Clustering of critical points in the Cosmic web

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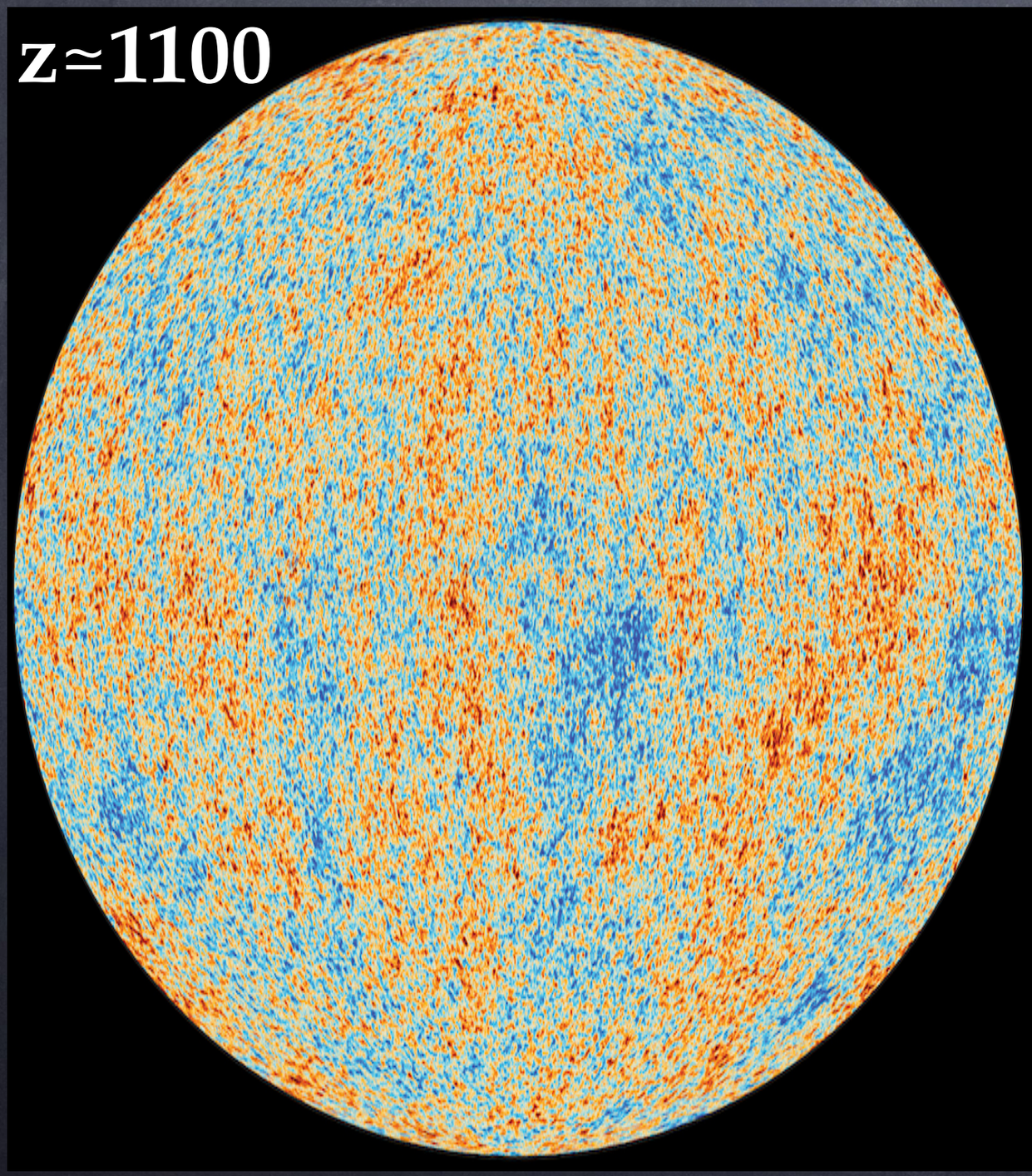
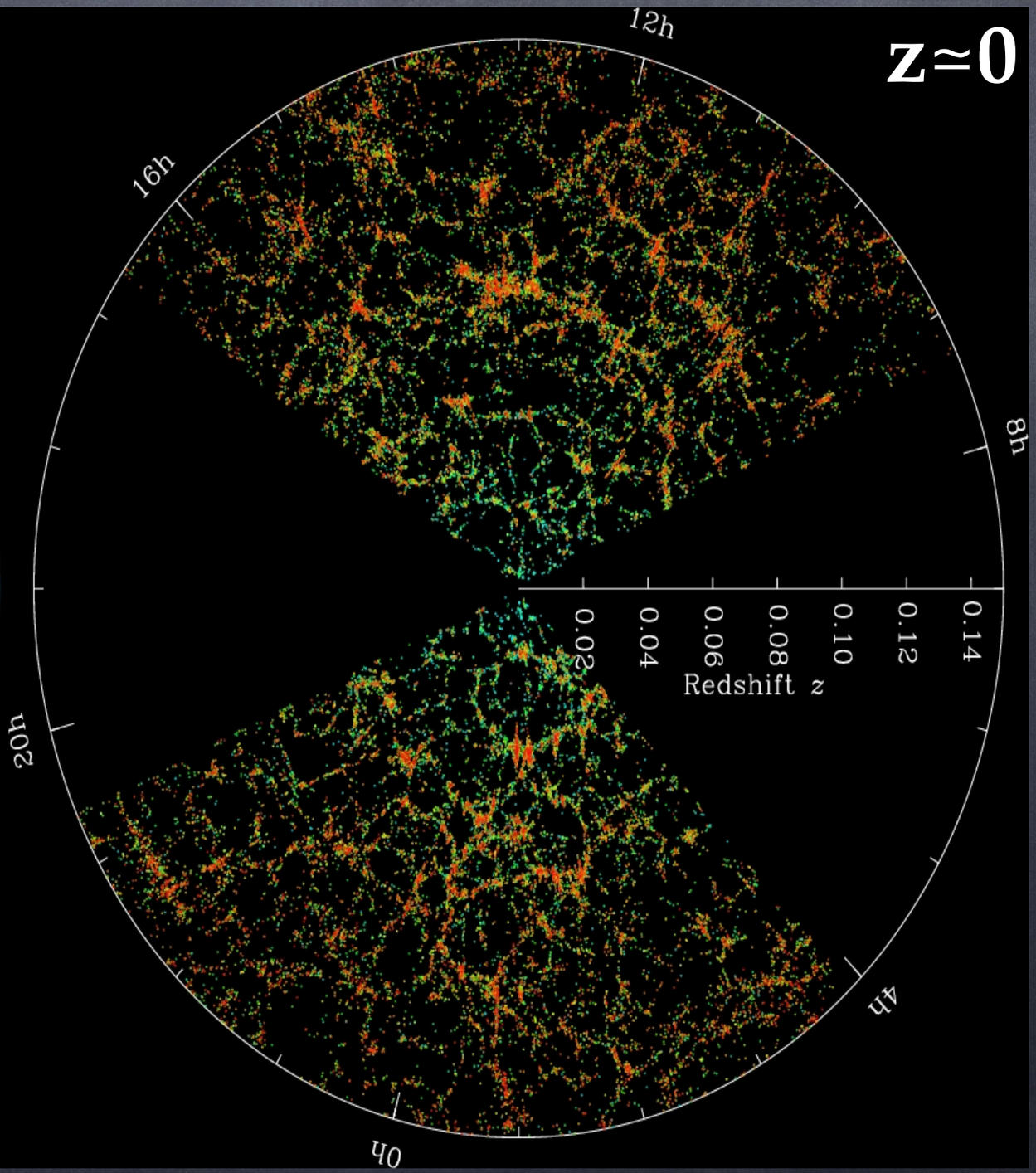
LSSs in our Universe



credit : Planck Collaboration, ESA



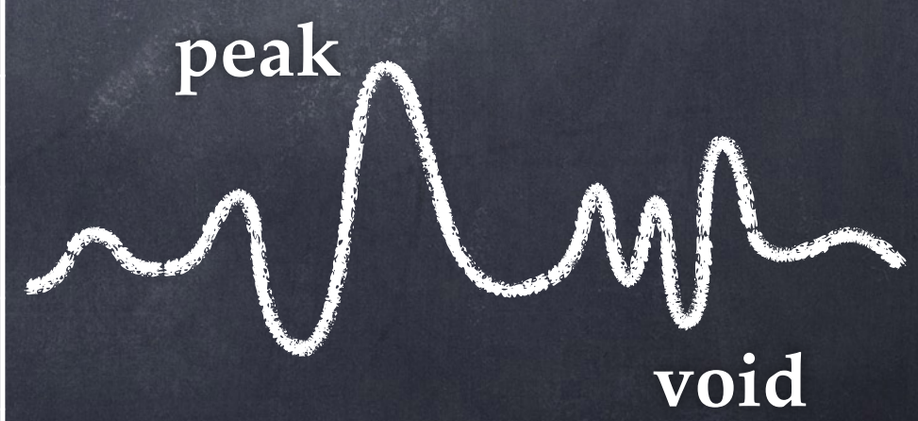
credit : M. Blanton, SDSS

Critical points (CPs)

- Points where the gradient of a (density) field vanishes, i.e. $\partial\rho/\partial x^i=0$ (Milnor 63; Bardeen+86; Arnold 06)

- Categorized depending on the eigenvalues of the density Hessian ($\partial^2\rho/\partial x^i\partial x^j$)

($\lambda_1 < \lambda_2 < \lambda_3$)	λ_1	λ_2	λ_3
peak (maximum)	-	-	-
filament (saddle)	-	-	+
wall (saddle)	-	+	+
void (minimum)	+	+	+



Significance of CPs

- **Topological objects of density fields**
- **Formation sites of physical structures**
  - **Peak-point** (maximum) **: Massive cluster** (Kaiser 84; Bardeen+86; Bond+91; Ludlow & Porciani 11)
  - **Void-point** (minimum) **: Cosmic void** (Bertschinger 85; Bardeen+86; Sheth & van de Weygaert 04; Colberg 05)
  - **Filament-point** (saddle) **: Large-scale filament** (Fard+19)
  - **Wall-point** (saddle) **: Sheet-like structure** (i.e. SGW)
- **Backbone of the Cosmic web**

(Sousbie+08, 09; Pogosyan+09; van de Weygaert+11; Gay+12; Codis+18)

Clustering of CPs

- Topology of density field
- Spatial organization of structures

Simulation data

- L_{box} = 500 Mpc/h
- $N_{particle} = 256^3$
- 532 realizations (Gadget 2)
- R_G = 6Mpc/h

