Magnetic Field Intensification and Small-scale Dynamo Action in Compressible Convection

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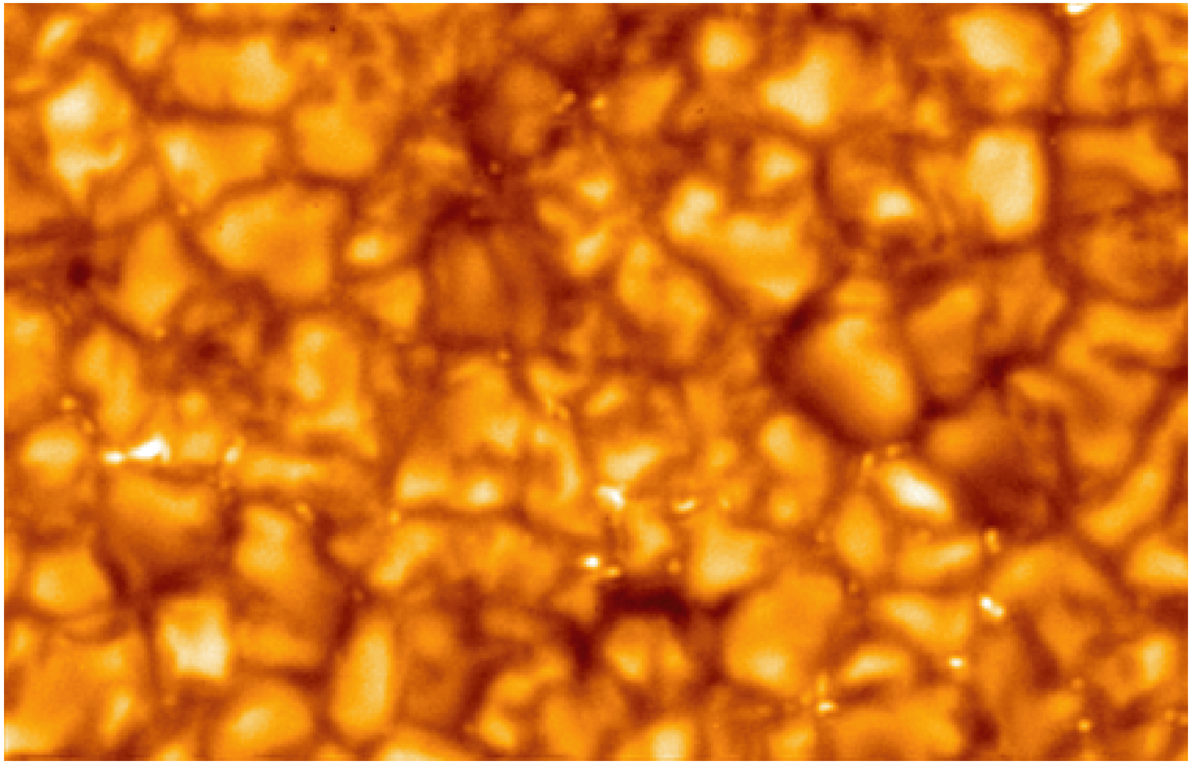
Observations of the solar photosphere

G Band image of a quiet region of the solar photosphere (Hinode).

Bright points correspond to regions of strong magnetic fields



- Intergranular lanes in the quiet Sun region often contain localised concentrations of magnetic flux (mixed polarities). Field strengths often exceed a kilogauss.

- What is the origin of these magnetic features?

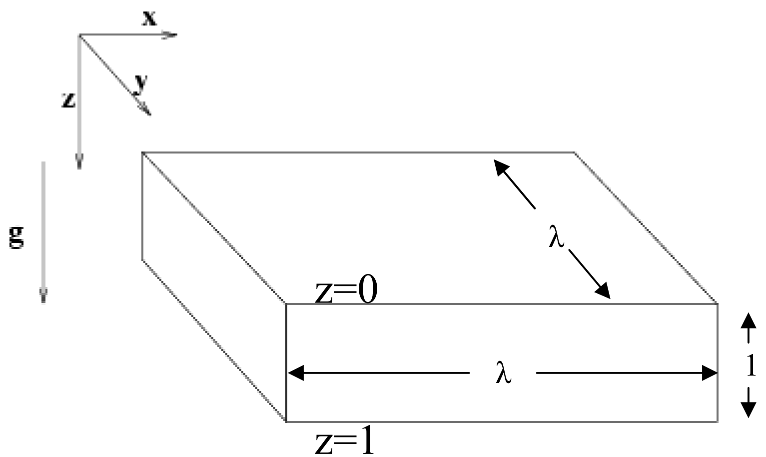
Model setup: governing equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \qquad P = \mathcal{R} \rho T$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla P + \rho g \hat{\mathbf{z}} + \frac{1}{\mu_0}(\nabla \times \mathbf{B}) \times \mathbf{B} + \mu \left[\nabla^2 \mathbf{u} + \frac{1}{3}\nabla(\nabla \cdot \mathbf{u})\right]$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \qquad \nabla \cdot \mathbf{B} = 0$$

$$\rho c_{\mathrm{v}} \left[\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T\right] = -P \nabla \cdot \mathbf{u} + K \nabla^2 T + Q_\nu + Q_\eta$$



Initially: Fully-developed hydrodynamic convection. Density and temperature vary by an order of magnitude across the layer.

$$\mathbf{B} = B_0(x, y)\hat{\mathbf{z}}$$

A horizontally-periodic Cartesian domain of unit depth (λ typically 4 or 8)

Upper and lower boundaries**:** Impermeable, stress-free, vertical field, fixed T

Model setup (cont.)

