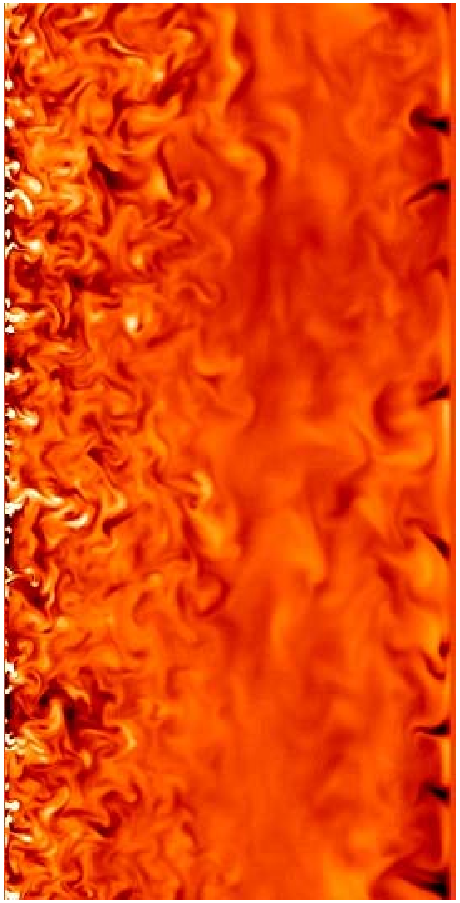

Dynamo Action in MRI-Driven Turbulence in a Cylindrical Annulus

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Fausto Cattaneo [1,2,3,4]

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"Magnetic Field Generation in Experiments, Geophysics and Astrophysics" Conference

Kavli Institute for Theoretical Physics, Santa Barbara, CA

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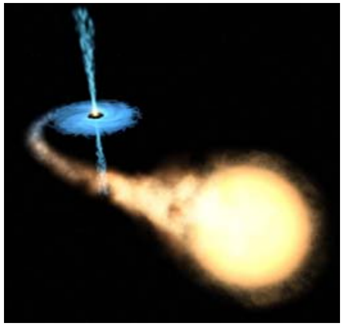
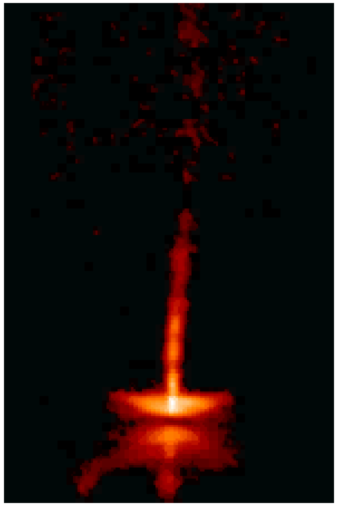
[4] Computation Institute, University of Chicago

Accretion

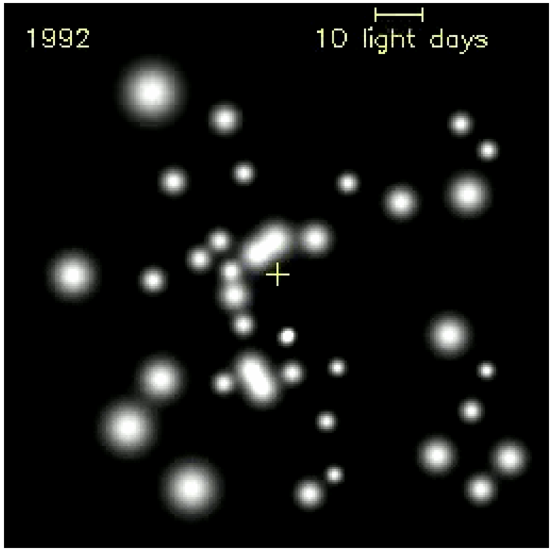


The process of accretion is required to explain

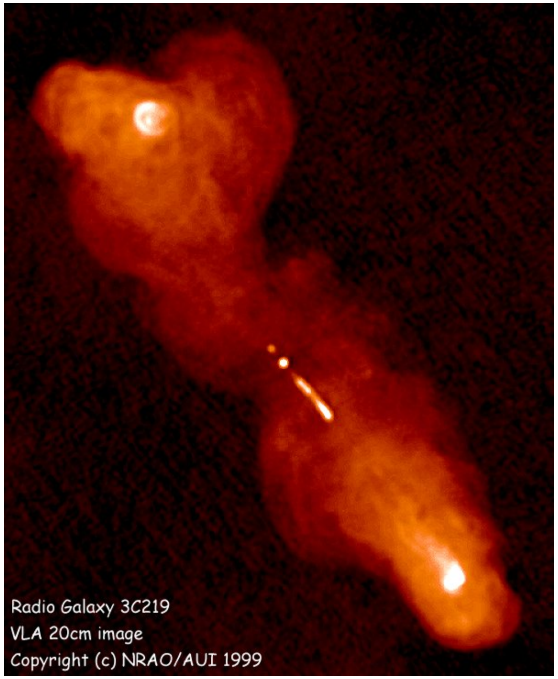
- The formation of compact objects: planets, stars, supermassive black holes
- The release of gravitational energy: Cataclysmic Binaries, Radio Galaxies, Active Galactic Nuclei



HH 30 (Hubble ST)



Sagittarius A* (ESO)

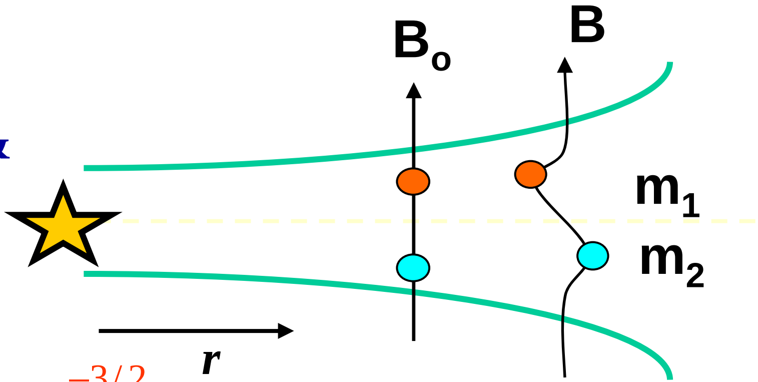


300 Kp

Circular Keplerian rotation $\Omega \propto r^{-3/2}$ **with angular momentum** $r^2\Omega \propto \sqrt{r}$ **increasing outward is (linearly) stable (for axisymmetric disturbances) due to Rayleigh criterion**

John William Strutt, third Baron Rayleigh

$\Rightarrow$ **Magneto-Rotational Instability (Balbus & Hawley 1991; Velikov 1959, Chandrasekhar 1960) for hydro-magnetic flows with angular velocity increasing inward** $\Omega \propto r^{-3/2}$



