16 March 2009
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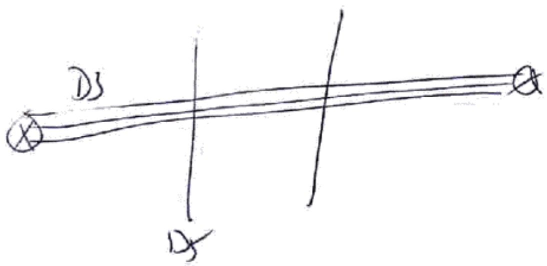
S-duality and boundary conditions, I

Half BPS, conformally invariant boundary conditions for $N=4$ SYM with ~~gauge~~ gauge group G.

Hanany - Witten:



NS5 $\leftrightarrow$ D5 produces 3D mirror symmetry.

$N=4$ SYM on $\mathbb{R}^3 \times I$ with boundary conditions at ends of interval

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3D mirror symmetry has a mathematical treatment:
Symplectic duality. $[C, H]$ dual hyperkähler cones

The boundary conditions are labelled by a triple (ρ, H, B).
where $\rho: SU(2) \to G$, H subgroup of G s.t. $[H, \rho] = 0$,
B is a 3D CFT with coupling to H.

$$(\rho, H, B) \leftrightarrow ({}^L\rho, {}^LH, {}^LB)$$

②

(The categorical framework here is a 3-category.)

The duality map is well understood for ρ = trivial, $H = G$. (and ${}^L\rho$ = trivial, ${}^LH = {}^LG$)

Less well understood are cases with ρ trivial, $H \neq G$.

Hard: general case.

For $U(1)$ gauge theory, S duality = Electromagnetic duality

$$F_A \leftrightarrow * F_{{}^LA}$$

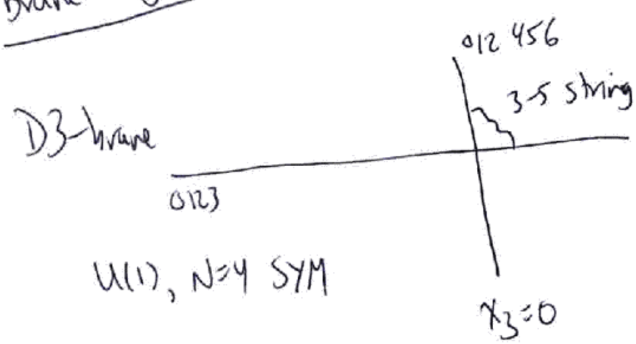
Dirichlet boundary condition: $\xleftrightarrow{\text{S-duality}}$ Neumann boundary condition:

$$F_{ij}\big|_{x_3=0} = 0 \qquad F_{3i}\big|_{x^3=0} = 0.$$
$$i,j = 0,1,2$$

$U(1)$ A $\qquad q \qquad \int |D_A q|^2 d^3x$

S-duality: $\int d^4x \, F \wedge * F + {}^LF \wedge F$, add Lagrange multiplier — but cannot do that if there is a (boundary) term involving A.

<u>brane engineering</u>



U(1) gauge theory with defect:

$$\int |D_A q|^2 d^3x$$

$x_3 = 0$

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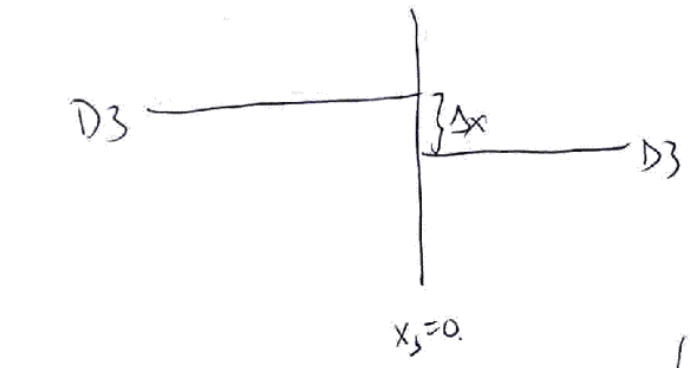
D3-branes can split/break



$\longleftrightarrow$ expectation value $q = q_0$

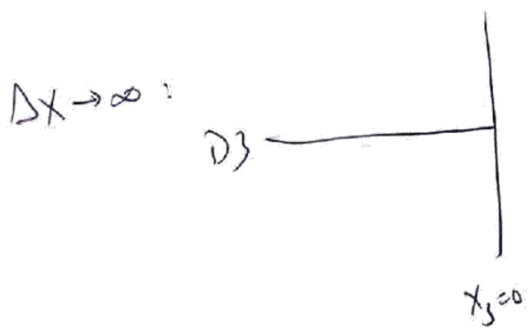
$(A_\mu, x^4, x^5, x^6, y^7, y^8, y^9)$

$F^+_{3i} - F^-_{3i} = J_i$

add superpartners of this eqn; get $X_+^{4,5,6} - X_-^{4,5,6} = \mu^{1,2,3} = q^\dagger \sigma^{1,2,3} q$

(hyperkähler moment maps)

$\Delta X \to \infty$:



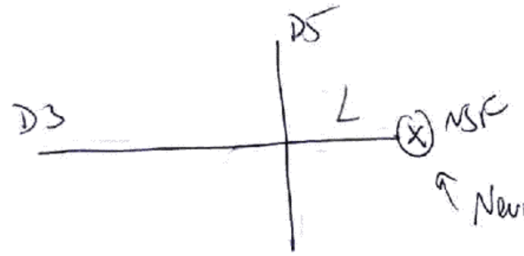
i.e. $q_0 \gg 0$ Higgs's the gauge group along body ...

