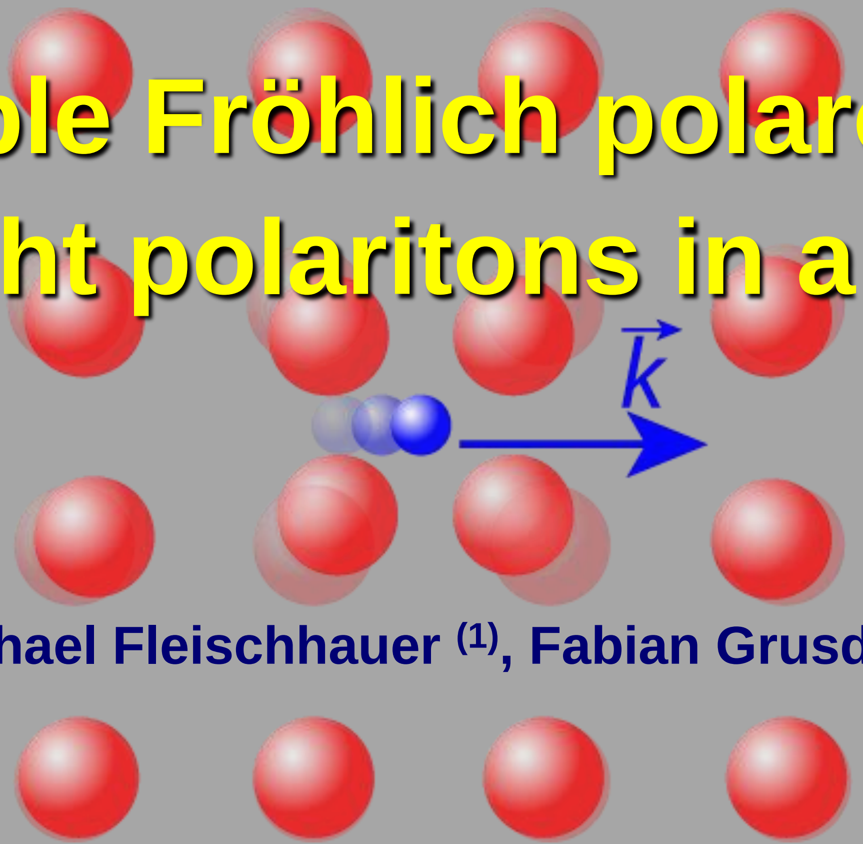
Tunable Fröhlich polarons of slow-light polaritons in a 2D BEC



Michael Fleischhauer [1], Fabian Grusdt [1,2]

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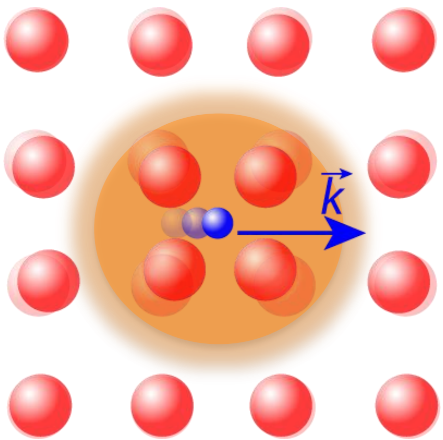
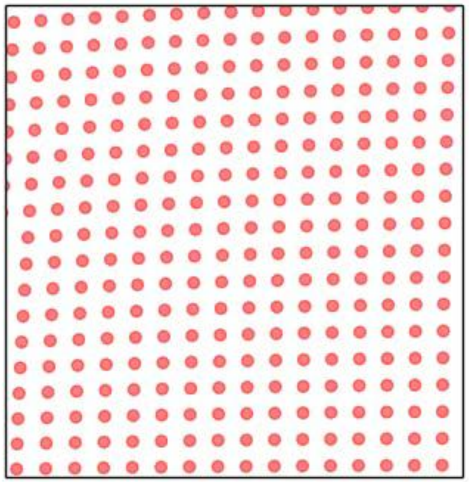
Dense light, KITP, Santa Barbara 06.10.2015

F. Grusdt, M. Fl.
arXiv 1507.08248

polarons in solids

- in solid-state systems

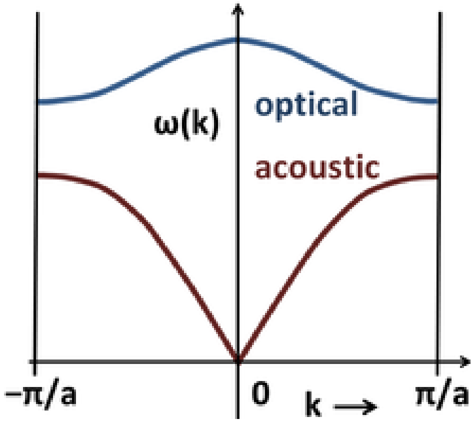




Fröhlich model, Adv. Phys. (1954)

$$H_0 = \int \mathrm{d}k \left(\omega_k \hat{a}_k^\dagger \hat{a}_k + \frac{k^2}{2M} \hat{\Psi}_k^\dagger \hat{\Psi}_k \right)$$

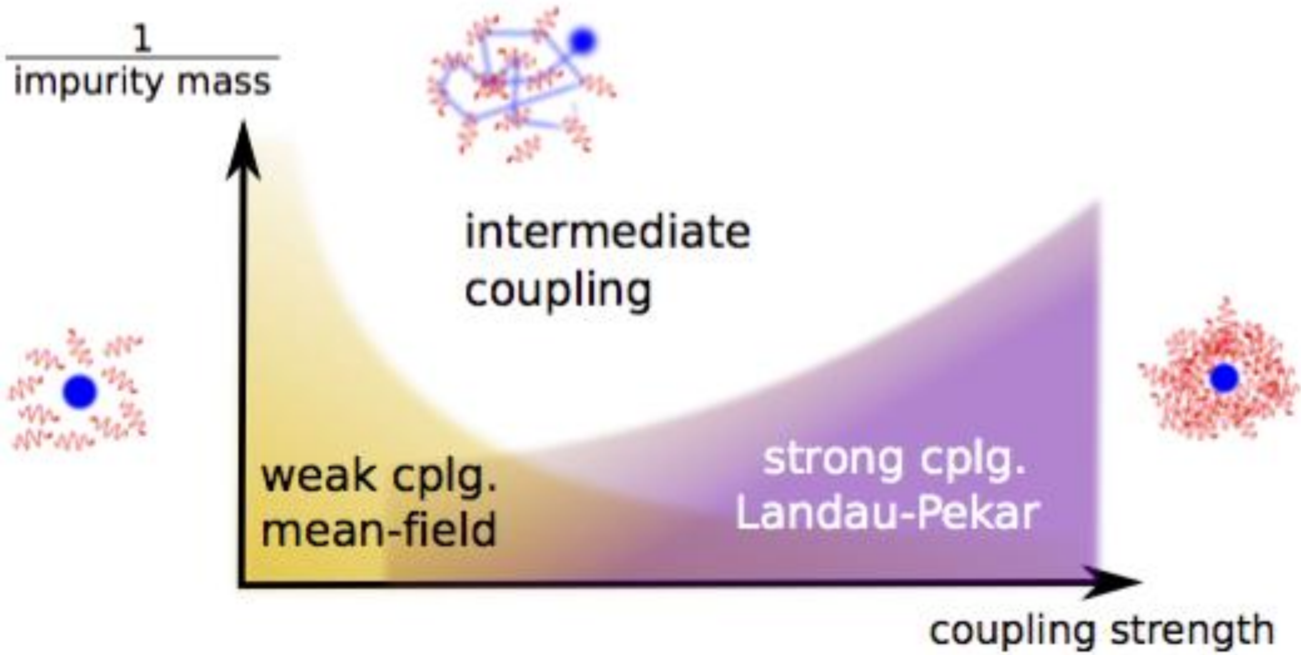
$$H_F = \int \mathrm{d}k \int \mathrm{d}r \, \hat{\Psi}^\dagger(r) \hat{\Psi}(r) \, V_k \, e^{ikr} \, \left(\hat{a}_k + \hat{a}_{-k}^\dagger \right)$$



