Self-consistent Modeling of Accretion with Magnetohydrodynamic Turbulence

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Statistical theory of MHD turbulence

1-point modeling of radial and ⊥ quantities for spherical geometry

applied to

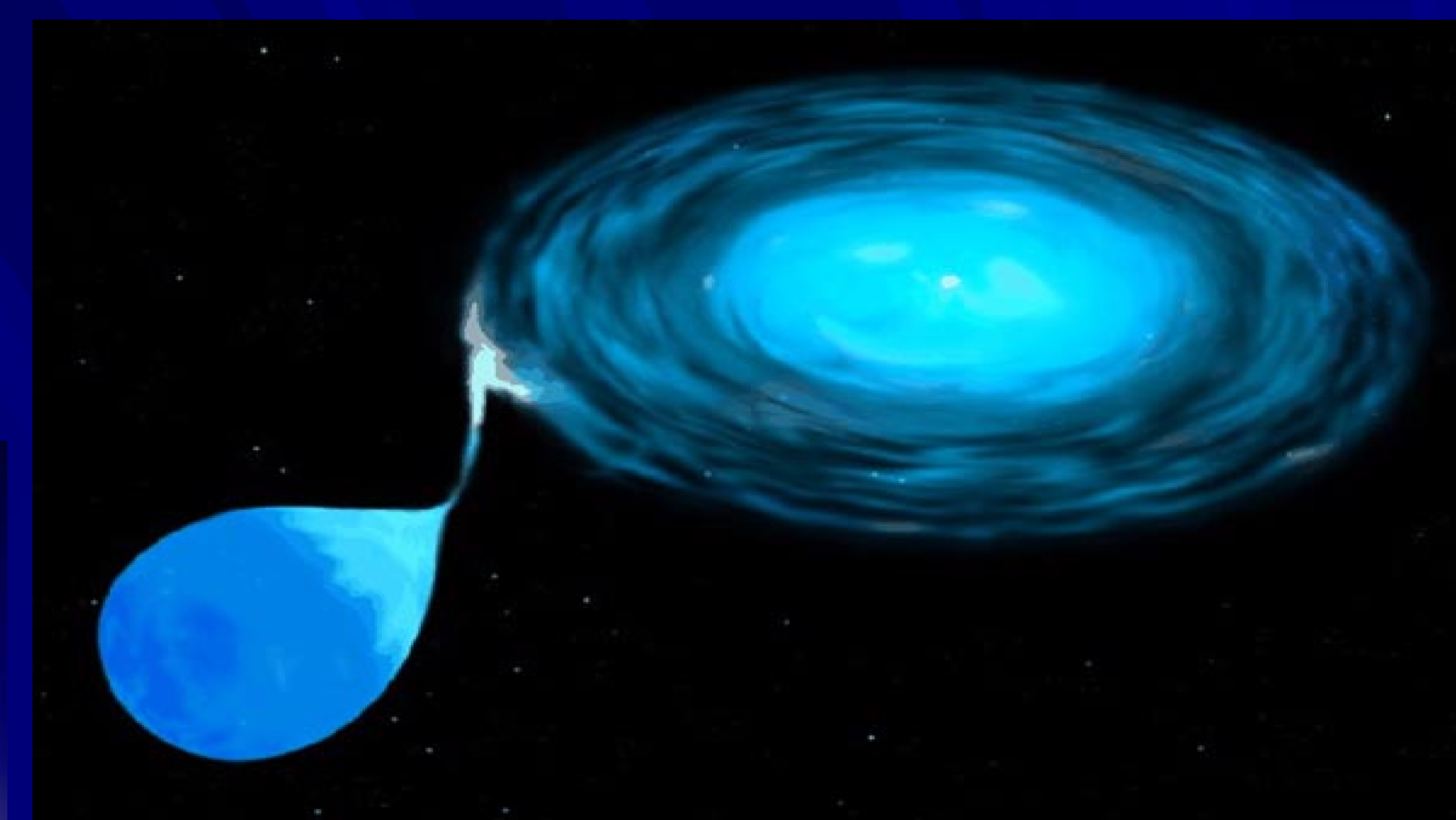
Accretion onto compact objects

onto the black hole in the Galactic Center

$$(u^2)'_t = \frac{\hat{c}_{uB} v_A^2 u - \hat{c}_{uu} u^3}{L}, \qquad (v_A^2)'_t = \frac{\hat{c}_{Bu} v_A^2 u - \hat{c}_{BB} v_A^3}{L}$$