ΛCDM cosmology

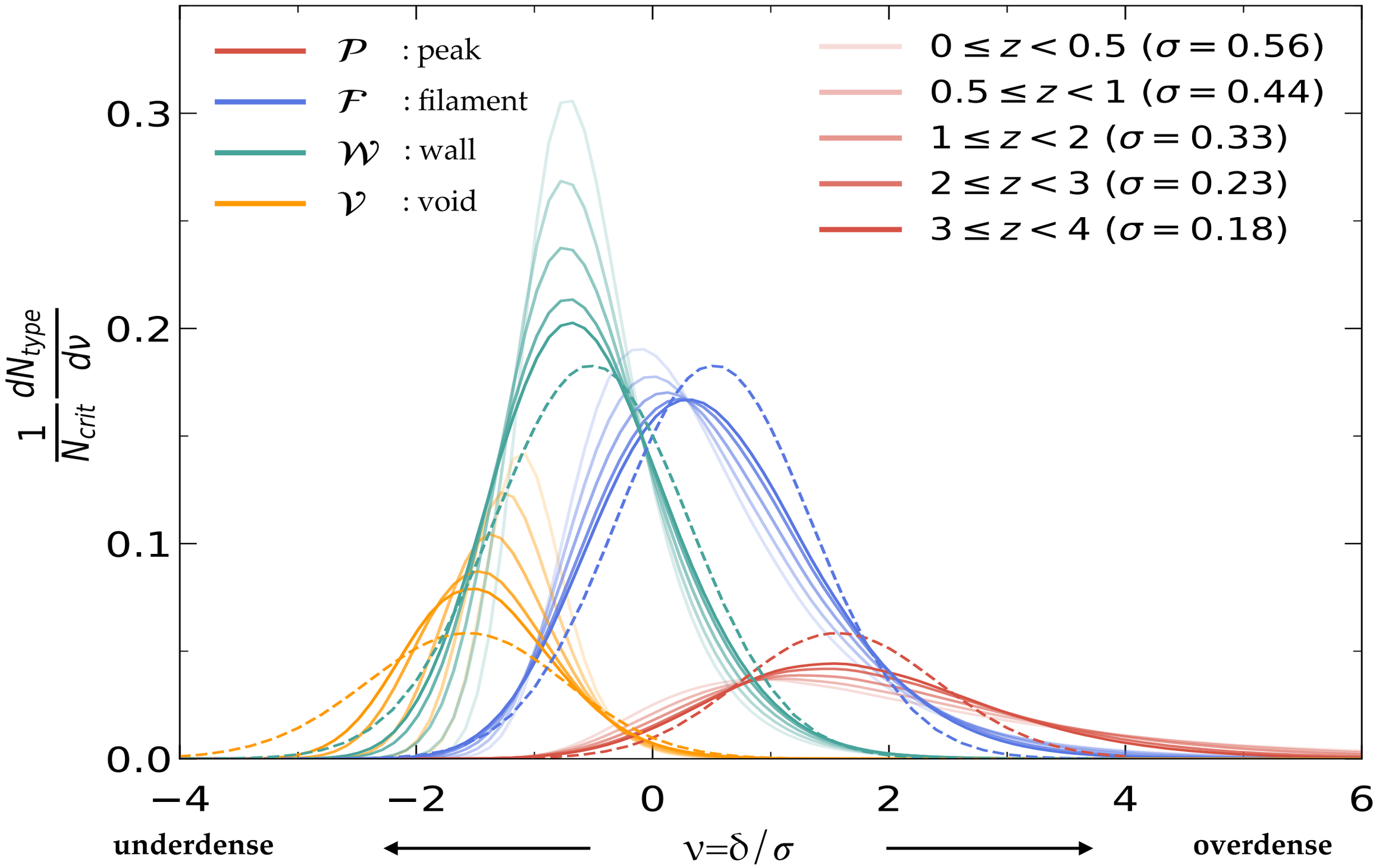
- Ω_b=0.04, Ω_m=0.24 Ω_Λ=0.76
- h=0.7, n_s=1, σ_8=0.8

CPs and LSSs

(in this setup)

- R_G = 6 Mpc/h (~$10^{15} M_\odot/h$)
- At z=0, highest 5% peaks can be associated with collapsed halos (M~$10^{15} M_\odot/h$).
- Different types of CPs are proxies of LSSs of the same kind (~$10^{15} M_\odot/h$).
- At higher redshifts, CPs track the progenitors of these structures.

Relative number counts of CPs



2pCFs of CPs

1) Four auto-correlation functions

2) Six cross-correlation functions

Threshold ν for CPs

more non-linear

Most overdense
Peaks &
Filaments

Most
underdense
Walls & Voids

type	rarity	$0 \le z < 0.5$ ($\sigma = 0.56$)	$0.5 \le z < 1$ ($\sigma = 0.44$)	$1 \le z < 2$ ($\sigma = 0.33$)	$2 \le z < 3$ ($\sigma = 0.23$)	$3 \le z < 4$ ($\sigma = 0.18$)	$z \to \infty$ ($\sigma \to 0$)
$\mathcal{P}$	20%	3.43	3.28	3.08	2.85	2.73	2.30
	15%	3.98	3.73	3.45	3.15	2.99	2.46
	10%	4.74	4.36	3.95	3.55	3.33	2.67
	5%	6.09	5.42	4.77	4.18	3.87	2.98
$\mathcal{F}$	20%	1.13	1.19	1.23	1.25	1.25	1.21
	15%	1.39	1.44	1.46	1.46	1.45	1.37
	10%	1.76	1.79	1.78	1.74	1.71	1.57
	5%	2.40	2.36	2.29	2.19	2.12	1.87
$\mathcal{W}$	20%	-0.90	-0.97	-1.04	-1.10	-1.14	-1.21
	15%	-0.96	-1.05	-1.13	-1.21	-1.26	-1.37
	10%	-1.03	-1.14	-1.24	-1.35	-1.41	-1.57
	5%	-1.12	-1.26	-1.39	-1.53	-1.62	-1.87
$\mathcal{V}$	20%	-1.24	-1.41	-1.60	-1.79	-1.91	-2.30
	15%	-1.27	-1.46	-1.66	-1.88	-2.01	-2.46
	10%	-1.31	-1.51	-1.74	-1.98	-2.13	-2.67
	5%	-1.37	-1.59	-1.85	-2.13	-2.30	-2.98

rarer

Rarity Dependence

Redshift Evolution

Auto-correlations of CPs

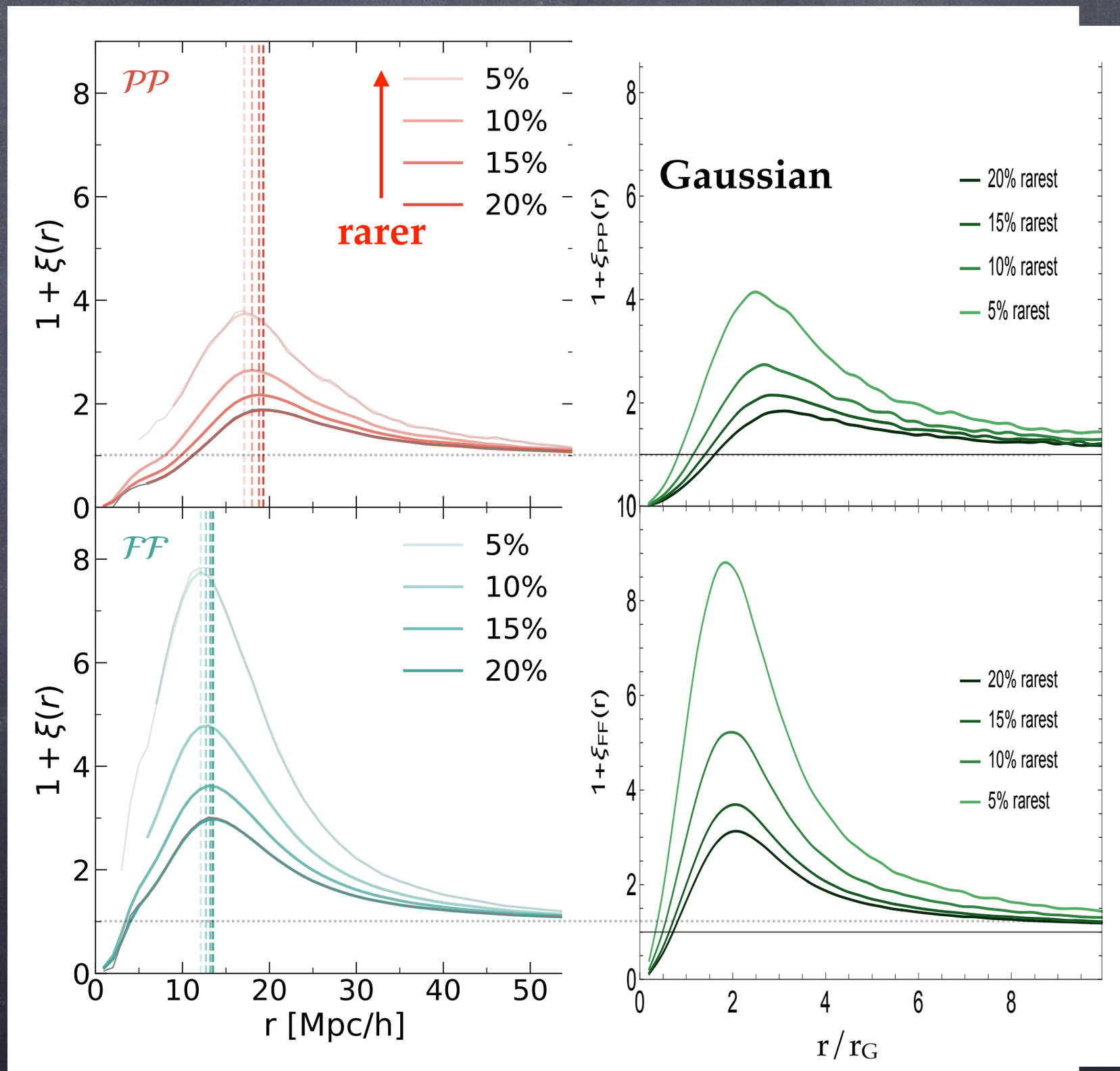
$$1+\xi_{ij}(r) = \frac{\langle C_i C_j \rangle}{\sqrt{\langle C_i R_j \rangle \langle C_j R_i \rangle}} \sqrt{\frac{N_{R_i} N_{R_j}}{N_{C_i} N_{C_j}}}$$

(Davis & Peebles 83)

1) Enhanced clustering

2) Decreasing maximum correlation radius
(Baldauf+16, Baldauf+20)

3) Narrower anti-clustering region (1+ξ<1)