Numerical method (Direct numerical simulation)

• Mixed finite-difference/pseudo-spectral scheme

• Horizontal derivatives evaluated in Fourier space

• Fourth order finite differences (either upwinded or centred, as appropriate) are used to calculate vertical derivatives

• Typical computational meshes use 256/512 points in each horizontal direction and > 100 points vertically

• Code parallelised using MPI

Key Parameters: (Photospheric estimates given in brackets)

Rayleigh number:	$Ra = 4 \times 10^5 \sim 300 Ra_{crit}$	(10^{16})
Reynolds number:	$Re \sim 150$	(10^{12})
Mag. Reynolds number:	$Rm \sim 60 - 700$	(10^6)
Prandtl number:	$\sigma = 1$	(10^{-7})
Mag. Prandtl number:	$Pm \sim 0.4 - 4.7$	(10^{-6})

Numerical results: Convective intensification

Initial magnetic field: $\mathbf{B} = B_0\hat{\mathbf{z}}$

B_0 is constant and small: $E_m \approx 0.002E_k$

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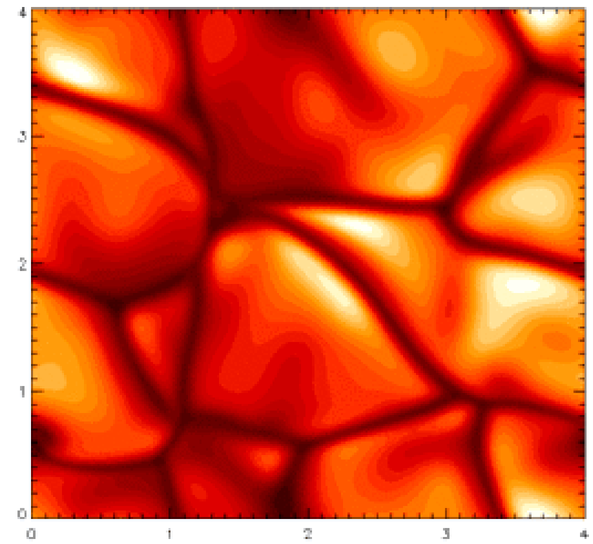
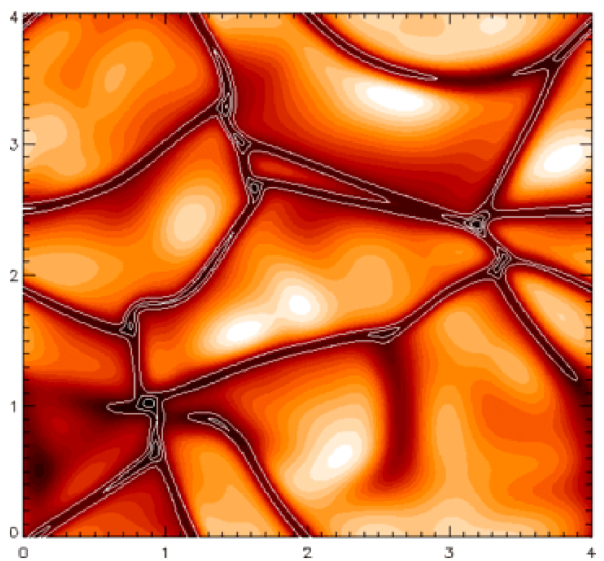
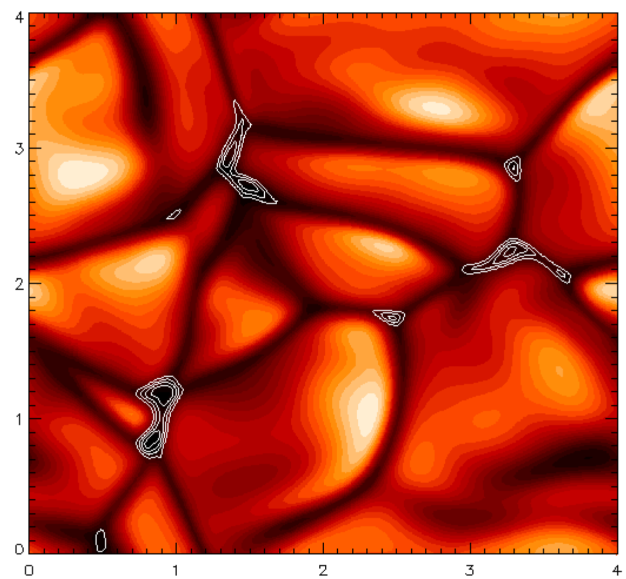


$Rm \sim 120 \quad Re \sim 150$

Early phase of evolution: **Flux expulsion** (e.g. Proctor and Weiss 1982). Leads to the accumulation of magnetic flux in the convective downflows…..





Contours of Bz overlaid upon contours of constant temperature, in a plane just below the upper surface

Far left: t=0.12

Left: t=1.61

Numerical results: Convective intensification (cont.)

Partial evacuation: High magnetic pressure inhibits converging convective flows - convective downflows drain fluid away from the upper layers of the magnetic feature. Leads to partial evacuation…..

