Photonic Quantum Hall Effect: Lessons From Chern Simons Perspective

Non-equilibrium dynamics of strongly interacting photons
October 7, 2015

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Observation of the Chern-Simons gauge anomaly
arXiv:1504.00369

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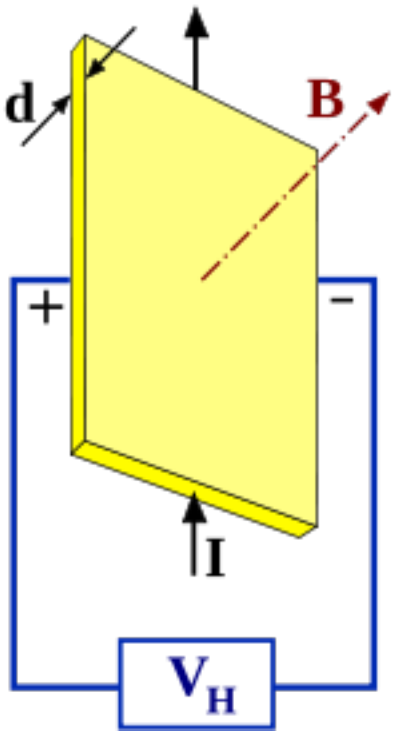
Review: Integer Quantum Hall effect



VOLUME 45, NUMBER 6 PHYSICAL REVIEW LETTERS 11 AUGUST 1980

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

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and

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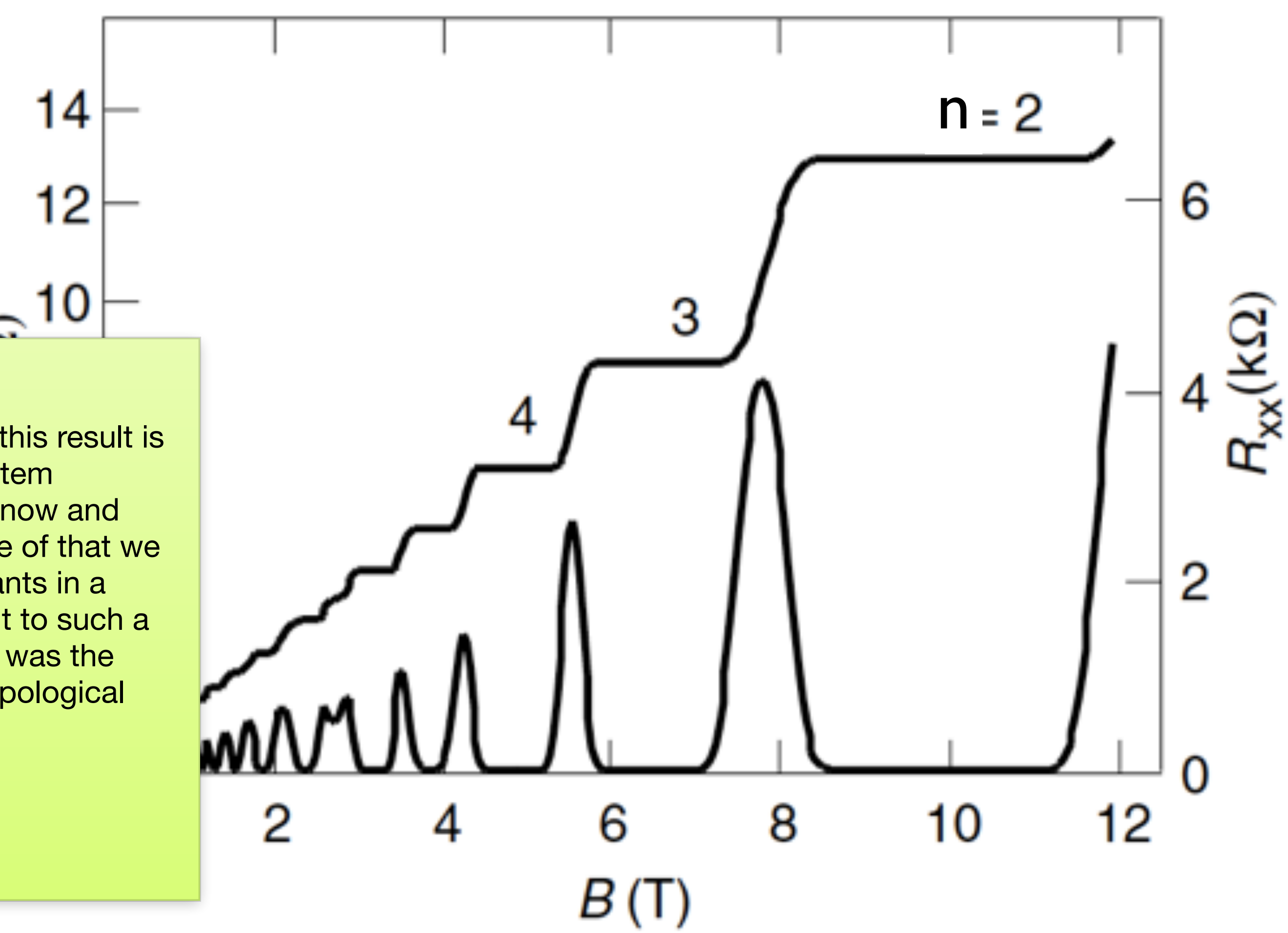
(Received 30 May 1980)

Measurements of the Hall voltage of a two-dimensional electron gas, realized with a silicon metal-oxide-semiconductor field-effect transistor, show that the Hall resistance at particular, experimentally well-defined surface carrier concentrations has fixed values which depend only on the fine-structure constant and speed of light, and is insensitive to the geometry of the device. Preliminary data are reported.

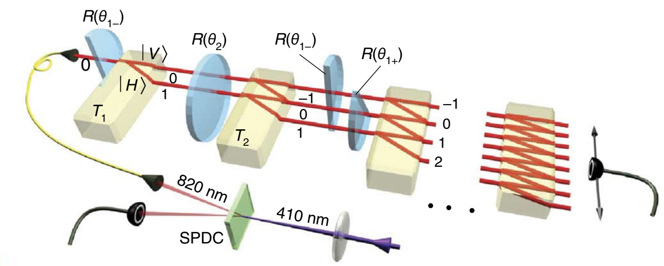
$$V_H = R_H I$$

$$\boxed{R_H = \frac{h}{ne^2}}$$

What is remarkable about this result is that there are so many system parameters that we dont know and cannot control. Irrespective of that we obtain fundamental constants in a macroscopic measurement to such a remarkable accuracy. This was the first hint that something topological must be involved.



Some recent experiments on synthetic gauge field with photons



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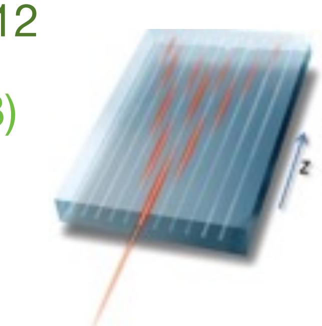
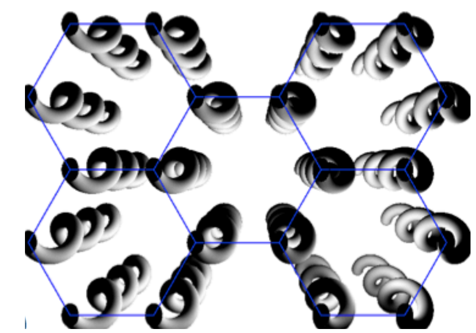
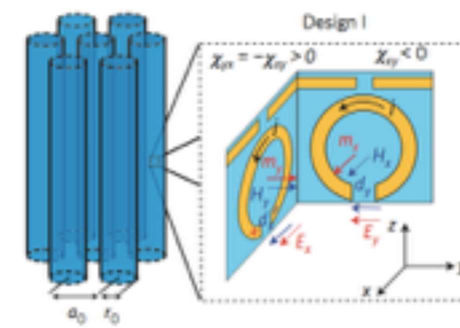
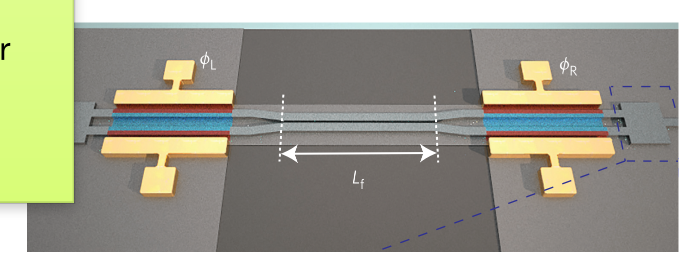
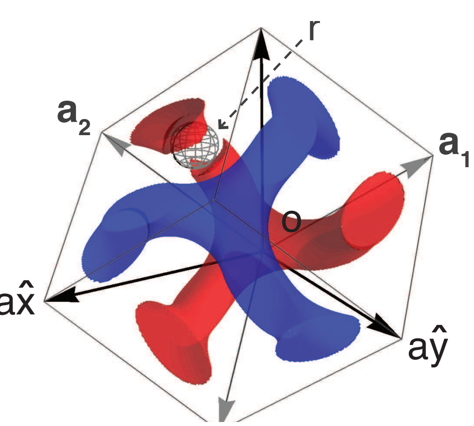