$$\hat{c}_{Bu} = 0.70, \quad \hat{c}_{BB} = 1.75$$
$$\hat{c}_{uu} = 1.14, \quad \hat{c}_{uB} = 0.09$$

– coefficients from numerical simulations of homogeneous turbulence. Sreenivasan 1995; Biskamp 2003; Schekochihin 2004

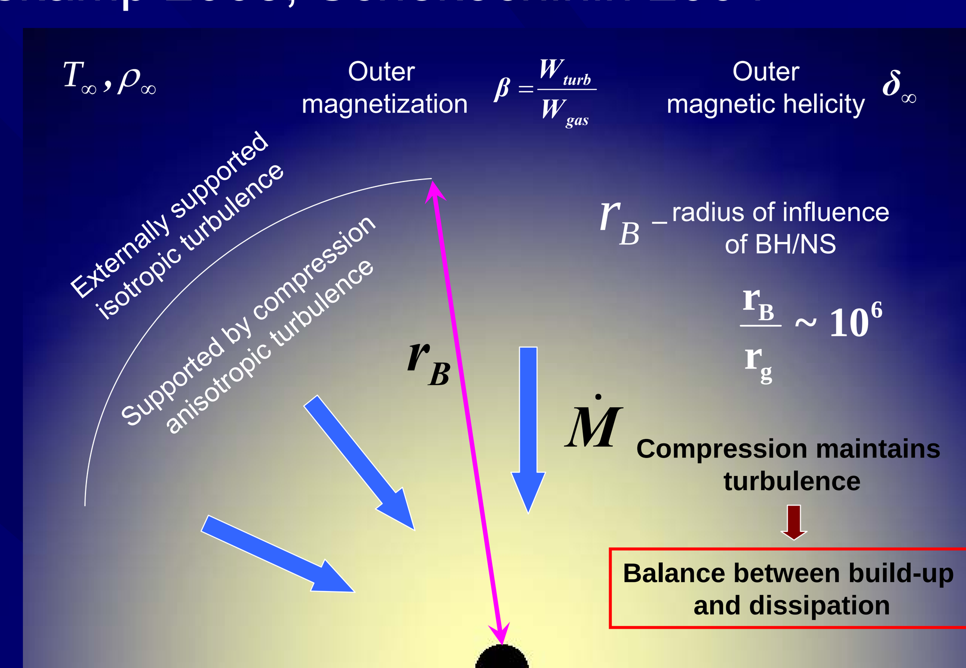


Then extended and, finally, phenomenologically includes:

- small-scale dynamo action,
- dissipation of magnetic and kinetic energies,
- return to isotropy (rate about dissipation rate),
- return to magnetic <-> kinetic equipartition,
- selective decay at constant magnetic helicity: $\exp(-H_M/(L \cdot E_M))$ suppression of energy decay rate.

Outer boundary:

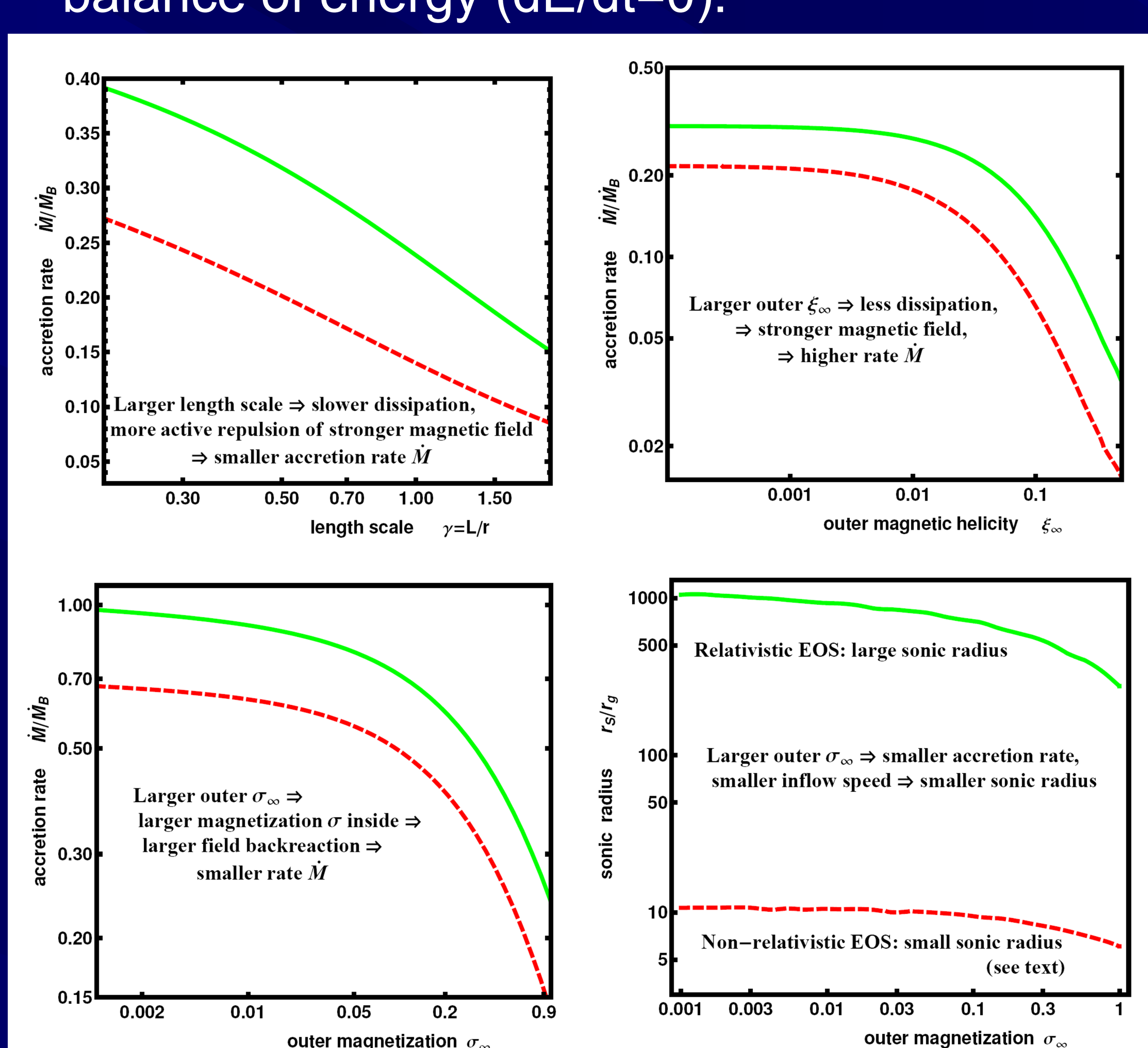
- homogeneous turbulence,
- source of stirring (equal energies $E_k = E_M$),
- balance of energy (dE/dt=0).

