Learning about simulations in space and time

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January 26, 2023

1. Learning about simulations in space and time
2. A lopsided instability in dark-matter halos

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1. How to compare theoretical dynamical predictions with simulations?
2. How to find dynamics in simulations that you don't know about to start?

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A new approach to an answer:

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$\Rightarrow$ Let the simulation speak for itself by identifying its own dynamics!

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Relevance to us @ Cosmic Web 2023

1. BFE+mSSA is a useful idea in many simulation contexts
2. DM halos couple strongly from the outside in affecting the disk
3. Features excited in the halo persist for a very long time

- Construct a pair of functions (u_i^{lm}, d_i^{lm}) that satisfy Poisson $\nabla^2 u_i^{lm} = 4\pi G d_i^{lm}$ and are complete with the scalar product

$$-\frac{1}{4\pi G}\int dr\, r^2 u_i^{lm\,*}(r) d_j^{lm}(r) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise.} \end{cases}$$

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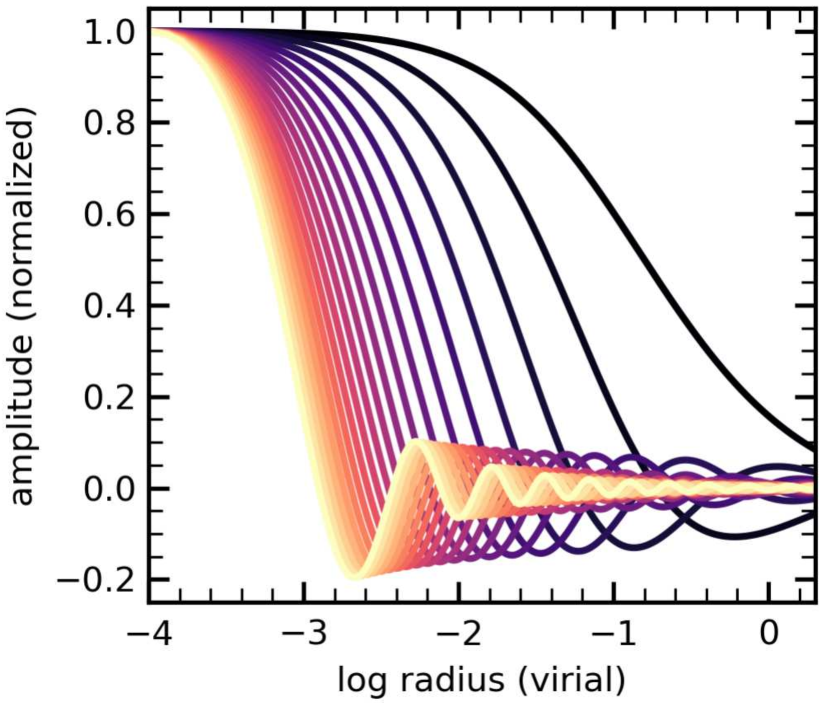

- Spherical basis eigenfunctions

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- The expansion coefficients are:

$$a_i^{lm} = -\frac{1}{4\pi G}\int_\Omega d\Omega Y_{lm}^*(\Omega) \int dr\, r^2 u_i^{lm\,*}(r)\rho(\mathbf{r})$$

- For pure points, the estimator is:

$$\tilde{a}_i^{lm} = -\frac{1}{4\pi G}\sum_{k=1}^{N} m_k Y_{lm}^*(\theta_k, \phi_k) u_i^{lm\,*}(r_k)$$

- Estimates of the total density and potential are:

$$\tilde{\Phi}(\mathbf{r}) = \sum_{lm} Y_{lm}(\theta,\phi) \sum_j \tilde{a}_j^{lm}(t) u_j^{lm}(r), \quad (1)$$

$$\tilde{\rho}(\mathbf{r}) = \sum_{lm} Y_{lm}(\theta,\phi) \sum_j \tilde{a}_j^{lm}(t) d_j^{lm}(r). \quad (2)$$

- An N-body simulation provides time series $\mathbf{a}_j^{lm}(t)$
- Gravitational potential energy is: $W = -\frac{1}{2}\sum_{lmj} |a_j^{lm}|^2$
- Can construct bases which look like any galaxy you want
- Can use any one set to get another via Gram-Schmidt

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[Golyandina & Zhigljavsky 2013]

How & why does this work?. . .

- Gravitational dynamics is a stochastic process that drives its evolution
- Describe the state of a galaxy using BFE: coefficients
- Karhunen–Loève theorem tells us that the temporal structure in coefficients can be represented by orthogonal functions

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Simulations can be converted orthogonal functions in space and time!

- Discover unknown dynamics and summarize known dynamics
- Enormous dimensional and data compression
  - 10 TB of phase space $\Longrightarrow$ 10 MB of functional data!

How to do it. . .

We embed the original time series into a sequence of lagged vectors:

$$A_i = (a_i, \ldots, a_{i+L-1})^\top, \quad i = 1 \ldots, K.$$

The trajectory matrix of the series A_N is

$$\mathbf{T} = [A_1 : \ldots : A_K]$$

$$= \begin{pmatrix} a_1 & a_2 & a_3 & \ldots & a_K \\ a_2 & a_3 & a_4 & \ldots & a_{K+1} \\ a_3 & a_4 & a_5 & \ldots & a_{K+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_L & a_{L+1} & a_{L+2} & \ldots & a_N \end{pmatrix}.$$

The rows and columns of $\mathbf{T}$ are subseries of the original series

- We can form the lag-covariance matrix:

$$\mathbf{C} = \frac{1}{K}\mathbf{T}^\top \cdot \mathbf{T}. \quad (3)$$

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$$\begin{aligned} \mathbf{\Lambda} &= \mathbf{E}^\top \cdot \mathbf{C} \cdot \mathbf{E} \qquad (4) \\ &= \frac{1}{K}\left(\mathbf{E}^\top \cdot \mathbf{T}\right) \cdot \left(\mathbf{E}^\top \cdot \mathbf{T}\right)^\top = \frac{1}{K}\mathbf{P} \cdot \mathbf{P}^\top \qquad (5) \end{aligned}$$

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- The columns of $\mathbf{P}$ are known as the principal components (uncorrelated, orthogonal by construction)
- Project the principle components back to the original basis and then diagonally average the result.

$$\tilde{\mathbf{T}}_k = \mathbf{P} \cdot \mathbf{E}^k \to \{\tilde{a}_j\}$$

Gaussian function $w = 5, N = 100, L = 20$

Time series is a Gaussian centered at $t = 20$ with width $\Delta t = 5$

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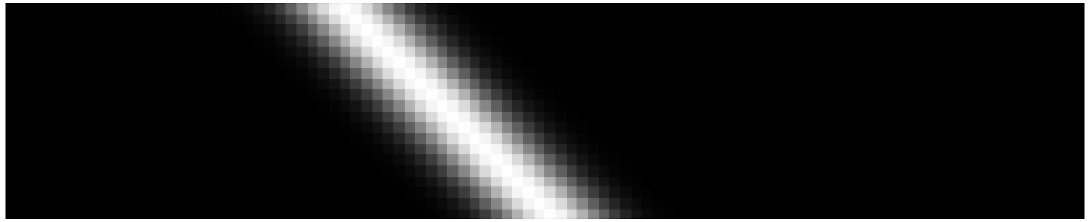
- The trajectory matrix is banded



