

# Topological Phases of Sound and Light

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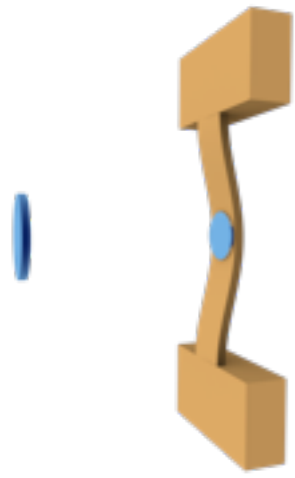
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Stefan Keßler, Steve Habraken

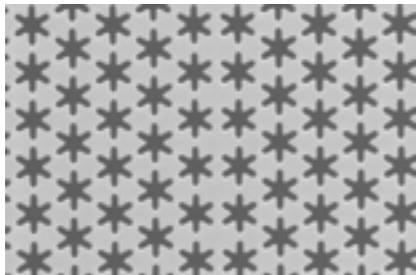


**DARPA  
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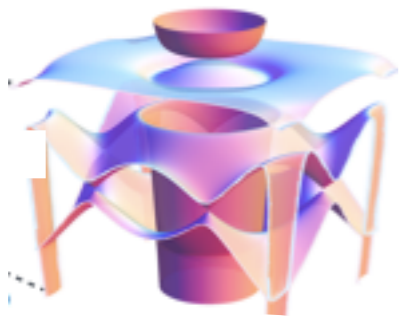




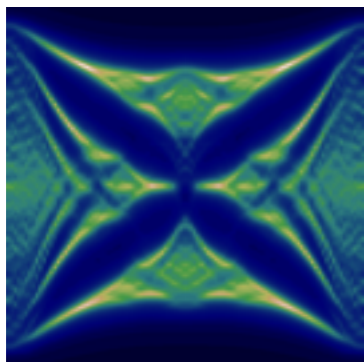
## Cavity Optomechanics



## Optomechanical Arrays



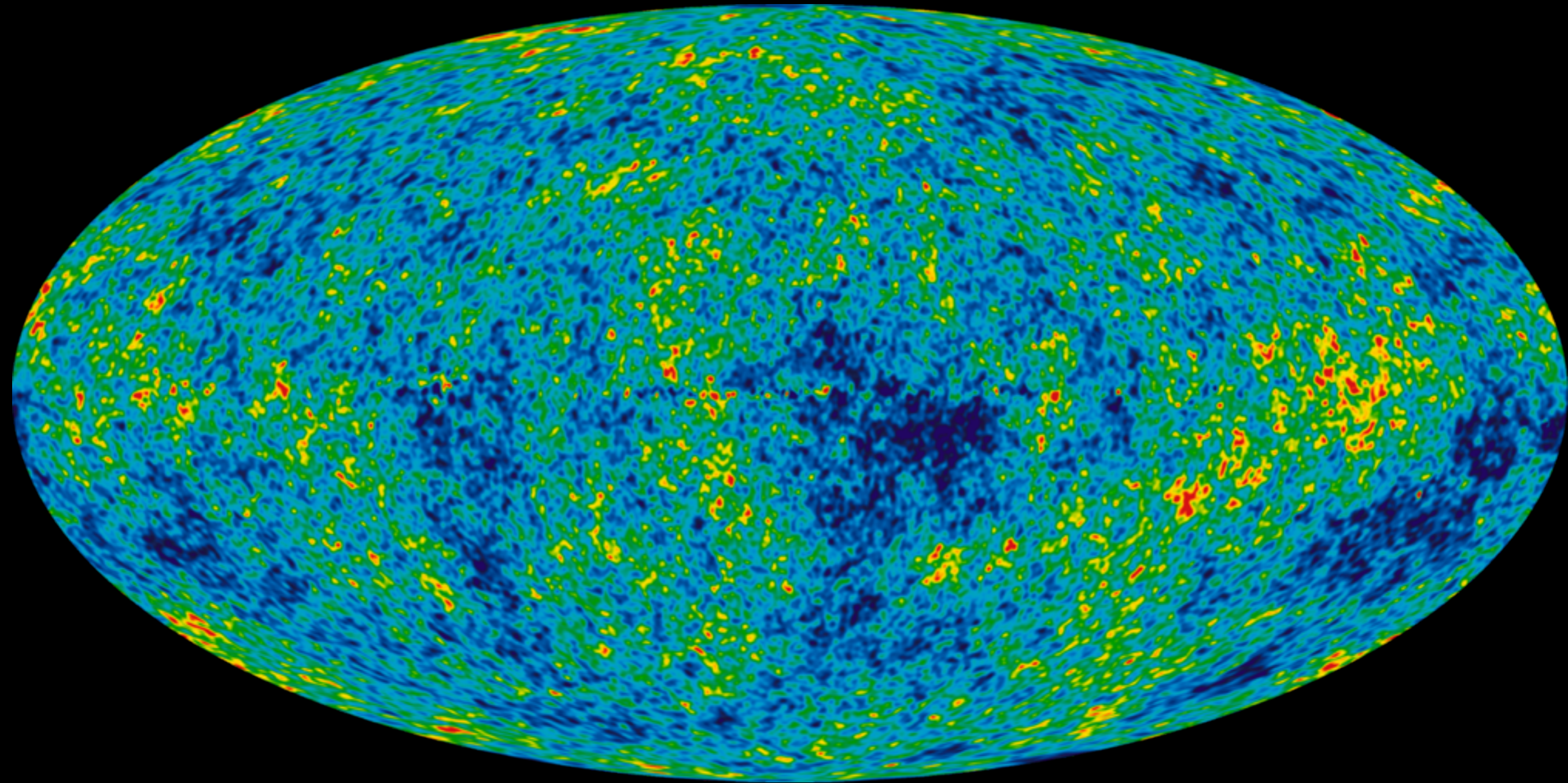
## Topological Phases of Sound (and Light)



## Dynamical Gauge Fields for the photons

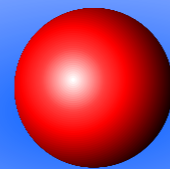


baryon-photon fluid: sound speed  $c/\sqrt{3}$





# Radiation forces

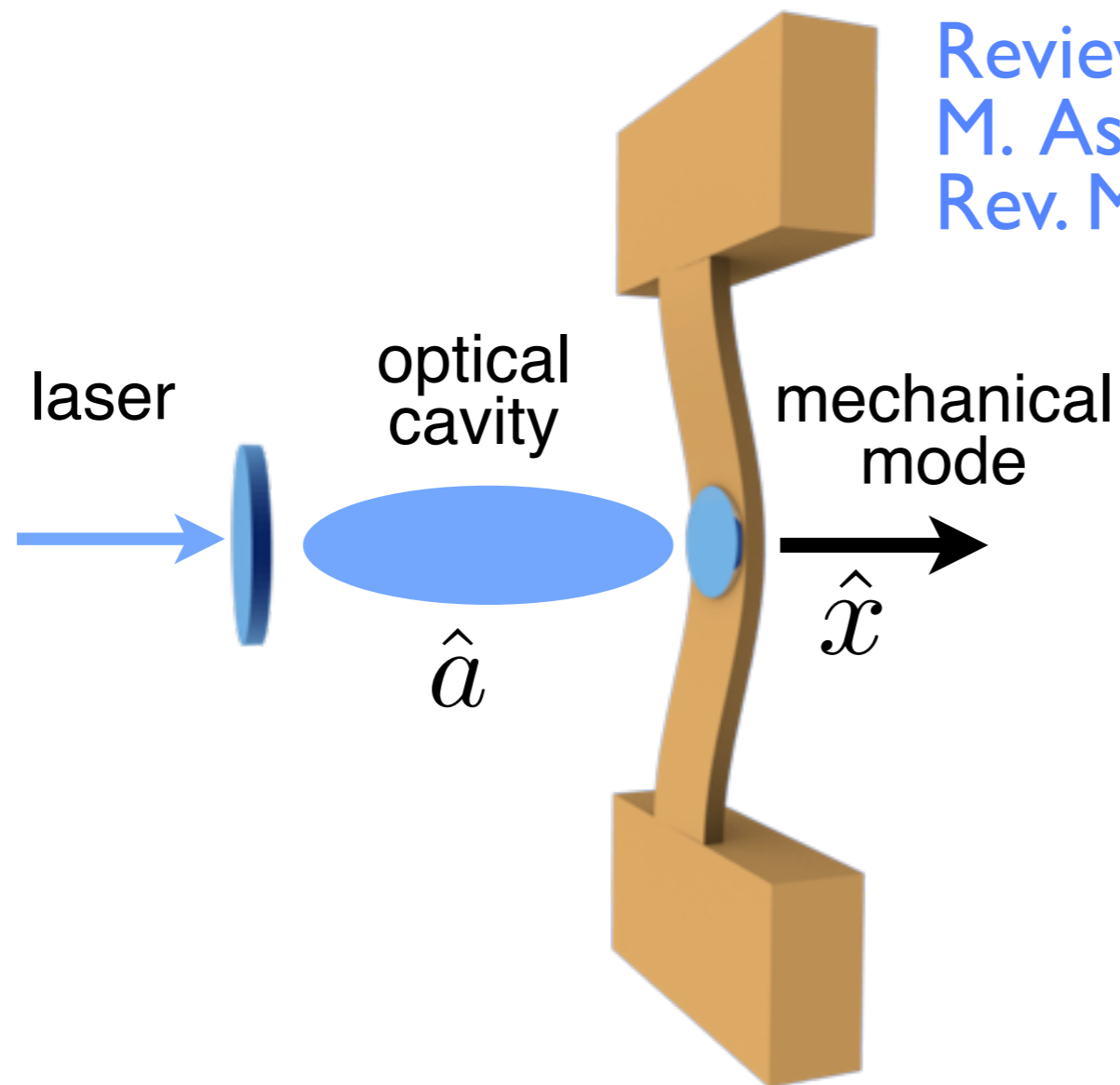


Trapping and cooling

- Optical tweezers
- Optical lattices



# Optomechanical Hamiltonian



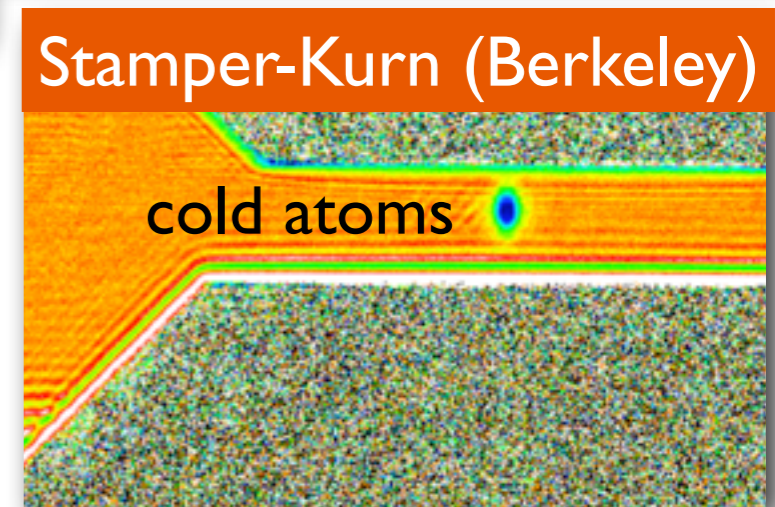
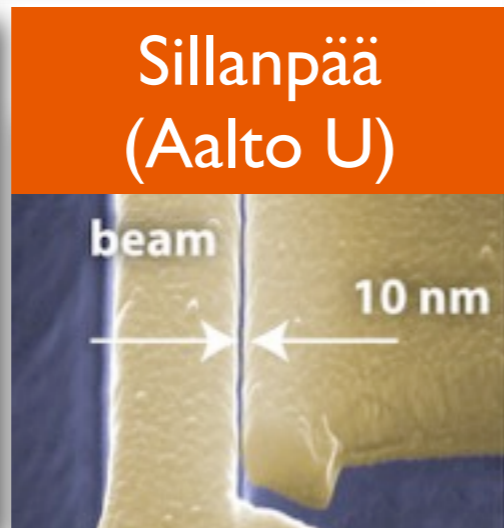
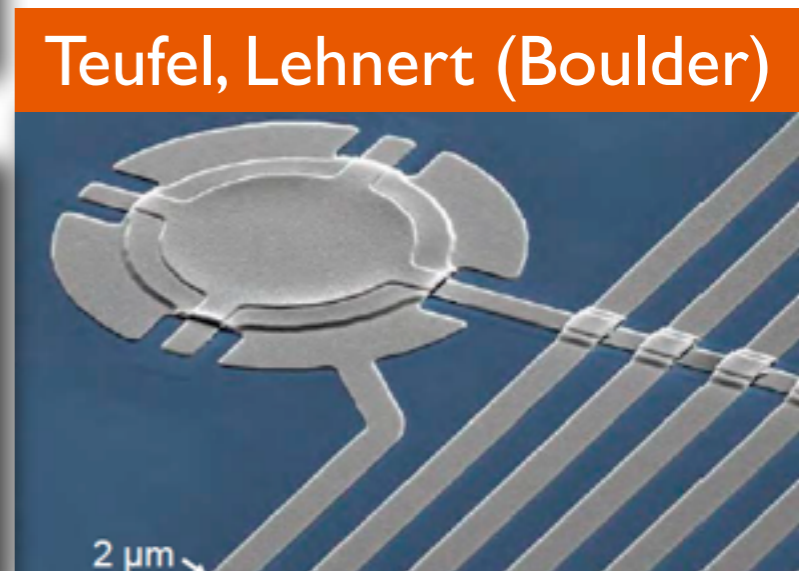
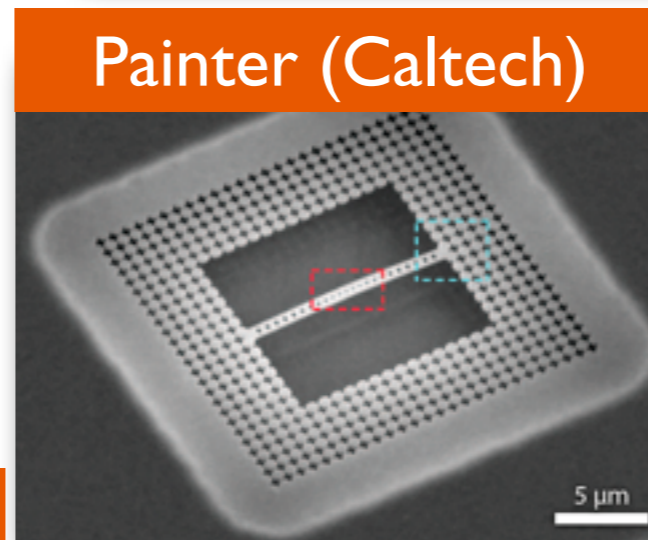
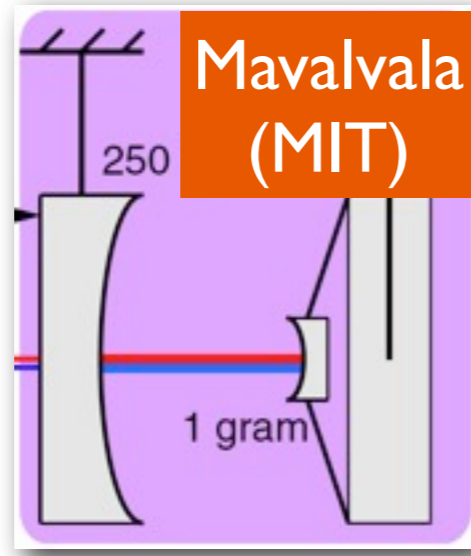
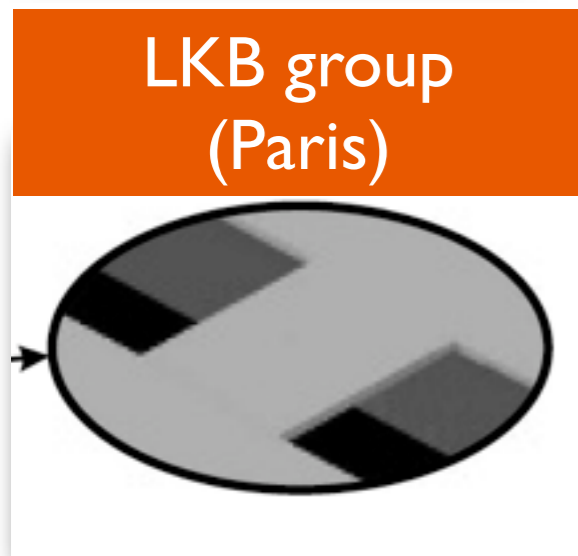
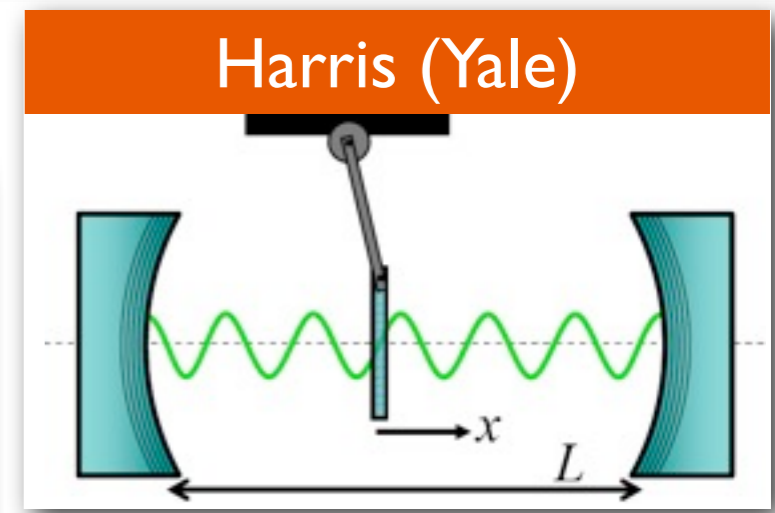
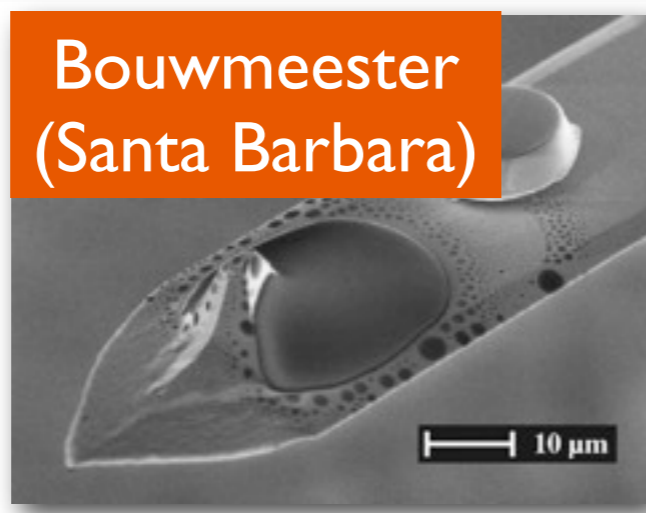
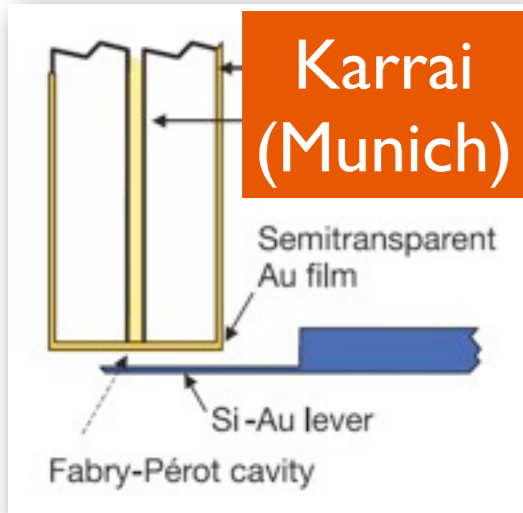
Review “Cavity Optomechanics”:  
M. Aspelmeyer, T. Kippenberg, FM  
Rev. Mod. Phys. **86**, 1391 (2014)

$$\hat{H} = \hbar\omega_{\text{cav}} \cdot (1 - \hat{x}/L)\hat{a}^\dagger \hat{a} + \hbar\Omega\hat{b}^\dagger \hat{b} + \dots$$

$$\hat{x} = x_{\text{ZPF}}(\hat{b} + \hat{b}^\dagger)$$

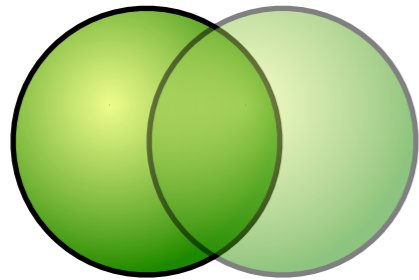
$$x_{\text{ZPF}} = \sqrt{\hbar/2m\Omega}$$

