Beyond Collisionless Dark Matter:
From a Particle Physics Perspective

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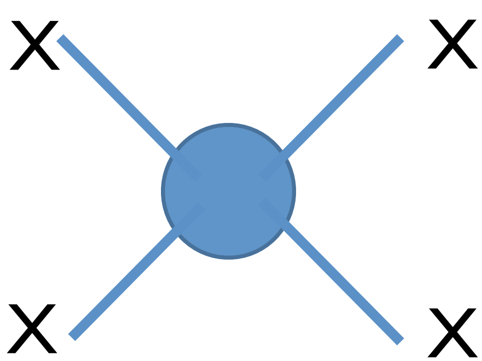
KITP Dark Matter Conference 2013



Feng, Kaplinghat, Tu, HBY (2009) JCAP
Feng, Kaplinghat, HBY (2009) PRL
Tulin, HBY, Zurek (2012) PRL
Tulin, HBY, Zurek (2013) PRD

Collisionless VS. Collisional

- Large scales: Great!
- Small scales (dwarf galaxies, subhalos)?

  cusp vs. core problem

  "too big to fail?" problem (Strigari, Peter, Dawson)

- These anomalies can be solved if DM is sufficiently self-interacting

  Recent simulations

  Harvard group: Vogelsberger, Zavala, Loeb (2012); Zavala, Vogelsberger, Walker (2012)
  UCI group: Rocha, Peter, Bullock, Kaplinghat, Garrison-Kimmel, Onorbe, Moustakas (2012); Peter, Rocha, Bullock, Kaplinghat (2012)

Astrophysics Summary

- Evidence for DM self-interactions on dwarf galaxy scales

$\sigma/m_X \sim 0.1 - 10\ cm^2/g$ for $v \sim 10$ km/s

- Constraints: elliptical halo shapes; evaporation of subhalos; core collapse; Bullet Cluster

$\sigma/m_X < 0.1 - 1\ cm^2/g$ for $v \sim 100$ km/s (MW)
and $v \sim 1000$ km/s (cluster) Peter, Rocha, Bullock, Kaplinghat (2012)

Challenges

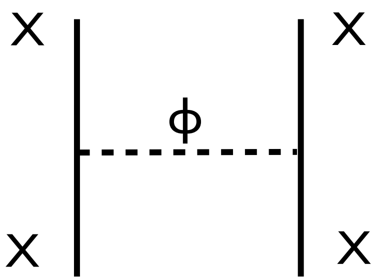
- A really large scattering cross section!

$\sigma \sim 1 cm^2\ (m_X/g) \sim 2\times10^{-24}\ cm^2\ (m_X/GeV)$ $\sigma_{EW} \sim 10^{-36}\ cm^2$

- How to avoid the constraints?

In particular, if $\sigma \sim$constant Spergel, Steinhardt (1999)

Particle Physics of Dark Forces



- A light force mediator is necessary

$$\sigma \approx 5 \times 10^{-23}\,\mathrm{cm}^2 \left(\frac{\alpha_X}{0.01}\right)^2 \left(\frac{m_X}{10\,\mathrm{GeV}}\right)^2 \left(\frac{10\,\mathrm{MeV}}{m_\phi}\right)^4$$

in the perturbative and small velocity limit

- With a light mediator, σ can depend on DM velocities
  - $m_\chi v \ll m_\phi$, σ~constant Spergel, Steinhardt(1999)
  - $m_\chi v \gg m_\phi$, $\sigma \sim v^{-4}$ Coulomb scattering our focus
  - $m_\chi v \sim m_\phi$, σ~constant-v^{-4} our focus
- σ can be enhanced on small scales and suppressed on large scales

Go beyond usual WIMPs

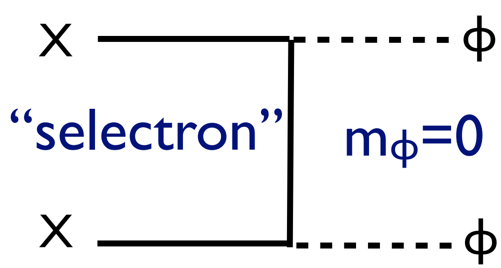
Models With Light Mediators

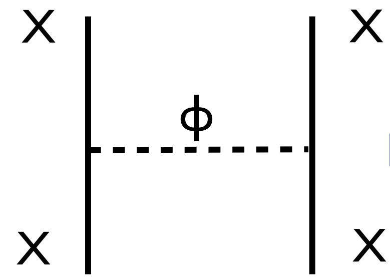
- Examples of models with light mediators

WIMPless miracle

$$\Omega_X \sim \frac{1}{\langle \sigma v \rangle} \sim \frac{m_X^2}{\alpha_X^2} \sim \frac{m_W^2}{\alpha_W^2}$$

Kumar, Feng (2008)

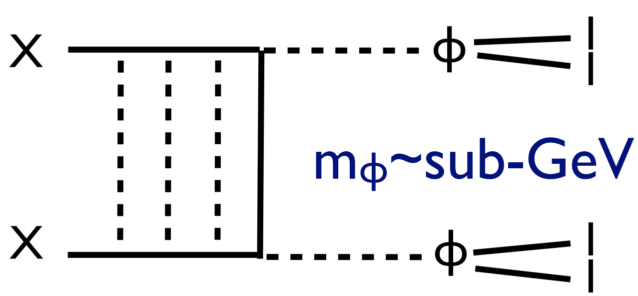


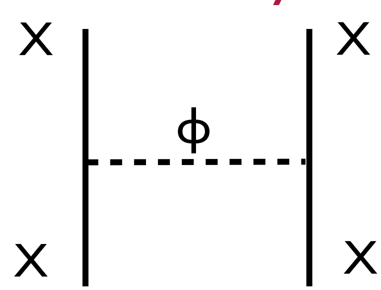


Feng, Tu, HBY (2008)

Ackerman, Buckley, Carroll, Kamionkowski (2008); Feng, Kaplinghat, Tu, HBY (2009)

The model motivated by the PAMELA anomaly





Feng, Kaplinghat, HBY (2009)
Fox, Buckley (2009)

Arkani-Hamed, Finkbeiner, Slatyer, Weiner (2008); Pospelov, Ritz (2008)

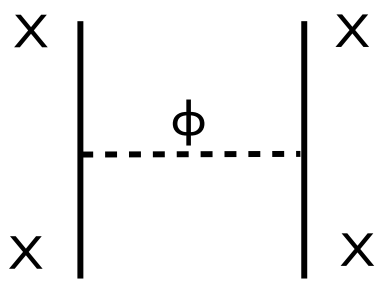
Asymmetric dark matter

$$\Omega_X / \Omega_B \sim 5$$

Nussinov (1985); Kaplan (1992)...

Kaplan, Luty, Zurek (2009)...

A General Study



$$\mathscr{L}_{\text{int}} = \begin{cases} g_X \bar{X} \gamma^\mu X \phi_\mu & \text{vector mediator} \\ g_X \bar{X} X \phi & \text{scalar mediator} \end{cases}$$

A Yukawa potential

$$V(r) = \pm \frac{\alpha_X}{r} e^{-m_\phi r}$$

$$\alpha_X = g_X^2/(4\pi)$$

$$\sigma_T = \int d\Omega \,(1 - \cos\theta)\, \frac{d\sigma}{d\Omega}$$

Map out the parameter space (m_X, m_ϕ, α_X)

- Solve small scale anomalies
- Avoid constraints on large scales
- Get the relic density right

Scattering with a Yukawa Potential

$$V(r) = \pm \frac{\alpha_X}{r} e^{-m_\phi r}$$

DM self-scattering

Perturbative (Born) regime ✓
$\alpha_X m_X / m_\phi \ll 1$

Feng, Kaplinghat, HBY (2009)

Nonperturbative regime
$\alpha_X m_X / m_\phi \gtrsim 1$

Classical regime
$m_X v / m_\phi \gg 1$

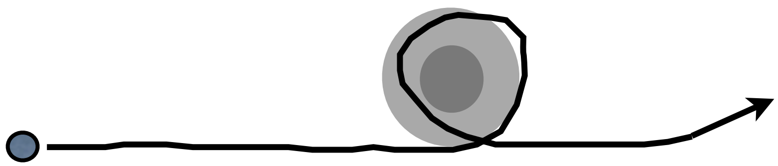
Resonant regime
$m_X v / m_\phi \lesssim 1$