$$\boxed{\begin{array}{c}(\rho=0, H=0, B=0) \text{ Dirichlet} \\ \Updownarrow \\ (\rho=0, H=U(1), B=0) \text{ Neumann}\end{array}}$$

(D3 ending on D5)

S-dual: D5 become NS5, symbol $\otimes$

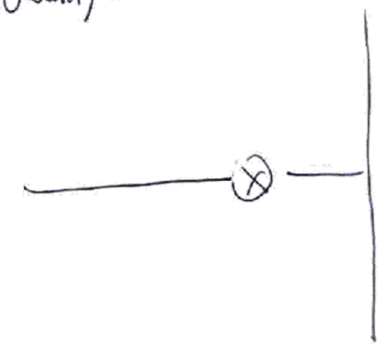
④



↑ Neumann boundary condition

↑ $x_3 = L$, hypermultiplet coupled to $N=4$ SYM.

duality:



Hanany-Witten effect: if you pull the NS5 through the D5, need a D3 brane connecting

$\Rightarrow$ this boundary condition is self-dual.

$$\left(\rho = 0,\ U(1),\ B = \mathbb{R}^4\right)$$

↑ hypermultiplet H

How to do this in field theory?

$N=4$ SYM with $G = U(1)$ and boundary condition $(\rho = 0, U(1), B = B_\rho)$

A = gauge field

↑ boundary theory

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Introduce $\tilde{A}$ at some distance, with $F_A = F_{\tilde{A}}$ on bdy.

Dualize: $U(1)$, ${}^L A$ and $* {}^L F_A = F_{\tilde{A}}$

(can be induced by $\int {}^L A \wedge d\tilde{A}$ term, for example)

Try to take the length L to 0

$$\begin{array}{c} (1) \xrightarrow{\text{coupled}} B_{\tilde{A}} \\ \Big\downarrow \text{IR} \\ {}^L B \end{array}$$

3D theory.

$(1) = \tilde{A} + \text{theory}^{U(1)}$ in 3D ...

kinetic term $(\partial_\mu \varphi + {}^L A_\mu)^2$

In 3D, dualize $F_{\tilde{A}} = * d\varphi$.

if no $B_{\tilde{A}}$ then φ gets a shift symmetry

Quantum corrections to Coulomb branch from B_A...

Example: $B_A = H/$ $\quad (1) - \boxed{1}$

naively $S^1 \times \mathbb{R}^3$ in Coulomb branch

in presence of hypermultiplet, $S^1 \times \mathbb{R}^3$ deformed to Taub-NUT.

in IR, Taub-NUT $\to \mathbb{R}^4$

(6)


Taub-Nut

Shift symmetry becomes a rotation in $\mathbb{H}$ in IR.

If we put more hypermultiplets in $B_A = \mathbb{H}^n$?

$(1)-\boxed{n}$

becomes $\mathbb{R}^4/\mathbb{Z}_n$.

i.e. $(\rho=0, U(1), B)$

$(1)-B$

$\downarrow IR$

${}^L B$ (new coupling comes from shifts of dual photon)

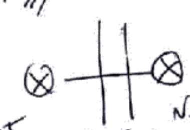
S-dual is $(\rho=0, U(1), {}^L B)$.

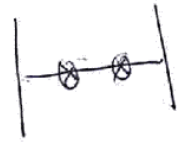
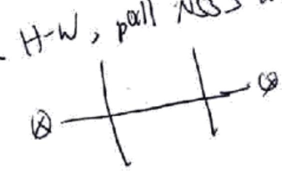
$(1)-\boxed{n}$ Higgs branch $\mathbb{H}^n /\!/\!/ U(1)$

Coulomb branch $\mathbb{R}^4/\mathbb{Z}_n$, $\mathbb{H}^n /\!/\!/ U(1)^{n-1}$.

3D $(1)-\boxed{2}$ What is the mirror?

Higgs branch is $\mathbb{H}^2 /\!/\!/ U(1)$.

Engineer with branes: 

S-duality.  or via H-W, pull NS5's out 

$\Rightarrow$ self-mirror.

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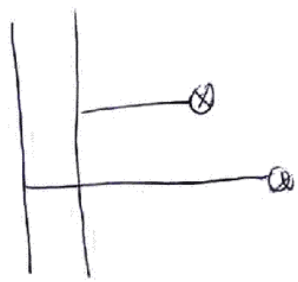
This can live on the wall between $SU(2)$ and ${}^L SU(2) = SO(3)$



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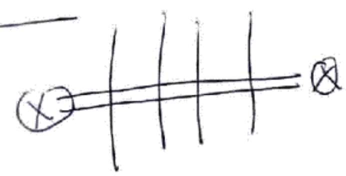
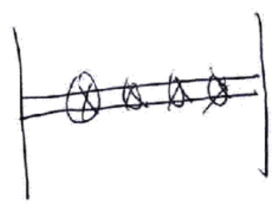
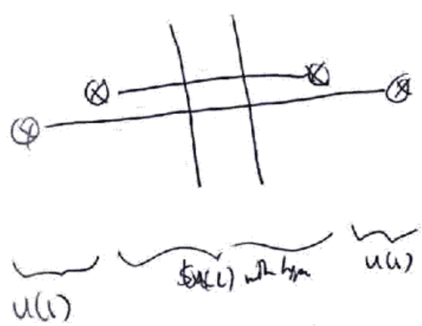
(2) − [4]



S-dual to:



or



(1) — (2) — (1)
$\quad\quad\;\;$ |
$\quad\quad\;$ [2]

makes the SU(2) symmetry manifest.