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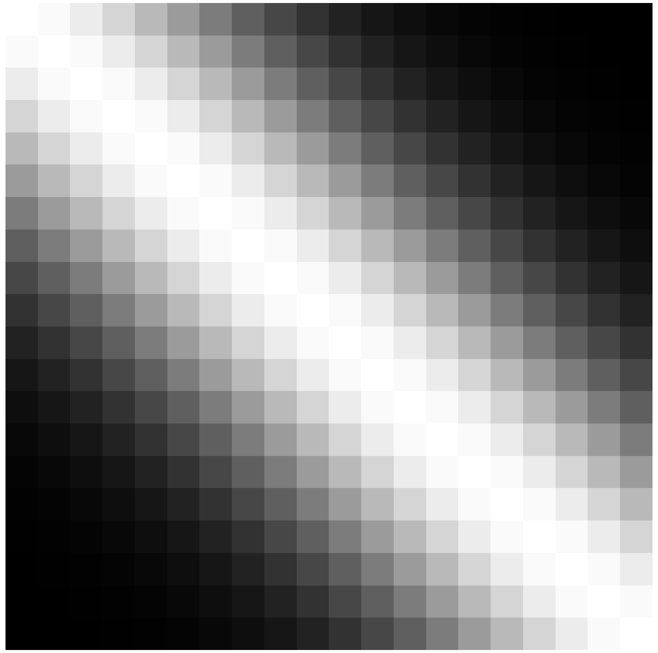

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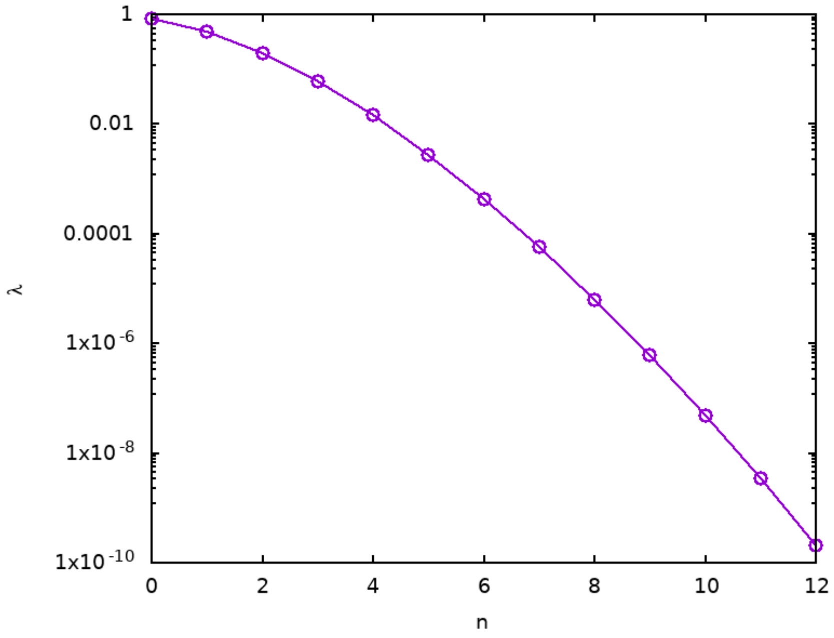

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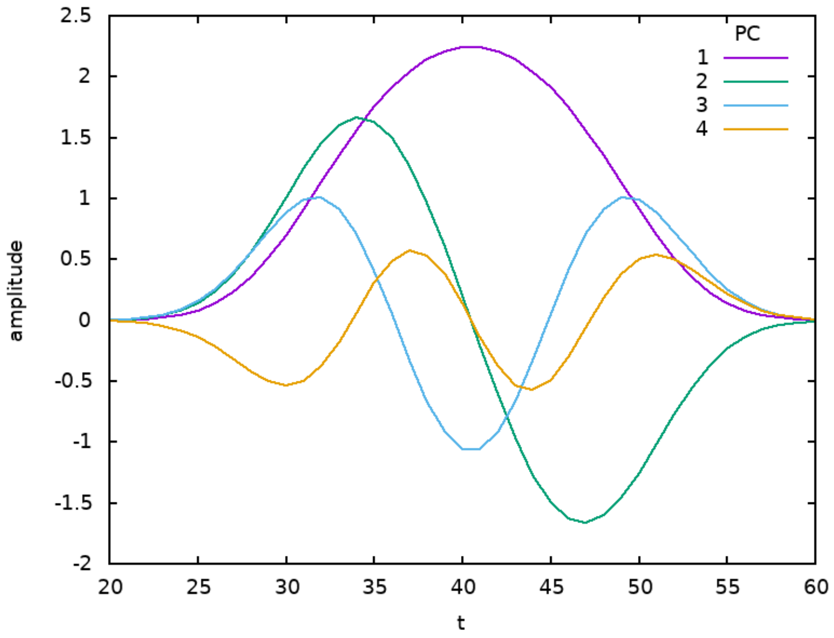
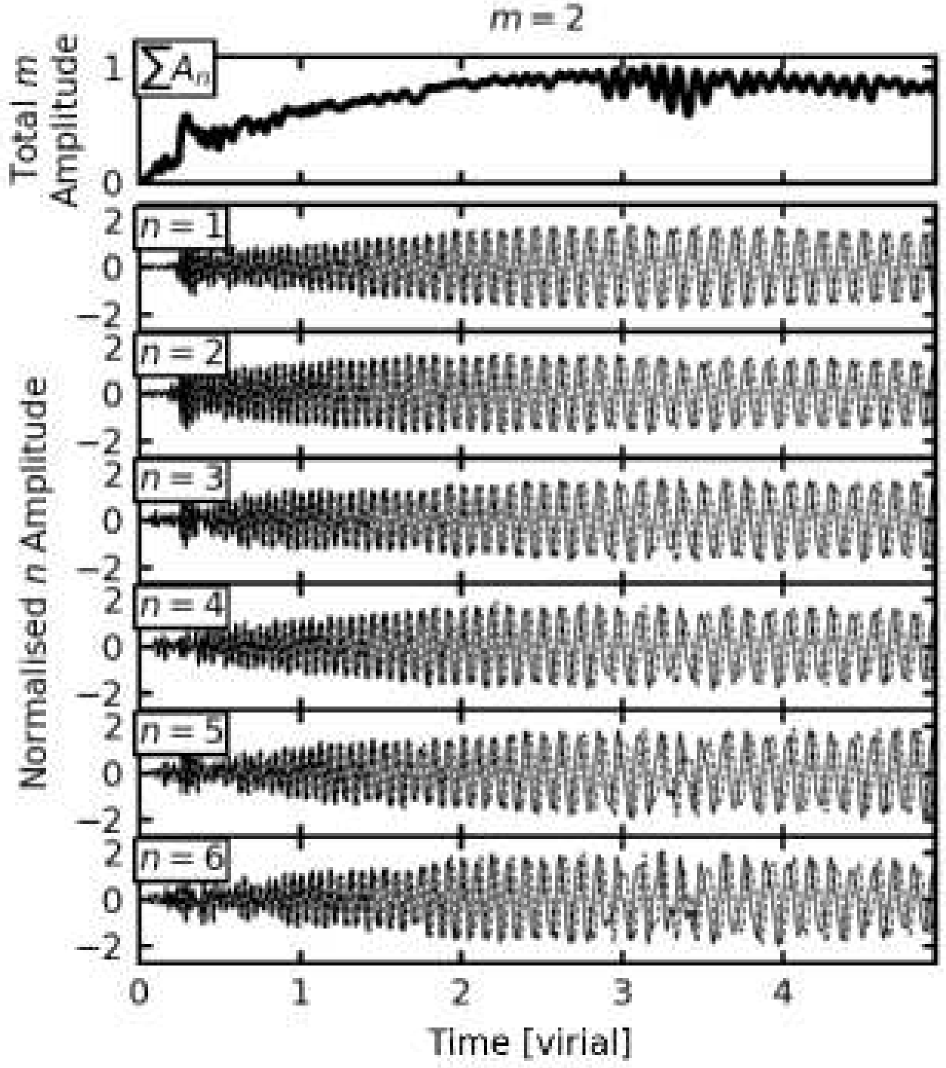
- Rapidly decreasing eigenvalues
- PCs