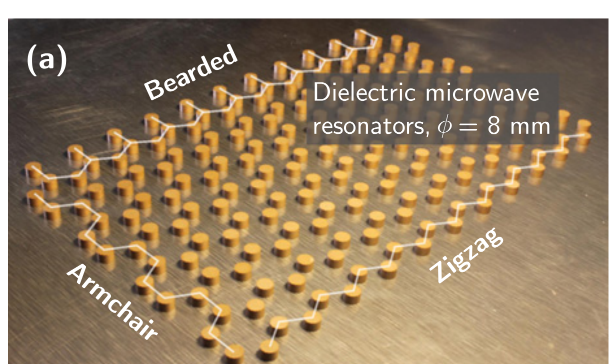


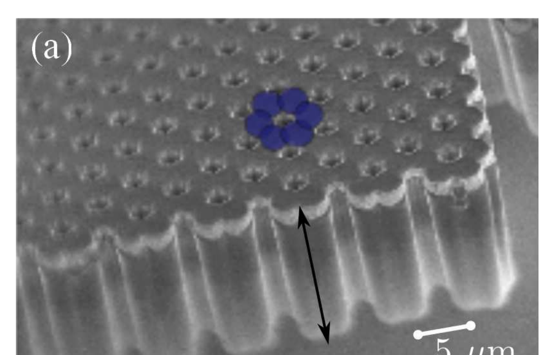
More recently several new platforms, more specifically optical systems have realized quantum hall effect.
The advantage of studying these new platforms is that each will provide different diagnostics to probe the topology of the state, which may not be feasible in more traditional condensed matter systems. Take for example Jon's talk yesterday.

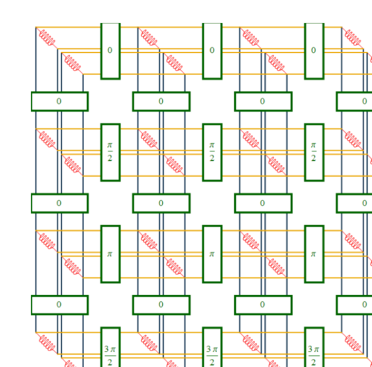






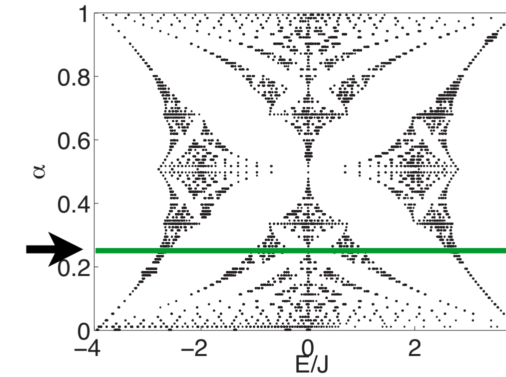






Observation and characterization of “topological” edge states

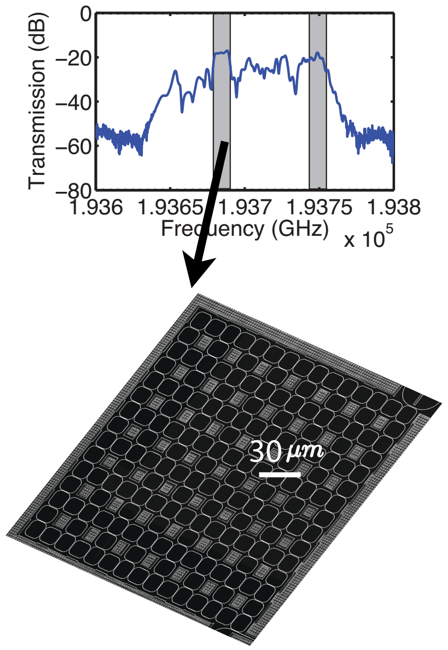
$$H_0 = -J\sum_{x,y} \hat{a}^\dagger_{x+1,y}\hat{a}_{x,y}e^{-i2\pi\alpha y} + \hat{a}^\dagger_{x,y}\hat{a}_{x+1,y}e^{i2\pi\alpha y} + \hat{a}^\dagger_{x,y+1}\hat{a}_{x,y} + \hat{a}^\dagger_{x,y+1}\hat{a}_{x,y}$$

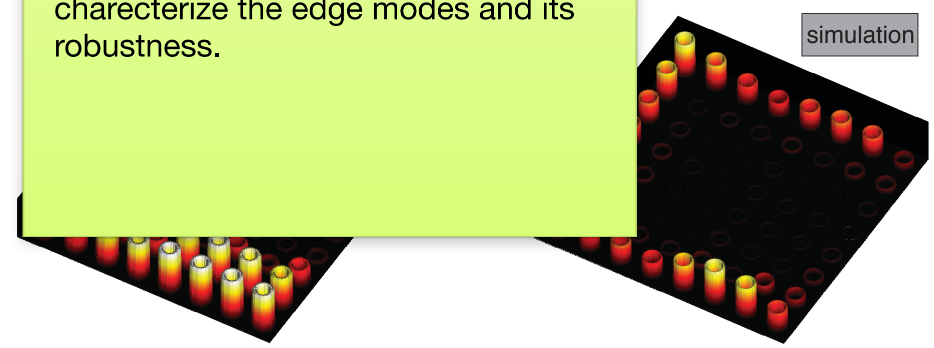


I would focus on what seems to be popularly known as Mohammad’s resonators. This platform has implemented Hofstadter Hamiltonian with a synthetic gauge field. Lot of progress has been made to charecterize the edge modes and its robustness.

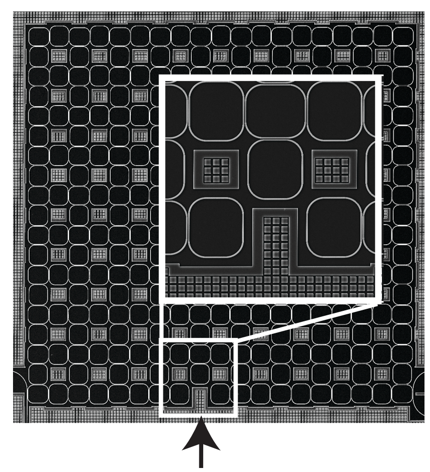
afezi, Mittal, Fan, Migdall, Taylor
Nature Photon 7, 1001 (2013)