# The zoo of experimental setups in cavity optomechanics, 2005-now

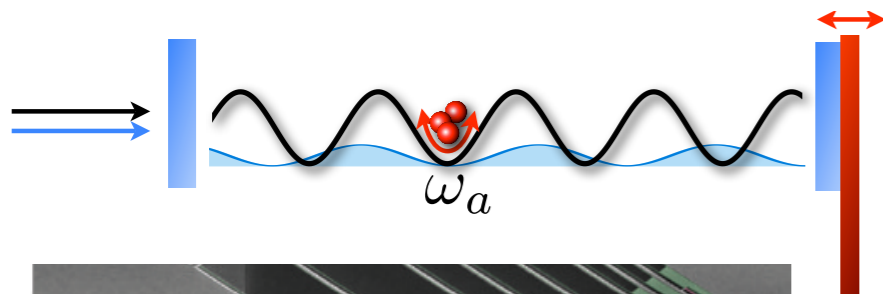




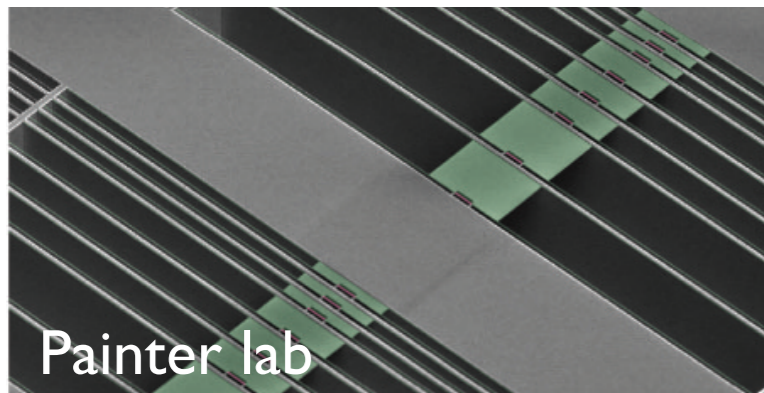
# Optomechanics: general outlook



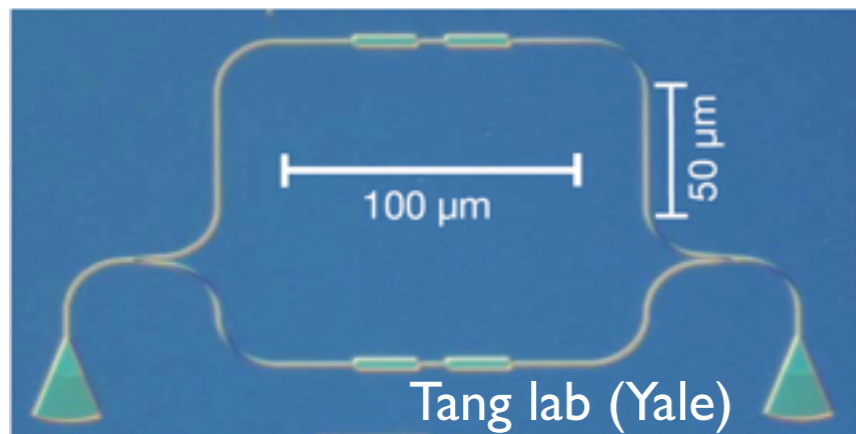
**Fundamental tests of quantum mechanics in a new regime:** entanglement with ‘macroscopic’ objects, unconventional decoherence? [e.g.: gravitationally induced?]



**Mechanics as a ‘bus’ for connecting hybrid components:** superconducting qubits, spins, photons, cold atoms, ....

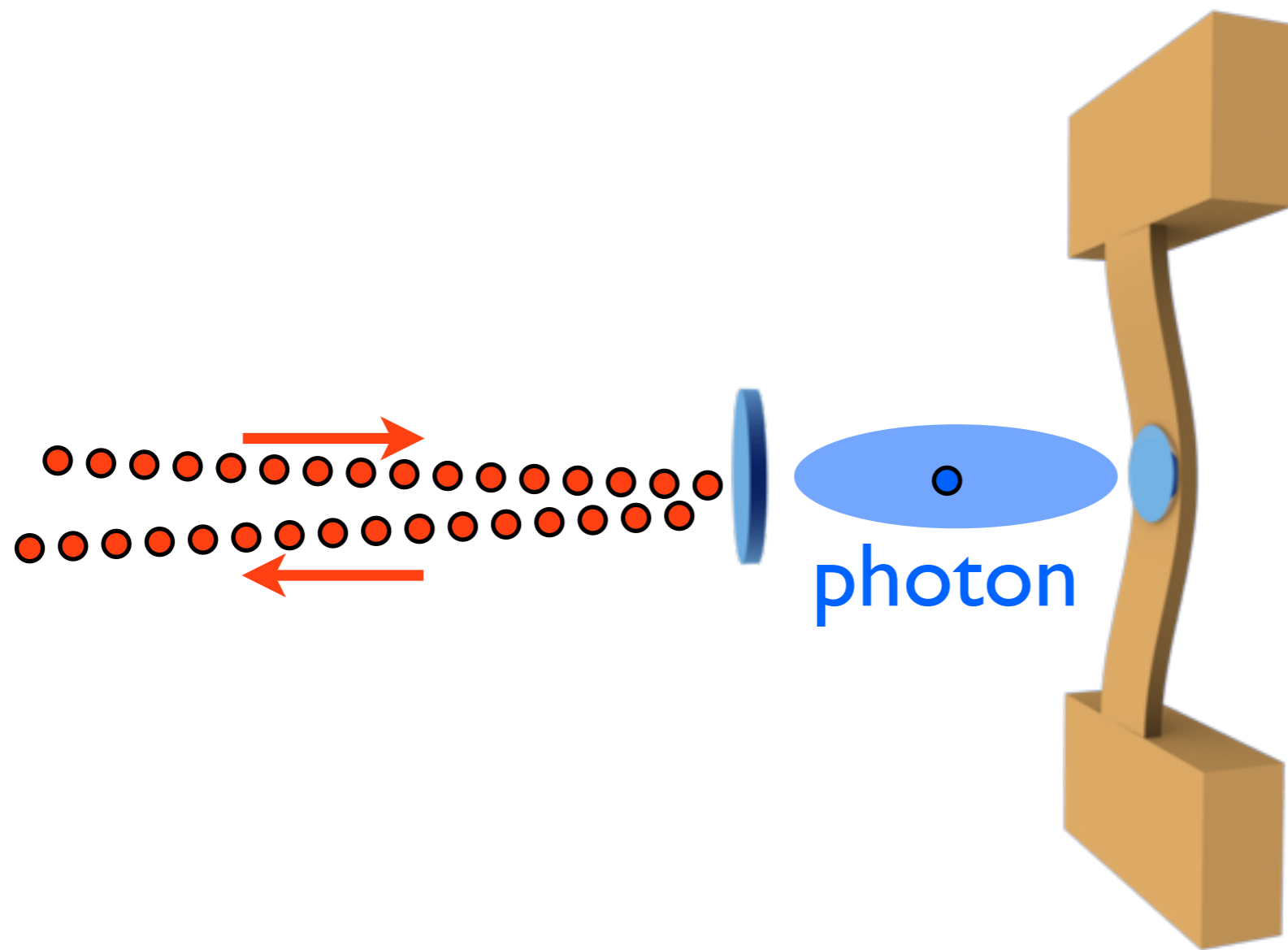


**Precision measurements**  
small displacements, masses, forces, and accelerations



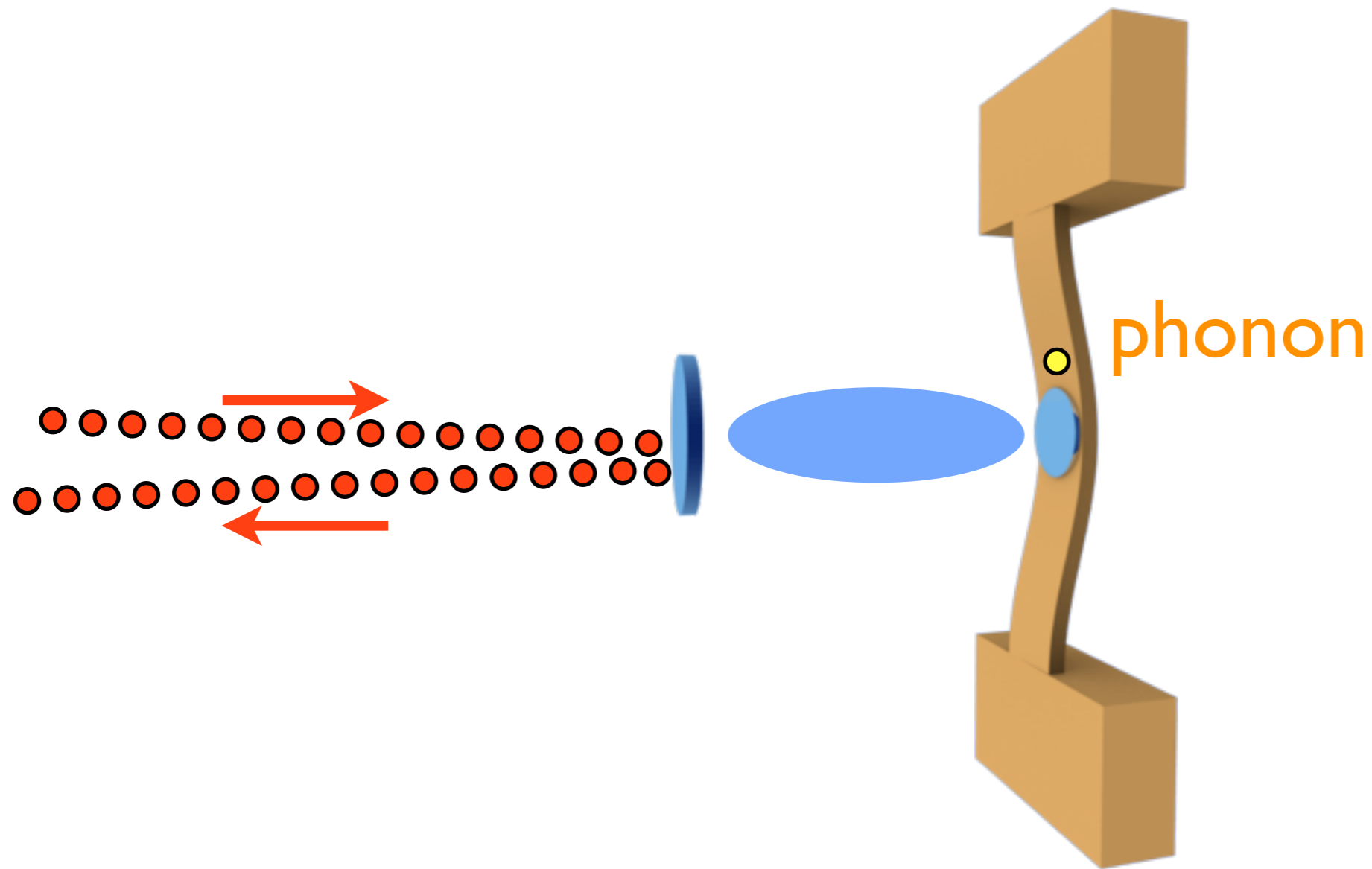
**Optomechanical circuits & arrays**  
Exploit nonlinearities for classical and quantum information processing, storage, and amplification; study collective dynamics in arrays

# Converting photons into phonons






# Converting photons into phonons



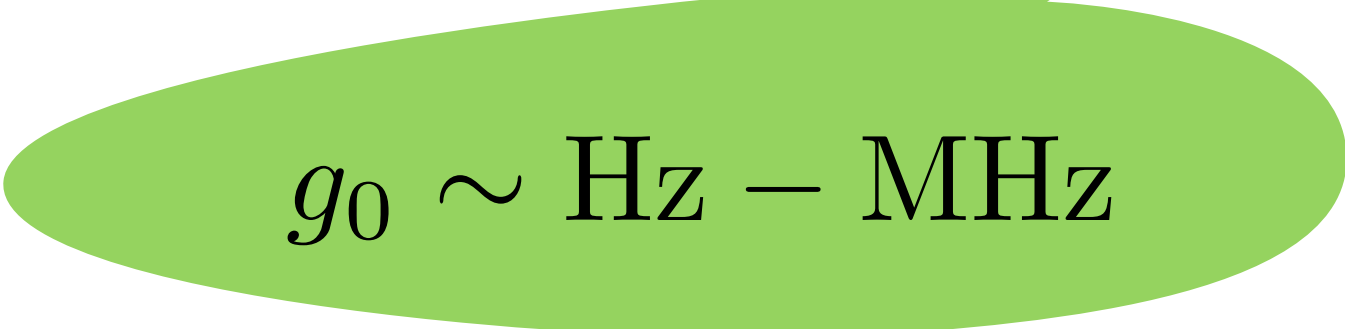
# “Linearized” Optomechanical Hamiltonian

$$\hbar g_0 \hat{a}^\dagger \hat{a} (\hat{b} + \hat{b}^\dagger)$$

$$\hat{a} = \alpha + \delta \hat{a}$$


$$\hbar g_0 (\alpha \delta \hat{a}^\dagger + \alpha^* \delta \hat{a}) (\hat{b} + \hat{b}^\dagger)$$

“laser-enhanced  
optomechanical coupling”:  $g = g_0 \alpha$

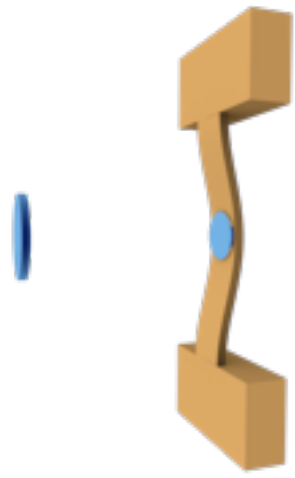

$$g_0 \sim \text{Hz} - \text{MHz}$$

bare optomechanical coupling  
(geometry, etc.: fixed!)