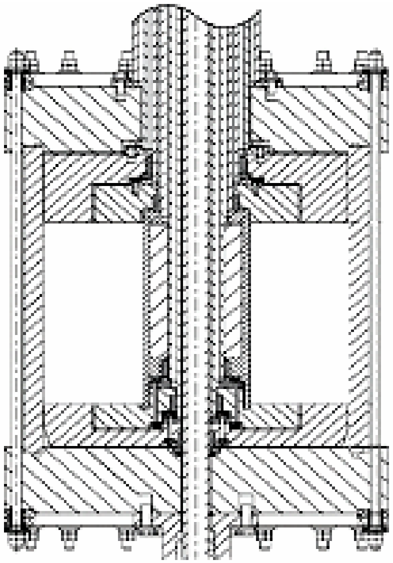
- **Saturation of MRI?**

Cylinders

- **Some basic features of the MRI can be studied by laboratory experiments using liquid metals (Na, Ga) confined between coaxial rotating cylinders**

- **Cylinders rotate so that basic state has circular streamlines and angular velocity increasing inward and angular momentum outward (Keplerian-like profile). Ideally,**

$$u_\theta = r\,\Omega_C(r) = r\left[\frac{R_1^2\Omega_1 - R_2^2\Omega_2}{R_1^2 - R_2^2}\right] + \frac{1}{r}\left[\frac{(\Omega_1 - \Omega_2)R_1^2 R_2^2}{R_1^2 - R_2^2}\right]$$



Princeton MRI liquid gallium experiment: H. Ji & J. Goodman (see also Schartman 2008)

Top and Bottom Boundaries

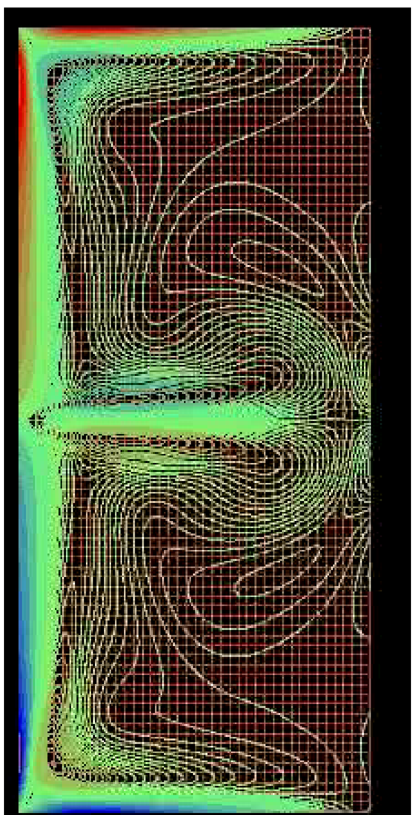
- **Periodic in the vertical: circular Couette (Keplerian-like) flow**

$$u_\theta = r\,\Omega_C(r) = r\left[\frac{R_1^2\Omega_1 - R_2^2\Omega_2}{R_1^2 - R_2^2}\right] + \frac{1}{r}\left[\frac{(\Omega_1 - \Omega_2)R_1^2 R_2^2}{R_1^2 - R_2^2}\right]$$

- **Experiments need vertical boundaries: lead to some form of Ekman flow**

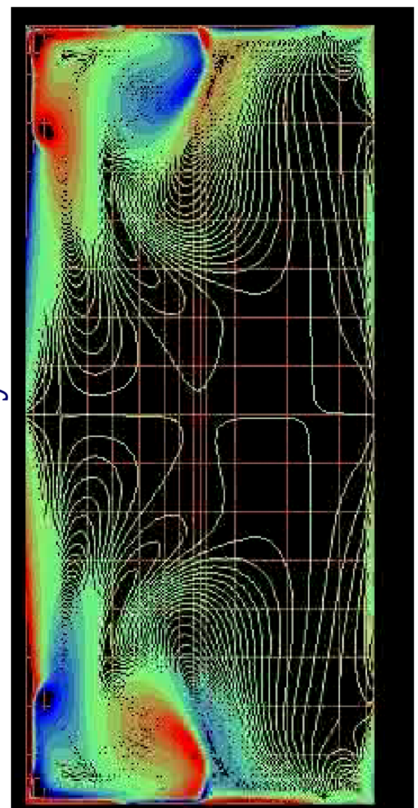
Azimuthal vorticity (*Re*=6200)

inner cylinder

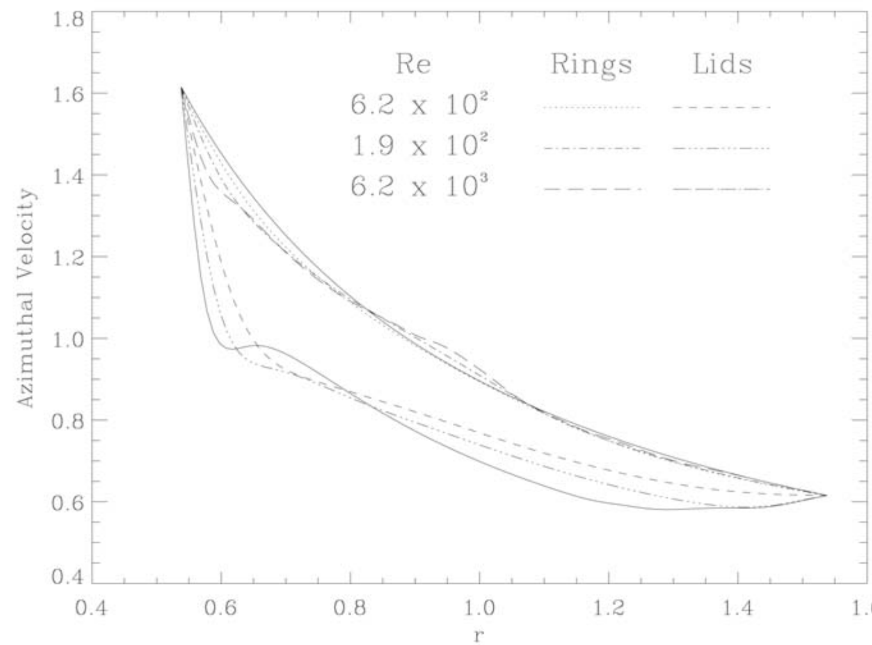


lids

inner cylinder



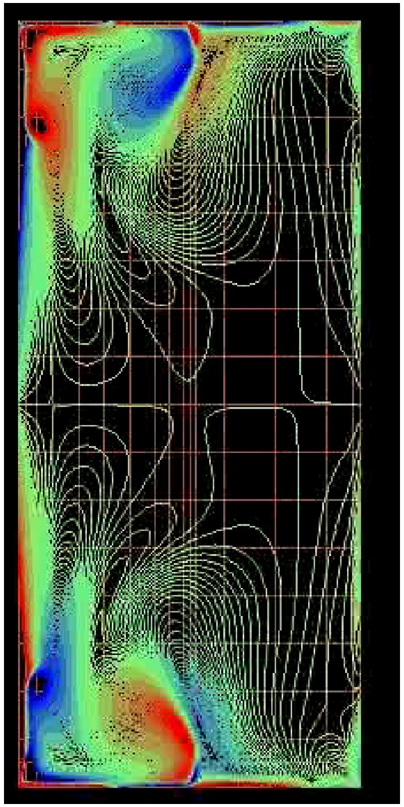
rings



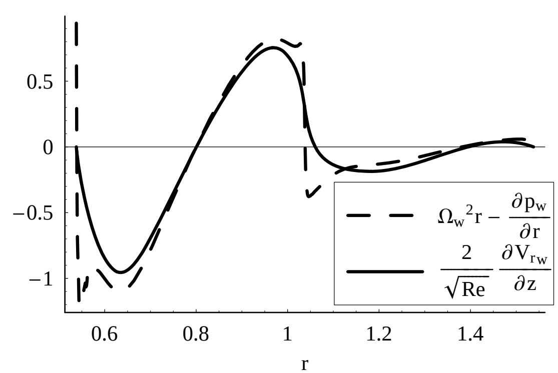
⇒ **Less distortion of Couette profile in case w/ rings due to the disruption of Ekman flow**

