Saturation and Reversals in Numerical Dynamos

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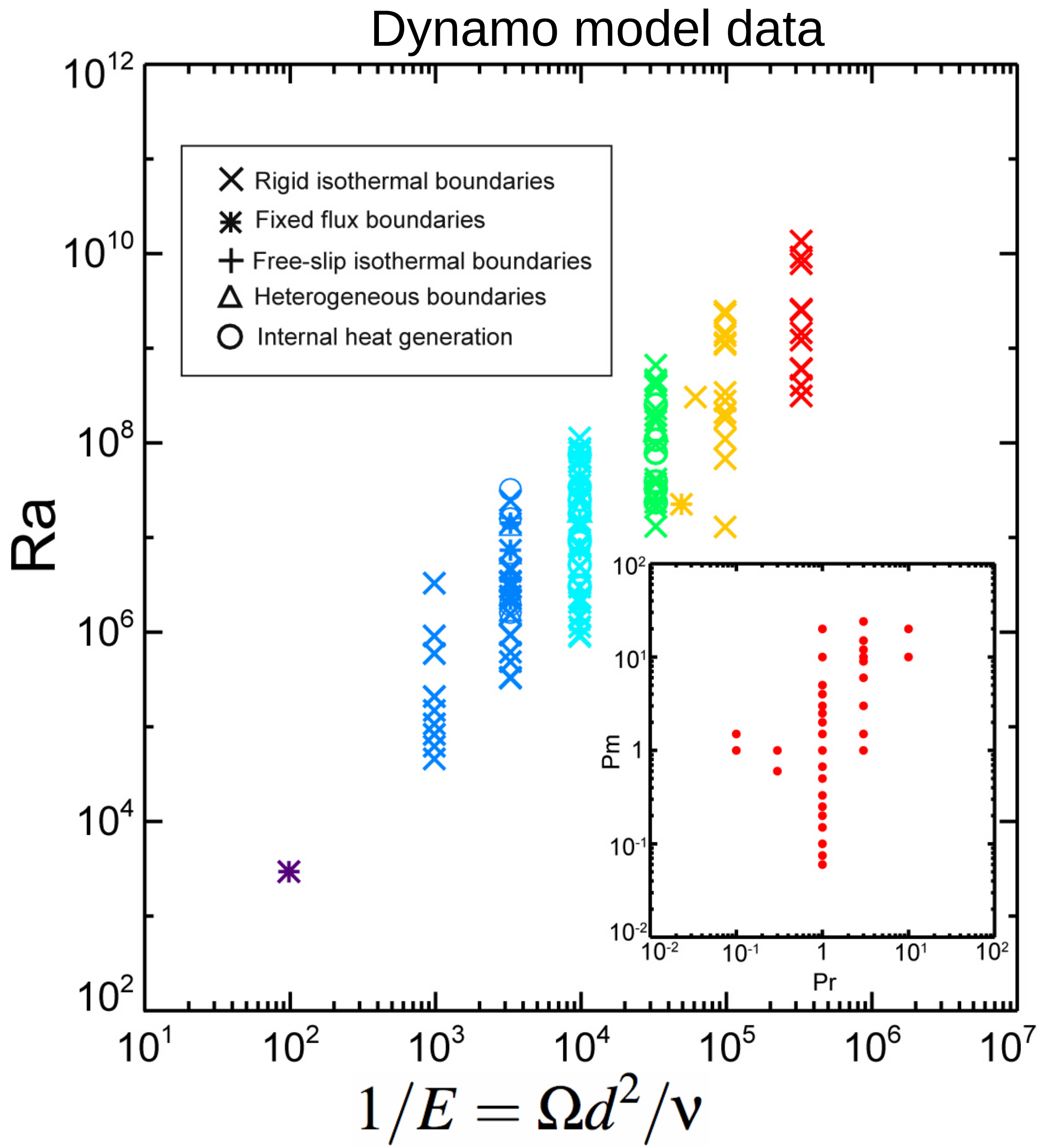
Topics

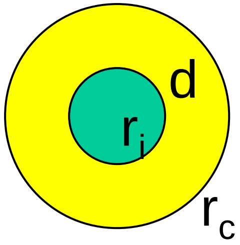
- Saturation magnetic field intensity in rotating spherical dynamos
- Low frequency variability in the geomagnetic field
- Low frequency variability and polarity reversals in simple geodynamo models

Magnetic Intensity Saturation in Rotating Spherical Shell Dynamos



Earth's Core:

$1/E \sim 10^{13}$
$Ra \sim 10^{18\text{-}20}$
$Pr \sim 0.1$
$Pm \sim 10^{-6}$
$r_i/r_c = 0.35$



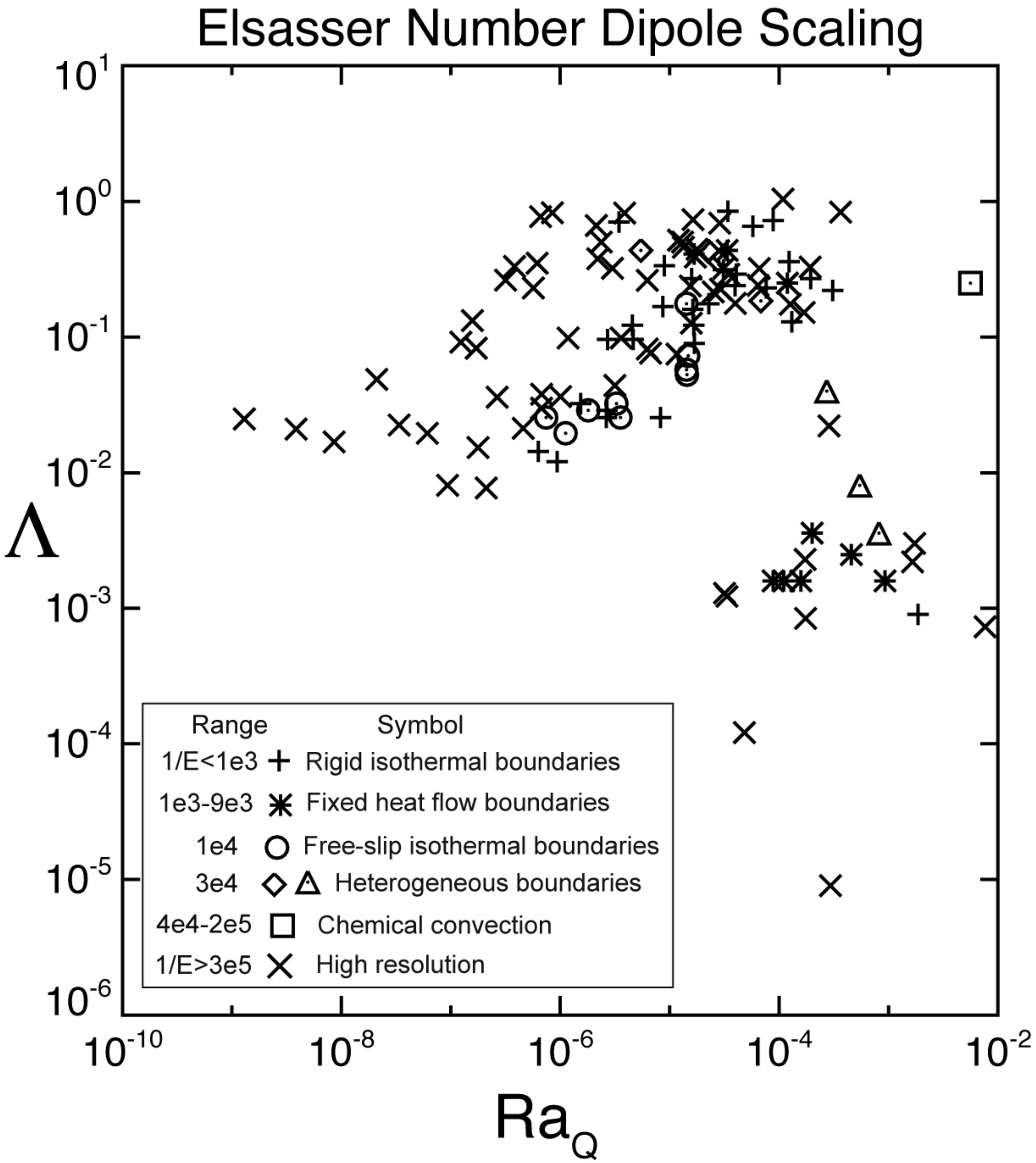
Olson & Christensen EPSL (2006)

