Nonlinear η_t and α tensors

MRI dynamos: → recordings in Princeton and here

Mean-field paradigm: linear → nonlinear

Turbulent diffusivity: a final frontier

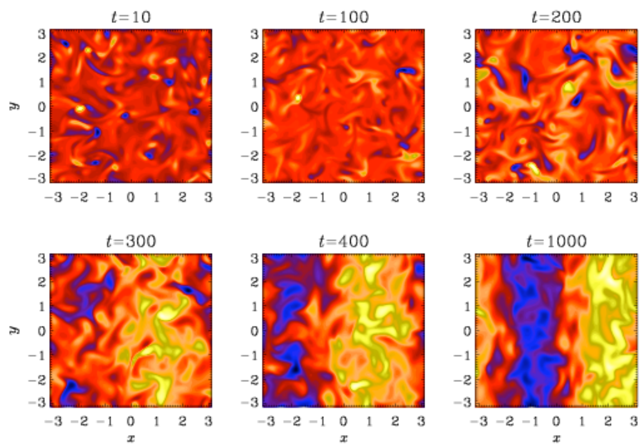
Axel Brandenburg (*Nordita, Stockholm*)

with Karl-Heinz Rädler, Matthias Rheinhardt,

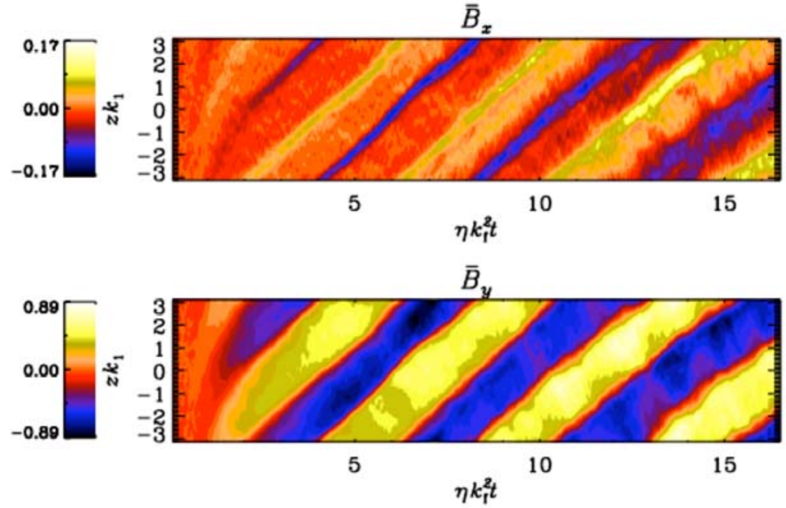
Kandaswamy Subramanian

Examples where α and η_t at work?

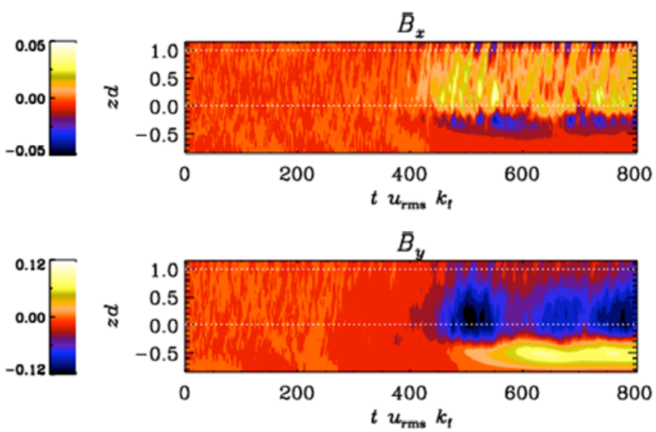
Helical turbulence (B_y)



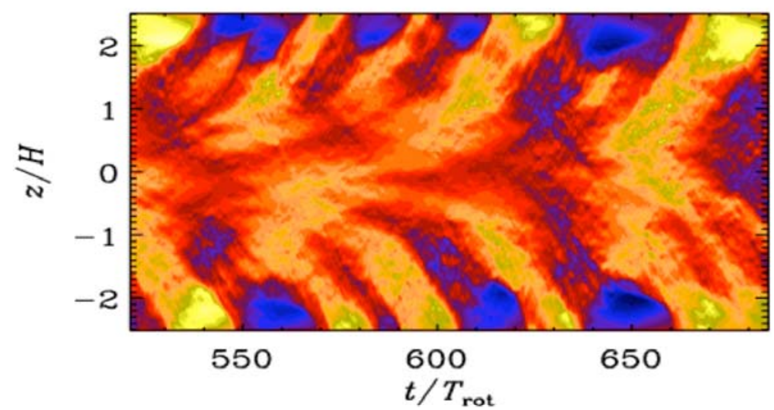
Helical shear flow turb.



Convection with shear



Magneto-rotational Inst.

