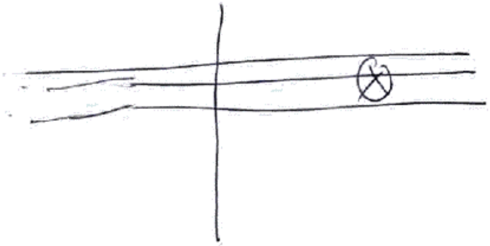
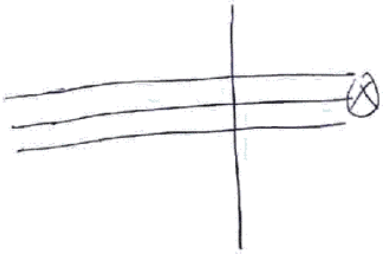
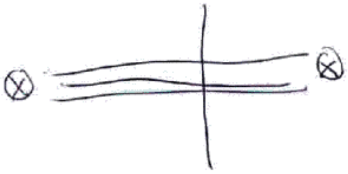
17 March 2009
D Gaiotto

S-duality and boundary conditions, II



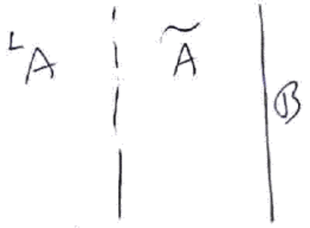




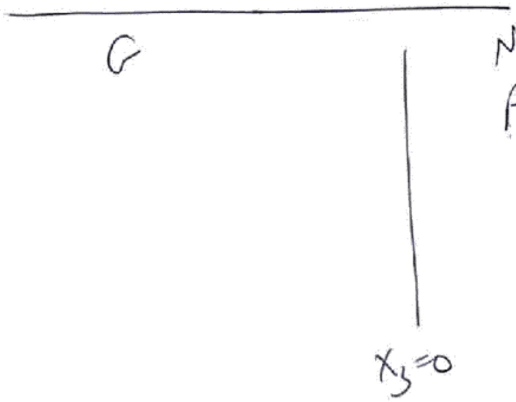
(ρ, H, B)

$$(\emptyset, U(1), B) \longleftrightarrow (\emptyset, U(1), {}^L B)$$

$$\begin{tikzcd} \text{①} - B \arrow[d, "IR"] & * \tilde{F} \arrow[d, "IR"] \\ {}^L B & J \end{tikzcd}$$



$\int {}^L A \wedge \tilde{F}$



Neumann

$F_{3i}|_\partial = 0$

$X|_\partial = 0$

$\partial_3 Y|_\partial = 0$

(gauge grp at boundary)

Dirichlet

$F_{ij}|_\partial = 0$

$\partial_3 X|_\partial = 0$

$Y|_\partial = 0$

(no gauge grp at boundy)

$OSp(4|4) \subset PSU(2,2|4)$

$Sp(4) = SO(3,2)$

$SU(2,2) \simeq SO(4,2)$

$SO(4) = SO(3) \times SO(3)$

$SU(4) \simeq SO(6)_R$

$Sp(4) \simeq SO(3,2)$

$x^1, x^2, x^3, \quad y^1, y^2, y^3$

$4, 5, 6 \quad 7, 8, 9$

D5 NS5

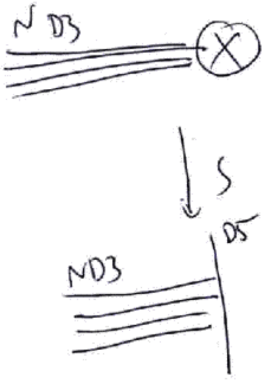
Dirichlet boundary for G admits global symmetry:

$X|_\partial \to g X|_\partial g^{-1}$; gives (boundary) current ${}^G J$.