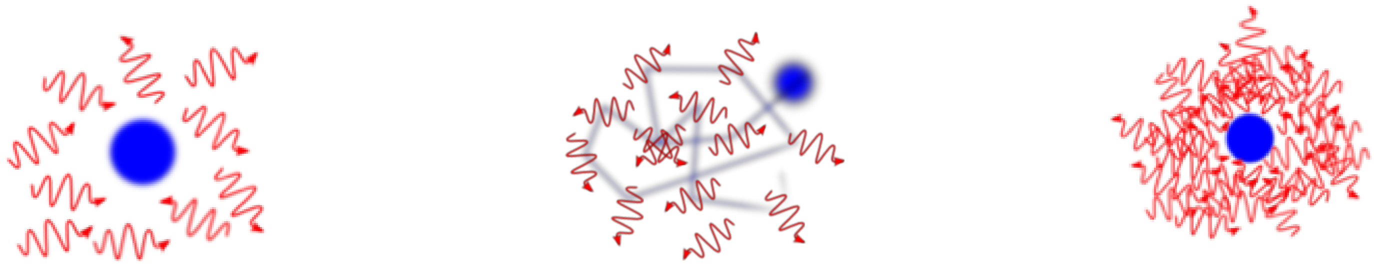
polarons in solids

coupling strength $\alpha \sim \left(\frac{E_{\mathrm{IH}}}{E_{\mathrm{ph}}}\right)^2$

impurity mass $\frac{1}{M}$



polarons in solids



coupling strength α →

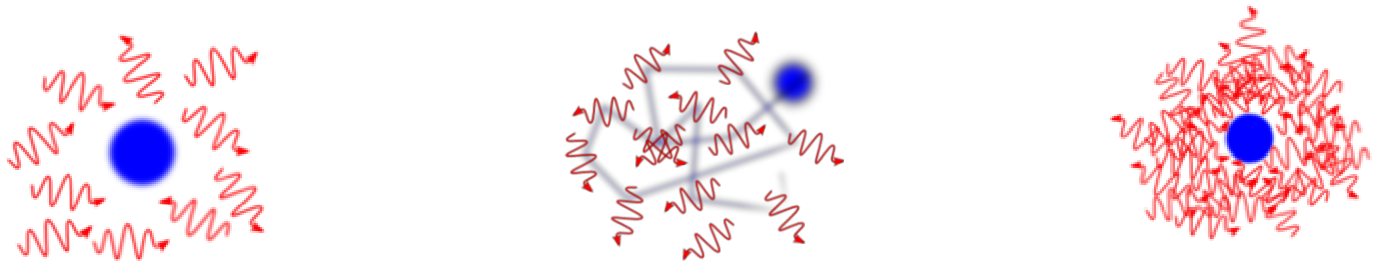
weak coupling | intermediate coupling | strong coupling

coherent displacement of phonon field

$$H = \int \mathrm{d}k \left[\omega_k \hat{a}_k^\dagger \hat{a}_k + V_k \left(\hat{a}_{-k}^\dagger + \hat{a}_k\right)\right] + \frac{1}{2M} \int \mathrm{d}q\, \hat{\Psi}_q^\dagger \hat{\Psi}_q \left(q - \int \mathrm{d}k\, k\, \hat{a}_k^\dagger \hat{a}_k\right)^2$$

phonon-phonon scattering

polarons in solids



coupling strength α

weak coupling

intermediate coupling

strong coupling

← Feynman variational approach →

perturbative
mean-field

Lee, Low, Pines,
Phys Rev (1953)

no phase transition

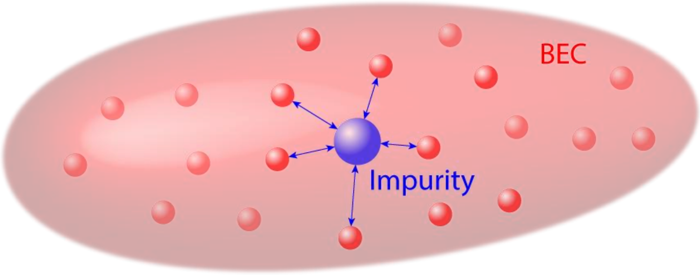
Gerlach, Löwen
RMP (1991)

self-trapping

Landau, Pekar
Zh. Exp. Th. Phyz.
(1946)

polarons in BEC

- impurity in a Bose-Einstein condensate



interaction with Bogoliubov phonons in unperturbed condensate

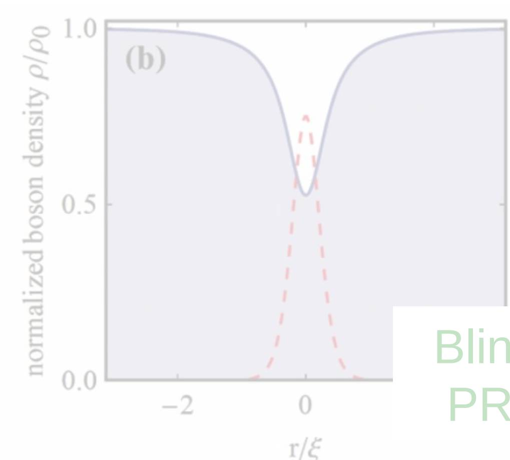
Fröhlich polaron

$$n_{\mathrm{ph}} \ll n_0$$

$$N_{\mathrm{ph}} = \int \mathrm{d}^3 k \left(\frac{V_k}{\omega_k} \right)^2 \sim \frac{n_0 g_{IB}^2}{c^2 \xi}$$

deformation of condensate

bubble polaron

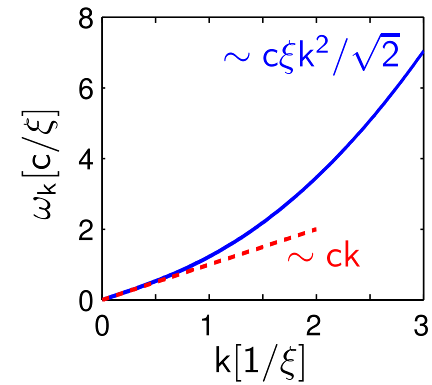


Blinova et al. PRA (2013)

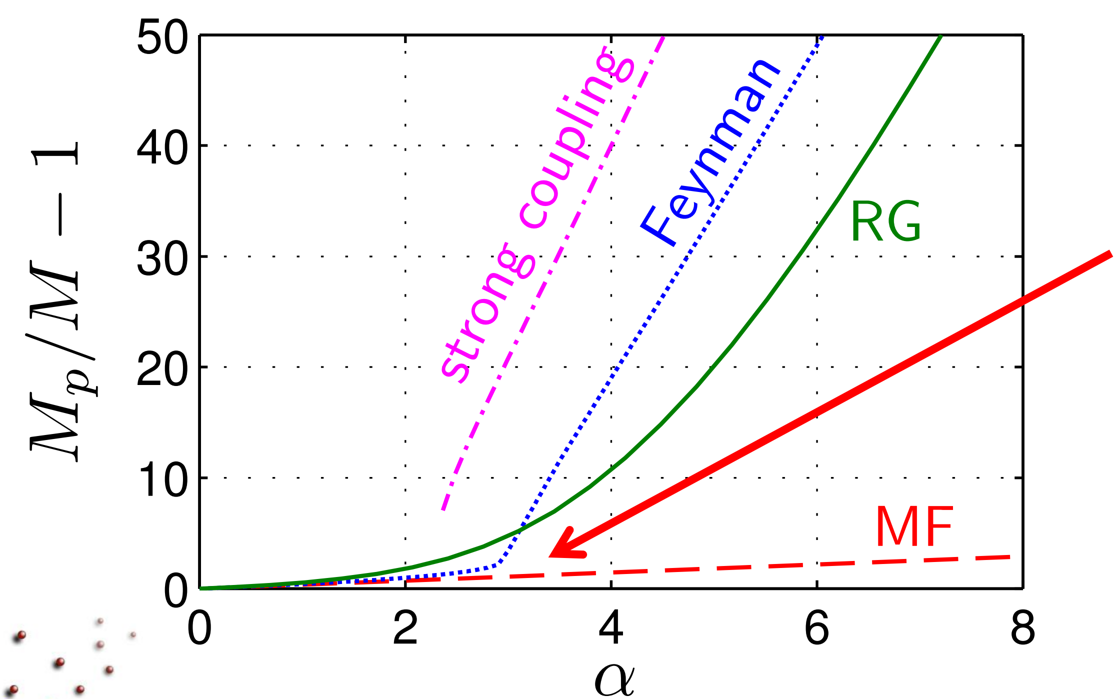
polarons in BEC

- phase transition vs. crossover ?