Exception: m_ϕ=0

Feng, Kaplinghat, Tu, HBY (2009)

Classical Regime

- Classical approximation from plasma physics



Charged-particle scattering in plasma

$$\pm \frac{\alpha_X}{r} e^{-m_\phi r} \qquad \alpha_X = \alpha_{\rm EM}$$

$$m_\phi = \text{Debye photon mass}$$

Attractive Khrapak et al. (2003) (2004)

$$\sigma_T^{\rm clas} \approx \begin{cases} \frac{4\pi}{m_\phi^2} \beta^2 \ln\left(1+\beta^{-1}\right) & \beta \lesssim 10^{-1} \\ \frac{8\pi}{m_\phi^2} \beta^2 / \left(1+1.5\beta^{1.65}\right) & 10^{-1} \lesssim \beta \lesssim 10^3 \\ \frac{\pi}{m_\phi^2} \left(\ln\beta + 1 - \frac{1}{2}\ln^{-1}\beta\right)^2 & \beta \gtrsim 10^3 \end{cases}$$

Repulsive

$$\sigma_T^{\rm clas} \approx \begin{cases} \frac{2\pi}{m_\phi^2} \beta^2 \ln\left(1+\beta^{-2}\right) & \beta \lesssim 1 \\ \frac{\pi}{m_\phi^2} \left(\ln 2\beta - \ln\ln 2\beta\right)^2 & \beta \gtrsim 1 \end{cases}$$

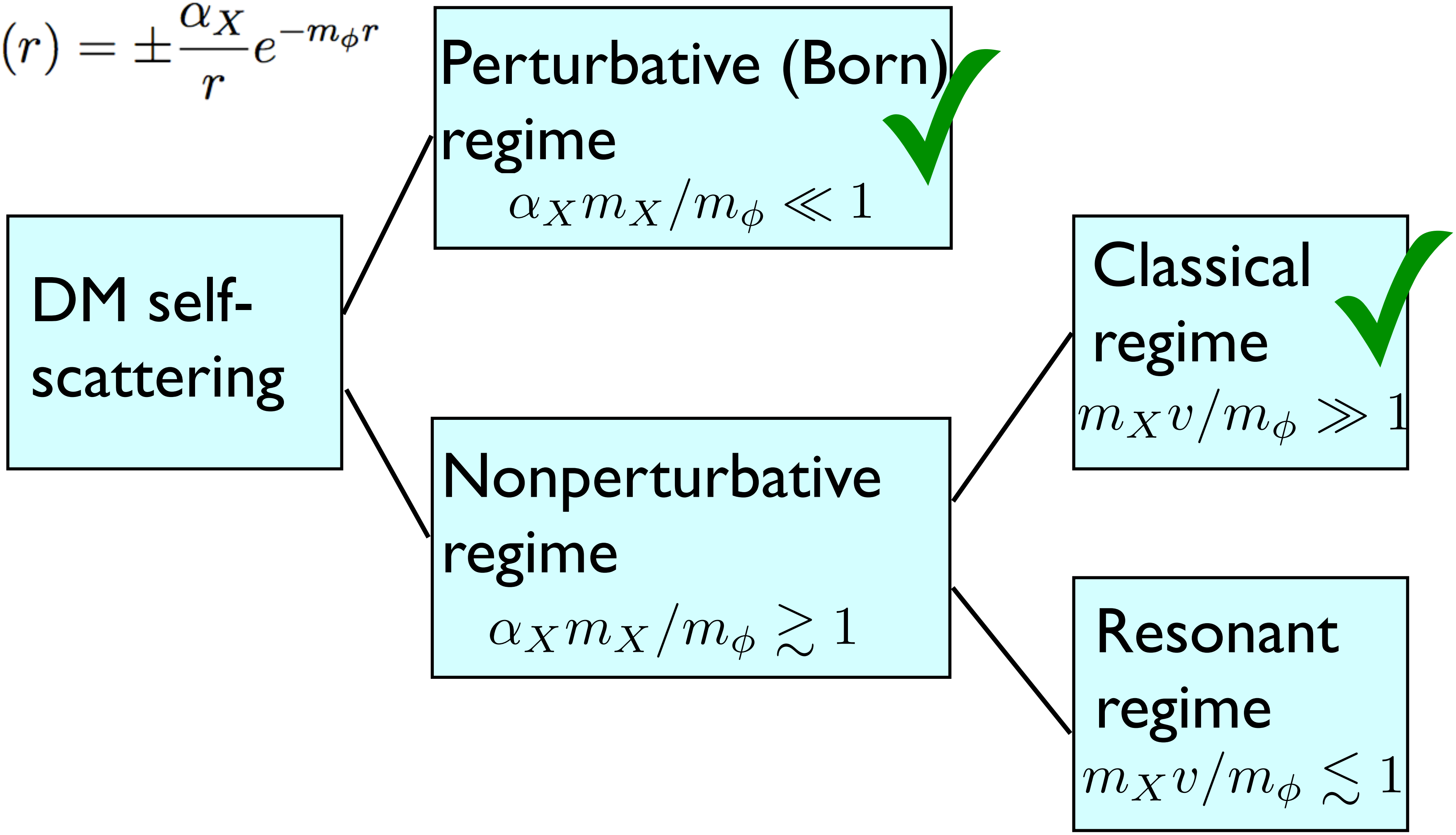
$$\beta \equiv 2\alpha_X m_\phi / (m_X v^2)$$

$\sigma_T \sim v^{-4}$ at large v
$\sigma_T \sim$const at small v (saturated)

Apply to DM: σ_T is enhanced on dwarf scales compared to larger scales
Feng, Kaplinghat, HBY (2009); Loeb, Weiner (2010); Vogelsberger, Loeb, Zavala (2012)...

Beyond Perturbation

$$V(r) = \pm \frac{\alpha_X}{r} e^{-m_\phi r}$$



Numerical Approach

- Quantum mechanics 101-partial wave analysis

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_\ell}{dr}\right)+\left(k^2-\frac{\ell(\ell+1)}{r^2}-2\mu V(r)\right)R_\ell=0$$

- Transfer cross section

$$\frac{d\sigma}{d\Omega}=\frac{1}{k^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}P_\ell(\cos\theta)\sin\delta_\ell\right|^2 \qquad \sigma_T = \int d\Omega\,(1-\cos\theta)\,\frac{d\sigma}{d\Omega}$$

$$\frac{\sigma_T k^2}{4\pi}=\sum_{\ell=0}^{\infty}[(2\ell+1)\sin^2\delta_\ell-2(\ell+1)\sin\delta_\ell\sin\delta_{\ell+1}\cos(\delta_{\ell+1}-\delta_\ell)]$$

Rearrange ell→ell+1

$$\frac{\sigma_T k^2}{4\pi}=\sum_{\ell=0}^{\infty}(\ell+1)\sin^2(\delta_{\ell+1}-\delta_\ell)$$

Both formulas are identical in the limit of ell→∞

But the second one converges much faster

Numerical Approach

- Partial wave analysis

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_\ell}{dr}\right)+\left(k^2-\frac{\ell(\ell+1)}{r^2}-2\mu V(r)\right)R_\ell=0$$