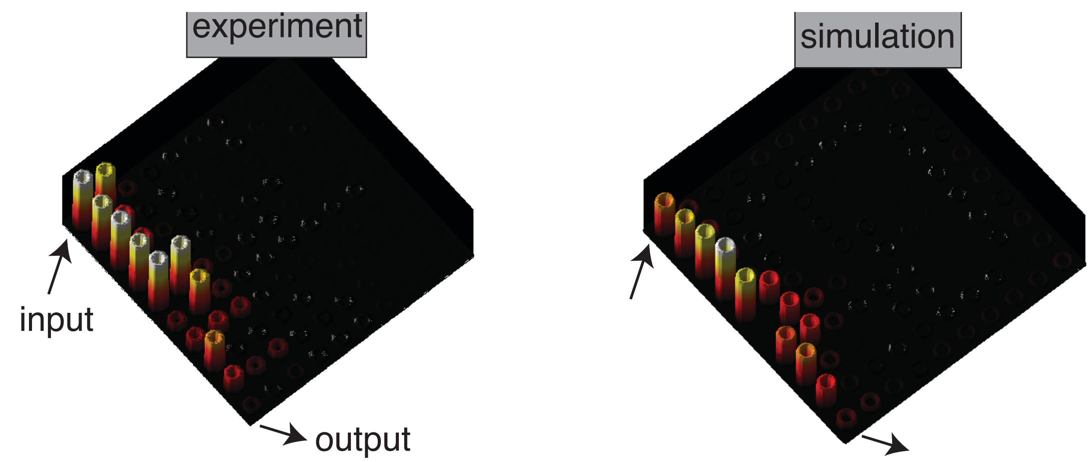
Observation of boundary modes



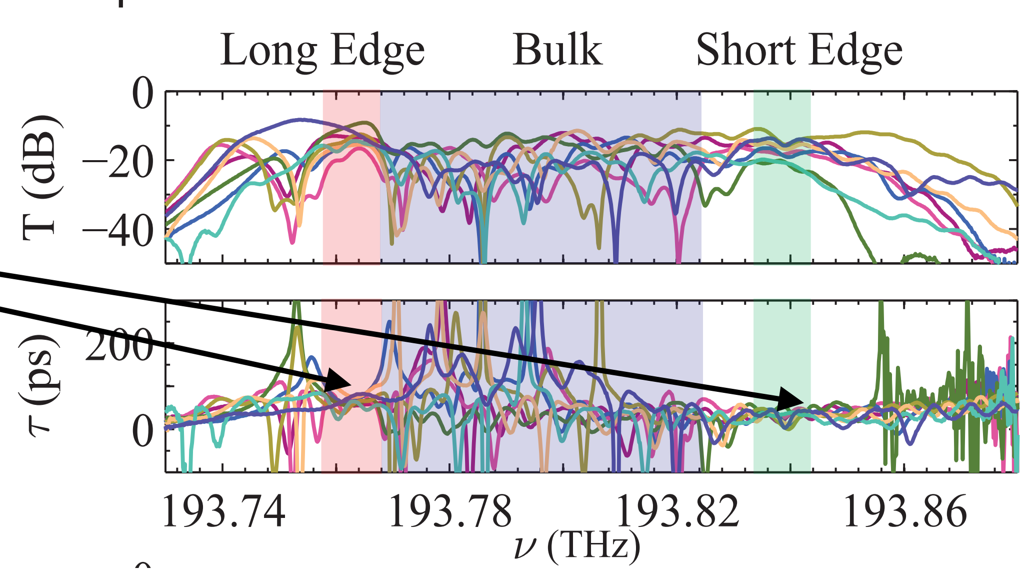


Robust against disorder (Absence of backscattering)





Statistical evidence of chiral (one way) propagation



M … zi.

But the hallmark feature of the topological phase is the topological invariant. Can we measure the topological invariant within this plat form?

In this talk

- Scheme to measure topological invariants

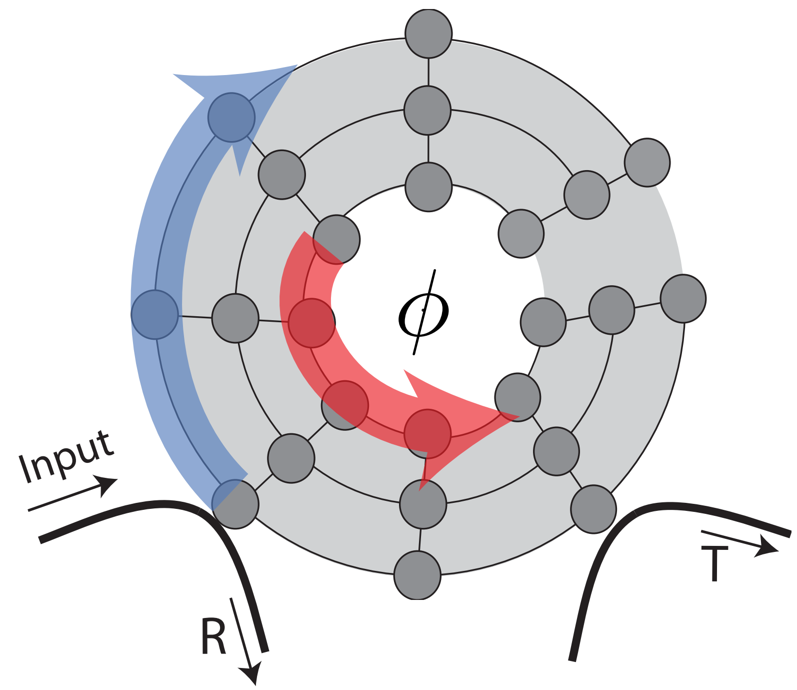
- Experimental observation of topological invariants

- Generalization to interacting many body topological states

Topological invariants with photonic resonators

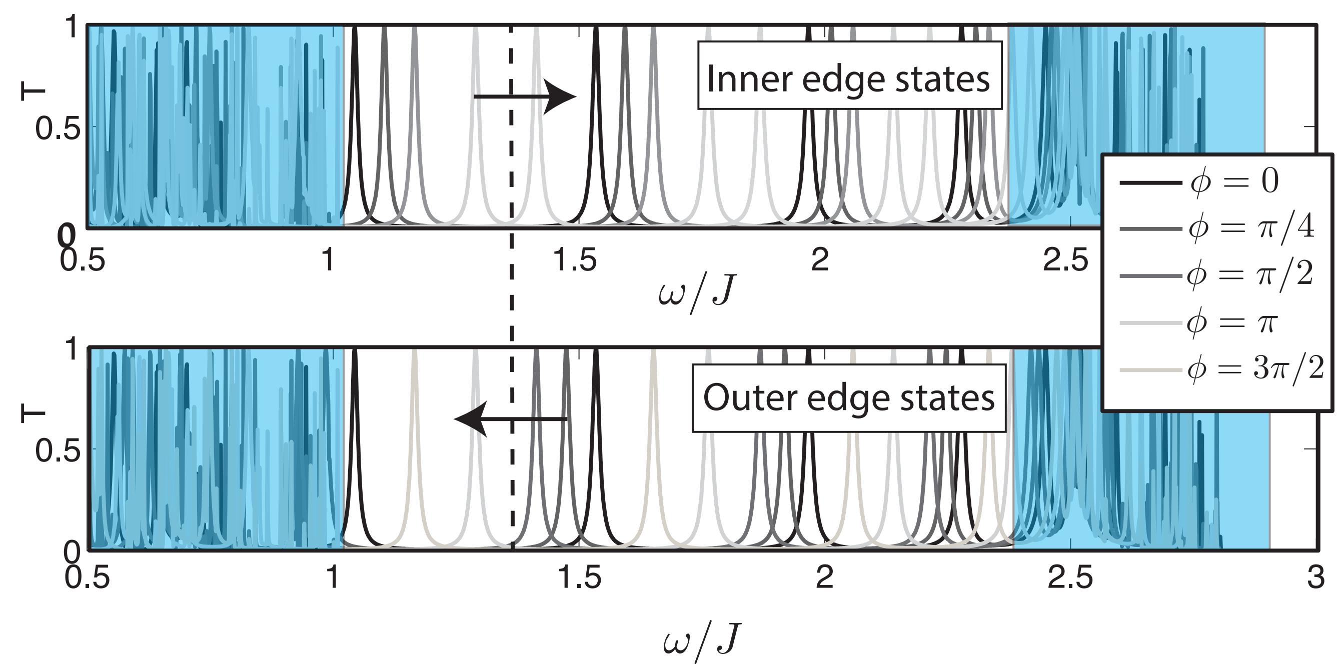
Laughlin's argument

A canonical way to think about topological invariant is to use Laughlin's though experiment specialized to the resonator system.