Λ = *constant*
saturation??

$$\Lambda = \frac{\sigma B_{rms}^2}{\rho \Omega}$$

$$Ra_Q = \frac{r_c F}{r_i d^2 \Omega^3}$$

F = buoyancy flux

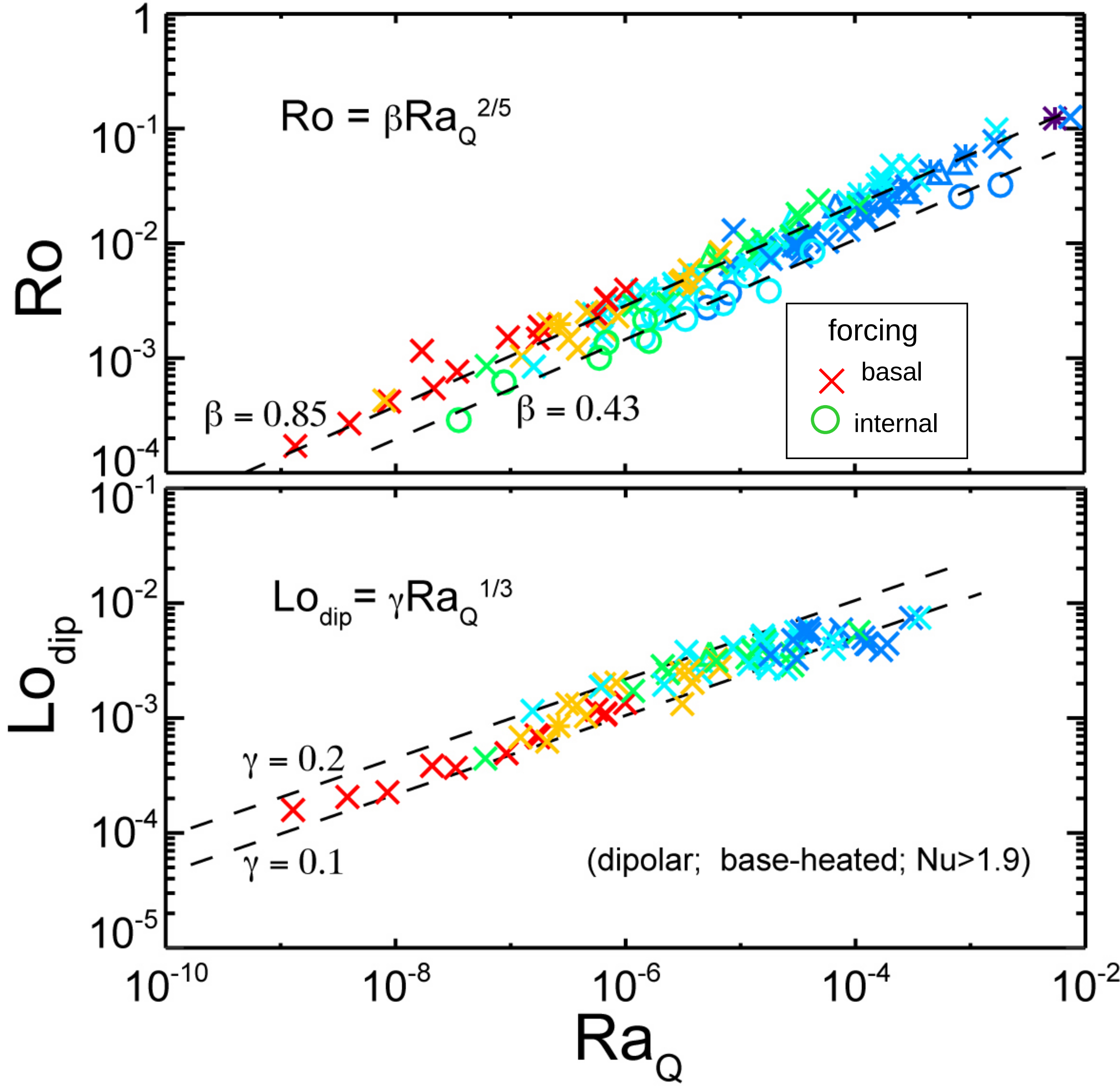


Buoyancy Flux Saturation

$$Ra_Q = \frac{r_c F}{r_i d^2 \Omega^3}$$

$$Ro = \frac{u_{rms}}{\Omega d}$$

$$Lo = \frac{B_{rms}}{\sqrt{\rho\mu_0}\Omega d} = (\Lambda Pm E)^{1/2}$$



dipole saturation: $Lo = \gamma Ra_Q^{1/3}\ ; \quad \gamma \simeq 0.1 - 0.2$

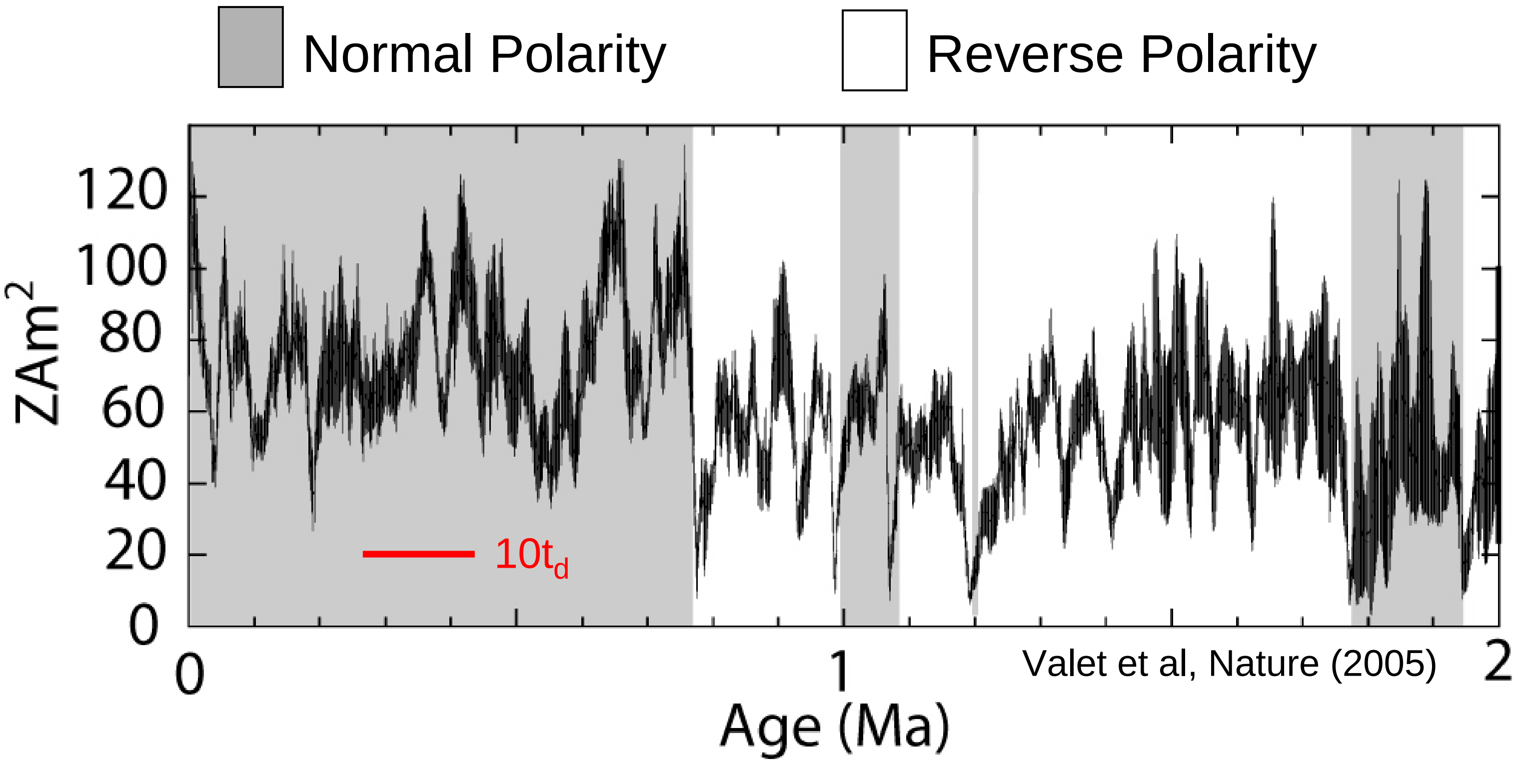
equivalent dipole moment: $M \simeq 4\pi r_c^3 \gamma (\rho/\mu_0)^{1/2} (Fd)^{1/3}$

for the geodynamo yields $M = 7\text{x}10^{22}$ Am^2 with $F = 2\text{x}10^{-13}$ m^{2}s^{-3}