- Boundary conditions $r \to \infty$

$$R_\ell(r) \to \sin(kr - \pi\ell/2 + \delta_\ell)/r$$

$$R_\ell(r) \to \cos\delta_\ell j_\ell(kr) - \sin\delta_\ell n_\ell(kr)$$ ✓

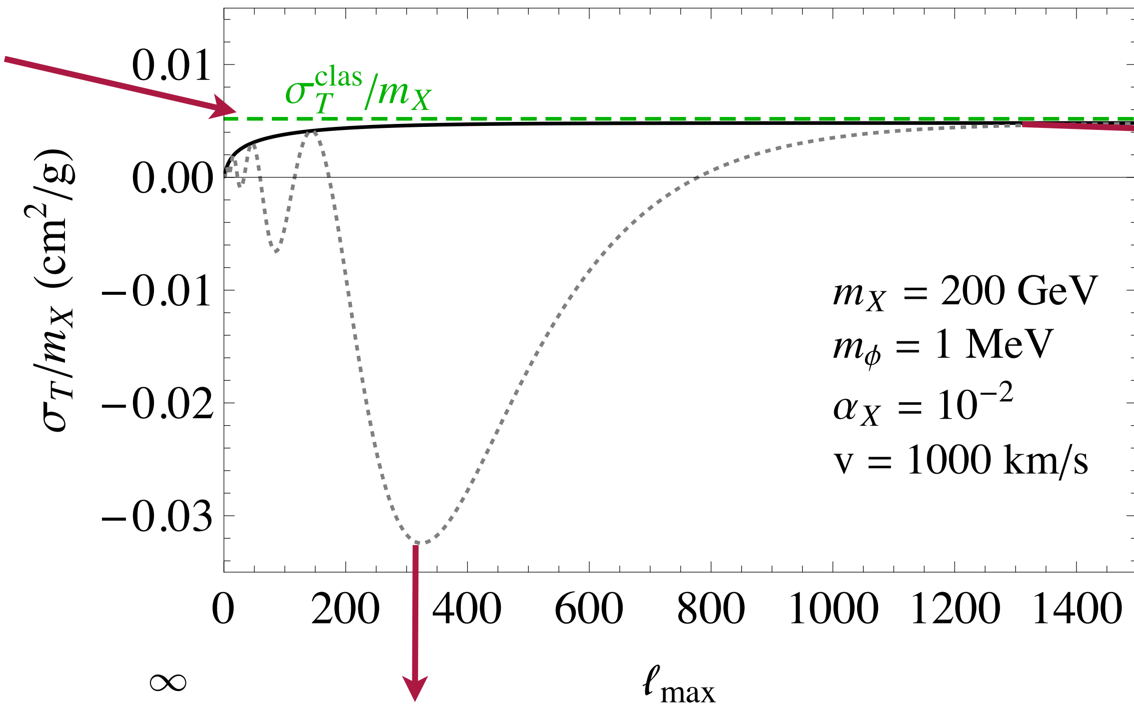
The second one is much more efficient

Numerical Approach

- Classical regime

Tulin, HBY, Zurek (2013)

Plasma



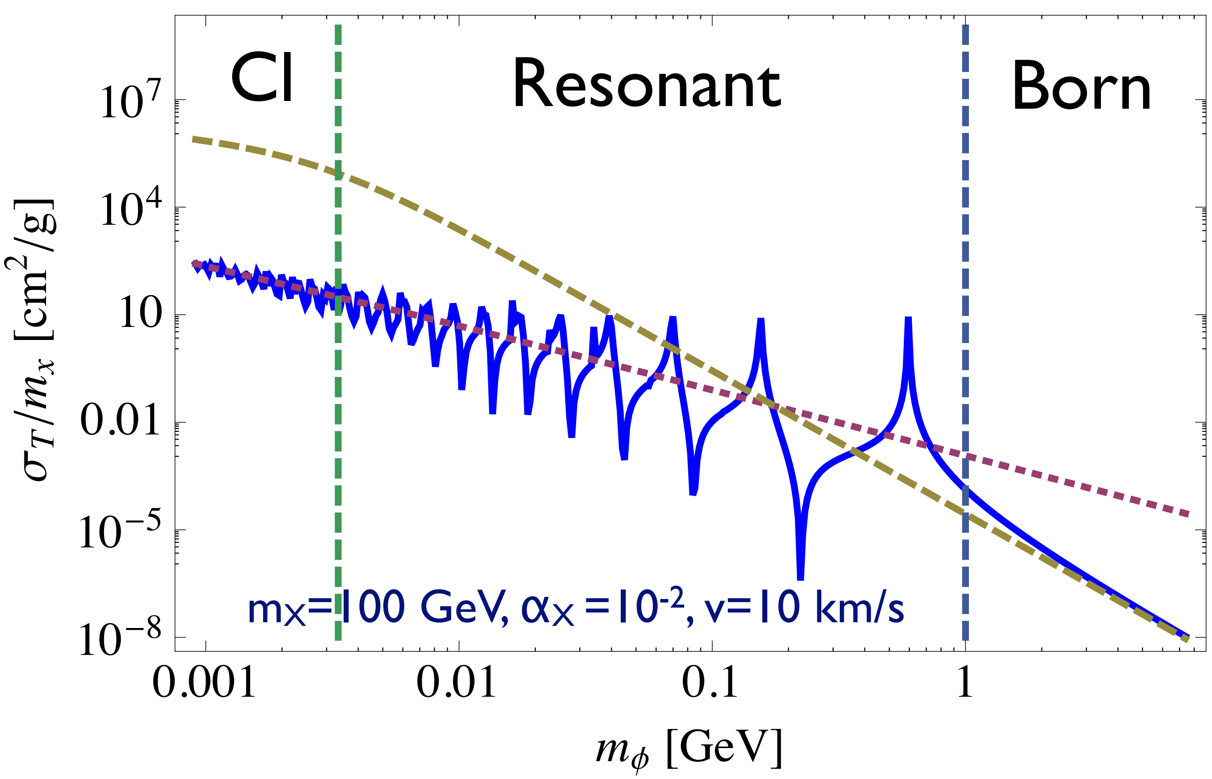
$$\sum_{\ell=0}^{\infty}(\ell+1)\sin^2(\delta_{\ell+1}-\delta_\ell)$$

$$\sum_{\ell=0}^{\infty}[(2\ell+1)\sin^2\delta_\ell - 2(\ell+1)\sin\delta_\ell\sin\delta_{\ell+1}\cos(\delta_{\ell+1}-\delta_\ell)]$$

