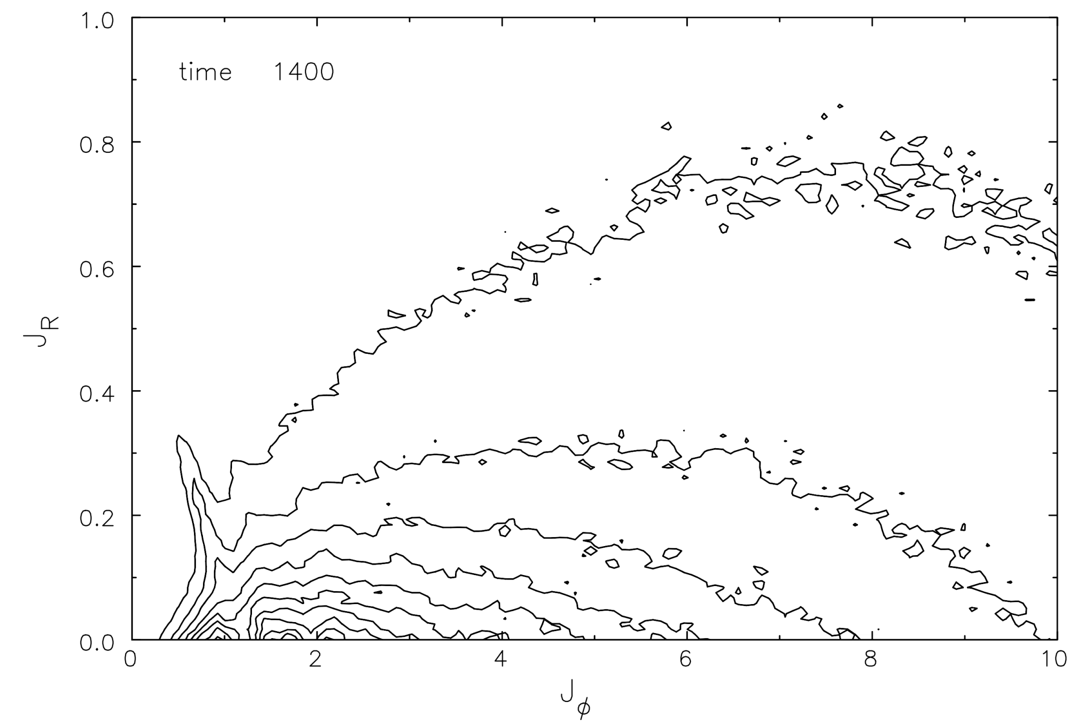


Sellwood (2012) disc

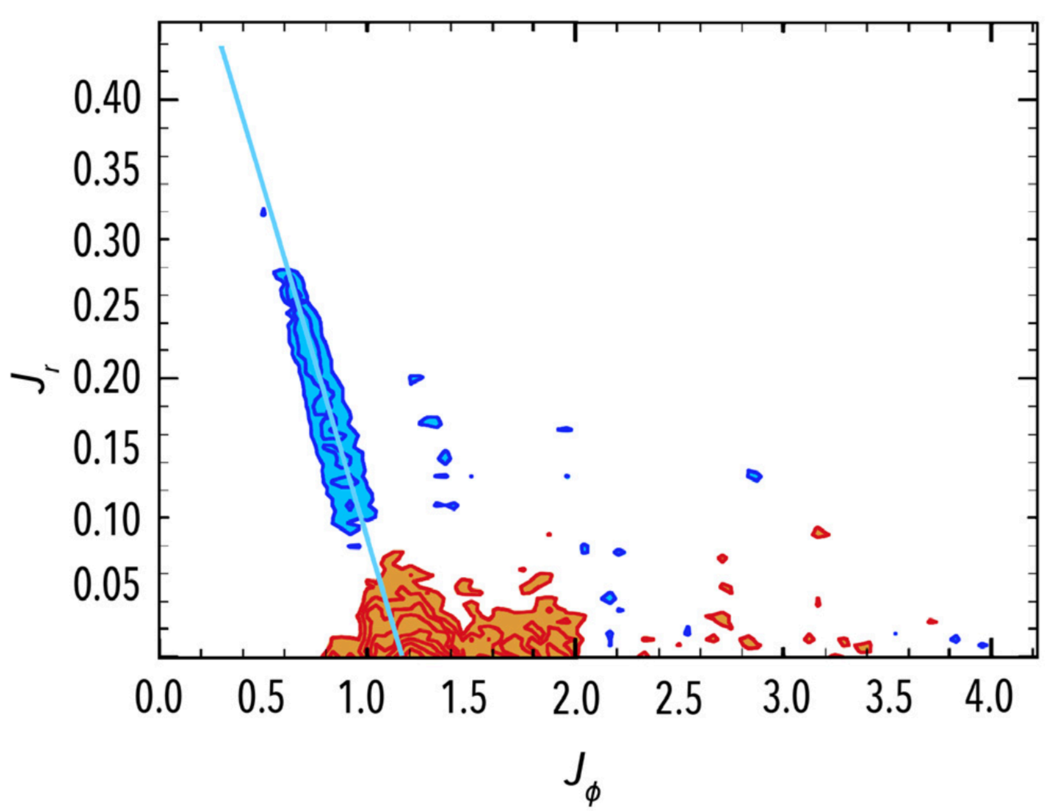
Initial DF *Sellwood (2012)*