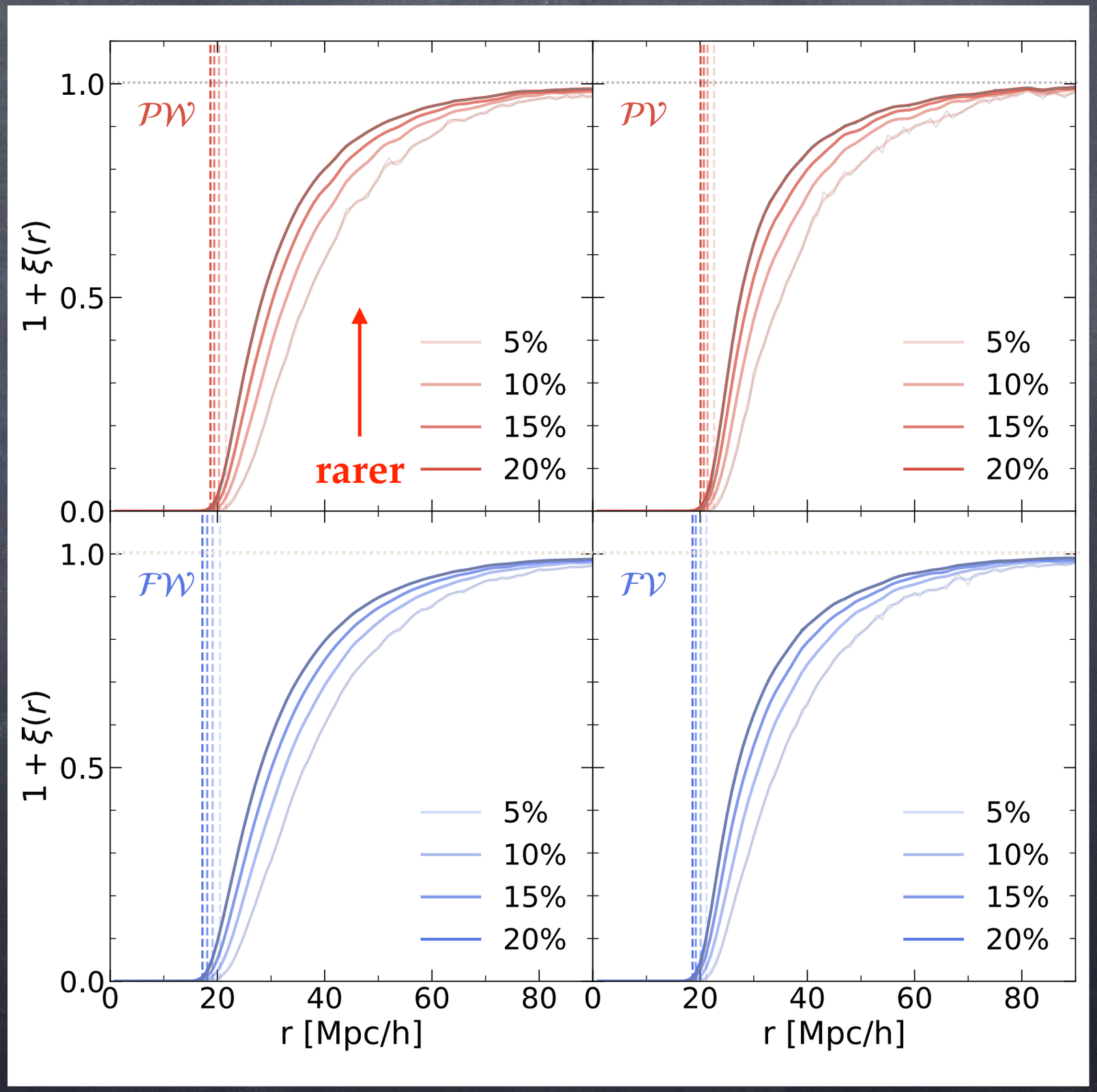


Cross-correlations of CPs

1) Anti-clustering ($1+\xi<1$)

2) Existence of exclusion zone ($1+\xi=0$)

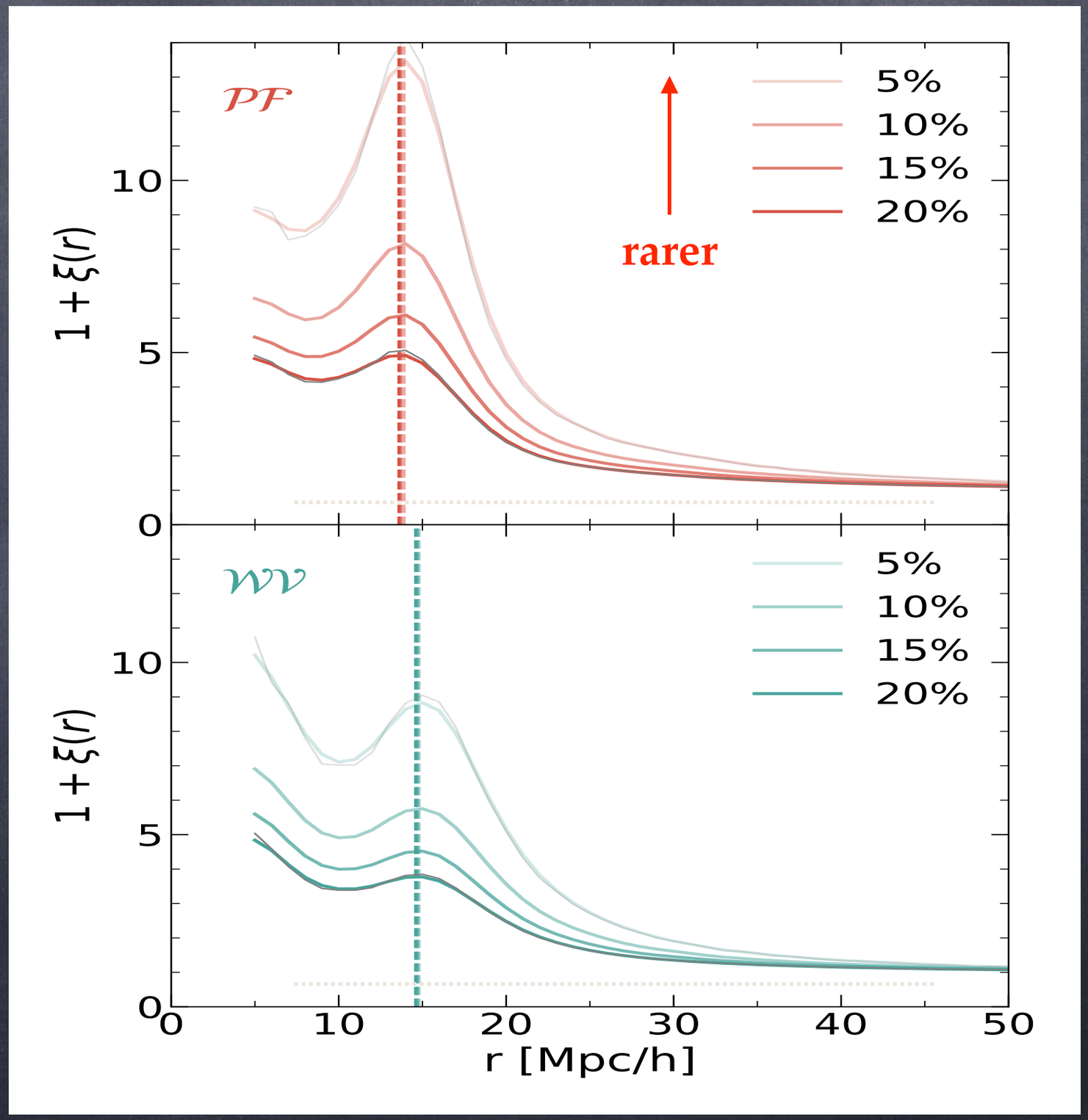
3) Monotonic increase without maximum



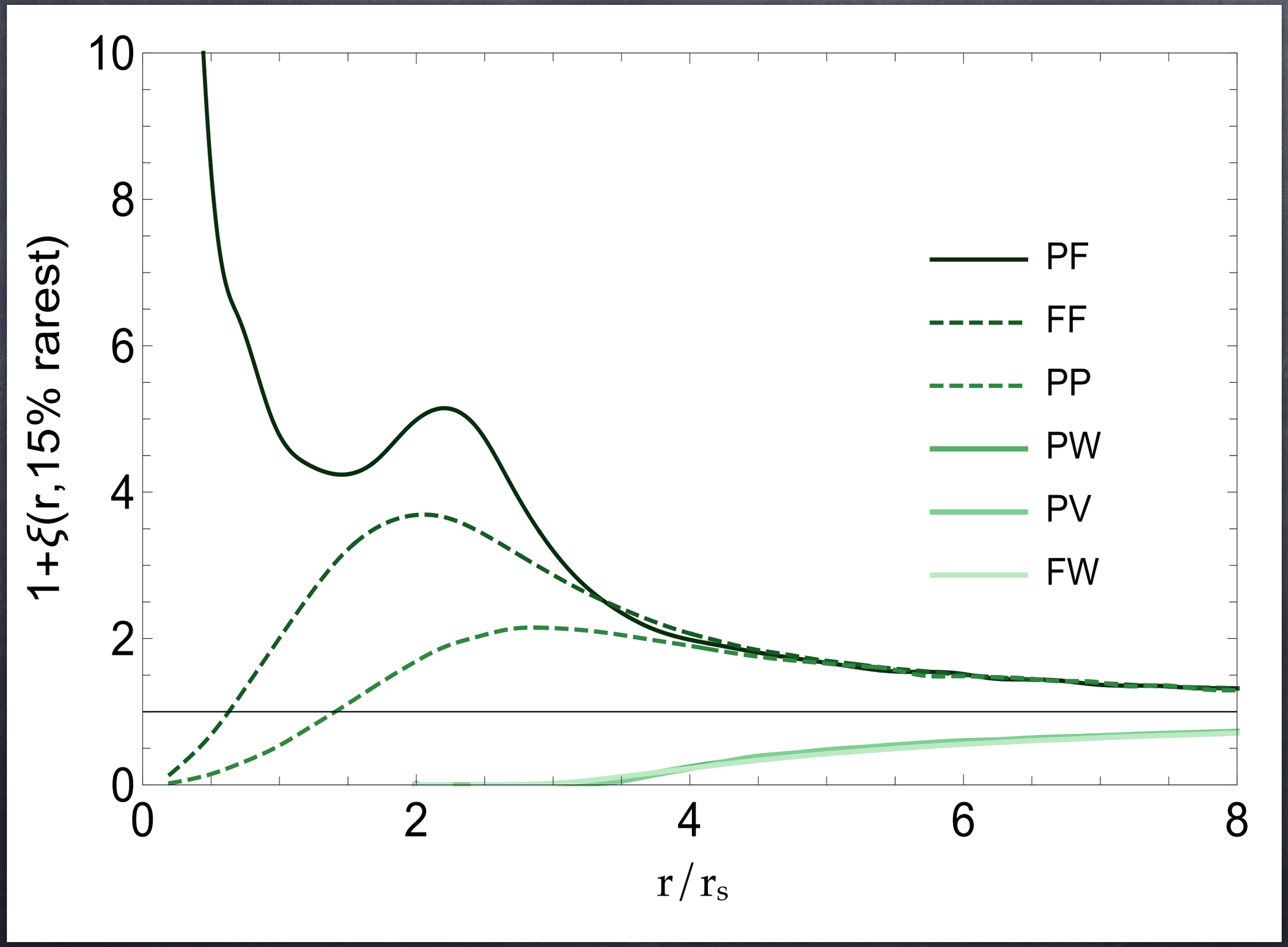
Cross-correlations of CPs

1) Diverging feature at r→0
(critical event; Cadiou+20)