Ekman flow disruption

inner cylinder



$\Omega_1 > \Omega_3 > \Omega_4 > \Omega_2$



- **Inward/outward Ekman flows are due to near-wall rotation momentum deficit/excess over centripetal pressure gradient (Obabko, Cattaneo & Fischer 2008)**

Vagn Walfrid Ekman

⇒ Disruption of Ekman circulation in the case of rings and therefore, decrease of associated angular momentum due to smaller $u_r u_\theta$ correlation

Experiments and simulations

Experiments:

- Can reach high Reynolds number (Re = O(10^6 - 10^7))
- Are stuck at low magnetic Reynolds number (Rm = O(10 - 10^2))
- Vertical boundaries confuse the issue
- Difficult to take measurements with high spatial resolution
- Can be run for a long time

Simulations:

- "Scenarios" (Leo Kadanoff) require validation
- Stuck at moderate Reynolds numbers (both kinetic and magnetic)
- 'No problem' with vertical boundary conditions
- Can measure anything (but getting harder with bigger supercomputers)
- Almost impossible to run for very long

$\Rightarrow$ In particular to MRI, it is extremely hard to conduct high Rm MRI experiment

$\Rightarrow$ FFT in time works better for experiments and FFT in space – for simulations

Problem Formulation

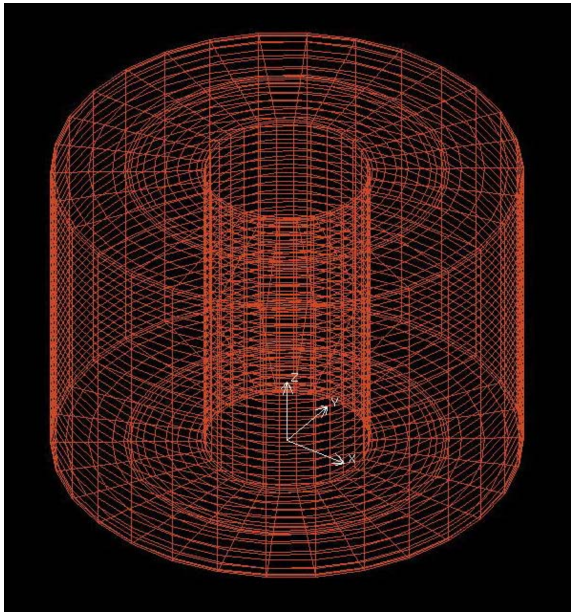
- **Incompressible viscous resistive MHD equations in cylindrical geometry of Princeton MRI liquid gallium experiment**

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = (\mathbf{B}\cdot\nabla)\mathbf{B} + \frac{1}{Re}\nabla^2\mathbf{u} - \nabla\left(p + \frac{\mathbf{B}^2}{2}\right)$$

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{B} = (\mathbf{B}\cdot\nabla)\mathbf{u} + \frac{1}{Rm}\nabla^2\mathbf{B}$$

$$\nabla\cdot\mathbf{u} = \nabla\cdot\mathbf{B} = 0$$

$$Re = \frac{(\Omega_1 R_1 - \Omega_2 R_2)(R_2 - R_1)}{\nu}, \quad Rm = Re\, Pm$$



Element boundary mesh

- **Boundary conditions:**

$$u_r = u_z = 0, \quad u_\theta = \Omega(r)\, r,$$

$$B_r = B_\theta = 0,$$

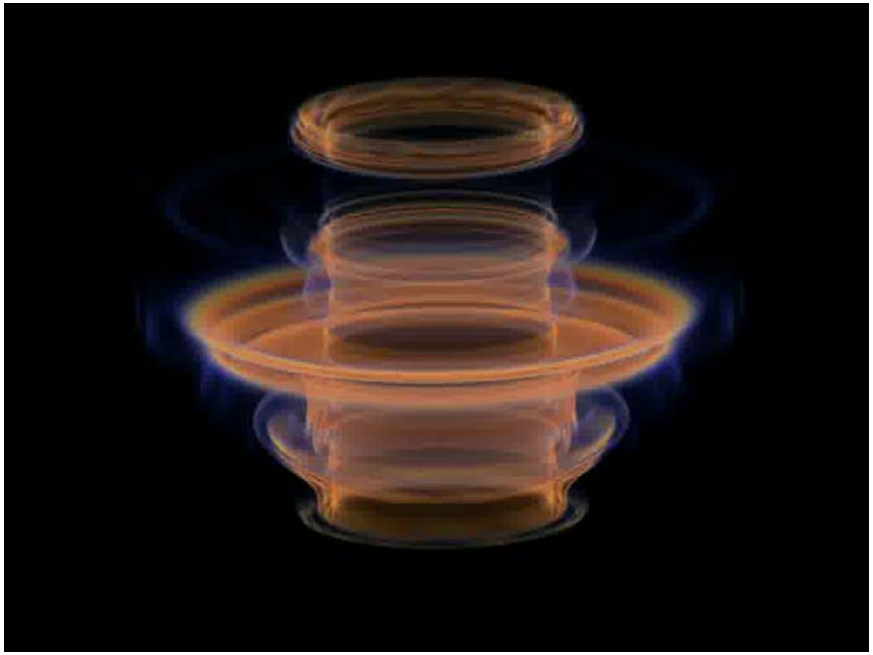
$$B_z\big|_{r=R_1,R_2} = B_o, \quad \frac{\partial B_z}{\partial z}\bigg|_{z=0,H} = 0$$

$$\Omega(r) = \begin{cases} \Omega_1 = 3.003, & r = R_1 = 0.538 \\ \Omega_2 = 0.400, & r = R_2 = 1.538 \\ \Omega_3, & R_1 < r < \frac{R_1+R_2}{2} \\ \Omega_4, & \frac{R_1+R_2}{2} < r < R_2 \end{cases}$$