Gerlach - Löwen proof does not hold here



mass of polaron vs. bare mass



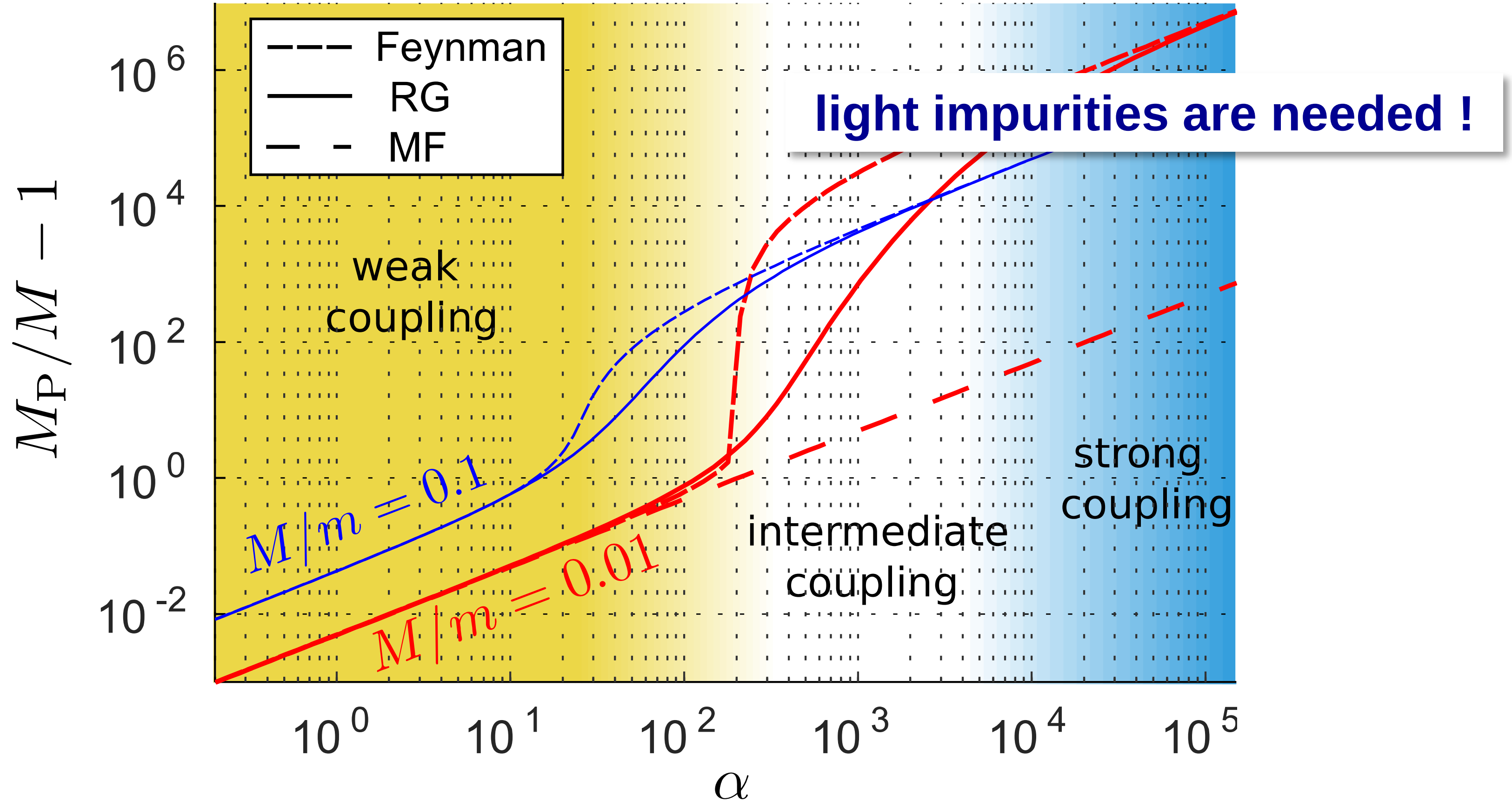
phase transition ?

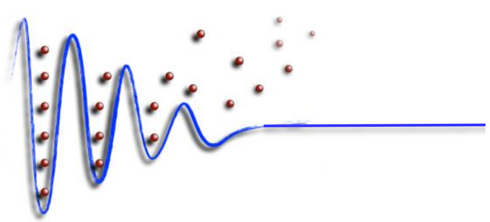
Grusdt et al.
Sci. Rep. 2015

Casteels et al.
PRA 2012

Fröhlich polarons in 2D BEC: extended RG

F. Grusdt, M. Fleischhauer, arXiv 1507.08248



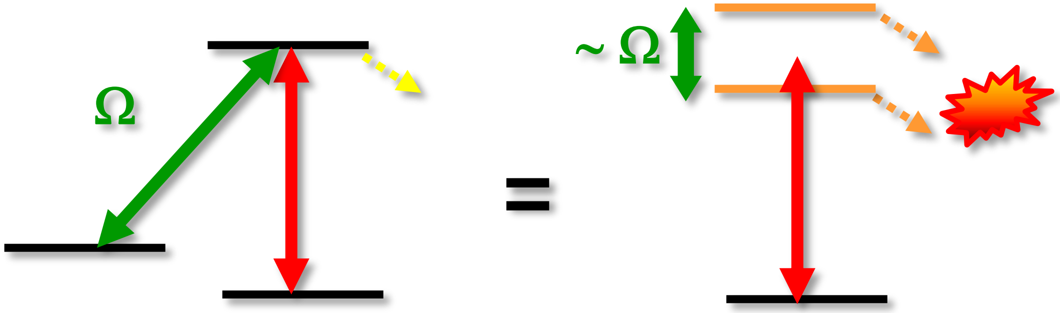


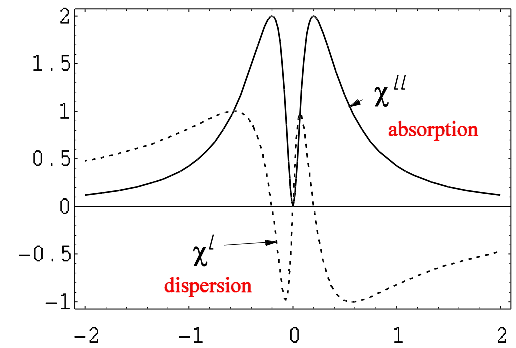
renormalization group approach

F. Grusdt, Y. Shchadilova, A. Rubtsev, E. Demler, Sci. Rep. (2015)

F. Grusdt, arXiv 1509.08974

Elm. Induced Transparency & slow light

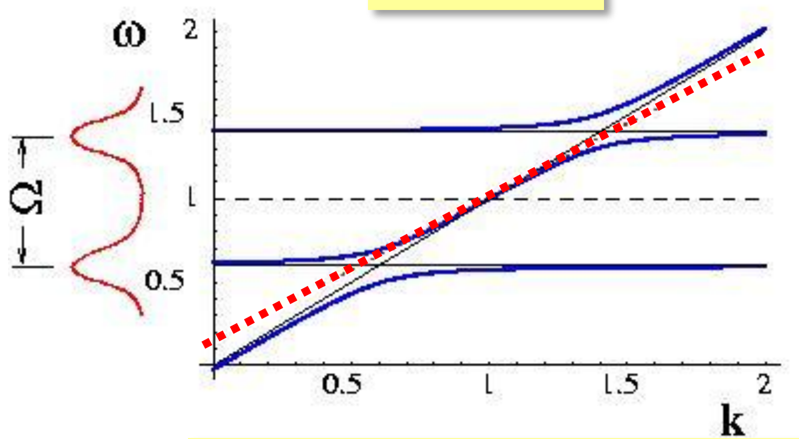




large Ω

dispersion

$\omega(k)$



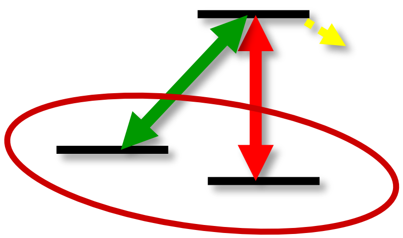
$$v_{\mathrm{gr}} = \frac{\mathrm{d}\omega}{\mathrm{d}k} = c_0 \cos^2\theta$$