We have confirmed the analytical formula from plasma physics

Numerical Approach

- All regimes



Solid: numerical; Dashed: Born; Dotted: plasma

In the resonant regime, the cross section can be enhanced or suppressed

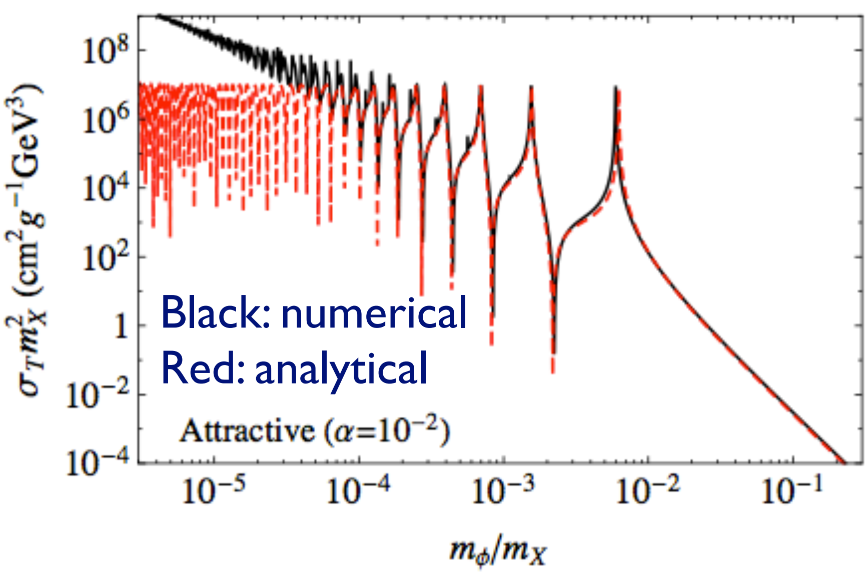
Analytical Approach

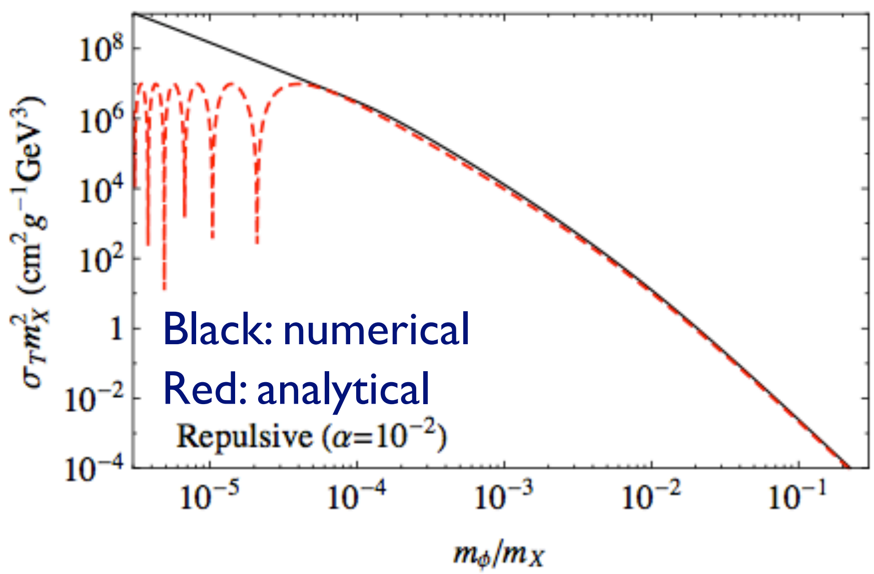
$$V(r) = \pm\frac{\alpha_X}{r}e^{-m_\phi r} \longrightarrow V(r) = \pm\frac{\alpha_X \delta\, e^{-\delta r}}{1-e^{-\delta r}} \qquad \delta = \kappa m_\phi \qquad \kappa \simeq 1.6$$

Hulthén potential

The Schrödinger equation is solvable analytically for ell=0

$$\sigma_T^{\text{Hulthén}} = \frac{16\pi}{m_X^2 v^2}\sin^2\delta_0 \qquad \delta_0 = \arg\left(\frac{i\,\Gamma\left(\frac{i m_X v}{\kappa m_\phi}\right)}{\Gamma(\lambda_+)\Gamma(\lambda_-)}\right), \quad \lambda_\pm \equiv \begin{cases} 1+\frac{i m_X v}{2\kappa m_\phi} \pm \sqrt{\frac{\alpha_X m_X}{\kappa m_\phi} - \frac{m_X^2 v^2}{4\kappa^2 m_\phi^2}} & \text{attractive} \\ 1+\frac{i m_X v}{2\kappa m_\phi} \pm i\sqrt{\frac{\alpha_X m_X}{\kappa m_\phi} + \frac{m_X^2 v^2}{4\kappa^2 m_\phi^2}} & \text{repulsive} \end{cases}$$

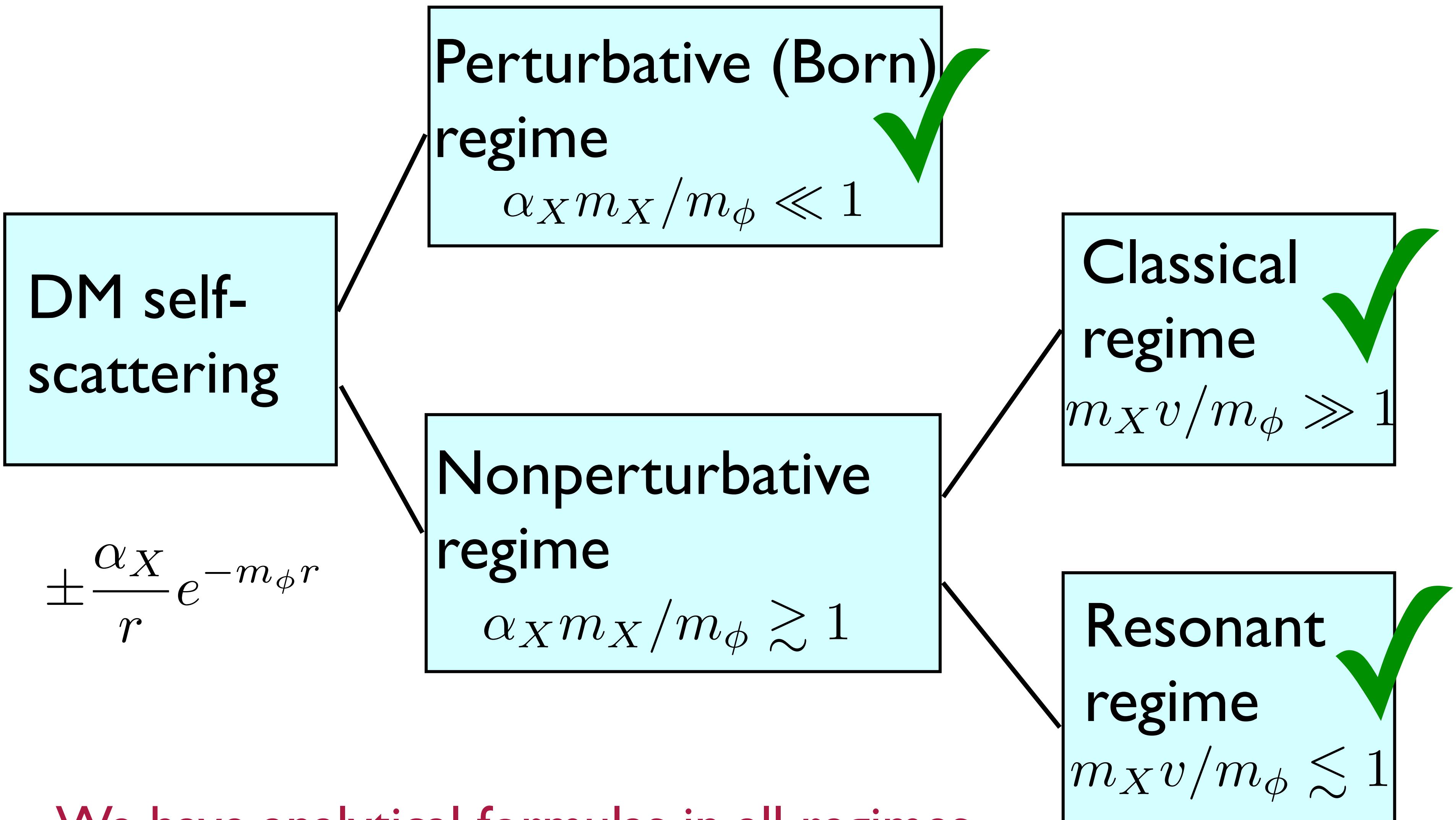




It is useful for simulations

Tulin, HBY, Zurek (2013)

Beyond Perturbation



We have analytical formulas in all regimes

Velocity Dependence

- σ_T has a rich structure

Tulin, HBY, Zurek (2012)