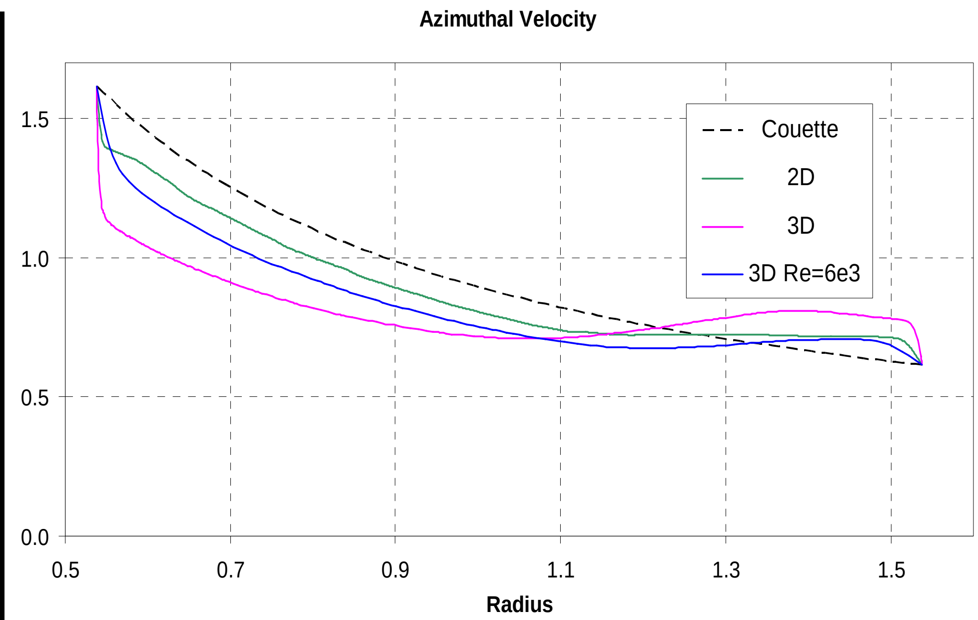
- **Solved numerically by MHD version of spectral element code Nek5000 optimized for highly parallel machines**

Axisymmetric vs 3D (z-periodic, Bz_0=0.05)

Square current: *Re*=6,000 *Rm*=3,000 * §



Re=60,000 *Rm*=30,000 ϒ



⇒ **Axisymmetric solution is strongly unstable to 3D perturbations (P_m = O(1))**

⇒ **Saturation both through dissipation and modification of background velocity for axisym / 3D toward constant azimuthal / constant angular velocity (cf. Julien & Knobloch 2005)**

* Acknowledge the use of resources of NERSC at Lawrence Berkeley National Laboratory (as INCITE 2005)

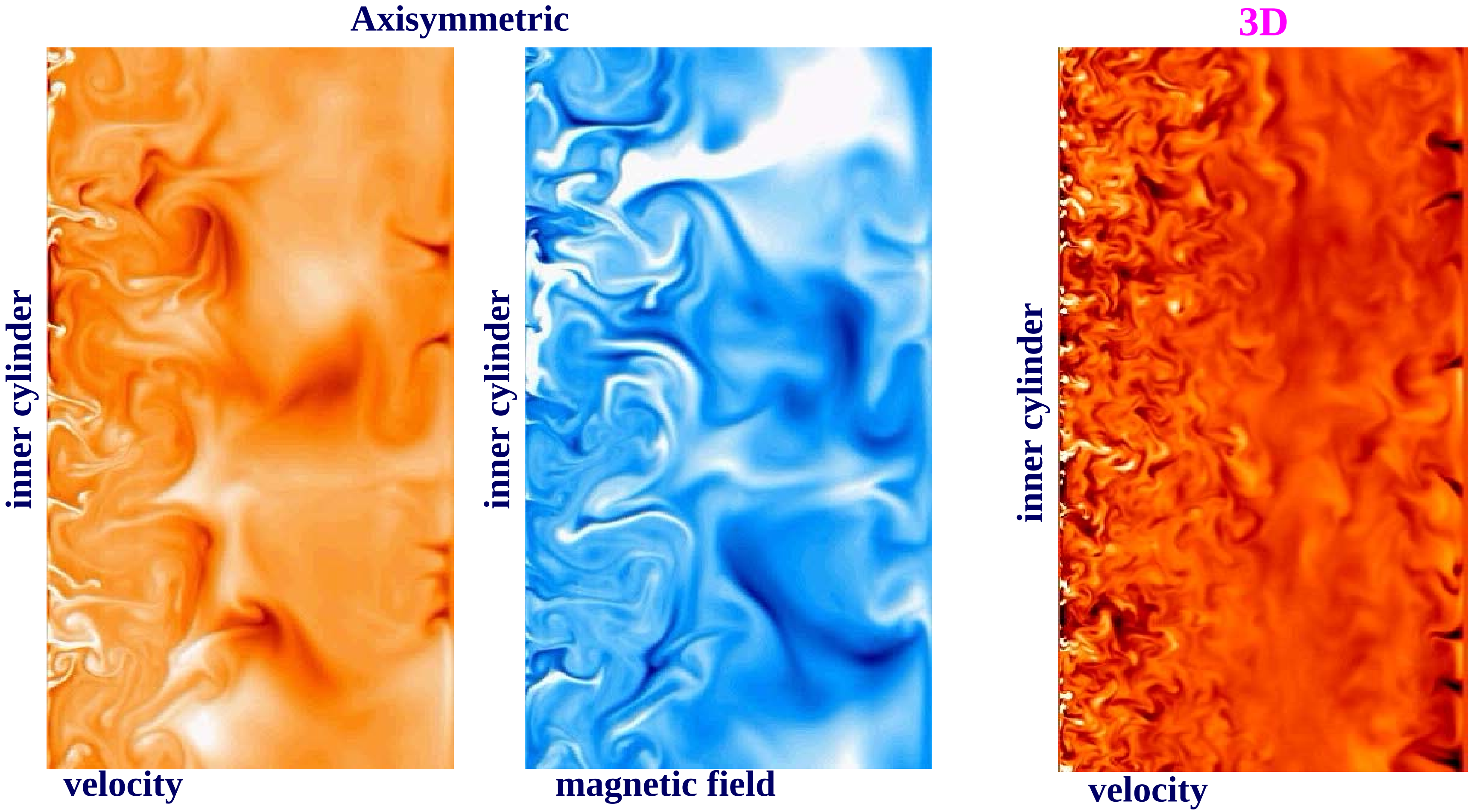
§ Acknowledge the help of NERSC Visualization Group, LBNL

ϒ Run time on 32,768 processors of Blue Gene Watson (BGW) was provided courtesy of the IBM Corporation & acknowledgement of the use of resources of Argonne Leadership Computing Facility operated by ANL

from MRI to MRI-driven turbulence

Look at strongly supercritical cases (*Re* = 60,000 - periodic conditions)

