Self sustaining cycles in accretion discs MHD turbulence

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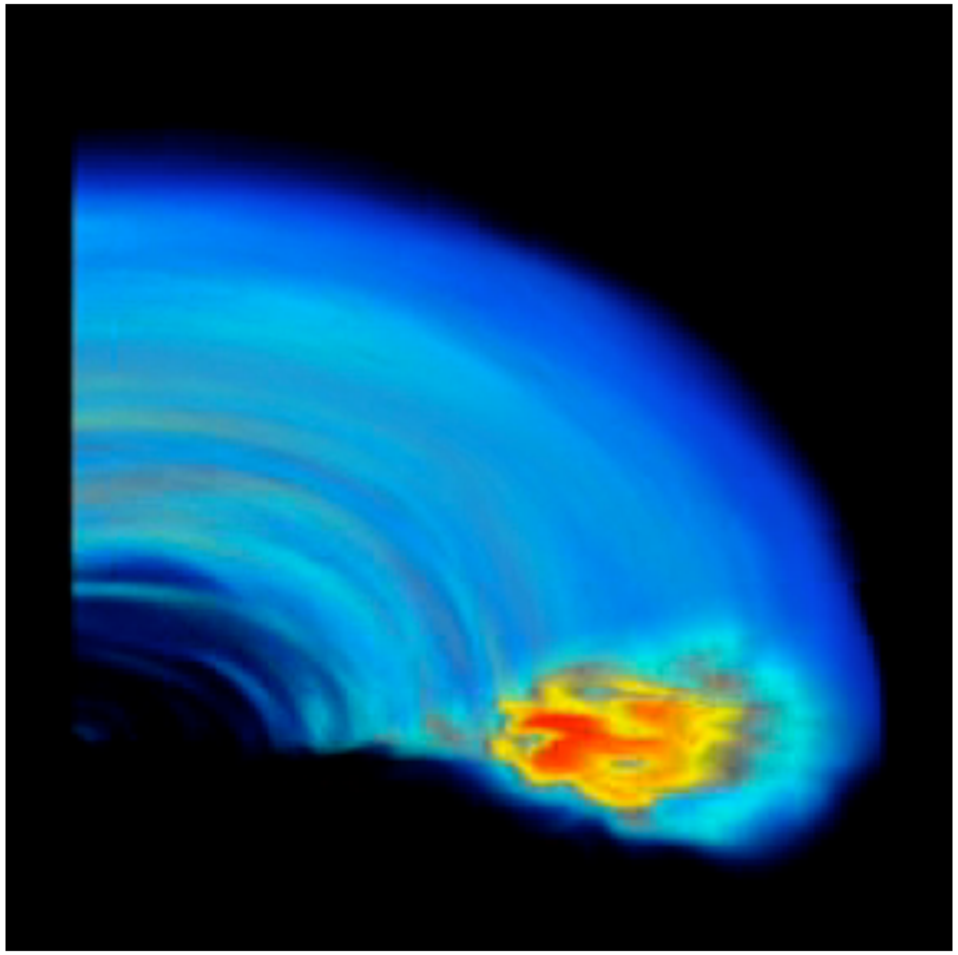
MHD Turbulence in discs

The magnetorotational instability (MRI) provides an efficient source of turbulence in accretion discs.

- Leads to a “strong” turbulent transport of angular momentum



Hawley & Balbus (2002)

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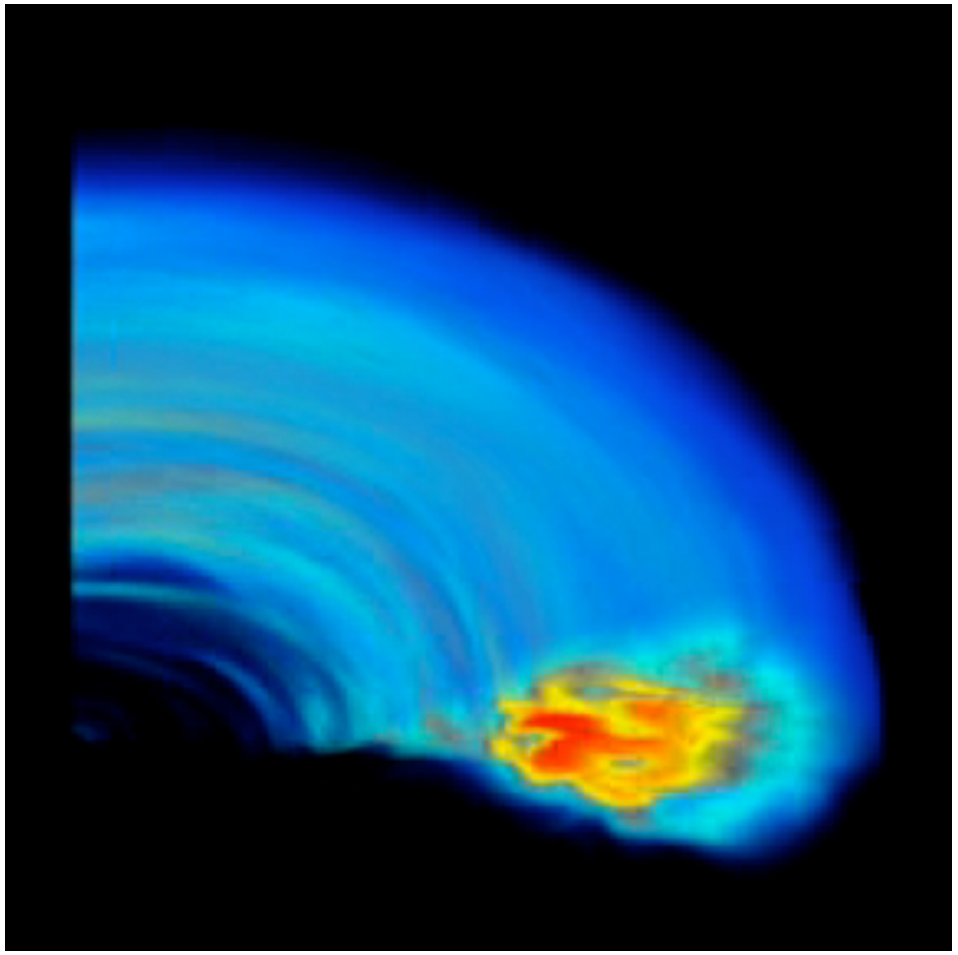

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The magnetorotational instability (MRI) provides an efficient source of turbulence in accretion discs.

- Leads to a "strong" turbulent transport of angular momentum

However

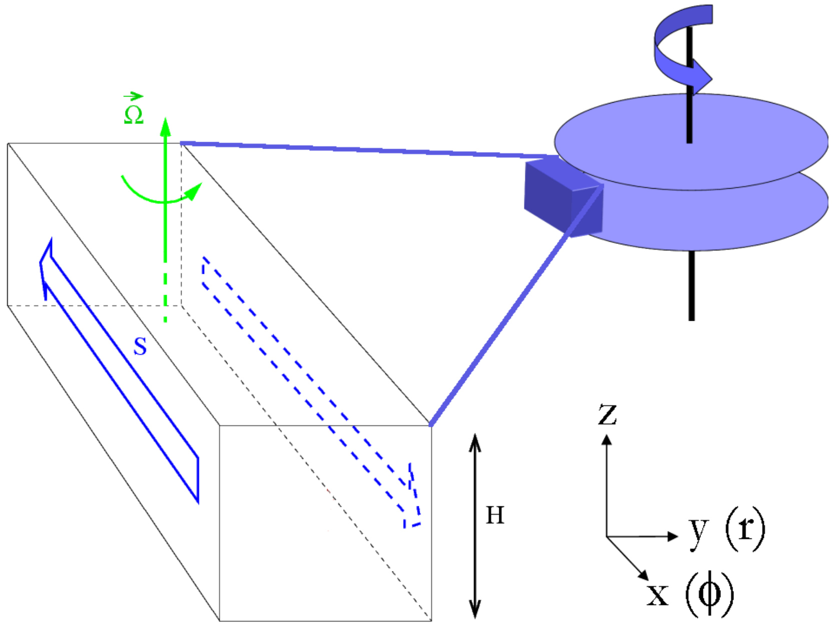
MHD turbulence can also be a source of large scale magnetic fields (numerous examples in this conference)

- Useful to understand disc winds & jets collimation
- Useful for star-disc interaction

Can we find & describe a "disc dynamo"?