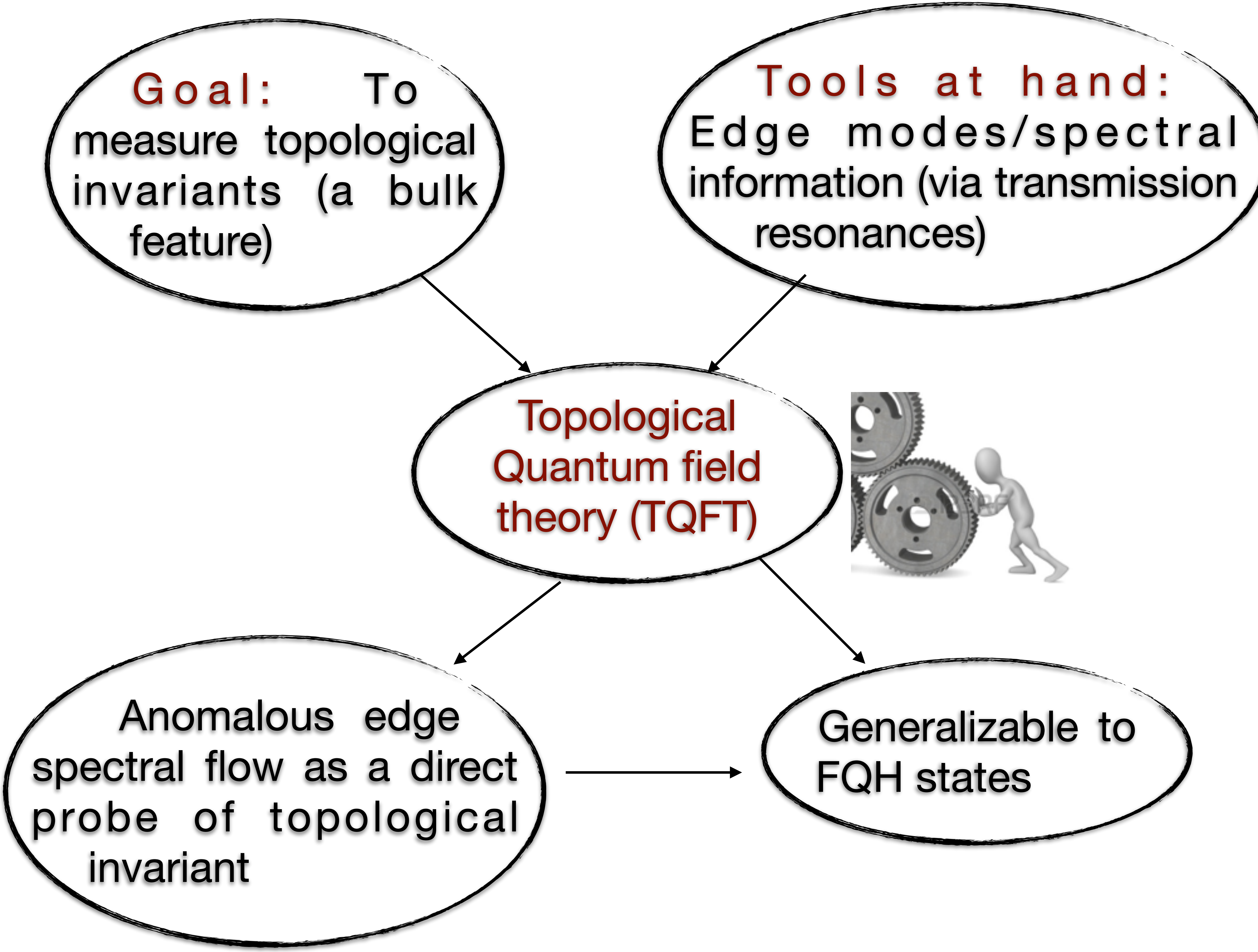
Hafezi, PRL 112, 210405 (2014)

Shift in transmission resonances



Threading unit flux requires manipulating every link resonator
Is there a simpler way?

see also: Ozawa et al. (2012),
Bardyn et al., Y. Chong
Kraus et al. (2012)


Goal: To measure topological invariants (a bulk feature)
Tools at hand: Edge modes/spectral information (via transmission resonances)
Topological Quantum field theory (TQFT)
Anomalous edge spectral flow as a direct probe of topological invariant
Generalizable to FQH states

Topological quantum field theory

Geometric skeleton of a more complex field theory

Microscopic description

TQFT description

Restrict this talk to U(1) Chern-Simons theory

Review: Effective gauge theories with gauge invariance

Effective EM actions for 3+1D in matter

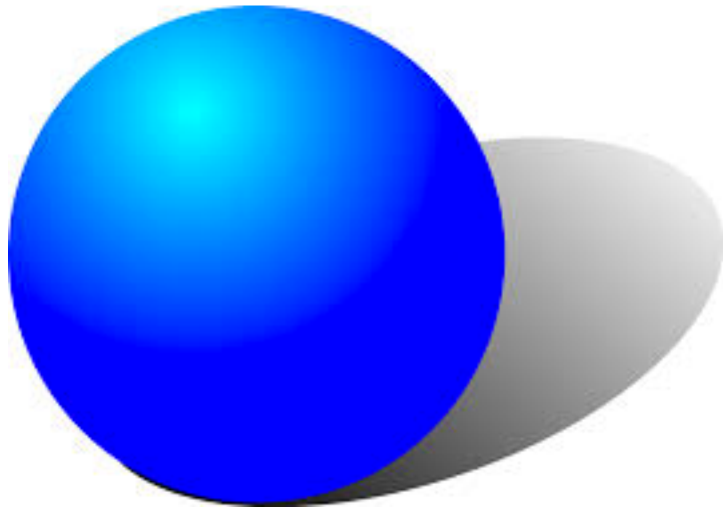
$$I_{em} = \int d^3x \; dt \; (a_{ij}E_iE_j + b_{ij}B_iB_j + c_iE_i + d_iB_i)$$

$$E = -\nabla\phi - \frac{\partial A}{\partial t}, \quad B = \nabla \times A$$

$$A'_i = A_i + \partial_i \Lambda \longrightarrow I_{em}(\mathbf{A}') = I_{em}(\mathbf{A})$$

Manifestly gauge invariant

$$J_i = -\frac{\delta I_{em}}{\delta A_i} \longrightarrow$$ Matter Maxwells equations



Chern-Simons Gauge Theory

2+1D one can write another action with an intricate gauge invariance

$$I_{CS}(\mathbf{A}) = \frac{k}{4\pi}\int_{\mathcal{M}^2\times\mathbb{R}} d^2x\ dt\ \epsilon^{ijk}A_i\partial_j A_k$$

$$A'_i = A_i + \partial_i\Lambda \longrightarrow \epsilon^{ijk}A'_i\partial_j A'_k = \epsilon^{ijk}A_i\partial_j A_k + \partial_i(\epsilon^{ijk}\Lambda\partial_j A_k)$$

$$I_{CS}(\mathbf{A}') = I_{CS}(\mathbf{A}) \mod 2\pi k\mathbb{Z}$$

Not quite gauge invariant, but $e^{iI_{CS}}$ is well defined if $k \in \mathbb{Z}$

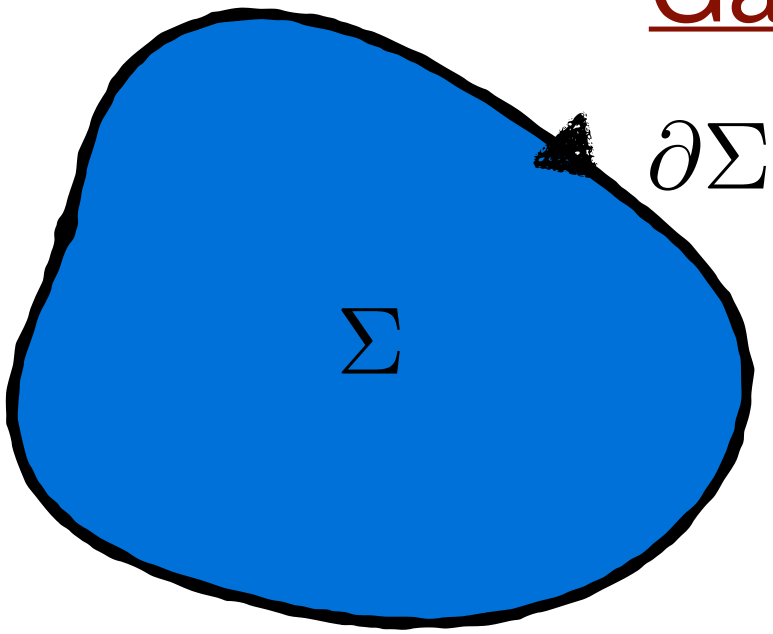
$$J_x = -\frac{\delta I_{CS}}{\delta A_x} = \frac{kE_y}{2\pi} \longrightarrow \sigma_{xy} = k\frac{e^2}{h}$$

Quantized Hall conductance!!!

One to One correspondence with TKNN argument

https://pitp2015.ias.edu/sites/pitp2015.ias.edu/files/WittenLecture2.pdf

Gauge invariance with a boundary



$$I_{CS}(\mathbf{A}) = \frac{k}{4\pi}\int_{\Sigma\times\partial\Sigma\times\mathbb{R}} d^2x\ dt\ \epsilon^{ijk}A_i\partial_j A_k$$

$$A_i' = A_i + \partial_i\Lambda \longrightarrow I_{CS}(\mathbf{A}') = I_{CS}(\mathbf{A}) + \frac{k}{4\pi}\int_{\partial\Sigma} dx\ dt\Lambda\epsilon^{jk}\partial_j A_k$$