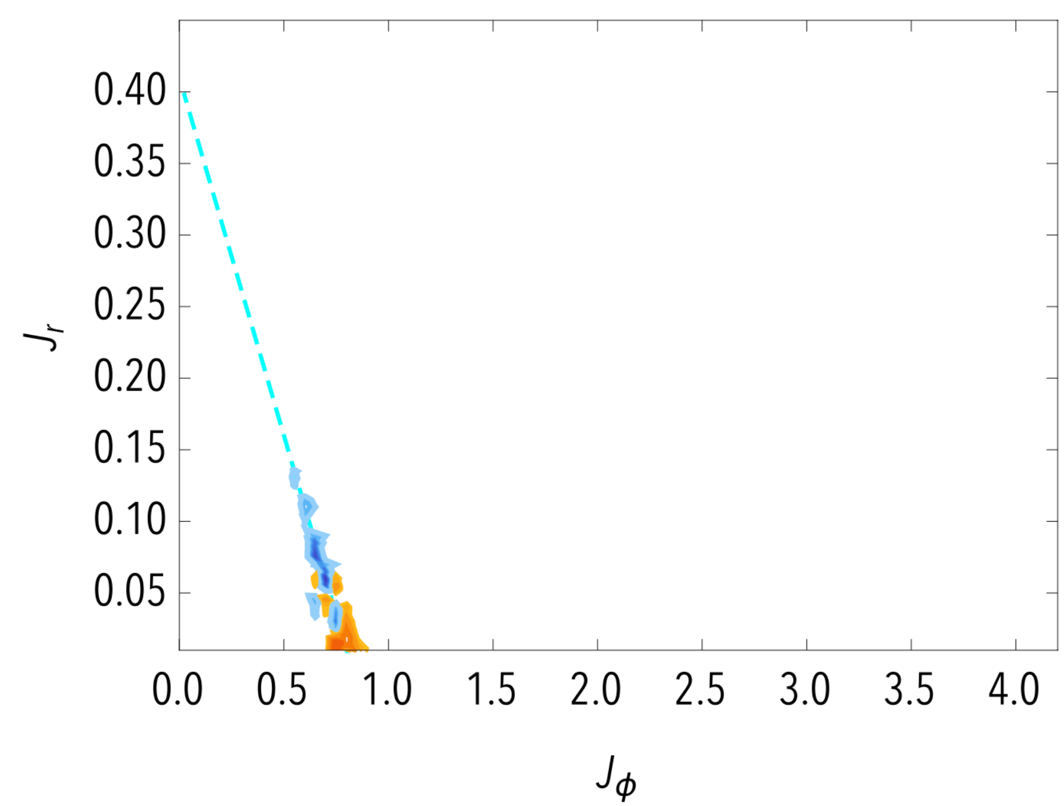
Ridge *Sellwood (2012)*



N-body *Sellwood (2012)*



Balescu-Lenard *Fouvry+(2015)*