Mike Petersen's thesis simulation



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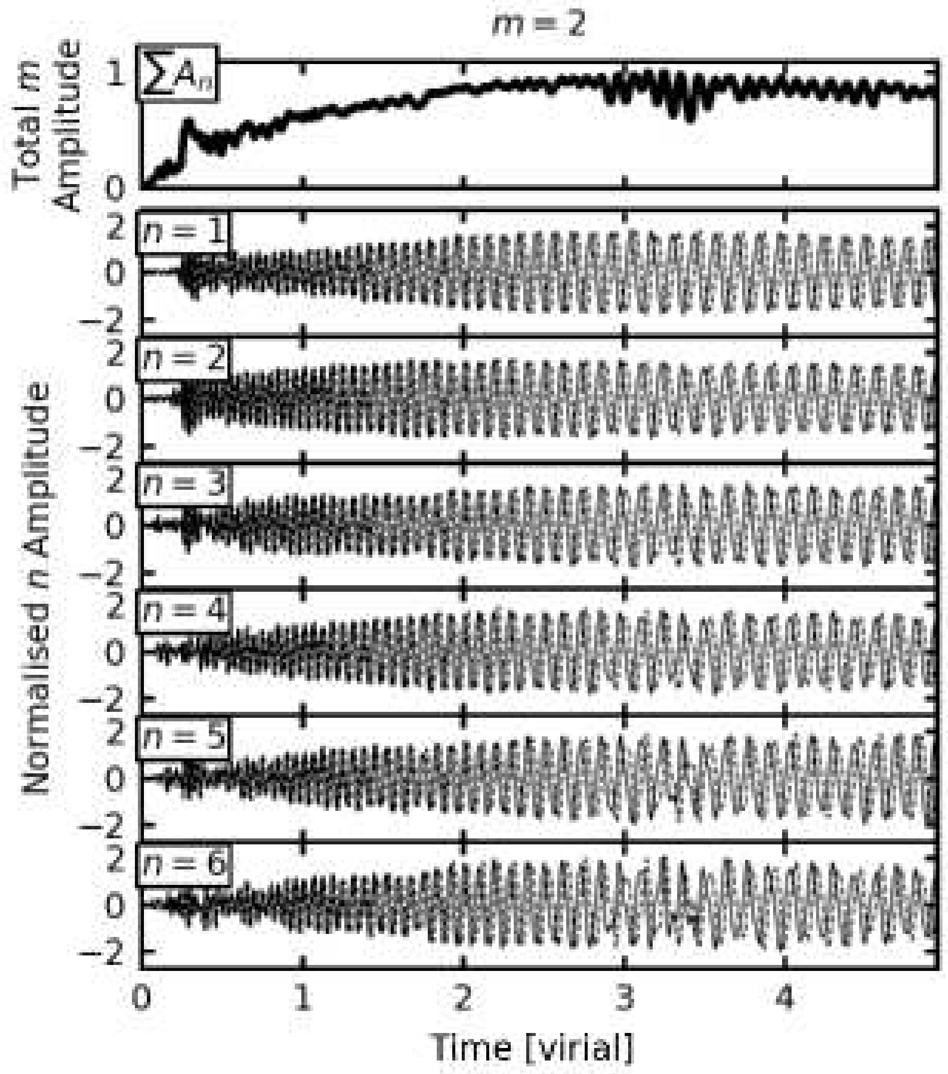
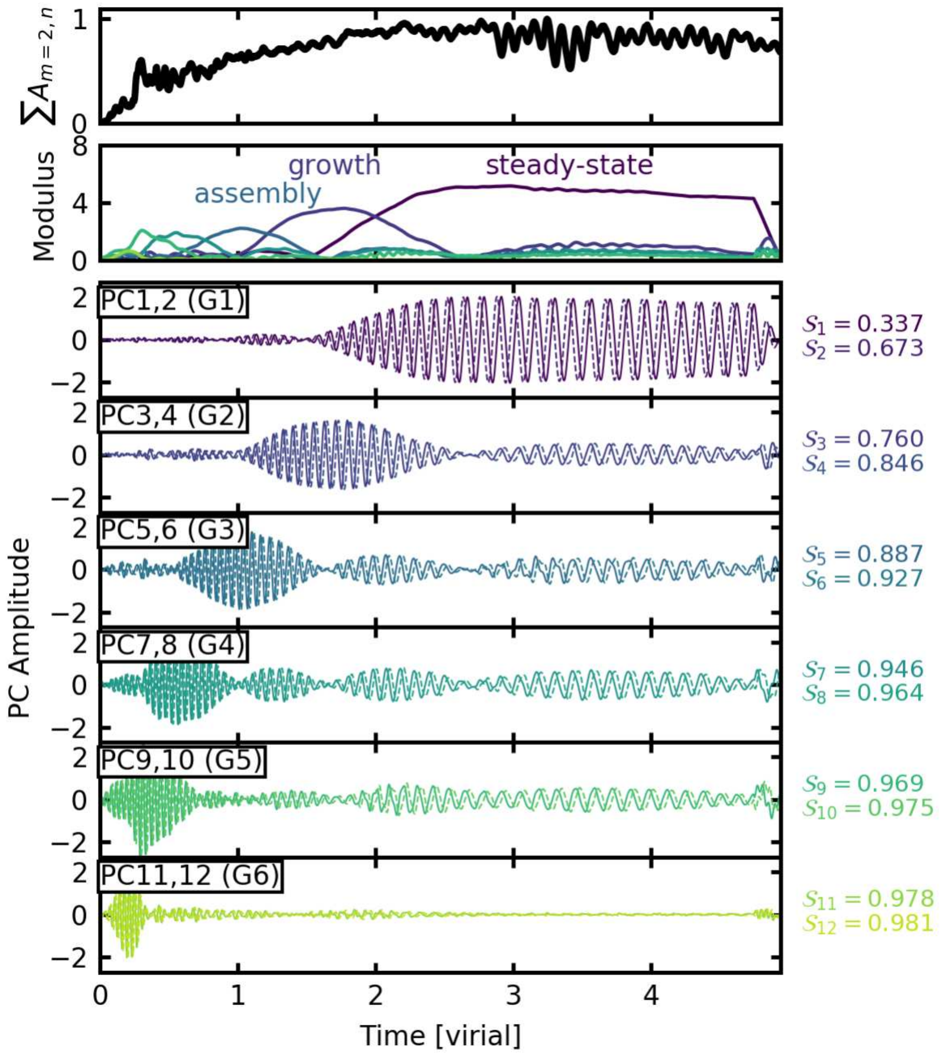
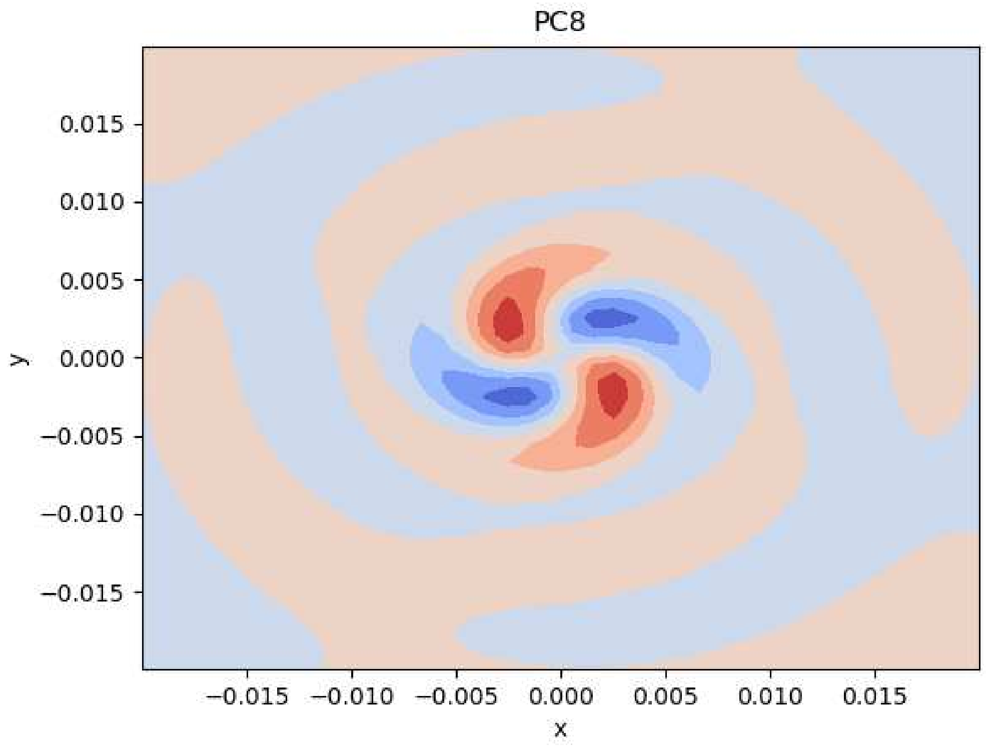