Gauge non-invariant due to boundary term

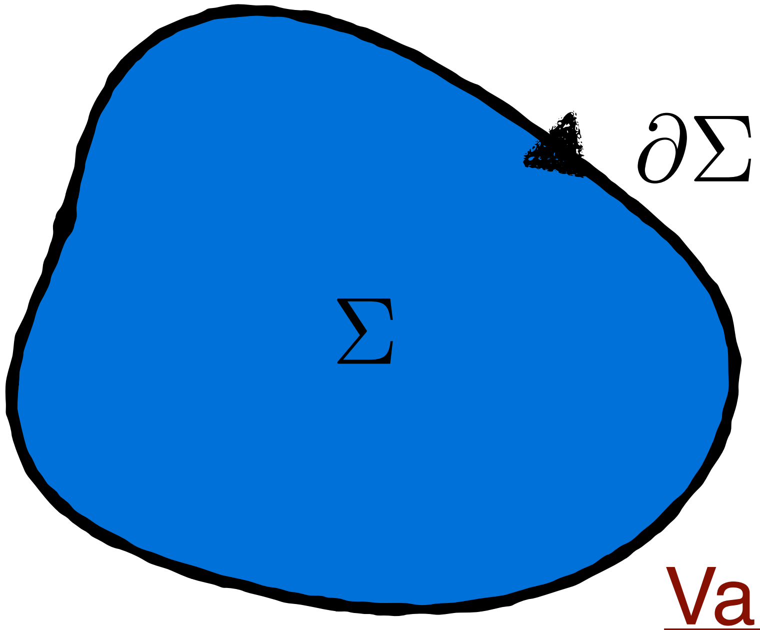
Only reconciliation $\quad I_{CS} = I^{\Sigma}_{CS}(\mathbf{A}) + I^{\partial\Sigma}(\phi, \mathbf{A}|_{\partial\boldsymbol{\Sigma}})$

Gauge invariance requires $I^{\partial\Sigma}(\phi, \mathbf{A}|_{\partial\boldsymbol{\Sigma}})$ to be chiral and gapless in ϕ

$I^{\partial\Sigma}(\phi, \mathbf{A}|_{\partial\boldsymbol{\Sigma}})$ is forbidden in 1+1D on its own

$I^{\Sigma}_{CS}(\mathbf{A})$ is forbidden in 2+1D on its own

But together they are well defined (bulk-edge correspondence)



Gauge invariance of CS theory

1) Leads to integer quantization of Hall current

2) Fixes effective boundary theory

Valid effective boundary theory: Chiral luttinger liquid (single edge mode)

$$H = \frac{v}{4\pi}\int_{\partial\Sigma}(\partial_x\phi - \eta_x)^2 dxdt \qquad [\phi(x), \partial_y\phi(y)] = 2\pi i\delta(x-y)$$

Bosonic field $\phi = \phi + 2\pi$

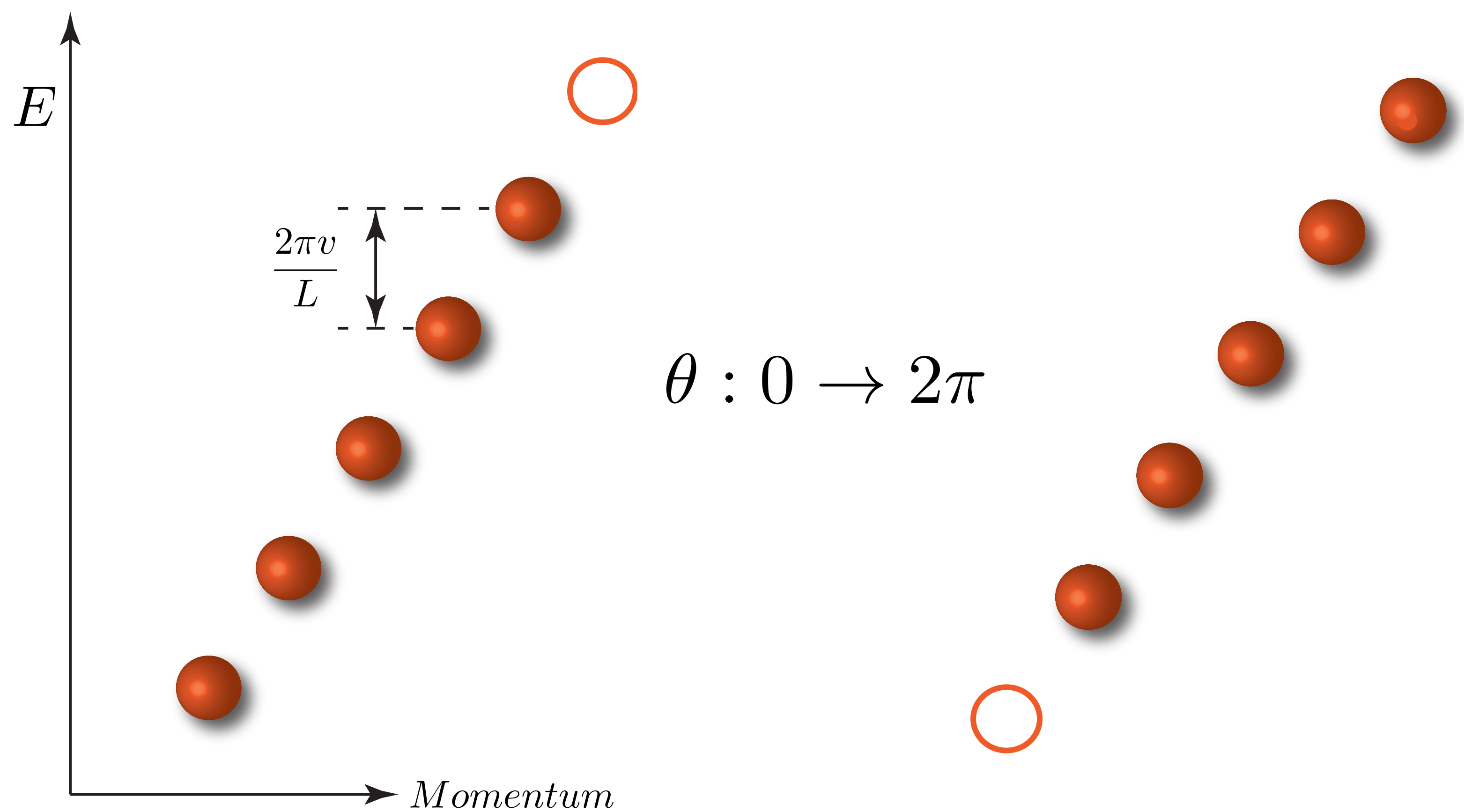
$$\eta_x = A|_{\partial\Sigma} = \frac{\theta}{L}, \quad k = 1$$

Canonical commutation directly leads to anomalous edge spectral flow

$$[H, O_n\{\phi\}] = E_n O_n\{\phi\} \longrightarrow E_n = \frac{2\pi v}{L}\left(n - \frac{\theta}{2\pi}\right)$$

Anomalous spectral flow at the edge

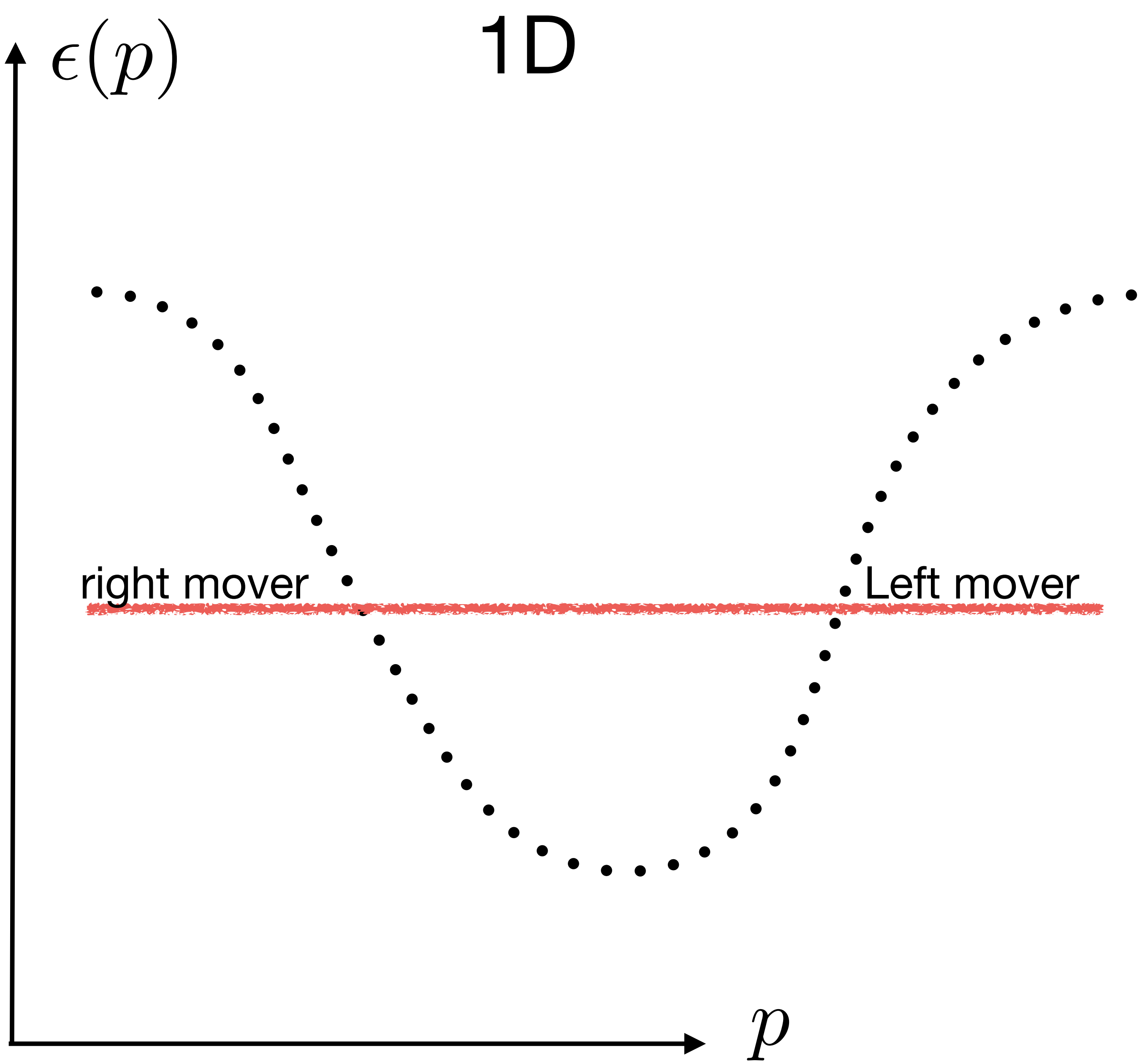
$$E_n = \frac{2\pi v}{L}\left(n - \frac{\theta}{2\pi}\right)$$



