New approaches to topological and non-reciprocal photonic states

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- Squeezing to producing new kinds of topological states (bosonic analogue of a topological superconductor?)
  - With V. Peano, M. Houde, C. Brendel and F. Marquardt; arXiv:1508.01383
- Reservoir engineering to make arbitrary interactions non-reciprocal
  - With A. Metelmann, PRX 5, 021025 (2015)
Motivation

• Can we realize “simple” topological states of photons (or phonons)?
  • e.g. Chern insulators, QHE-like states (quadratic Hamiltonians!)

• Why?
  • Robust generation of protected edge modes, “one-way” waveguides
    (recent review: Lu et al, Nat. Photon. 2014)

Basic physics...

\[ \hat{H} = -J \sum_{\langle i,j \rangle} \left( e^{i\phi_{ij}} \hat{c}_i^\dagger \hat{c}_j + h.c. \right) \]

• Topology encoded in single-particle wavefunctions
• No essential difference for bosons or fermions

Possible Realization of Directional Optical Waveguides in Photonic Crystals
with Broken Time-Reversal Symmetry

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We show how, in principle, to construct analogs of quantum Hall edge states in “photonic crystals”
made with nonreciprocal (Faraday-effect) media. These form “one-way waveguides” that allow electromagnetic energy to flow in one direction only.

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Particle non-conserving terms

- What if we consider a more general quadratic Hamiltonian?
  - Allow particle-number non-conserving terms…

\[ \hat{H} = \sum_{k, n} \varepsilon_n[k] b_{k, n}^\dagger b_{k, n} + \sum_{k, n, n'} \left( \lambda_{nn'}[k] b_{k, n}^\dagger b_{-k, n'}^\dagger + h.c. \right) \]

- Fermions?
  - Generic Hamiltonian of a superconductor
  - “Anomalous” terms can themselves lead to topological structures
    - e.g., spinless p-wave superconductor

\[ H_{\text{BdG}} = \frac{1}{2} \sum_{p} \Psi_p^\dagger \begin{pmatrix} \epsilon(p) & 2i\Delta' \sin p_x + i \sin p_y \\ -2i\Delta' \sin p_x - i \sin p_y & -\epsilon(p) \end{pmatrix} \Psi_{p'} \]

- Majorana modes at boundaries…
- Treat BdG wavefunctions like ordinary single particle wavefunctions…

\[ \sum_j |u_j[k]|^2 + |v_j[k]|^2 = 1 \]

\[ A_l = i \langle k_l | \nabla_k | k_l \rangle \]
Particle non-conserving terms

• What if we consider a more general quadratic Hamiltonian?
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\[ \hat{H} = \sum_{k,n} \varepsilon_n[k] b_{k,n}^\dagger b_{k,n} + \sum_{k,n,n'} (\lambda_{nn'}[k] b_{k,n}^\dagger b_{-k,n'}^\dagger + h.c.) \]

• Fermions?
  • Generic Hamiltonian of a topological superconductor
  • “Anomalous” terms can directly give rise to topological structures
    • Non-trivial winding of order parameter

(Ryu, Schneider, Furusaki and Ludwig, NJP 2010)
Topological photonic superconductors?

- What if we consider a more general quadratic Hamiltonian?
  - Allow particle-number non-conserving terms...
    \[ \hat{H} = \sum_{k,n} \varepsilon_n[k] b^\dagger_{k,n} b_{k,n} + \sum_{k,n,n'} (\lambda_{nn'}[k] b^\dagger_{k,n} b^\dagger_{-k,n'} + h.c.) \]

- Bosons?
  - No longer identical to the fermionic problem...
    - Pairing terms not constrained by Pauli principle
    - Possibility of instabilities (⇒ usual classification fails?)
    - Definition of a Chern number?
      - No trivial mapping to single particle wavefunctions

- Topological phases?
- Protected edge modes?
  - Transport properties?