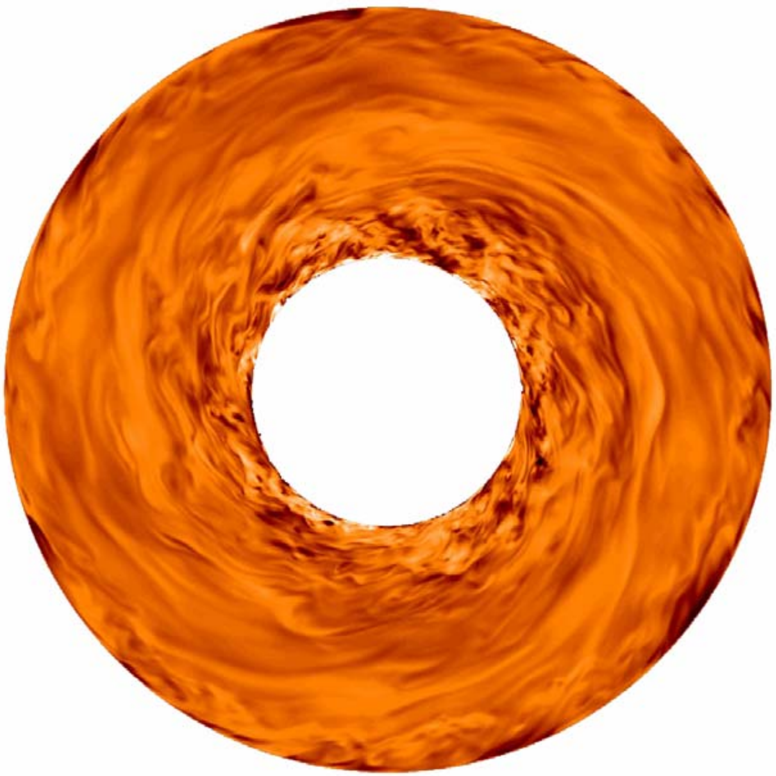
- **Fluctuations of azimuthal quantities:**

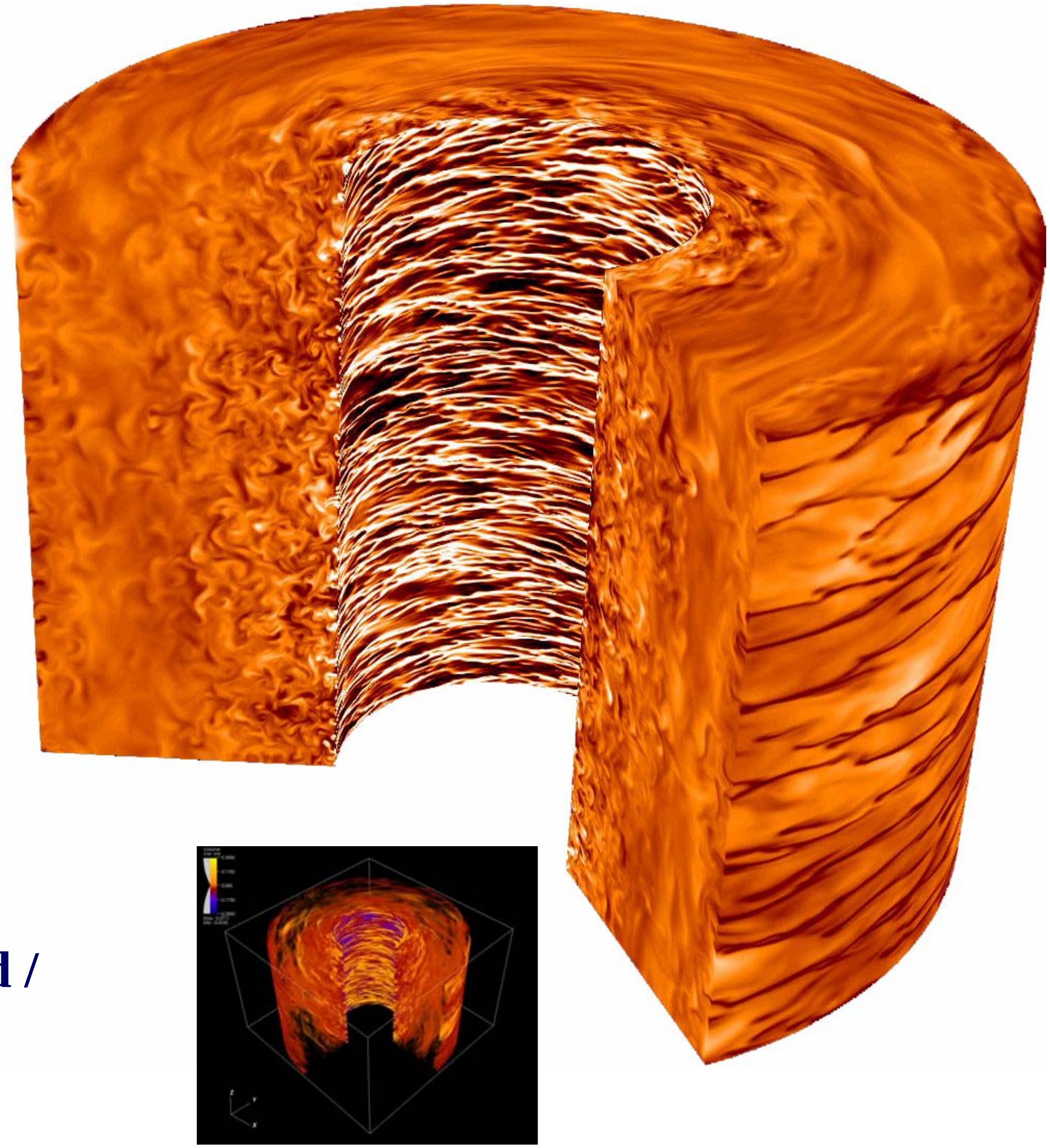


⇒ Qualitatively looks similar to (penetrative) convection

⇒ Torque increase: 5 (axisym) and 20 (3D)