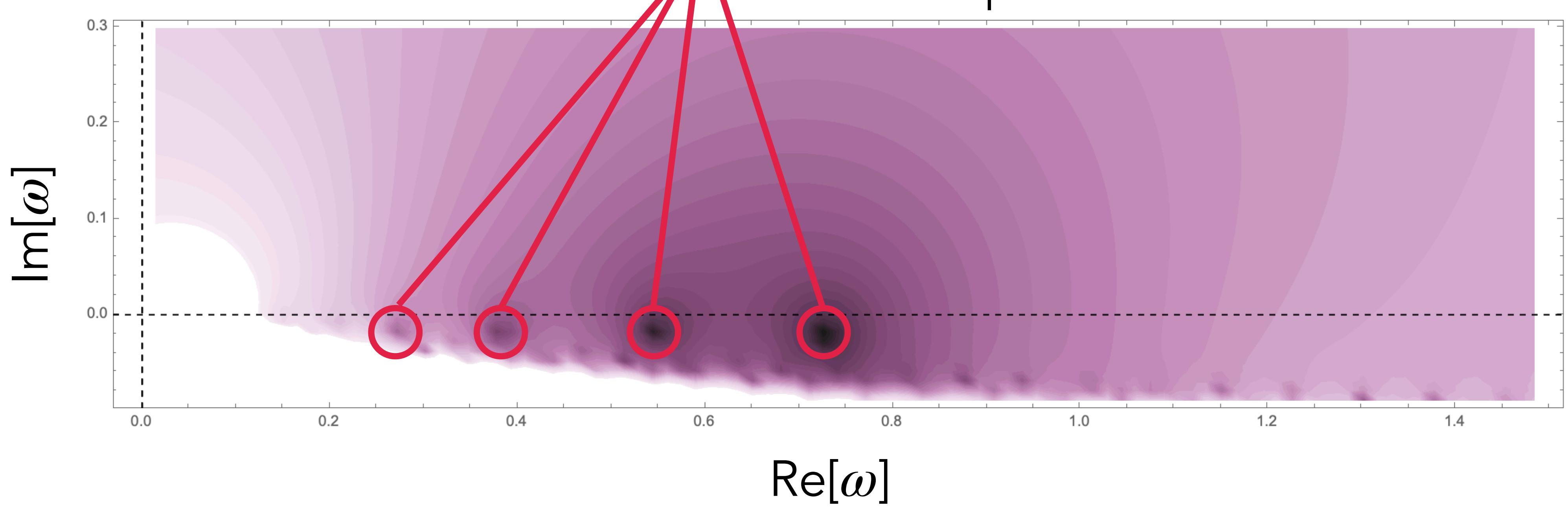
Sellwood (2012) disc

?

Particle-particle interaction (Balescu-Lenard)