Related work:
- Shindou et al, PRB 2013 (magnons)
- Brandes et al, arXiv.1503.02503
- Bardyn et al, arXiv.1503.08824 (weakly interacting condensates)
Simple model

• Bosons hopping on Kagome lattice:

\[ \hat{H} = -J \sum_{\langle i,j \rangle} \left( \hat{c}^\dagger_i \hat{c}_j + h.c. \right) \]

• No flux, no anomalous terms
  • Three bands, no gaps
• Realizations?
  • Arrays of superconducting microwave resonators
    (Koch et al PRA 2010)
  • Arrays of defect cavities in a photonic crystal
    (e.g. Painter group)
Simple model

• Bosons hopping on Kagome lattice:

\[ \hat{H} = -J \sum_{\langle i,j \rangle} \left( e^{i\phi_{ij}} \hat{c}_i^\dagger \hat{c}_j + h.c. \right) \]

• No flux, no anomalous terms
  • Three bands, no gaps

• Add hopping phases
  • Break TRS, anomalous QHE (Ohgushi, Murakami, Nagaosa, PRB 2000)
  • Gaps, non-zero Chern numbers
  • Protected edge states

(Petrescu & LeHur, PRA 2012)
Break TRS via parametric driving

\[ \hat{H} = \hat{H}_0 + \hat{H}_{\text{par}} \]

\[ \hat{H}_0 = \sum_j \omega_0 \hat{a}_j^\dagger \hat{a}_j - J \sum_{\langle ij \rangle} \hat{a}_i^\dagger \hat{a}_j \]

\[ \hat{H}_{\text{par}} = -\frac{\nu_{\text{on}}}{2} \sum_j e^{i\phi_j} \hat{a}_j^\dagger \hat{a}_j^\dagger - \frac{\nu_{\text{off}}}{2} \sum_{\langle ij \rangle} e^{i\phi_{ij}} \hat{a}_i^\dagger \hat{a}_j^\dagger + \text{h.c.} \]

- “Pairing terms” break both time-reversal symmetry and charge conservation
  - Keep C3 rotation symmetry for simplicity
  - Pairing terms have a phase-winding in real space

\[ \phi_\alpha = 0, \pm 2\pi / 3 \]
Break TRS via parametric driving

\[ \hat{H} = \hat{H}_0 + \hat{H}_{\text{par}} \]

\[ \hat{H}_0 = \sum_i \omega_0 \hat{a}_i^{\dagger} \hat{a}_i - J \sum_{\langle ij \rangle} \hat{a}_i^{\dagger} \hat{a}_j \]

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- “Pairing terms” break both time-reversal symmetry and charge conservation (keep C3 rotation symmetry)
- **Focus on regime where system remains stable without dissipation** \((\nu < \omega_0 - 4J)\)
Physical realization?

\[
\hat{H}_{\text{par}} = -\frac{\nu_{\text{on}}}{2} \sum_j e^{i\phi_j} \hat{a}_j \hat{a}_j^\dagger - \frac{\nu_{\text{off}}}{2} \left| i j \right\rangle e^{i\phi_{ij}} \hat{a}_i \hat{a}_j^\dagger + h.c.
\]

- Array of defect cavities in a nonlinear photonic crystal ($\chi_2$ material)
- Phase control: pump e.g. with three lasers…..

\[
\hat{H}_{\text{NL}} = \Lambda \hat{a}^\dagger \hat{a}^\dagger \hat{b} + h.c.
\approx \Lambda \hat{a}^\dagger \hat{a}^\dagger \langle \hat{b} \rangle + h.c.
\]

- Other possibilities?
  - Optomechanical arrays?
  - Arrays of nonlinear superconducting cavities…
Band structure: bulk

- Parametric driving opens gaps…

\[ J = 0.02 \omega_0 \]

\[ \nu_{\text{on}} = -0.09 \omega_0 \]

\[ \nu_{\text{off}} = 0.22 \omega_0 \]
Band structure: bulk

- Parametric driving opens gaps...

\[ J = 0.02 \omega_0 \]

\[ \nu_{\text{on}} = -0.09 \omega_0 \]

\[ \nu_{\text{off}} = 0.22 \omega_0 \]
Band structure: strip geometry
Defining a topological invariant

\[ \hat{H}_{\text{par}} = -\frac{\nu_{\text{on}}}{2} \sum_{j} e^{i \phi_{j}} \hat{a}_{j} \hat{a}_{j}^{\dagger} - \frac{\nu_{\text{off}}}{2} \sum_{\langle ij \rangle} e^{i \phi_{ij}} \hat{a}_{i} \hat{a}_{j}^{\dagger} + \text{h.c.} \]