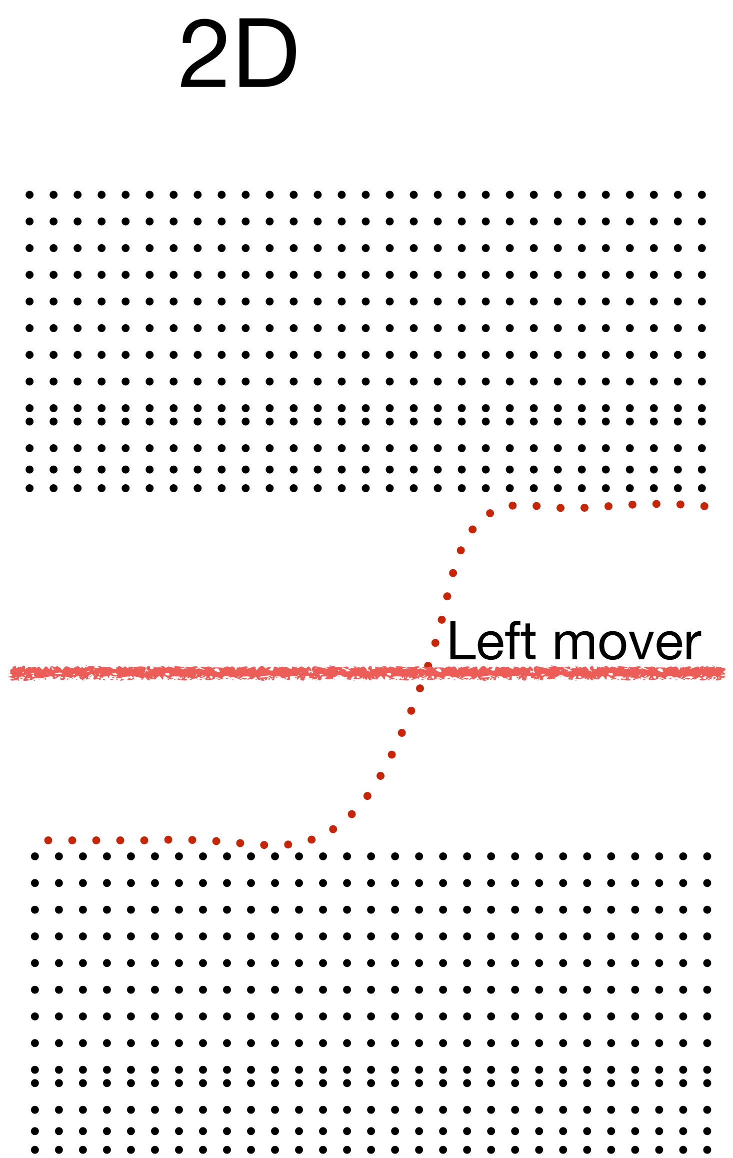
Shift of the edge spectrum ⟷ Gauge non-invariance of edge

“Integerness" of shift ⟷ $k \in \mathbb{Z}$

What is anomalous?(Microscopic viewpoint)



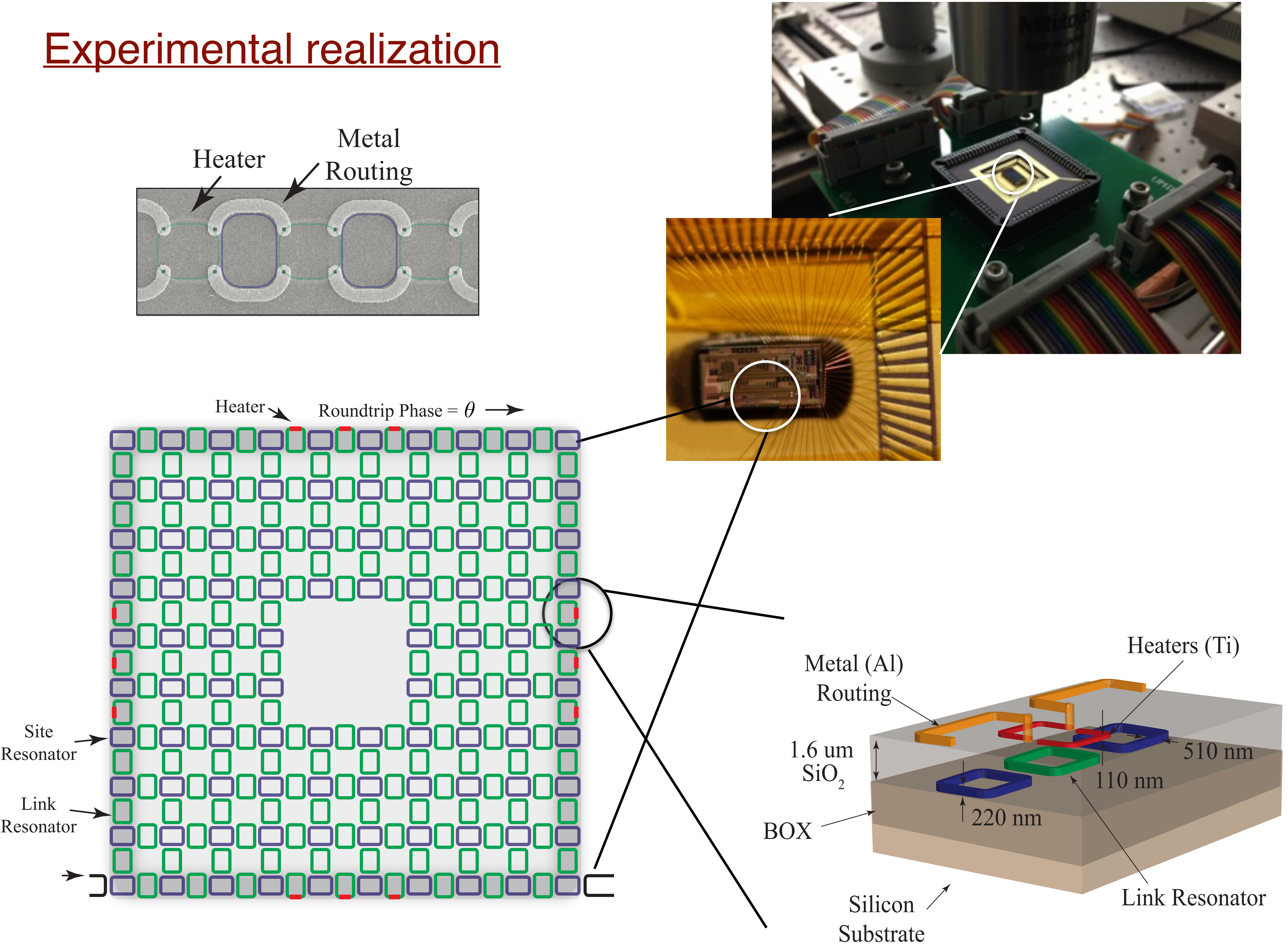
LM and RM cannot exist on their own in 1D