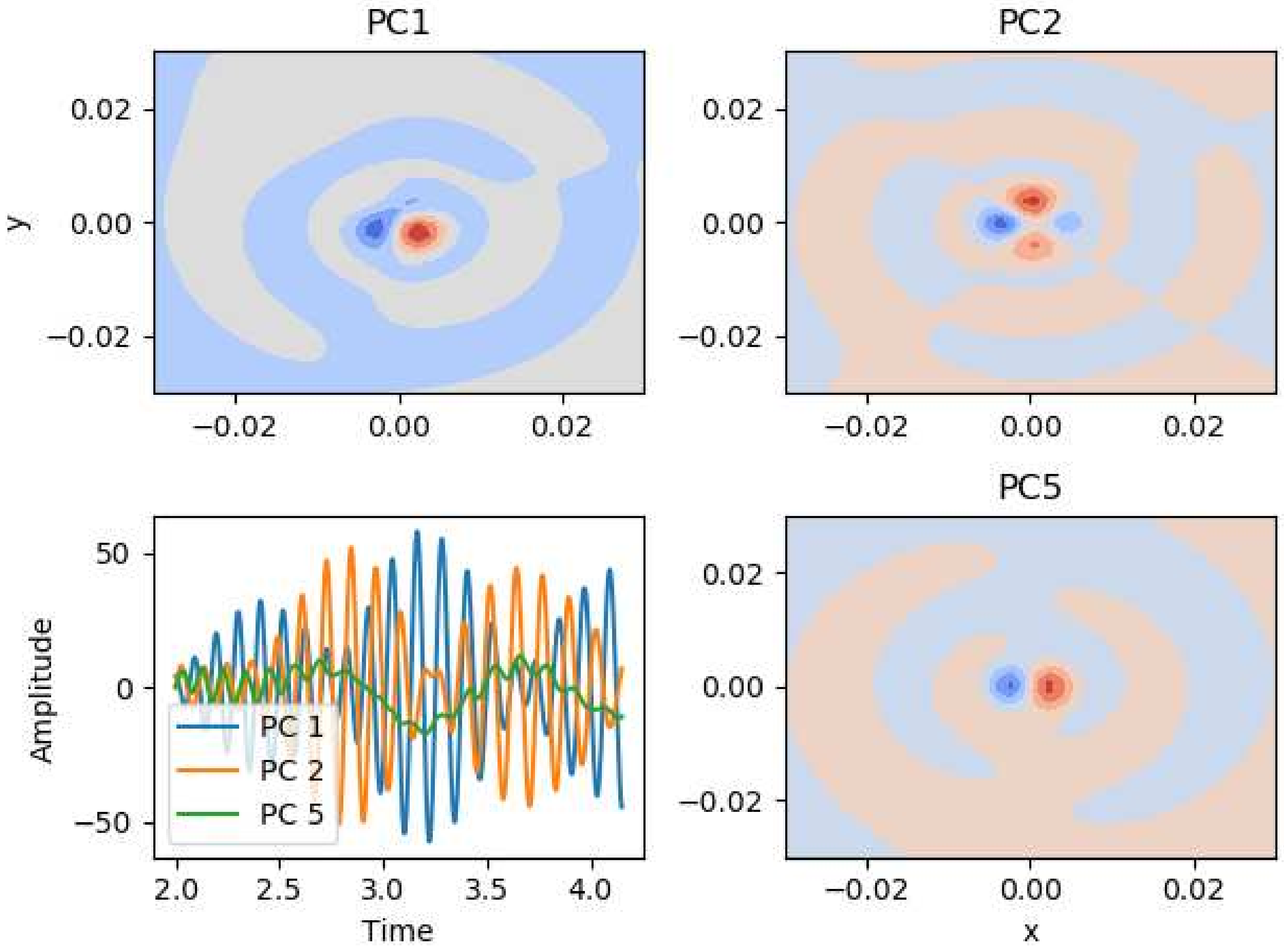


- Amplitude of first 6 PC groups

Mike's thesis simulation



- $\approx 1\%$ of the energy of PC1, $\approx 3\%$ of amplitude
- Dominated by early evolution (shown here at late time, T=2.5)
- The pattern speed different than those of the bar
- Non-steady owing to the quasi-periodic excitation and damping



- Combined m = 1 and m = 2 MSSA analysis

1. Global functional: $f(E)$ only where $\rho(r) = \int d^3v f(E)$
   - Stable for $f(E)$, $df(E)/dE < 0$

     [Antonov (1962, 1973), Goodman (1988)]

2. Dispersion relations: find self-similar collective modes $\propto e^{(i\Omega+\gamma)t}$
   - Instabilities $\boxed{\gamma > 0}$ for $f(E, J)$
     - Radial orbit instability (ROI)

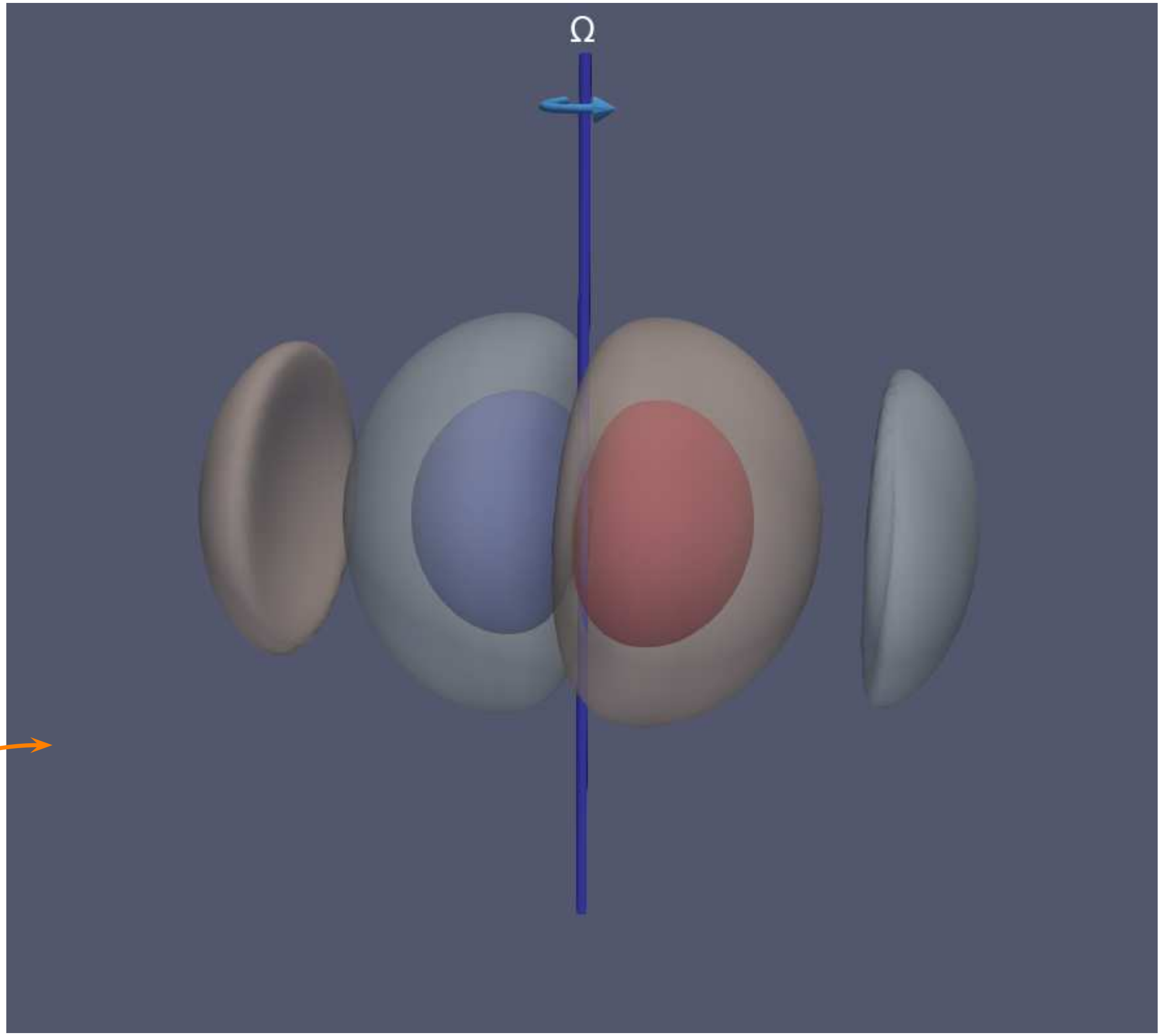
     [Palmer & Papaloizou 1987, Saha 1991, Weinberg 1991, Saha 1992,...see B&T for more]
   - Weakly-damped $\boxed{\gamma < 0}$
     - $l = 1$ seiche mode

     [Weinberg 1994, Heggie et al. 2020, Hamilton 2021, Fouvry & Prunet, 2022]