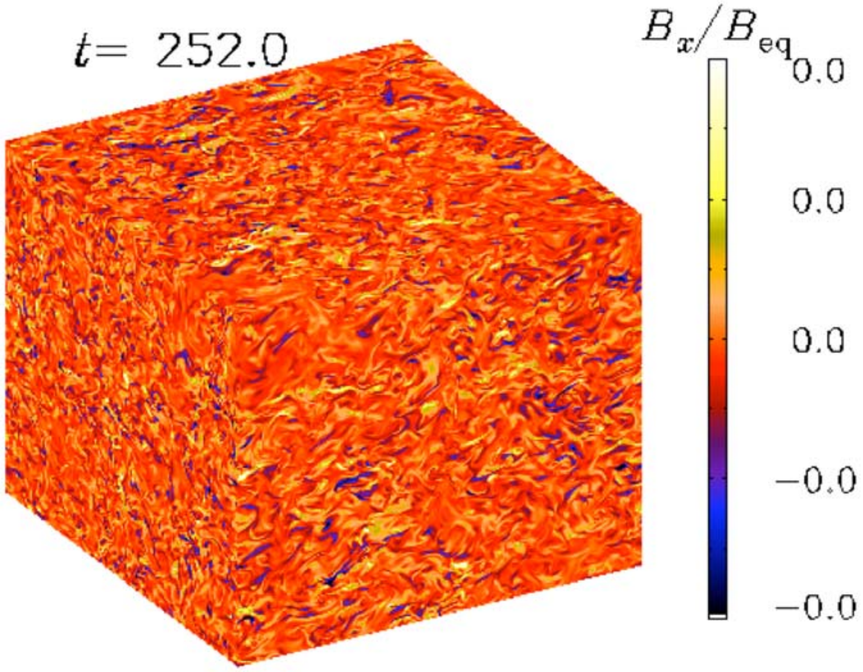


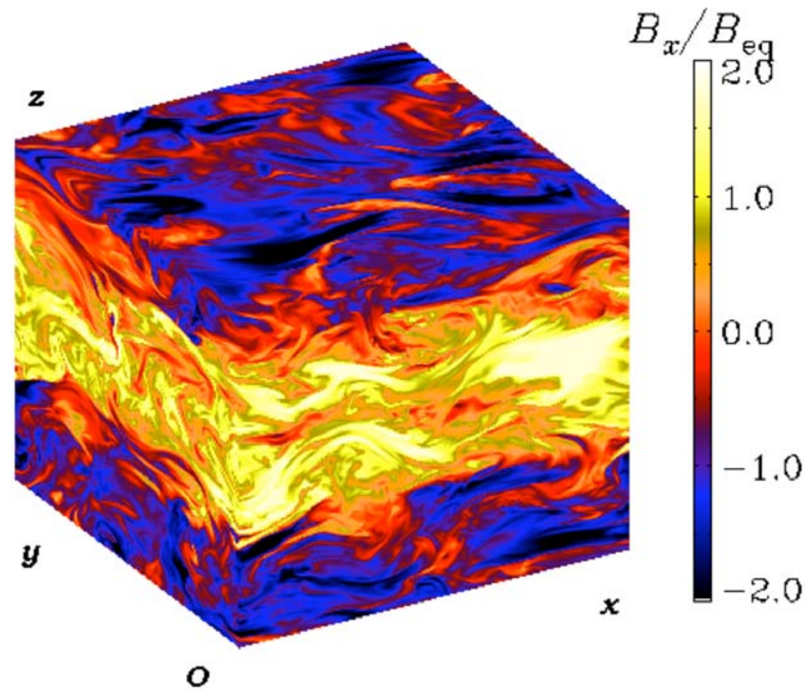
$$\omega = \eta_t k_1^2$$

$$c = \eta_t k_1$$

Käpyla et al (2008)

Dynamo in kinematic stage – no large-scale field?