- Can’t directly take BdG wavefunctions ~ single-particle wavefunction
- Back to basics: consider Berry phase of QP states

1. \( H[k] \): effective 3-mode parametric amplifier problem
   1. Ground state: \( |\Omega[\vec{k}]\rangle \)
   2. QP state: \( |n, \vec{k}\rangle = \hat{\beta}^{\dagger} [\vec{k}, n] |\Omega[\vec{k}]\rangle \)

\[ C_l = -\frac{1}{2\pi} \int_{BZ} d^2 k (\nabla_k \times A_1(k)) \cdot \mathbf{e}_z \]
Defining a topological invariant

\[ \hat{H}_{\text{par}} = -\frac{\nu_{\text{on}}}{2} \sum_j e^{i\phi_j} \hat{a}_j^\dagger \hat{a}_j^\dagger - \frac{\nu_{\text{off}}}{2} \sum_{\langle ij \rangle} e^{i\phi_{ij}} \hat{a}_i^\dagger \hat{a}_j^\dagger + \text{h.c.} \]

- Can’t directly take BdG wavefunctions \( \sim \) single-particle wavefunction
- Back to basics: consider Berry phase of QP states

1. \( H[k] \): effective 3-mode parametric amplifier problem
   1. Ground state: \( |\Omega[\vec{k}]\rangle \)
   2. QP state: \( |n, \vec{k}\rangle = \hat{\beta}^\dagger [\vec{k}, n] |\Omega[\vec{k}]\rangle \)

2. Berry’s phase for a single quasiparticle:

\[ \vec{A}[^{\vec{k}}] = i \langle n, \vec{k} | \vec{\nabla} | n, \vec{k} \rangle \]
\[ = \vec{A}[^{\vec{k}}]_{\text{gnd}} + \vec{A}[^{\vec{k}}]_{\text{qp}} \]
\[ \vec{A}[^{\vec{k}}]_{\text{qp}} = i \left( \sum_j u_j^* \vec{\nabla} u_j - \sum_j v_j^* \vec{\nabla} v_j \right) \]

(see also Shindou et al, PRB 2013)
Chern numbers

\[ \hat{H}_{\text{par}} = -\frac{\nu_{\text{on}}}{2} \sum_j e^{i\phi_j} \hat{a}_j^\dagger \hat{a}_j^\dagger - \frac{\nu_{\text{off}}}{2} \sum_{\langle ij \rangle} e^{i\phi_{ij}} \hat{a}_i^\dagger \hat{a}_j^\dagger + \text{h.c.} \]

- Phases with “Standard” Chern numbers
  - \([-1, 0, 1]\)
  - \([1, 0, -1]\)

- Phases not found in particle-conserving model:
  - \([-2, 1, 1]\)
  - \([1, -4, 3]\)
  - \([1, -2, 1]\)
  - \([-2, -1, 3]\)
  - \([-4, 3, 1]\)

- NB: Chern numbers of “particle” bands must sum to zero (no zero modes)
Effective model

- Simpler structure: diagonalize on-site Hamiltonian

\[ \hat{H} = \sum_{j} \tilde{\omega} \hat{\alpha}_j^\dagger \hat{\alpha}_j - \sum_{\langle j, l \rangle} \tilde{J}_{jl} \hat{\alpha}_j^\dagger \hat{\alpha}_l - \left( \frac{\tilde{\nu}}{2} \sum_{\langle j, l \rangle} \hat{\alpha}_j^\dagger \hat{\alpha}_l^\dagger + h.c. \right) \]

- Berry connection invariant under local squeezing transformation
- Reduced number of dimensionless parameters:

\[ \tilde{\nu}, \tilde{\omega}, \tilde{\phi}_{AB} \]
Transport

\[ \hat{H}_{\text{par}} = -\frac{\nu_{\text{on}}}{2} \sum_j e^{i\phi_j} \hat{a}_j^{\dagger} \hat{a}_j^{\dagger} - \frac{\nu_{\text{off}}}{2} \sum_{\langle ij \rangle} e^{i\phi_{ij}} \hat{a}_i^{\dagger} \hat{a}_j^{\dagger} + \text{h.c.} \]

- As usual, edge states mediate chiral, protected transport
  - BUT: elastic and inelastic transport channels!