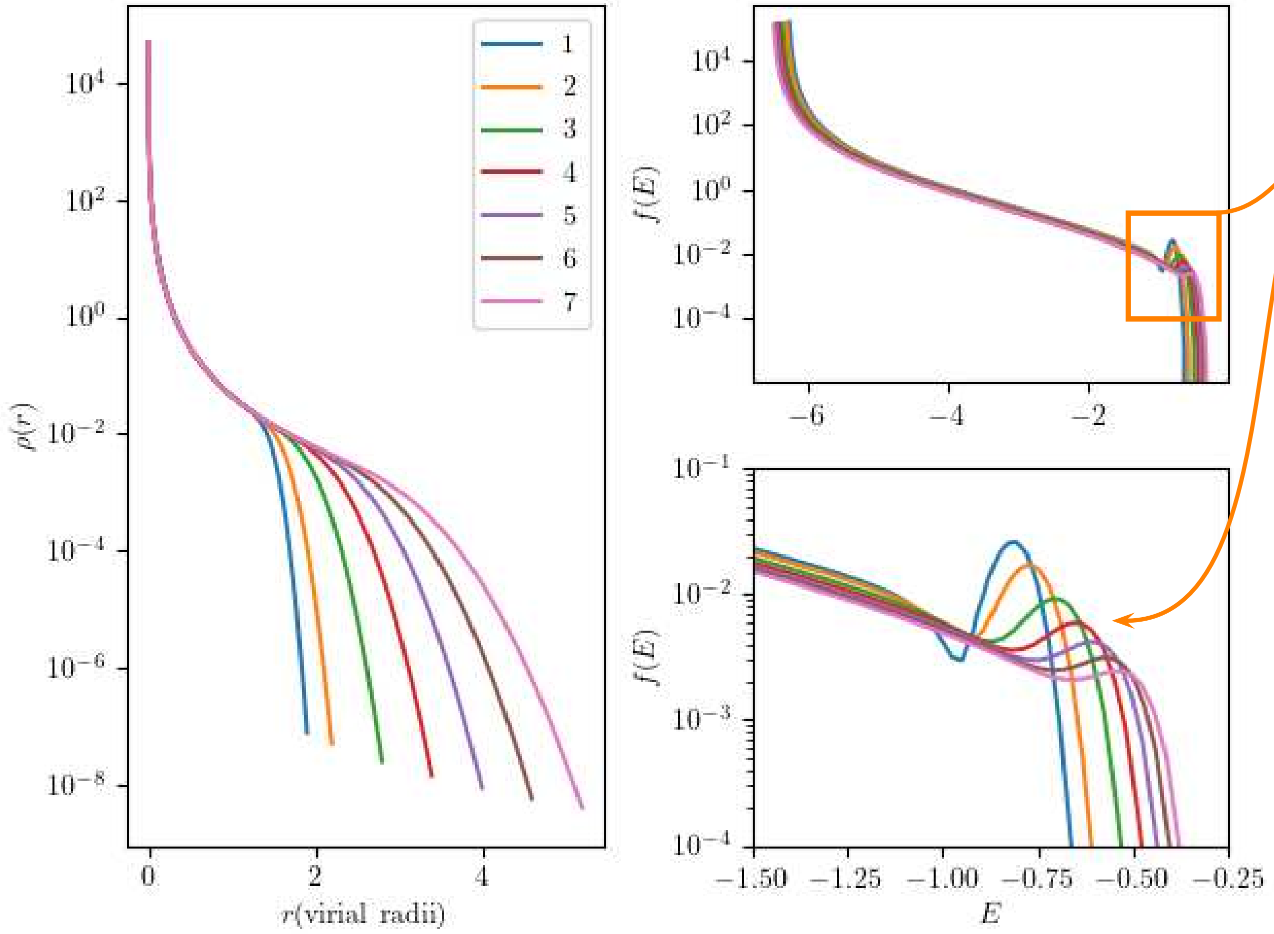
How/why do damped modes work?

- Response correlates apocenter positions
- Pattern speed is very slow: $\Omega \ll \frac{v_c}{r}$
- Couples only to orbits in outskirts
- Damps or grows depending on $\partial f / \partial \mathbf{I}$
- For many finite models: loses angular momentum slowly

Perturbation theory



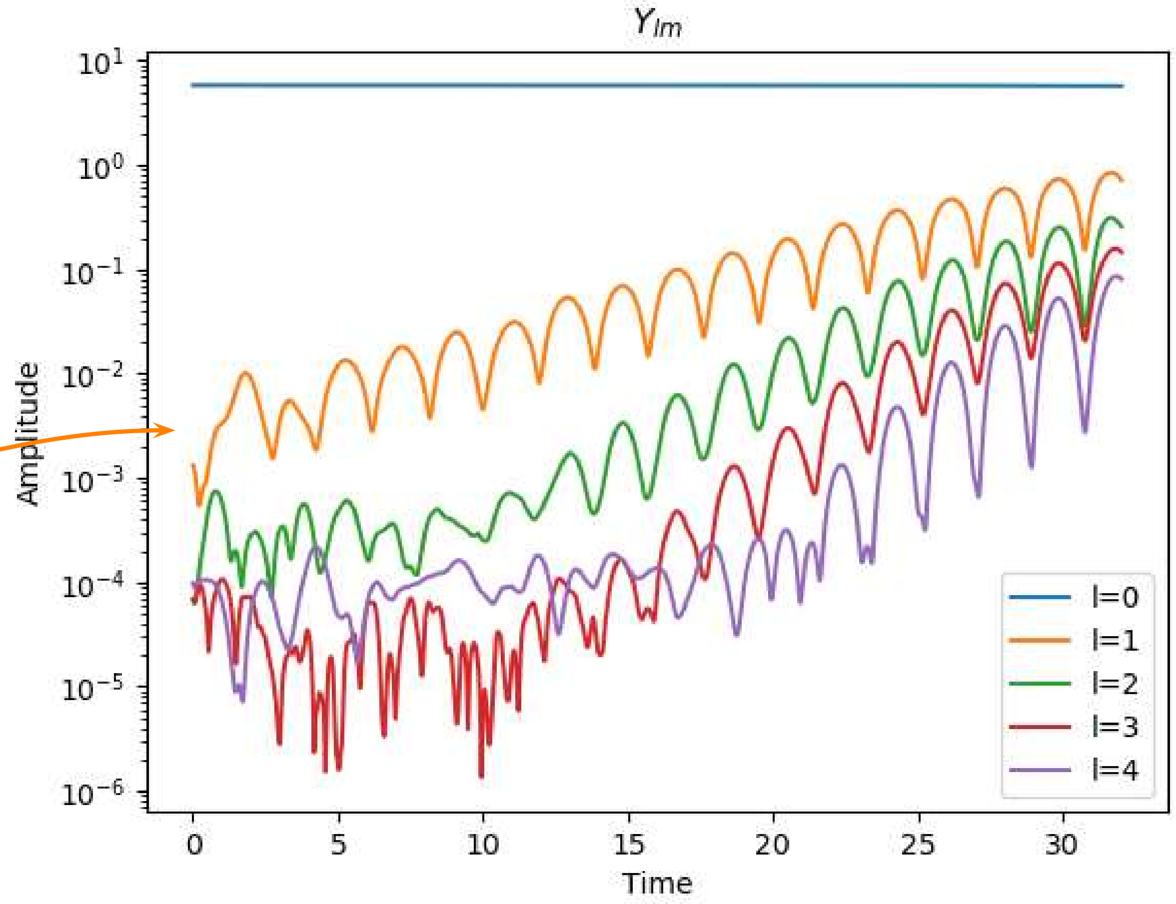
Tweak the NFW profile by smooth truncation