Local model & Numerical method

$$
\begin{aligned}
\partial_t \mathbf{u} + Sy\partial_x \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} &= -\nabla\psi - 2\Omega u_x \mathbf{e_y} + (2\Omega - S)u_y \mathbf{e_x} \\
&\quad + \mathbf{B}\cdot\nabla\mathbf{B} + \nu\Delta\mathbf{u} \\
\partial_t \mathbf{B} + Sy\partial_x \mathbf{B} &= SB_y \mathbf{e_x} + \nabla\times(\mathbf{u}\times\mathbf{B}) + \eta\Delta\mathbf{B} \\
\nabla\cdot\mathbf{u} &= 0 \\
\nabla\cdot\mathbf{B} &= 0
\end{aligned}
$$



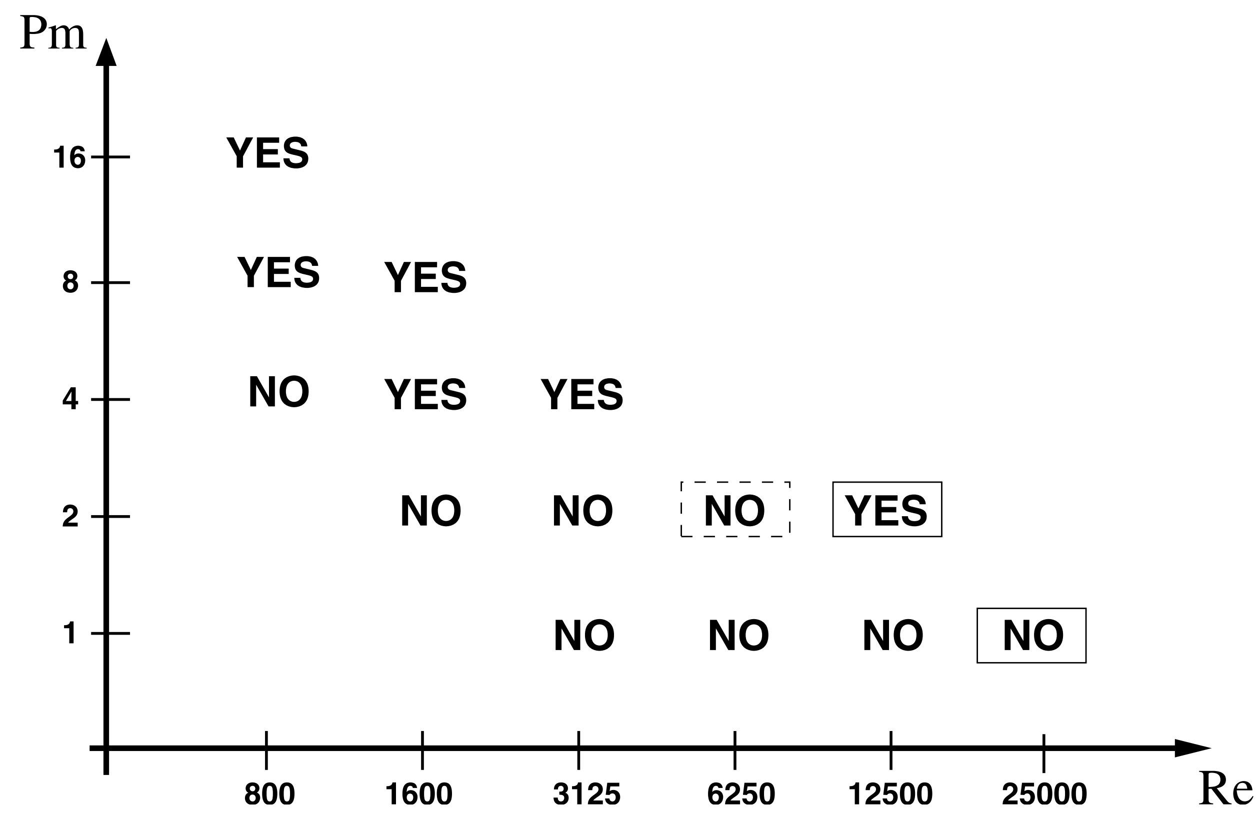
Local model (shearing box)

- Rotating sheared flow
- Incompressible approximation
- Periodic (Φ,z) and shear-periodic (r) boundary conditions
- Numerics: 3D Spectral (Fourier) method with remap (e.g. Umurhan & Regev 2004)

Zero mean field boxes: open questions

Formally no linear instability for the case without a mean field

- How turbulence is maintained ?
- How do we explain/extend the Pm dependancy ?



Fromang et al. (2007)

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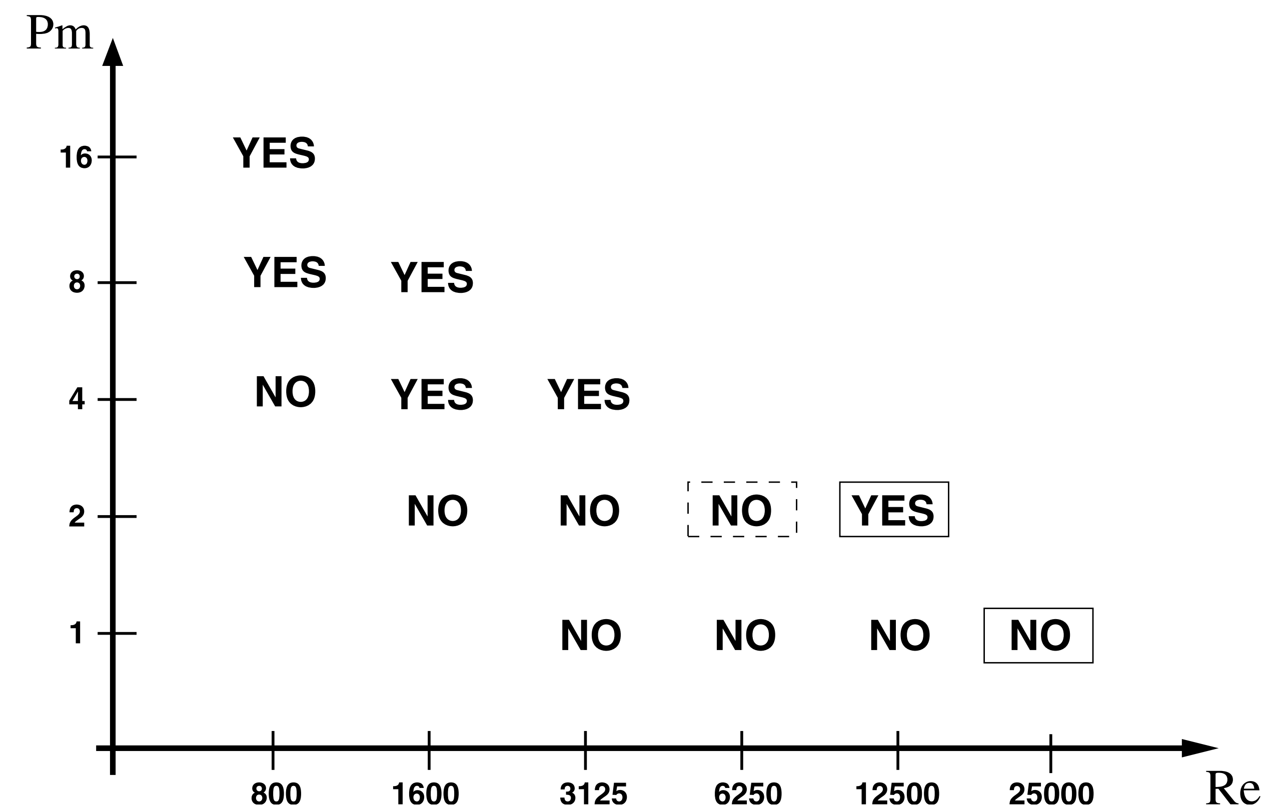
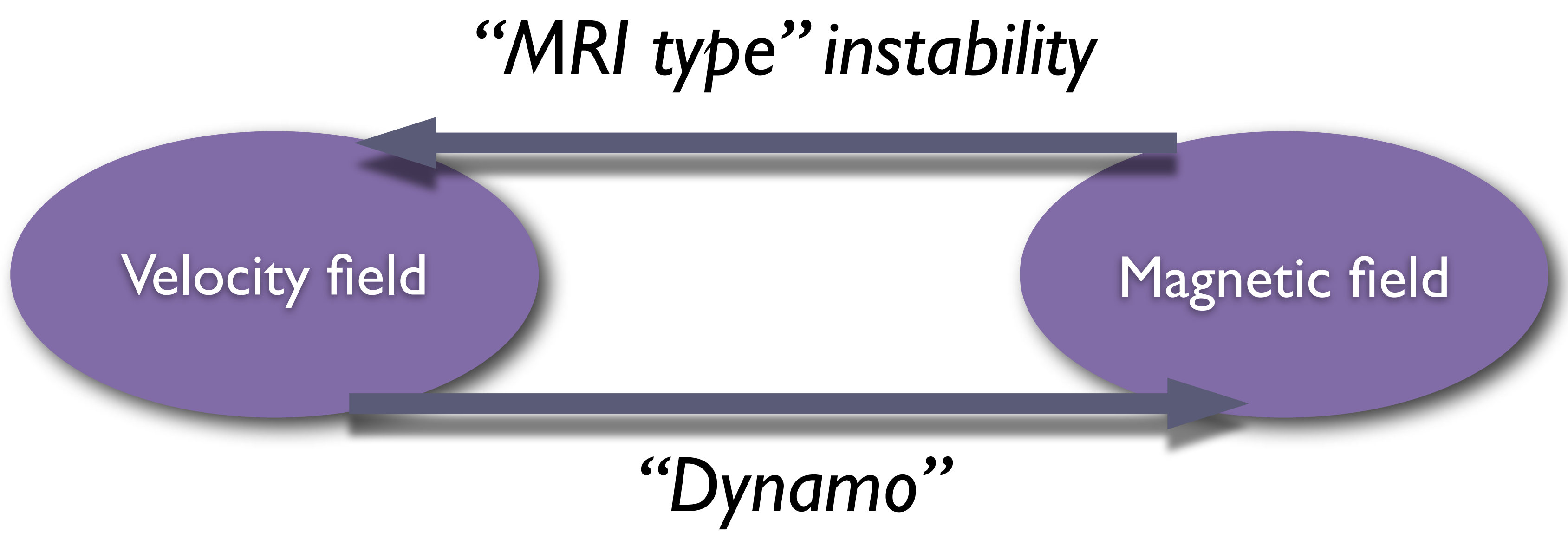