Born regime: σ_T~const below MW scales

Classical regime: σ_T increases on small scales

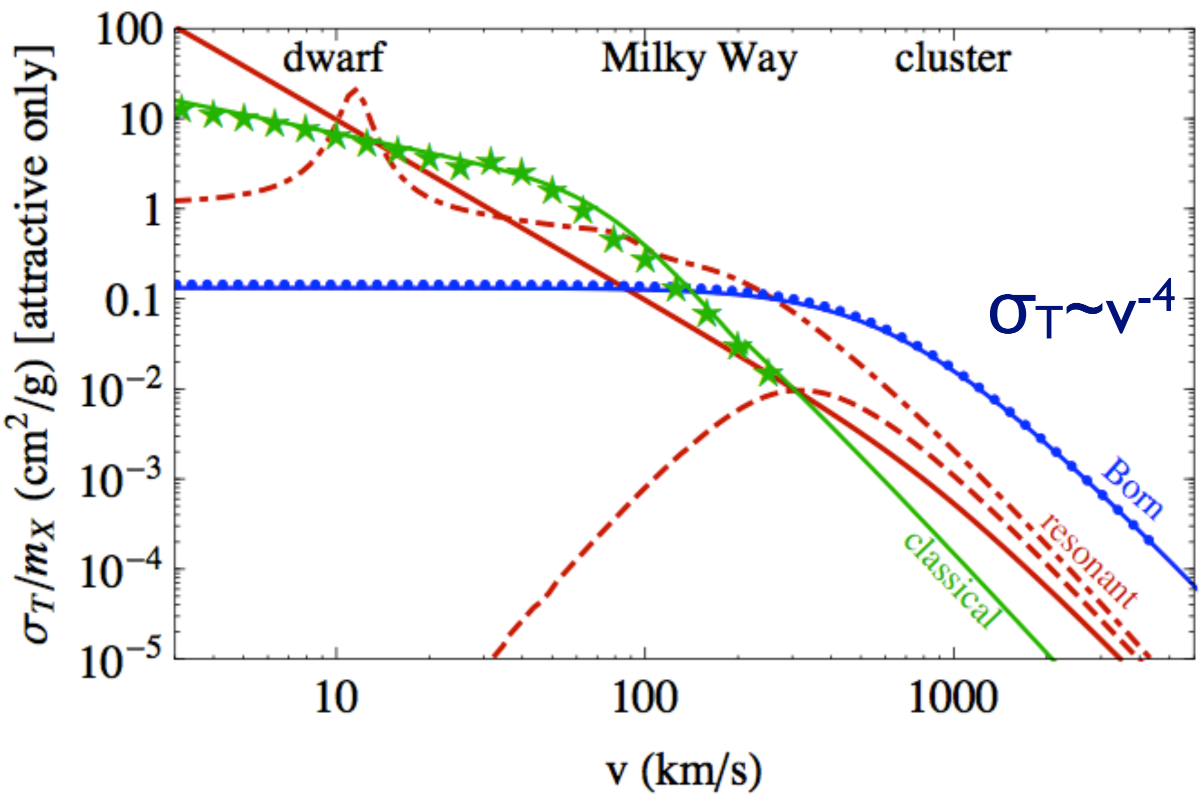
★: numerical

Resonant regime:

s-wave: $\sigma_T \sim v^{-2}$

p-wave

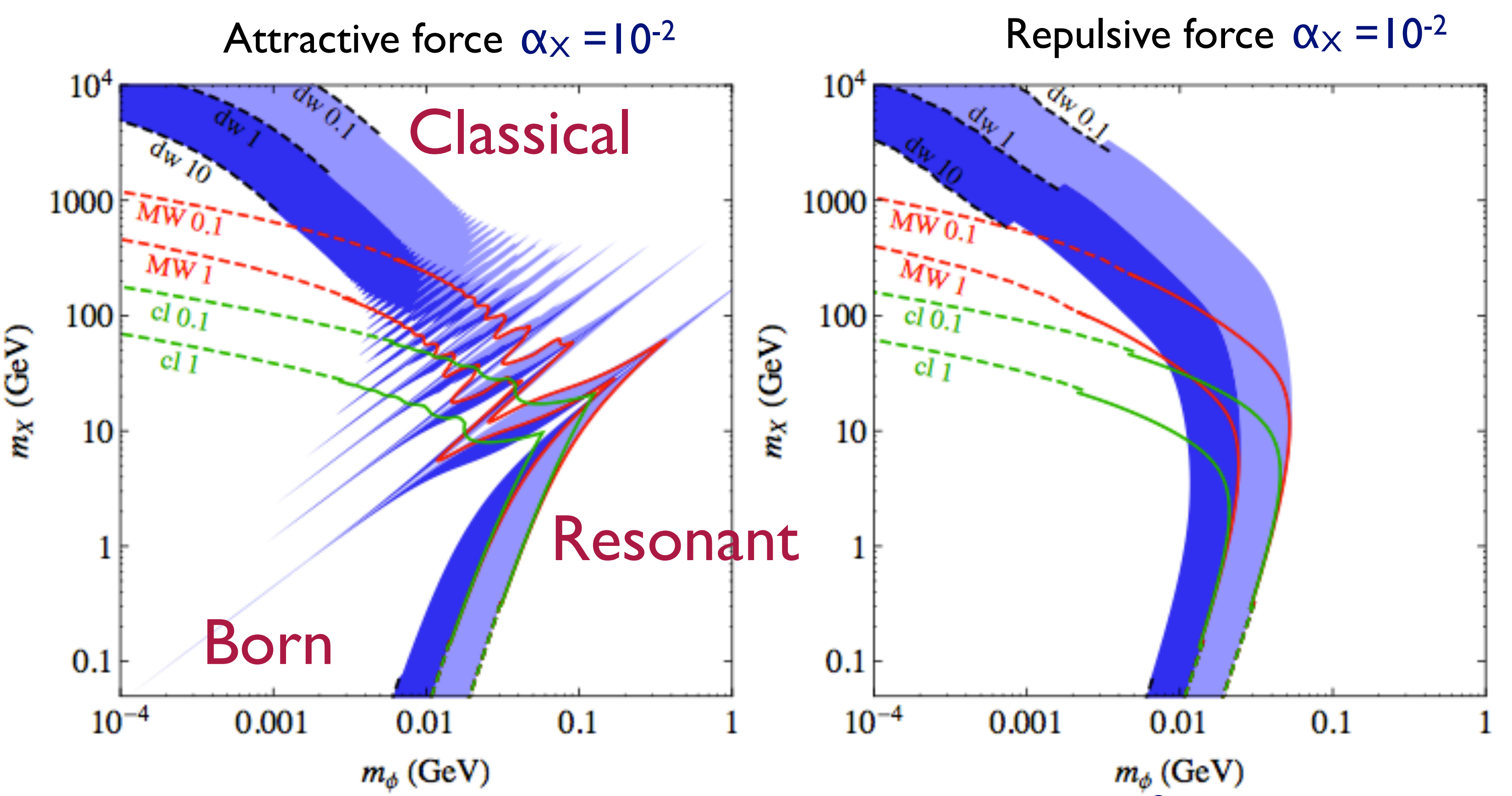
anti-resonance



$\sigma_T \sim v^{-4}$

- In many cases, σ_T is enhanced on dwarf scales
- This helps us avoid constraints on MW and cluster scales