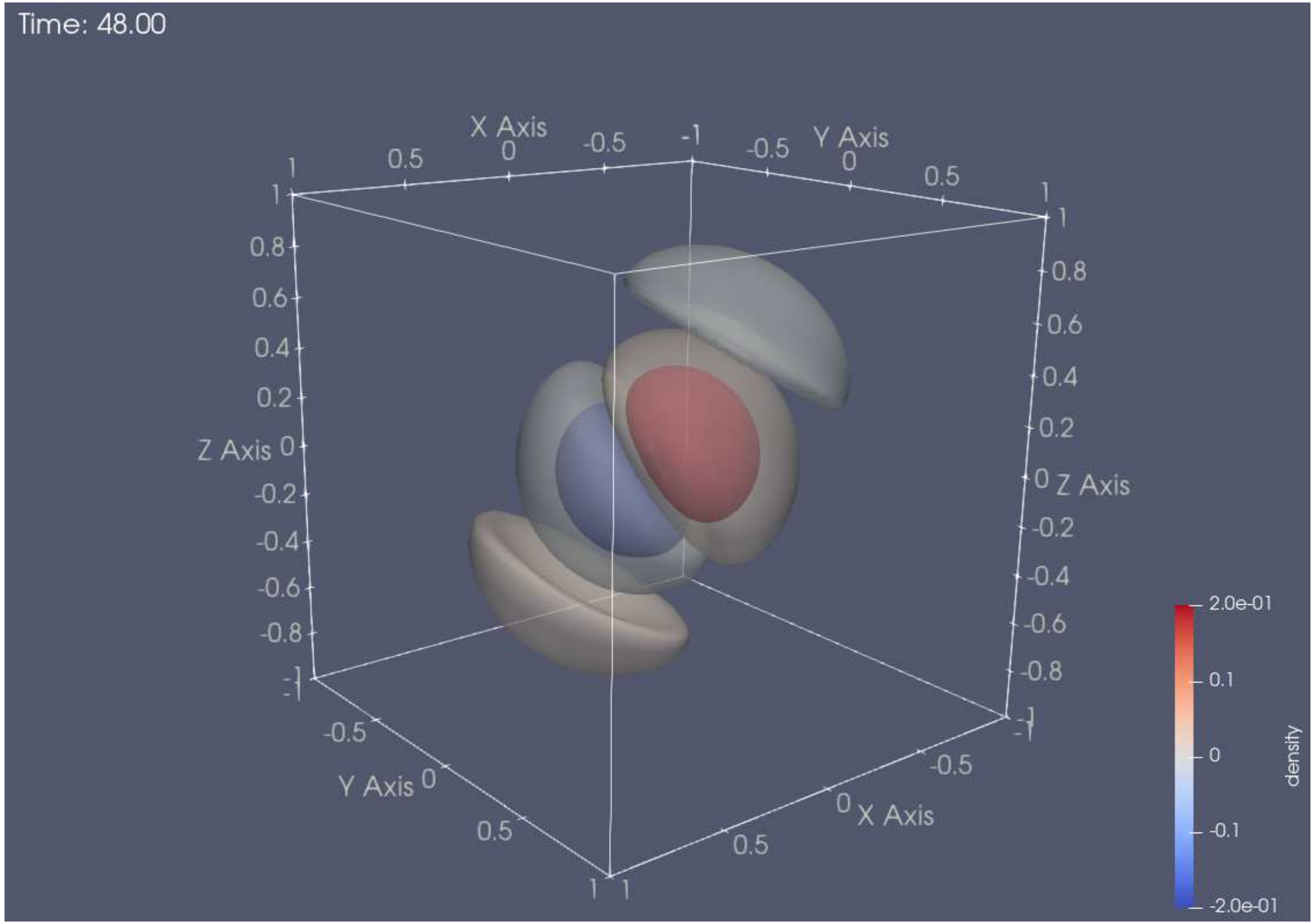
- Causes an inflection in DF
- Dispersion relation: 'bump on tail' converts damped to growing mode
- Mode in Hernquist profile: damps
    - Too little mass in outskirts
- Mode in truncated isothermal: grows
    - Heavy outskirts

N-body simulations for insight

- N-body simulations using EXP, $N = 20,000,000$ [Petersen et al. 2022]
  - Biorthogonal expansion Poisson solver, GPU
  - Public release with Python bindings soon with mSSA
- Spectral analysis of the coefficients time series (M-SSA) [Weinberg & Petersen 2021, Johnson et al. 2023]
- Exponential growth of $l = 1$
- Same behavior with Gadget-2
- Linear instability verified by response theory