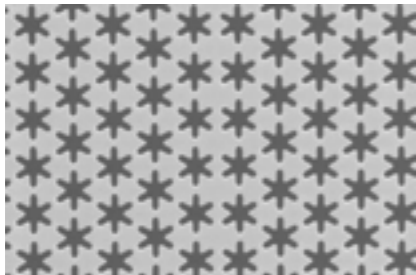

$$\alpha$$

laser-driven  
cavity amplitude  
tuneable! **phase!**

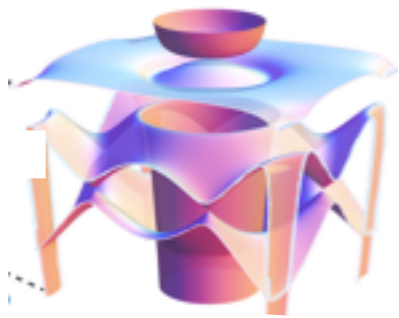




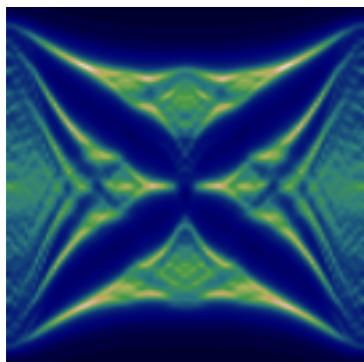
## Cavity Optomechanics



## Optomechanical Arrays

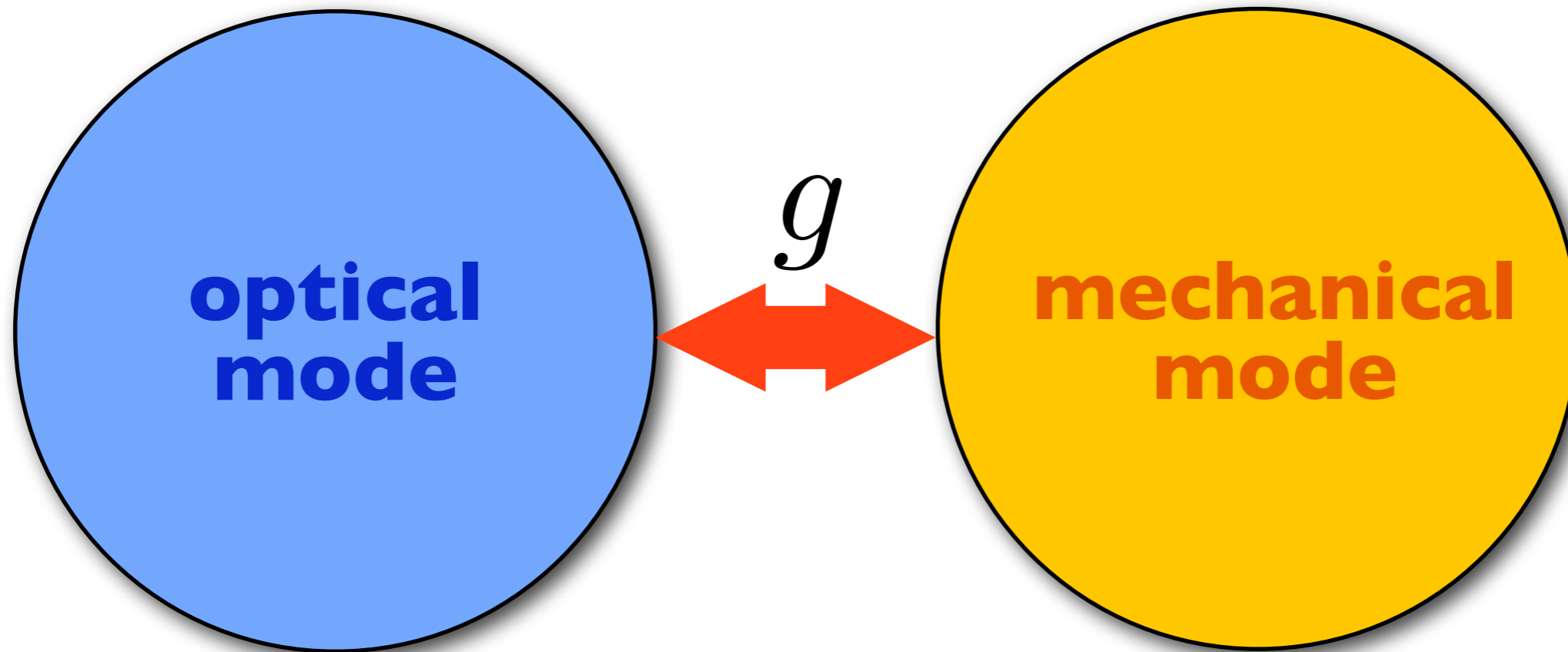


## Topological Phases of Sound (and Light)



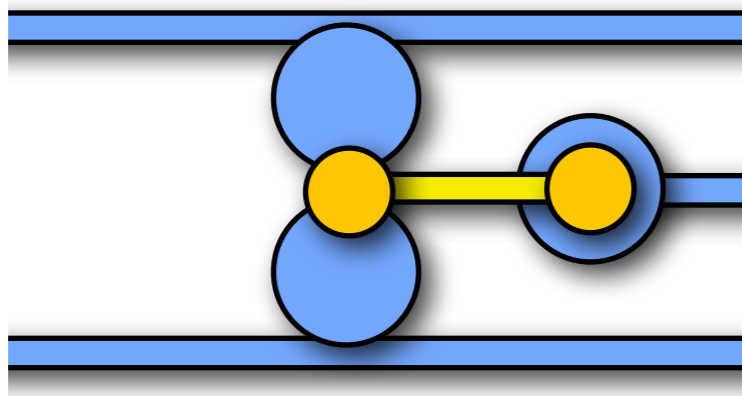
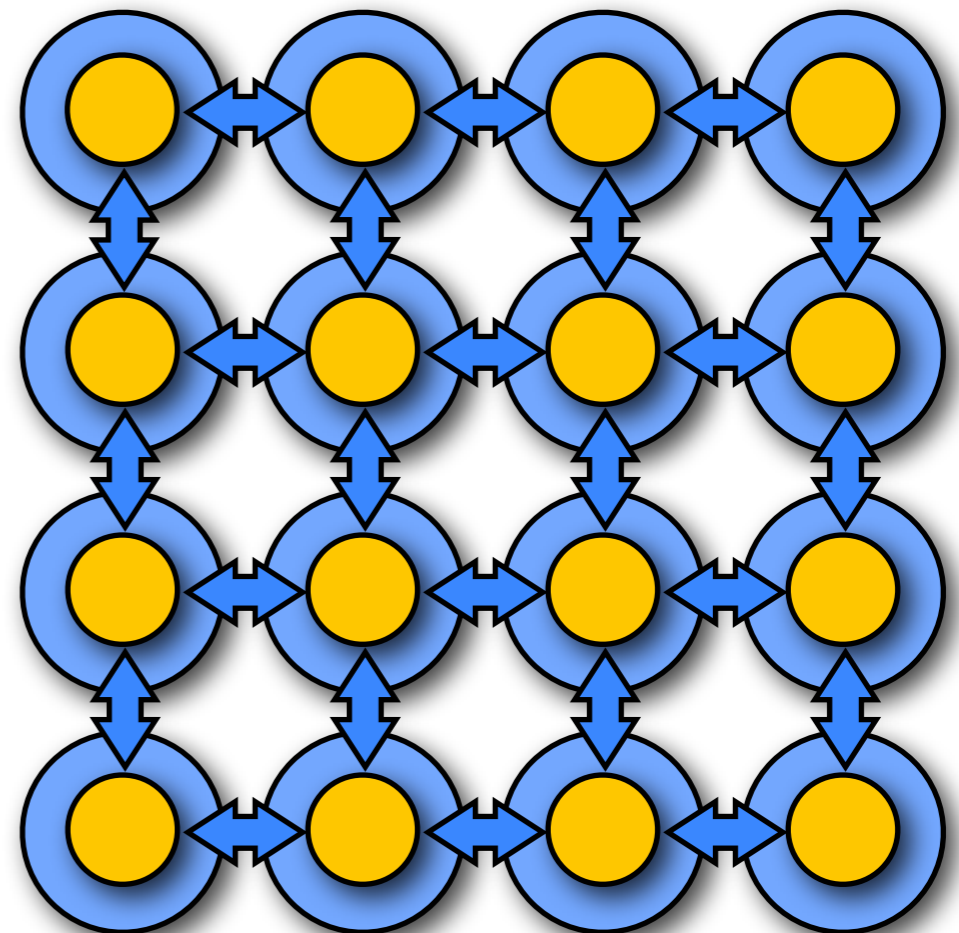
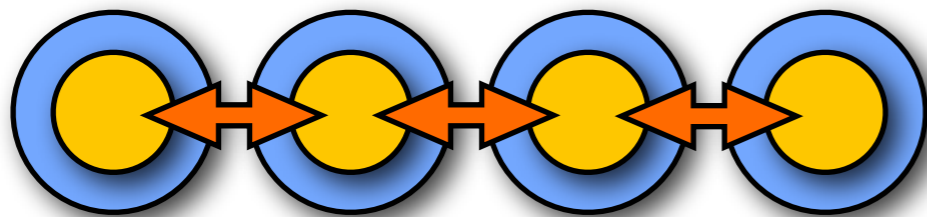
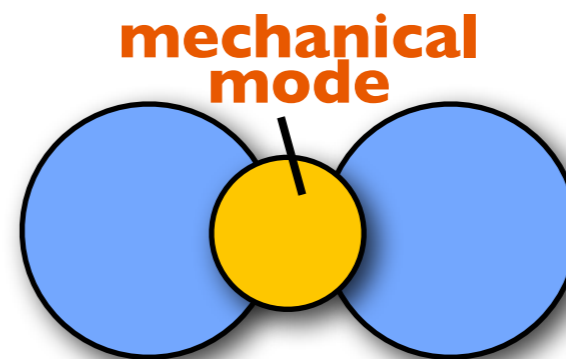
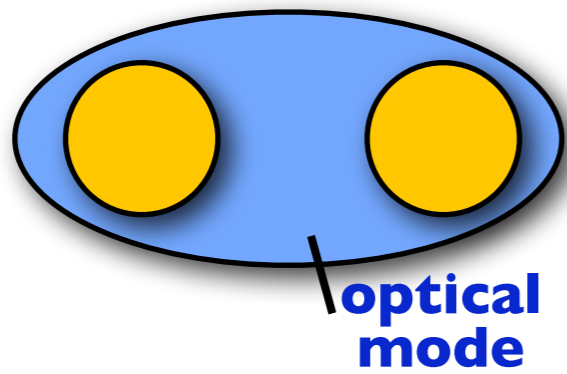
## Dynamical Gauge Fields for the photons

# Single-mode optomechanics



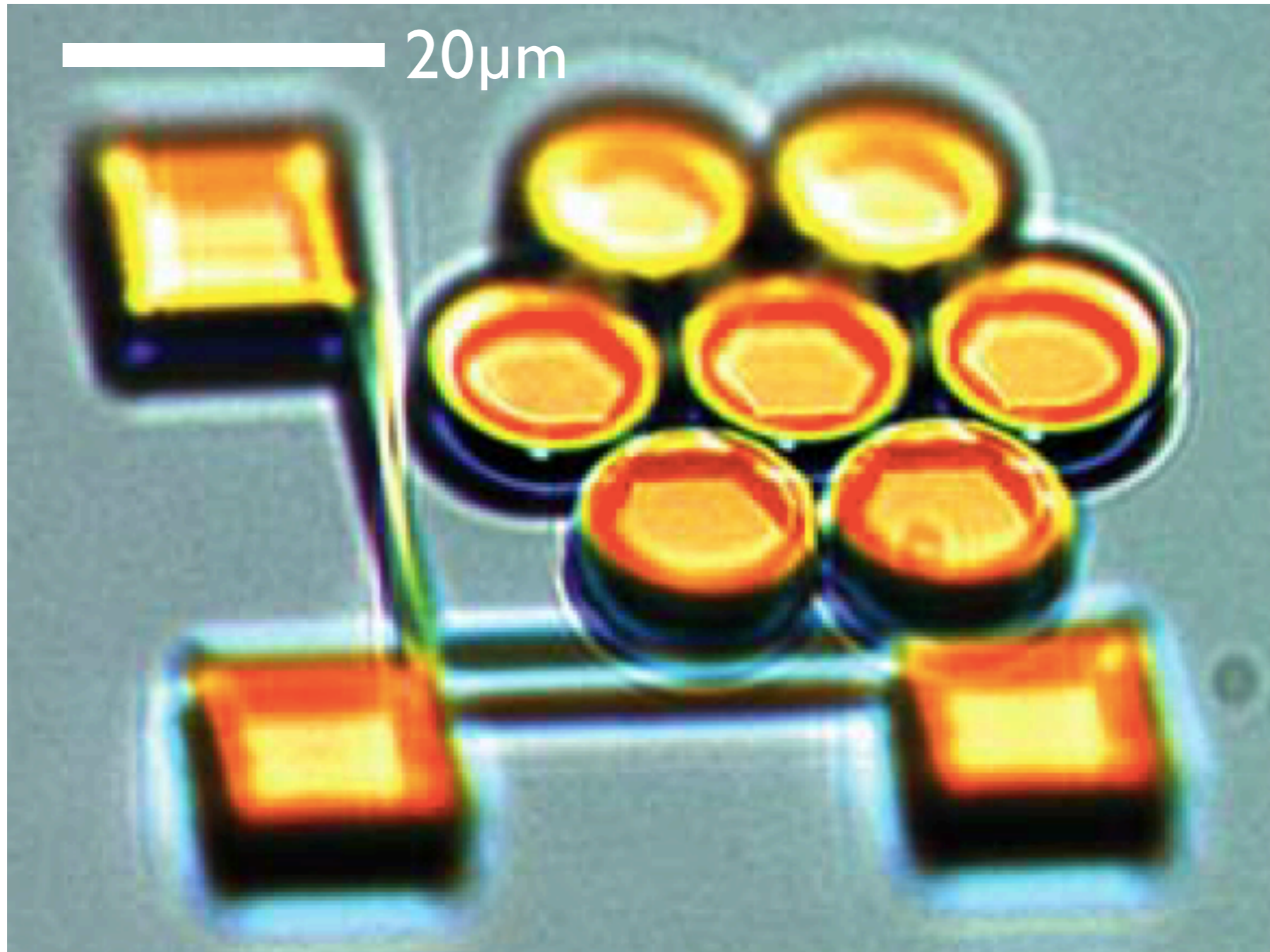
- ✓ displacement sensing
- ✓ cooling
- ✓ strong coupling
- ✓ self-oscillations (limit cycles)

# Many modes





# First realizations

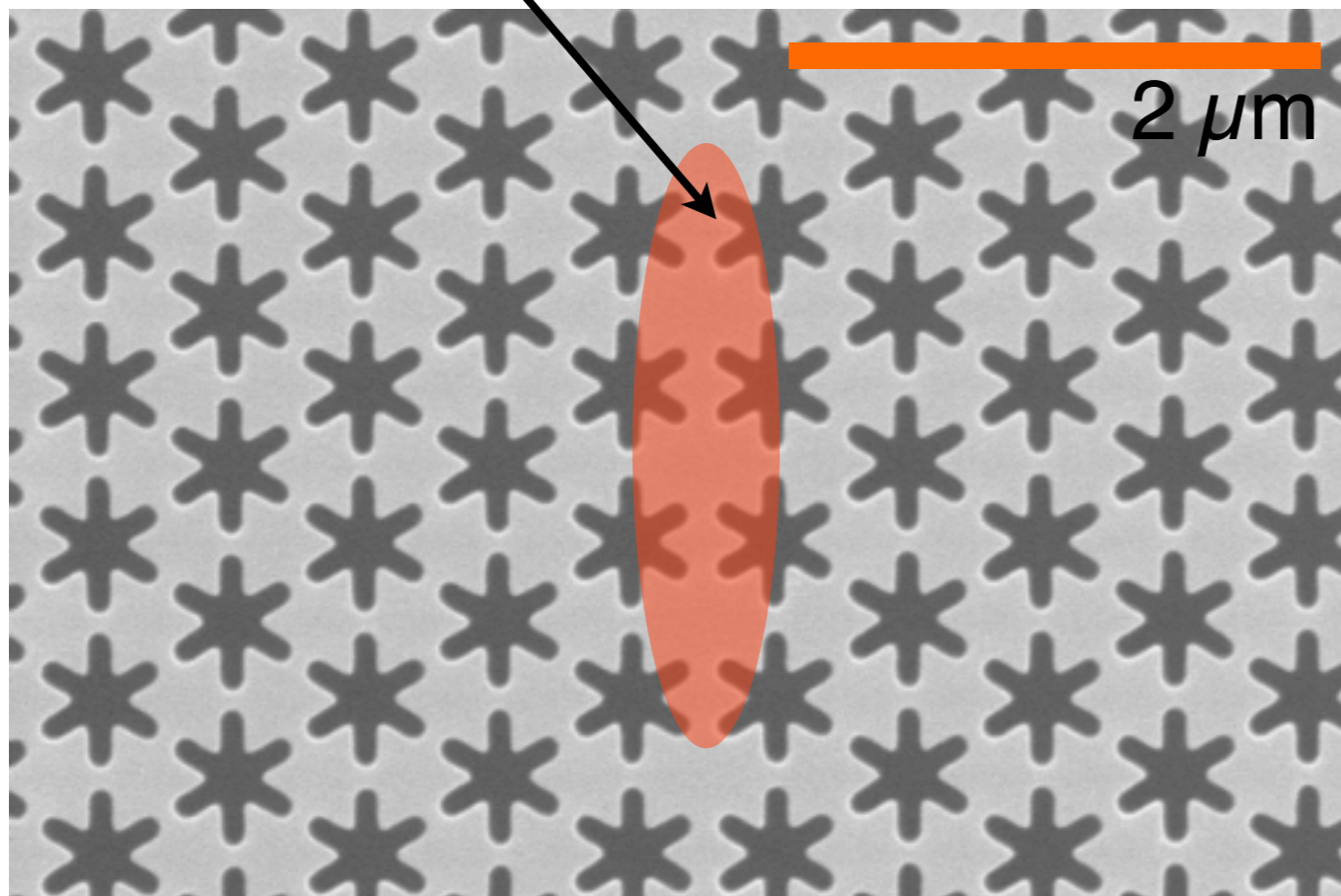


Lipson group, Cornell arXiv:1505.02009 (synchronization)