$$\cos^2\theta = \frac{\Omega^2}{\Omega^2 + g^2 N}$$

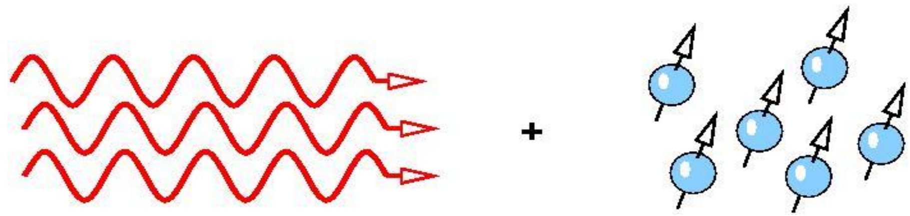
dark-state polaritons

- dark-state polariton

$$\hat{\Psi} = \cos\theta\,\hat{E} - \sin\theta\,\hat{S}$$



$$\hat{S}$$



$$v_{\rm gr} = c_0\,\cos^2\theta$$

$$\cos^2\theta = \frac{\Omega^2}{\Omega^2 + g^2 N}$$

$$\left[\frac{\partial}{\partial t} + v_{\rm gr}\frac{\partial}{\partial z} - \frac{i}{2m_{\|}}\frac{\partial^2}{\partial z^2} - \frac{i}{2m_{\perp}}\Delta_{\perp}\right]\hat{\Psi} = 0$$

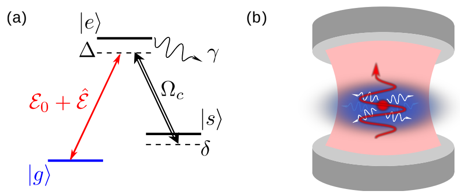
$$m_{\perp} = \frac{\hbar k}{v_{\rm gr}} = m\frac{v_{\rm rec}}{v_{\rm gr}}$$

- bright-state polariton (decays)

$$\hat{\Phi} = \cos\theta\,\hat{S} + \sin\theta\,\hat{E}$$

M.Fl., M. Lukin, PRL **84,** 5094 (2000); PRA **65**, 022314 (2002)

dark-state polariton impurity



- free Hamiltonian

$$\hat{\mathcal{H}}_0 = \int d^2\vec{r}\, \Big\{ -\hat{\psi}_g^\dagger \frac{\nabla^2}{2m}\hat{\psi}_g + \hat{\psi}_e^\dagger \left(-\frac{\nabla^2}{2m} - i\gamma + \Delta_0 \right) \hat{\psi}_e + $$

$$+ \hat{\psi}_s^\dagger \left(-\frac{\nabla^2}{2m} + \delta_0 \right) \hat{\psi}_s + \hat{\mathcal{E}}^\dagger \left(-\frac{\nabla^2}{2M_{\mathrm{ph}}} - i\kappa \right) \hat{\mathcal{E}} \Big\}$$

$$M_{\mathrm{ph}} = \frac{\hbar k}{c_0}$$

- atom-light interaction

$$\hat{\mathcal{H}}_{\mathrm{al}} = \int d^2\vec{r}\, \left\{ g_{2\mathrm{D}} \hat{\psi}_e^\dagger \hat{\psi}_g \hat{\mathcal{E}} + \Omega_c \hat{\psi}_e^\dagger \hat{\psi}_s + \mathrm{h.c.} \right\}$$

- BEC in ground state

$$\hat{\psi}_g \rightarrow \sqrt{n_0}$$

- dark-state polariton in a BEC

$$\hat{\Psi} = \sin\theta\, \hat{\psi}_s - \cos\theta\, \hat{\mathcal{E}}$$

$$\tan\theta = \frac{g_{2\mathrm{D}}\sqrt{n_0}}{|\Omega_c|}$$

dark-state polariton impurity

$$\hat{\mathcal{H}} = \int d^2\vec{k}\Big\{\hat{\Psi}_k^\dagger\hat{\Psi}_k\nu_k^{\mathrm{DSP}} + \hat{\Phi}_k^\dagger\hat{\Phi}_k\nu_k^{\mathrm{BSP}} + \hat{\psi}_e^\dagger\hat{\psi}_e\left(\frac{k^2}{2m} - i\gamma + \Delta_0\right) +$$

$$+\hat{\psi}_e^\dagger\hat{\Phi}_k\Omega_{\Phi e} + \hat{\Psi}_k^\dagger\hat{\Phi}_k\Omega_{\Psi\Phi}(k) + \mathrm{h.c.}\Big\}$$

bright-excited state coupling

non-adiabatic coupling

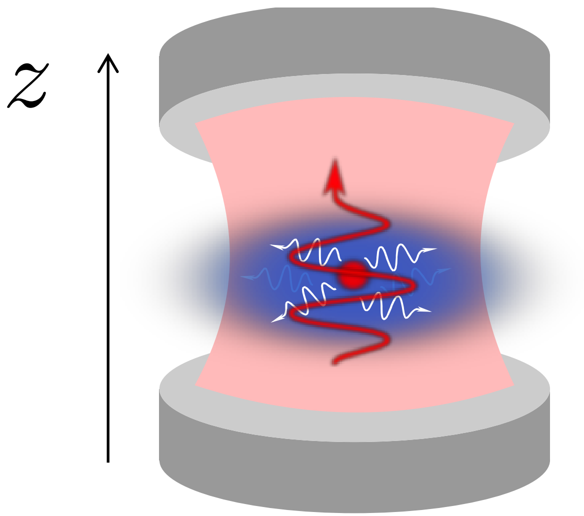
$$\Omega_{\Phi e} = \sqrt{\Omega_c^2 + g_{2\mathrm{D}}^2 n_0}$$

$$\Omega_{\Psi\Phi} = \cos\theta\sin\theta\left(\frac{k^2}{2m} - \frac{k^2}{2M_{\mathrm{ph}}} + i\kappa + n_{\mathrm{C}}\, g_{gg}^{2\mathrm{D}} - \mu_{\mathrm{BEC}}\right)$$

$$\Omega_{\Phi e} \gg \Omega_{\Psi\Phi}$$

dark-state polariton impurity

- problem: $v_{\mathrm{gr}} = c_0 \cos^2\theta \gg c$ DSP is super-sonic



2D confinement of light

2D confinement of BEC

to avoid interaction- induced scattering $\hat{\Psi} \rightarrow \hat{\Phi}$

- Frank-Condon reduction

$$g_{\mathrm{2D}} = g \int_0^L dz \left(\phi_e^0(z)\right)^* \phi_g^0(z) \mathcal{E}_z^0(z) \qquad \tan\theta = \frac{g_{\mathrm{2D}}\sqrt{n_0}}{|\Omega_c|}$$

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dark-state polariton impurity