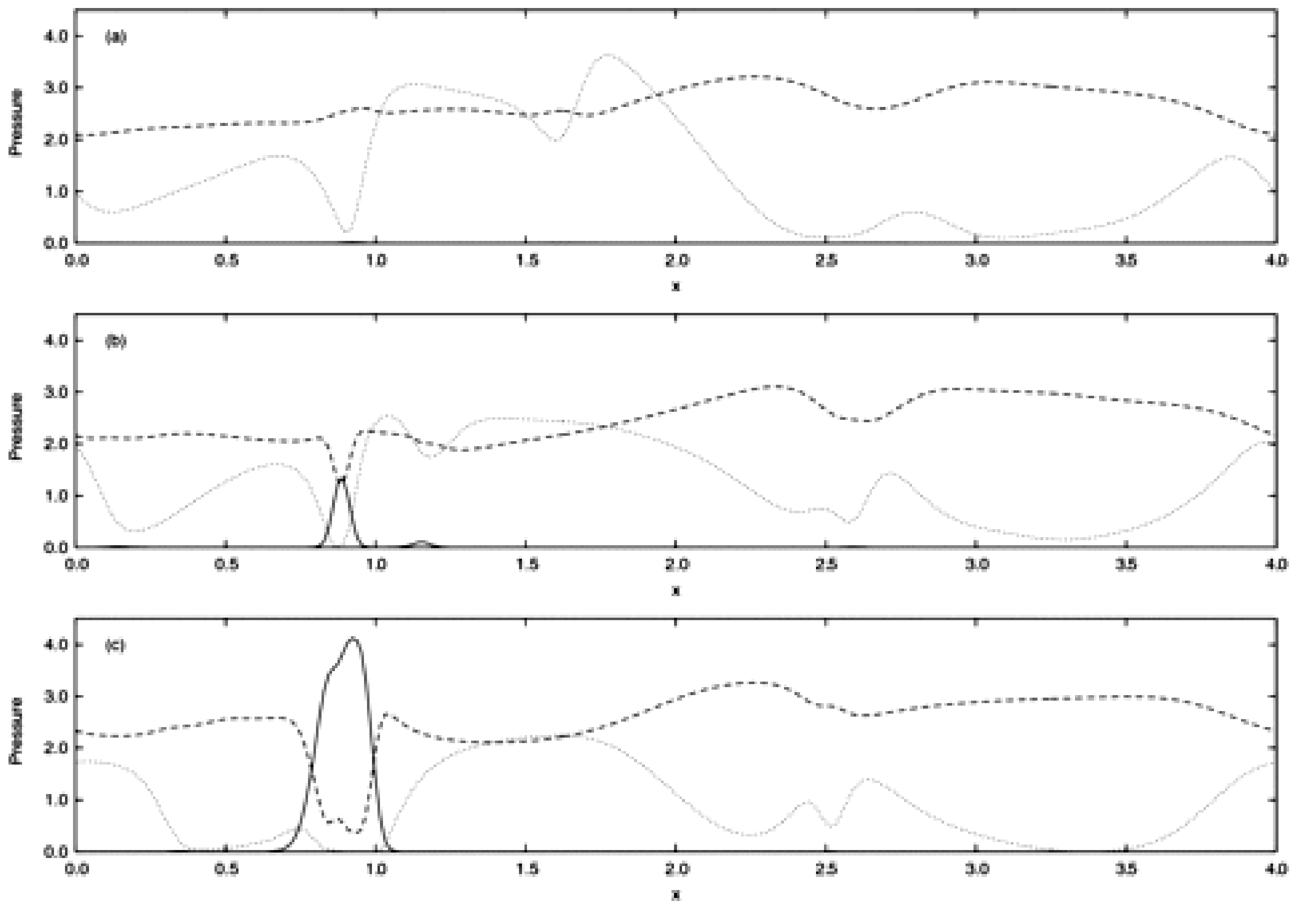
Right: Pressure distributions along a horizontal cut through the top of a magnetic feature at t=0.12 (top), t=0.61 (middle), t=1.61 (bottom).

Solid line: P_mag

Dashed line: P_gas

Dotted line: P_dyn

Note high P_mag…



(B et al. 2008 MNRAS)

Numerical results: Convective intensification (cont.)

• Partial evacuation due to convective downflows important for field intensification - local magnetic energy density exceeds mean kinetic energy density. Similar to "convective collapse" scenario (e.g. Parker 1978; Spruit 1979 and others..). Also seen in other numerical studies… (e.g. Vögler et al. 2005; Stein & Nordlund 2006…)

• This process could explain the appearance of kilogauss-strength magnetic features at the solar photosphere (very difficult without partial evacuation).

However…..

This process of partial evacuation → important implications for numerical scheme…

Alfvén speed, $V_A \sim \frac{B_o}{\sqrt{\rho}}$ →

Coefficient of thermal diffusion $\sim \frac{\kappa}{\rho}$ →

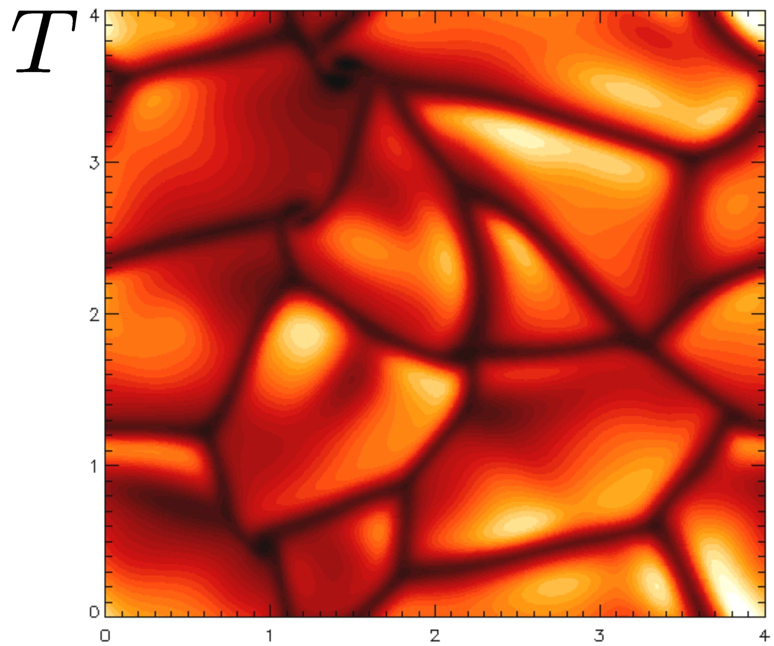
Both become very large in partially-evacuated concentrations of magnetic flux