# Optomechanical crystals

= free-standing photonic crystal structures (Painter group)

localized optical and vibrational (GHz) mode



## advantages:

tight vibrational confinement:  
high frequencies, small mass  
(stronger quantum effects)

tight optical confinement:  
large optomechanical  
coupling  
(100 GHz/nm)

integrated on a chip

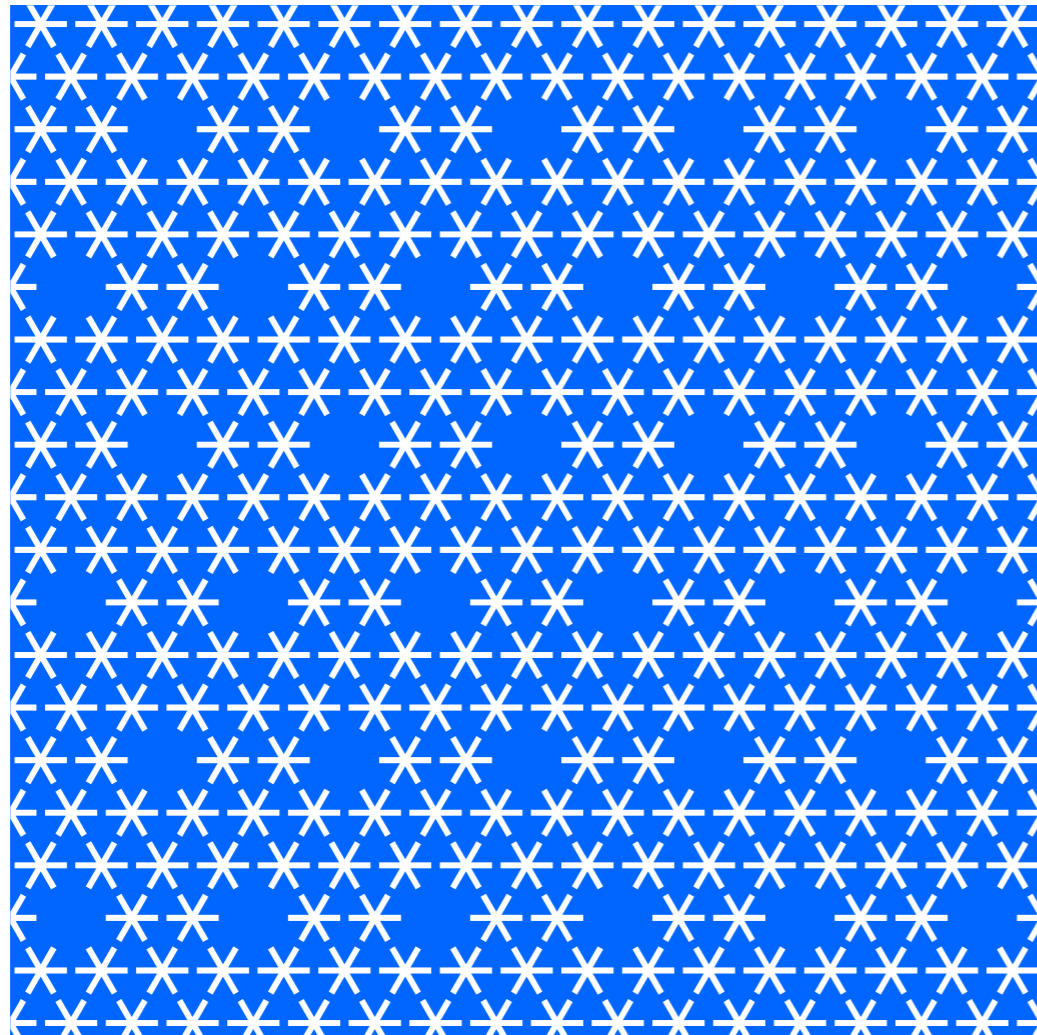
Safavi-Naeini et al PRL 2014

Eichenfield et al Nature 2009

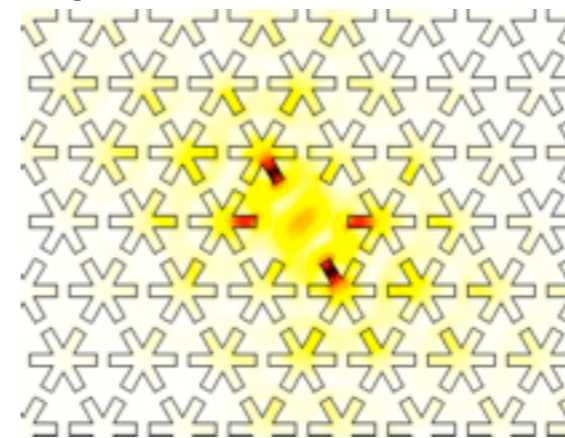


# Optomechanical arrays

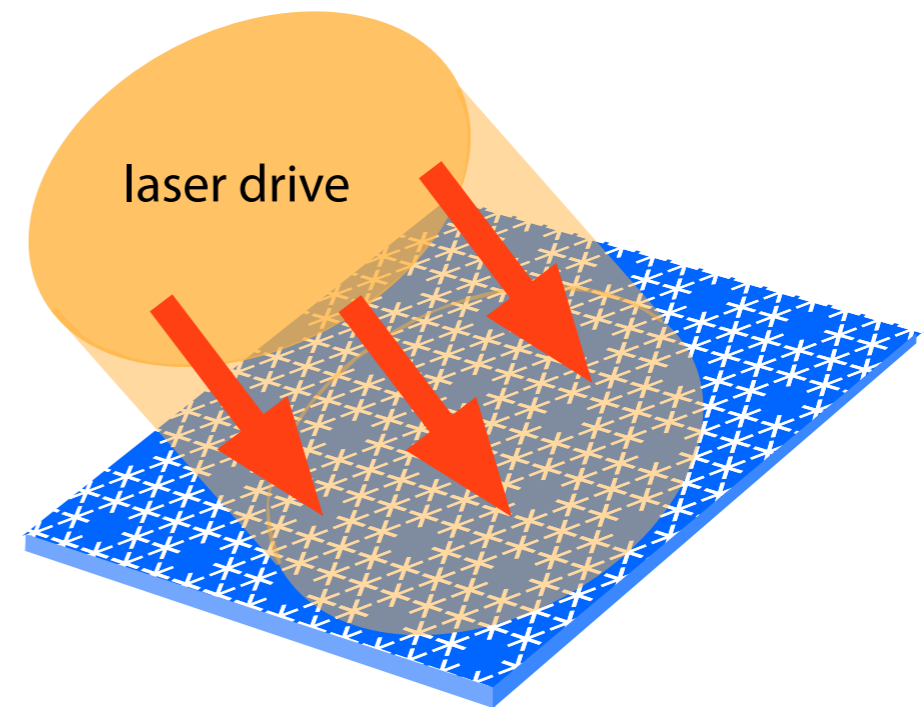
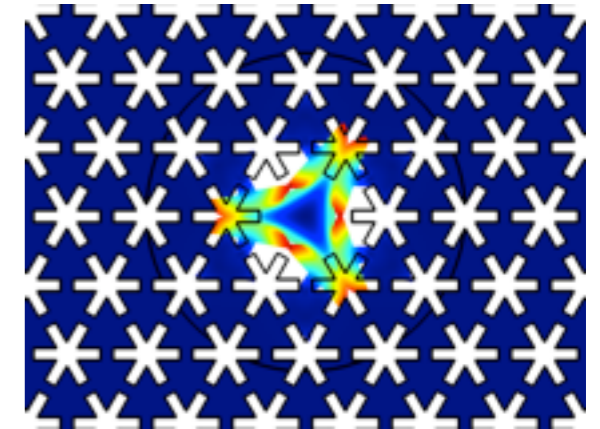
Optomechanical array: Many coupled optomechanical cells



optical mode



mechanical mode

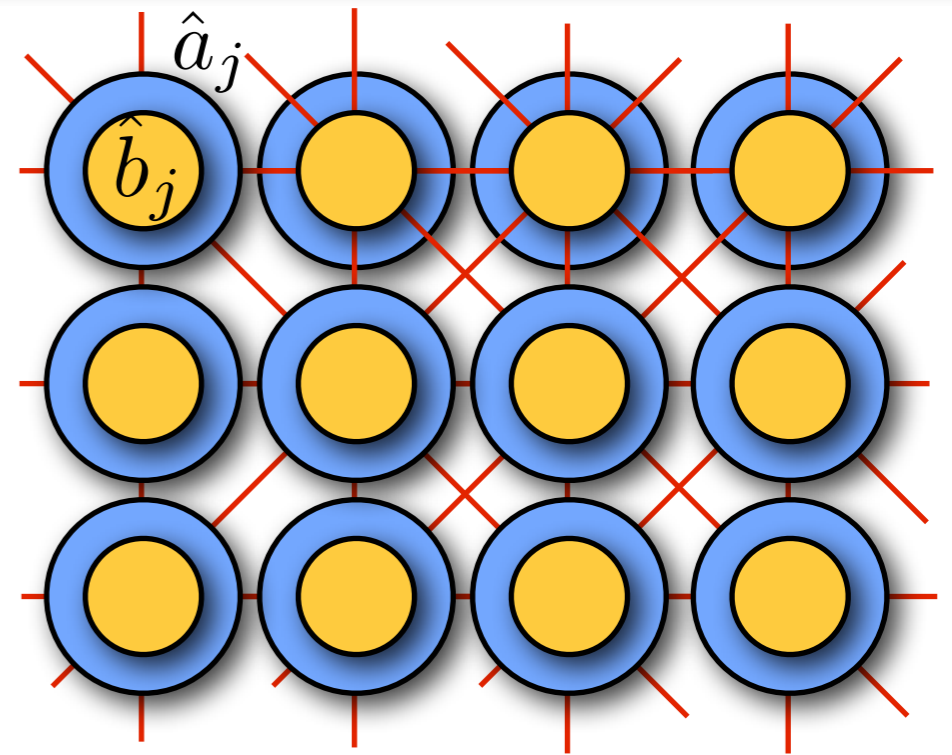


Possible design based on “snowflake” 2D optomechanical crystal (Painter group), here: with suitable defects forming a superlattice (array of cells)



# Modeling an optomechanical array

Tight-binding model for photons & phonons hopping and interacting on a lattice



laser detuning

$$\Delta = \omega_L - \omega_{\text{opt}}$$

each cell:

$$\hat{H}_{\text{om},j} = -\Delta \hat{a}_j^\dagger \hat{a}_j + \Omega \hat{b}_j^\dagger \hat{b}_j - g_0 (\hat{b}_j^\dagger + \hat{b}_j) \hat{a}_j^\dagger \hat{a}_j + \alpha_L (\hat{a}_j^\dagger + \hat{a}_j)$$

optomech. interaction
laser drive

$$\hat{H}_{\text{int}} = - \mathbf{J} \sum_{\langle i,j \rangle} (\hat{a}_i^\dagger \hat{a}_j + \hat{a}_i \hat{a}_j^\dagger) - \mathbf{K} \sum_{\langle i,j \rangle} (\hat{b}_i^\dagger \hat{b}_j + \hat{b}_i \hat{b}_j^\dagger)$$

optical coupling  
(photon tunneling)
mechanical coupling  
(phonon tunneling)

# Optomechanical Arrays

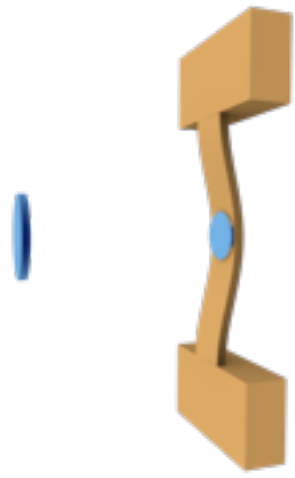
global view:

light-tunable metamaterial for photons & phonons

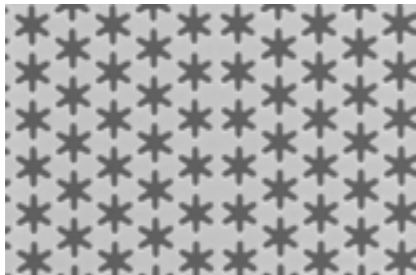


similar in spirit:  
optical lattices  
nonlinear optical materials

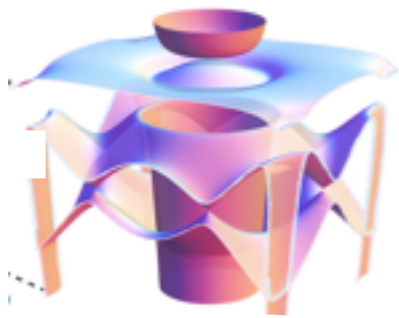
conceptually simple: one material, with holes



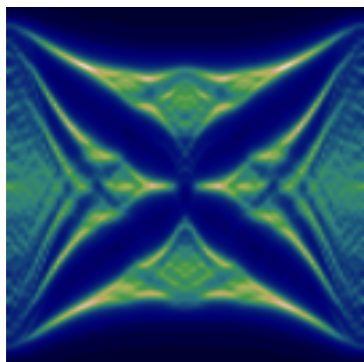
## Cavity Optomechanics



## Optomechanical Arrays



## Topological Phases of Sound (and Light)



## Dynamical Gauge Fields for the photons

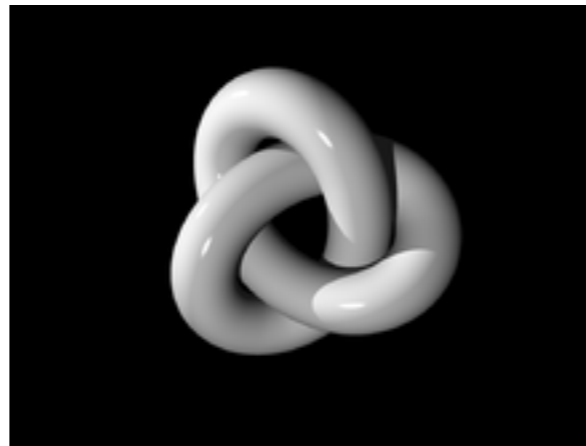


# Topological Materials

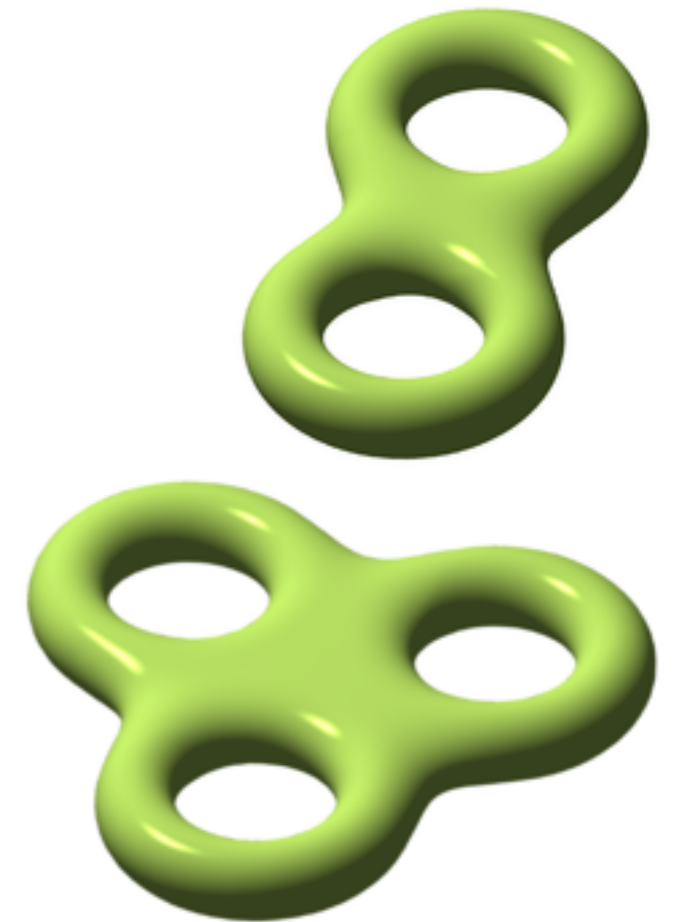
Topological properties:  
robust against smooth changes!



Möbius strip



knots



n-fold torus

superfluid  
vortex



# Topological Materials

Waves can show topological robustness!

review: Hasan, Kane RMP 2010

Quantum Hall Effect (Chern number = conductance)

Topological Insulators:

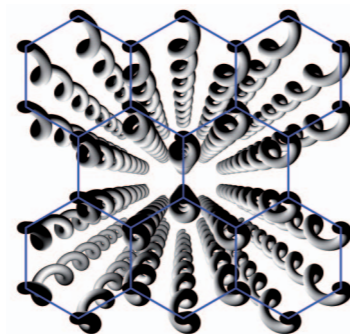
2D topological insulators, e.g. HgTe

3D topological insulators, e.g. BiSe

Other than electronic systems?

Proposals/first experiments for:  
atoms, ions, photons, magnons

cold atoms experiment: G. Jotzu et al. (Esslinger group), Nature 2014



photons:  
Khanikaev, ..., Shvets, Nature Materials 2012

Rechtsman, ..., Szameit Nature 2013

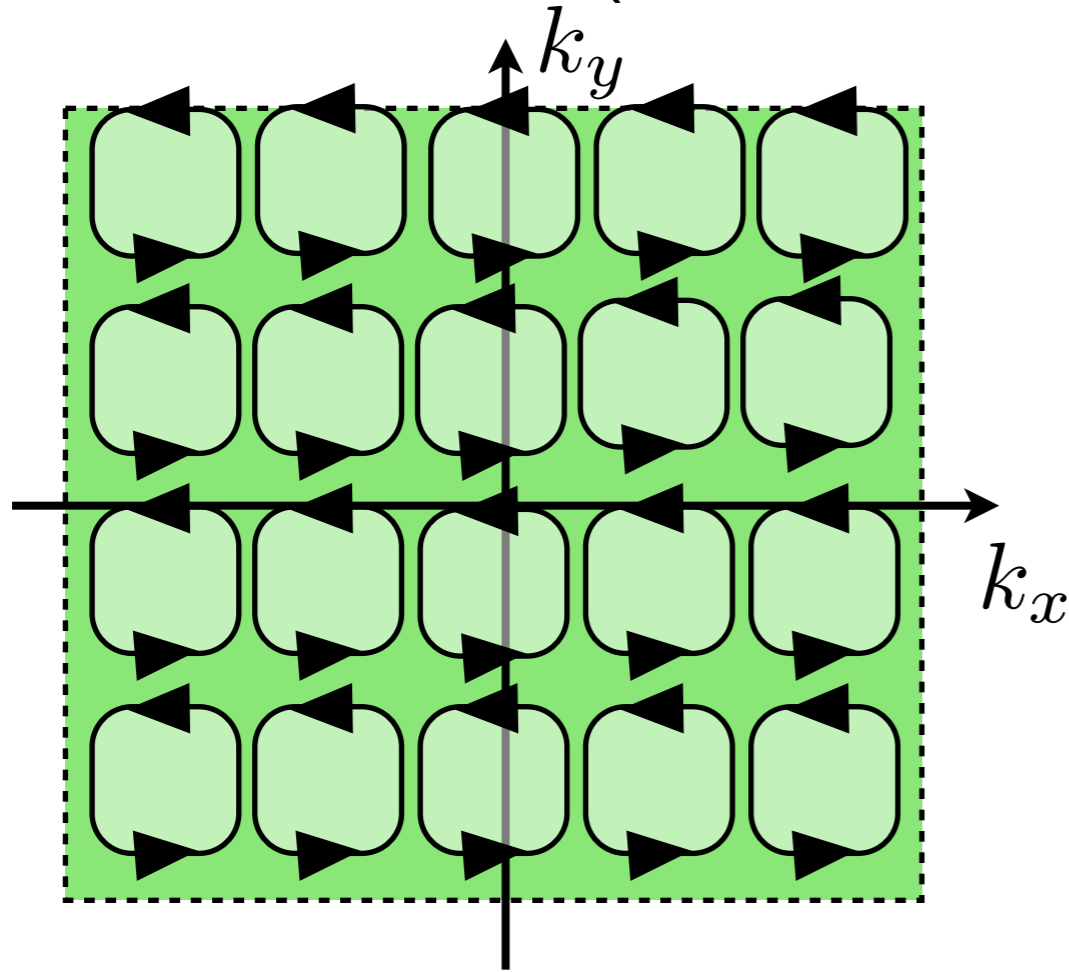
Mittal, ..., Hafezi PRL 2014

...

magnons: Zhang et al. 2013, Shindou et al 2013, Romhanyi et al 2015, ...