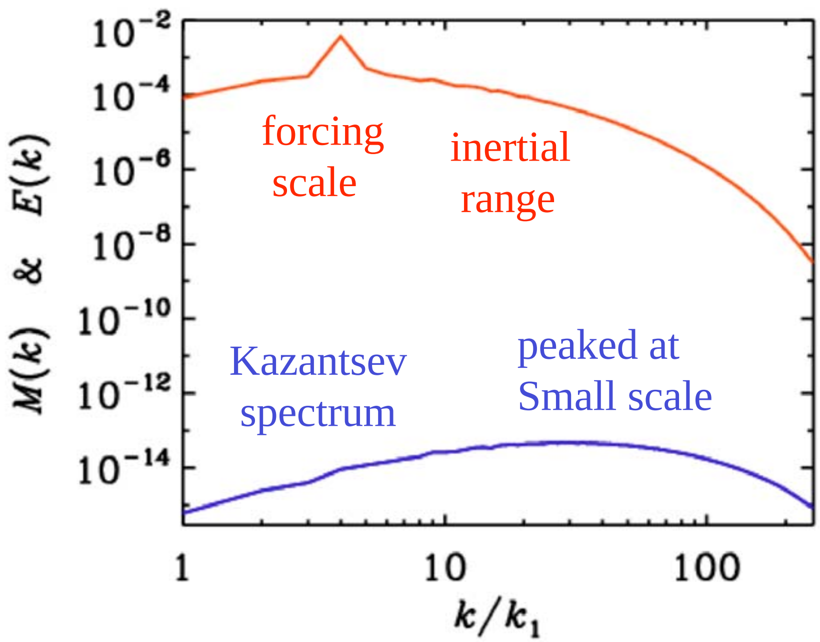




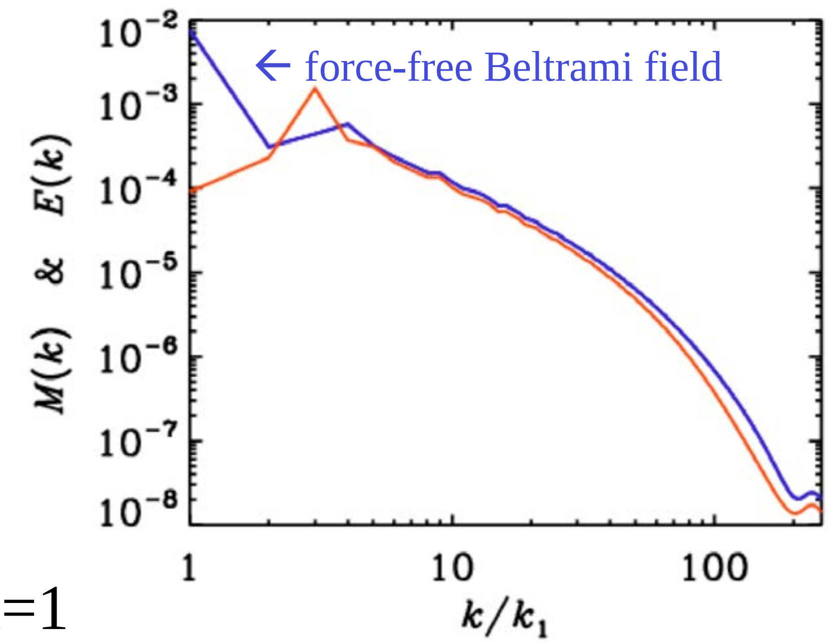
Fully helical turbulence, periodic box, resistive time scale!

Large-scale dynamo = nonlinear?

No kinematic stage of large-scale dynamo?



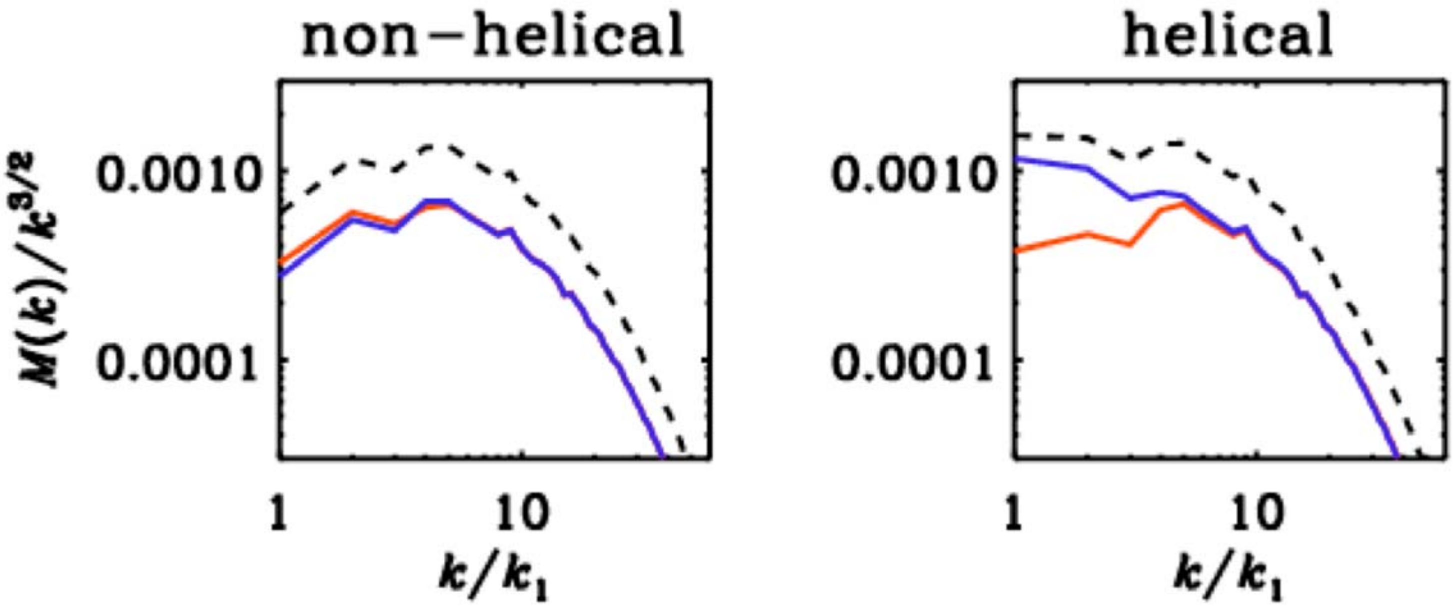
Pm=1



Large-scale field only during nonlinear stage!