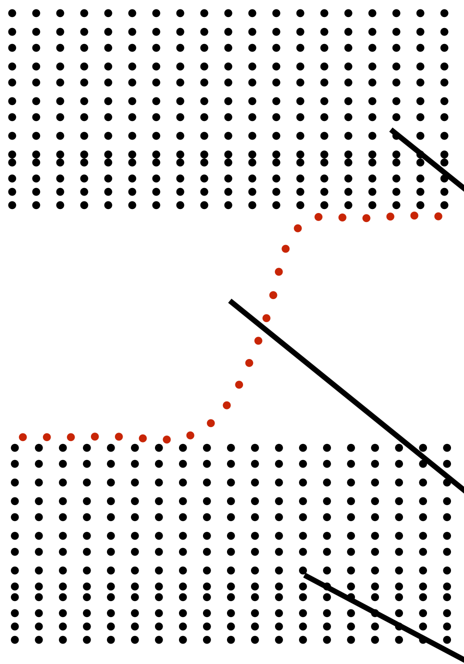
LM/RM can exist on their own at the boundary of 2D with Topological order

- Scheme to measure topological invariants
- Experimental observation of topological invariants
- Generalization to interacting many body topological states

Experimental realization

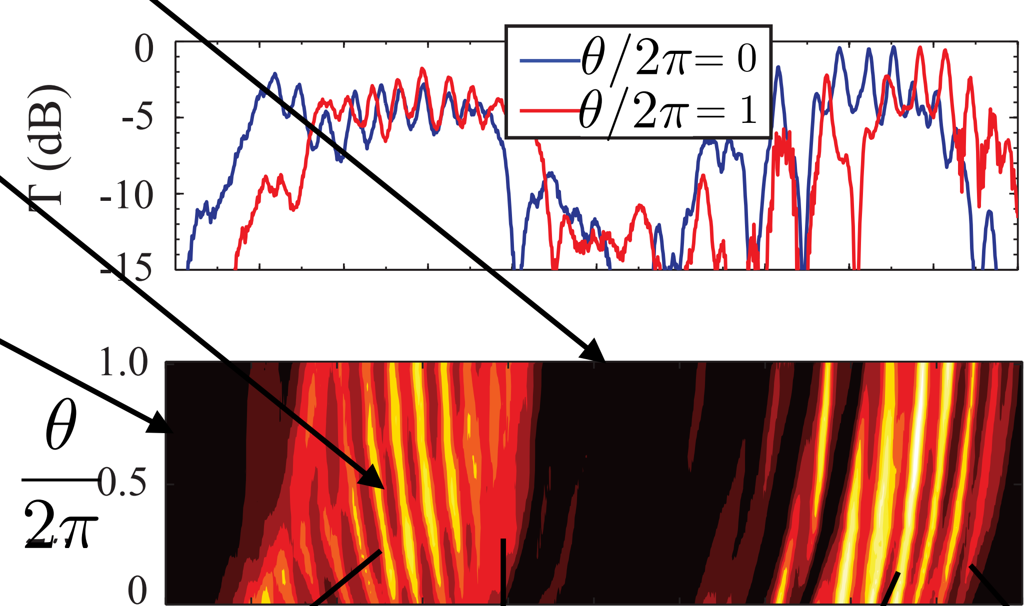


First observation of anomalous spectral flow !!!

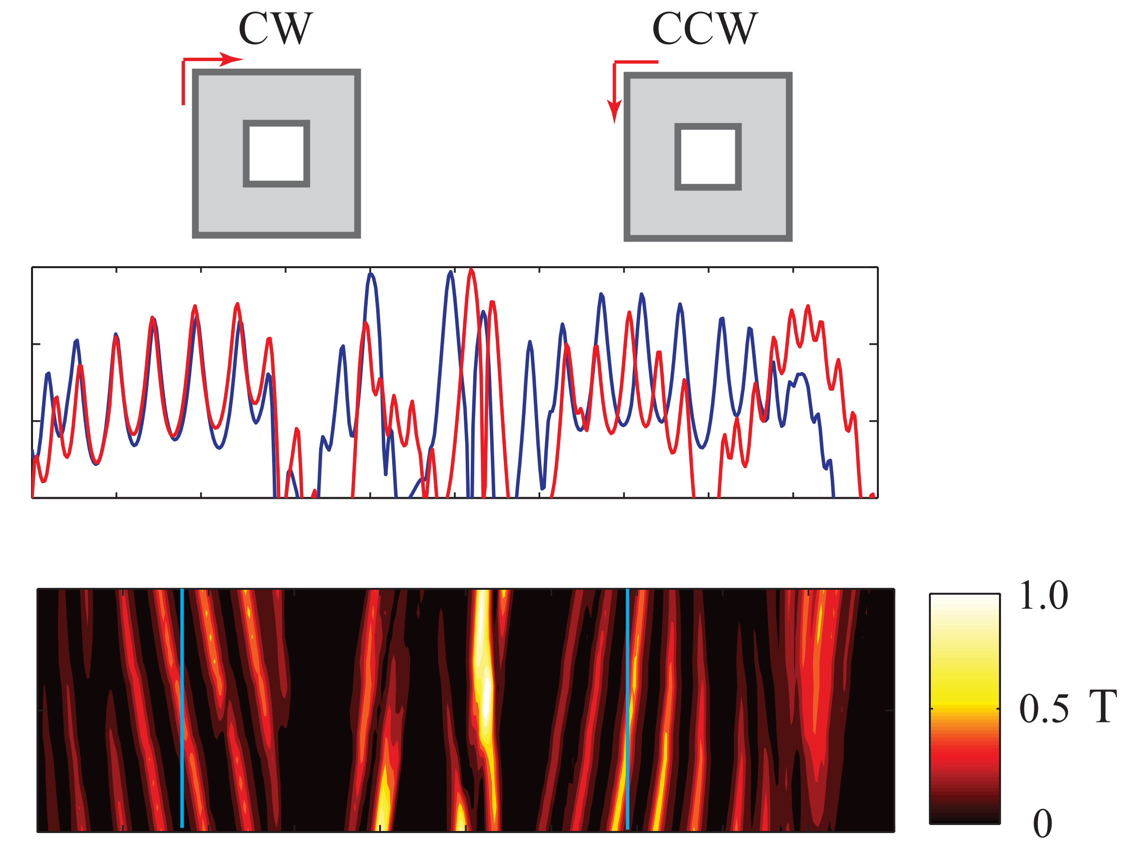


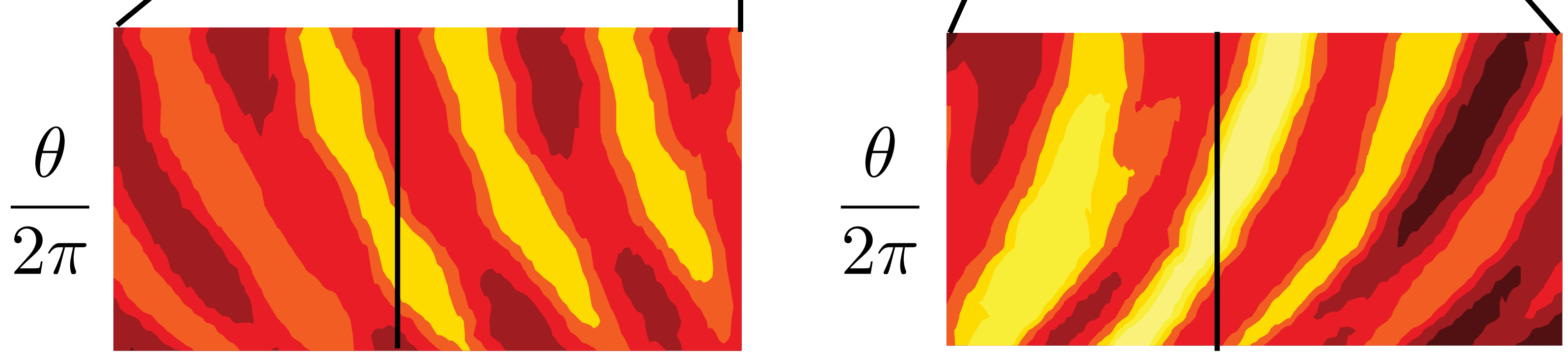
gauge at outside edge

Measured



Simulated





k = +1.0(1)