Fromang et al. (2007)

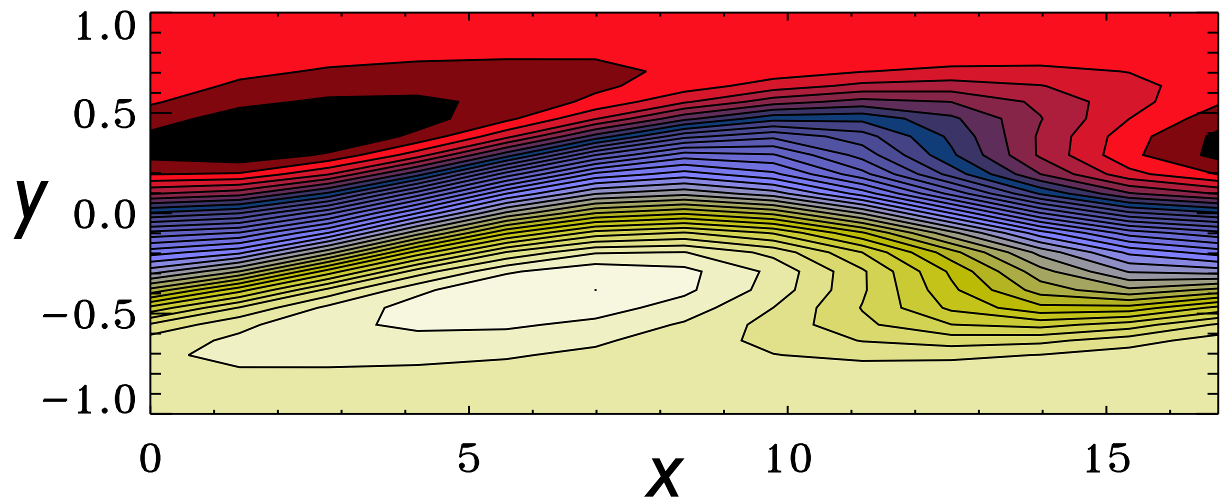
Phenomenological picture:



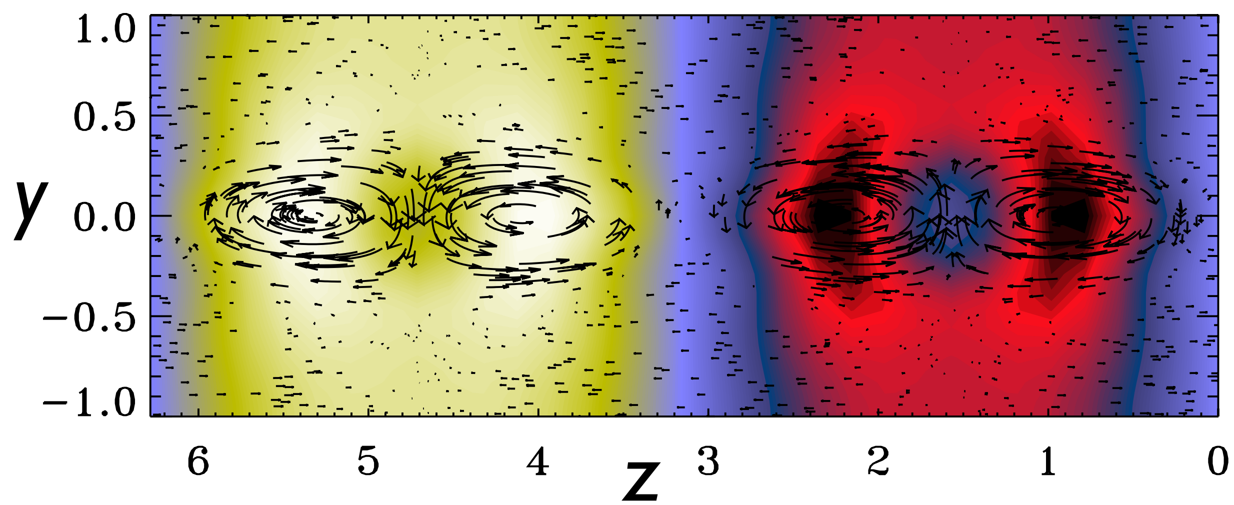
The quest for a non linear mechanism

Non linear steady solution (Rincon et al. 2007) in Couette flows (no slip, perfectly conducting walls in the y=r direction)