Can we identify large-scale dynamo during kinematic stage?

… yes, with red/blue goggles



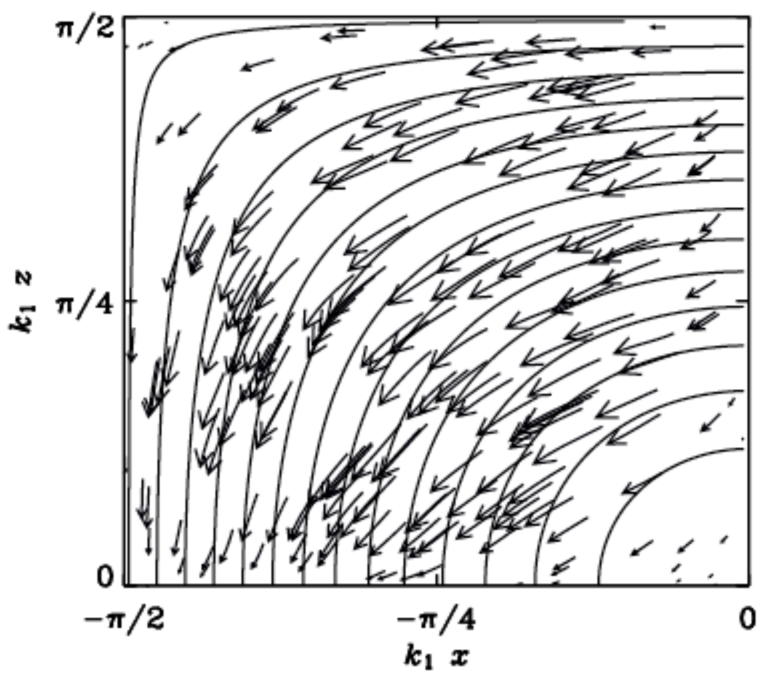
Chandrasekhar-Kendall decomposition

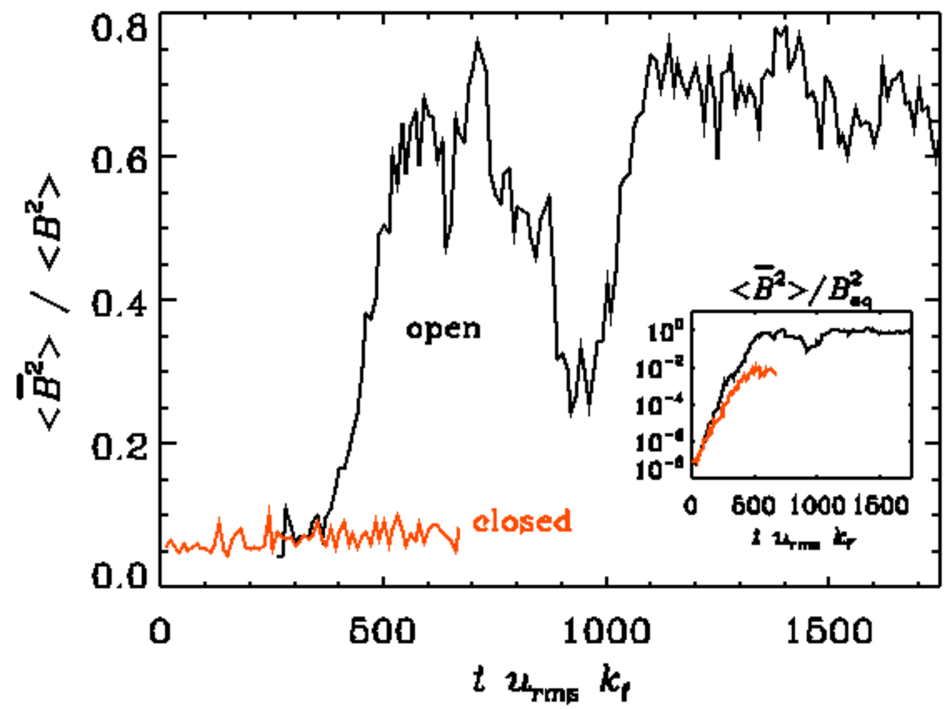
Brandenburg, Dobler, & Subramanian (2002)

Brandenburg & Subramanian (2005)

Nonlinear stage: consistent with ...

$$\alpha = \frac{\alpha_K + R_m \left[\left(\eta_t \overline{\mathbf{J}} \cdot \overline{\mathbf{B}} - \tfrac{1}{2} k_f^{-2} \nabla \cdot \overline{\mathbf{F}}_C^{SS} \right) / B_{eq}^2 - \dfrac{\partial \alpha / \partial t}{2 \eta_t k_f^2} \right]}{1 + R_m \overline{\mathbf{B}}^2 / B_{eq}^2}$$