Time interval	Average dipole moment	Reference
160 a	$8.12 \times 10^{22}\ Am^2$	Jackson et al. (2000)
7 Ka	6	Korte and Constable (2005)
10 Ka	8.75 ± 1.6	Valet et al. (2005)
15-50 Ka	4.5	Merrill and McElhinny (1998)
300 Ka	8.4 ± 3.1	Selkin and Tauxe (2000)
800 Ka	7.5 ± 1.5	Valet et al. (2005)
0.8-1.2 Ma	5.3 ± 1.5	Valet et al. (2005)
0.3-5 Ma	5.5 ± 2.4	Juarez and Tauxe (2000)
0.5-4.6 Ma	3.6 ± 2	Yamamoto and Tsunakawa (2005)
5 Ma	7.4 ± 4.3	Kono and Tanaka (1995)
0.3-300 Ma	4.6 ± 3.2	Selkin and Tauxe (2000)

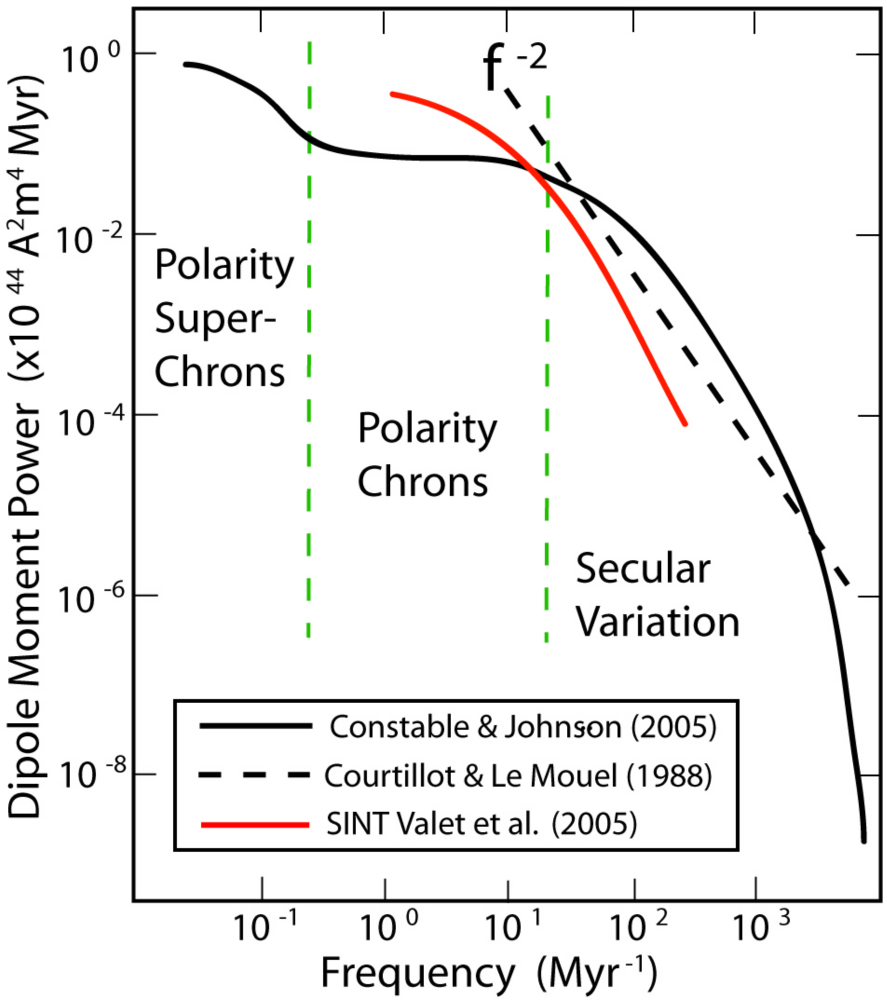
lots of variability!