2) Appearance of local maximum



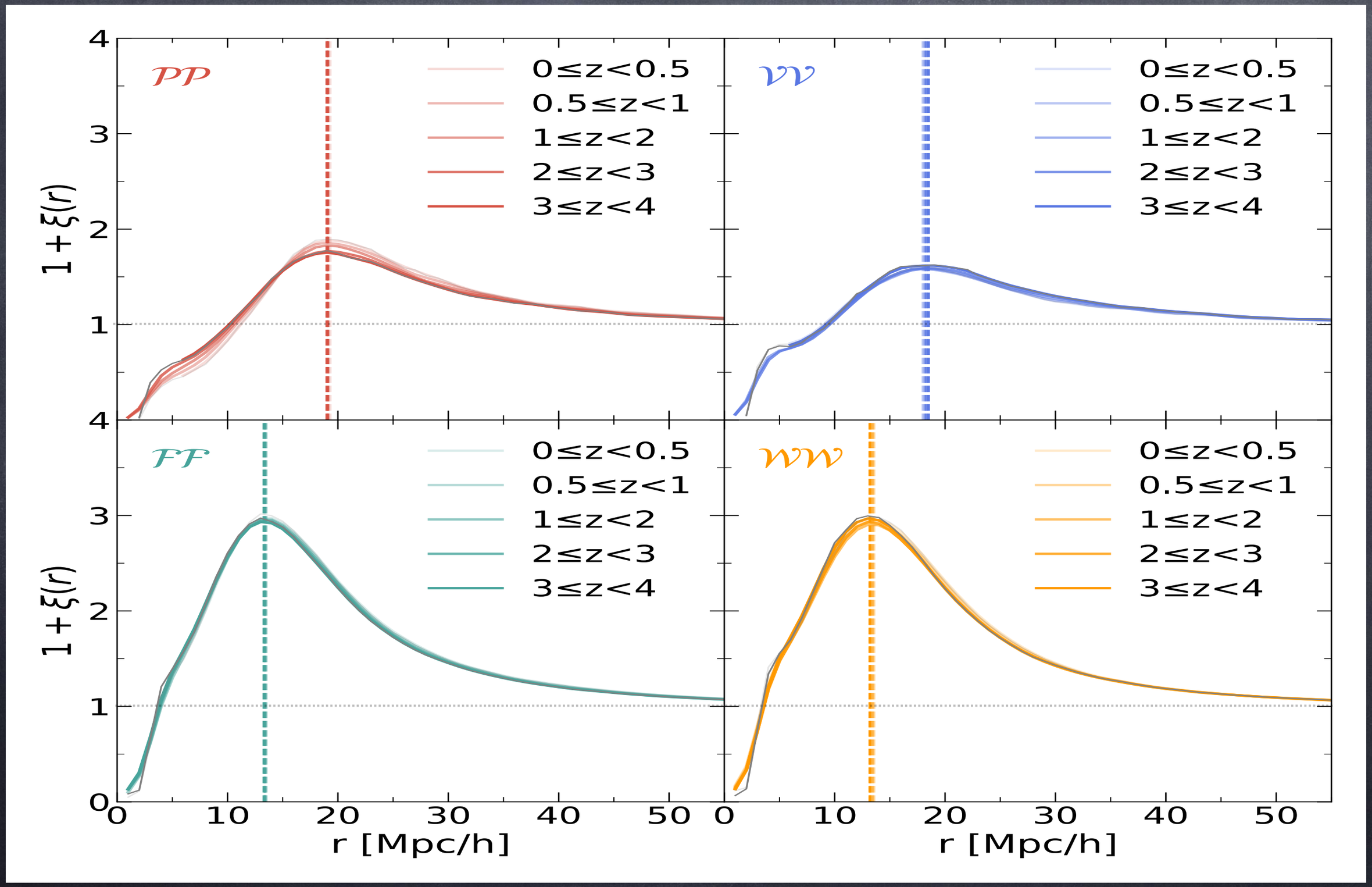
Gaussian predictions



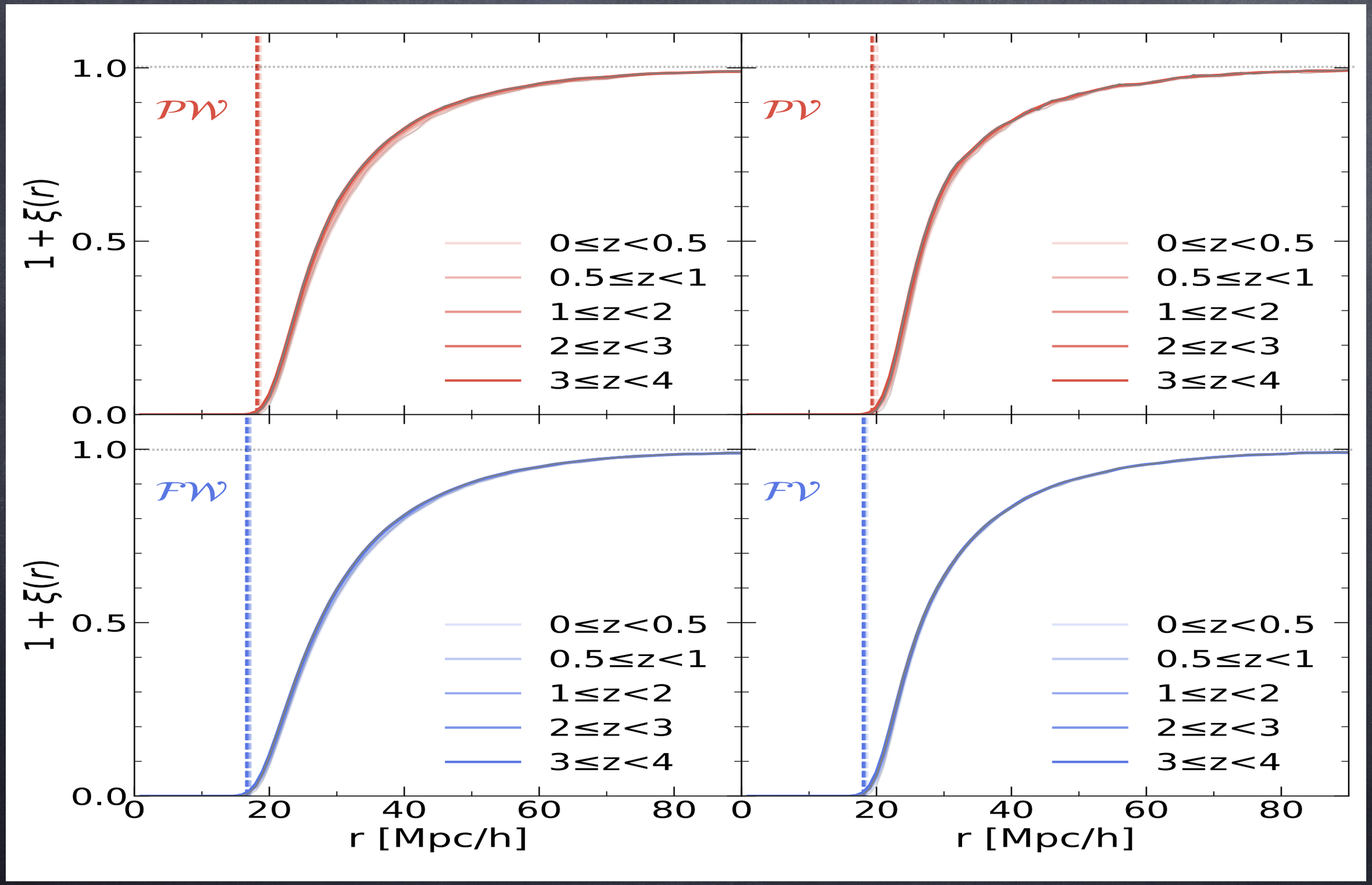
Rarity Dependence

Redshift Evolution

Auto-correlations of CPs



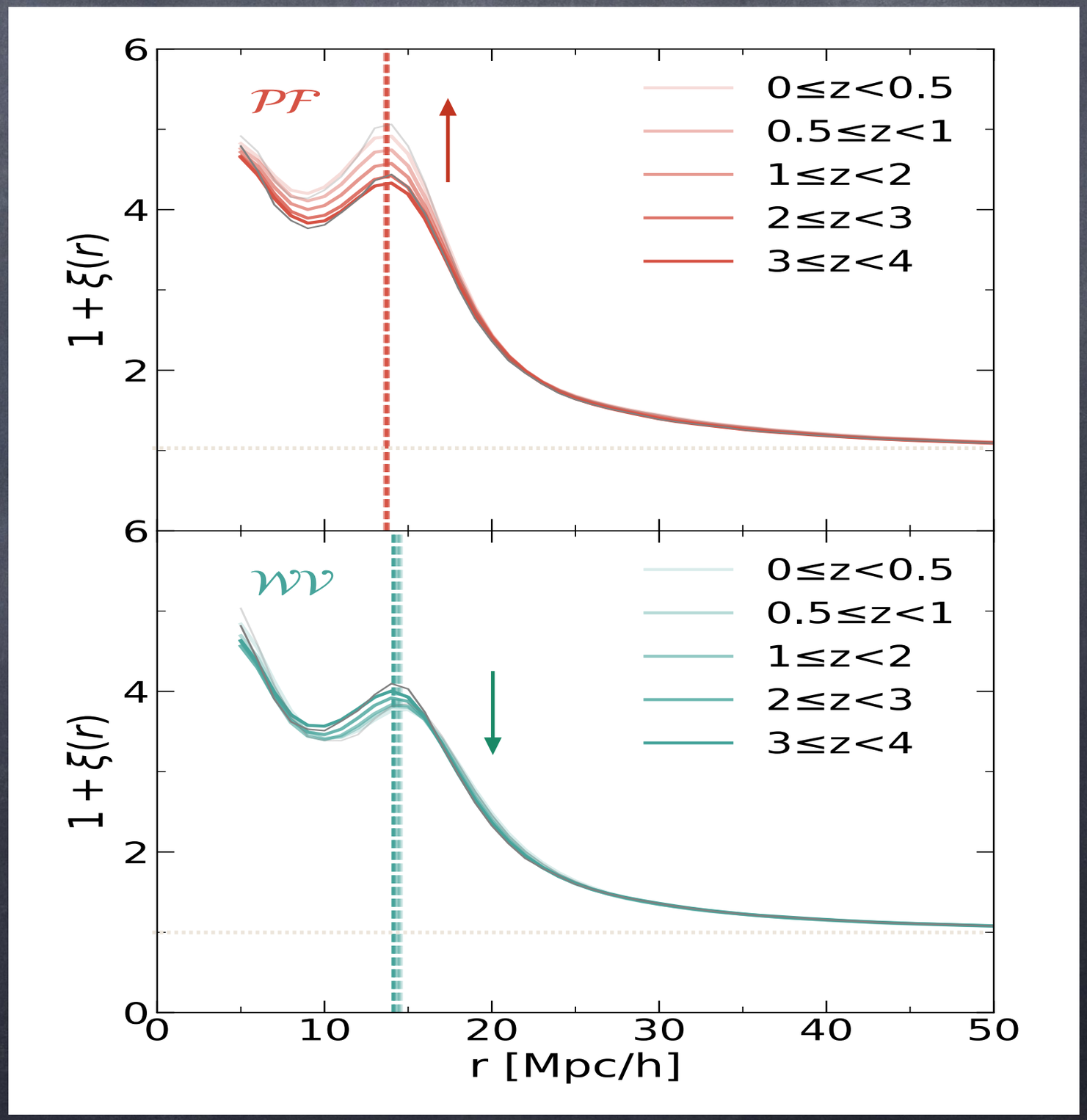
Cross-correlations of CPs