MRI-Driven Turbulence: u'_θ

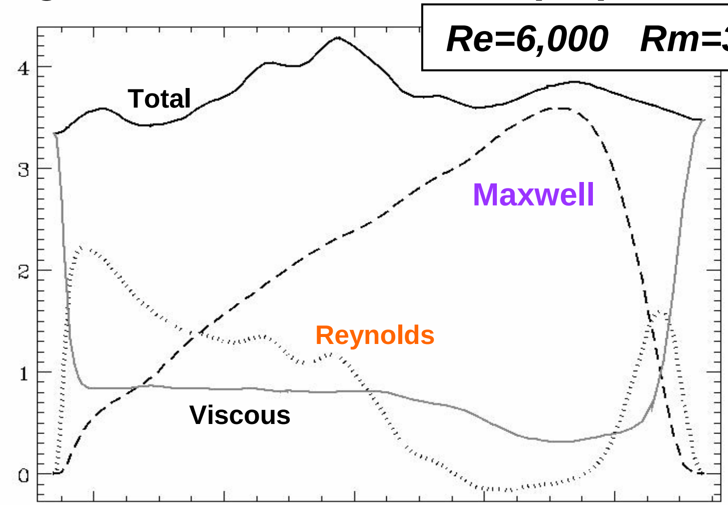


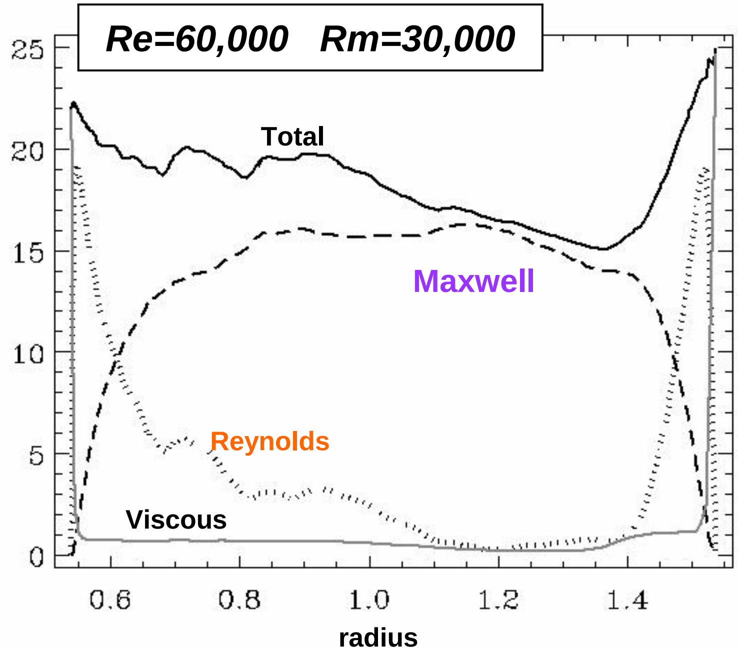


⇒ **Streaks of high and low speed / angular momentum**

Angular Momentum Transport

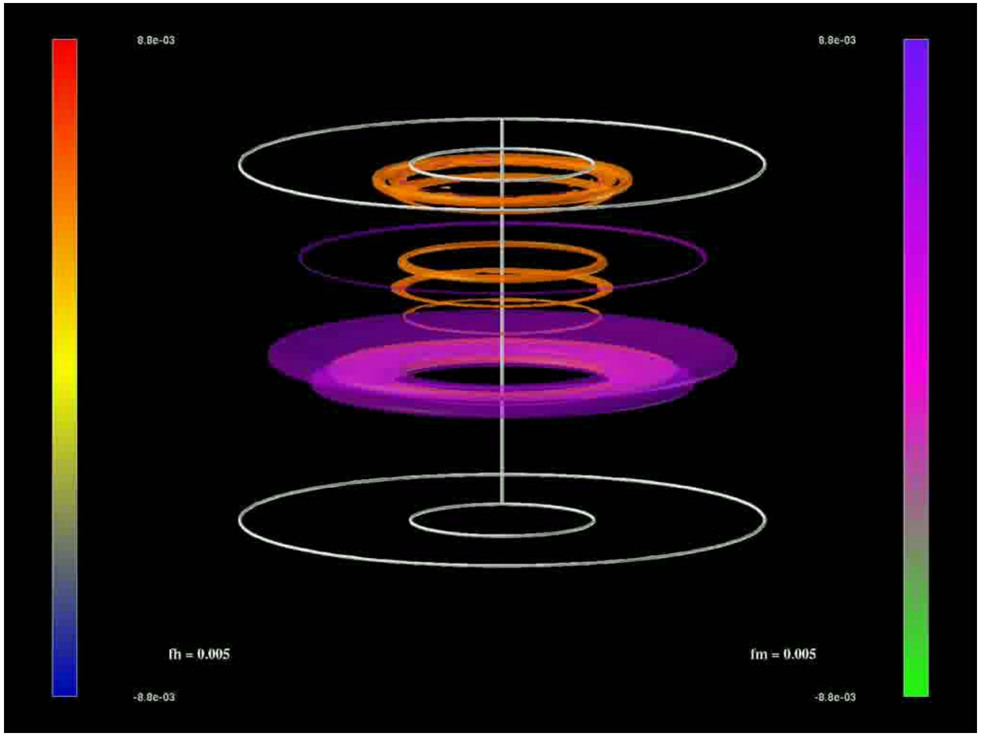
Angular momentum fluxes (x r)