B_z plot in the (x,y) =(ϕ,r) plane



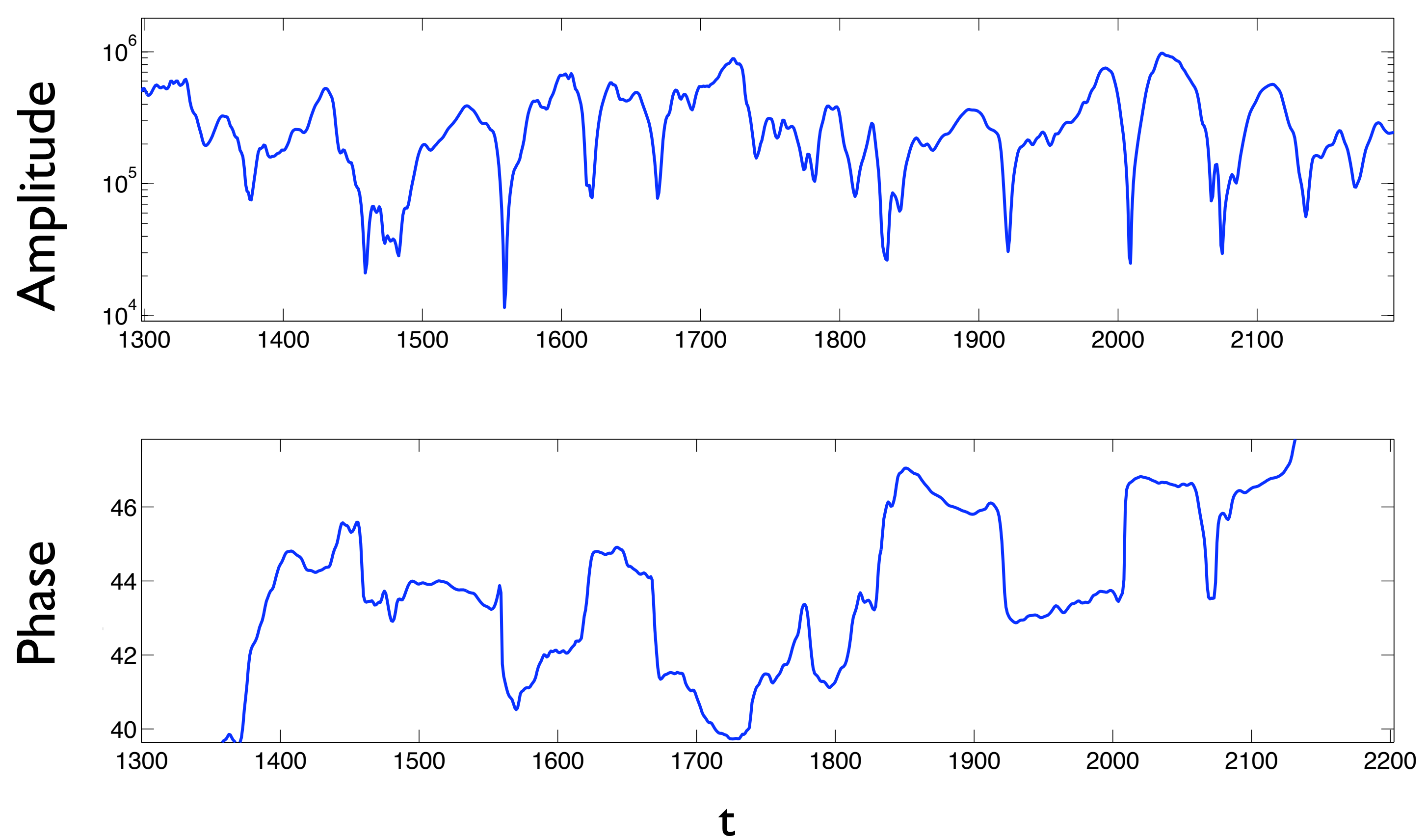
B plot in the (z,y) =(z,r) plane

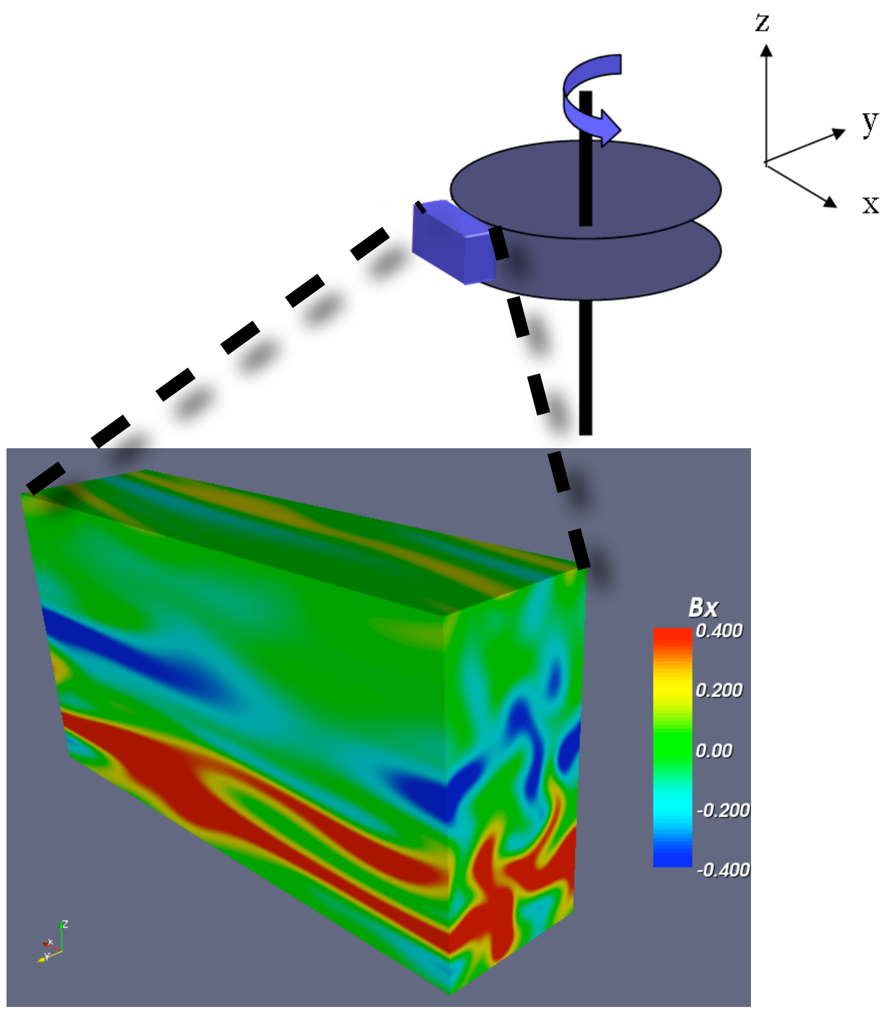


- Non axisymmetric solution
- Involves a large scale $B_\phi(z)$
- *Global solution, related to the presence of walls !*

An azimuthal field cycle in shearing boxes?

- Zero mean field shearing boxes simulations seem to show a strong azimuthal field with a vertical structure
- Fourier Analysis of $B_\phi(\mathbf{k_0} = \frac{2\pi}{L_z}\mathbf{e_z})$ shows regeneration cycles with T~50 S^{-1}