# Topological Bandstructures

Chern number = (sum of Berry phases across Brillouin zone)/ $2\pi$



Chern number =

$$\frac{1}{2\pi} \int dk_x dk_y \vec{\nabla} \times \langle \Psi_k | \vec{\nabla} | \Psi_k \rangle$$

Chern number = integer! topologically robust!



# Edge States

Chern=0

'trivial' band  
insulator

Chern=1

topologically  
nontrivial band  
insulator

Chiral Edge State

The diagram consists of two vertical rectangular regions. The left region is a lighter yellow color and contains the text 'Chern=0' and ''trivial' band insulator'. The right region is a darker yellow color and contains the text 'Chern=1' and 'topologically nontrivial band insulator'. A vertical black line with three upward-pointing arrowheads separates the two regions. At the bottom of this line, the text 'Chiral Edge State' is written.

# Edge States

Chern=0

'trivial' band insulator

Chern=1

topologically nontrivial band insulator

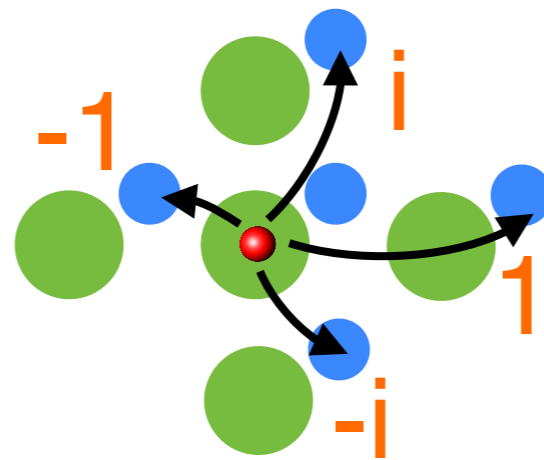
Chiral Edge State

The diagram shows a vertical interface between two materials. The left material is a 'trivial' band insulator with Chern=0. The right material is a topologically nontrivial band insulator with Chern=1. A single black line with arrows pointing upwards represents a chiral edge state that exists only at the interface between the two materials.

# Phonon Topological Materials

What about topological transport of phonons?

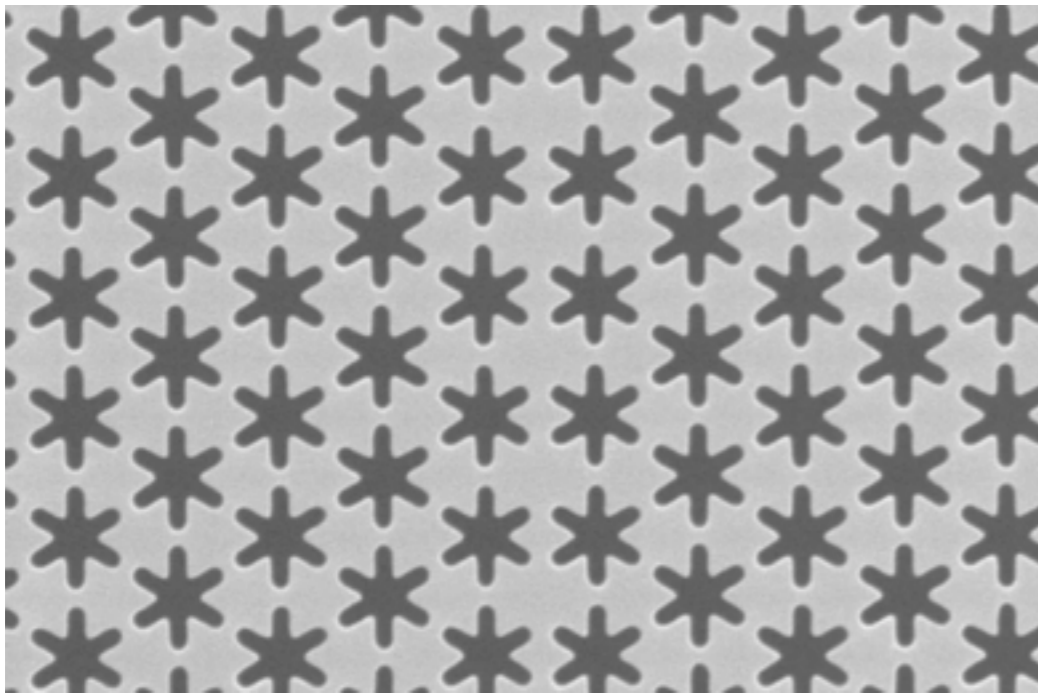
Engineer non-reciprocal phases for phonon transport!



# Topological Phases of Sound and Light

What about topological transport of phonons?

Need:



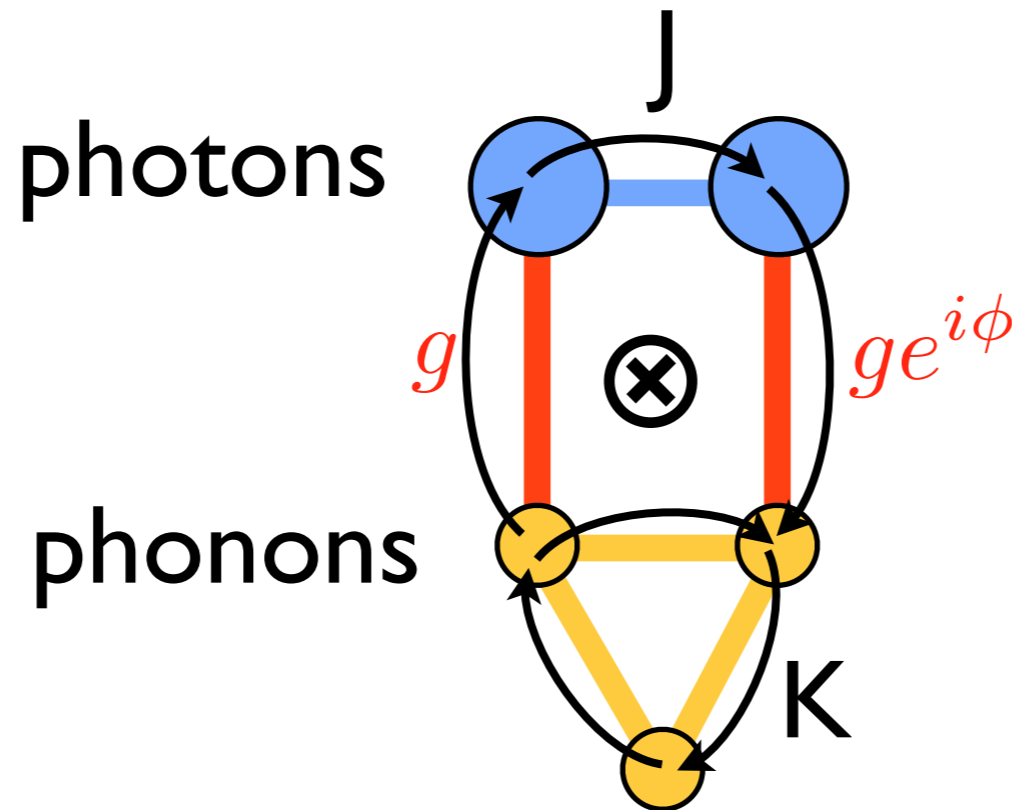
Dielectric  
(with the right  
pattern of holes)



One Laser  
(with the right  
pattern of phases)



# Gauge fields for phonons

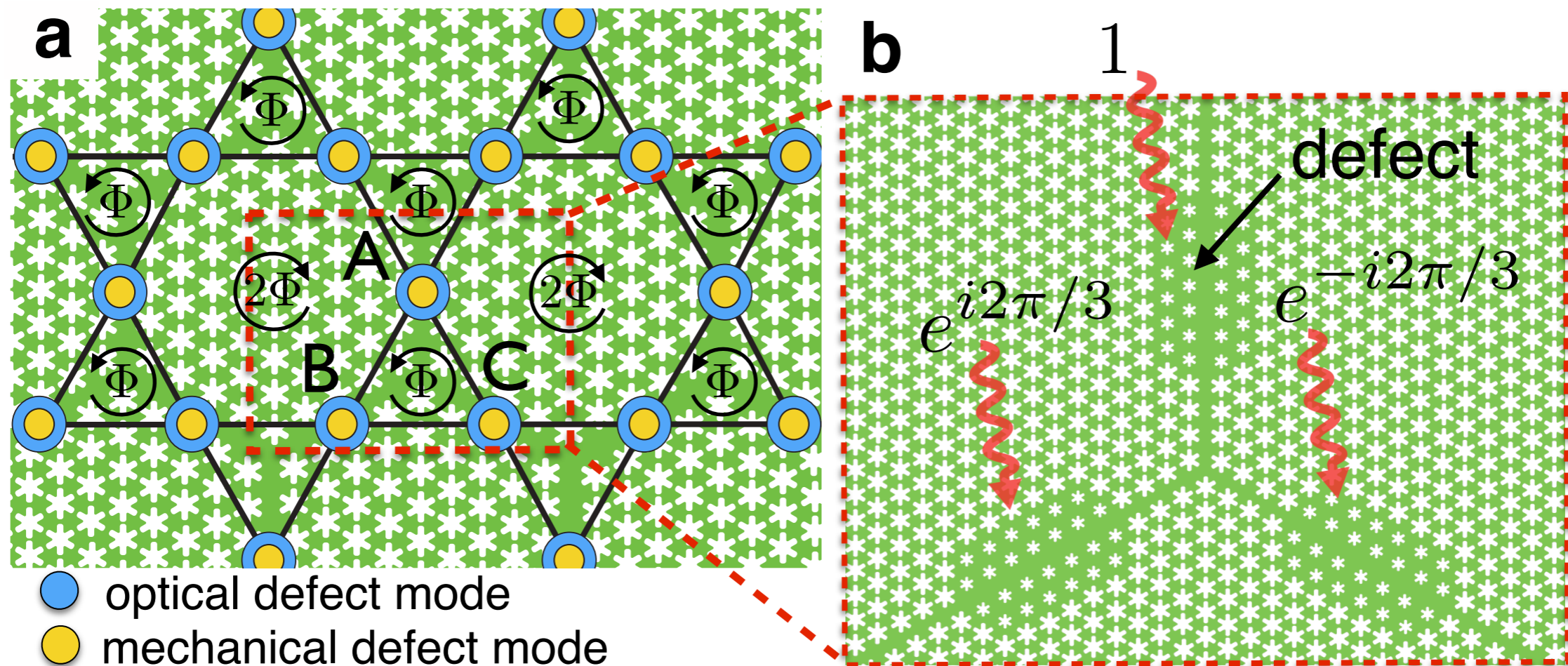


(works best for phonons, due to  $K \ll J$ )

first such scheme: “phonon circulator”,  
Habraken, Stannigel, Lukin, Zoller, and Rabl, *New Journal of Physics*, 14, 115004 (2012)

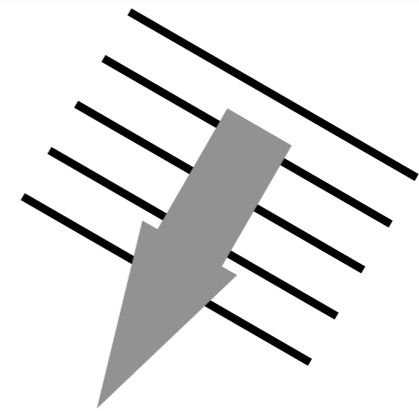
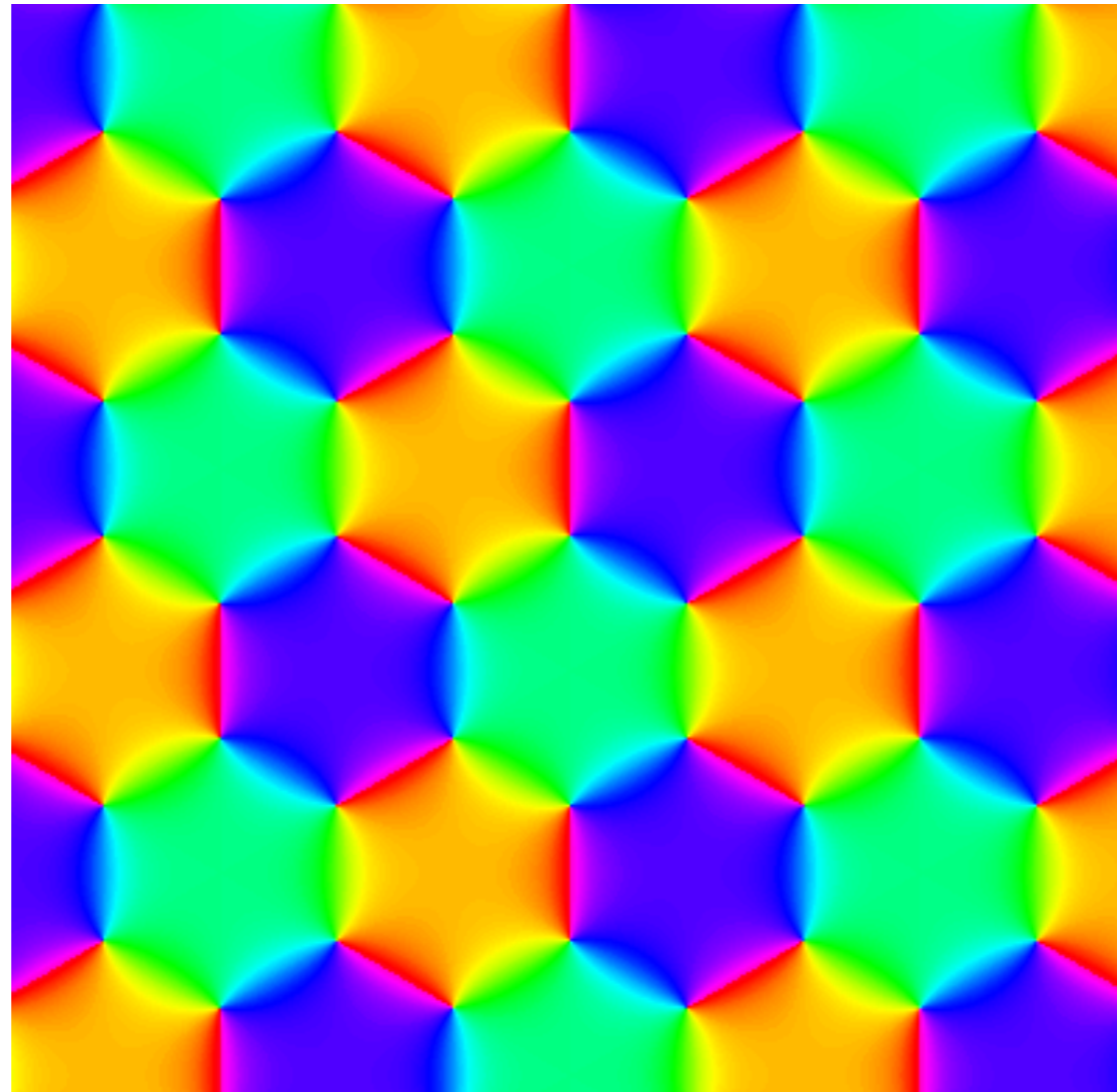
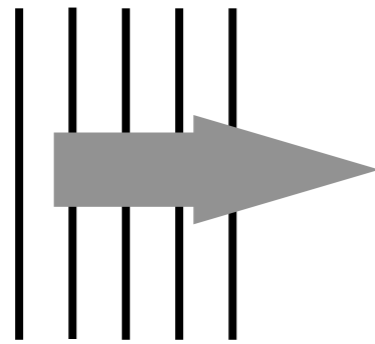
# Topological Phases of Sound and Light in an Optomechanical Array