The time-scales associated with thermal diffusion and alfvénic disturbances therefore become very small → critical time-step for the stability of the (explicit) numerical scheme becomes very small…. → Significantly increased runtime!

Numerical results: Small-scale dynamos

What happens if there is no net flux across the domain?

$$\mathrm{B} = \epsilon \cos(2\pi x/\lambda) \cos(2\pi y/\lambda)\hat{\mathbf{z}}$$



Key Parameter: $$Rm = \frac{U_{rms} d}{\eta}$$

Magnetic Reynolds number must be large enough that inductive effects due to the flow outweigh magnetic diffusion. For a given flow, can vary Rm by varying η.....

Previous studies:

• Boussinesq convection (Cattaneo 1999 + follow-up papers)

• Compressible convection - less well understood, although see LES simulations by Vögler & Schüssler (2007) and calculations by Brummell (and collaborators) in the weakly superadiabatically-stratified regime.

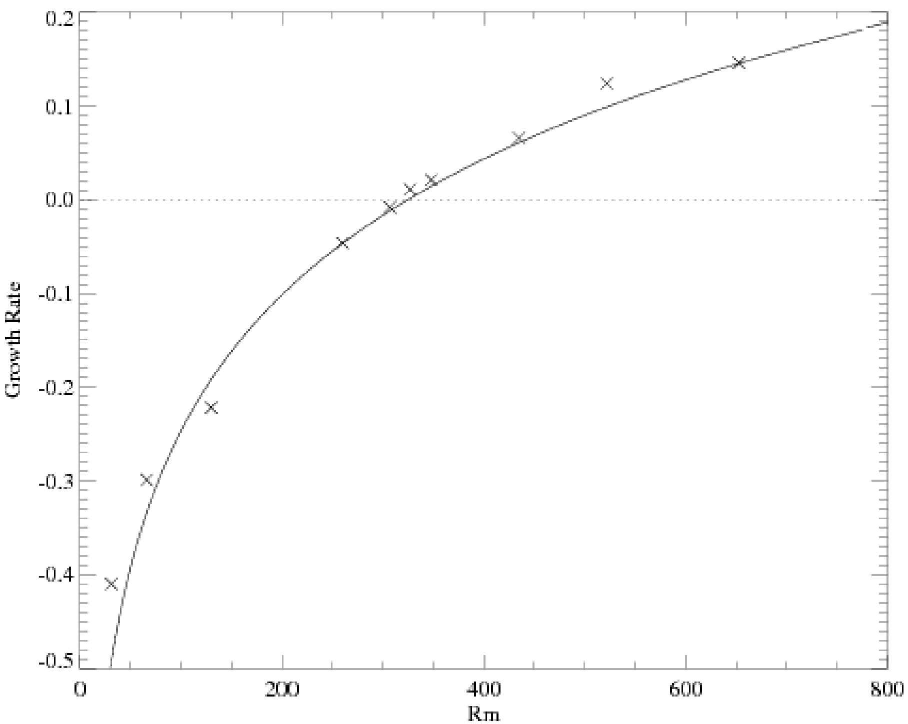
Numerical results: Small-scale dynamos (cont.)

Focus initially upon the **kinematic** regime - no Lorentz feedback upon the flow.

Right: Dynamo growth rate as a function of Rm

$$Rm_{crit} \sim 325$$

Growth rate $\approx 0.21 \log\left(\frac{Rm}{Rm_{crit}}\right)$



Logarithmic fit for the growth rate similar to low Pm result of Rogachevski & Kleeorin (1997)

Magnetic Prandtl number: There has been some debate regarding the viability of forced small-scale dynamos at low magnetic Prandtl number (e.g. Boldyrev & Cattaneo 2004; Schekochihin et al. 2005), although Iskakov et al (2007) may have recently resolved this issue in favour of low Pm dynamos…..