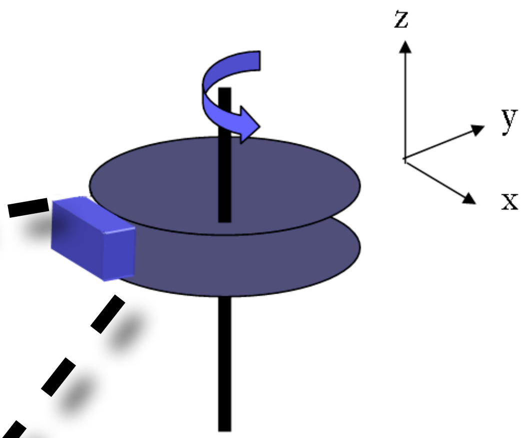


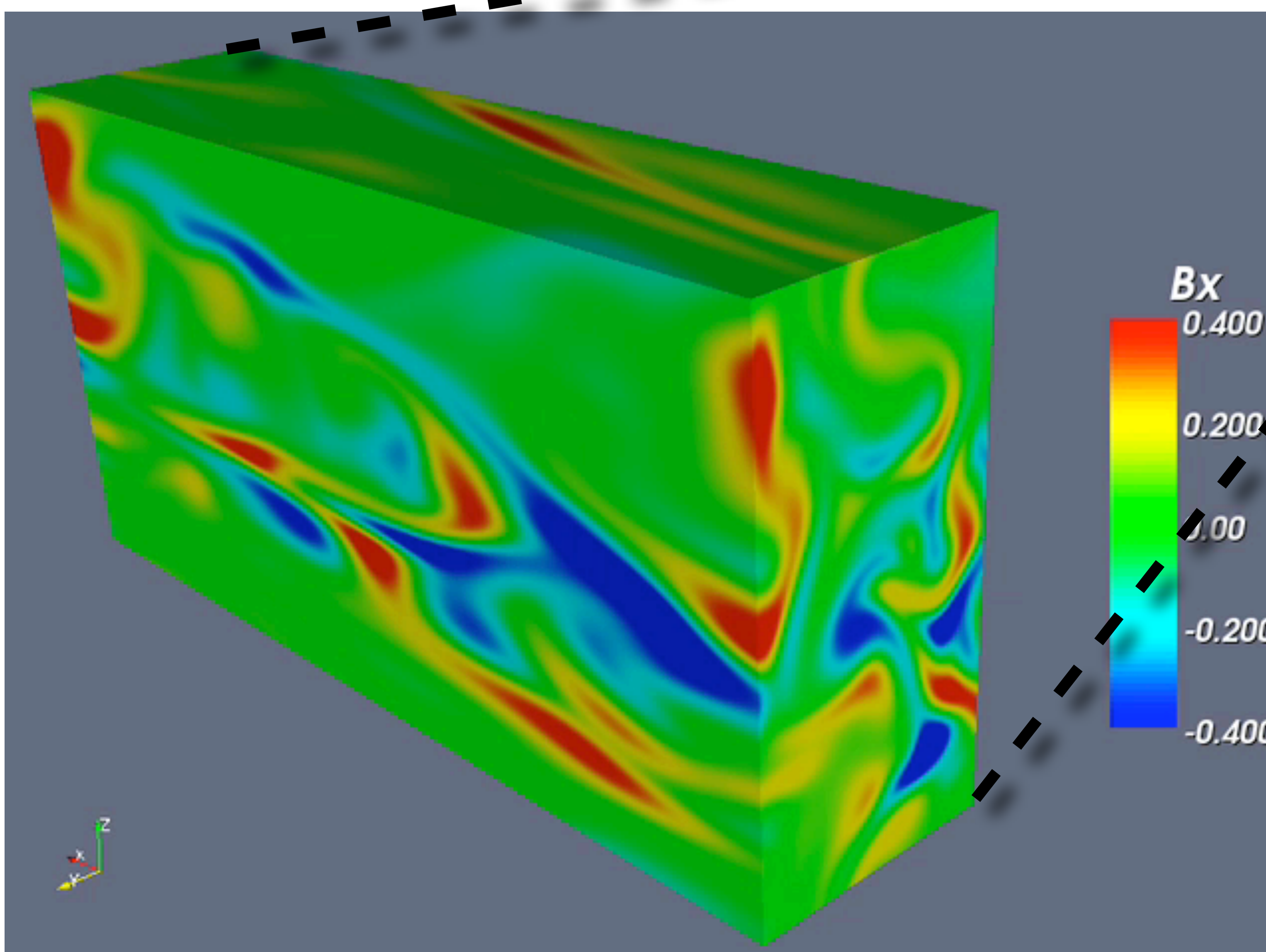


Behaviour during one cycle

Simulation in a "tall" box (L_z=$2L_r$)

Re=1600, Pm=4 for 50 S^{-1}





3D Plot of B_Φ

Studying one cycle...

- Budget for a pure exp(ik₀z) mode :

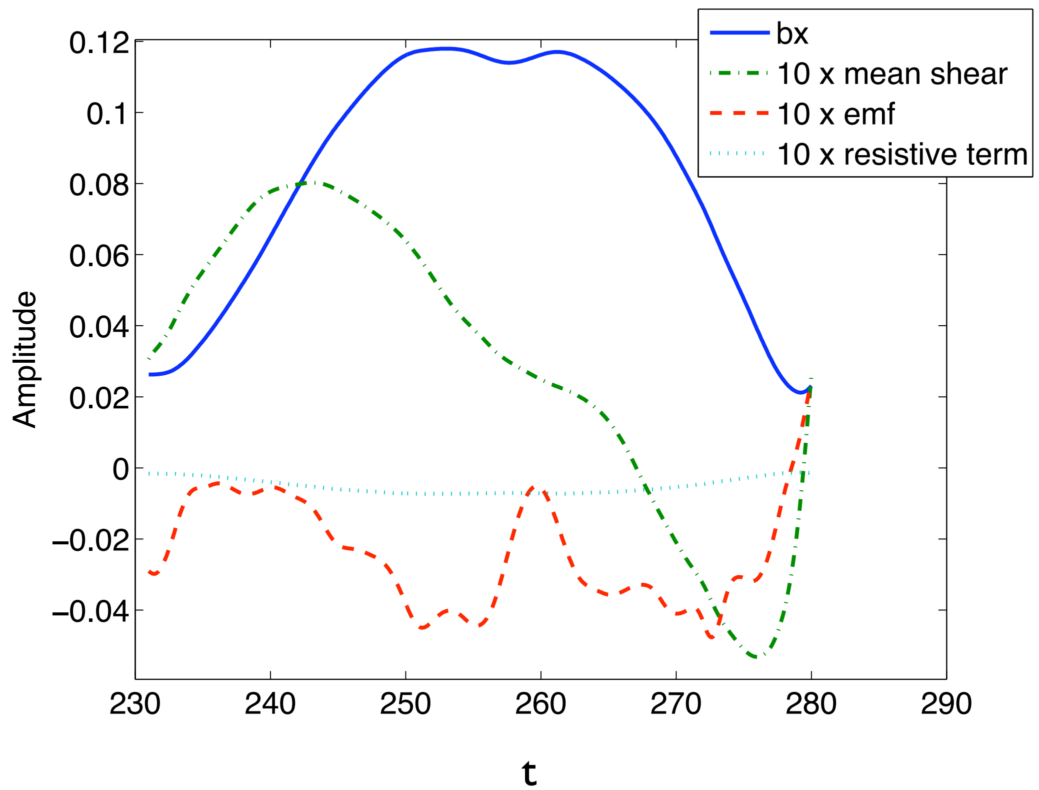
$$\frac{\partial \widehat{B}_\phi}{\partial t} = S\widehat{B}_r - ik_0\widehat{E}_r - \eta k_0^2 \widehat{B}_\phi$$

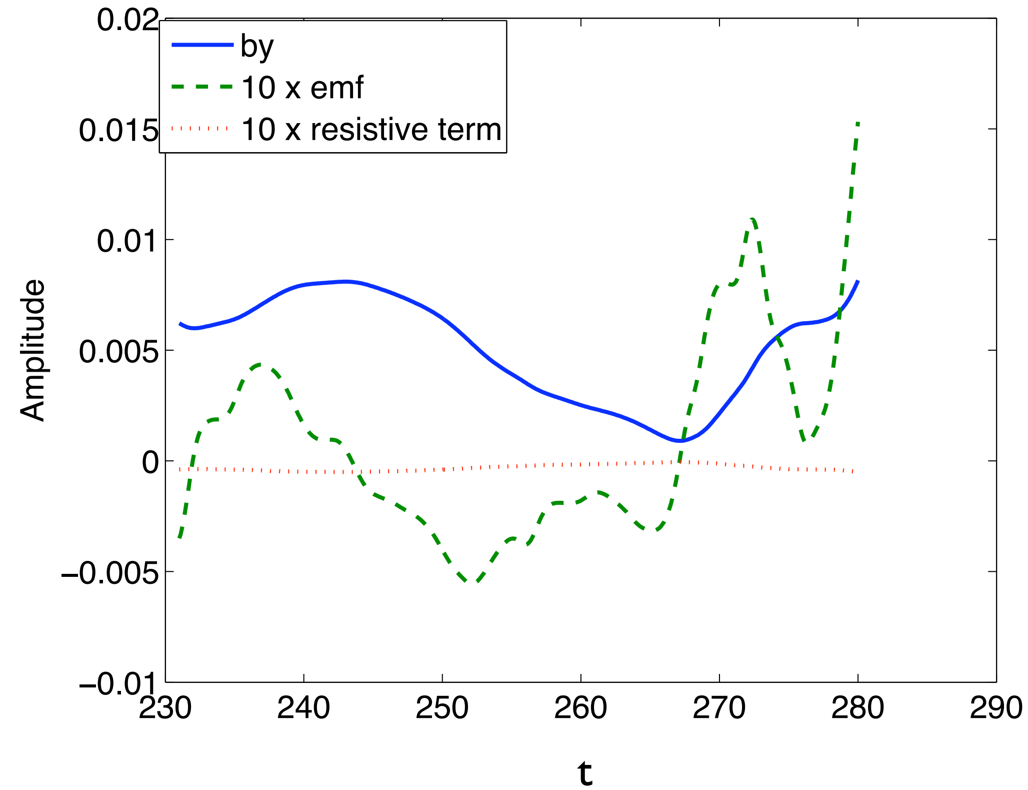
$$\frac{\partial \widehat{B}_r}{\partial t} = ik_0\widehat{E}_\phi - \eta k_0^2 B_r$$