S-Dual cannot just be Neumann – need extra ~~things~~ theory T(G).
on boundary, with a G-action (from duality) and ${}^L G$-action. (to couple)

↑ Coulomb branch ↑ Higgs branch

What is the S-dual of Neumann?



produces Neumann b.c. for U(N) SYM on D3-brane

$$X^i \sim \frac{R^i}{x_3}$$

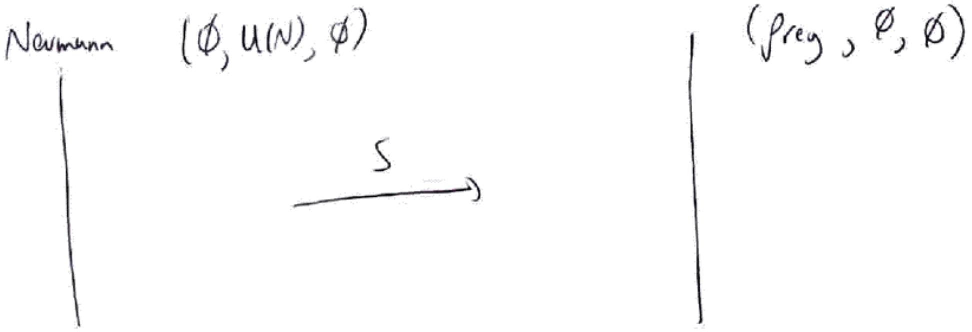
$$D_3 X^1 = [X^2, X^3]$$
$$D_3 X^2 = [X^3, X^1]$$
$$D_3 X^3 = [X^1, X^2]$$

(Nahm equations)

$$R^1 = [R^2, R^3], \ ---$$

(monopole physics)

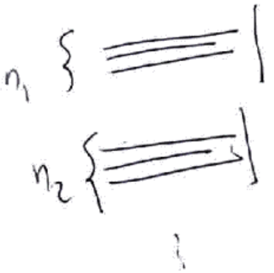
R^i N-dimensional irrep of SU(2)



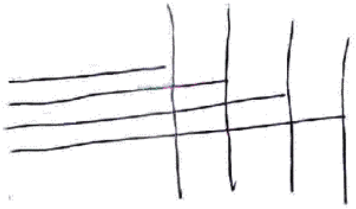
$(\rho_{reg}, \emptyset, \emptyset)$

$(\emptyset, G, \emptyset)$

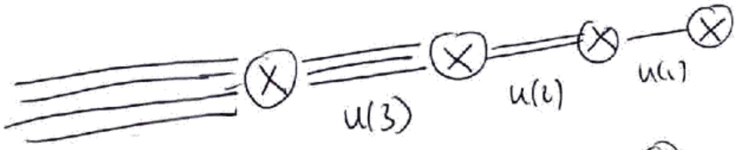
$\rho = \sum (n_i)$



$\rho = \sum 1 \Rightarrow$ every D3 brane ends on its own D5 brane.



The S-dual is



↓IR

T(U(4))

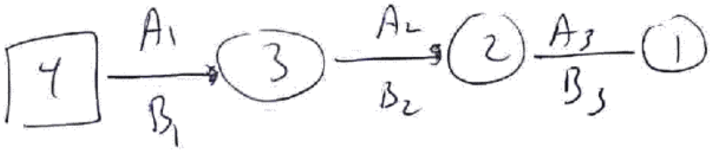
Self-mirror quiver.

⑤

In this theory, dim Higgs $= 4\times3+3\times2+2\times1-3^2-2^2-1^2=6$

dim Coulomb $=1+2+3=6$

(hyperkähler dimension)



$$M = A_1 B_1$$
$$M^2 = A_1 B_1 A_1 B_1 = A_1 A_2 B_2 B_1$$
$$M^3 = A_1 A_2 A_3 B_3 B_2 B_1$$
$$M^4 = 0.$$

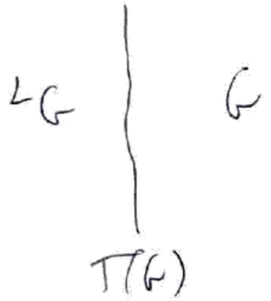
Higgs $=$ Nil$(GL(4))$

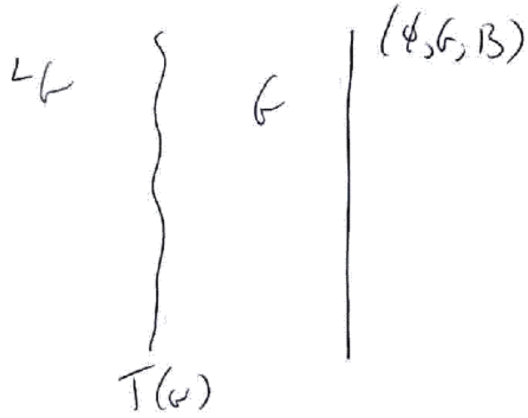
$T(G)$ has Higgs $=$ Nil$({}^L G_{\mathbb{C}})$