## Kagome Optomechanical Array

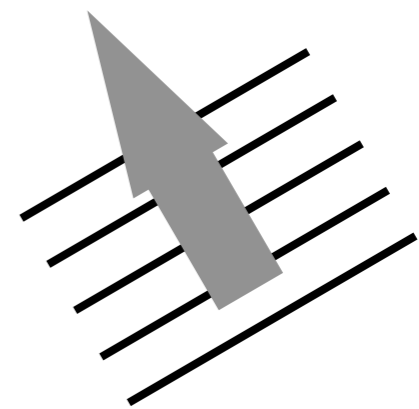


laser-field with different phases on sites A,B,C

# Creating an optical phase pattern

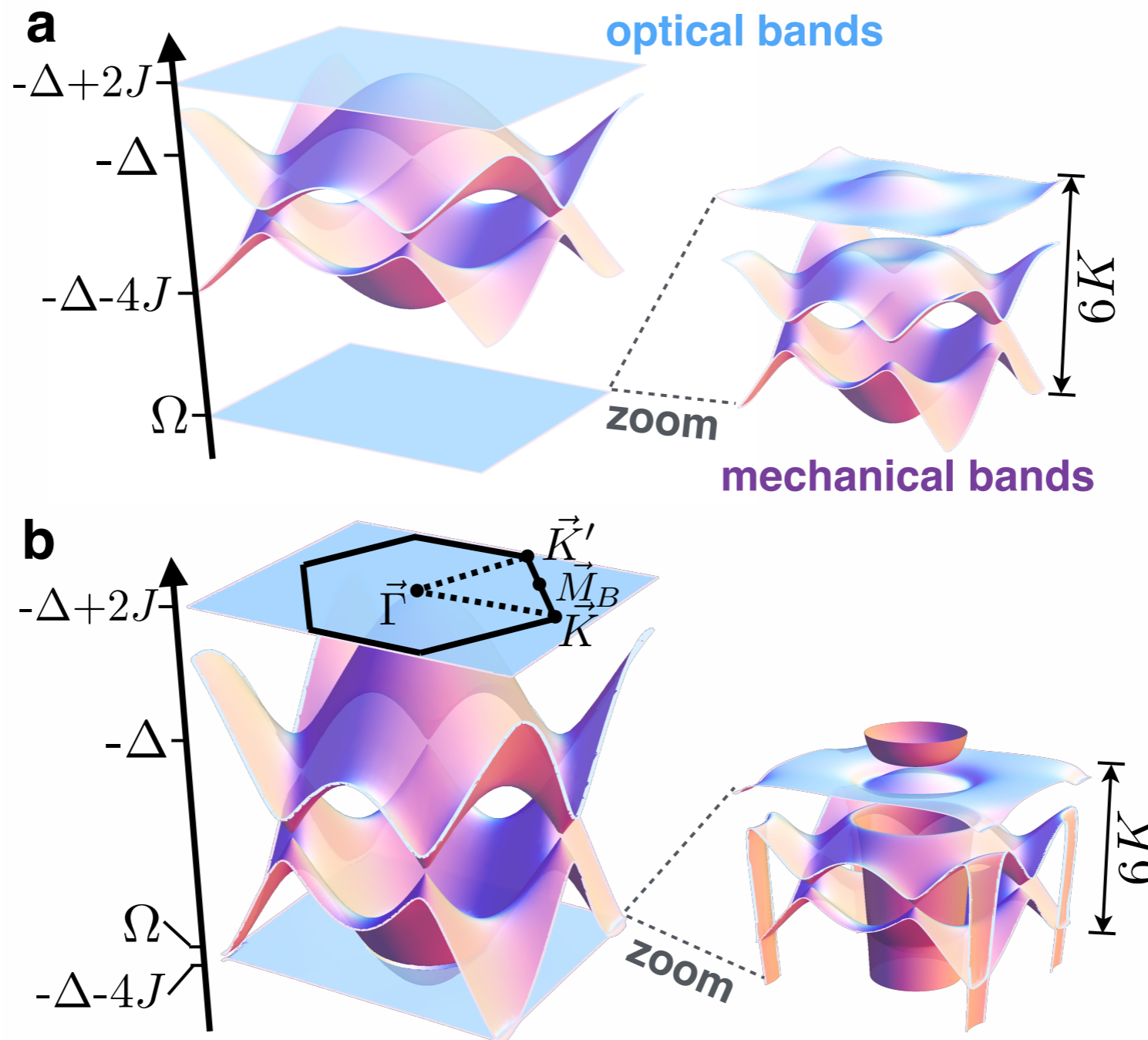


(optical phase)



# Topological Phases of Sound and Light in an Optomechanical Array

Vittorio Peano, Christian Brendel, Michael Schmidt, and Florian Marquardt, PRX 2015

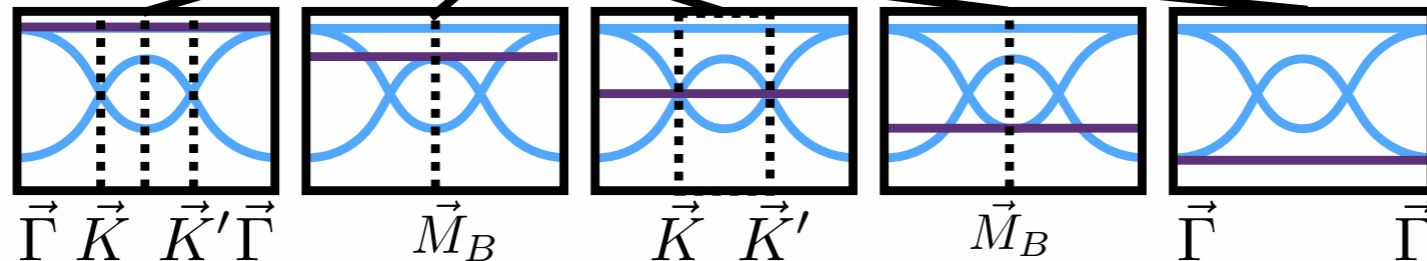
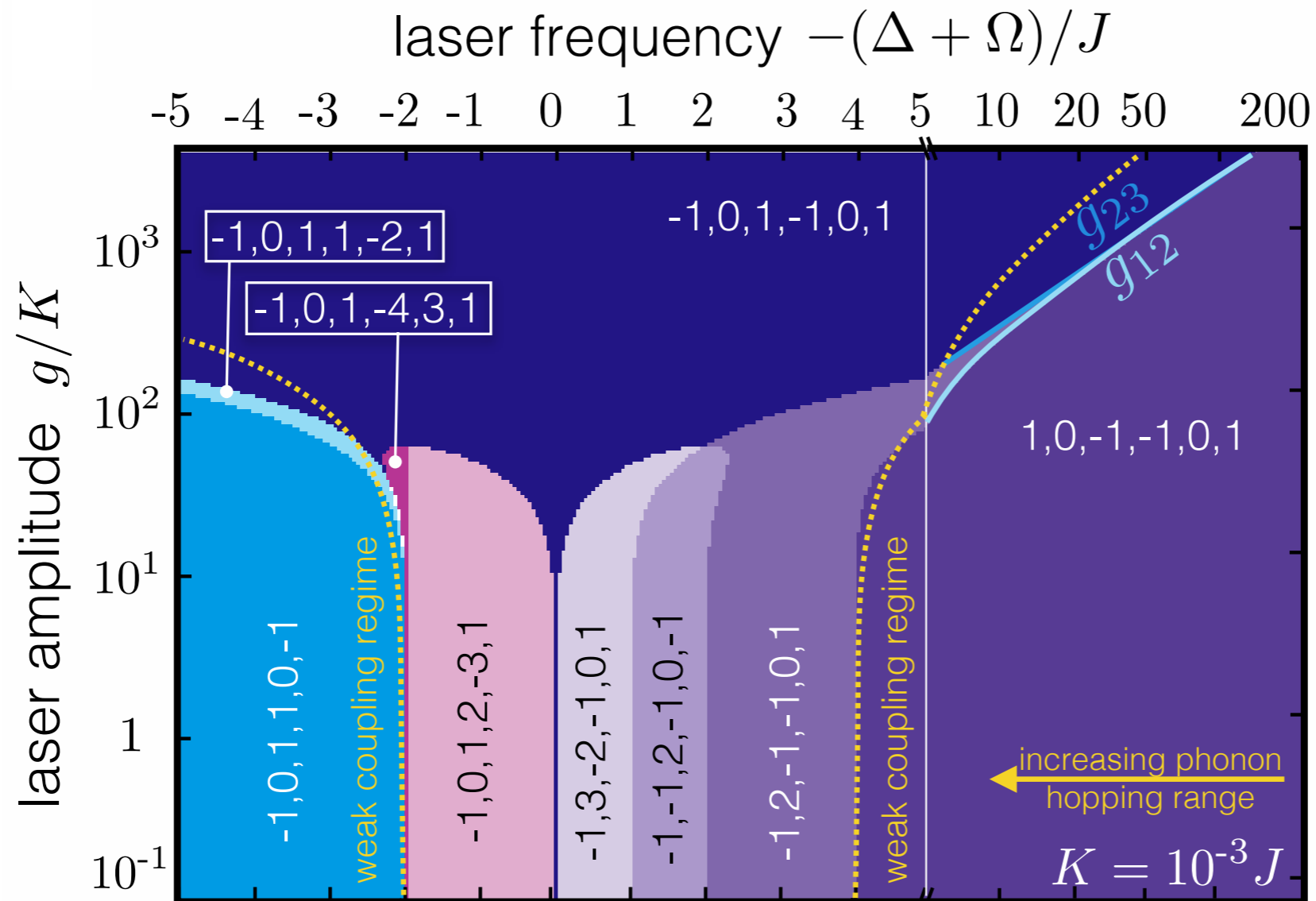


**a** “weak coupling”: light field modifies phonon hopping  
**b** “strong coupling”: photon and phonon bands mix



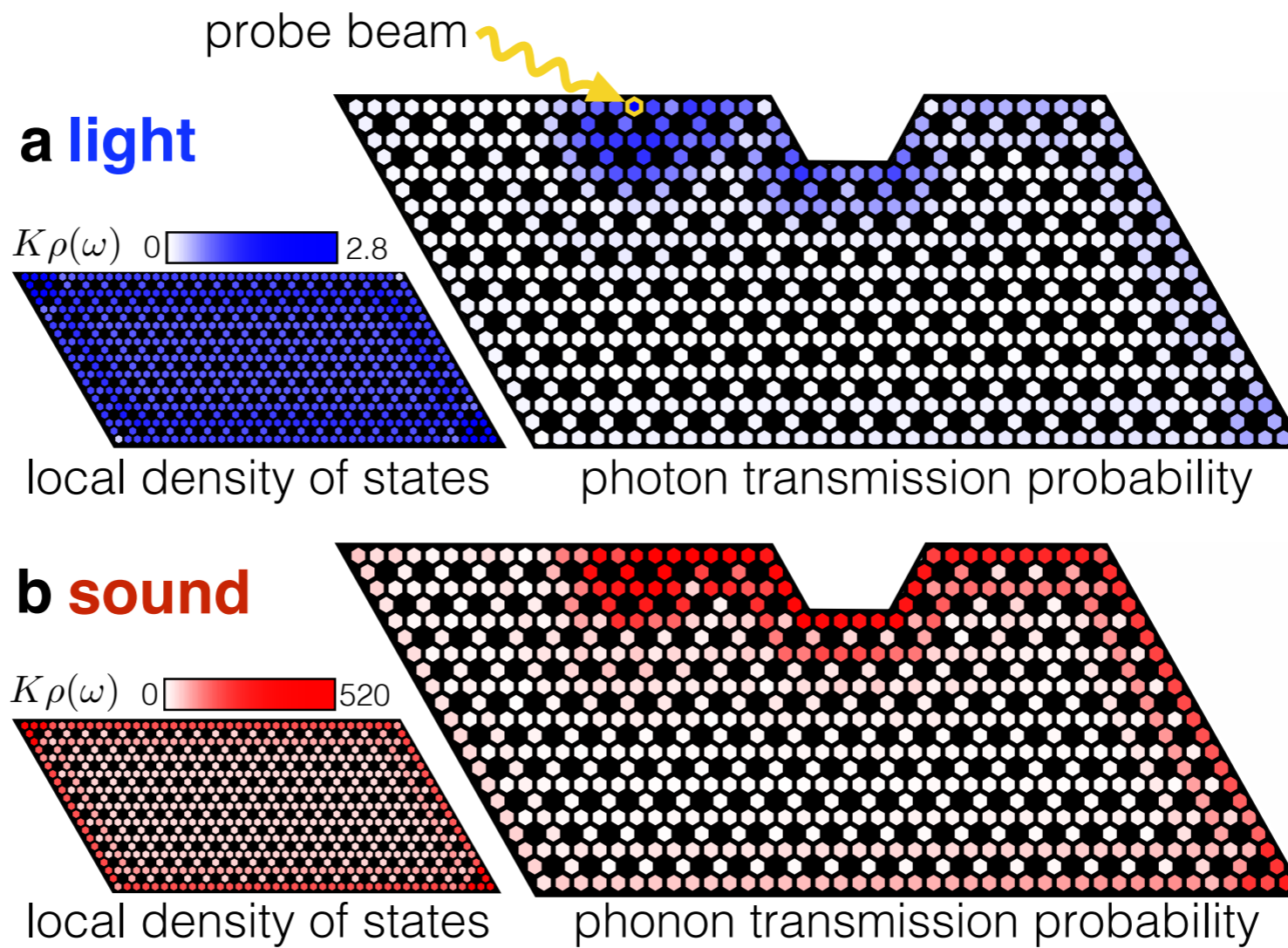
# Topological Phases of Sound and Light in an Optomechanical Array

## Topological Phase Diagram

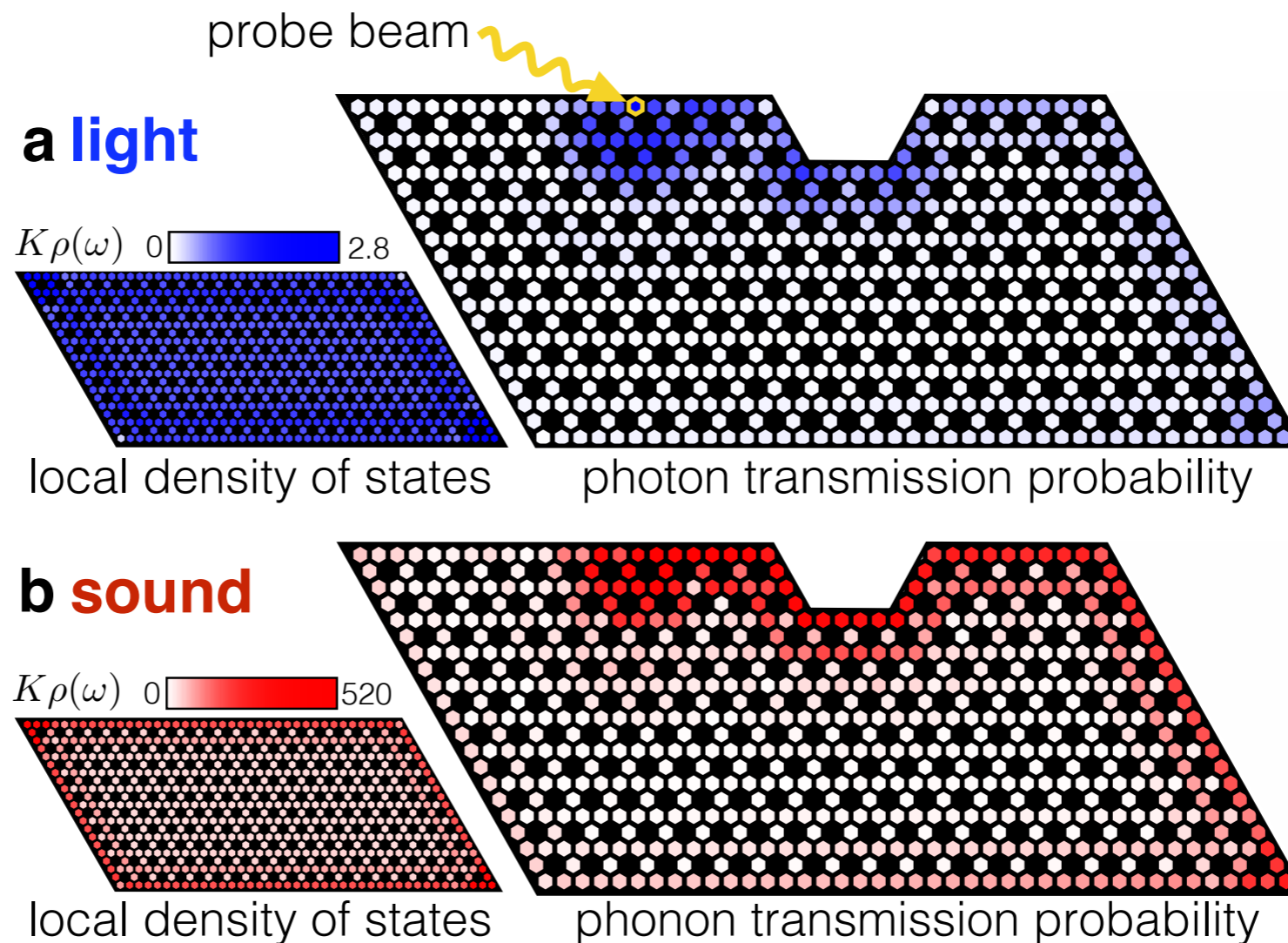


[Chern numbers of bands]

# Robust chiral transport of phonons



# Robust chiral transport of phonons



## Challenges (for optomechanical crystals)

fabrication disorder: current 1%

– need to reduce by factor 100 (postprocessing)

intensity requirement: ca.  $10^5$ - $10^6$  circulating photons

– OK, but large (optimize, improve coupling  $g_0$ )

# Features

- **Topologically protected transport of phonons in the solid state**

compare... coupled pendula

[Süsstrunk, Huber Science '15](#)

coupled gyroscopes

[Nash,...,Irvine, arXiv:1504.03362](#)

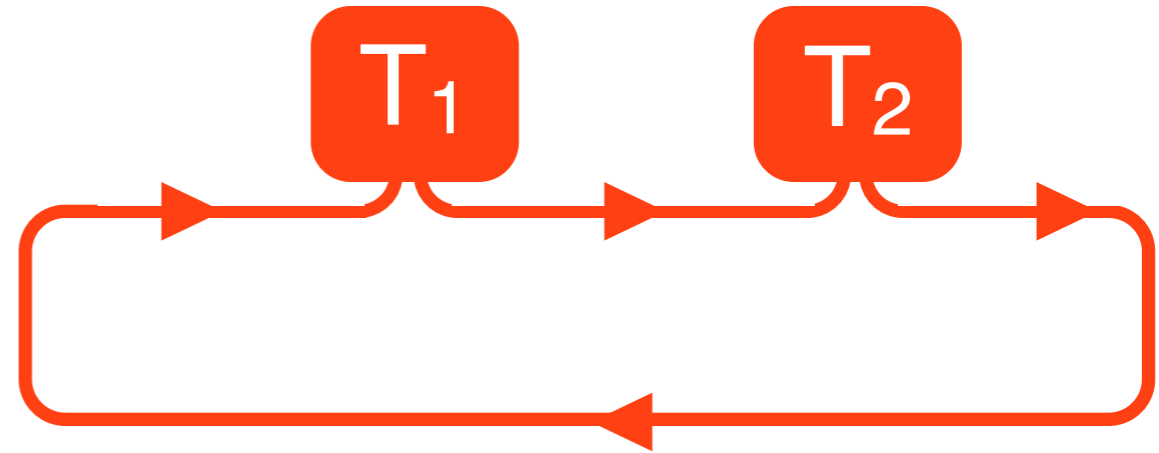


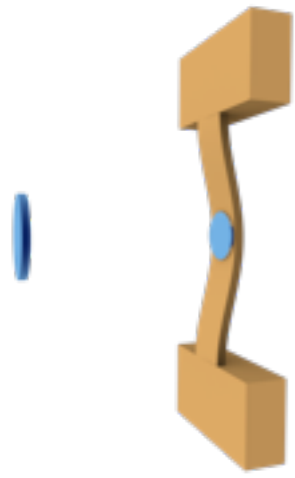
- **Here:** nanostructure, tuneable
- Full optical control and readout
- Arbitrary domains