- free polariton hamiltonian

$$\hat{\mathcal{H}} = \int d^2\vec{k}\ \hat{\Psi}_k^\dagger \hat{\Psi}_k \nu_k^{\rm DSP}$$

polariton dispersion

$$\nu_k^{\rm DSP} = \frac{k^2}{2M} - i\kappa \cos^2\theta + \mu_{\rm DSP}$$

cavity decay of elm. part

- impurity mass

$$\frac{1}{M} = \frac{\sin^2\theta}{m} + \frac{\cos^2\theta}{2M_{\rm ph}}$$

$$M_{\rm ph} = \frac{\hbar k}{c_0}$$

Fröhlich polarons of dark-state polaritons

- Interactions with Bogoliubov phonons

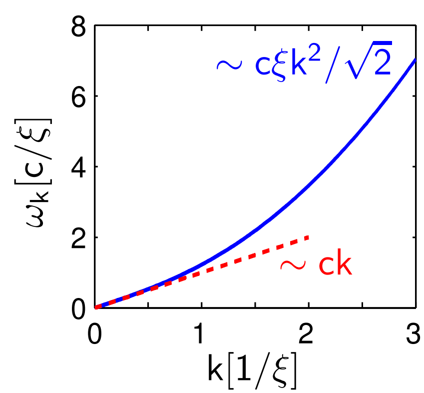
$$\hat{\psi}_g(\vec{r}) \to \sqrt{n_0} + \sum_{\vec{k}\neq 0} \frac{e^{i\vec{k}\cdot\vec{r}}}{L}\left(u_k \hat{a}_k - v_k \hat{a}^\dagger_{-k}\right)$$

- Fröhlich Hamiltonian

$$V_k = \sqrt{\alpha}\frac{c}{2\sqrt{\pi}}\left(\frac{k^2\xi^2}{2+k^2\xi^2}\right)^{1/4}$$

$$\hat{\mathcal{H}}_{\mathrm{F}} = \int d^2\vec{k}\left\{\omega_k \hat{a}^\dagger_k \hat{a}_k + \hat{\Psi}^\dagger_k \hat{\Psi}_k \nu^{\mathrm{DSP}}_k + \int d^2\vec{r}\;\hat{\Psi}^\dagger(\vec{r})\hat{\Psi}(\vec{r}) e^{i\vec{k}\cdot\vec{r}} V_k \left(\hat{a}_k + \hat{a}^\dagger_{-k}\right)\right\}$$

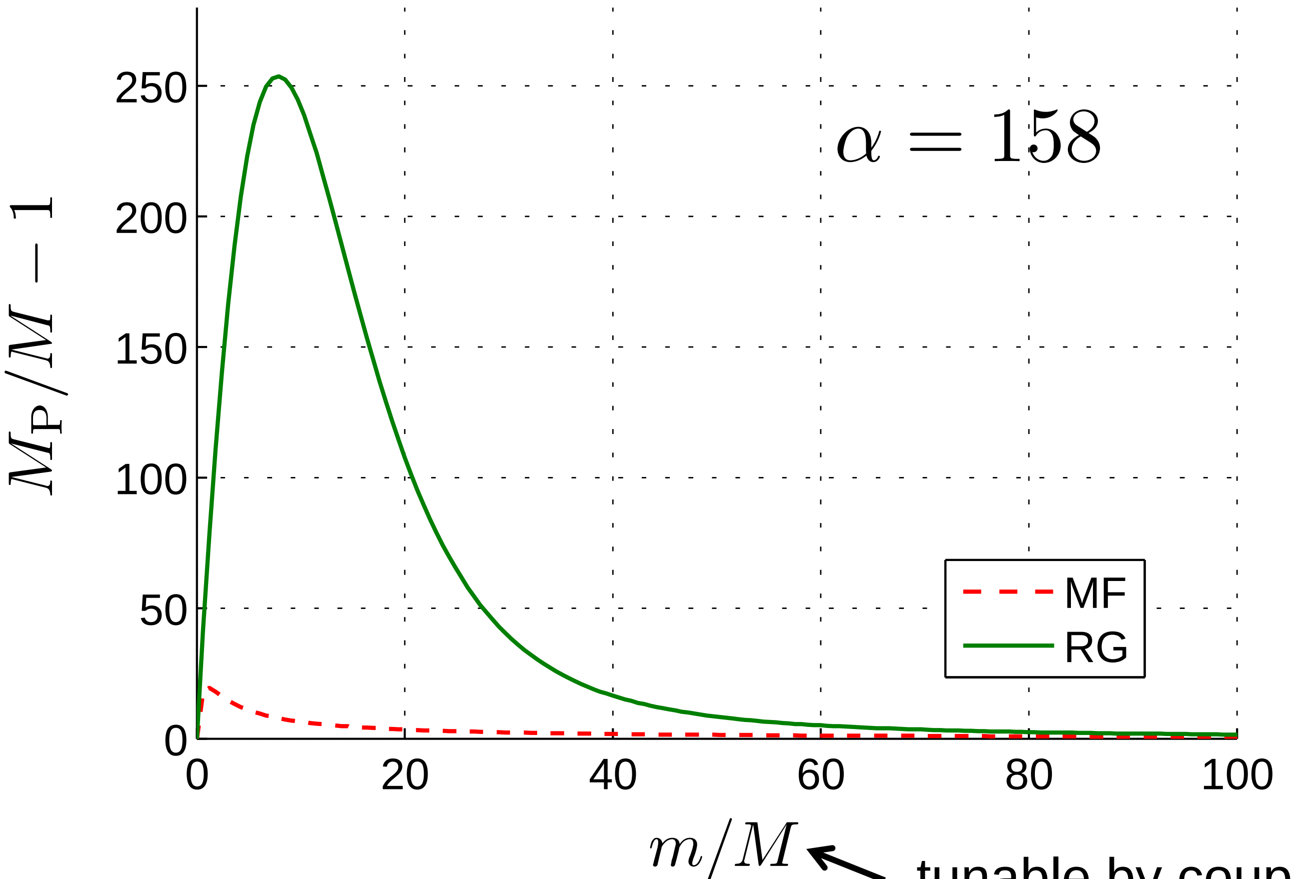
$$\omega_k = ck\sqrt{1+(\xi k)^2/2}$$



- Interaction strength

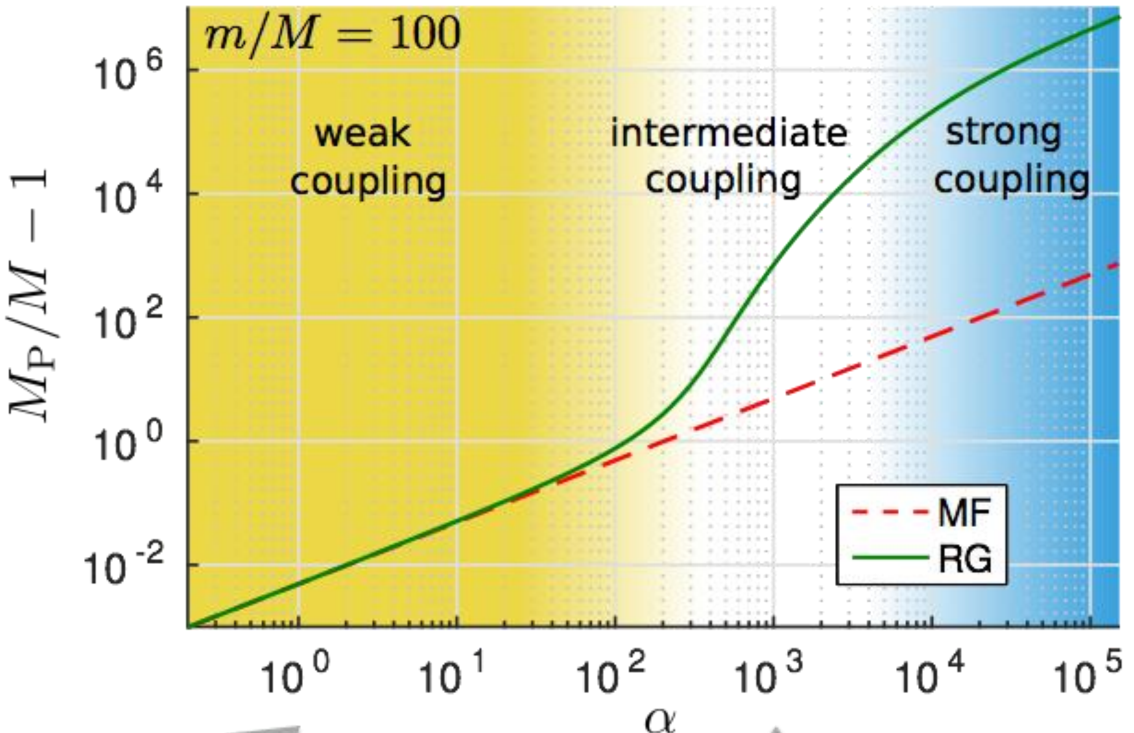
$$\alpha = \sin^4\theta\,(g^{\mathrm{2D}}_{gs})^2/\pi c^2$$

polaronic mass renormalization



tunable by coupling laser

phase diagram



tunable via Feshbach res.