- Recall: parametric driving at freq. \( \omega_L \)
  - This was shifted to zero in our rotating frame
Transport

- Edge states mediate chiral, protected “inelastic” transport
  - Inelastic transport dominates for driving of “hole” bands
- Amplifying behaviour possible (even without entering unstable regime)

\( \omega_0 = 1, J = 0.02, \nu_{on} = 0.4, \nu_{off} = 0.02, \kappa = 0.001 \)
Classification

\[ \hat{H}_{\text{par}} = -\frac{\nu_{\text{on}}}{2} \sum_j e^{i\phi_j} \hat{a}_j \hat{a}_j^{\dagger} - \frac{\nu_{\text{off}}}{2} \sum_{\langle ij \rangle} e^{i\phi_{ij}} \hat{a}_i \hat{a}_j^{\dagger} + h.c. \]

• Can we use standard classification of fermionic non-interacting Hamiltonians?
  • Standard route (Ryu et al, NJP 2010):
    • Consider disorder problem
      (ensemble of random matrices with given symmetry)
    • Look at corresponding NLSM, possibility of adding topological terms
  • Issue here:
    • Cannot formulate disorder problem the same way
      • Condition of stability \( \rightarrow \) matrix elements of \( \hat{H} \) correlated
    • Standard symmetry classes of limited used
      (Guararie and Chalker, PRL 2002, PRB 2003)
Nonreciprocal transport?

• Scattering description:

\[
\begin{bmatrix}
    a_{L,\text{out}} \\
    a_{R,\text{out}}
\end{bmatrix} =
\begin{pmatrix}
    0 & 0 \\
    s_{21} & 0
\end{pmatrix}
\begin{bmatrix}
    a_{L,\text{in}} \\
    a_{R,\text{in}}
\end{bmatrix}
\]

• Tunneling interaction:

\[
\hat{H} = t_{21} \hat{d}_2 \hat{d}_1 + t_{12} \hat{d}_1^{\dagger} \hat{d}_2
\]

Want: \( t_{12} \neq t_{21} \)

• Asymmetric phases ~ TRS breaking
• Asymmetric magnitudes = ????

• Motivation for reciprocity-breaking interactions?
  • Devices: isolators, circulators, \textbf{directional amplifiers}, …
  • Without magnetic materials / magneto-optic effects?
  • A route to new kinds of quantum phases?
Dissipative quantum interactions?

- Goal: make a given interaction “one-way” (non-reciprocal)  
  (A. Metelmann and AC, PRX 2015)

\[ \hat{H}_{coh} = \lambda \left( \hat{O}_1 \hat{O}_2 + h.c. \right) \]

- General solution:
  - use additional interaction mediated by a dissipative reservoir

\[ \frac{d}{dt} \hat{\rho} = -i[\hat{H}_{coh}, \hat{\rho}] + \Gamma \left( \hat{z} \hat{\rho} \hat{z}^\dagger - \frac{1}{2} \{ \hat{z}^\dagger \hat{z}, \hat{\rho} \} \right) \]

\[ \hat{z} = \hat{O}_1 + e^{i\phi} \hat{O}_2^\dagger \]

- Makes any interaction unidirectional...
  - Applications: non-reciprocal photonic devices
  - Non-reciprocal many-body interactions?
Dissipative quantum interactions?

- E.g.: Reservoir-engineered isolators & amplifiers (A. Metelmann and AC, PRX 2015)

- Bandwidth of non-reciprocity set by response time of bath
- Generalization of "cascaded quantum systems" theory
Conclusions

- Parametric driving as a route to new kinds of topological states
  (Peano, Houde, Brendel, Marquardt & AC, arXiv:1508.01383)

\[
\hat{H}_{\text{par}} = -\frac{\nu_{\text{on}}}{2} \sum_j e^{i\phi_j} \hat{a}_j \hat{a}_j - \frac{\nu_{\text{off}}}{2} \sum_{\langle ij \rangle} e^{i\phi_{ij}} \hat{a}_i \hat{a}_j + \text{h.c.}
\]

- Edge states which facilitate protected & directional inelastic transport and even amplification

- Reservoir engineering as a route to non-reciprocal interactions
  (A. Metelann and AC, PRX 2015)

- A route to new kinds of non-reciprocal devices
- A route to new kinds of interacting photonic phases?