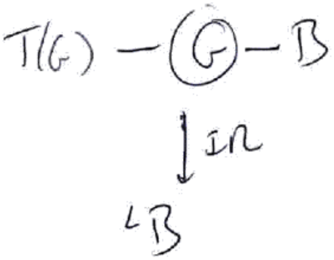
Coulomb $=$ Nil$(G_{\mathbb{C}})$

$g^2(x_3)$

in IR, get a domain wall:



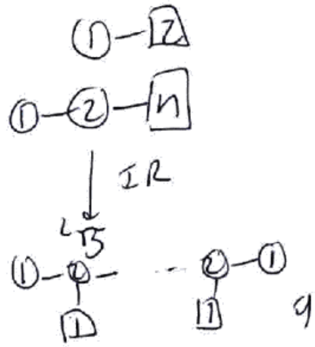




$(\phi, {}^LG, {}^LB)$ if IR flow is smooth

e.g. $B = \mathbb{H}^{2n}$, ~~$G = U(N)$~~ $G = U(2)$.

$T(U(2))$ is coupled theory

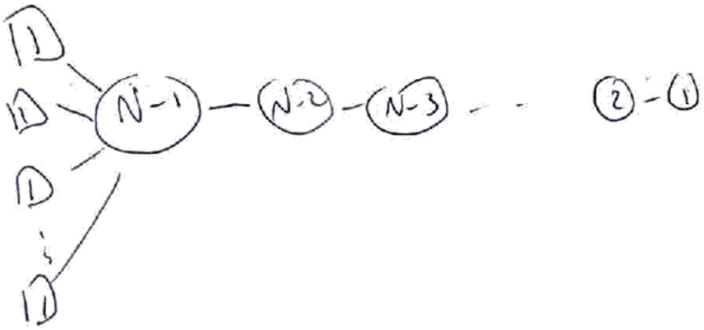


mirror

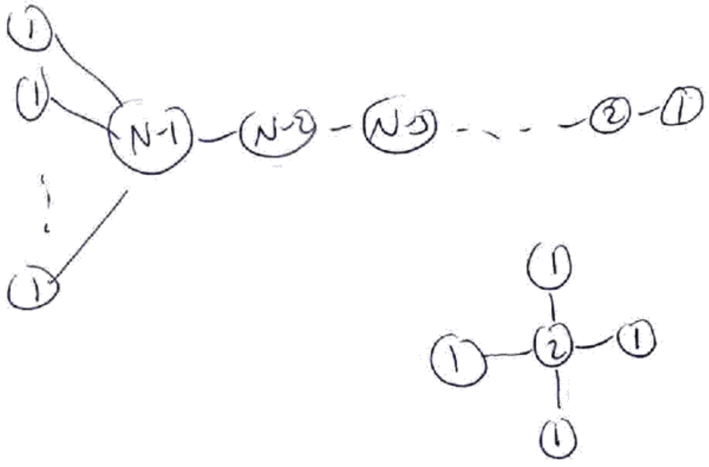
if $n > 2$

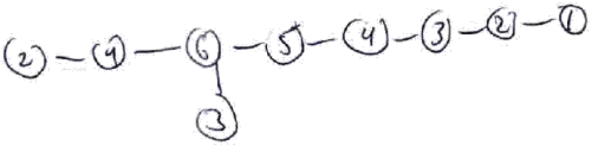
$${}^L\chi^{\vec{N}} = {}^L G_{\mu}^{\vec{N}}$$

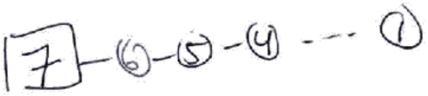
$U(N)$	$(\emptyset, U(1)^N, \emptyset)$

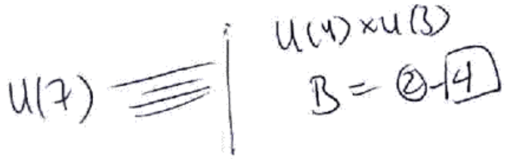


couple to $U(1)^N$:









$$X = \begin{pmatrix} 0 & * & * & & \\ * & 0 & * & & \\ * & * & 0 & & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix}$$

$$D_3 X^i = \epsilon^{ijk}[X^j, X^k], \quad X|_{x_3 \sim}$$