Quantum fluids of light
first successes and many exciting perspectives

Iacopo Carusotto
INO-CNR BEC Center and Università di Trento, Italy

Onur Umucalilar (now Koc)  Tomoki Ozawa  Hannah Price  Grazia Salerno
José Lebreuilly  Pierre-Elie Larré  Pjotrs Grisins

In collaboration with:

- C. Ciuti  (MPQ, Paris 7)
- M. Wouters  (Univ. Antwerp)
- A. Bramati, E. Giacobino  (LKB, Paris)
- M. H. Szymanska  (Univ. College London)

- T. Volz, M. Kroner, A. Imamoglu  (ETHZ)
- A. Amo, J. Bloch, H.-S. Nguyen  (LPN-CNRS)
- O. Zilberberg  (ETHZ)
- N. Goldman  (UL Brussels)
- D. Faccio  (Heriot-Watt, UK)
Why not hydrodynamics of light?

Light field/beam composed by a huge number of photons
- in vacuo photons travel along straight line at $c$
- (practically) do not interact with each other
- in cavity, collisional thermalization slower than with walls and losses

$\Rightarrow$ optics typically dominated by single-particle physics

In photonic structure:
- $\chi^{(3)}$ nonlinearity $\rightarrow$ photon-photon interactions
- Spatial confinement $\rightarrow$ effective photon mass

$\Rightarrow$ collective behaviour of a quantum fluid

Many experiments so far:
BEC, superfluidity, synthetic magnetism, Chern insulators...

In this talk: a few selected topics
Review of BEC/superfluidity, quantum hydrodynamics, IQH in 4D photonic lattices, conservative photon fluids
Standing on the shoulders of giants

Laserlight—First Example of a Second-Order Phase Transition Far Away from Thermal Equilibrium*

R. Graham and H. Haken
I. Institut für theoretische Physik der Universität Stuttgart
Received April 23, 1970

We solve the functional Fokker-Planck equation established in a previous paper in the vicinity of laser threshold. The stationary solution is obtained explicitly in the form $P = N \exp [-\varphi(\bar{u}, \bar{u}^*)]$. $\varphi$ has exactly the same form as the Ginzburg-Landau expression for the free energy of a superconductor, if the pair wave function is replaced by the electromagnetic field amplitude $\bar{u}$. This gives us the key for a thermodynamic reinterpretation of all laser phenomena.

In particular the laser threshold appears as a second-order phase transition in all details. It is indicated that our theory provides a new formalism also for the Ginzburg-Landau theory.

Vortices and Defect Statistics in Two-Dimensional Optical Chaos

F. T. Arecchi, (1) G. Giaconcelli, P. L. Ramazza, and S. Residori
Istituto Nazionale di Ottica, Largo E. Fermi, 6, 50125 Firenze, Italy
(Received 1 April 1991)

We present the first direct experimental evidence of topological defects in nonlinear optics. For increasing Fresnel numbers $F$, the two-dimensional field is characterized by an increasing number of topological defects, from a single vortex, up to a large number of vortices with zero net topological charge. At variance with linear scattering from a fixed phase plate, here the defect pattern evolves in time according to the nonlinear dynamics. We assign the scaling exponents for the mean number of defects, their mean separation, and the charge unbalance as functions of $F$, as well as the correlation time of the defect pattern.

Optique/Optics


And of course many others:
Coullet, Gil, Rocca, Brambilla, Lugiato...
Part 0:

A primer to BEC and superfluidity in semiconductor microcavities
Planar DBR microcavity with QWs

- **DBR**: stack $\lambda/4$ layers (e.g. GaAs/AlAs)
- Cavity layer $\rightarrow$ confined photonic mode, delocalized along 2D plane:
  \[
  \omega_C(k) = \omega_C^0 \sqrt{1 + k^2 / k_z^2}
  \]
- e-h pair in QW: sort of H atom. **Exciton**
- bosons for $n_{\text{exc}} a^2_{\text{Bohr}} \ll 1$ (verified by QMC)
- Excitons delocalized along cavity plane.
  Flat exciton dispersion $\omega_x(k) \approx \omega_x$
- Optical $\chi^{(3)}$ from exciton collisions

**Exciton radiatively coupled to cavity photon at same in-plane $k$**

**Bosonic superpositions of exciton and photon**, called **polaritons**

**Two-dimensional gas of polaritons**

Small effective mass $m_{\text{pol}} \approx 10^{-4} m_e$ $\rightarrow$ originally promising for BEC studies
- **Exciton** $\rightarrow$ interactions. **Photons** $\rightarrow$ radiative coupling to external world
How to create and detect the photon gas?

Pump needed to compensate losses: stationary state is NOT thermodynamical equilibrium

- **Coherent laser** pump: directly injects photon BEC in cavity, may lock BEC phase
- **Incoherent** (optical or electric) pump: BEC transition similar to laser threshold spontaneous breaking of U(1) symmetry

Classical and quantum correlations of in-plane field directly transfer to emitted radiation
2006 - Photon/polariton Bose-Einstein condensation

But also differences due to non-equilibrium:

- BEC @ \( k \neq 0 \rightarrow \) volcano effect
- T-reversal broken \( \rightarrow n(k) \neq n(-k) \)
- interesting questions about thermalization

Many features very similar to atomic BEC

Photon/polariton BEC closely related to laser operation in VCSELs

Quantized vortices
K. Lagoudakis et al.