Cross-correlations of CPs

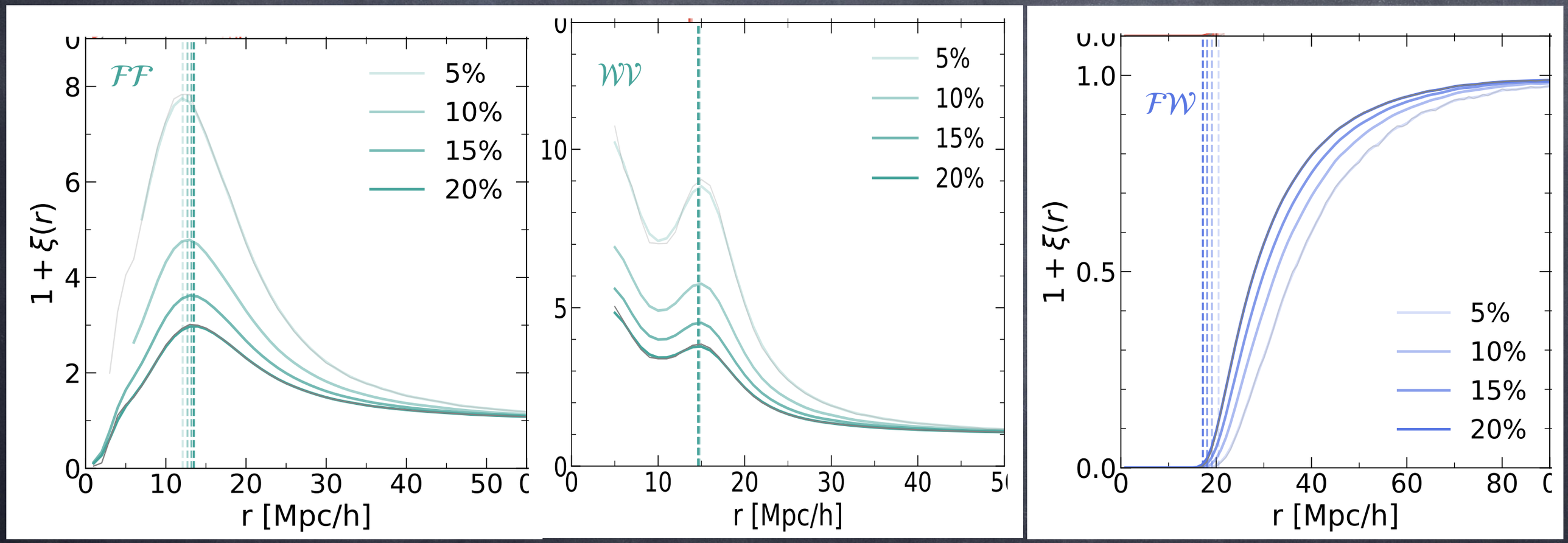
1) Growing local maximum for PF

2) Decaying local maximum for WV



Summary I

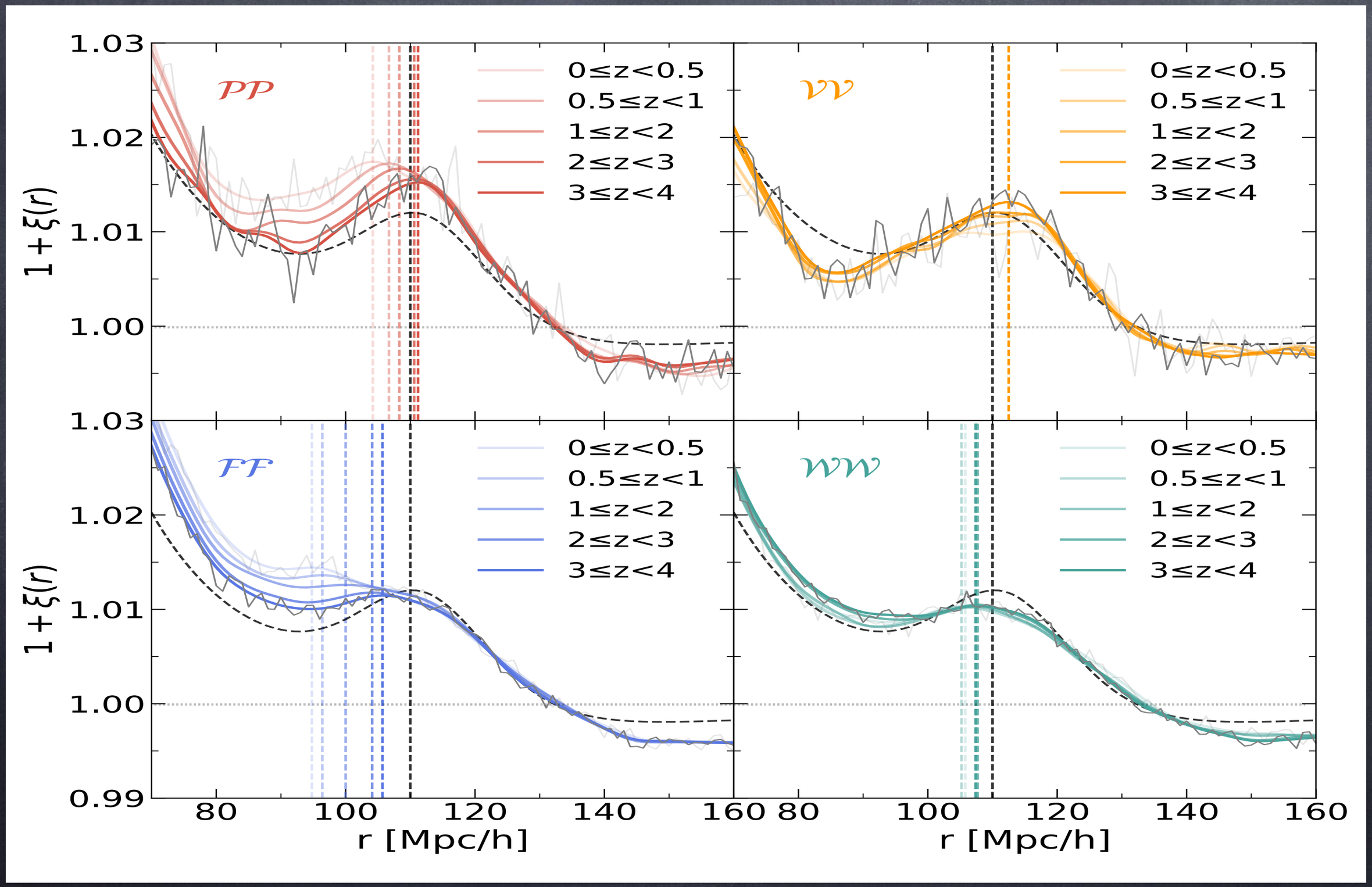
On small scales



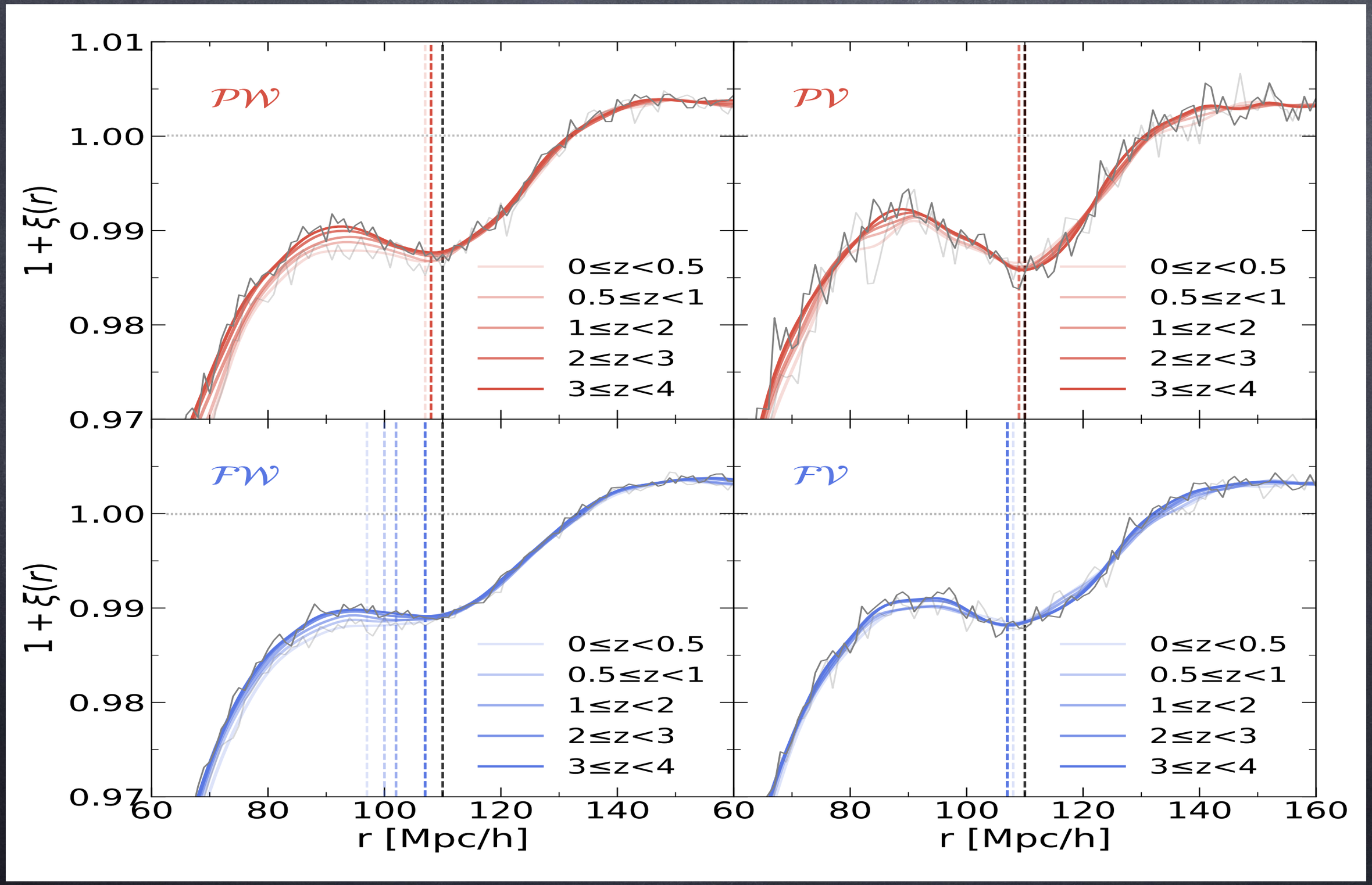
Maximum correlation

Exclusion zone