Low Frequency Geomagnetic Dipole Moment Variability

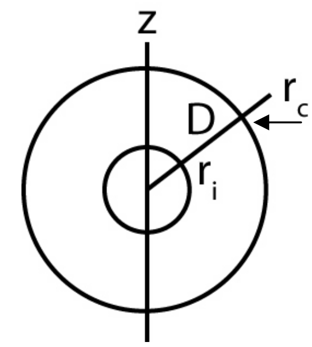


- “virtual” dipole moment
- obtained from marine sediment magnetization
- polarity transitions & excursions follow dipole collapses
- dipole free decay time in the core: t_d ~ 20 kyr

Geomagnetic Dipole Frequency Spectrum



Numerical Dynamos with Low Frequency Variability



Navier-Stokes

$$E\left(\frac{\partial u}{\partial t} + u \cdot \nabla u - \nabla^2 u\right) + 2\hat{z} \times u + \nabla P = EPr^{-1}Ra\frac{r}{r_o}\chi + Pm^{-1}(\nabla \times B) \times B$$

Induction

$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) + Pm^{-1}\nabla^2 B$$

Continuity

$$\nabla \cdot (u, B) = 0$$

Light Elements Transport

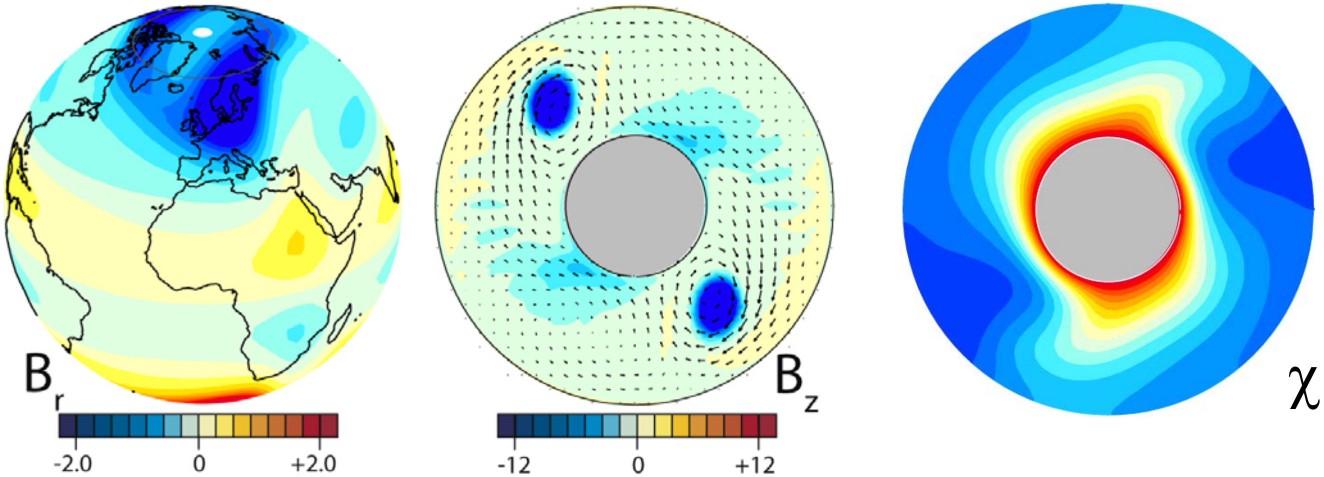
$$\frac{\partial \chi}{\partial t} + u \cdot \nabla \chi = Pr^{-1}\nabla^2 \chi - 1$$

$$r_o = \frac{r_c}{D} \qquad [t] = \frac{D^2}{\nu} \qquad [u] = \frac{\nu}{D}$$

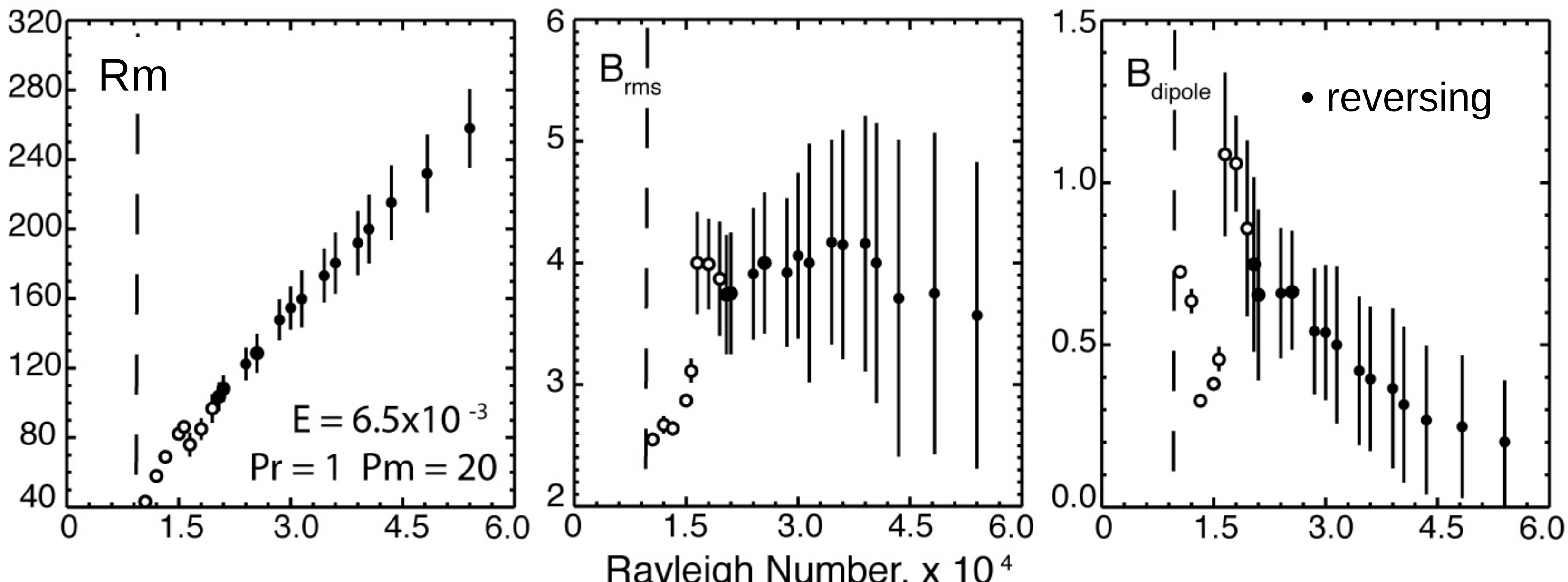
$$[B] = \sqrt{\frac{\rho\Omega}{\sigma}} \qquad [\chi] = \left(\frac{D^2}{\nu}\right)\dot{\chi}_o$$

$$Ra = \frac{\beta g_o D^5 \dot{\chi}_o}{\kappa \nu^2}$$

$$E = \frac{\nu}{\Omega D^2} = 6.5 \times 10^{-3} \qquad Pr = \nu/\kappa = 1 \qquad Pm = \nu/\eta = 20$$