Suppressed fluctuations
A. Baas et al., PRL 96, 176401 (2006)

Interference
Richard et al., PRL 94, 187401 (2005)
Figure from LKB-P6 group:

**2009-10 - Superfluid hydrodynamics**

**Oblique dark solitons** →

A. Amo, et al., Science 332, 1167 (2011)

← **Turbulent behaviours**

A. Amo, et al., Science 332, 1167 (2011)

**Hydrodynamic nucleation** → **of vortices**

Nardin et al., Nat. Phys. 7, 635 (2011)

Role of interactions crucial in determining regimes as a function of $v/c_s$
Part I: *Quantum hydrodynamics*

Artificial black holes in atom and photon fluids

*The (hopefully forthcoming) tale of Navier and Stokes meeting Heisenberg at Hawking's place*
• Horizon region separating:
  
  sub-sonic (i.e. sub-fishic) flow (upstream)

  from super-sonic (i.e. super-fishic) flow (downstream)

• Excitations (i.e. fish) in super-sonic (i.e. super-fishic) region
  can not travel (i.e. swim) back through horizon

What happens with quantum sound ?

Hawking radiation of acoustic phonons ?

Unruh, PRL 1981
Analog Hawking radiation

Unruh PRL '81:

Sound propagation on superfluid ↔ light propagation on space-time with curved metric determined by density $n(x)$ and velocity $v(x)$ fields

$$ds^2 = G_{\mu \nu} dx^\mu dx^\nu = \frac{n(x)}{c_s(x)} \left[ -c_s(x)^2 dt^2 + (d\vec{x} - \vec{v}(x)dt)(d\vec{x} - \vec{v}(x)dt) \right]$$

Wave equation for superfluid phase

$$\frac{1}{\sqrt{-G}} \partial_\mu \left[ \sqrt{-G} G^{\mu \nu} \partial_\nu \right] \varphi(x, t) = 0$$

Once quantized → quantum field theory in a curved space time

Astrophysical black holes: Hawking emission from horizon at

$$T_H = \frac{\hbar c^3}{8 \pi k_B G M}$$

Acoustic black holes:

- emit Hawking radiation of phonons at
  $$T_H = \frac{\hbar}{4 \pi k_B c_s} \left[ \frac{d}{dx} \left( c_s^2 - v^2 \right) \right]_H$$
- in nK range for μm-sized ultracold atomic BECs (not so bad...)
- much higher for superfluids of light thanks to small photon mass
  as first proposed by F. Marino, PRA 78, 063804 (2008)
What about acoustic horizons in fluids of light?

Polariton-polariton interactions

- Bogoliubov phonon dispersion on top of polariton condensate

Pump at an angle

- finite in-plane wavevector, so condensate is flowing

Tailored pump spot + Defect

→ Horizon with large surface gravity

Hawking emission

- phonons on photon fluid
- correlations of emitted light

D. Gerace and IC, PRB 86, 144505 (2012); similar results in Solnyshkov et. al., PRB 84, 233405 (2011)
Very recent experimental results @ LPN

BH created!

The hunt for Hawking radiation is now open!!

How to detect Hawking radiation?

Density-density correlation function

\[ G^{(2)}(x, x') = \frac{\langle : n(x) n(x') : \rangle}{\langle n(x) \rangle \langle n(x') \rangle} \]

Prediction of gravitational analogy:

→ Entanglement in Hawking pairs gives long-range in/out correlations

\[ G_2(x, x') = 1 - \frac{\xi_1 \xi_2}{16\pi c_1 c_2} \frac{k^2}{\sqrt{n^2 \xi_1 \xi_2}} \frac{c_1 c_2}{(c_1 - v)(v - c_2)} \cosh^{-2} \left[ \frac{k}{2} \left( \frac{x}{c_1 - v} + \frac{x'}{v - c_2} \right) \right] \]

→ Allows to isolate Hawking phonons from background of incoherent thermal phonons

→ the “Balbinot-Fabbri” moustache

Ab initio numerics: Wigner-Monte Carlo

At t=0, homogeneous system:
- Condensate wavefunction in plane-wave state
- Quantum + thermal fluctuations in plane wave Bogoliubov modes
- Gaussian $\alpha_k$, variance $<|\alpha_k|^2> = [2 \tanh(E_k / 2k_BT)]^{-1} \to \frac{1}{2}$ for $T \to 0$.

$$\psi(x, t=0) = e^{i k_0 x} \left[ \sqrt{n_0 + \sum_k \left( u_k e^{i k x} \alpha_k + v_k e^{-i k x} \alpha^*_k \right)} \right]$$

At later times: conservative (for atoms!) evolution under GPE

$$i \hbar \frac{\partial}{\partial t} \psi(x) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x) \psi(x) + g(x) |\psi(x)|^2 \psi(x)$$

Expectation values of observables:
- Average over noise provides symmetrically-ordered observables

$$\langle \psi^*(x) \psi(x') \rangle_W = \frac{1}{2} \langle \hat{\psi}^\dagger(x) \hat{\psi}(x') + \hat{\psi}(x') \hat{\psi}^\dagger(x) \rangle_Q$$