$M_\mathrm{p} - M \propto \alpha$

$M_\mathrm{p} - M \not\propto \alpha$

$M_\mathrm{p} - M \propto \alpha$

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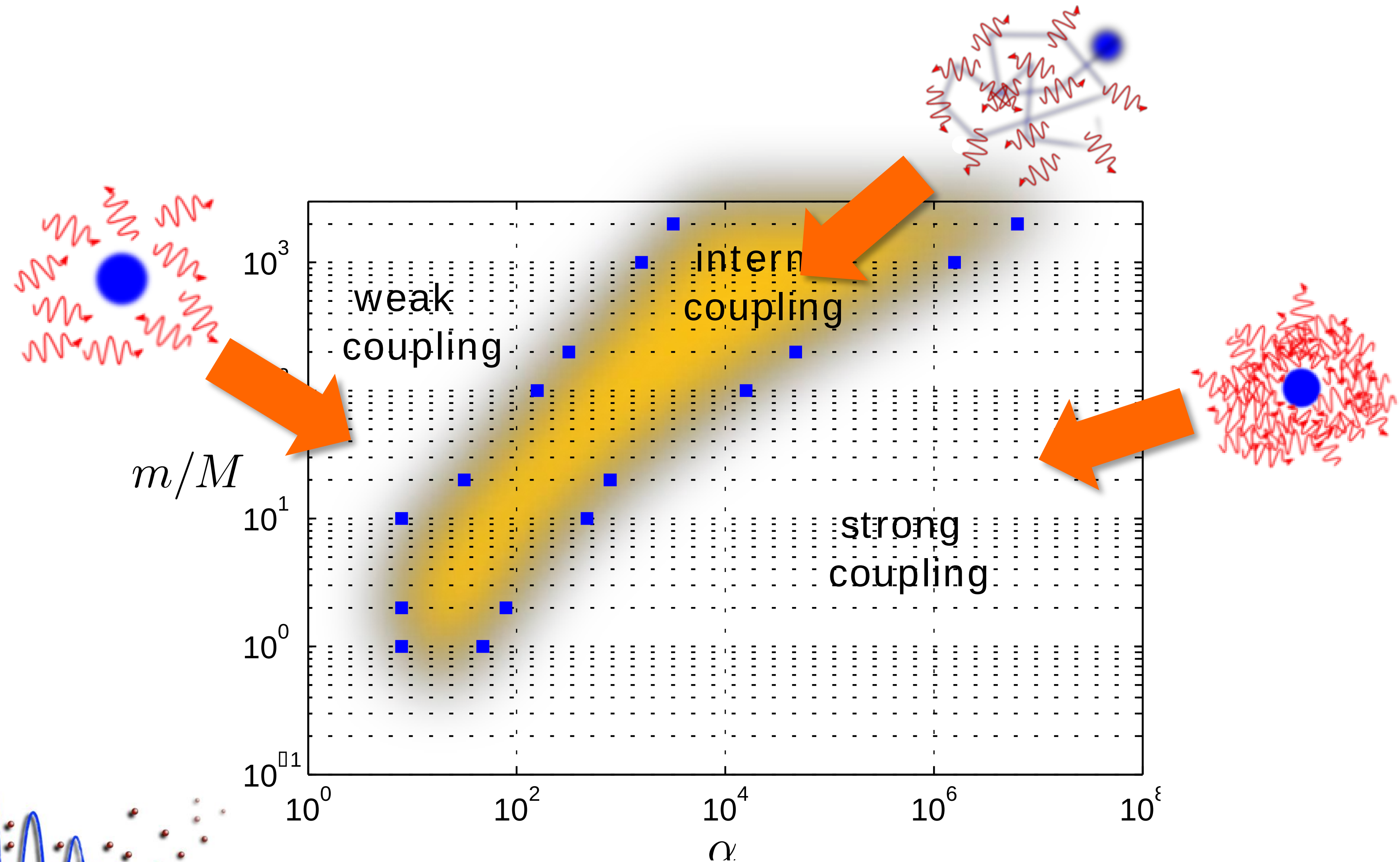
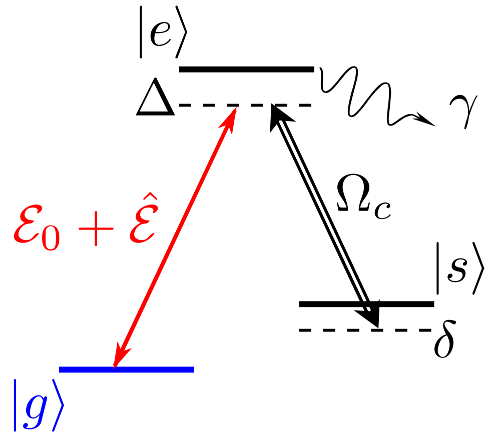
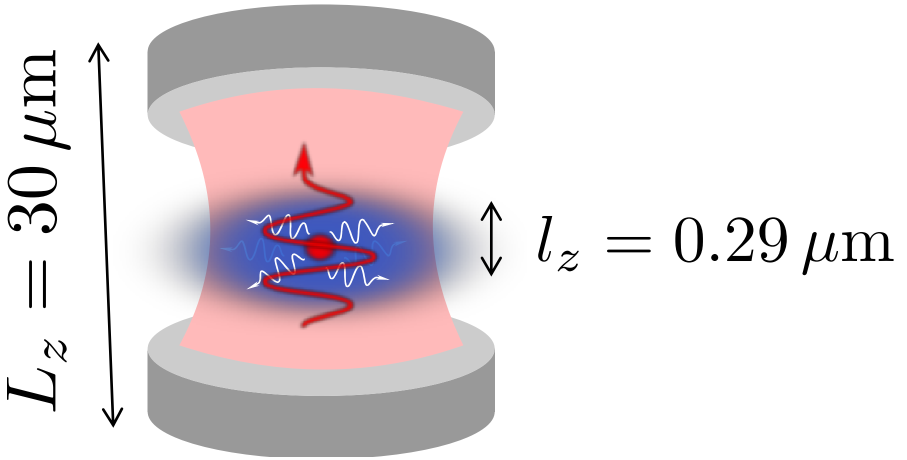
phase diagram