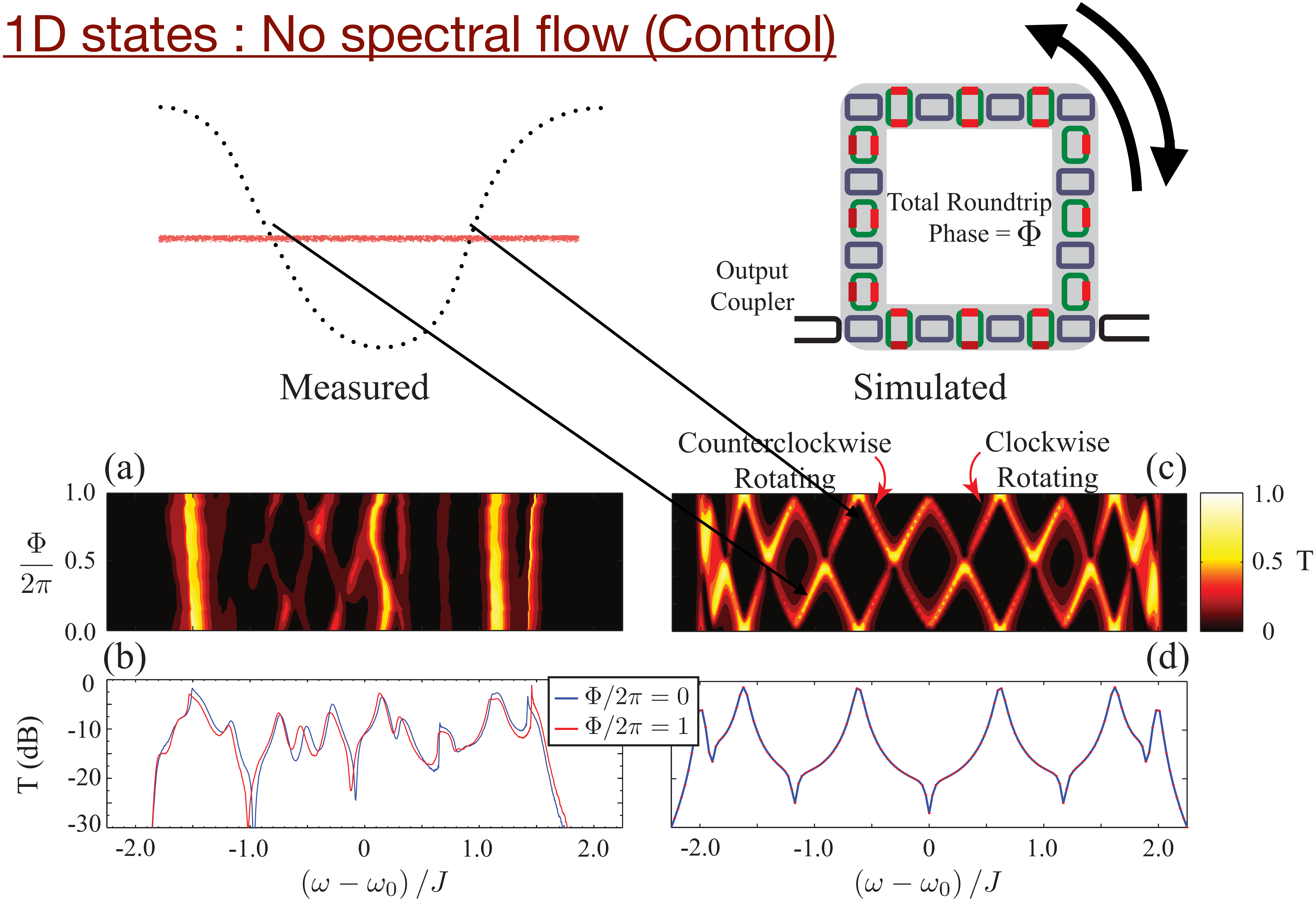
k = −1.0(2)

Topological invariant from the edge spectral flow

$$k = \pm 1$$

Mittal, SG, Vaezi, Fan, Hafezi, arXiv:1504.00369

1D states : No spectral flow (Control)



Mittal, SG, Vaezi, Fan, Hafezi, arXiv:1504.00369

- » Scheme to measure topological invariants

- » Experimental observation of topological invariants

- » Generalization to interacting many body topological states

Fractional Quantum Hall state of light

Gauge field + interaction (k-body)

↓

Fractional quantum Hall states

↓

Topological Quantum field theory (TQFT)

Kapit et al. PRB (2013),PRX (2014)

Generalization of CS to FQH states (e.g. Laughlin 1/2 state)

$$I_{CS}(\mathbf{A}) = \frac{k}{4\pi}\int_{\mathcal{M}^2\times\mathbb{R}} d^2x\ dt\ \epsilon^{ijk} a_i \partial_j a_k - \frac{1}{2\pi}\int_{\mathcal{M}^2\times\mathbb{R}} d^2x\ dt\ \epsilon^{ijk} A_i \partial_j a_k$$

Emergent U(1) gauge field "a" that constitutes intrinsic topological order

Bulk:

$$I_{CS}^{eff}(\mathbf{A}) = \frac{1}{4\pi k}\int_{\mathcal{M}^2\times\mathbb{R}} d^2x\ dt\ \epsilon^{ijk} A_i \partial_j A_k \qquad \sigma_{xy} = \frac{e^2}{kh}$$

Boundary:
(Single mode edge)

$$H = \frac{vk}{4\pi}\int_{\partial\Sigma} (\partial_x\phi - \eta)^2 dx dt \qquad [\phi(x), \partial_y\phi(y)] = \frac{2\pi i}{k}\delta(x-y)$$

$$\phi = \phi + \frac{2\pi}{k}$$

Anomalous spectral flow:

$$[H, O_n\{\phi\}] = E_n O_n\{\phi\}$$

quasiparticle spectrum!

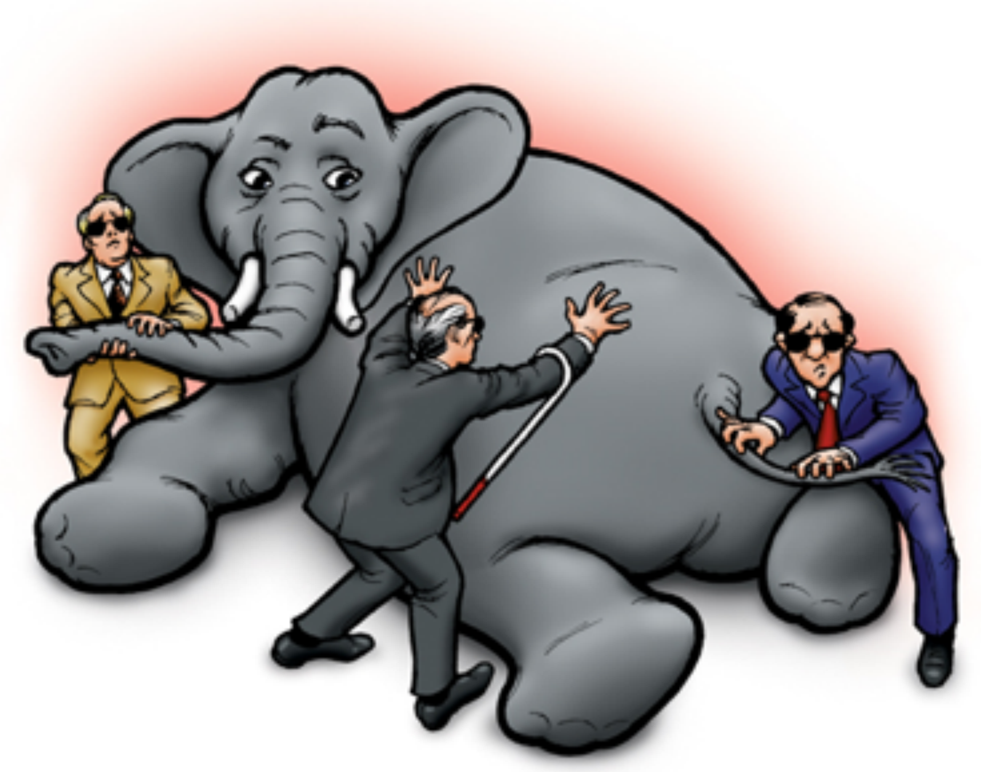
$$E_n = \frac{2\pi v}{L}\left(n - \frac{\theta}{2\pi k}\right)$$

$k = 2$ for Laughlin 1/2

Mittal, SG, Vaezi, Fan, Hafezi, arXiv:1504.00369

Measuring spectral flow for Laughlin 1/2 state

$$E_n = \frac{2\pi v}{L}\left(n - \frac{\theta}{2\pi k}\right)$$



Program

1) Preparation.

2) Observe boundary modes.

3) Selective gauging of boundary modes. (What is gauged? quasiparticles or fundamental particles!!!)

4) Anomalous edge spectral flow would be a window into the bulk state with strongly correlated topological order

Mittal, SG, Vaezi, Fan, Hafezi, arXiv:1504.00369

Collaborators

- **<u>Sunil Mittal</u>** (UMD NIST JQI)
- A. Vaezi (Cornell -> Stanford)
- J. Fan (NIST)
- Mohammad Hafezi (UMD, NIST, JQI)

Thanks you for your attention!