Equivalent to Bogoliubov, but can explore longer-time dynamics

Density correlations in atomic gas: the movie

A snapshot of density correlations

Density plot of: \( (n \xi_1) \ast [G^{(2)}(x,x') - 1] \)

(ii) Dynamical Casimir emission

(iii) Hawking in / out “Balbinot-Fabbri moustache”

(i) Many-body antibunching

(iv) Hawking in / in

Hawking emission in driven-dissipative photon fluids

- **Wigner-MC simulation with driving/losses:**
  \[ i \, dE = \left( \omega_o - \frac{\hbar \nabla^2}{2m} + V_{\text{ext}} + g |E|^2 - \frac{i}{2} \gamma \right) E \, dt + F_{\text{ext}}(x, t) \, dt + dW \]

- **Near-field emission pattern from wire:**
  Correlation function of *intensity noise* at different positions \((x, x')\)

- **Signature of Hawking radiation processes:**
  "Balbinot-Fabbri" correlation tongues
  Conversion of zero-point fluctuations into *correlated pairs of Bogoliubov phonons* propagating away from horizon

- **In optics language:**
  parametric emission of entangled photons
  flow+horizon play role of pump
  photons dressed by fluid into *phonons*

- **Proposed experiment:**
  - steady state under cw pumping
  - collect near-field emission
  - measure *intensity noise*
  - integrate over long time to extract signal out of shot noise

D. Gerace and IC, PRB 86, 144505 (2012)
Pump-probe detection of (classical) HR

- CW probe at $\omega_{\text{probe}}$, frequency resolved detect at $\omega_{\text{probe}}$ and FWM signal @ $2\omega_{\text{pump}} - \omega_{\text{probe}}$

- Stimulated Hawking on mode $d_{\text{2out}}$ → peak in angular distribution

- Scattering matrix $S(\omega)$:
  $T_{\text{H}}/\omega$ scaling @ low $\omega$ → signature of thermal Hawking emiss.

- In contrast to pulse expt, no need for temporal resolution

Expt with surface waves on water (Weinfurtner, Unruh, PRL 2010) appears not conclusive as no horizon present, new expt in progress (Rousseaux)
How to assess quantum nature of HR?

- Signal in density/intensity correlations **reinforced** at finite $T$ by **stimulated Hawking emission**.  
  → **Not a signature of quantum origin of emission**

- **Peres-Horodecki criterion** for entanglement in bipartite systems  
  → **correlations of quadratures of phonon operators** on either side of the horizon

- **Phonon wavepacket operators** localized in **real- and momentum spaces**

- To measure phonon quadratures with spatial and spectral selectivity:  
  → **optomechanical coupling**, measure **cavity frequency shift**

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Finazzi, IC, *Entangled phonons in atomic Bose-Einstein condensates*, PRA 2014
Is HR in microcavities a quantum process?

Unavoidable losses of microcavity device generate excitations in the fluid:
lost photon → creates hole → Bogoliubov excitation

Spurious excitations up to $\omega \sim gn$, comparable to $T_H$


Stimulate Hawking processes giving rise to “thermal” Hawking signal:
- **Density correlation signal** reinforced
- **Quantum entanglement** still present?
  - How to detect it?
  - Hong-Ou-Mandel on emitted light spatially + wavevector-selected?
Part II: Synthetic gauge fields and quantum magnetism with light
Photonic (Chern) topological insulator

MIT '09, Soljacic group
Original proposal Haldane-Raghu, PRL 2008

Magneto-optical photonic crystals for μ-waves
T-reversal broken by magnetic elements

Band with non-trivial Chern number:

→ chiral edge states within gaps

➢ unidirectional propagation
➢ immune to back-scattering by defects

Wang et al., Nature 461, 772 (2009)
Synthetic gauge fields for photons

2D lattice of coupled cavities with tunneling phase

\[
H = \sum_i \hbar \omega_i \hat{a}_i^\dagger \hat{a}_i - \hbar J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j e^{i\phi_{ij}} + \sum_i \hbar F_i(t) \hat{a}_i^\dagger + \text{h.c.}
\]

- Floquet bands in helically deformed waveguide lattices → Rechtsman/Segev
- silicon ring cavities → Hafezi/Taylor (JQI)
- electronic circuits with lumped elements → J. Simon (Chicago)
- related: honeycomb potential for polaritons → A. Amo/J.Bloch (LPN)

Hafezi et al., Nat. Phot. 7, 1001 (2013)
Hofstadter butterfly and chiral edge states

2D square lattice of coupled cavities at large magnetic flux

- eigenstates organize in bulk Hofstadter bands

- Berry connection in k-space: $A_{n,k} = i \langle u_{n,k} | \nabla_k u_{n,k} \rangle$

Bulk-edge correspondence:

$A_{n,k}$ has non-trivial Chern number

$\rightarrow$ chiral edge states within gaps

- unidirectional propagation
- (almost) immune to scattering by defects
- T-reversal not broken, 2x pseudo-spin bands with opposite Chern

Hafezi et al., Nat. Phot. 7, 1001 (2013)
How to observe topological properties of bulk?