Dark Force Parameter Space

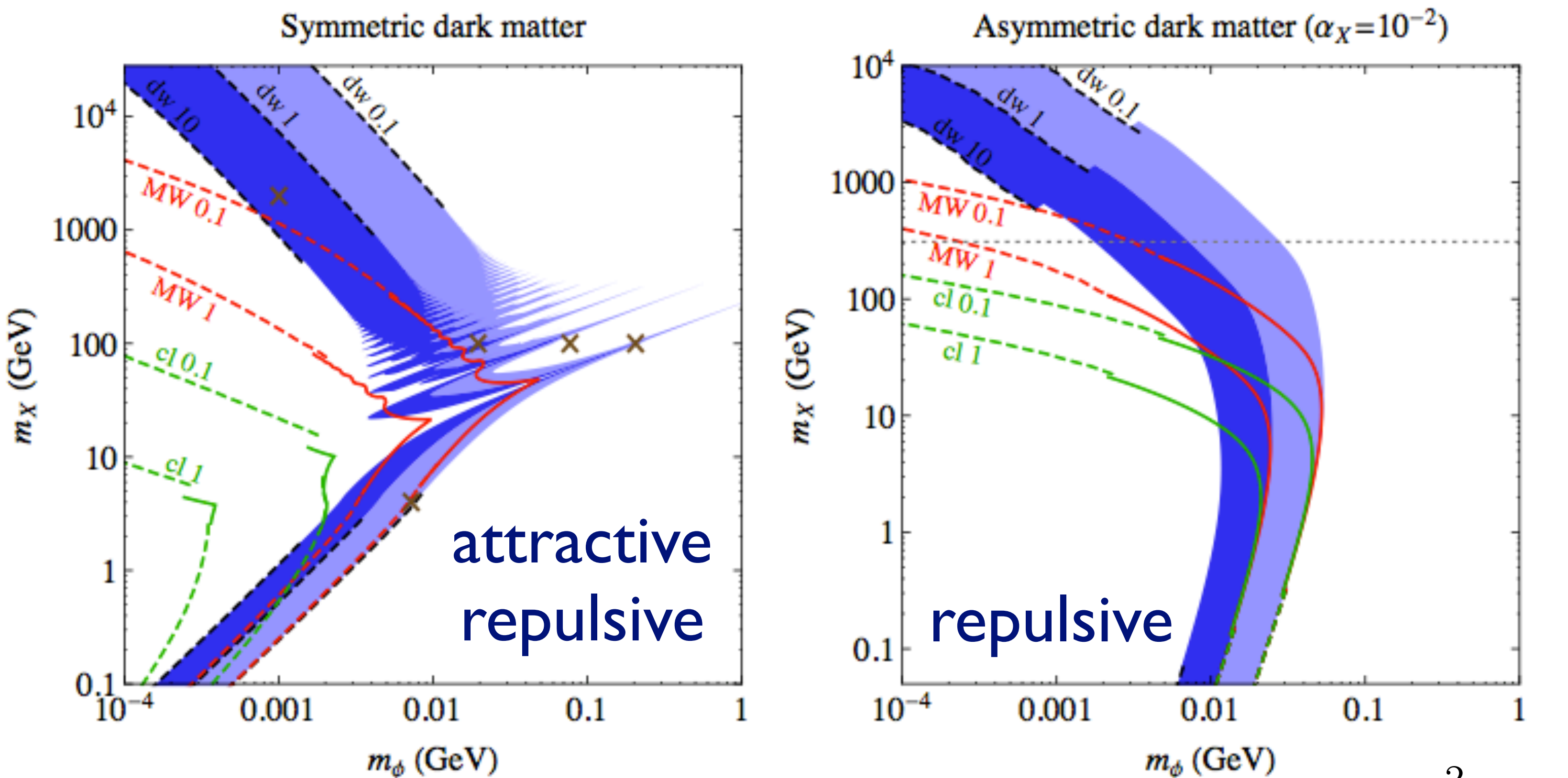


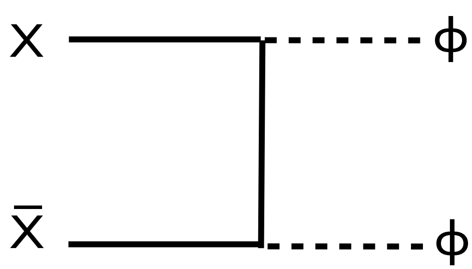
Contours show $\langle\sigma_T\rangle/m_X$ in cm^2/g

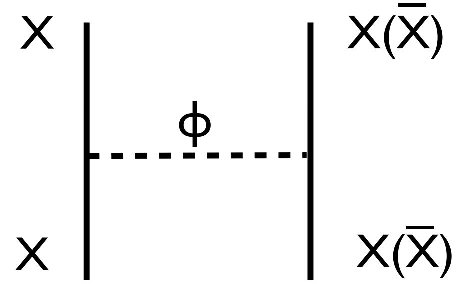
dw: dwarf (10 km/s)
MW: Milky Way (200 km/s)
cl: cluster (1000 km/s)

Blue region: Explain small scale anomalies

A Unified Model







Fix α_X

$$\langle \sigma v \rangle = \frac{\pi \alpha_X^2}{m_X^2}$$

$$\simeq 6 \times 10^{-26}\ \mathrm{cm}^3/\mathrm{s}$$

$$\gtrsim 6 \times 10^{-26}\ \mathrm{cm}^3/\mathrm{s}$$

Tulin, HBY, Zurek (2012)

Implications

- Indirect detection

Particle physics

$$\frac{d\Phi(b,\ell)}{dE} = \frac{\langle\sigma_A v\rangle}{2}\frac{J(b,\ell)}{J_0}\frac{1}{4\pi m_\chi^2}\frac{dN_\gamma}{dE}$$

$$J(b,\ell) = J_0 \int dx\; \rho^2(r_{\rm gal}(b,\ell,x))$$

Astrophysics

Implications

- Indirect detection

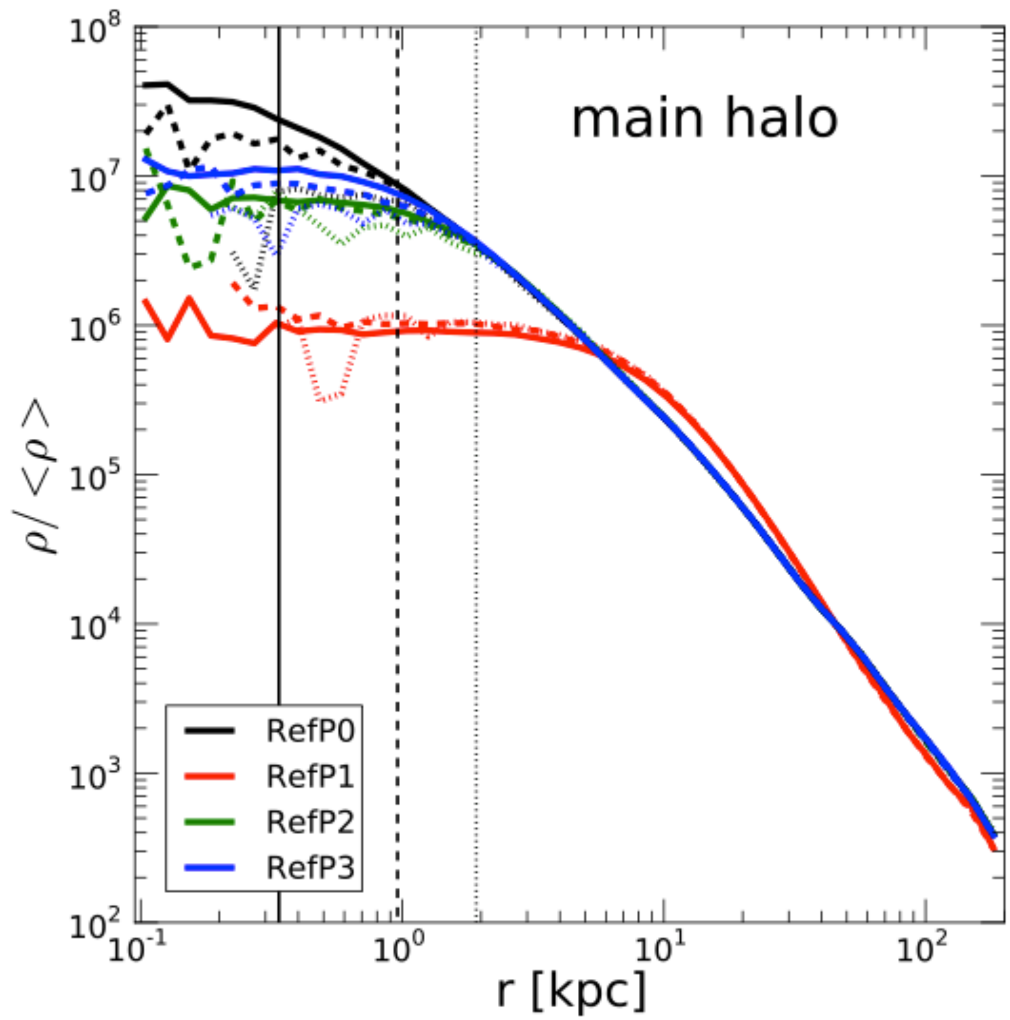
Vogelsberger, Zavala, Loeb (2012)

Name	Type	σ_T^{max}/m_χ [cm^2 g^{-1}]	v_{max} [km s^{-1}]
RefP0	CDM	/	/
RefP1	SIDM (ruled out)	10	/
RefP2	vdSIDM (allowed)	3.5	30
RefP3	vdSIDM (allowed)	35	10

$$J(b,\ell) = J_0 \int dx\, \rho^2(r_{gal}(b,\ell,x))$$

also depends on particle physics parameters (m_χ, m_ϕ, α_χ)