Mode reconstruction



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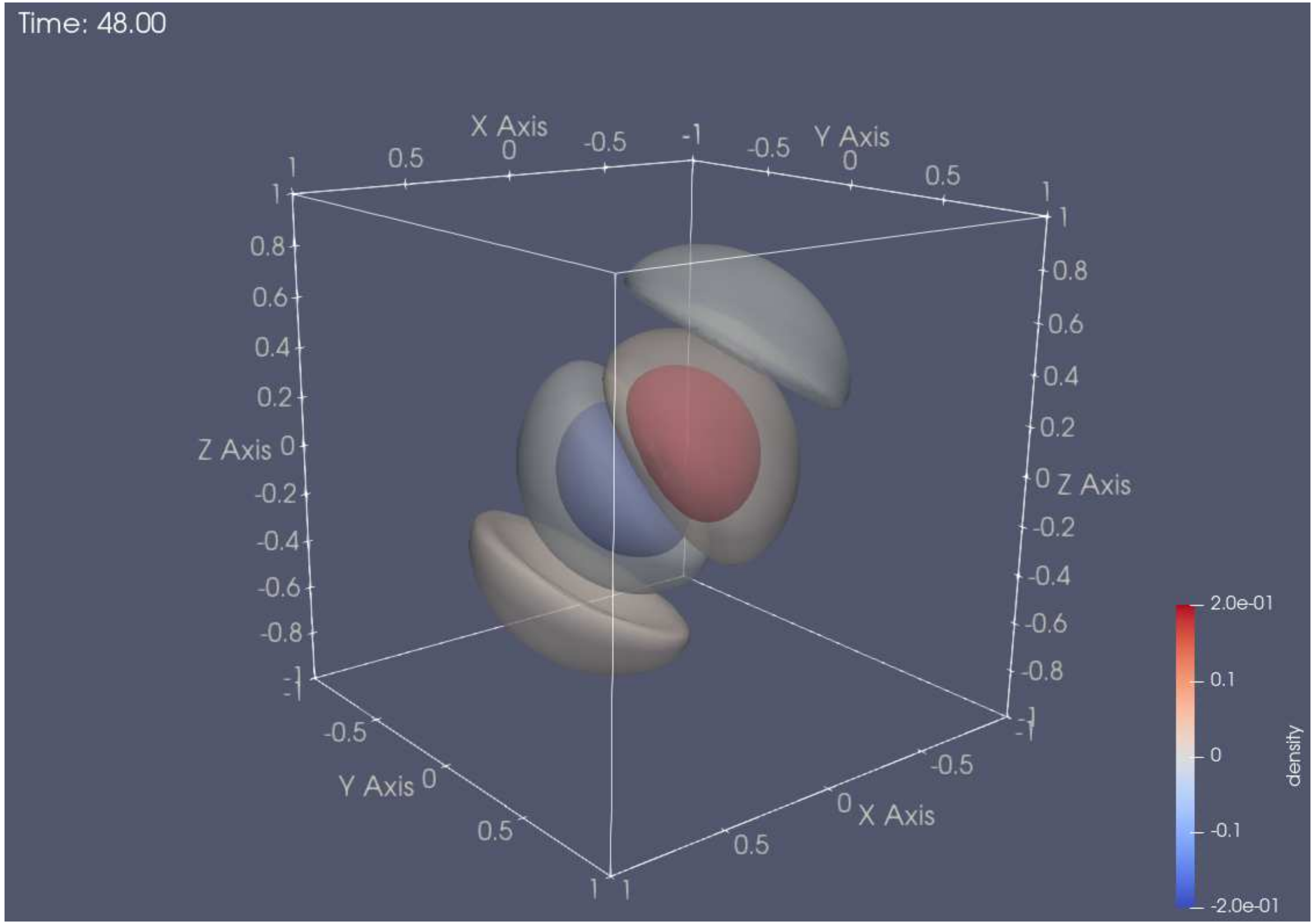
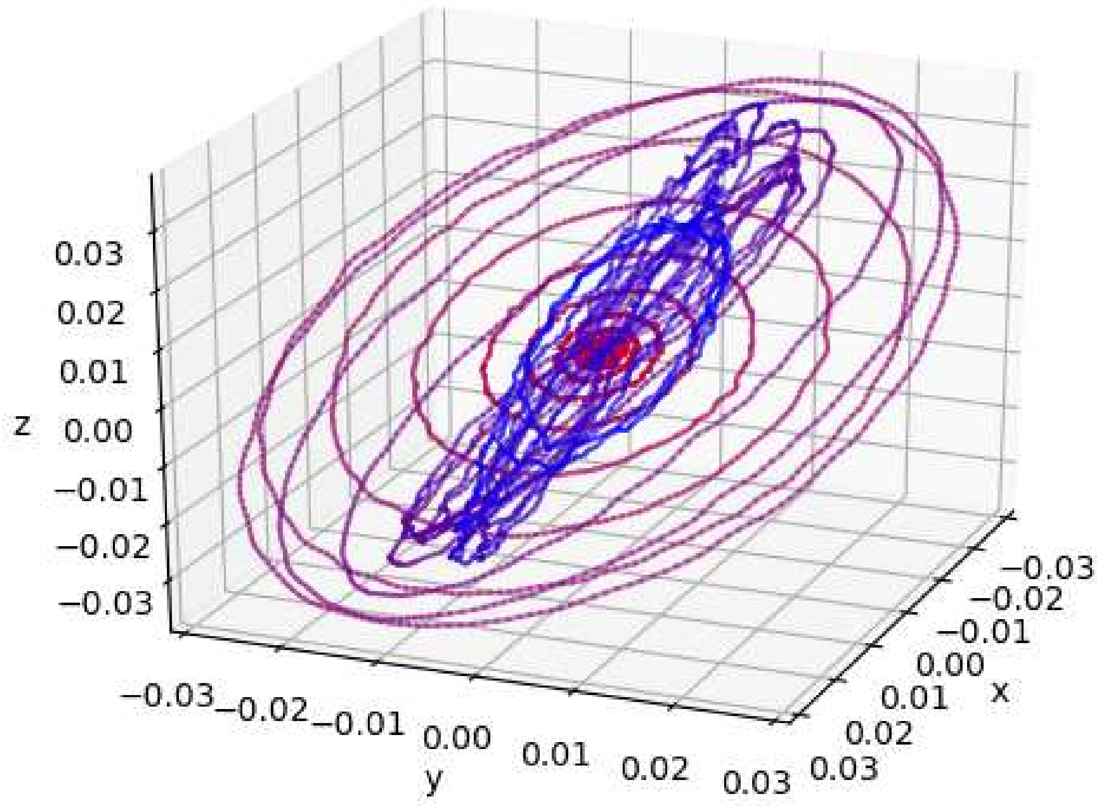
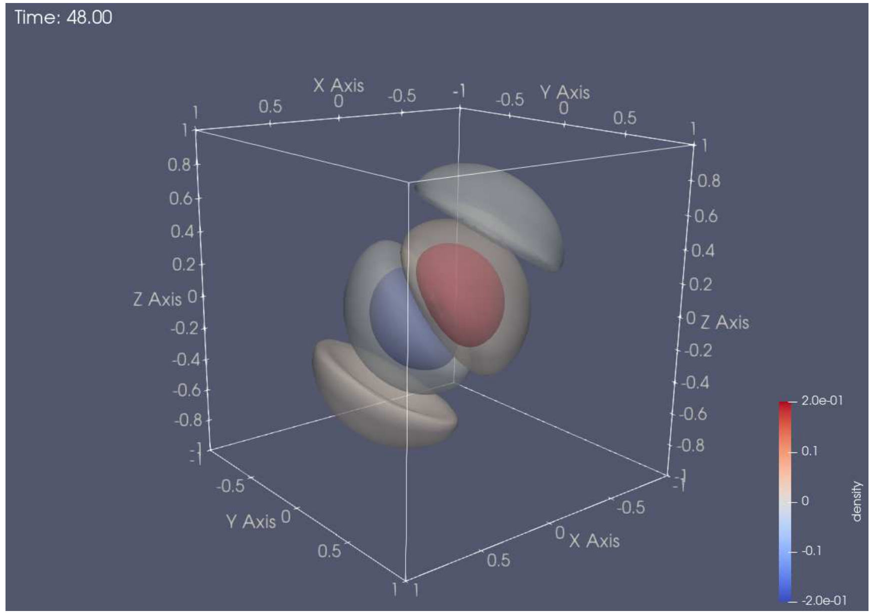


- Affect on disk?



- ☐ Cusp location
- ☐ Proxy for disk center



Take-aways

- $l = 1$ analog of radial-orbit instability
- Radial angle becomes bunched
- Eccentric orbits feed the main mode
- 1-orbit fit: low L, 16% of halo mass
- $l = 1$ is important even when damped

- DM profiles can have unstable $l = 1$ modes
  - Outer halo profile matters!
  - Confirmed by linear response theory (arXiv:2209.06846)
- Character of the mode
  - $l = 1$ analog of the radial orbit instability
  - Orbits become bunched in radial phase
- Observational signatures
  - Persistent dipole distortions in DM (seen in photometry!)
  - May be excited by minor mergers, outer halo features, filaments(?)