Brandenburg (2005, ApJ)

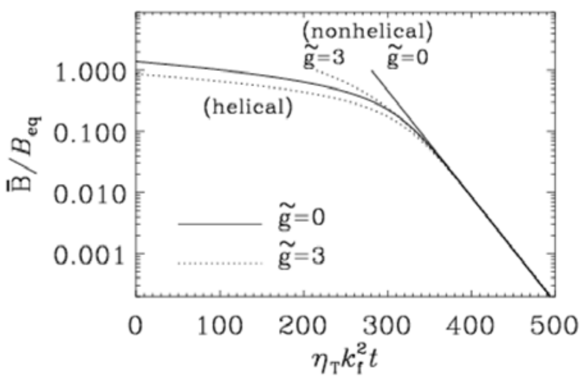
$$\overline{\mathbf{F}}_C^{SS} = C_{VC} \left(\overline{\mathrm{S}} \overline{\mathbf{B}} \right) \times \overline{\mathbf{B}}, \qquad \overline{\mathrm{S}}_{ij} = \tfrac{1}{2} \left(\overline{U}_{i,j} + \overline{U}_{j,i} \right)$$

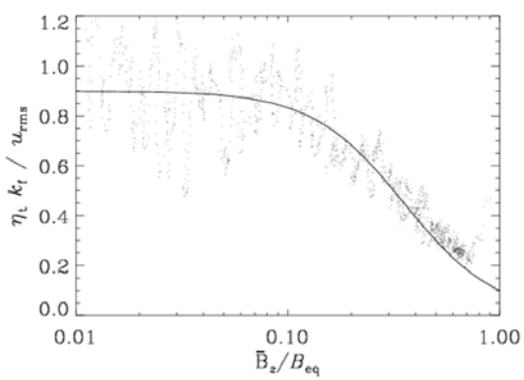
Quenching of η_t??

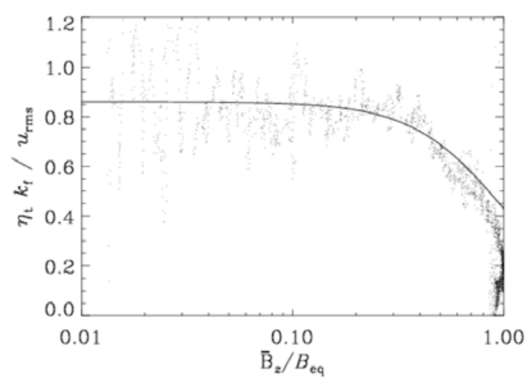
Yousef et al.
(2003, A&A)

$$\alpha = \frac{\alpha_K + R_m \left[\left(\eta_t \overline{\mathbf{J}} \cdot \overline{\mathbf{B}} - \frac{1}{2} k_f^{-2} \nabla \cdot \overline{\mathbf{F}}_C^{SS} \right) / B_{eq}^2 - \frac{\partial \alpha / \partial t}{2 \eta_t k_f^2} \right]}{1 + R_m \overline{\mathbf{B}}^2 / B_{eq}^2}$$

$$R_m \rightarrow \infty : \quad \alpha = \eta_t \overline{\mathbf{J}} \cdot \overline{\mathbf{B}} / \overline{\mathbf{B}}^2 = \eta_t k_m$$







$$\eta_t = \frac{\eta_{t0}}{1 + g \left| \overline{\mathbf{B}} / B_{eq} \right|}, \quad g = 3$$

Calculate full α_{ij} and η_{ij} tensors

Response to arbitrary mean fields

$$\frac{\partial \mathbf{b}^{pq}}{\partial t} = \nabla \times \left(\overline{\mathbf{U}} \times \mathbf{b}^{pq} + \mathbf{u} \times \overline{\mathbf{B}}^{pq} + \mathbf{u} \times \mathbf{b}^{pq} - \overline{\mathbf{u} \times \mathbf{b}^{pq}} \right) + \eta \nabla^2 \mathbf{b}^{pq}$$

Calculate

$$\overline{\mathcal{E}}^{pq} = \overline{\mathbf{u} \times \mathbf{b}^{pq}}$$

$$\overline{\mathcal{E}}_j^{pq} = \alpha_{ij} \overline{B}_j^{pq} + \eta_{ijk} \overline{B}_{j,k}^{pq}$$

$$\overline{\mathbf{B}}^{11} = \begin{pmatrix} \cos kz \\ 0 \\ 0 \end{pmatrix}, \quad \overline{\mathbf{B}}^{21} = \begin{pmatrix} \sin kz \\ 0 \\ 0 \end{pmatrix}, \quad \ldots$$

Example:

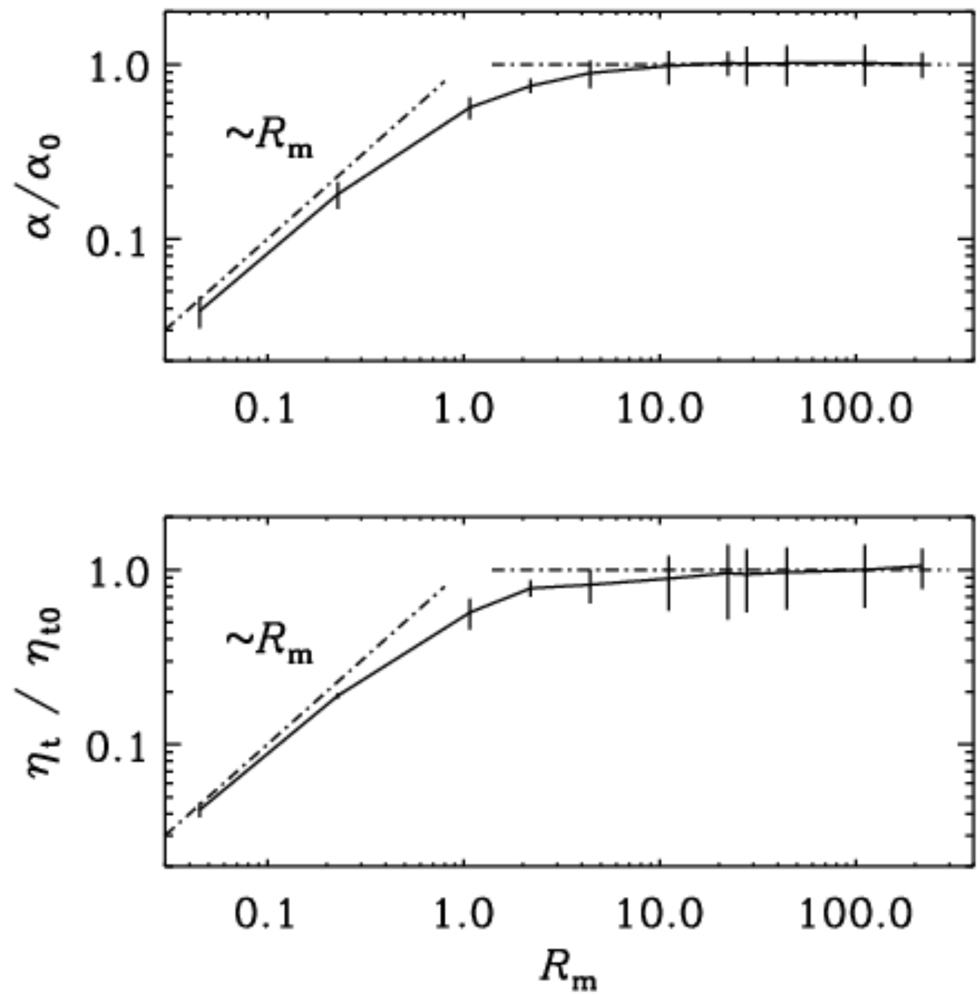
$$\overline{\mathcal{E}}_1^{11} = \alpha_{11} \cos kz - \eta_{113} k \sin kz$$

$$\overline{\mathcal{E}}_1^{21} = \alpha_{11} \sin kz + \eta_{113} k \cos kz$$

$$\begin{pmatrix} \alpha_{11} \\ \eta_{113} k \end{pmatrix} = \begin{pmatrix} \cos kz & \sin kz \\ -\sin kz & \cos kz \end{pmatrix} \begin{pmatrix} \overline{\mathcal{E}}_1^{11} \\ \overline{\mathcal{E}}_1^{21} \end{pmatrix}$$

$$\begin{pmatrix} \eta_{11}^* & \eta_{12}^* \\ \eta_{21}^* & \eta_{22}^* \end{pmatrix} = \begin{pmatrix} \eta_{123} & -\eta_{113} \\ \eta_{223} & -\eta_{213} \end{pmatrix}$$

Kinematic α and η_t independent of Rm (2…200)



$$\alpha_0 = -\tfrac{1}{3}\tau\left\langle \mathbf{\grave{u}} \cdot \mathbf{u} \right\rangle$$

$$\eta_0 = \tfrac{1}{3}\tau\left\langle \mathbf{u}^2 \right\rangle$$

$$\tau = \left(u_{\mathrm{rms}} k_{\mathrm{f}}\right)^{-1}$$

$$\alpha_0 = -\tfrac{1}{3} u_{\mathrm{rms}}$$

$$\eta_0 = \tfrac{1}{3} u_{\mathrm{rms}} k_{\mathrm{f}}^{-1}$$

Sur et al. (2008, MNRAS)