Variability Statistics



Olson, PNAS (2007)

Time Series

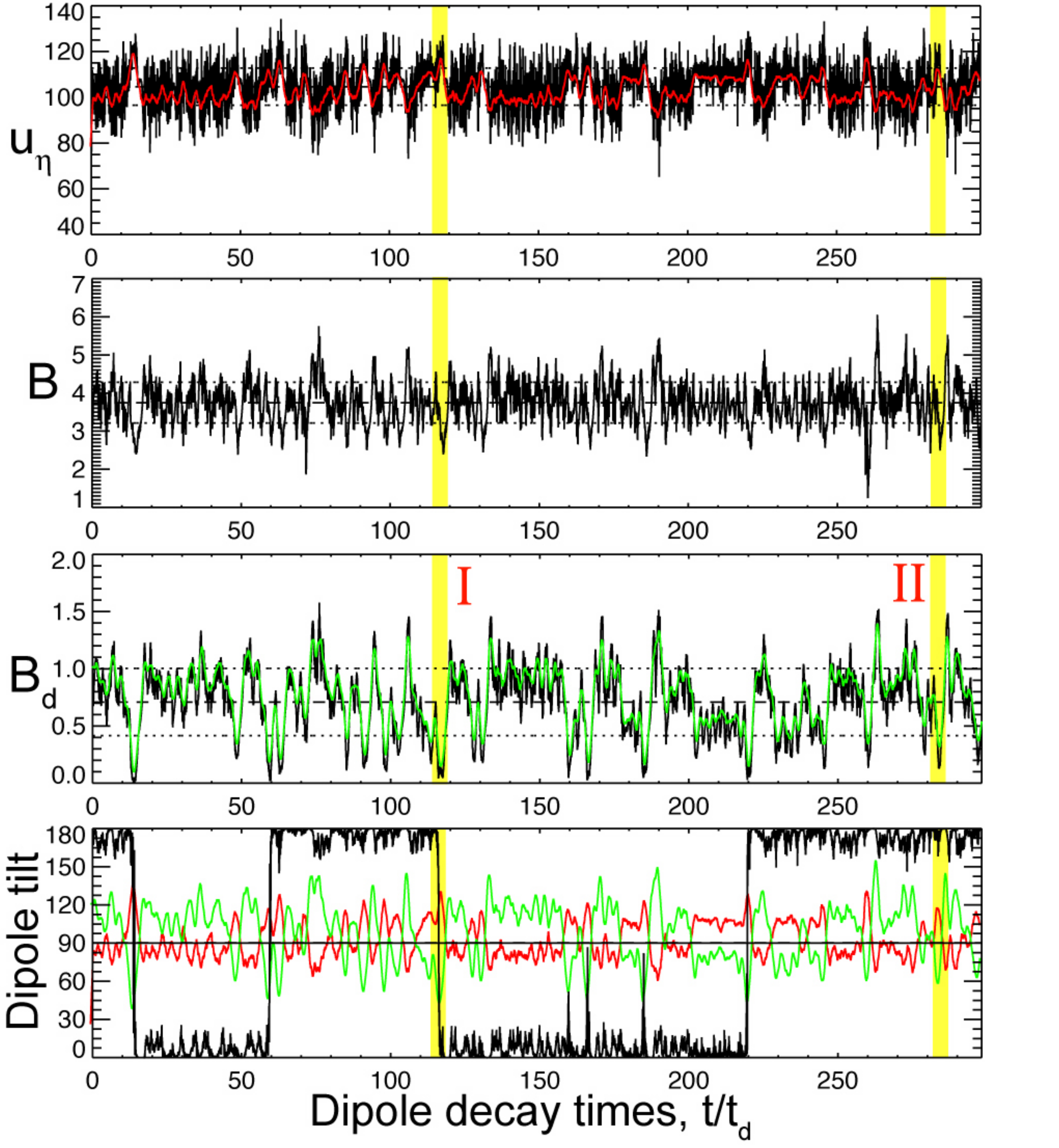
$$Ra = 1.9 \times 10^4$$

new scaling:

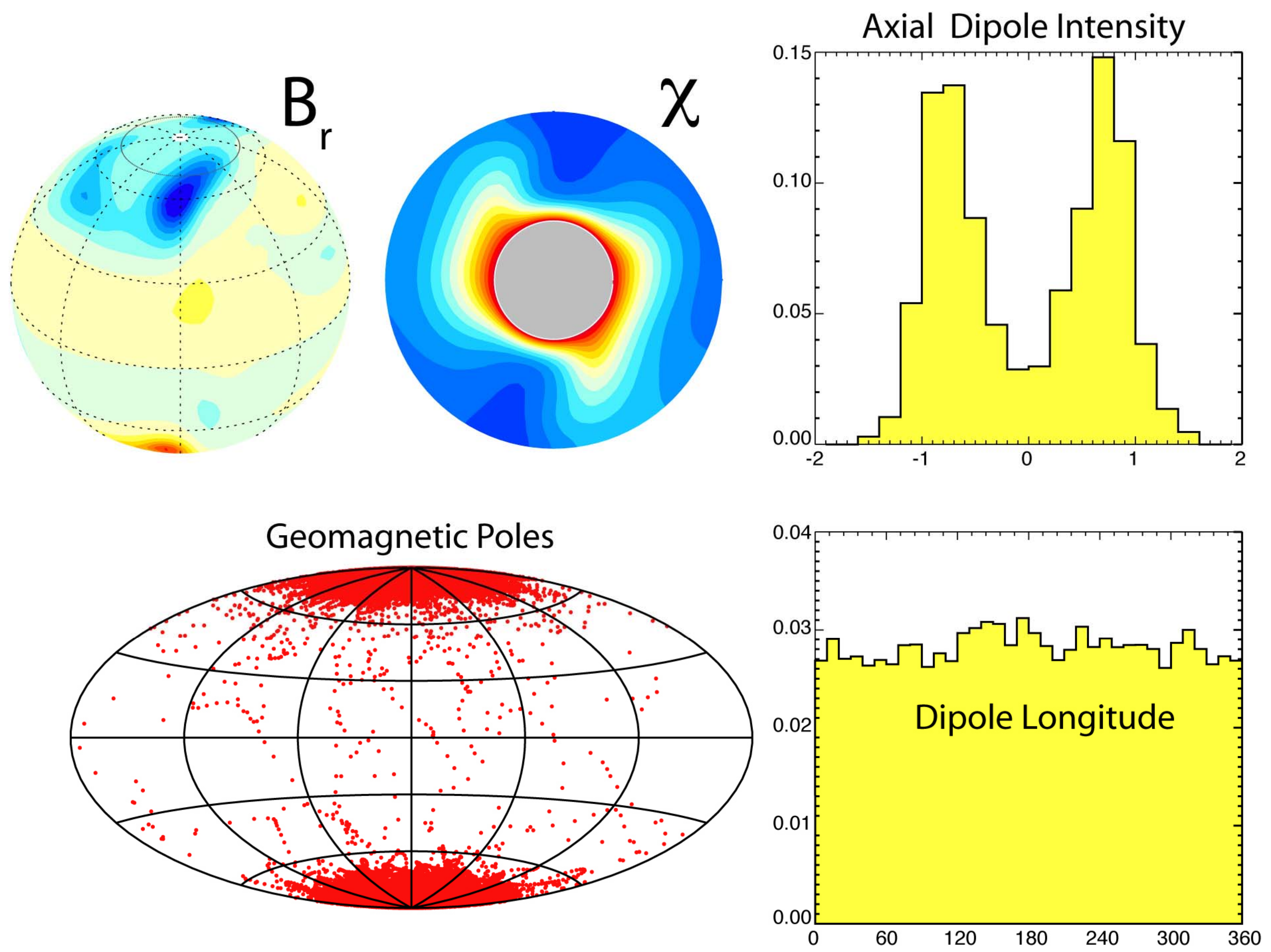
$$[u]_\eta = \frac{\eta}{D}$$

$$[B] = \sqrt{\frac{\rho\Omega}{\sigma}}$$

$$[t]_d = \frac{r_c^2}{\pi^2\eta}$$

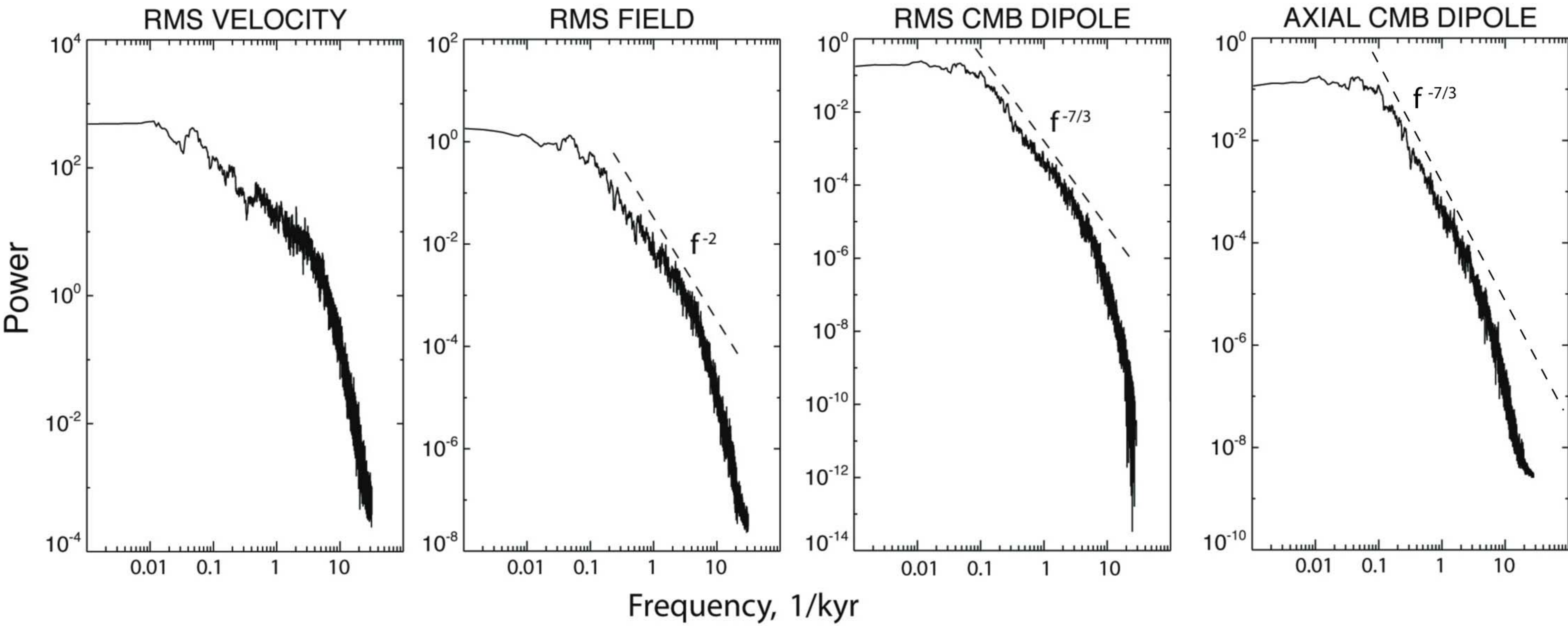


Olson, Driscoll & Amit , PEPI (submitted)