with $\widehat{\mathbf{E}} = \overbrace{(\mathbf{u} \times \mathbf{B})}_{k_0}$

Involves coupling between various other modes

One B_ϕ cycle from a numerical simulation

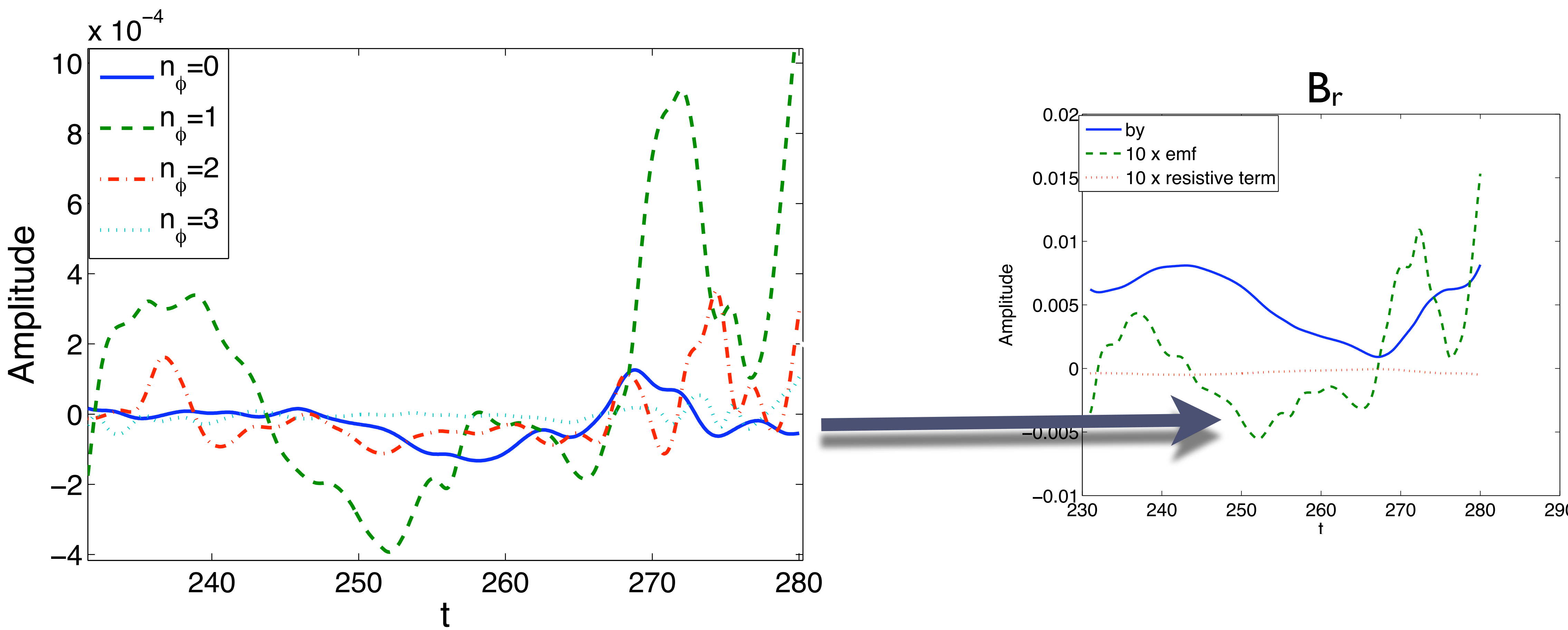




- E_r acts always as a resistive term
- E_ϕ is responsible for the cyclic behaviour

Non axisymmetric origin of the emfs

- Which waves contribute to the EMFs?
- Contribution of non axisymmetric wave numbers n_ϕ to E_ϕ



- E_ϕ comes from the coupling of the *largest* non axisymmetric modes
- Same conclusion for E_r (not shown)

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- The "largest" linear shearing waves are written:

$$\begin{aligned} \mathbf{u} &= \bar{\mathbf{u}}(z)\exp[i(k_\phi x + k_r(t) y)] \\ \mathbf{b} &= \bar{\mathbf{b}}(z)\exp[i(k_\phi x + k_r(t) y)] \end{aligned} \quad \textit{with} \quad \begin{aligned} k_\phi &= 2\pi/L_\phi \\ k_r &= -Stk_\phi \end{aligned}$$

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- To quantify the "quasi linear" feedback, we compute the correlations

$$\begin{aligned} \Gamma_\phi &= \int B_\phi(-\partial_z E_r)\,dz \\ \Gamma_r &= \int B_\phi \partial_z E_\phi\,dz \end{aligned}$$

Having in mind the large scale field equations :

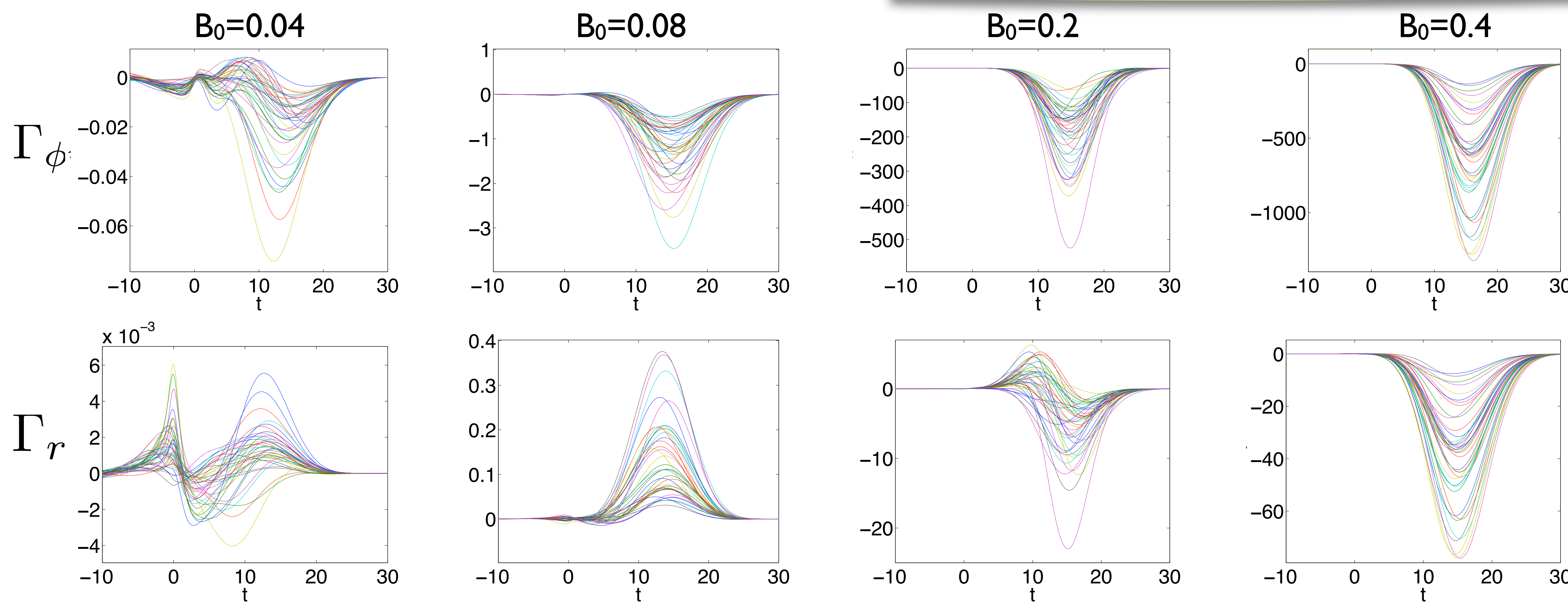
$$\begin{aligned} \partial_t B_\phi &= SB_r - \partial_z E_r \\ \partial_t B_r &= \partial_z E_\phi \end{aligned}$$

Shearing waves linear response (II)

IT'S MRI!
Transient amplification

Result from a linear computation with a ra

$$\mathbf{B} = B_0 \cos(k_0 z)\mathbf{e}_\phi$$



- The shearing waves always have a resistive effect on B_ϕ
- The correlation Γ_r get reversed for strong enough B_ϕ

A Toy model

- We assume a "turbulent resistivity" closure model for the emfs, following the linear properties of the shearing waves

$$\begin{aligned}\partial_t B_\phi(t) &= S B_r(t) - \beta k_0^2 B_\phi(t-t_L)\\ \partial_t B_r(t) &= \gamma k_0^2 B_\phi(t-t_L)\end{aligned}$$