From linear to nonlinear

$$\frac{\partial \boldsymbol{U}}{\partial t} = -\boldsymbol{U}\cdot\boldsymbol{\nabla}\boldsymbol{U} - c_s^2\boldsymbol{\nabla}\ln\rho + \boldsymbol{f} + \rho^{-1}\left(\boxed{\boldsymbol{J}\times\boldsymbol{B}} + \boldsymbol{\nabla}\cdot 2\rho\nu\mathbf{S}\right),$$

$$\frac{\partial \ln\rho}{\partial t} = -\boldsymbol{U}\cdot\boldsymbol{\nabla}\ln\rho - \boldsymbol{\nabla}\cdot\boldsymbol{U},$$

$$\boxed{\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{U}\times\boldsymbol{B} - \mu_0\eta\boldsymbol{J},}$$

$$\frac{\partial \boldsymbol{a}^{pq}}{\partial t} = \overline{\boldsymbol{U}}\times\boldsymbol{b}^{pq} + \boldsymbol{u}\times\overline{\boldsymbol{B}}^{pq} + \boldsymbol{u}\times\boldsymbol{b}^{pq} - \overline{\boldsymbol{u}\times\boldsymbol{b}^{pq}} - \mu_0\eta\boldsymbol{j}^{pq},$$

Mean and fluctuating
U enter separately

$$\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}$$

Use vector potential

$$\mathbf{B} = \nabla\times\mathbf{A}$$

$$\mathbf{b}^{pq} = \nabla\times\mathbf{a}^{pq}$$

Nonlinear α_{ij} and η_{ij} tensors

$$\alpha_{ij} = \alpha_1 \delta_{ij} + \alpha_2 \hat{B}_i \hat{B}_j$$

$$\eta_{ij} = \eta_1 \delta_{ij} + \eta_2 \hat{B}_i \hat{B}_j$$

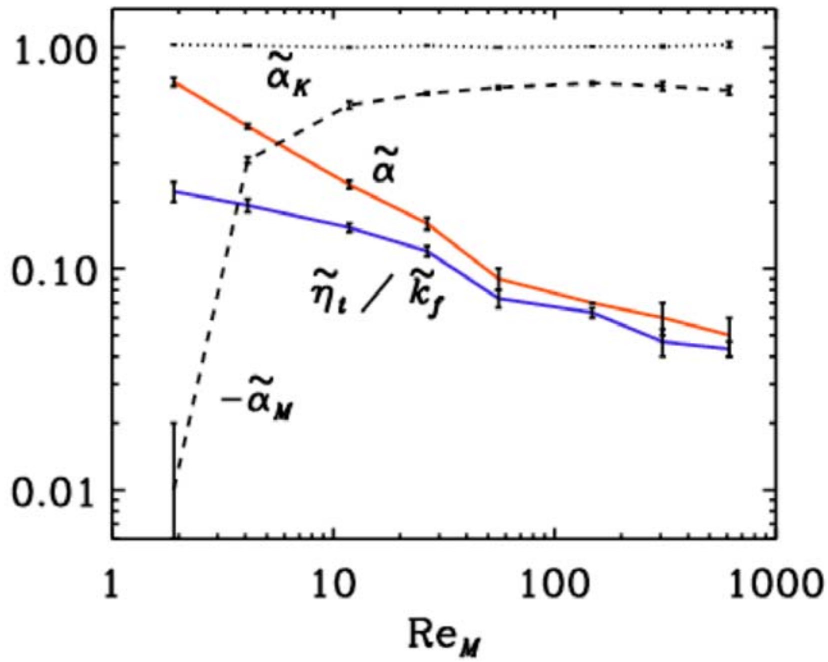
Consider steady state to avoid dα/dt terms

Expect:

$$\begin{aligned} \lambda &= \alpha k_1 - (\eta + \eta_t) k_1^2 \\ &= (\alpha_1 + \alpha_2) k_1 - (\eta + \eta_1 + \eta_2) k_1^2 \\ &= 0 \end{aligned}$$

λ=0 (within error bars) → consistency check!

R_m dependence for $B \sim B_{eq}$

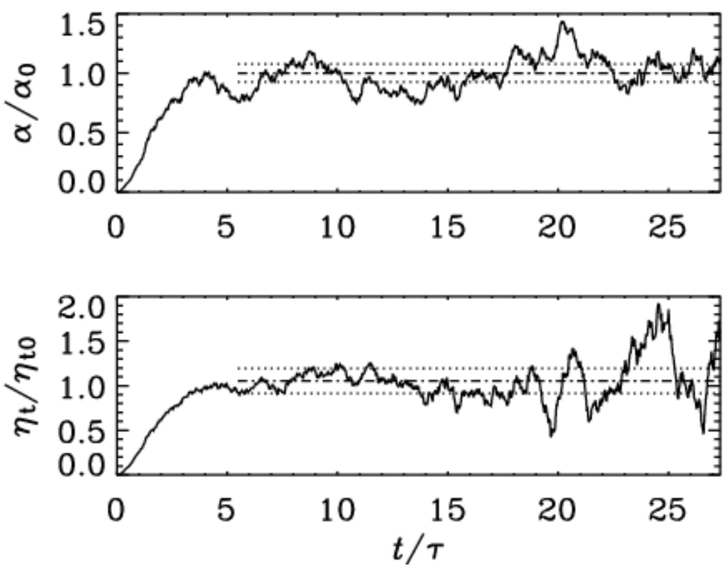


(i) λ is small → consistency
(ii) α_1 and α_2 tend to cancel
(iii) making α small
(iv) η_2 small