Lattice at strong magnetic flux, e.g. $\alpha = 1/3$

Band dispersion

Berry curvature

\[ \Omega_n(k) = \nabla_k \times A_{n,k} = \nabla_k \times [i\langle u_{n,k} | \nabla_k u_{n,k} \rangle] \]

Semiclassical eqs. of motion:

\[ \hbar \dot{k}_c(t) = eE, \]
\[ \hbar \dot{r}_c(t) = \nabla_k \mathcal{E}_{n,k} - eE \times \Omega_n(k) \]

Magnetic Bloch oscillations display a net lateral drift

- Initial photon wavepacket injected with laser pulse
- spatial gradient of cavity frequency $\rightarrow$ uniform force
- Berry curvature $\rightarrow$ sort of $k$-space magnetic field

Figures from Cominotti-IC, EPL 103, 10001 (2013).
First proposal in Dudarev, IC et al. PRL 92, 153005 (2004). See also Price-Cooper, PRA 83, 033620 (2012).
**Experiments with atoms**

Semiclassical equations of motion

\[ \hbar \dot{k}_e(t) = eE, \]
\[ \hbar \dot{r}_e(t) = \nabla_{k} \varepsilon_{n,k} + eE \times \Omega_{n}(k) \]

Slow momentum shift under uniform force $eE$ (as in Bloch oscillations)

Berry curvature of band $\Omega_{n} \rightarrow$ lateral displacement in space of atomic cloud

Anomalous Hall effect vs. Integer Quantum Hall effect
Photonic system

Cavity lattice geometry → promising in view of interacting photon gases, but radiative losses.

Short time to observe BO's, but experiment @ non-eq steady state even better

Coherent pumping \( H_d = \sum_i F_i(t) \hat{b}_i + F_i^*(t) \hat{b}_i^\dagger + \text{losses at rate } \gamma \)

Pump spatially localized on central site only:
- couples to all k's within Brillouin zone
- resonance condition selects specific states

In the presence of force \( F \):

motion in BZ → lateral drift in real space by Berry curvature

\[
\hbar \dot{k}_c(t) = eE,
\]
\[
\hbar \dot{r}_c(t) = \nabla_k E_{n,k} - eE \times \Omega_n(k)
\]

Detectable as lateral shift of intensity distribution by \( \Delta x \) perpendicular to \( F \)

T. Ozawa and IC, Anomalous and Quantum Hall Effects in Lossy Photonic Lattices, PRL (2014)
More quantitatively

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<th>4th</th>
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(a) Lowest band of $\alpha = 1/3$

(b) Middle band of $\alpha = 1/5$

Low loss  ($\gamma < \text{bandwidth}$)  $\rightarrow \Delta x = F \Omega(k_0) / 2\gamma$  (anomalous Hall eff.)

Large loss  ($\text{bandwidth} < \gamma < \text{bandgap}$)  $\rightarrow \Delta x = q \text{Chern} / 2\pi\gamma$  (integer-QH)

**Integer quantum Hall effect** for photons  (in spite of no Fermi level)

Photon phase observable $\Rightarrow$ expts sensitive to **gauge-variant** quantities!!

What happens with harmonic trap?

What are the quantum mechanical eigenstates of a harmonically trapped HH model?

Straightforward in optics under coherent pump:
- each absorption peak → an eigenstate
- coherent pump frequency selects a single state
  - near-field image → real-space eigenfunction
  - far-field emission → k-space eigenfunction

Toroidal Landau levels in momentum space

Absorption spectrum

Price, Ozawa, IC, Quantum Mechanics Under a Momentum Space Artificial Magnetic Field, PRL 2014
Part III: Towards higher dimensions
What about higher dimensions?

Generalize of semiclassical equations to 4D:

\[
\begin{align*}
\dot{r}^\mu(k) &= \frac{\partial E(k)}{\partial k^\mu} - \dot{k}_\nu \Omega^{\mu\nu}(k), \\
\dot{k}_\mu &= -E_\mu - \dot{r}^\nu B_{\mu\nu},
\end{align*}
\]

Integrate current over filled bands:

- **2D quantized Hall current** depends on 1\textsuperscript{st} Chern number

  \[ j^y = \frac{E_x}{(2\pi)^2} \int T^2 \Omega d^2 k = \frac{\nu_1}{2\pi} E_x \]

  analogous to \( j^y = \nu \frac{e^2}{h} \) well known in IQHE

- **4D magneto-electric response** depends on 2\textsuperscript{nd} Chern number (non-zero in \( d\geq 4 \))

  \[ j^\mu = E_\nu \frac{1}{(2\pi)^4} \int_{\mathbb{T}^4} \Omega^{\mu\nu} d^4 k + \frac{\nu_2}{4\pi^2} \varepsilon^{\mu\alpha\beta\nu} E_\nu B_{\alpha\beta} \]

  \[ \nu_2 = \frac{1}{4\pi^2} \int_{\mathbb{T}^4} \Omega^{xy} \Omega^{zw} + \Omega^{wx} \Omega^{yz} + \Omega^{zx} \Omega^{yw} d^4 k \]

How to create 4D system with atoms?