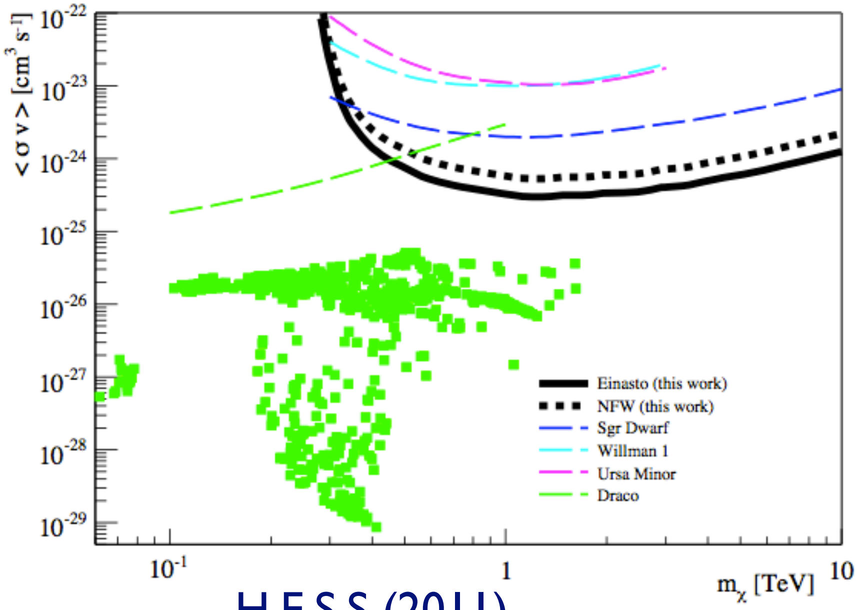
Kaplinghat, Linden, HBY work in progress



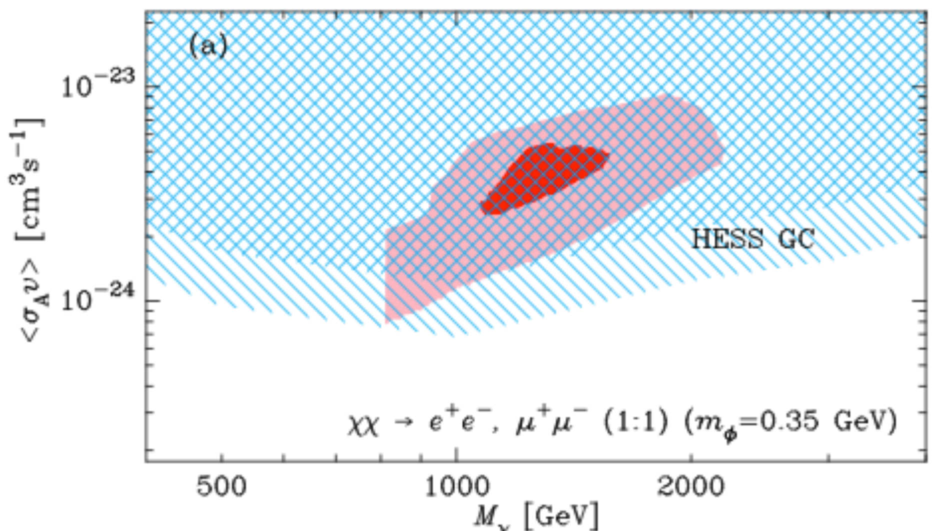
Baryons? (Brooks)

Implications

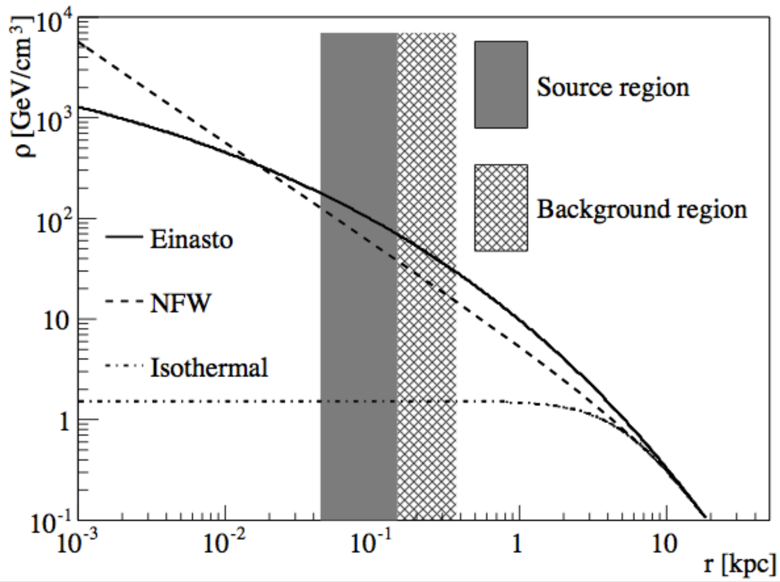
- Constraints from indirect detection



H.E.S.S (2011)



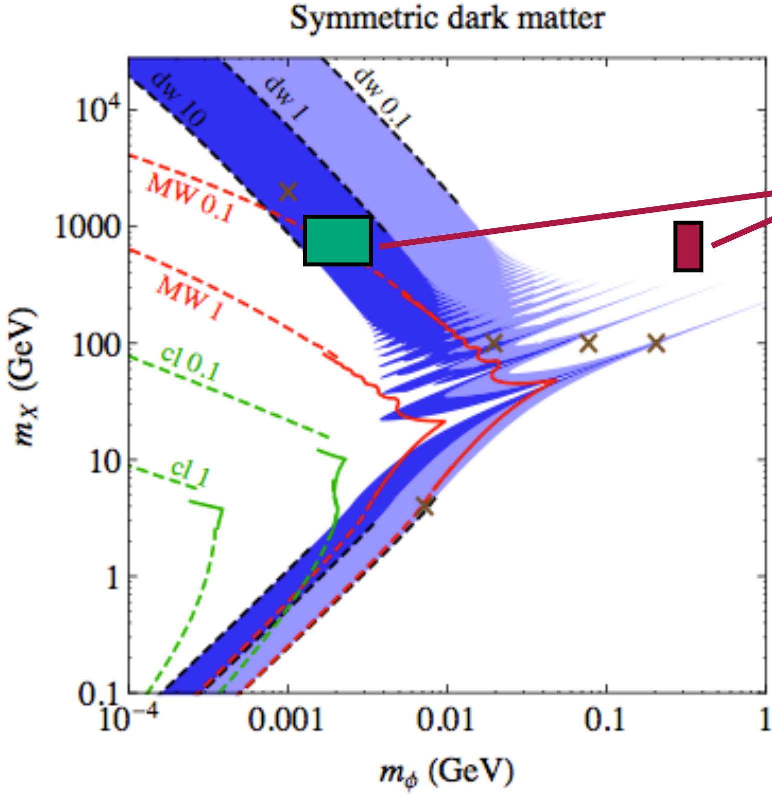
Abazajian, Harding (2012)

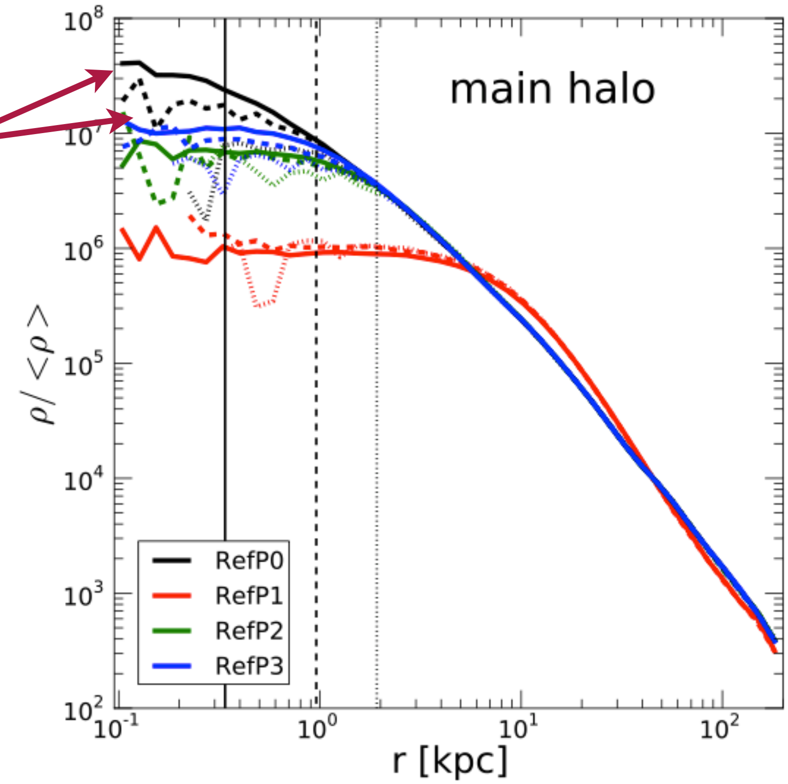


- A cored-isothermal profile with a constant-density core that extends at or beyond ~450 pc, NO constraint
- The background subtraction region would have an identical annihilation signal as the signal region