Reynolds and Maxwell stress fluxes of AM

$$r\,u'_\theta\,u_r \qquad -r\,B_\theta\,B_r$$

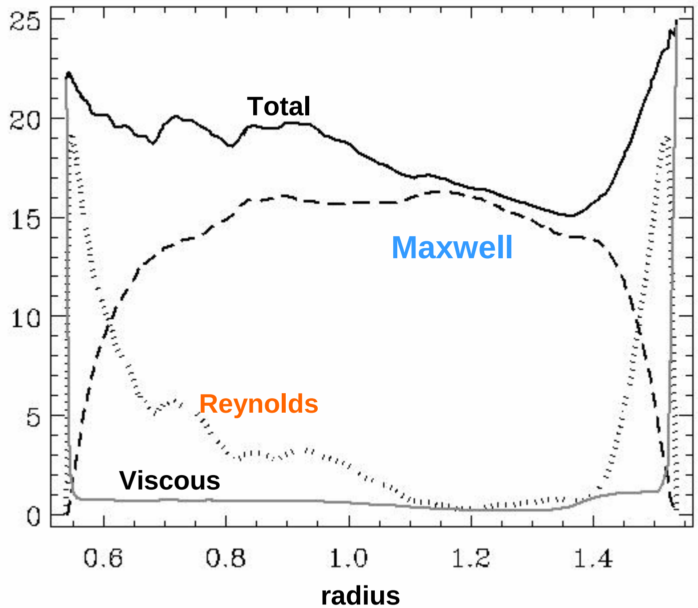


⇒ **Reynolds stress flux confinements to cylinder "boundary layers"**

⇒ **Maxwell stress flux domination**

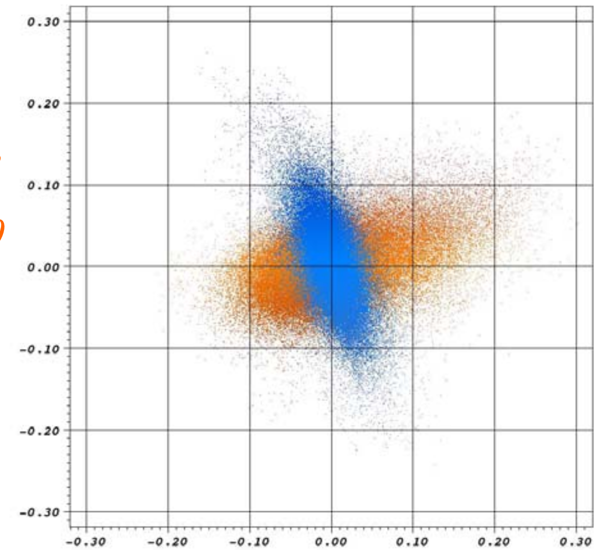
Angular Momentum Transport

$r\, u'_\theta\, u_r$ $\quad -r\, B_\theta B_r$



u'_θ

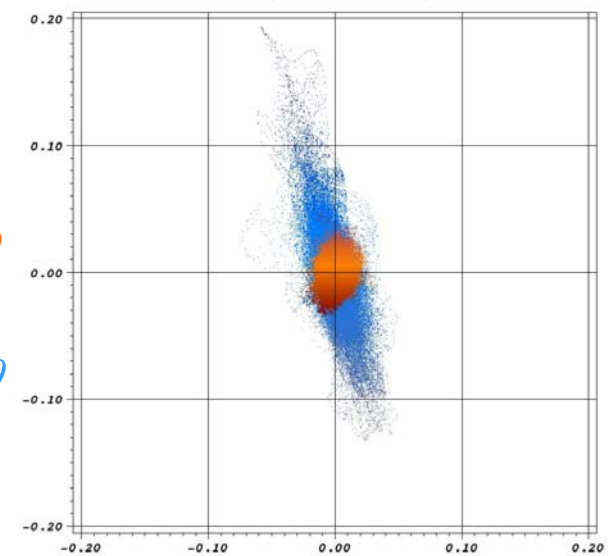
B_θ



u_r $\quad B_r$

u'_θ

B_θ

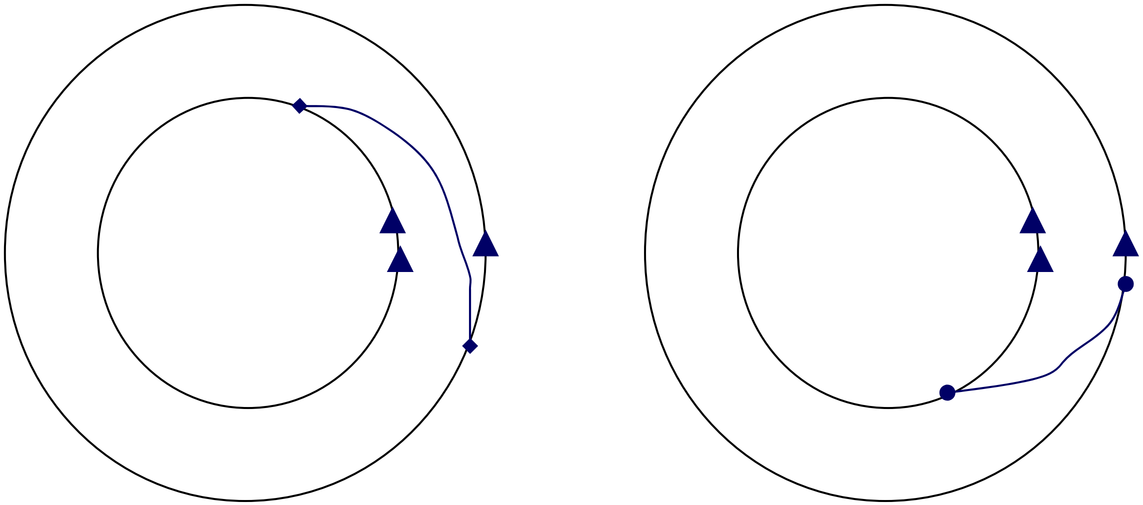


- **Maxwell stress flux domination**

$\Rightarrow$ **due to the correlation of B_r and B_θ correlation ($B_r * B_\theta$<0) for AM transport outward**

B_r and B_θ Correlation ($B_r * B_\theta < 0$)

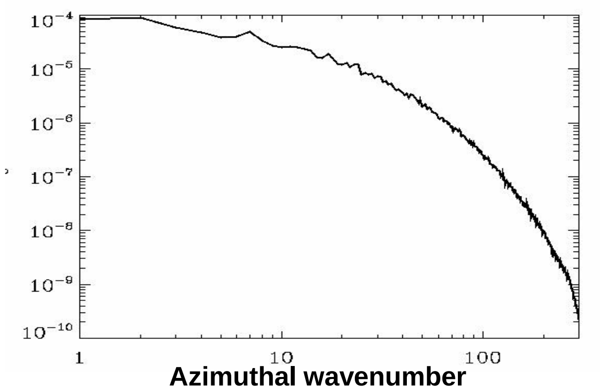
- **Maxwell stress flux domination due to the correlation of B_r and B_θ**

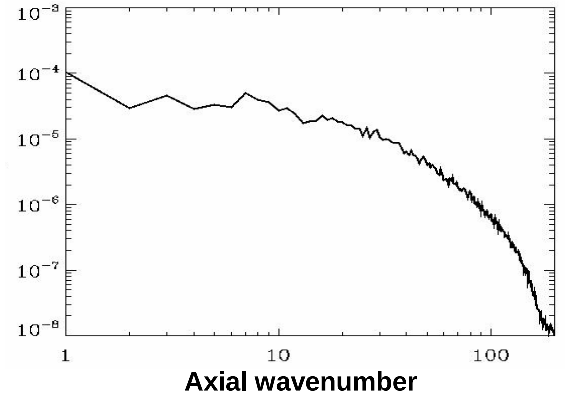


- **Angular momentum is carried outwards (inwards) by magnetic fluctuations that correspond to *winding* (*unwinding*) spirals -i.e. getting longer (shorter)**
- **In a random circular sheared motion there are more winding than unwinding spirals (can wind forever; can only unwind for a finite time)**
- **If angular velocity increases inwards (due to shear) Maxwell stress will carry angular momentum outwards (kinematic effect)**

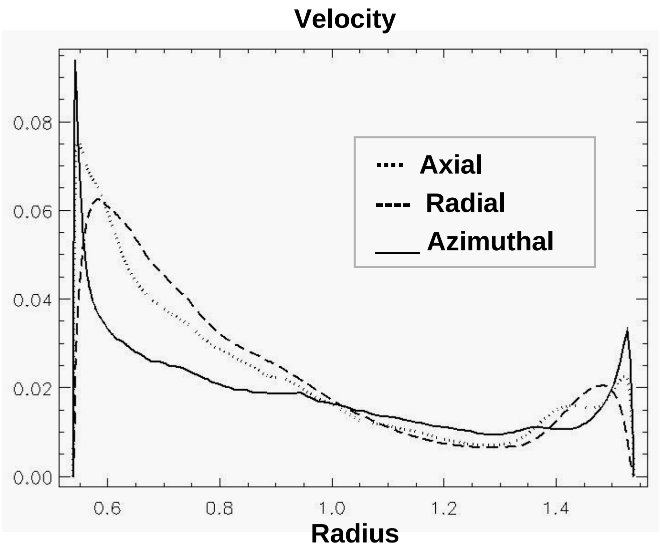
Fluctuations & Spectra

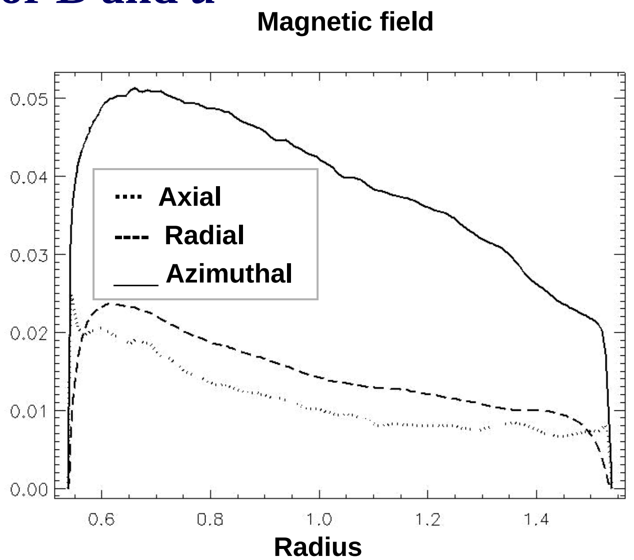
- **Spectra for 3D solutions are moderately flat at small wavenumbers**
  - **Magnetic energy**