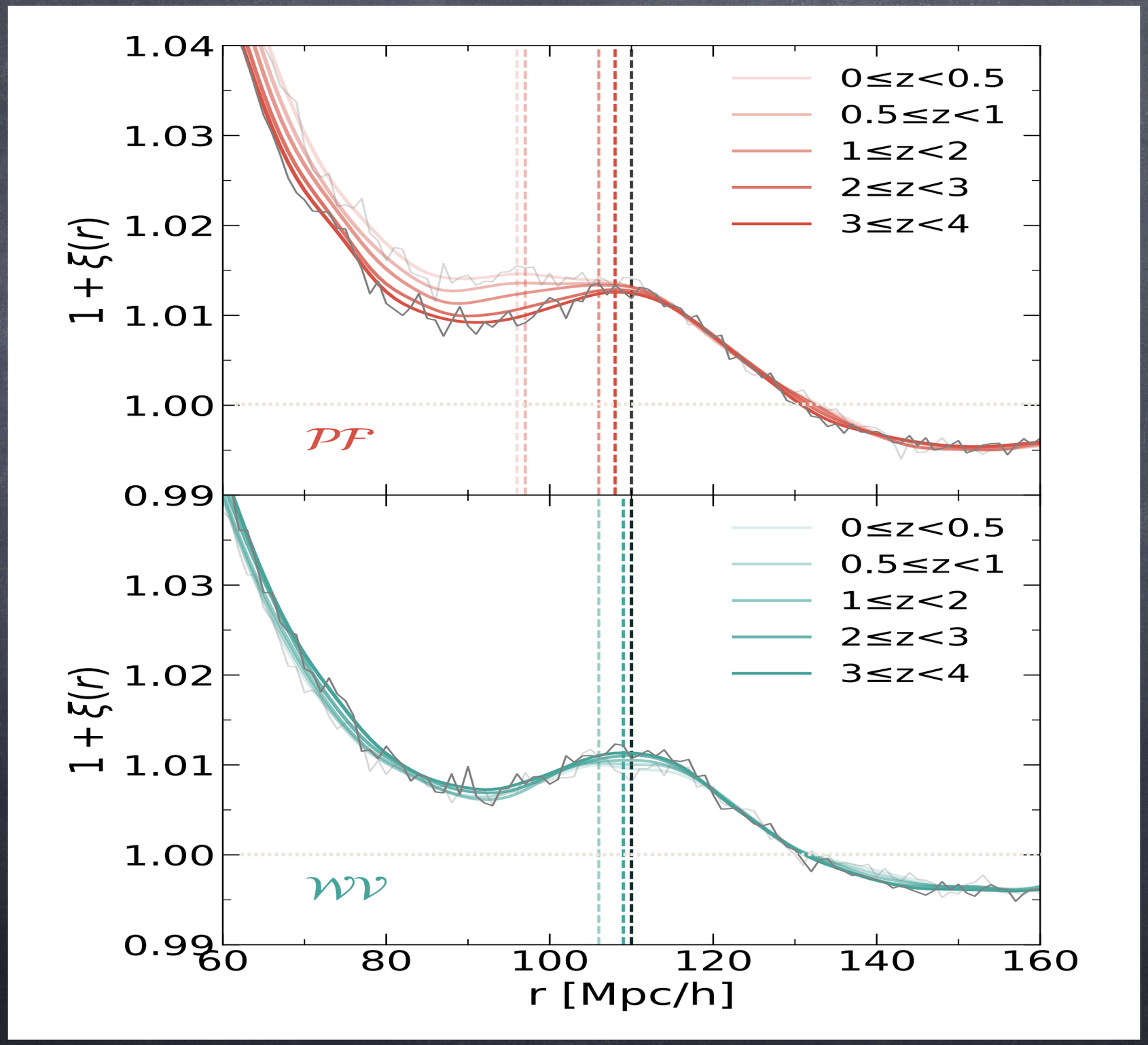
Auto-correlations of CPs



Cross-correlations of CPs

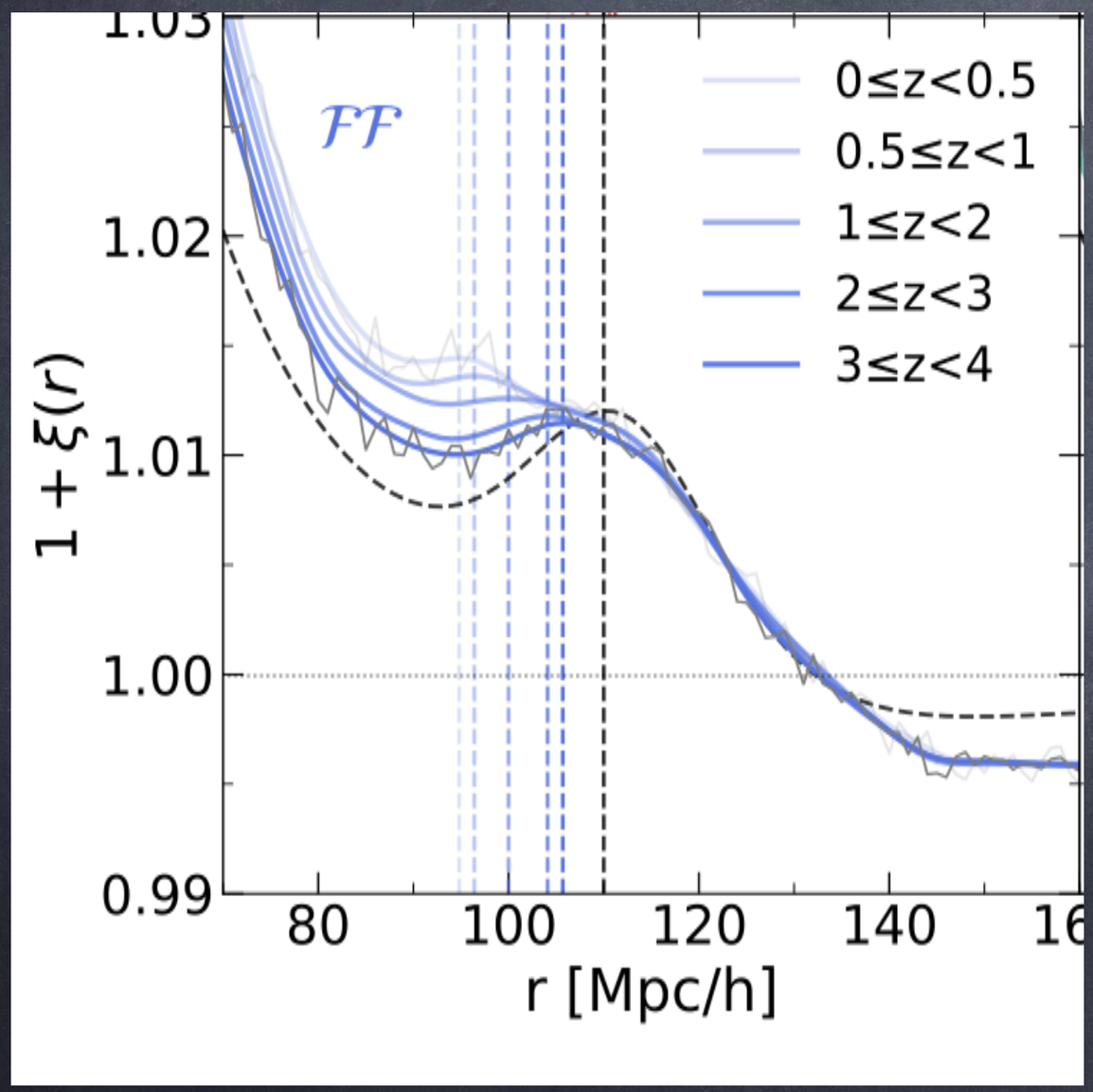


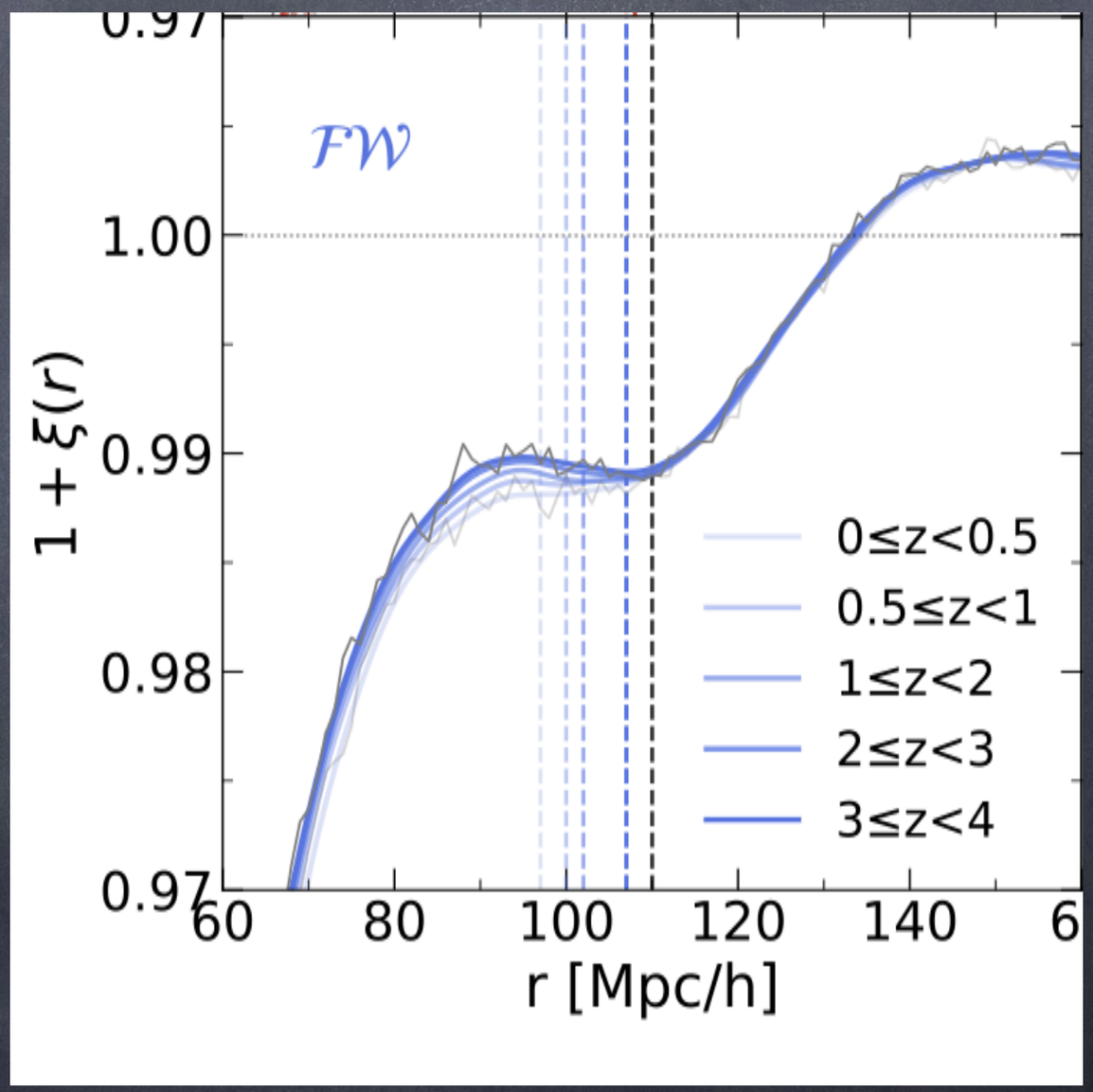
Cross-correlations of CPs



Summary II

On large scales





BAO bump

BAO dip

Spatial organization of critical points?