with

$$\gamma = \gamma_0 \left[1 - \frac{|B_\phi(t-t_L)|}{B_{\mathrm{Rev}}}\right]$$

Lag time $t_L \sim S^{-1}$

$B_{Rev} \sim 0.1$

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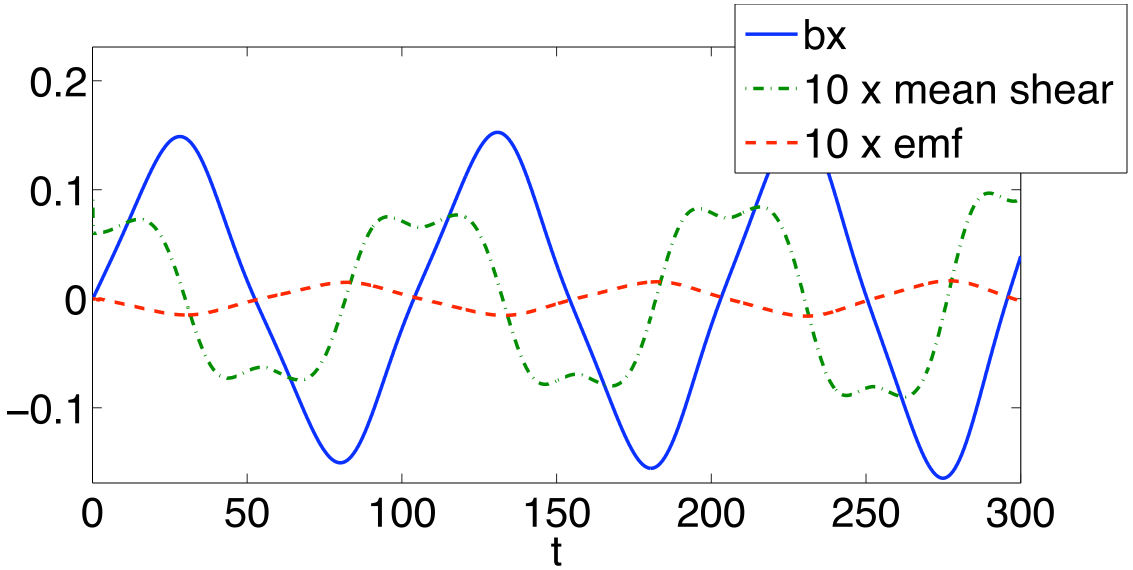

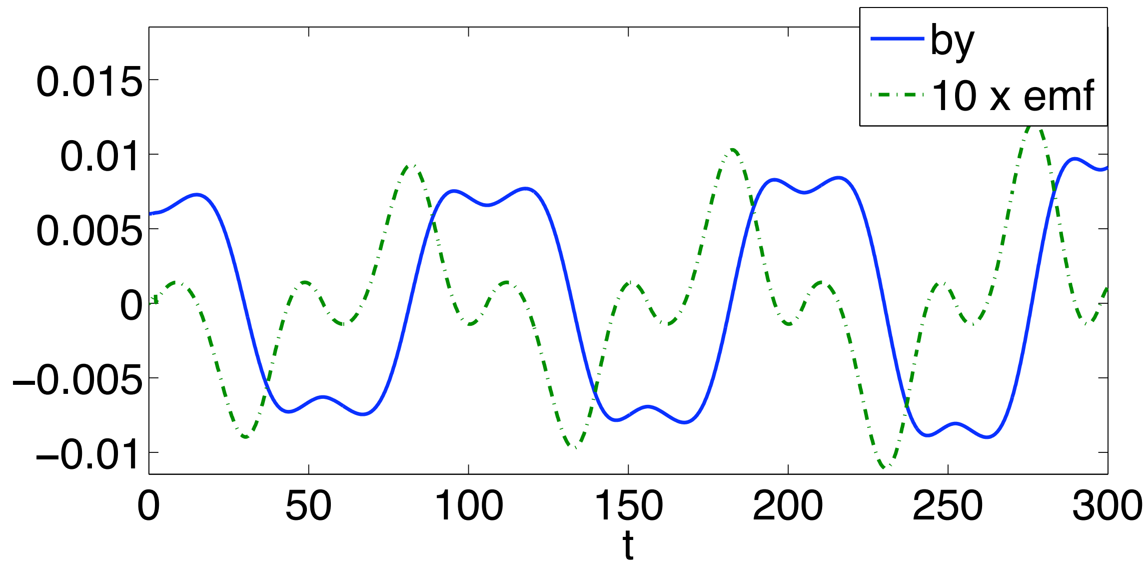

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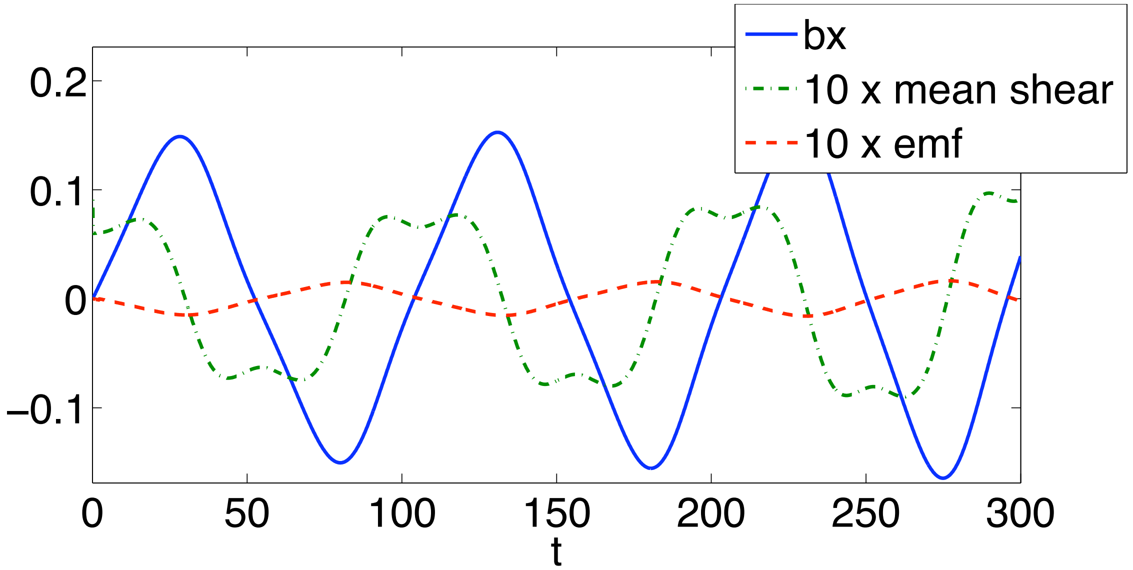
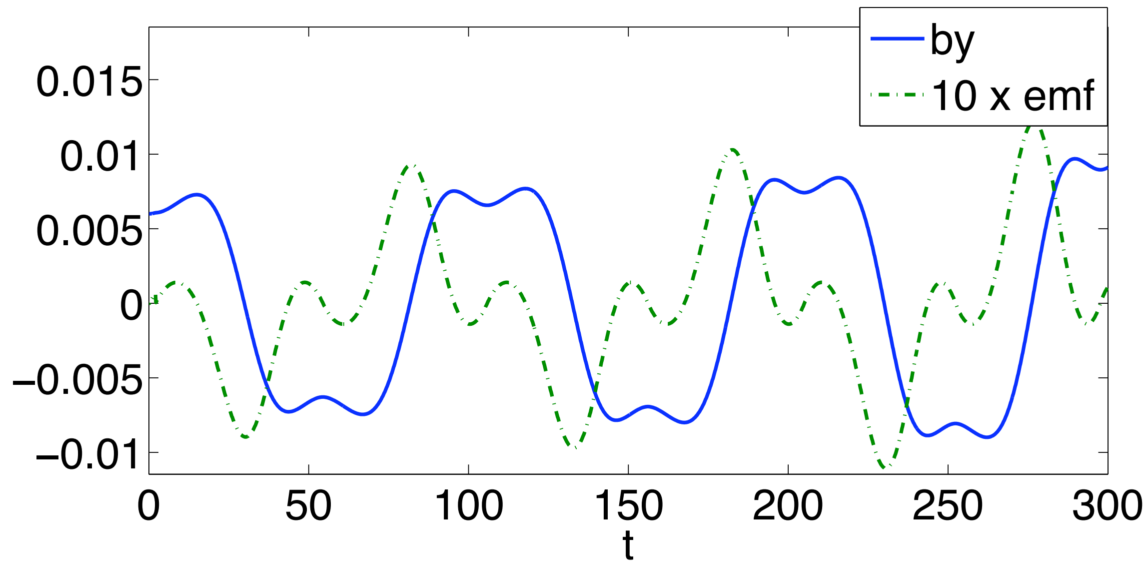
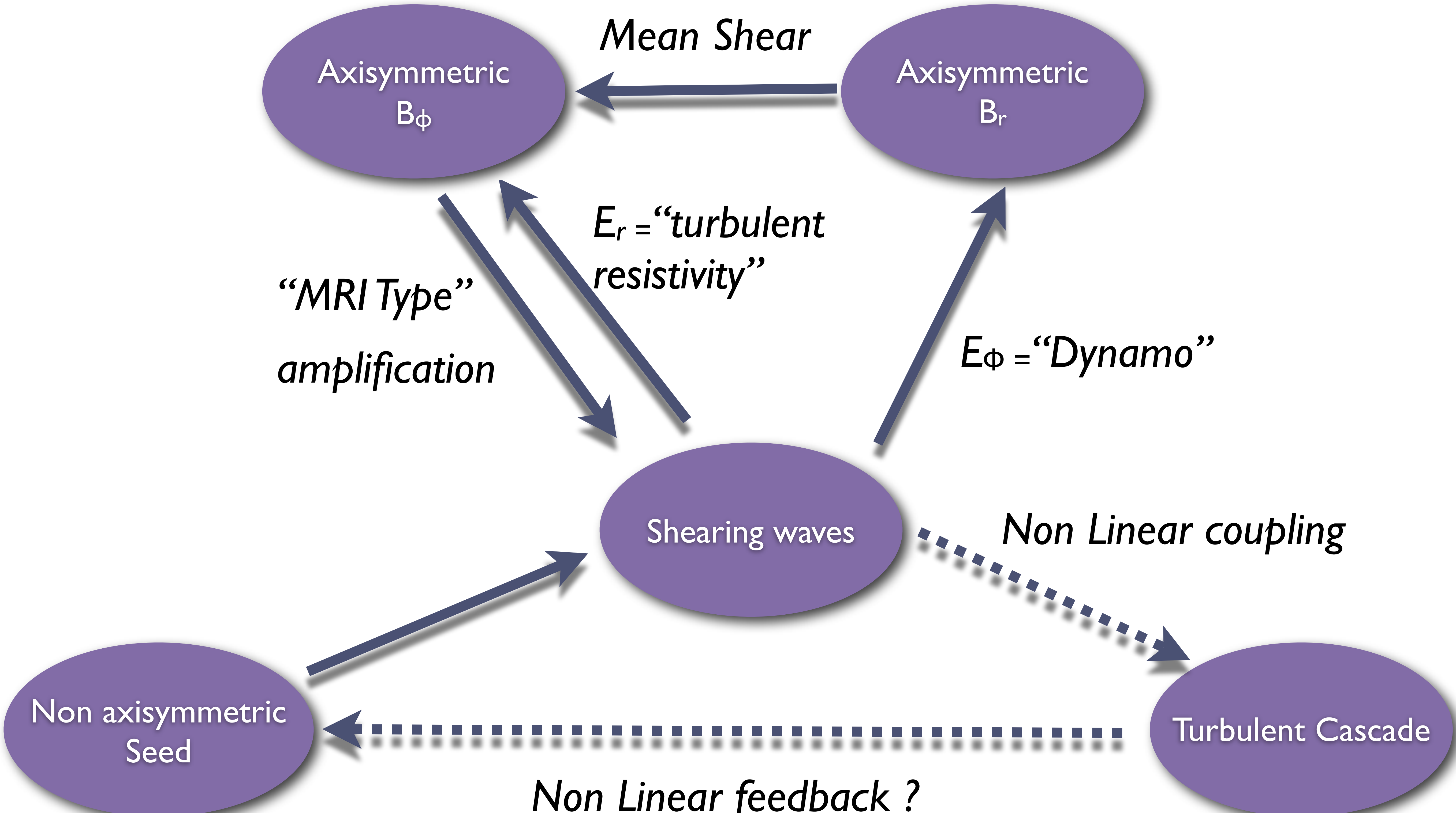