Internal state $\rightarrow$ Synthetic dimension $w$

Raman processes give tunneling along $w$
Spatial phase of Raman beams give Peierls phase
Synthetic magnetic field in $xw, yw, zw$ planes

First experimental realization:
- 1+1 dimens. using 3 spin states
- Cyclotron + Reflection on edges

Stuhl et al., arXiv:1502.02496
Numerical validation of 4D QH effect

Numerical simulation of full wave equation
Add external E and B fields
Results in good agreement with semiclassics
Allowed E,B enough to see the effect

\[ j^\mu = E_\nu \frac{1}{(2\pi)^4} \int_{T^4} \Omega^{\mu\nu} d^4k + \frac{\nu_2}{4\pi^2} \varepsilon^{\mu\alpha\beta\nu} E_\nu B_{\alpha\beta} \]

\[ \nu_2 = \frac{1}{4\pi^2} \int_{T^4} \Omega^{xy}\Omega^{zw} + \Omega^{wx}\Omega^{yz} + \Omega^{zx}\Omega^{yw} d^4k \]

**How to create synthetic dimensions for photons?**


Different modes of ring resonators → **synthetic dimension** \( w \)

**Tunneling along synthetic \( w \):**

- strong beam *modulates resonator* \( \varepsilon_{ij} \) at \( \omega_{\text{FSR}} \) via optical \( \chi^{(3)} \)
- neighboring modes get *linearly coupled*
- phase of modulation → **Peierls phase** along synthetic \( w \)

Peierls phase along \( x,y,z \) → Hafezi's ancilla resonators

Extends Fan's idea of synthetic gauge field via time-dependent modulation (Nat. Phys. 2008)
1+1 array: chiral edge states & optical isolation

1 (physical) + 1 (synthetic) dimensions: Hofstadter model
- Bulk topological invariant → Chern number
  - measured via Integer Quantum Hall effect

- Chiral states on edges:
  - Physical edges along x
  - Synthetic edges via design of $\varepsilon(\omega)$
    (e.g. inserting absorbing impurities in chosen sites)
    → topologically protected optical isolator

4D magneto-electric response
Nonlinear integer QH effect

Lateral shift of photon intensity distribution in response to external synth-E and synth-B:

➢ only present with both E & B
➢ proportional to $2^{\text{nd}}$ Chern

\[
j^\mu = E_\nu \frac{1}{(2\pi)^4} \int_{T^4} \Omega^{\mu\nu} d^4 k + \frac{\nu_2}{4\pi^2} \varepsilon^{\mu\alpha\beta\nu} E_\nu B_{\alpha\beta}
\]

\[
\nu_2 = \frac{1}{4\pi^2} \int_{T^4} \Omega^{x y} \Omega^{z w} + \Omega^{w x} \Omega^{y z} + \Omega^{z x} \Omega^{y w} d^4 k
\]

Part IV: Quantum fluids of light with a conservative dynamics
**Field equation of motion**

**Planar microcavities & cavity arrays**

**Propagating geometry**

\[ i \frac{dE}{dt} = \left( \omega_o - \frac{\hbar \nabla^2}{2m} + V_{ext} + g |E|^2 + \frac{i}{2} \left( \frac{P_0}{1 + \alpha |E|^2} - \gamma \right) \right) E + F_{ext} \]

Pump needed to compensate losses: driven-dissipative dynamics in real time stationary state ≠ thermodyn. equilibrium

Driven-dissipative CGLE evolution

Quantum correl. sensitive to dissipation

Monochromatic beam
Incident beam sets initial condition @ z=0
Conservative GPE paraxial propagation

\[ i \frac{d\hat{E}}{dz} = \left( -\frac{\hbar \nabla^2_{xy}}{2\beta} + V_{ext} + g \hat{E}^* \hat{E} \right) \hat{E} \]

- \( V_{ext} \), g proportional to \(-(\varepsilon(r)-1)\) and \( \chi^{(3)} \)
- Mass → diffraction (xy)
How to include quantum fluctuations beyond MF

Requires going beyond monochromatic beam and explicitly including physical time

Gross-Pitaevskii-like eq. for propagation of quasi-monochromatic field

\[
i \frac{\partial E}{\partial z} = -\frac{1}{2\beta_0} \left( \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} \right) - \frac{1}{2D_0} \frac{\partial^2 E}{\partial t^2} + V(r)E + g|E|^2E
\]

Propagation coordinate \( z \rightarrow \) time
Physical time \( \rightarrow \) extra spatial variable, dispersion \( D_0 \rightarrow \) temporal mass

Upon quantization \( \rightarrow \) conservative many-body evolution in \( z \):

\[
i \frac{d}{dz} |\psi> = H |\psi>
\]

with

\[
H = N \iiint dx \, dy \, dt \left[ \frac{1}{2\beta_0} \nabla \hat{E}^\dagger \nabla \hat{E} - \frac{D_0}{2} \frac{\partial \hat{E}^\dagger}{\partial t} \frac{\partial \hat{E}}{\partial t} + V \hat{E}^\dagger \hat{E} + \hat{E}^\dagger \hat{E} \hat{E} \right]
\]

Same \( z \) commutator

\[
[\hat{E}(x, y, t, z), \hat{E}^\dagger(x', y', t', z)] = \frac{c\hbar \omega_0 v_0}{\epsilon} \delta(x-x') \delta(y-y') \delta(t-t')
\]

Dynamical Casimir emission at quantum quench (I)

Air / nonlinear medium interface
→ sudden jump in interaction constant when moving along $z$

Monochromatic wave
Normal incidence
Weakly nonlinear medium
→ Weakly interacting Bose gas at rest

Propagation along $z$
→ conservative quantum dynamics

Mismatch of Bogoliubov ground state in air and in nonlinear medium
→ emission of phonon pairs at opposite $k$ on top of fluid of light

Observables:

- Far-field → correlated pairs of photons at opposite angles
- Near-field → peculiar pattern of intensity noise correl.