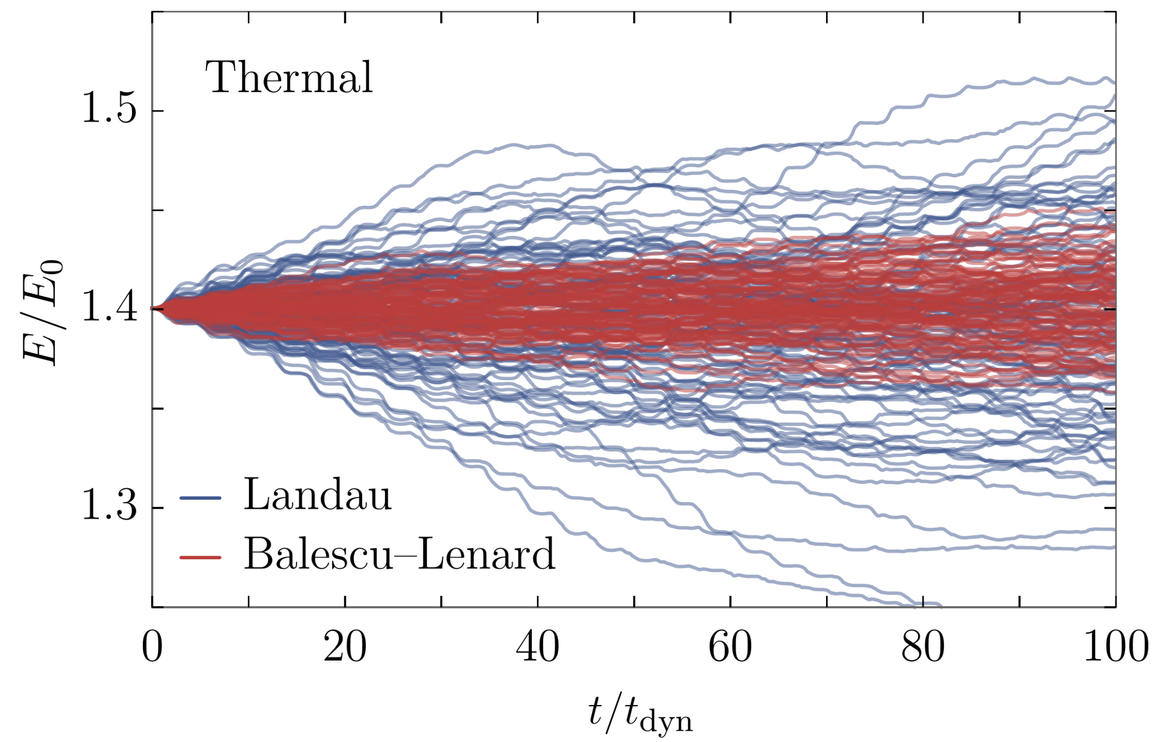
Wave-particle interaction (Quasi-Linear Theory)

Collective effects « amplitude »