Anisotropic contribution to cross-correlation

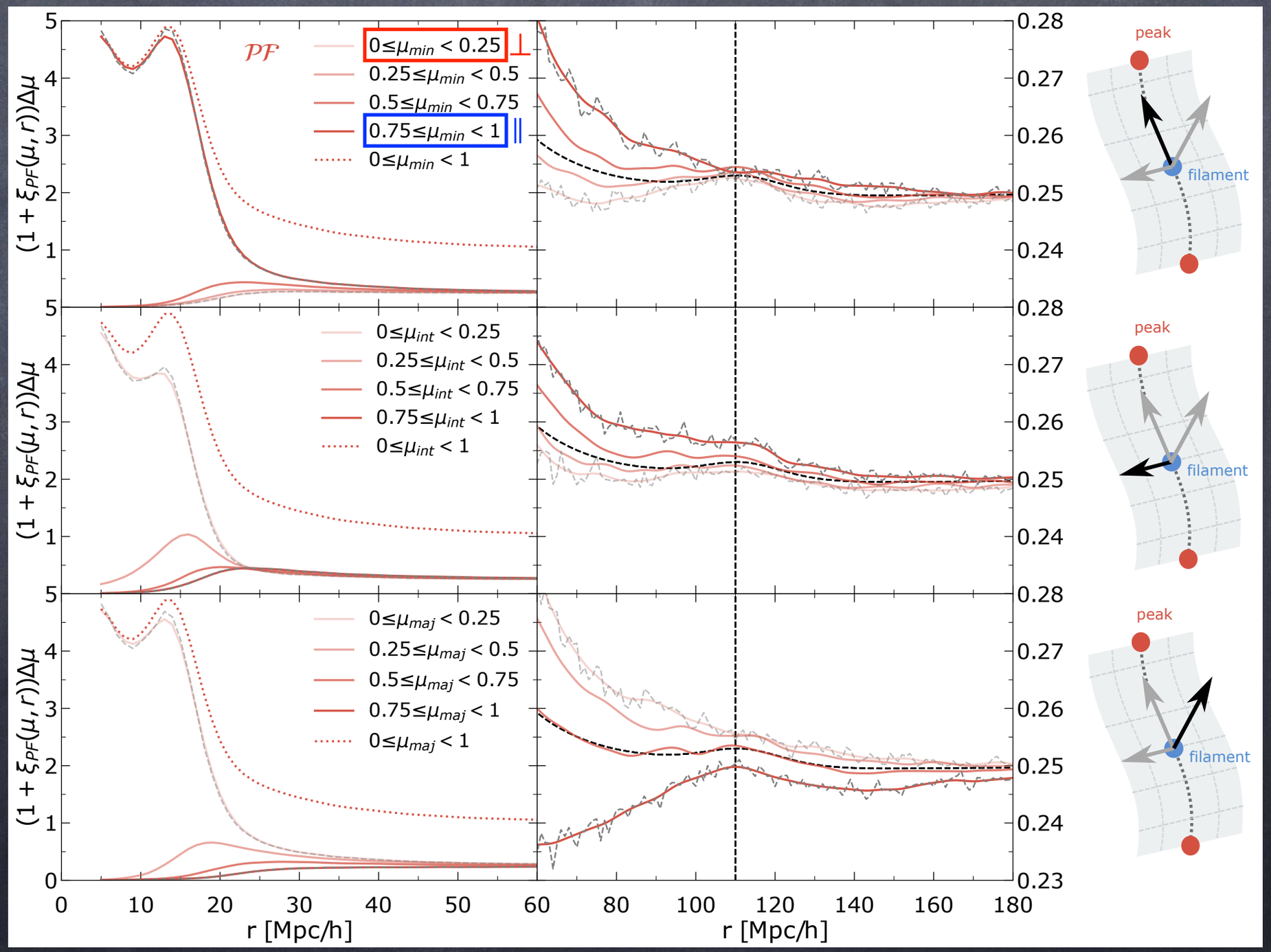
$(\lambda_1 < \lambda_2 < \lambda_3)$

Direction of increasing density ($\lambda_3 > 0$)

(Bond+96; Codis, Pichon & Pogosyan+15; Kraljic+18; Musso+18;)

$\lambda_2 < 0$

$\lambda_1 < 0$



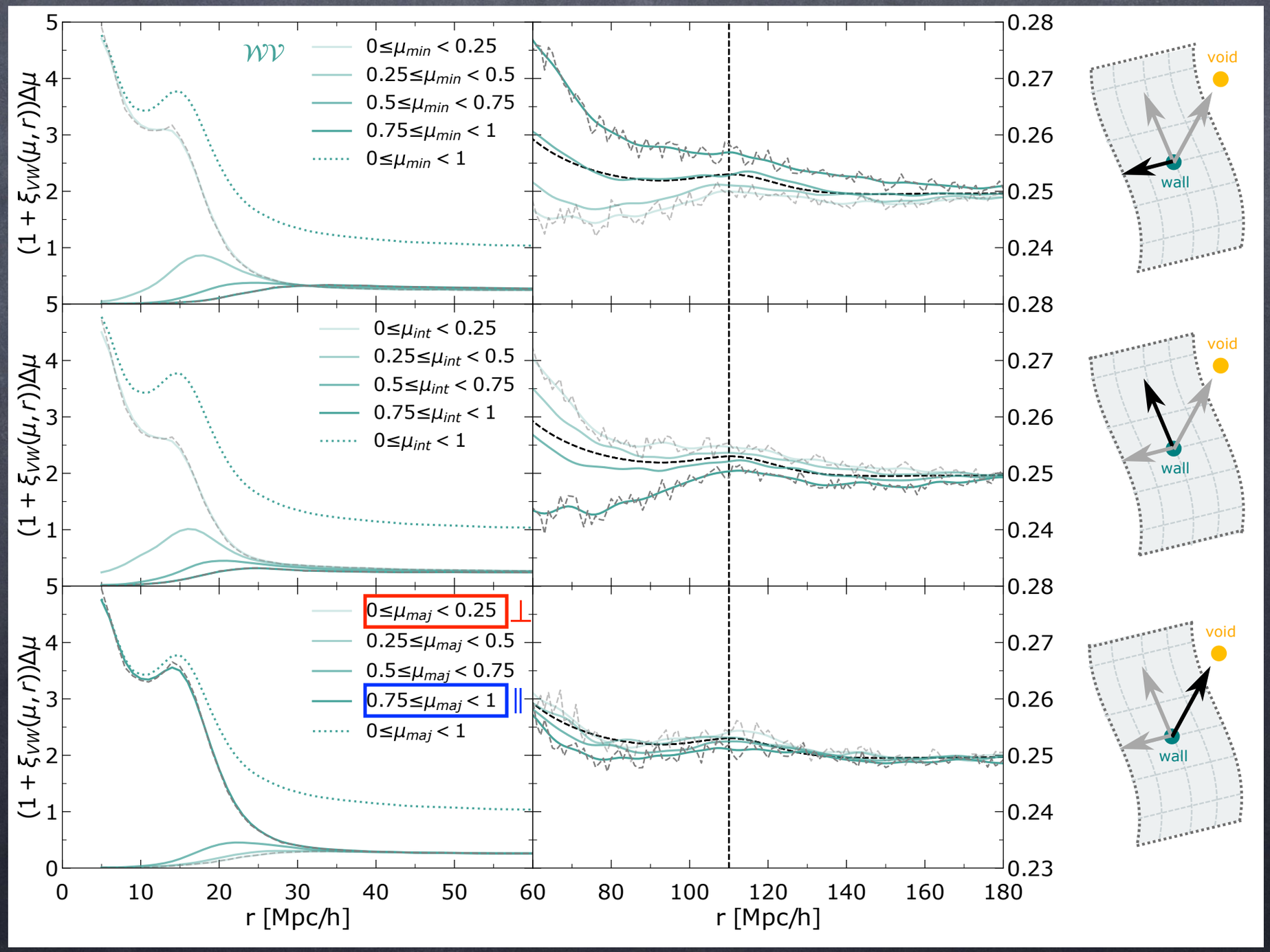
Anisotropic contribution to cross-correlation