First peak propagates at the speed of sound $c_s$

*May simulate dynamical Casimir effect & fluctuations in early universe*

Pimp & probe expt for speed of sound $c_s$ in spatial (xy) and temporal (t) directions:

- $c_s^{xy}$ (Heriot-Watt – Vocke et al. Optica '15)
- $c_s^t$ (Trento, in progress)

Quantum dynamics most interesting in strongly nonlinear media, e.g. Harvard expts with Rydberg polaritons

Conclusions and perspectives

Dilute photon gas

2000-6 → BEC in exciton-polaritons gas in semiconductor microcav.

GP-like equation

2008-10 → superfluid hydrodynamics effects observed

2009-13 → synthetic gauge field for photons and topologically protected edge states observed.

- Optical microcavity systems are unavoidably lossy → driven-dissipative, non-equilibrium dynamics not always a hindrance for many-body physics, but can be turned into great advantage!
- Bulk cavityless geometries: paraxial propagation → conservative dynamics time plays role of third dimension; useful to study quantum quenches, thermalization, etc.

Many questions still open:
- quantum hydrodynamic states of photon fluid, e.g. analog Hawking emission of phonon pairs
- Integer QH effects in high-dimensional photonics
- critical properties of transition in 2D → BKT or not to BKT

Challenging perspectives on a longer run:
- strongly correlated photon gases → Tonks-Girardeau gas in 1D necklace of cavities
- with synthetic gauge field → Laughlin states, quantum Hall physics of light
- Theoretical challenge → how to create and control strongly correlated many-photon states?
- more complex quantum Hall states: non-Abelian statistical phases. Integrated platform for topologically protected states to store and process quantum information?
If you wish to know more...


Or, even better, visit us in Trento!!
carusott@science.unitn.it

Save the date: May 8th-12th, 2017
2nd Workshop on Strongly Correlated Fluids of Light and Matter
Cargese, Corse
Part II: Quantum fluids of light with a conservative dynamics
Field equation of motion

**Planar microcavities & cavity arrays**

- Pump needed to compensate losses: driven-dissipative dynamics in real time
- Stationary state ≠ thermodynamic equilibrium

**Driven-dissipative CGLE evolution**

\[
i \frac{dE}{dt} = \left\{ \omega_0 - \frac{\hbar \nabla^2}{2m} + V_{\text{ext}} + g |E|^2 + \frac{i}{2} \left( \frac{P_0}{1 + \alpha |E|^2} - \gamma \right) \right\} E + F_{\text{ext}}
\]

- Quantum correl. sensitive to dissipation

**Propagating geometry**

- Monochromatic beam
- Incident beam sets initial condition @ z=0
- Conservative GPE paraxial propagation

\[
i \frac{d\hat{E}}{dz} = \left\{ -\frac{\hbar \nabla_{xy}^2}{2\beta} + V_{\text{ext}} + g \hat{E}^\dagger \hat{E} \right\} \hat{E}
\]

- \( V_{\text{ext}}, g \) proportional to \(-(\varepsilon(r)-1)\) and \( \chi^{(3)} \)
- Mass → diffraction (xy)
A few remarkable recent experiments

Dispersive superfluid-like shock waves


Bogoliubov dispersion of collective excitations


Chiral edge states in (photonic) Floquet topological insulator

Frictionless flow of superfluid light (I)

All superfluid light experiments so far:

- Planar microcavity device with stationary obstacle in flowing light
- Measure response on the fluid density/momentum pattern
- Obstacle typically is defect embedded in semiconductor material
- Impossible to measure mechanical friction force exerted onto obstacle

Propagating geometry more flexible:

- Obstacle can be solid dielectric slab with different refractive index
- Immersed in liquid nonlinear medium, so can move and deform
- Mechanical force measurable from magnitude of slab deformation

Frictionless flow of superfluid light (II)

Numerics for propagation GPE of monochromatic laser:

\[ i \partial_z E = -\frac{1}{2\beta} \left( \partial_{xx} + \partial_{yy} \right) E + V(r) E + g |E|^2 E \]

with \( V(r) = -\beta \Delta \varepsilon(r)/(2\varepsilon) \) with rectangular cross section and \( g = -\beta \chi^{(3)}/(2\varepsilon) \)

For growing light power, superfluidity visible:
- Intensity modulation disappears
- Suppression of opto-mechanical force

Fused silica slab: deformation almost in the \( \mu \)m range

Condensation of classical waves

Initial noisy configuration

Evolution during propagation → classical GPE

Thermalizes to classical distribution with Rayleigh-Jeans $1/k^2$ high-momentum tail

Strong cut-off dependence
What about quantum effects?
How to recover Planckian?