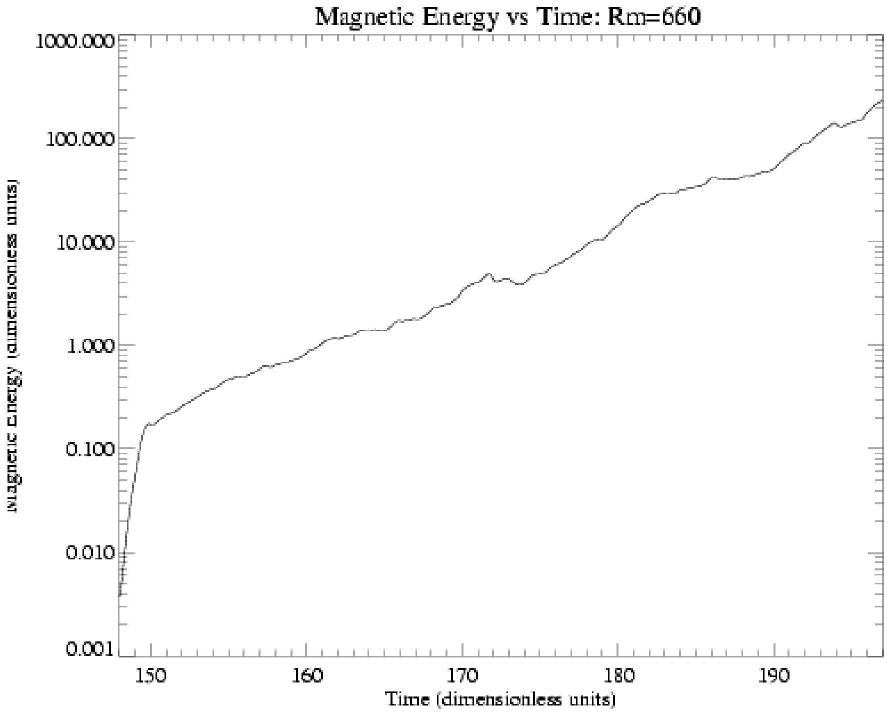
For this set of parameters: $Pm \sim 2$ when $Rm \sim Rm_{crit}$

Numerical results: Small-scale dynamos (cont.)

A kinematic dynamo: Turn off Lorentz force. Depending on Rm, magnetic energy either grows or decays exponentially

$$\lambda = 4 \quad Rm \sim 660 \quad Re \sim 150$$

Numerical resolution: $256 \times 256 \times 160$



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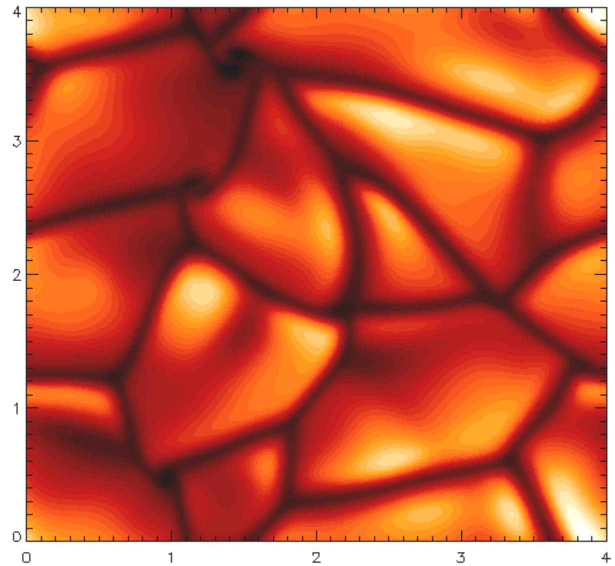
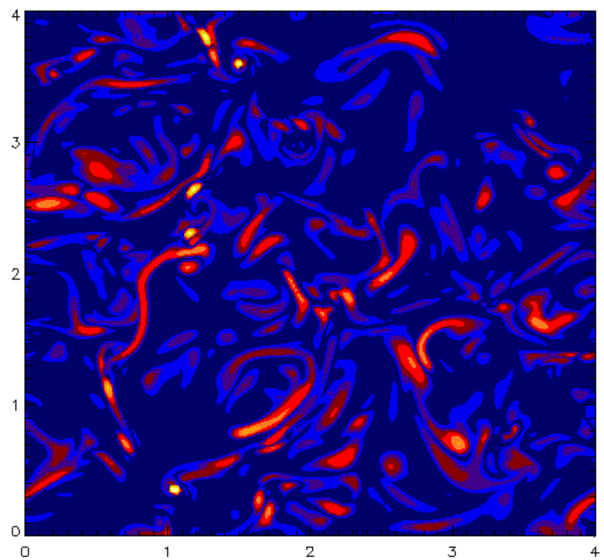