- **Fluctuations have comparable magnitude both for *B* and *u***

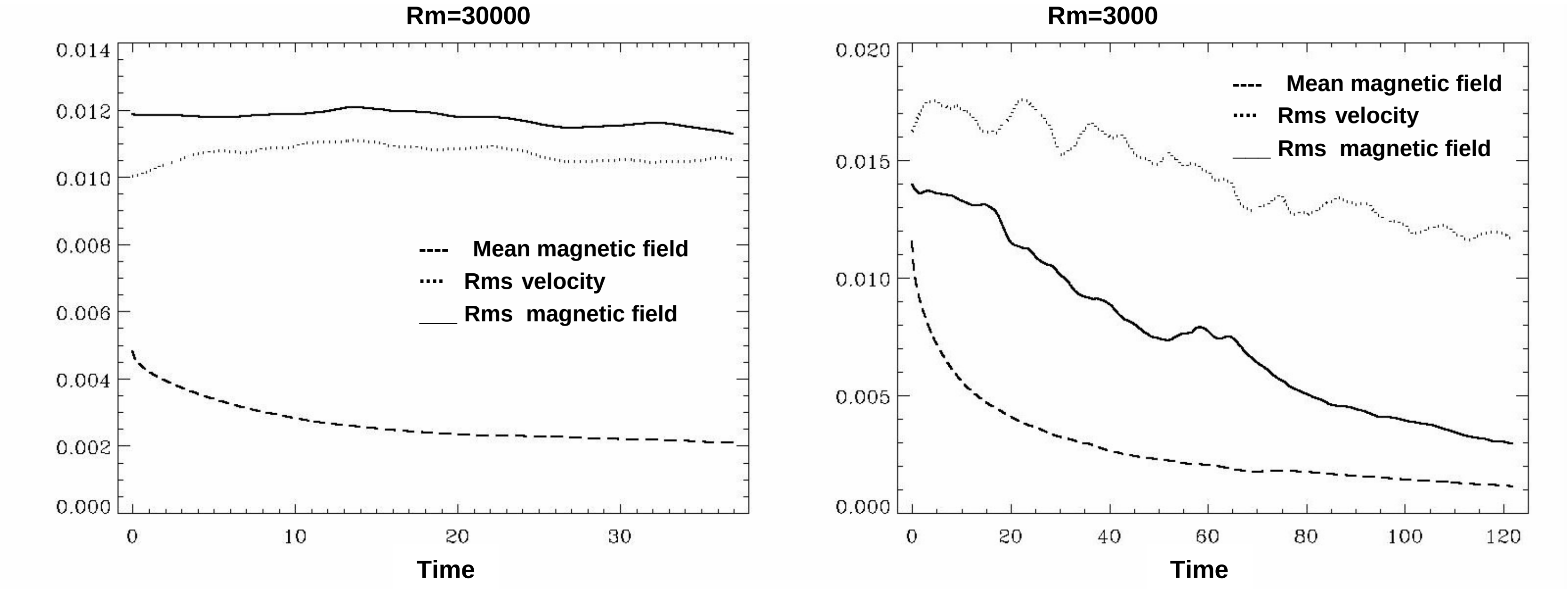




$\Rightarrow$ **Turbulent** $Rm \leq 600$

Fluctuations and dynamo

- **If the external axial field is switched off at the boundary, the averaged field decays but fluctuations survive for Rm=30000**



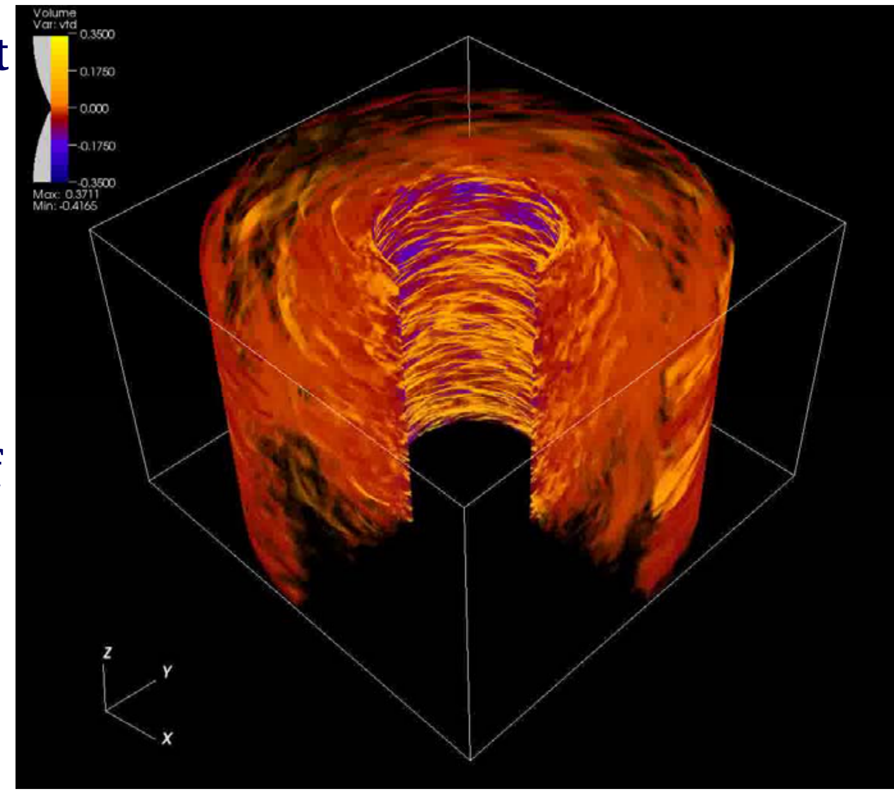
- **Probably, the case with Rm=30000 is a dynamo**

  $\Rightarrow$ Small-scale dynamo (with turbulent Rm $\leq$ 600)

$\Rightarrow$ The MRI-driven turbulence becomes self-sustained at high enough *Rm* / *Re* and regenerates magnetic field necessary for its own existence independently of the initial field that induced MRI in the first place

Conclusions

- A “scenario” of MRI-driven turbulence (MRIDT) provides very attractive rationale for enhanced angular momentum transport
  - In turbulent regime MRIDT might act as a (small-scale) dynamo



- MRIDT angular momentum transport
  - is dominated by Maxwell stresses due to negative correlation of radial and azimutal magnetic field fluctuations (kinematic effect)

- MRI saturates with highly 3D state of MRIDT through the dissipation and modification of the background velocity toward solid body rotation (cf. axisymmetric cases: constant azimuthal velocity)

- Future work on simulations of flows with smaller magnetic Prandtl number