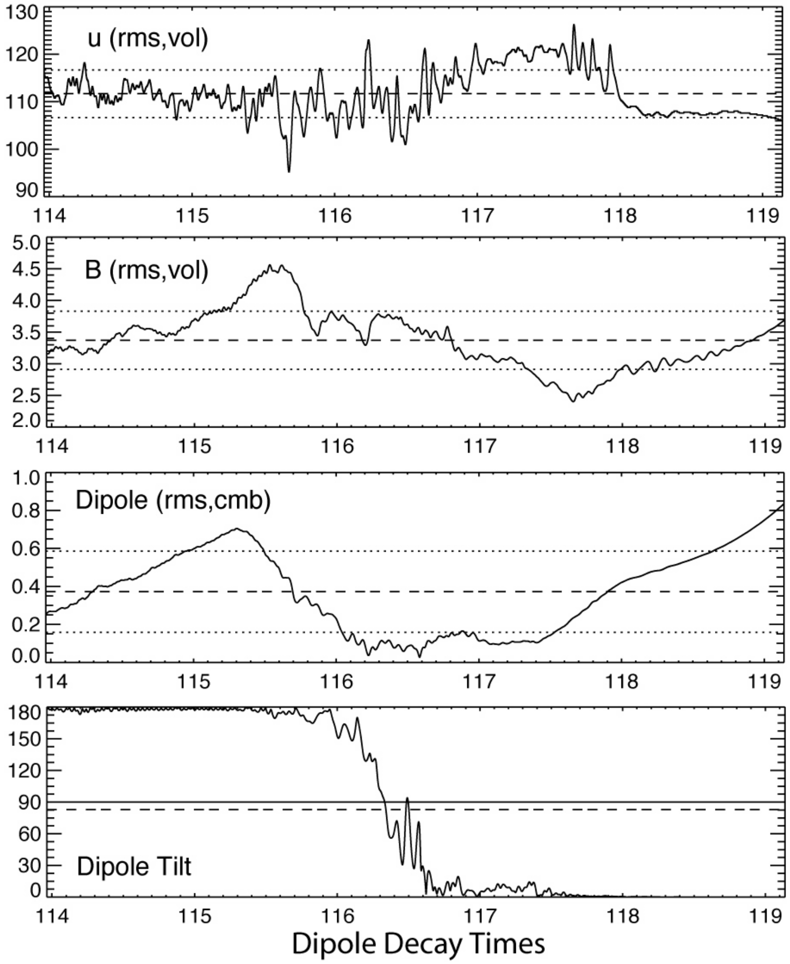
B
r
χ
Axial Dipole Intensity
0.15
0.10
0.05
0.00
-2
-1
0
1
2
Geomagnetic Poles
0.04
0.03
0.02
0.01
0.00
Dipole Longitude
0
60
120
180
240
300
360

Frequency Spectra

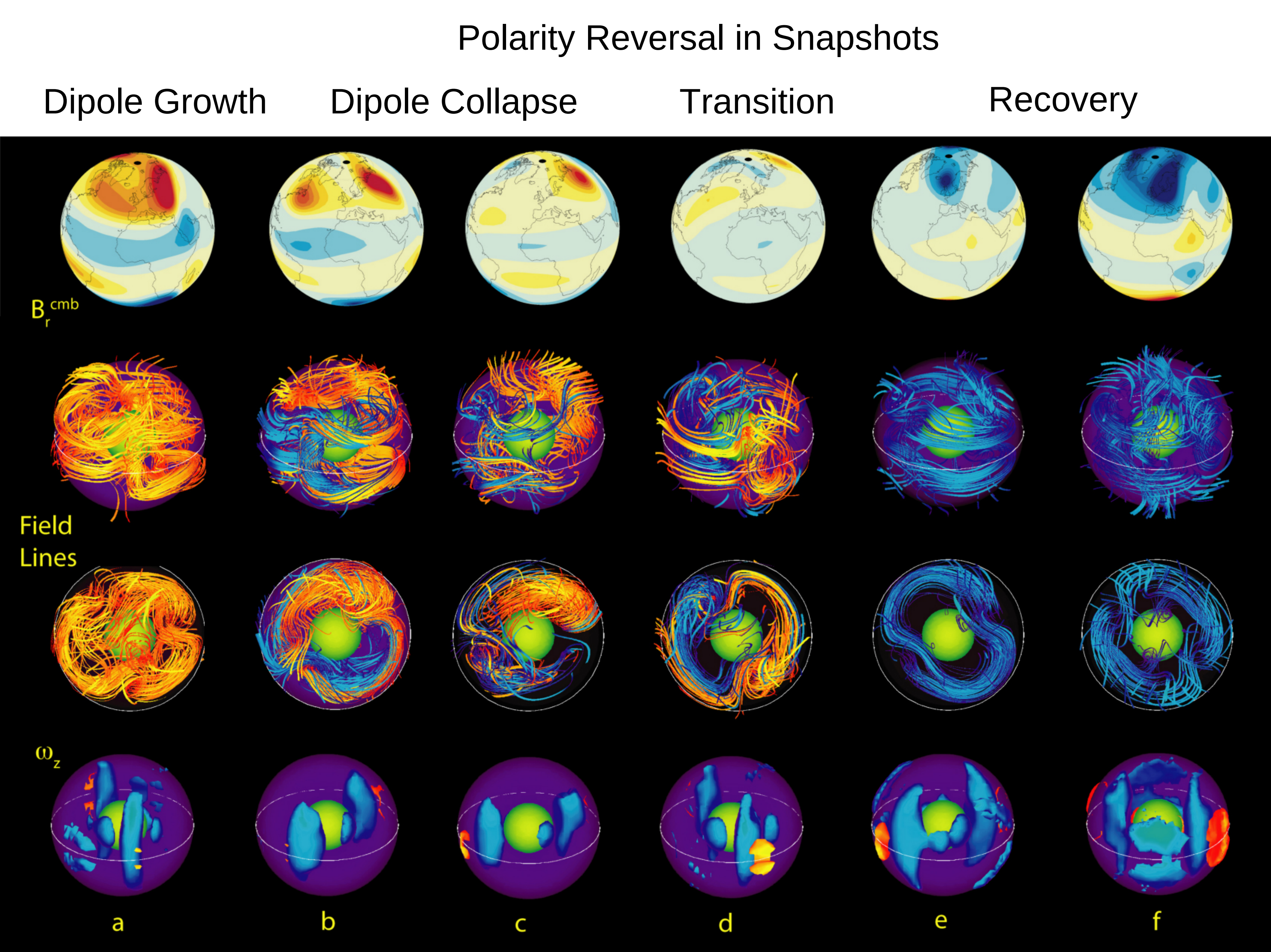


(for t_d = 20 kyr)