B^2



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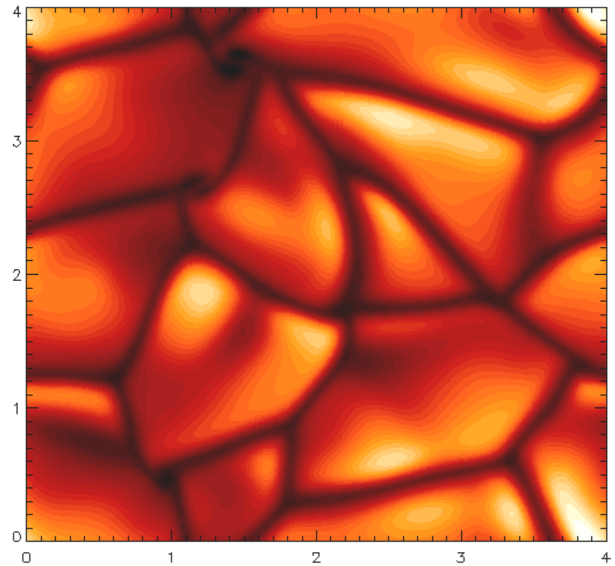
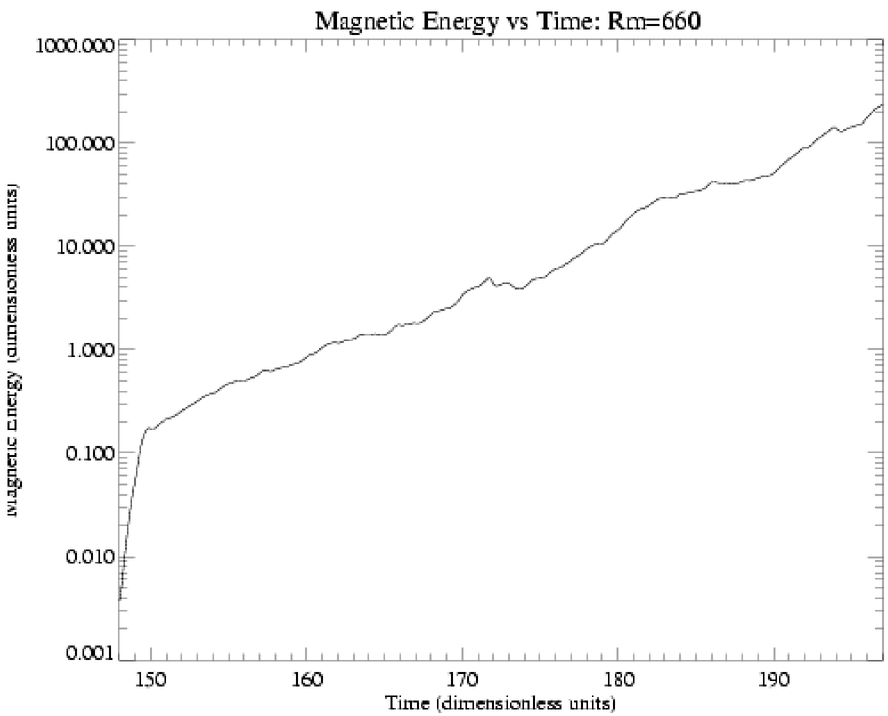
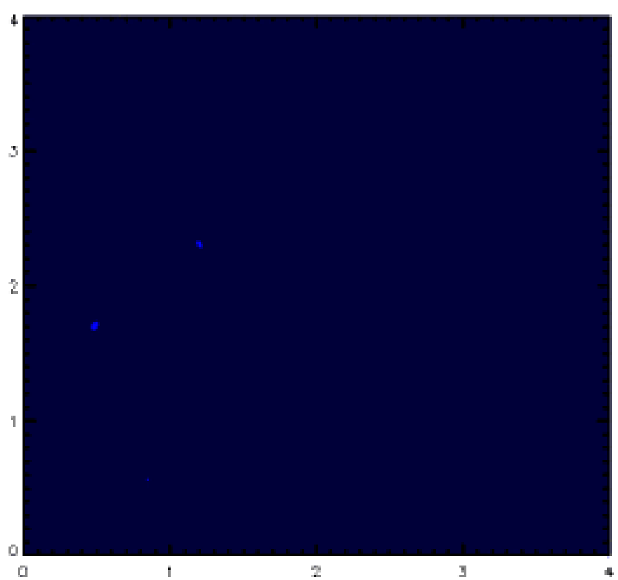




B^2



Numerical results: Small-scale dynamos (cont.)

Effect of increasing the box size: What happens to these kinematic dynamos if we increase the aspect ratio from λ=4 to λ=8?

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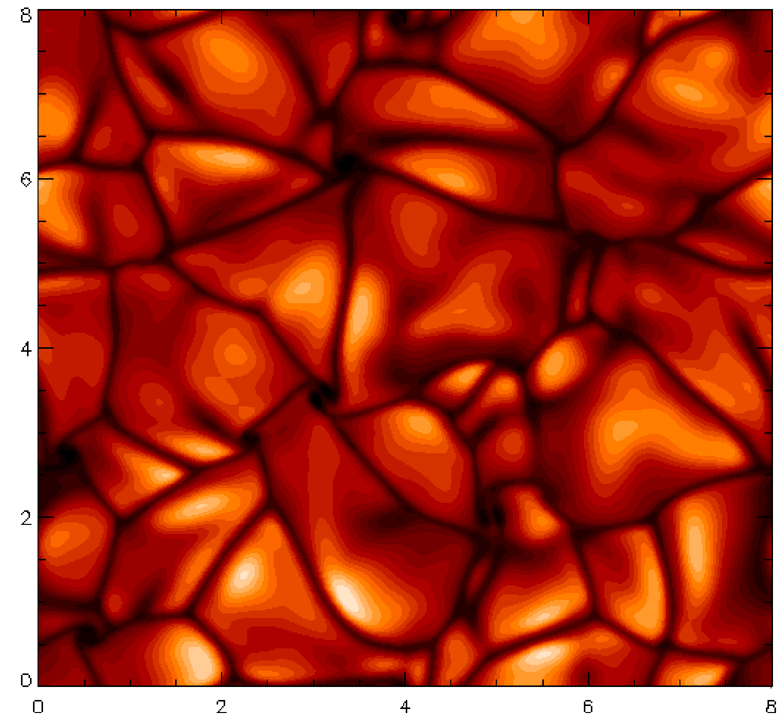
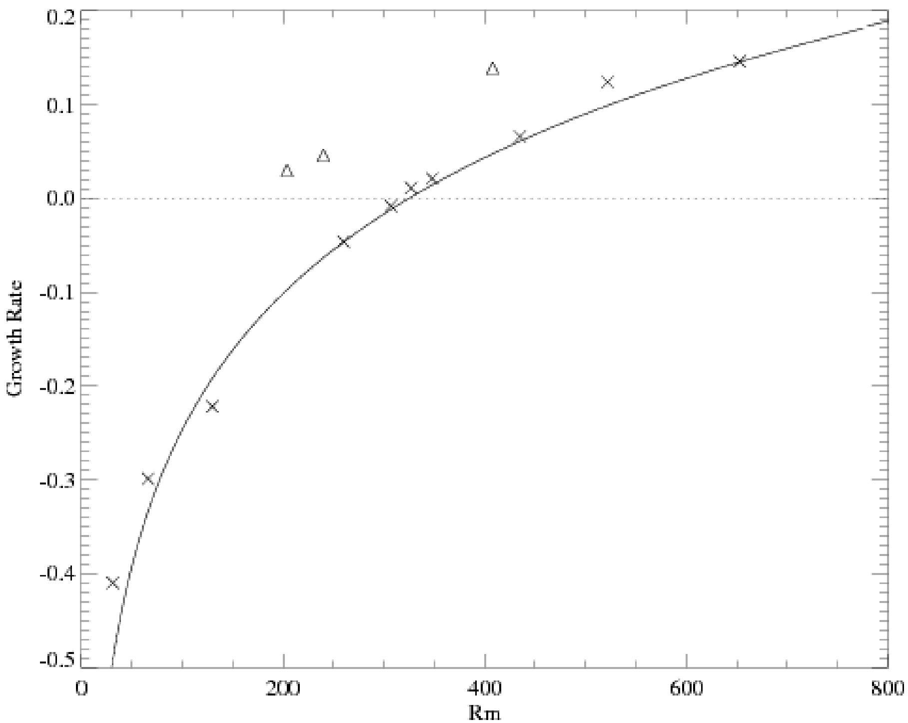