some numbers

^{87}Rb





$|g\rangle = |F = 1, m_F = -1\rangle \quad |s\rangle = |F = 2, m_F = 1\rangle$

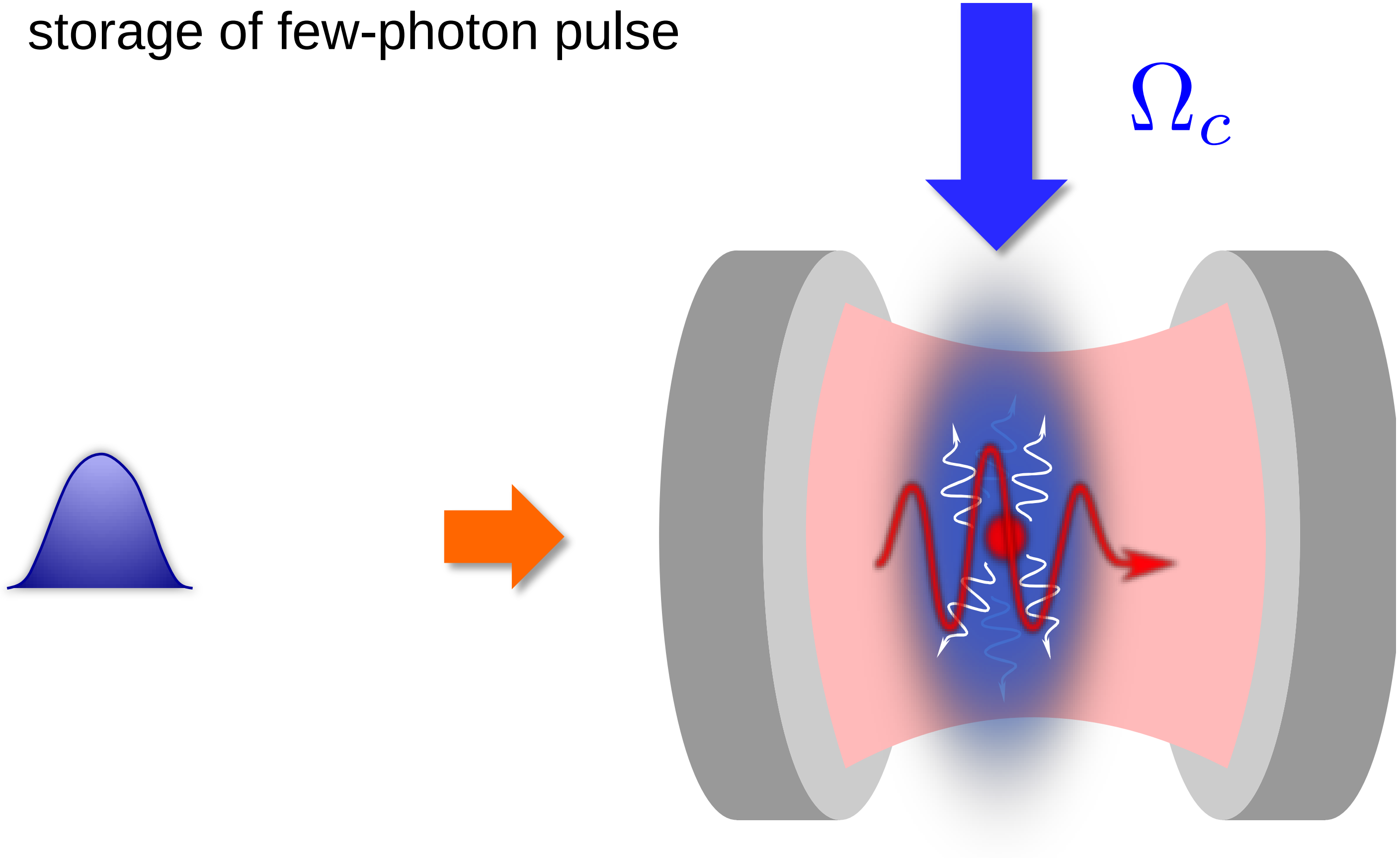
$a_{gg} = 100.4\ a_0$ $\quad a_{gs} = 98\ a_0$ $\quad n_0 = 100\,\mu\text{m}^{-2}$

$\Omega_c = 20\ \times\ 2\pi\, 10^3 \text{s}^{-1} \quad M/m \approx 2 \quad \alpha_{\text{max}} \approx 1.4$

$\Omega_c = 20\ \times\ 2\pi\, 10^5 \text{s}^{-1} \quad M/m \approx 10^{-4} \quad \alpha_{\text{max}} \approx 5 \times 10^3$

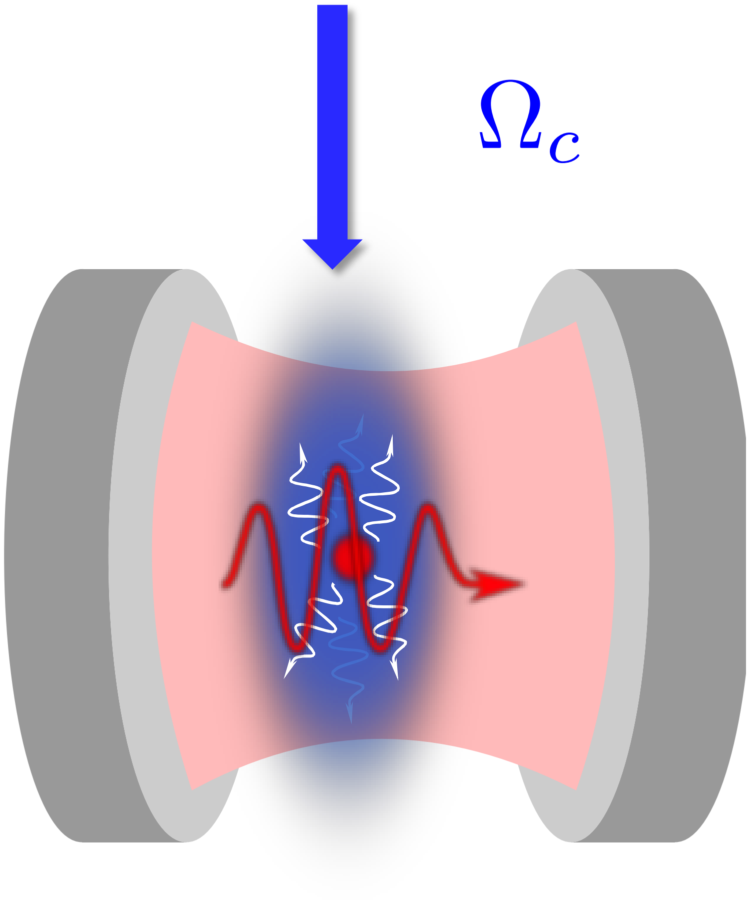
preparation of DSP polarons

storage of few-photon pulse



preparation of DSP polarons

partial retrieval



detection of DSP polarons

curved mirrors → harmonic confinement potential
excite finite transverse k-values

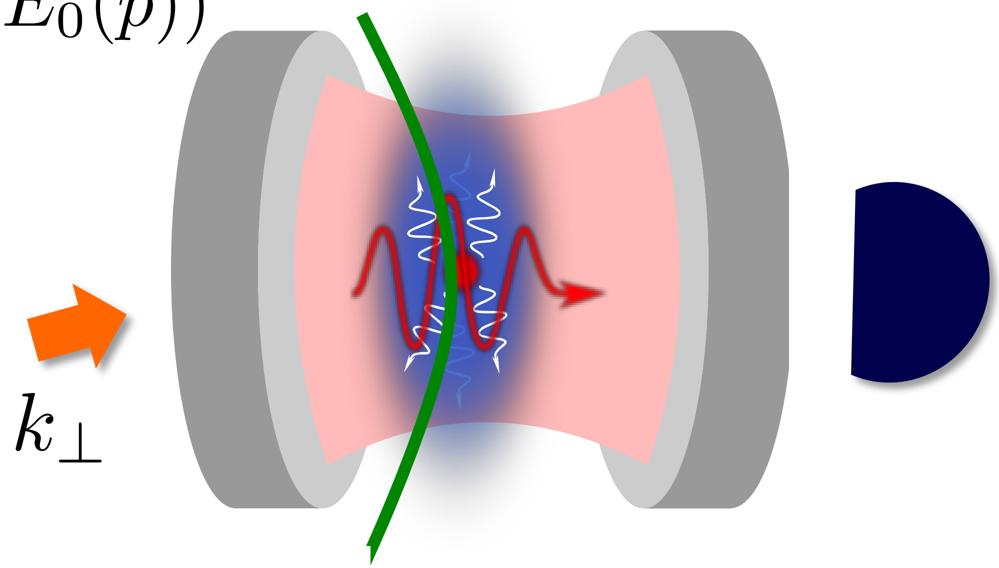
absorption spectrum → momentum resolved spectral function