Sun et al., Nature Physics 8, 470 (2012)
How to include quantum fluctuations beyond MF

Requires going beyond monochromatic beam and explicitly including physical time

Gross-Pitaevskii-like eq. for propagation of quasi-monochromatic field

\[ i \frac{\partial E}{\partial z} = - \frac{1}{2\beta_0} \left( \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} \right) - \frac{1}{2D_0} \frac{\partial^2 E}{\partial t^2} + V(r)E + g|E|^2E \]

Propagation coordinate \( z \rightarrow \) time
Physical time \( \rightarrow \) extra spatial variable, dispersion \( D_0 \rightarrow \) temporal mass

Upon quantization \( \rightarrow \) conservative many-body evolution in \( z \):

\[ i \frac{d}{dz} |\psi> = H |\psi> \]

with

\[ H = N \iiint dx \, dy \, dt \left[ \frac{1}{2\beta_0} \nabla \hat{E}^\dagger \nabla \hat{E} - \frac{D_0}{2} \frac{\partial \hat{E}^\dagger}{\partial t} \frac{\partial \hat{E}}{\partial t} + V \hat{E}^\dagger \hat{E} + \hat{E}^\dagger \hat{E}^\dagger \hat{E} \hat{E} \right] \]

Same \( z \) commutator

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May simulate dynamical Casimir effect & fluctuations in early universe

First peak propagates at the speed of sound $c_s$

Value of $c_s$ different in spatial (xy) and temporal (t) directions

Experiments on-going to measure $c_s^{xy}$ (Heriot-Watt) and $c_s^t$ (Trento)

Quantum dynamics even more interesting in strongly nonlinear media

A quite generic quantum simulator

Quantum many-body evolution in $z$:

$$i \frac{d}{dz} |\psi> = H |\psi> \quad \text{with:} \quad H = N \iint dx \, dy \, dt \left[ \frac{1}{2\beta_0} \nabla \hat{E}^\dagger \nabla \hat{E} - \frac{D_0}{2} \frac{\partial \hat{E}^\dagger}{\partial t} \frac{\partial \hat{E}}{\partial t} + V \hat{E}^\dagger \hat{E} + \hat{E}^\dagger \hat{E} \hat{E}^\dagger \hat{E} \right]$$

- Physical time $t$ plays role of extra spatial coordinate
- Same $z$ commutator:
  $$[\hat{E}(x,y,t,z), \hat{E}^\dagger(x',y',t',z)] = \frac{c \hbar \omega_0 v_0}{\epsilon} \delta(x-x') \delta(y-y') \delta(t-t')$$

Clever design of $V(x,y,z) \rightarrow$ simulate wide variety of physical systems:

- Floquet topological insulators
- Arbitrary splitting/recombination of waveguides $\rightarrow$ quench of tunneling
- In addition to photonic circuit $\rightarrow$ many-body due to photon-photon interactions
- On top of moving fluid of light $\rightarrow$ simulate general relativistic QFT


Rechtsman et al., Nature 2012
Part II: Toroidal Landau levels
Berry curvature & quantum mechanics

Chang-Niu's semiclassical equations of motion:

\[ \hbar \dot{k}_c(t) = eE, \]
\[ \hbar \dot{r}_c(t) = \nabla_k \varepsilon_{n,k} - eE \times \Omega_n(k) \]

Can be derived from quantum Hamiltonian

\[ H = E_n(p) + W[r + A_n(p)] \quad \text{with} \quad W(r) = -eE r \]

Similar to minimal coupling \( H = e \Phi(r) + [p - eA(r)]^2 / 2m \) with \( r \leftrightarrow p \) exchanged

Physical position \( r_{ph} = r + A_n(p) \) \( \leftrightarrow \) physical momentum \( p - eA(r) \)

Berry connection \( A_n(p) \) \( \leftrightarrow \) magnetic vector potential \( A(r) \)

Berry curvature \( \Omega_n(p) = \text{curl}_p A_n(p) \) \( \leftrightarrow \) magnetic field \( B(r) = \text{curl}_r A(r) \)

Band dispersion \( E_n(p) \) \( \leftrightarrow \) scalar potential \( e \Phi(r) \)

Trap energy \( W(r) \) \( \leftrightarrow \) kinetic energy \( p^2/2m \)


and references therein (starting from Karplus-Luttinger 1954)
Harper-Hofstadter model + harmonic trap

Magnetic flux per plaquette $\alpha = 1/q$:
- for large $q$, bands almost flat $E_n(p) \approx E_n$
- lowest bands have $C_n = -1$ and almost uniform Berry curvature $\Omega_n = a^2/2\pi\alpha$

Within single band approximation:

Momentum space magnetic Hamiltonian $H = E_n(p) + k[r + A_n(p)]^2/2$
equivalent to quantum particle in constant $B$: $H = e \Phi(r) + [p - e A(r)]^2 / 2m$

Mass fixed by harmonic trap strength $k$

- Landau Levels spaced by “cyclotron” $\rightarrow k |\Omega_n|$
- Global (toroidal) topology of FBZ matters!! Degeneracy of LLs reduced to $|C_n|$

Of course, if:
- Too small $\alpha$ / too strong trap $\rightarrow$ band too close for single band approx
- Too large $\alpha$ / too weak trap $\rightarrow$ effect of $E_n(p)$ important
Numerical spectrum

Landau levels of lowest HH band crossing with Landau levels of second HH band

$\alpha \to 0$ harmonic trap states (band gap too small)

Landau levels of lowest HH band

9\textsuperscript{th} and 48\textsuperscript{th} state for $\alpha = 1/11$

eigen-functions recover

$\beta = 8$ Landau level on torus

for 1\textsuperscript{st} and 2\textsuperscript{nd} HH bands.