- Higher growth rates
- Lower Rm_{crit}



Periodic boundary conditions for smaller box artificially increases the mixing of opposing magnetic polarities - lower growth rate for the kinematic dynamo

Numerical results: Small-scale dynamos (cont.)

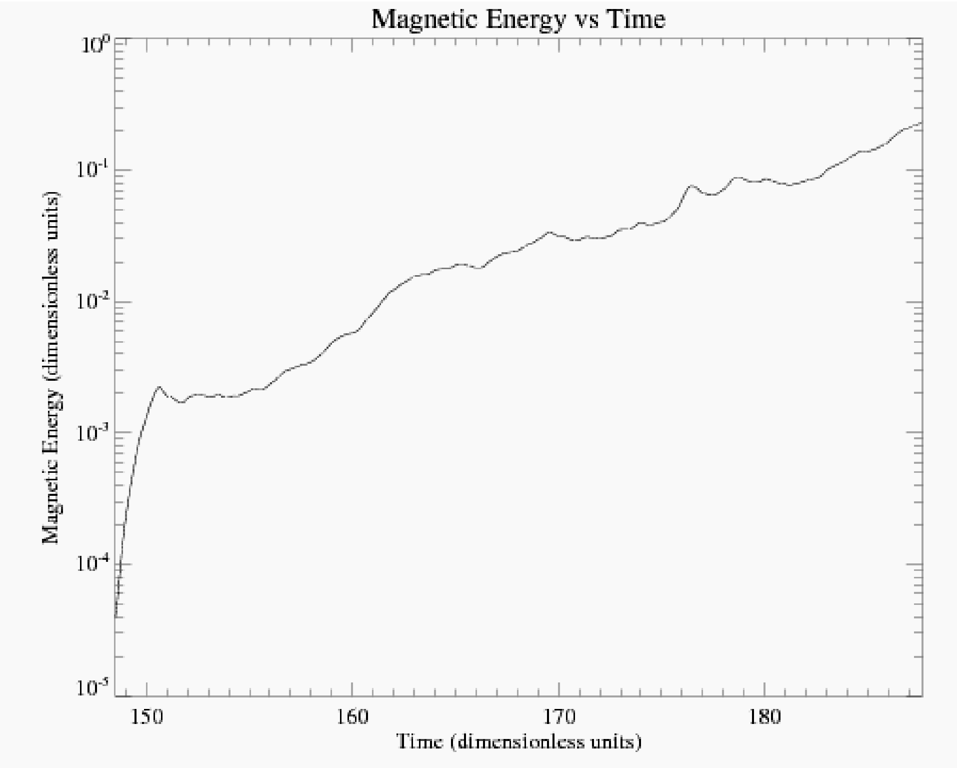
All the results so far are for **kinematic** dynamos - what about the nonlinear case?

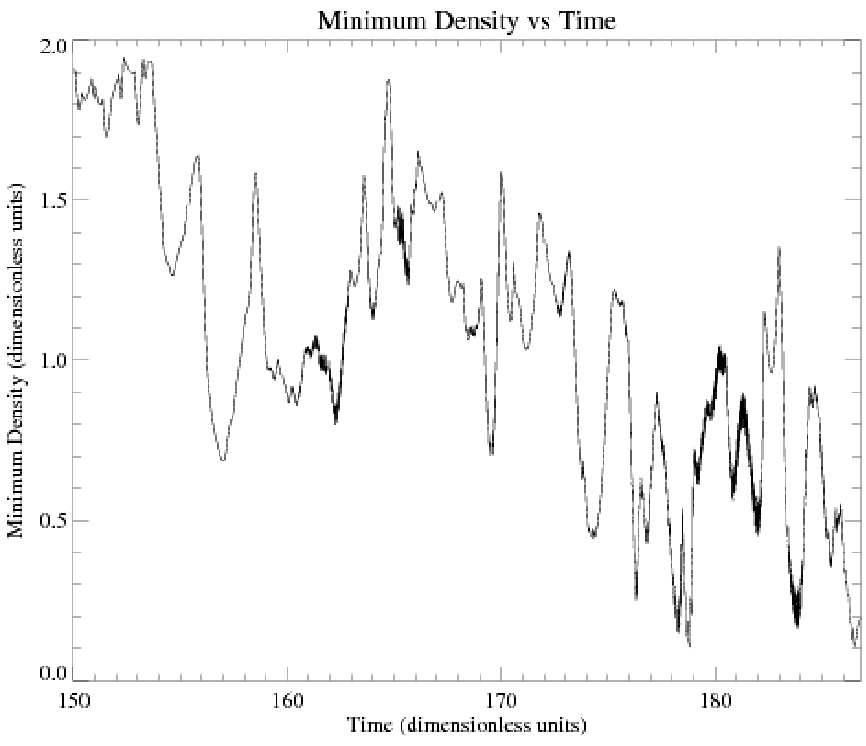
$$\lambda = 4 \quad Rm \sim 520 \quad Re \sim 150$$