$$I(\omega, p) = I_{\rm coh}(\omega, p) + I_{\rm incoh}(\omega, p)$$

$$I_{\rm coh}(\omega, p) = Z\,\delta\,(\omega - E_0(p))$$

Z = quasi-particle weight

$E_0(p)$ = polaron energy

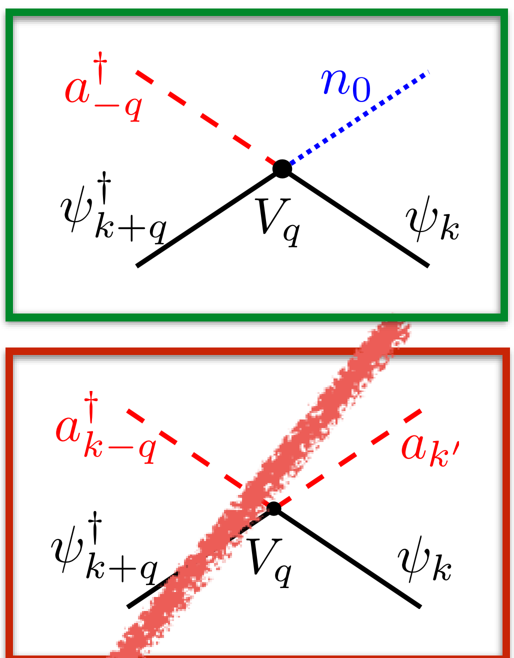


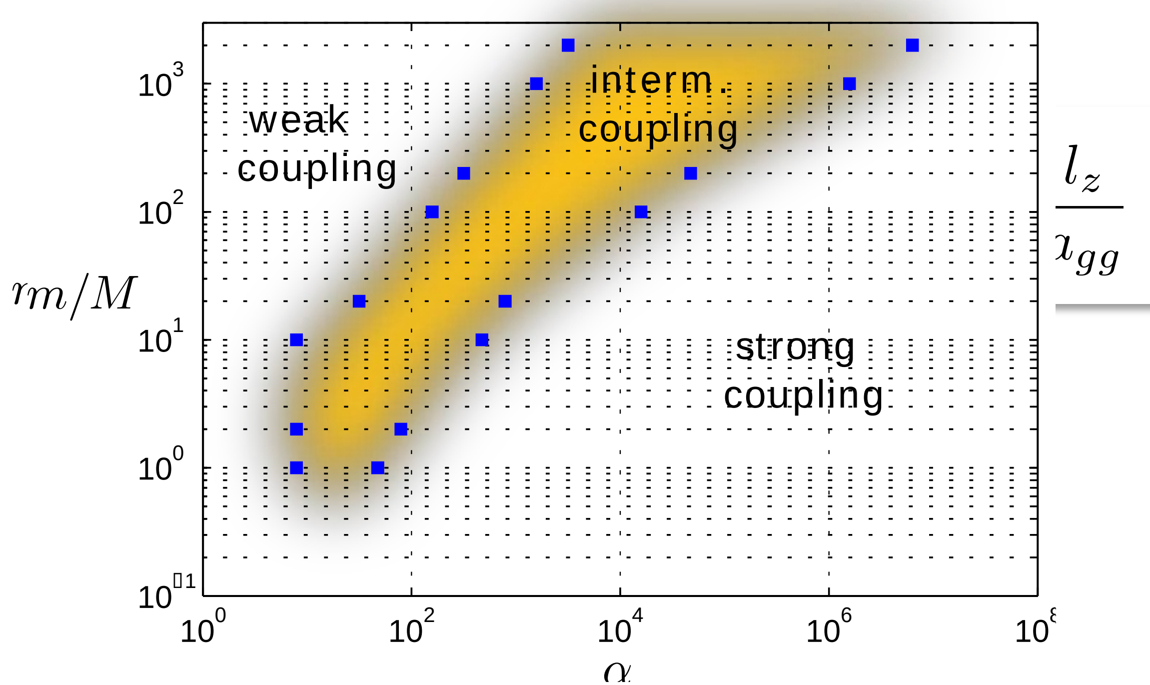
$$p \sim k_\perp$$

validity of Fröhlich model

- validity of Fröhlich model $\epsilon = \sqrt{n_{\mathrm{ph}}/n_0} \ll 1$

$$\alpha < \alpha_{\max} = \frac{\epsilon_{\max}^2}{g_{gg}^2}\left(\frac{1}{m} + \frac{1}{M}\right) = \frac{\epsilon_{\max}^2}{\sqrt{8\pi}}\frac{l_z}{a_{gg}}\left(1 + \frac{m}{M}\right)$$





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summary

- nature of self-trapping transition of BEC polarons is unclear
  - Feynman: sharp transition for small impurity masses
  - RG: extended intermediate coupling regime
- Dark-state polaritons in 2D cavity + 2D BEC = polaron with widely tunable parameter
- Detection of momentum-resolved spectral function, polaron mass etc.
- Self-trapping transition in steady-state of open system?
- Detection of beyond-Fröhlich effects, bubble polarons

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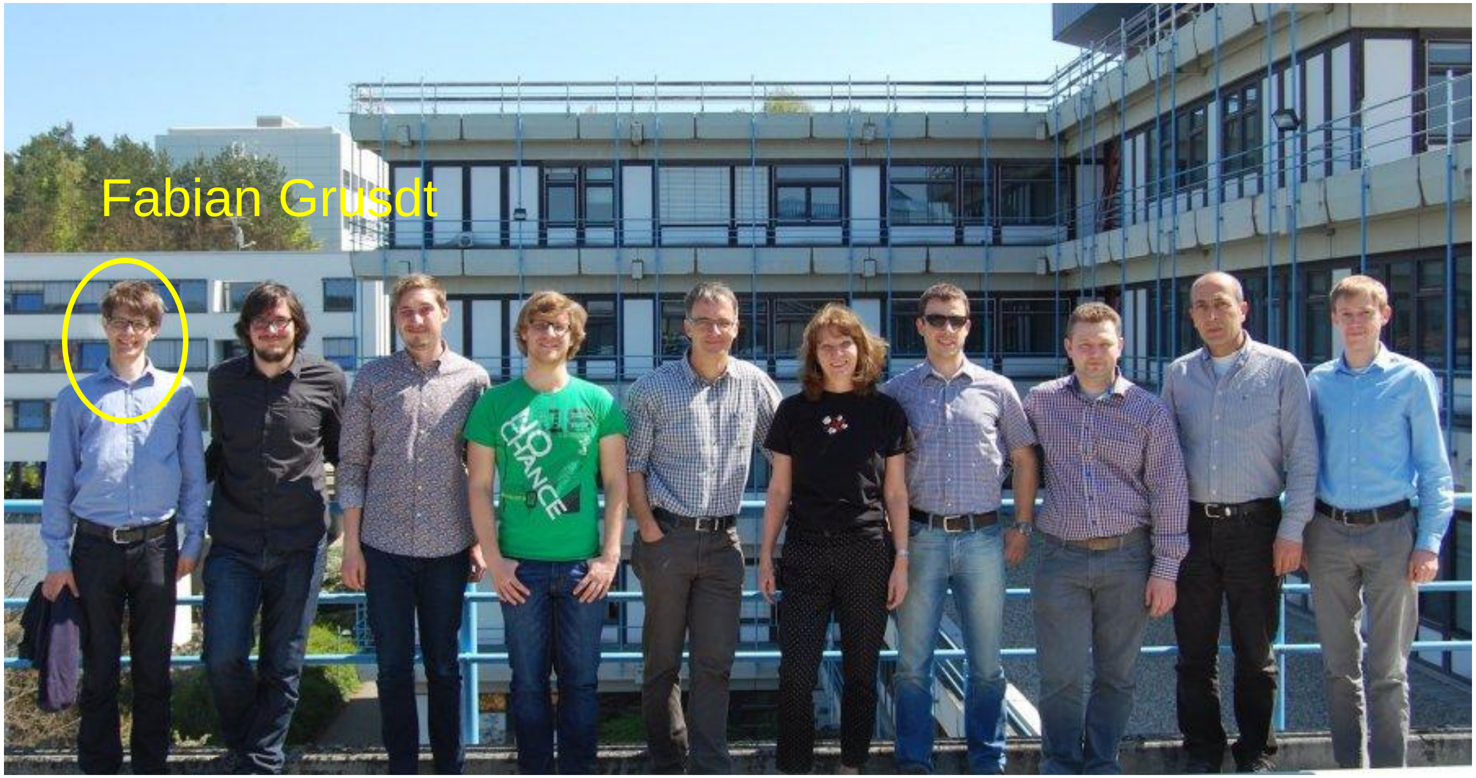
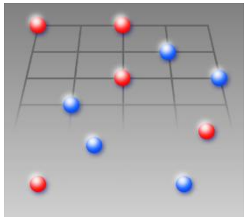
thanks to





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OPTIMAS light-spin-matter

Eugene Demler (Harvard)
Yulia Shadilova (Harvard)

Thanks !