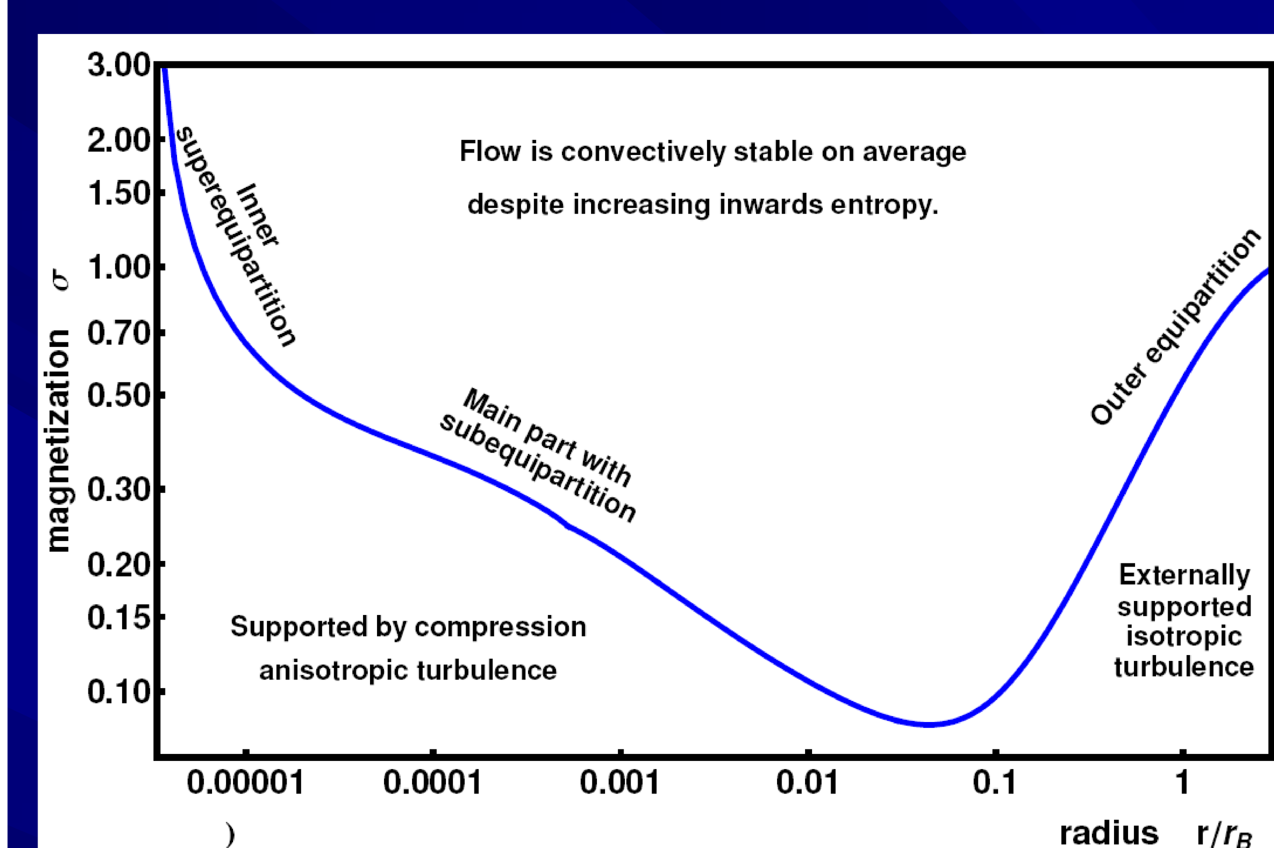


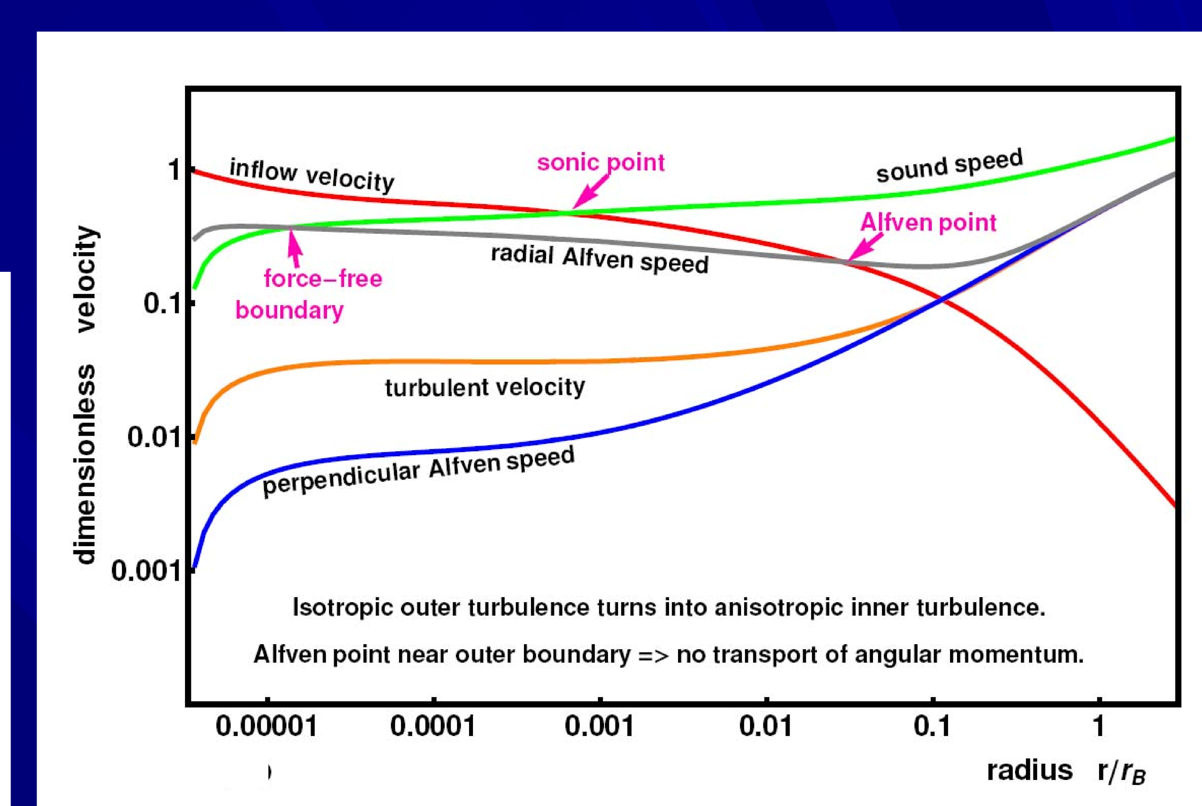
Shcherbakov 2008, ApJS accepted

- Find outer density, temperature from observations
- Infer plasma magnetization

Search for maximum accretion rate $\dot{M}$







No self-similar solution.
No equipartition between thermal and turbulent E.
No effect of finite magnetic helicity.
Agrees with DNS (Igumenshchev 2006).

1-point modeling of 2-nd moments

2+ point self-consistent modeling

Better statistical theory?

Algebraic modeling

- Suitable for engineering calculations
- No general form of the equations
- Cannot be extrapolated
- Coefficients from DNS that it tries to explain

Speziale 1991; Perot, Chartrand 2005

1-point approximations

- Good for engineering calculations
- General form exists
- Fixed coefficients
- General form contains higher derivatives
- Stability questions

Leslie 1973; Yoshizawa 2003

Assumed locality of certain interactions

DIA models (2-point)

- No adjustable parameters
- Good performance in isotropic case
- Misses some coherent effects (dynamo)
- Rather involved computationally