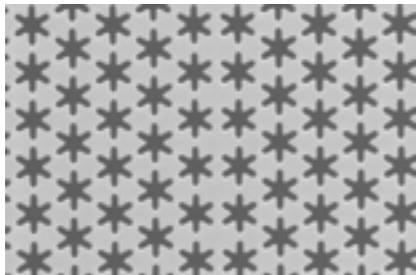
# Features

- study one-way phonon transport
- Time-dependent control: quenches, dynamical reconfiguration of edge states
- Photon/phonon polariton transport
- Classical nonlinear dynamics
- Thermalization in chiral edge states
- Quantum nonlinear dynamics: for larger  $g_0$

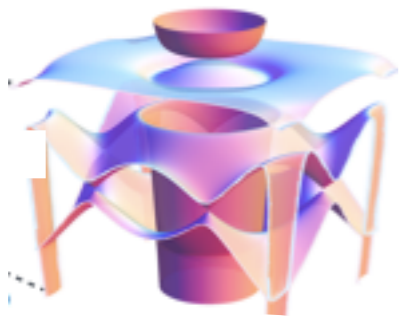




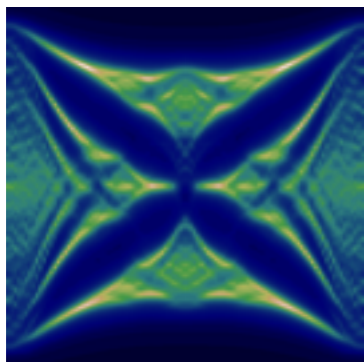
## Cavity Optomechanics



## Optomechanical Arrays



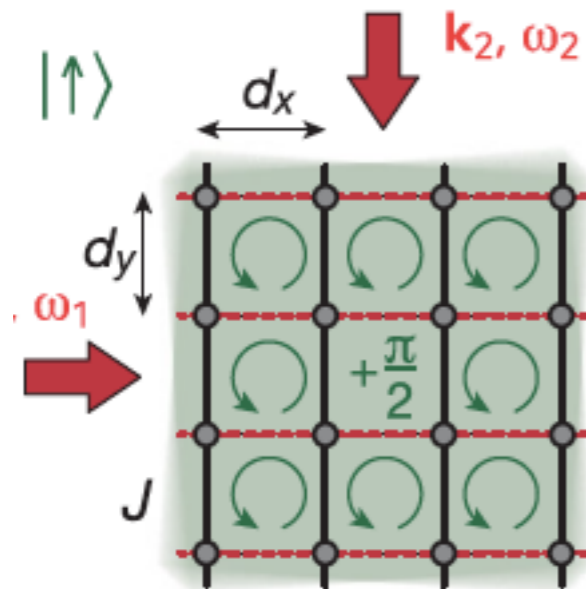
## Topological Phases of Sound (and Light)



## Dynamical Gauge Fields for the photons

# Synthetic magnetic fields

## neutral atoms



cold atom realizations:  
Aidelsburger et al (Bloch group) 2013,  
Miyake et al (Ketterle group) 2013

## photons

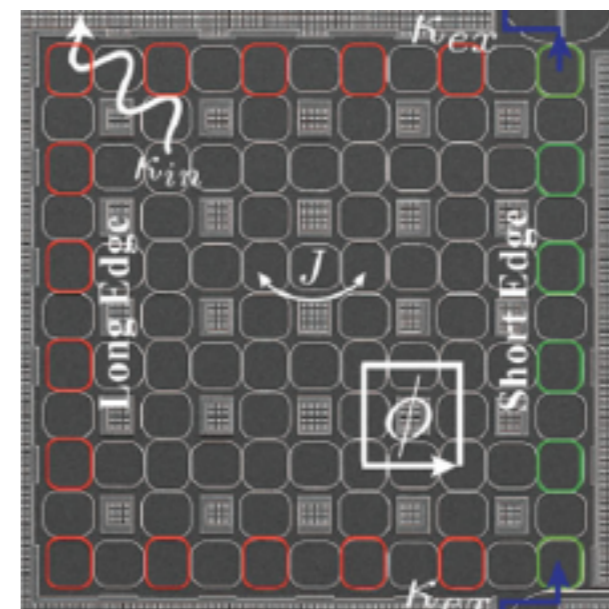
proposals:

Umucalilar and Carusotto, PRA 2011  
circularly refractive medium

Hafezi, Demler, Lukin, Taylor, Nature  
Physics 2011

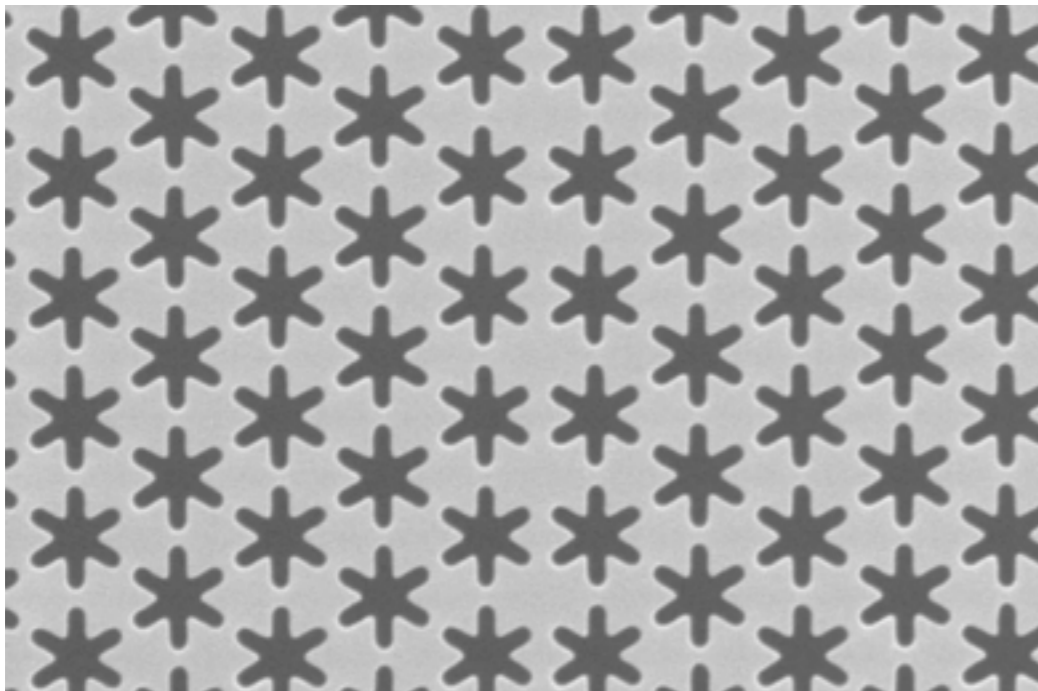
tuneable: Fang, Yu, Fan Nature  
Photonics 2012; proposed electrical  
modulation of refractive index

Mittal, ..., Hafezi PRL 2014  
coupled ring resonators



# Artificial magnetic fields for photons

Need:



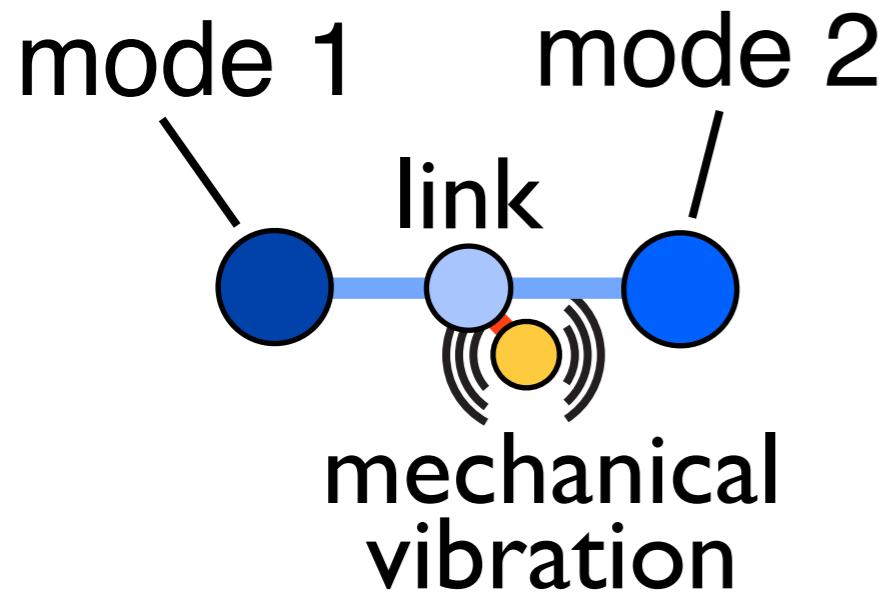
Dielectric  
(with the right  
pattern of holes)



Two Lasers  
(with the right  
pattern of phases)



# Phonon-assisted photon tunneling



vibration leads to modulation of effective photon tunnel coupling between mode 1 and 2

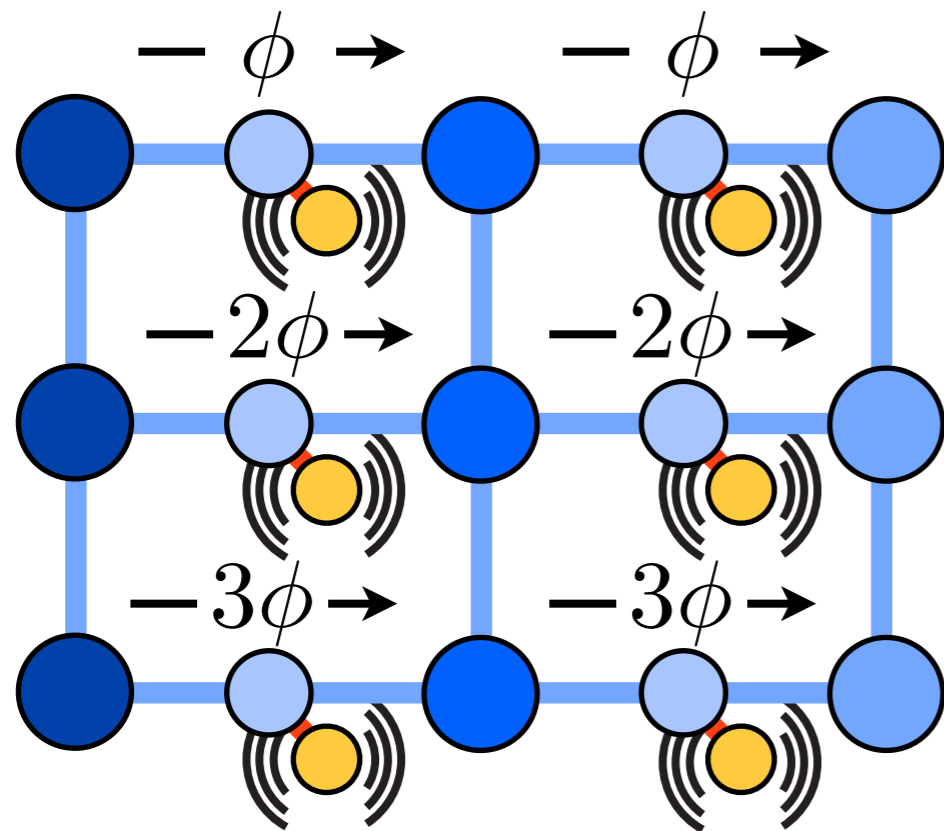
$$\omega = \omega_2 - \omega_1$$

$$2 \cos(\omega t + \phi) (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) \approx e^{i(\omega t + \phi)} \hat{a}_1^\dagger \hat{a}_2 + e^{-i(\omega t + \phi)} \hat{a}_2^\dagger \hat{a}_1$$

**non-reciprocal phase!**

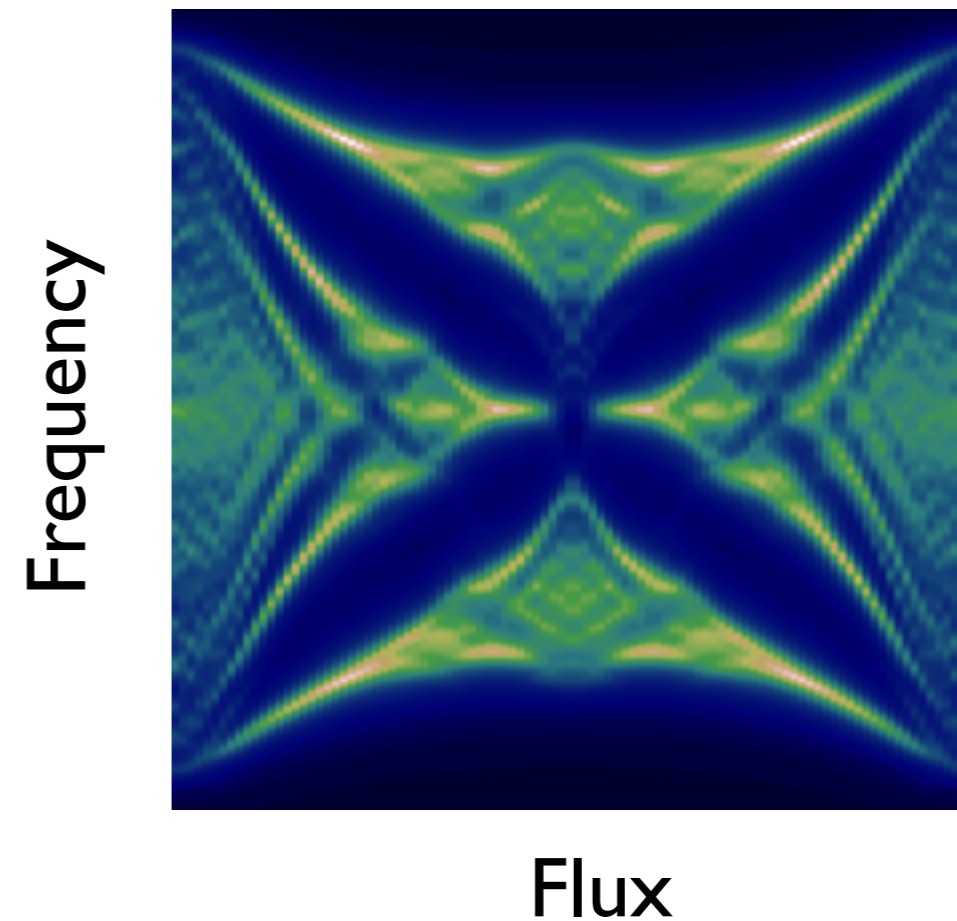
(similar to Fan group proposal, but mechanical vibration instead of electrical modulation)

# Artificial magnetic fields for photons



arbitrary optical re-  
configuration of magnetic  
field distribution

Hofstadter butterfly spectrum



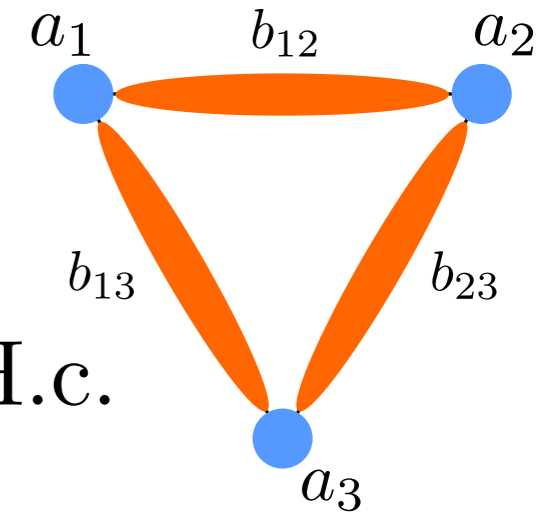
M. Schmidt, S. Keßler, V. Peano,  
O. Painter, F. Marquardt  
Optica 2015

# Dynamical Gauge Fields

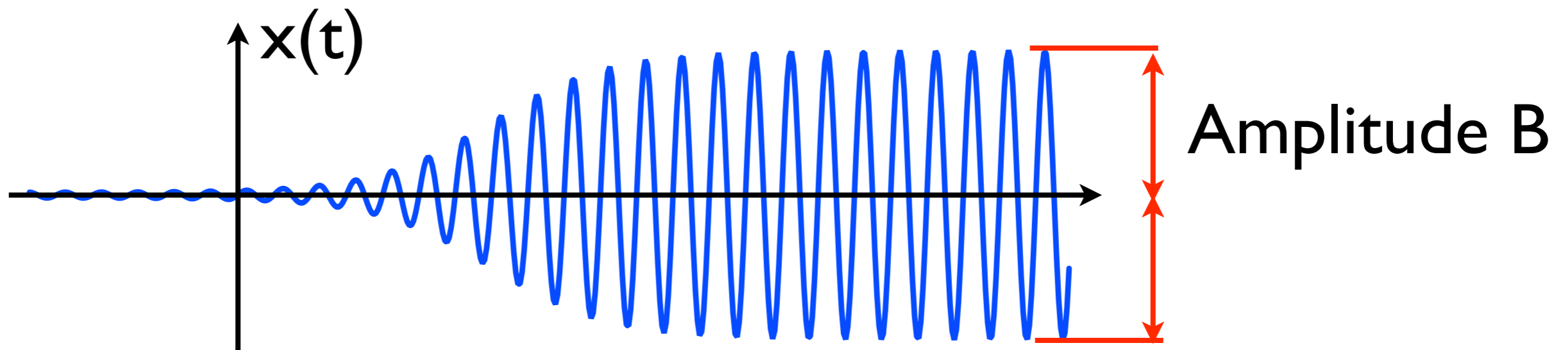
Optomechanical Hamiltonian:

phonon-assisted photon hopping

$$H = \sum_j \nu_j a_j^\dagger a_j + \sum_l \omega_l b_l^\dagger b_l + \sum_{\langle i,j \rangle} J_{i,j} b_{i,j} a_j^\dagger a_i + \text{H.c.}$$