Numerical resolution: $512 \times 512 \times 160$

Below left: Magnetic energy against time

Below right: Minimum density against time

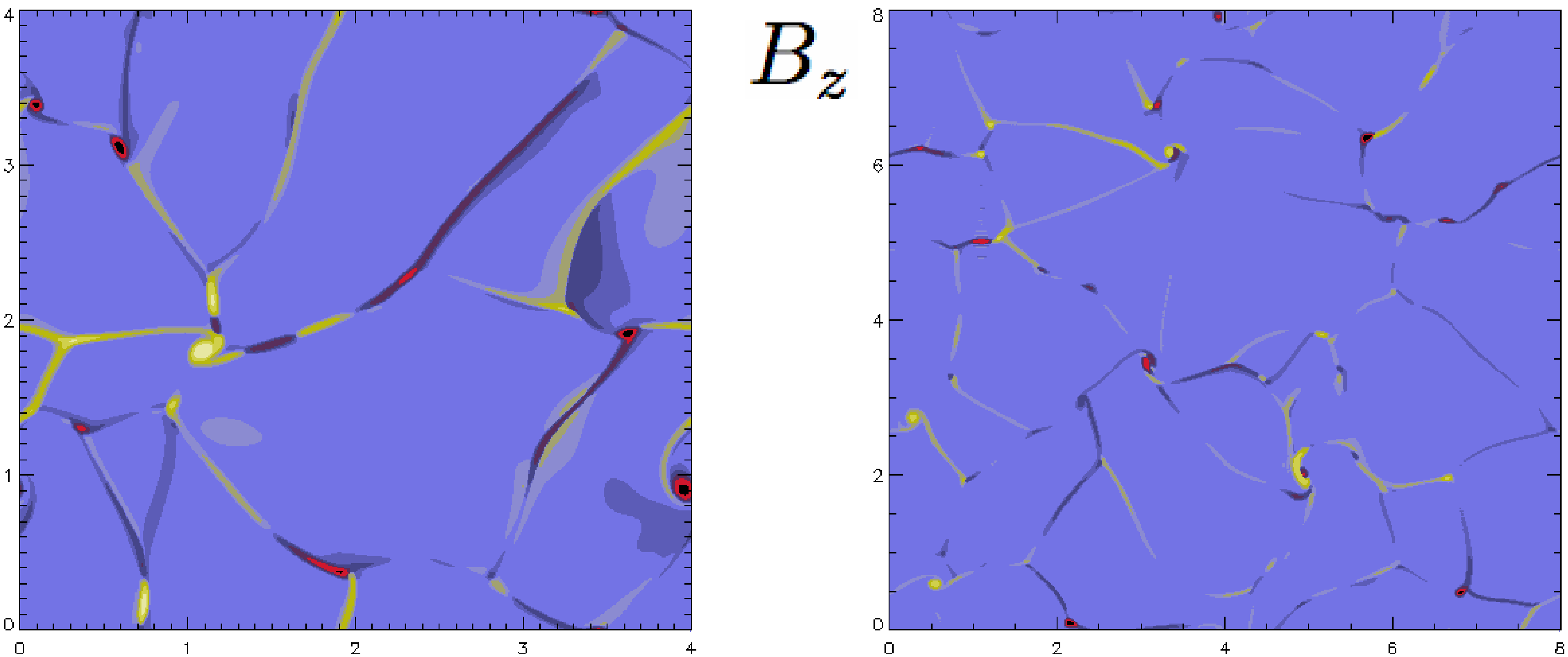




Numerical results: Small-scale dynamos (cont.)

• Magnetic energy vs. time plot suggests that dynamo is still in the kinematic phase. However, the minimum density plot implies that the **localised** magnetic feedback is extremely significant

• Partial evacuation implies that the time-step gets **extremely** small - the calculation grinds to a halt!



$\lambda = 4, Rm \sim 520$ $\lambda = 8, Rm \sim 240$

Summary and conclusions

• **Non-zero net flux:** Seed magnetic field can be amplified to super-equipartition field strengths by flux expulsion and subsequent convective intensification (“convective collapse”-like process). The surrounding convection has a significant confining influence upon the magnetic region.

• **No net flux:** At modest Re, relatively modest values of Rm are needed in order to drive a convective dynamo (Pm of order 2). The kinematic growth rate of the dynamo apparently has a logarithmic dependence upon Rm. Larger boxes appear lead to larger kinematic growth rates and lower critical Rm. Local nonlinear effects rapidly lead to the partial evacuation of the magnetic regions, even whilst the global magnetic energy is still growing exponentially….

Problems…..

• Time-stepping **extremely** (prohibitively!) expensive when partially evacuated regions form - possibly calls for AMR, artificial density “floor”, (semi)-implicit treatment of offending terms?

• Unrealistic parameter regime for the solar photosphere….