Implications

- Indirect detection





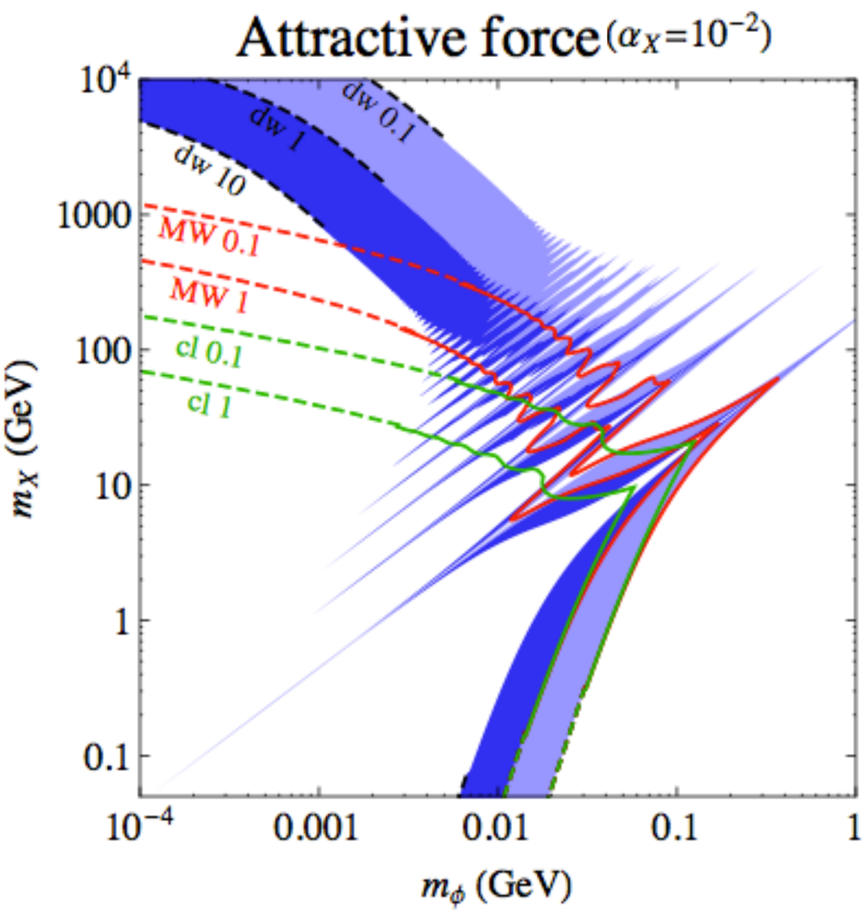
Vogelsberger, Zavala, Loeb (2012)

Green region

- DM self-interactions lead to a core for r less than ~1kpc
- Constraints can be lifted

Experimental Tests

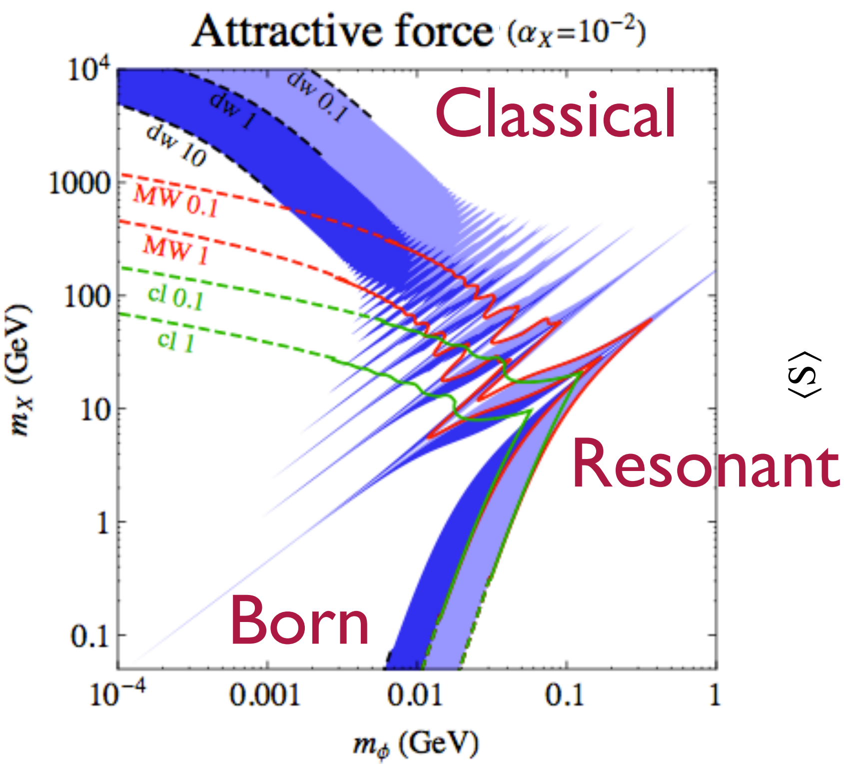
- DM density profiles on different scales

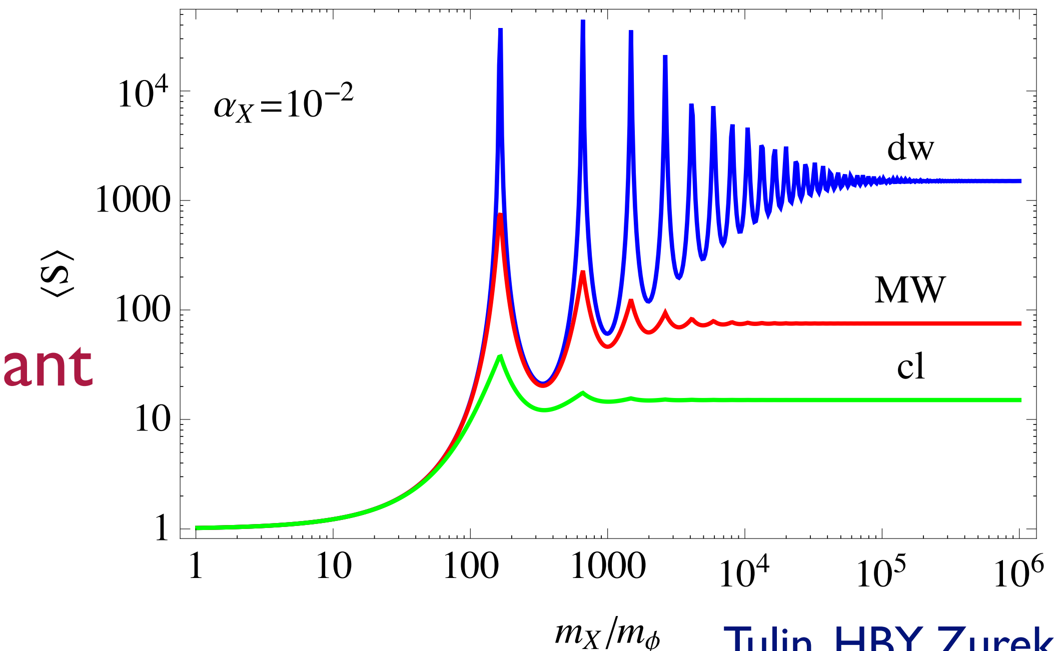


- In the Born regime, σ_T does not depend on DM velocities
- If we also observe DM cores in clusters, the Born regime is preferred

Experimental Tests

- Implications for indirect detection





Tulin, HBY, Zurek (2013)

- The light mediator can also lead to Sommerfeld enhancements for DM annihilation
- The resonant conditions are the same for both scattering and annihilation

$\langle S\rangle_{dw}/\langle S\rangle_{MW}$

Born regime: O(1)

Resonant regime: O(100)

Classical regime: O(10)

Conclusions

- In many DM models, DM is necessary self-interacting
- We have solved the scattering problem with a Yukawa potential completely
- Light dark forces can (with one coupling α_X)
  - Explain anomalies on dwarf galaxy scales
  - Satisfy bounds on Milky Way and cluster scales
  - Provide the correct DM relic density
- Implications for indirect/direct detection