Only difference is Bloch function
Part IV:

Strongly interacting photons: from Tonks-Girardeau gas to Fractional Quantum Hall states
**Photon blockade**

Cavity array + strong nonlinearity, e.g. via Rydberg atoms

**Bose-Hubbard model:**

\[ H_0 = \sum_i \hbar \omega_0 \hat{b}_i^{\dagger} \hat{b}_i - \hbar J \sum_{\langle i, j \rangle} \hat{b}_i^{\dagger} \hat{b}_j + \hbar \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) \]

- single-mode cavities at \( \omega_0 \). Tunneling coupling \( J \)
- Polariton interactions: on-site repulsion \( U \)

If \( U \gg \gamma, J \), coherent pump resonant with \( 0 \rightarrow 1 \) transition, but not with \( 1 \rightarrow 2 \) transition.

Photon blockade \( \rightarrow \) Effectively impenetrable photons

- Incident laser: coherent external driving \( H_d = \sum_i F_i(t) \hat{b}_i + h.c. \)
- Weak losses \( \gamma \ll J, U \) \( \rightarrow \) Lindblad terms in master eq.
  determine non-equilibrium steady-state
- Strong number fluctuations \( \rightarrow \) dramatic effect on MI, but....
Impenetrable “fermionized” photons in 1D necklaces

Many-body eigenstates of Tonks-Girardeau gas of impenetrable photons

Coherent pump selectively addresses specific many-body states

Transmission spectrum as a function pump frequency for fixed pump intensity:
- each peak corresponds to a Tonks-Girardeau many-body state $|q_1, q_2, q_3...>$
- $q_i$ quantized according to PBC/anti-PBC depending on $N=\text{odd/even}$
- $U/J \gg 1$: efficient photon blockade, impenetrable photons.

N-particle state excited by N photon transition:
- Plane wave pump with $k_p=0$: selects states of total momentum $P=0$
- Monochromatic pump at $\omega_p$: resonantly excites states of many-body energy $E$ such that $\omega_p = E / N$

See also related work D. E. Chang et al, Nature Physics (2008)
State tomography from emission statistics

Finite $U/J$, pump laser tuned on two-photon resonance
- intensity correlation between the emission from cavities $i_1$, $i_2$
- at large $U/\gamma$, larger probability of having $N=0$ or 2 photons than $N=1$
  - low $U<<J$: bunched emission for all pairs of $i_1$, $i_2$
  - large $U>>J$: antibunched emission from a single site
    positive correlations between different sites
- Idea straightforwardly extends to more complex many-body states.
Photon blockade + synthetic gauge field = QHE for light

Bose-Hubbard model:

\[ H_0 = \sum_i \hbar \omega \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j e^{i \Phi_{ij}} + \hbar \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) \]

gauge field gives phase in hopping terms

with usual coherent drive and dissipation → look for non-equil. steady state

Transmission spectra:

• peaks correspond to many-body states
• comparison with eigenstates of \( H_0 \)
• good overlap with Laughlin wf (with PBC)

\[ \psi_l(z_1, ..., z_N) = N_L F_{CM}^{(l)}(Z) e^{-\pi \alpha \sum_i |z_i|^2} \times \prod_{i<j}^{N} \left( \frac{1}{2} \right) \left( \frac{z_i - z_j}{L} | i \right) 2 \]
• no need for adiabatic following, etc....

See also related work by Cho, Angelakis, Bose, PRL 2008; Hafezi et al. NJP 2013; arXiv:1308.0225
Tomography of FQH states

Homodyne detection of secondary emission

→ info on many-body wavefunction

\[ \langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle \]

Note: optical signal gauge dependent, optical phase matters!

Non-trivial structure of Laughlin state compared to non-interacting photons

A simpler design: rotating photon fluids

Rotating system at angular speed $\Omega$. No need for cavity array

$\text{Coriolis } F_c = -2m\Omega \times v$

$\text{Lorentz } F_L = e v \times B$

Rotating photon gas injected by LG pump with finite orbital angular momentum

Strong repuls. interact., e.g. layer of Rydberg atoms

Resonant peak in transmission due to Laughlin state:

$$\psi(z_1, \ldots, z_N) = e^{-\sum_i |z_i|^2/2} \prod_{i<j} (z_i - z_j)^2$$

Overlap measured from quadrature noise of transmitted light

$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X^{(i)}_0 X^{(j)}_0 \rangle - \langle X^{(i)}_{\pi/2} X^{(j)}_{\pi/2} \rangle + i \langle X^{(i)}_0 X^{(j)}_{\pi/2} \rangle + i \langle X^{(i)}_{\pi/2} X^{(j)}_0 \rangle$$

Anyonic braiding phase

- LG pump to create and maintain quantum Hall liquid

- Localized repulsive potentials in trap:
  → create quasi-hole excitation in quantum Hall liquid
  → position of holes adiabatically braided in space

- Anyonic statistics of quasi-hole: many-body Berry phase $\phi_{\text{Br}}$ when positions swapped during braiding

- Berry phase extracted from shift of transmission resonance while repulsive potential moved with period $T_{\text{rot}}$ along circle
  $$\phi_{\text{Br}} \equiv (\Delta\omega_\infty - \Delta\omega_o) T_{\text{rot}} \left[ \frac{2\pi}{2\pi} \right]$$

- so far, method restricted to low particle number