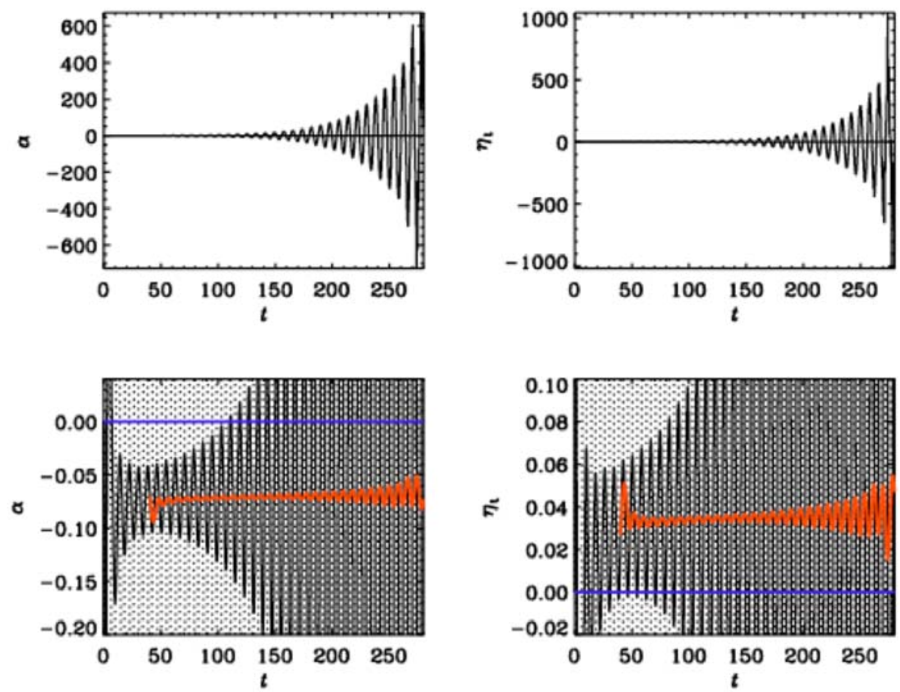
Run	Re_M	$\tilde B^2$	$\tilde b^2$	$\tilde\alpha$	$\tilde\eta_t$	$\tilde\eta$	$\tilde\lambda$	$-\tilde\alpha_2$	$-\tilde\eta_2$	$\tilde\alpha_{rms}$	$\tilde\eta_{rms}$	$+\tilde\alpha_K$	$-\tilde\alpha_M$	$\Delta\tilde t$
A	2	0.0	0.0	0.70 ± 0.03	0.67 ± 0.07	1.57	-0.14 ± 0.01	0.04 ± 0.05	-0.02 ± 0.06	0.09	0.12	1.03	0.01	150
B	4	0.9	0.4	0.44 ± 0.01	0.58 ± 0.04	0.73	0.00 ± 0.00	0.33 ± 0.02	-0.11 ± 0.03	0.10	0.21	1.02	0.31	422
C	12	1.7	0.7	0.24 ± 0.01	0.46 ± 0.02	0.25	0.00 ± 0.00	0.37 ± 0.02	-0.04 ± 0.01	0.09	0.16	1.00	0.55	601
D	30	1.9	0.8	0.16 ± 0.01	0.36 ± 0.02	0.11	-0.00 ± 0.01	0.37 ± 0.02	0.03 ± 0.03	0.07	0.14	1.02	0.62	350
E	60	2.0	0.8	0.09 ± 0.01	0.22 ± 0.02	0.05	0.00 ± 0.01	0.33 ± 0.01	0.05 ± 0.01	0.09	0.22	1.00	0.66	711
F	150	2.0	0.9	0.07 ± 0.00	0.19 ± 0.01	0.02	0.01 ± 0.01	0.24 ± 0.05	0.08 ± 0.01	0.07	0.16	1.01	0.69	225
G	300	1.8	0.9	0.06 ± 0.00	0.15 ± 0.00	0.01	0.01 ± 0.01	0.21 ± 0.02	0.05 ± 0.02	0.06	0.16	1.01	0.66	177
H	600	1.8	0.9	0.05 ± 0.01	0.13 ± 0.01	0.005	0.01 ± 0.04	0.14 ± 0.05	0.04 ± 0.01	0.05	0.10	1.03	0.64	44

Cooperation with small-scale dynamo

At large Rm α and η_t develop divergent fluctuations,
but the running mean is well defined



turbulence



Roberts flow

time averaging and regular restarting of b^{pq} required