$(\lambda_1 < \lambda_2 < \lambda_3)$

$\lambda_3 > 0$

$\lambda_2 > 0$

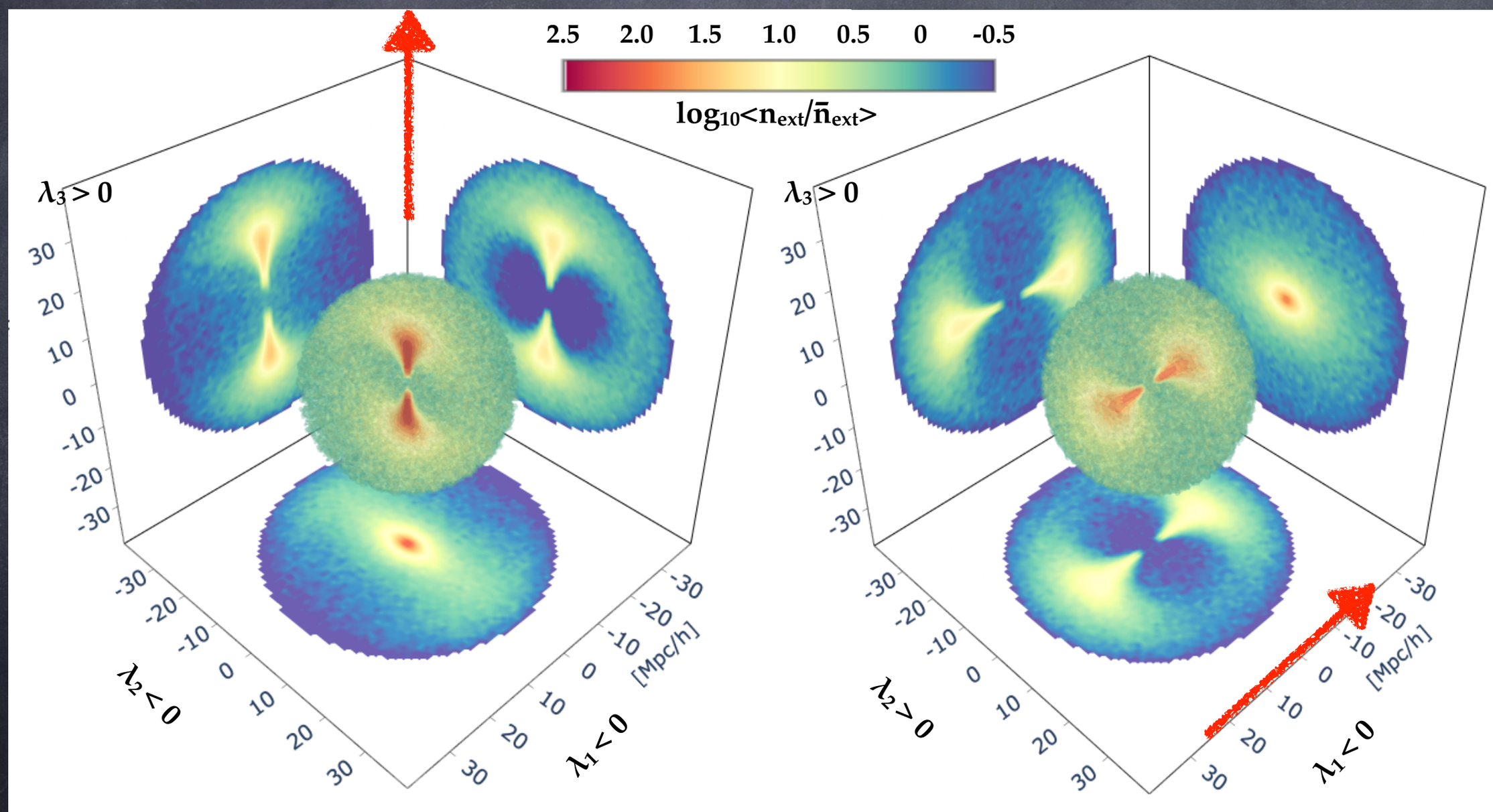
Direction of decreasing density $(\lambda_1 < 0)$



Anisotropic clustering of CPs

$$\frac{\langle n_{\rm ext}|{\rm sad}\rangle}{\overline{n}_{\rm ext}}(\mathbf{r}) \equiv \frac{\langle n_{\rm sad}(\mathbf{r}_{\rm sad})n_{\rm ext}(\mathbf{r}_{\rm sad}+\mathcal{R}\cdot\mathbf{r}))\rangle_{\mathbf{r}_{\rm sad}}}{\overline{n}_{\rm ext}\overline{n}_{\rm sad}}$$

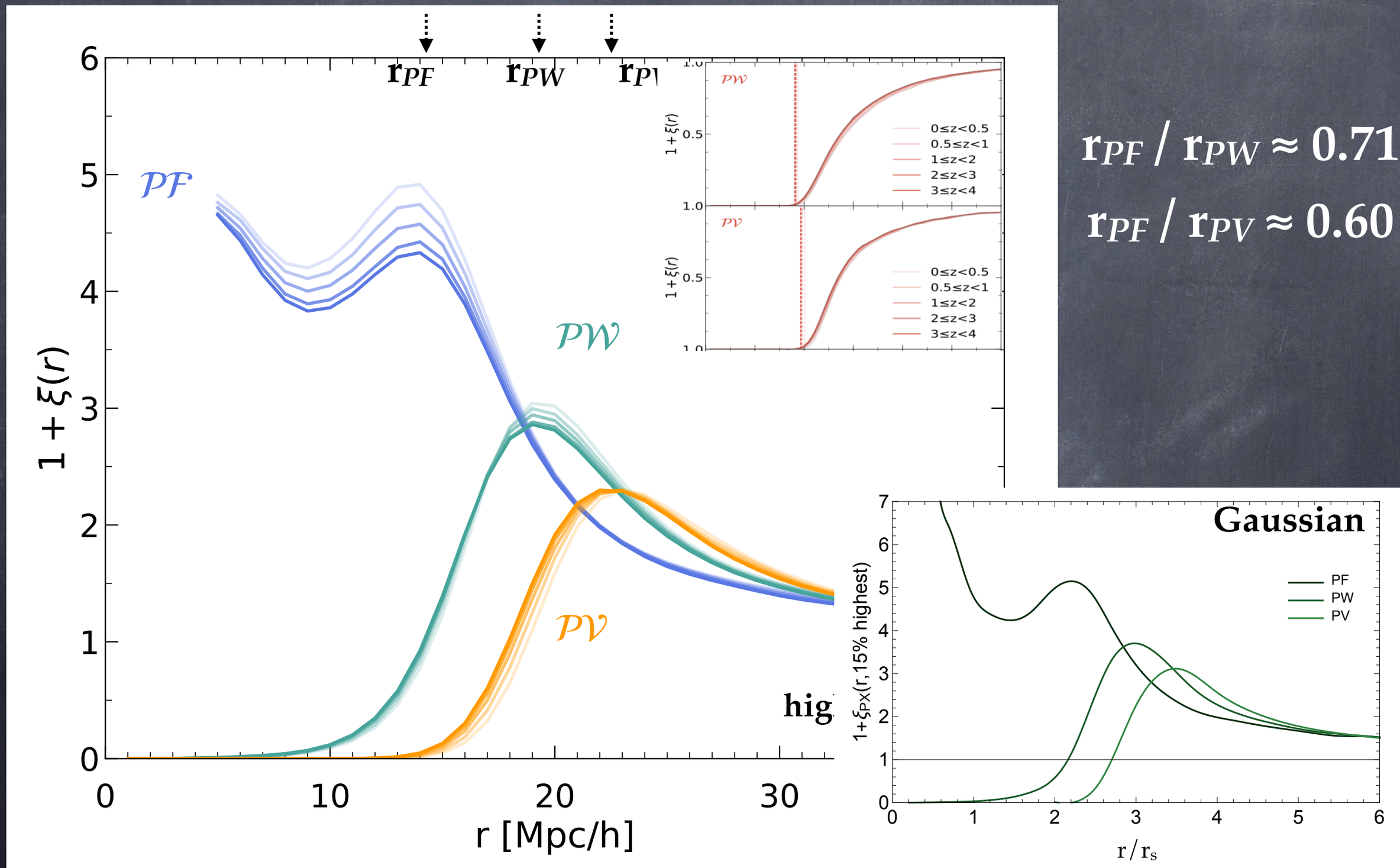
$(\lambda_1 < \lambda_2 < \lambda_3)$



Peaks around filaments

Voids around walls

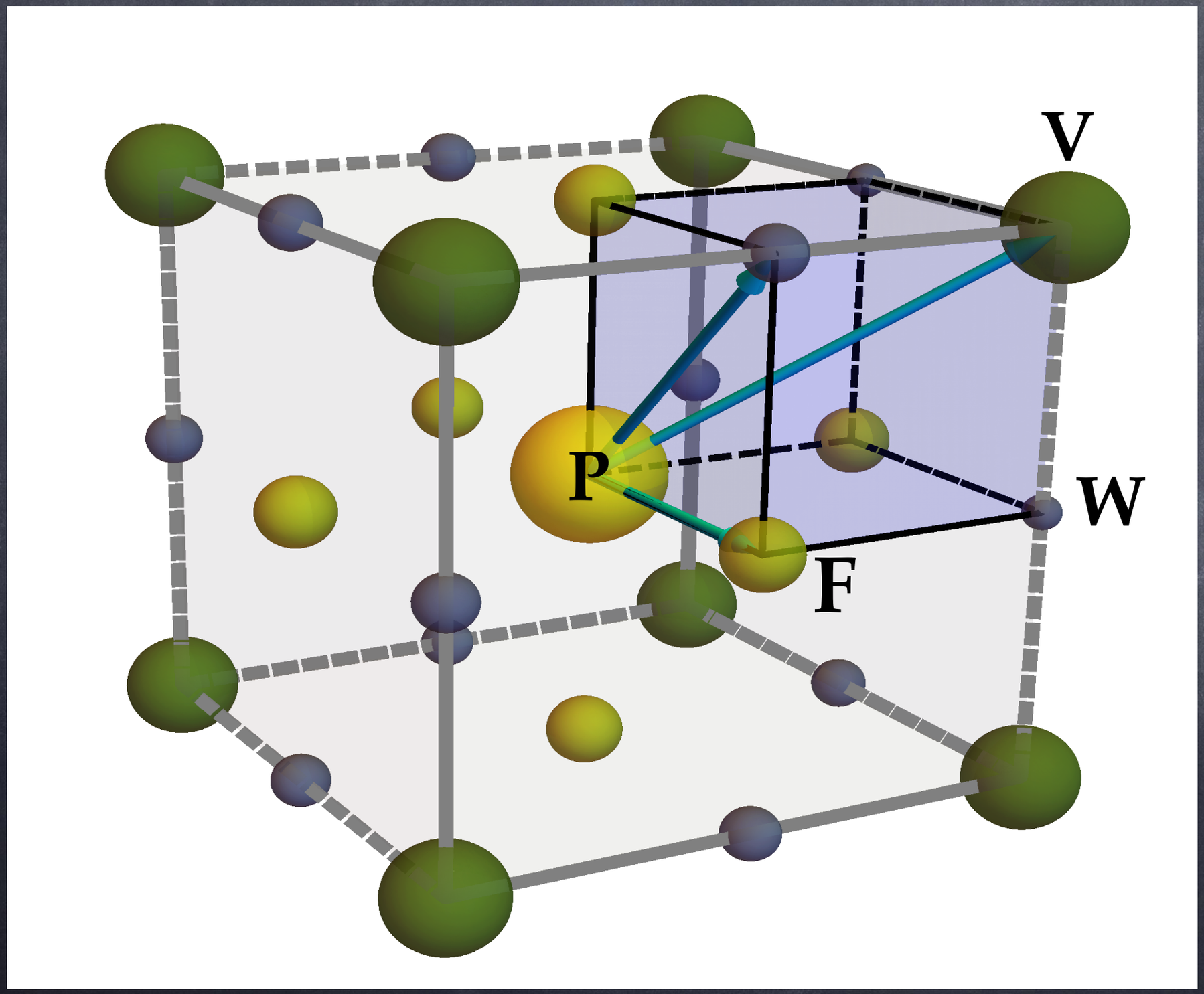
Radii at maximum correlations



$r_{PF} / r_{PW} \approx 0.71$

$r_{PF} / r_{PV} \approx 0.60$

Cosmic crystal structure



$r_{PF} / r_{PW} = 1/2^{0.5}$
$\simeq 0.71$

$r_{PF} / r_{PV} = 1/3^{0.5}$
$\simeq 0.58$

Summary

- CP clustering preserves the initial condition.
- CP clustering has characteristic clustering scales.