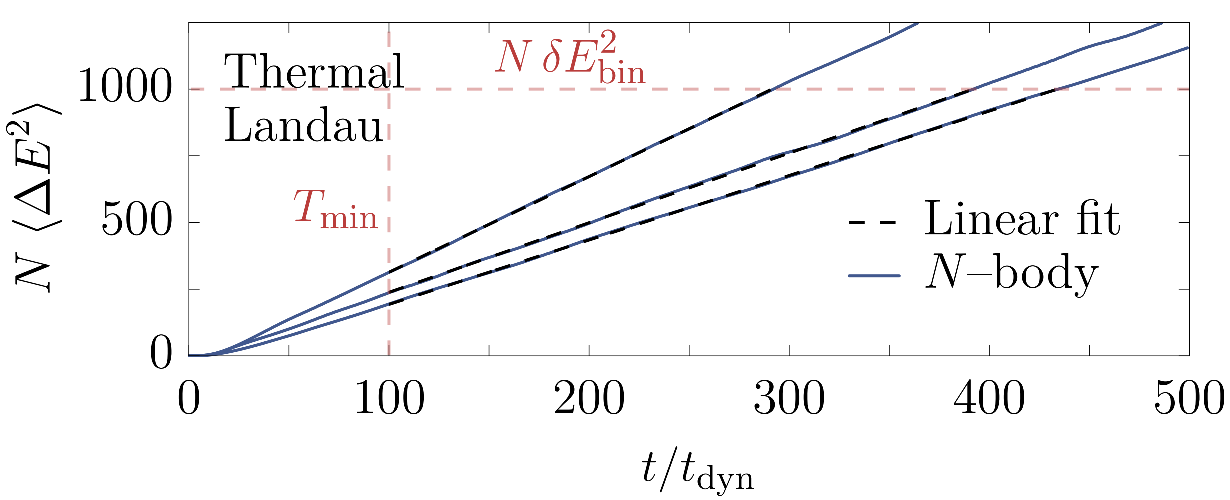


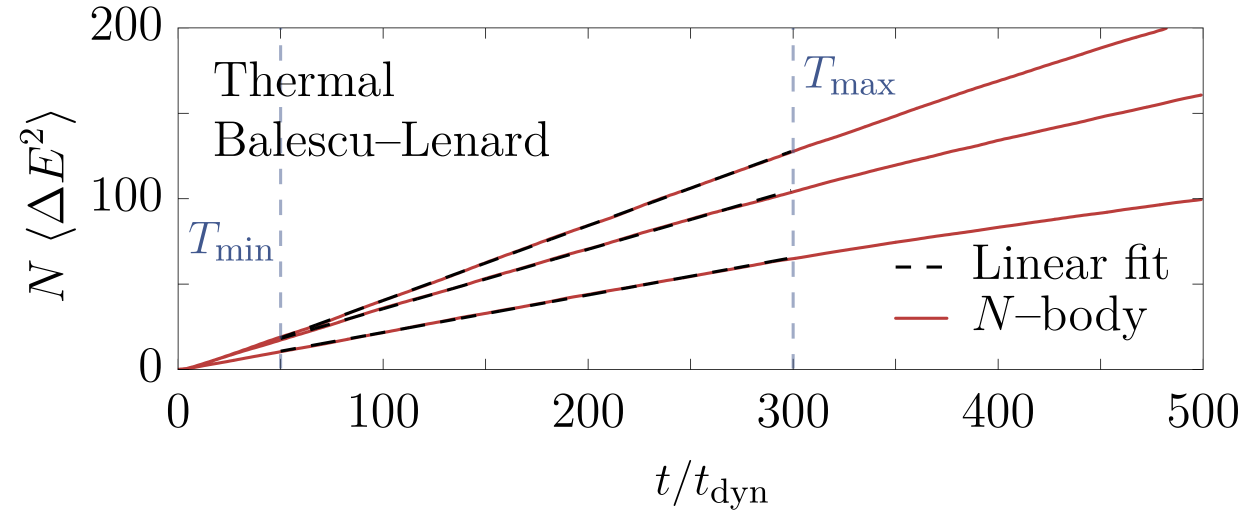
Back-up slides

N-body measurements



$$D(J) = \lim_{T\to+\infty} \frac{\left\langle \Delta J(T)^2 \right\rangle}{T}$$





Inhomogeneous Balescu-Lenard equation

$$\frac{\partial F_0}{\partial t} = 2\pi^2 m \frac{\partial}{\partial J} \sum_{n,n'} n \int \mathrm{d}J' \, \delta_{\mathrm{D}}(n\Omega - n'\Omega') \, |\psi^{\mathrm{d}}_{nn'}(J, J', n\Omega)|^2 \left(n \frac{\partial}{\partial J} - n' \frac{\partial}{\partial J'} \right) F_0(J) \, F_0(J')$$

F_0 Mean-field orbit population in action space J

$m \propto 1/N$ Individual mass of particles

$\frac{\partial}{\partial J}$ Divergence of a flux

$\sum_{n,n'}$ Scan over resonances

$\int \mathrm{d}J'$ Scan over orbits

$\delta_{\mathrm{D}}(n\Omega - n'\Omega')$ Resonance condition

$|\psi^{\mathrm{d}}_{nn'}(J, J', n\Omega)|^2$ « Dressed » coupling

Coupling coefficients

$$\psi^{\mathrm{d}}_{nn'}(J, J', \omega) = -\sum_{p,q} \psi^{(p)}_{n}(J) \left[\mathbf{I} - \mathbf{M}(\omega)\right]^{-1}_{pq} \psi^{(q)*}_{n'}(J')$$

Bi-orthogonal basis

Fourier Transform in angle θ of bi-orthogonal basis elements $\left(\rho^{(p)}, \psi^{(p)}\right)$

$$\int \mathrm{d}x\, \rho^{(p)}(x)\, \psi^{(q)*}(x) = -\, \delta^{p}_{q}$$

$$\psi^{(p)}(x) = \int \mathrm{d}x'\, \rho^{(p)}(x')\, U(x, x')$$

Response Matrix

$$\mathbf{M}[F_0](\omega)$$

$$\mathbf{M}_{pq}(\omega) = -\, 2\pi \sum_{n} \int_{\mathscr{L}} \mathrm{d}J\, \frac{n\, \partial F_0/\partial J}{n\Omega - \omega}\, \psi^{(p)*}_{n}(J)\, \psi^{(q)}_{n}(J)$$