- Cycles are reproduced with T~50 S^{-1}
- No "alpha effect" used in this description

Summary : MRI-Dynamo cycle



Conclusions

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- The MRI naturally provides a (non linear) dynamo feedback when a non homogeneous B_ϕ is imposed (Lesur & Ogilvie 2008)
- Explains the existence of a large scale cycle, with no dependance on the dissipation scales (should work even with very large Re, Rm)
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However

- No precise understanding of the origin of the non axisymmetric seed
- How the Prandtl number enters this problem ?
- Is this mechanism able to generate a global magnetic field ?

Extras

Studying one cycle (phases)...

- Budget for a pure exp(ik₀z) mode :

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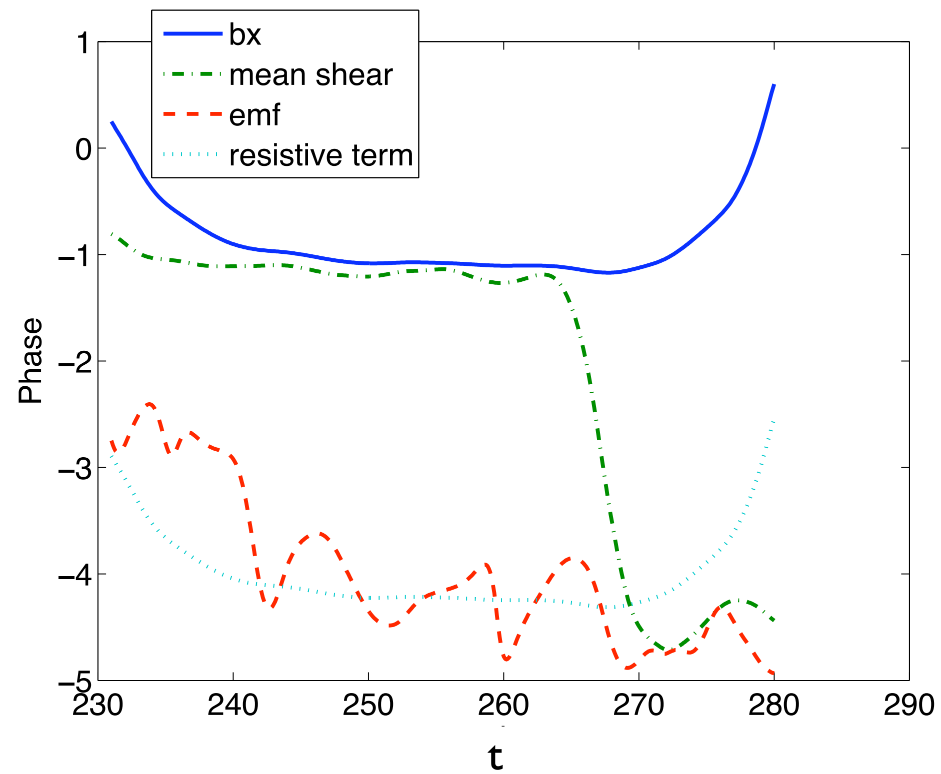

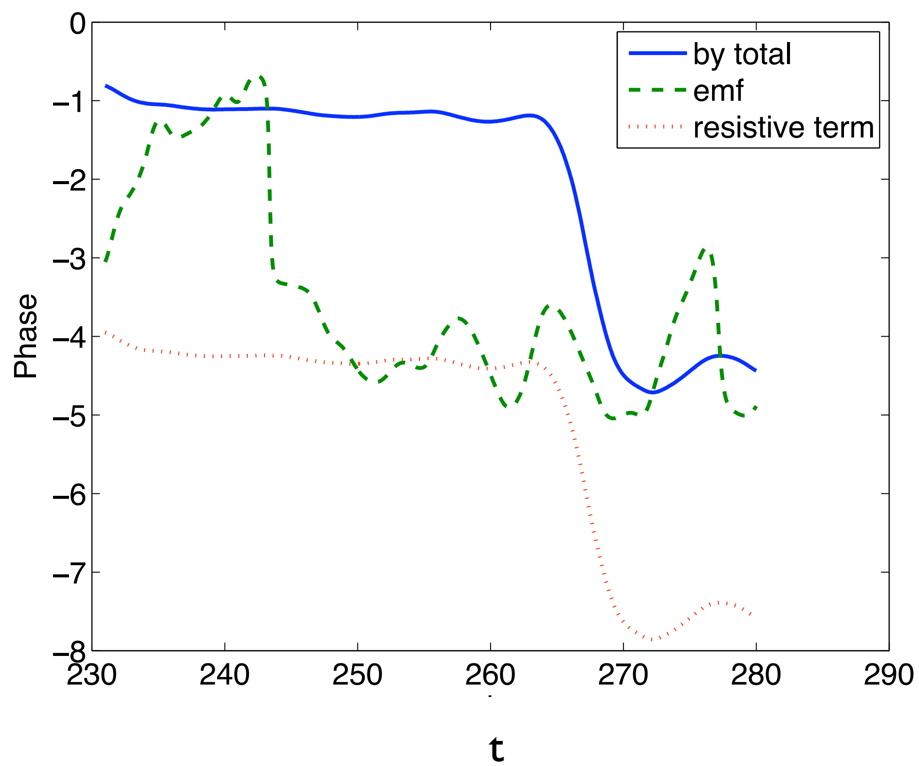

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