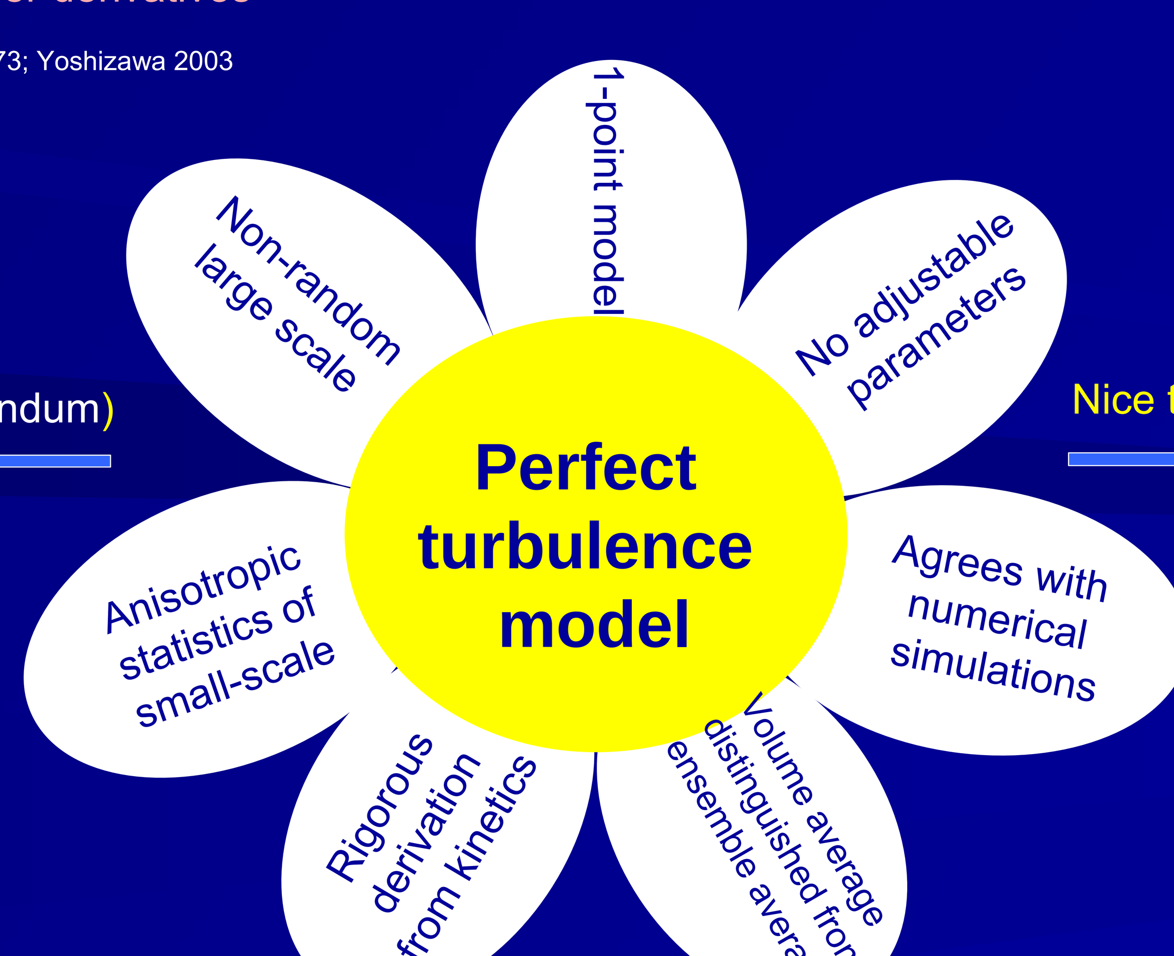
Kraichnan 1959, 1965 etc.

Higher orders MSR etc.

- Include all effects (possibly in finite order)
- Intractable numerically
- Analytical derivation is ambiguous?
- Not many researchers are comfortable with field theory

Martin, Siggia, Rose 1973

Empiric vertex renormalization
Regularized DIA models
O'Kane, Frederiksen 2004



But (de omnibus dubitandum)

Critique

- Derivation should start from the kinetic theory
- Separation of scales may not be rigorous
- 1-point model may simply be insufficient
- Why bother doing analytics?
- May require new mathematics
- The compact closed form may not exist

Nice try (per aspera ad astra)

Two-Scale DIA

- 1-point closure – easy to integrate
- Large scale fields are explicitly treated (dynamo)
- Coefficients are known to good accuracy
- Demonstrated agreement w/ solar wind turbulence
- Eulerian DIA is employed
- Small-scale variable is considered isotropic
- Sharp cut-off in spectrum of small-scale variable

Yoshizawa 1984, 2003; Yokoi et al. 2006, 2007

Polarized radiative transfer ⟹ diagnostics of magnetic field

Shcherbakov 2008, ApJ submitted