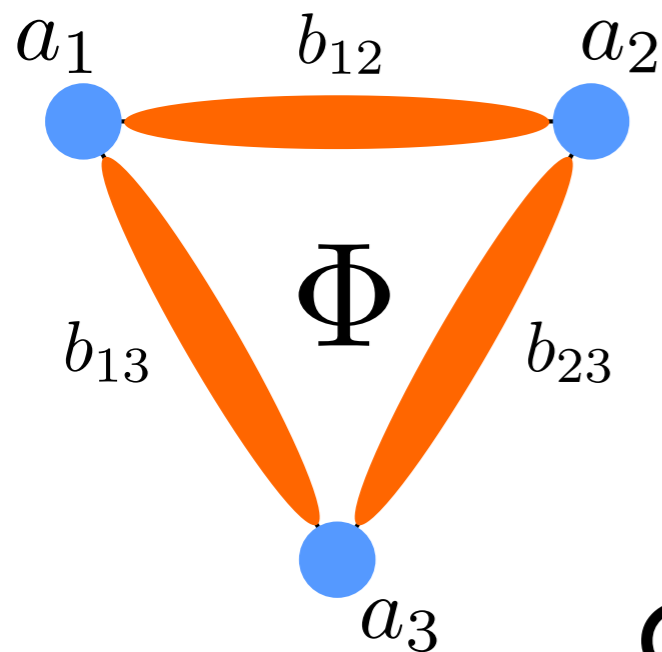


Mechanical oscillation: now self-oscillations (pumped by blue-detuned laser), instead of externally driven



S. Walter and FM, in preparation

# Gauge-invariant dynamics



amplitude and phase:

$$a_j = A_j e^{i\theta_j} \quad b_{ij} = B_{ij} e^{i\phi_{ij}}$$

gauge-invariant flux (seen by the photons)

$$\Phi = \phi_{13} + \phi_{21} + \phi_{32}$$

Classical nonlinear dynamics:

mechanical phase dynamics

$$\dot{\phi}_{ij} = -\omega_{ij} - \frac{J_{ij}}{B_{ij}} A_i A_j \cos(\phi_{ij} + \theta_{ij})$$

optical phase dynamics

$$\dot{\theta}_i = -\nu_i - \sum_{j \neq i} J_{ij} B_{ij} \frac{A_j}{A_i} \cos(\phi_{ij} + \theta_{ij})$$

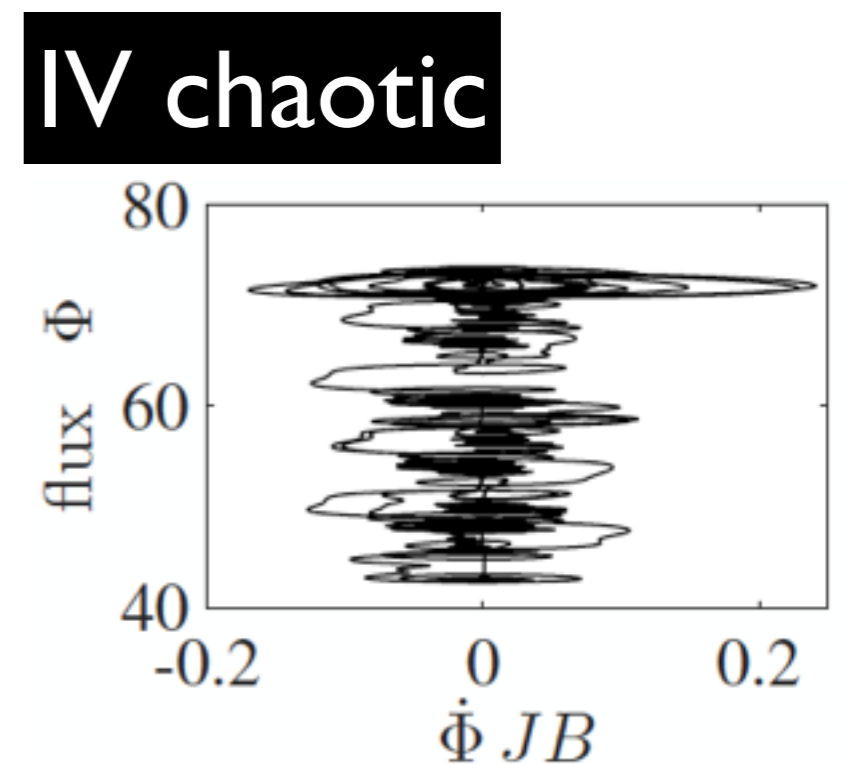
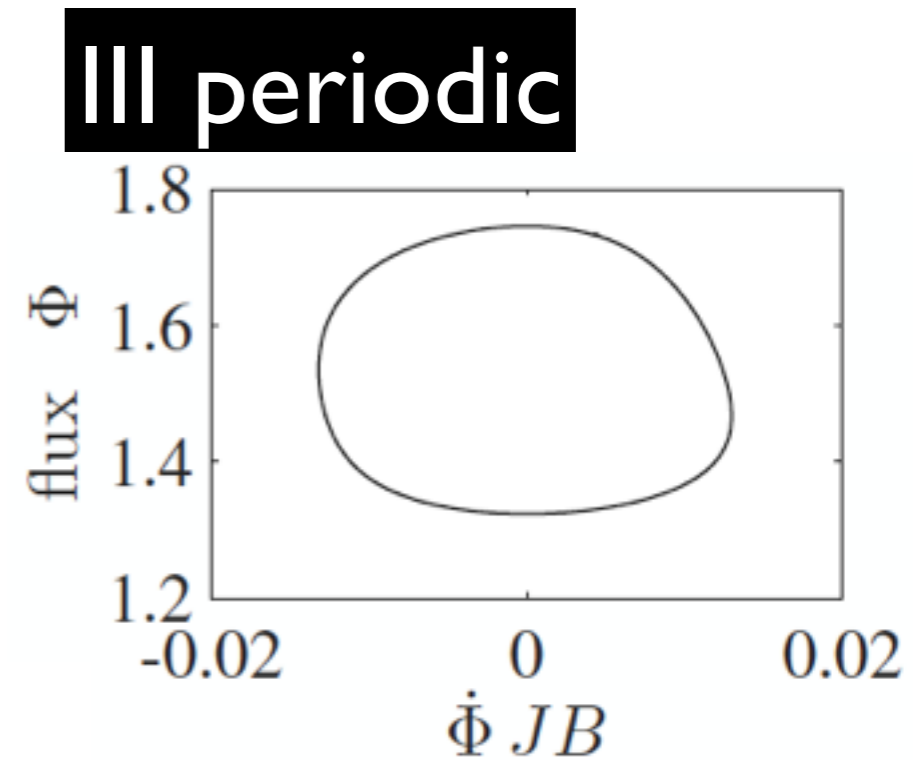
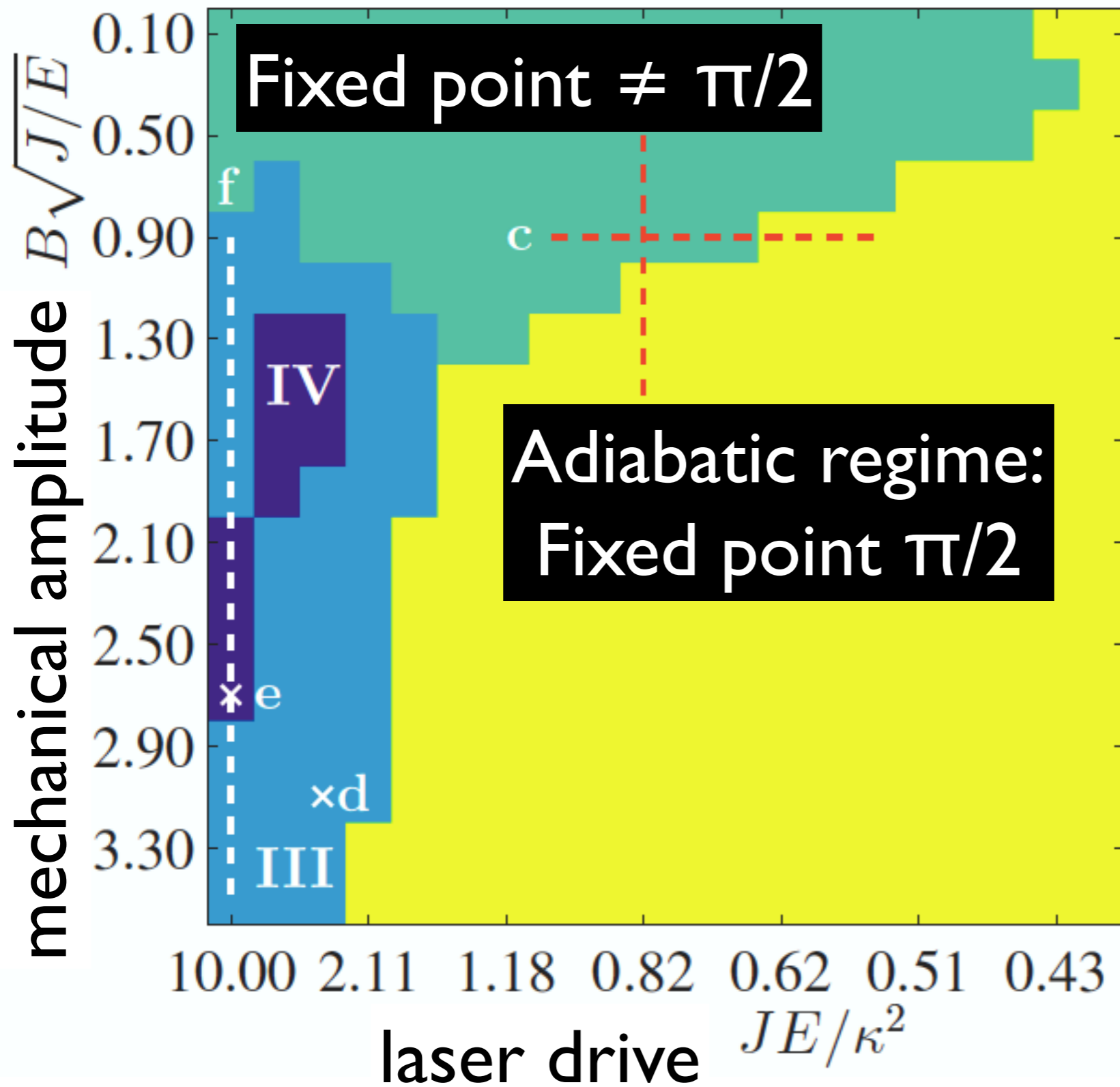
gauge transformation

$$\phi'_{ij} = \phi_{ij} + (\chi_j - \chi_i)$$

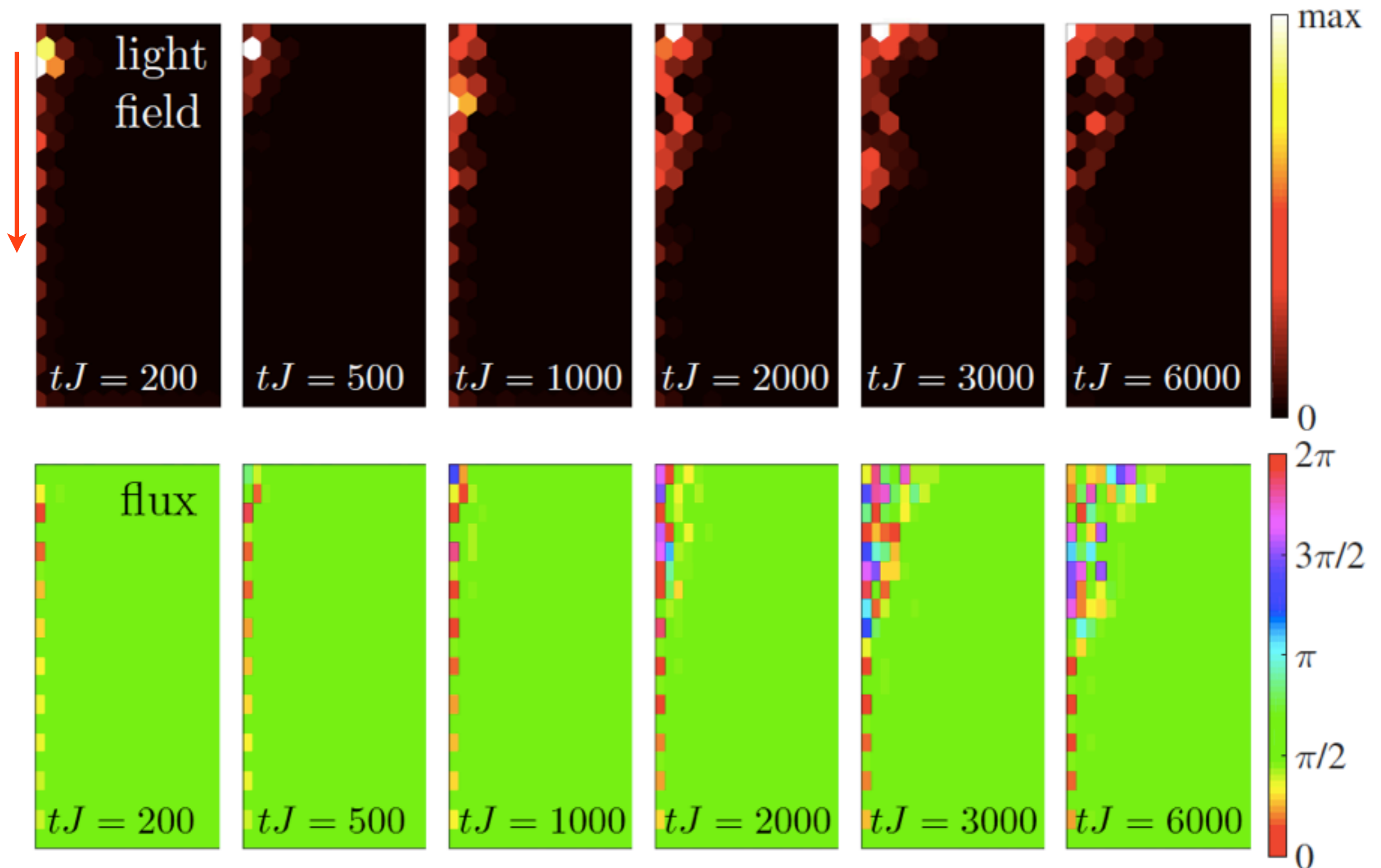
$$\theta'_i = \theta_i + \chi_i$$



# Flux dynamics (3-site model)



# Lattice: Photon flow reshapes flux



S. Walter and FM, in preparation

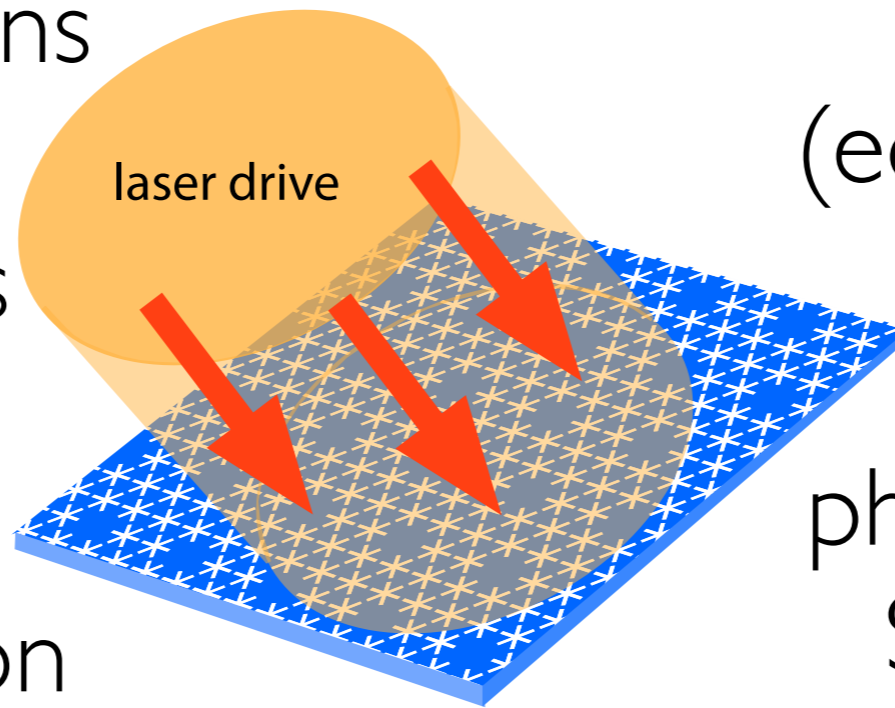
# Optomechanical Arrays: Future possibilities

Synthetic magnetic fields  
for photons/phonons

Dirac Physics and  
other band structures

Synchronization and  
Pattern Formation

Quantum Information  
Processing



Topological Phases

Transport  
(edge states/wires)

Nonequilibrium  
dynamics/Quench  
physics/Thermalization

Strongly Correlated  
Quantum Physics?

**All-optical control/  
readout**

“Topological Phases of Sound and Light”,  
Vittorio Peano, Christian Brendel, Michael Schmidt, and FM,  
Phys. Rev. X **5**, 031011 (2015)

“Optomechanical creation of magnetic fields for photons on a lattice”,  
M. Schmidt, S. Keßler, V. Peano, O. Painter, FM  
Optica **2**, 635 (2015)

**more: see Oskar Painter’s talk this afternoon!**