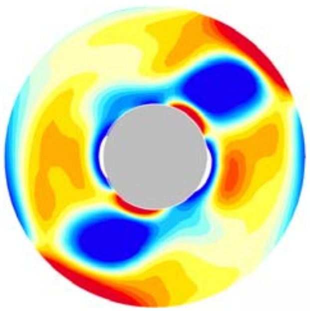
Dipole Collapse & Polarity Reversal



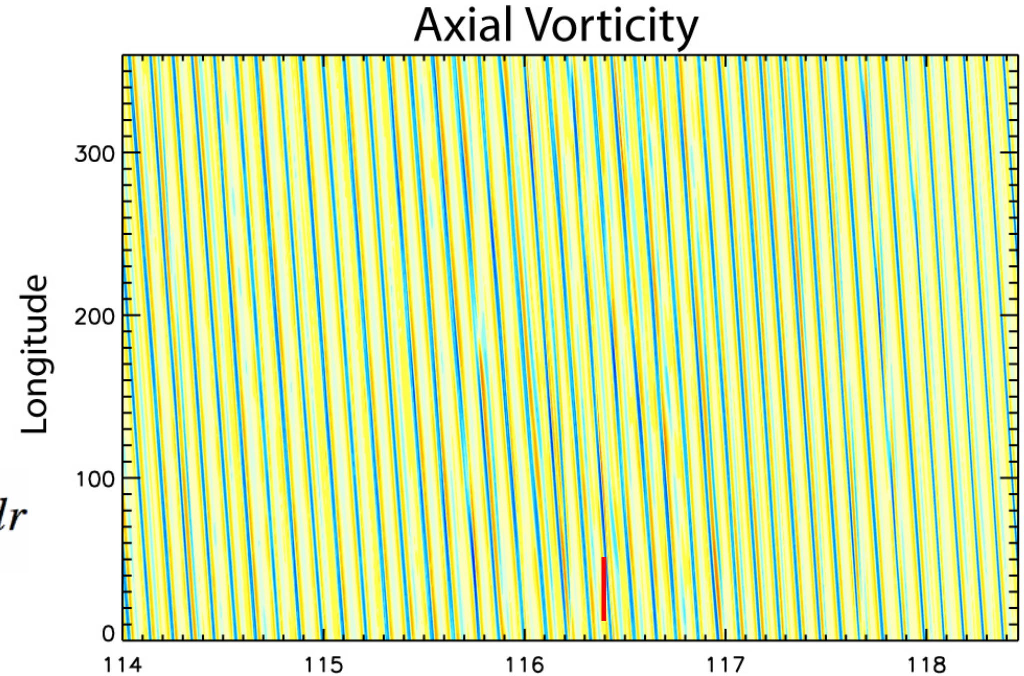
QuickTime™ and a
Sorenson Video 3 decompressor
are needed to see this picture.

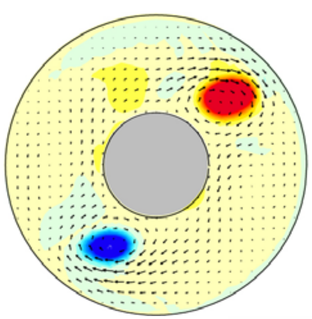

Polarity Reversal in Snapshots
Dipole Growth
Dipole Collapse
Transition
Recovery
B r cmb
Field Lines
ω z
a
b
c
d
e
f

Streak Diagrams

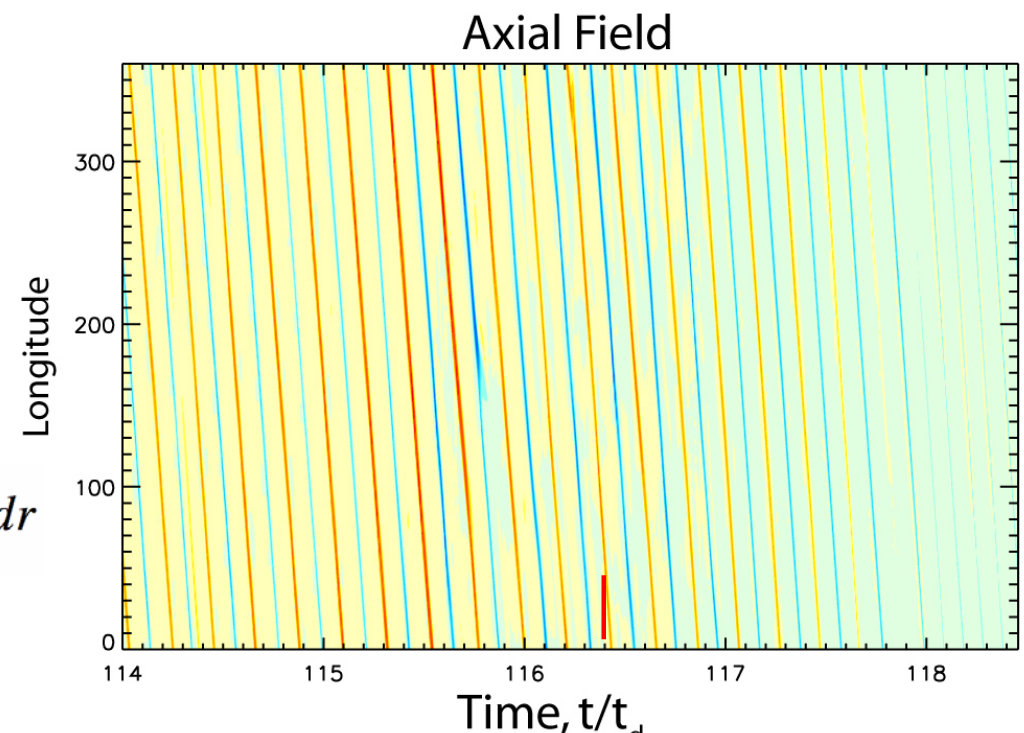


$$S_\omega(\phi,t) = \int_{r_i}^{r_o} \omega_z(z=0)\,r\,dr$$



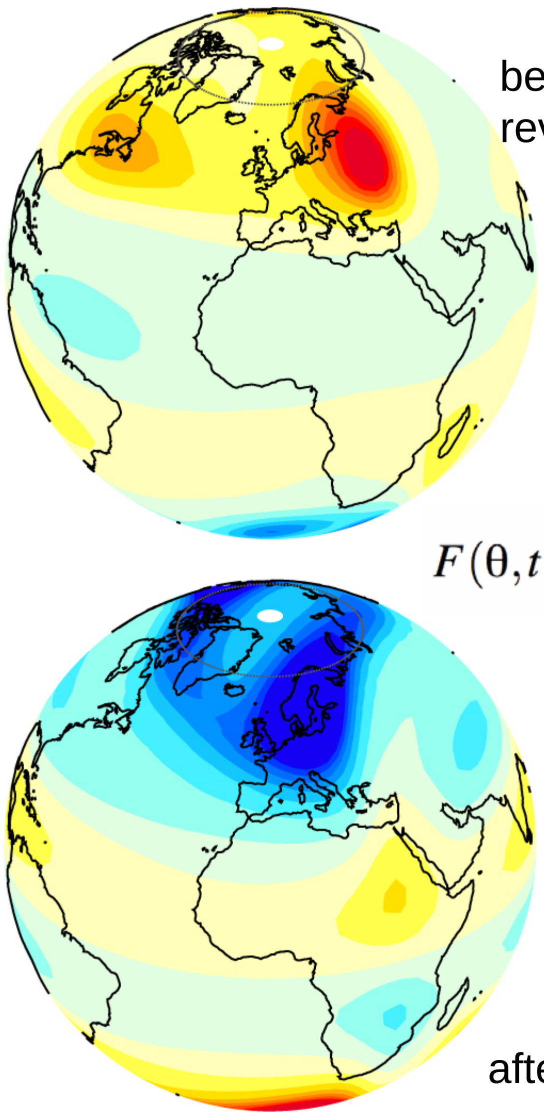


$$S_B(\phi,t) = \int_{r_i}^{r_o} B_z(z=0)\,r\,dr$$



Butterfly Diagrams

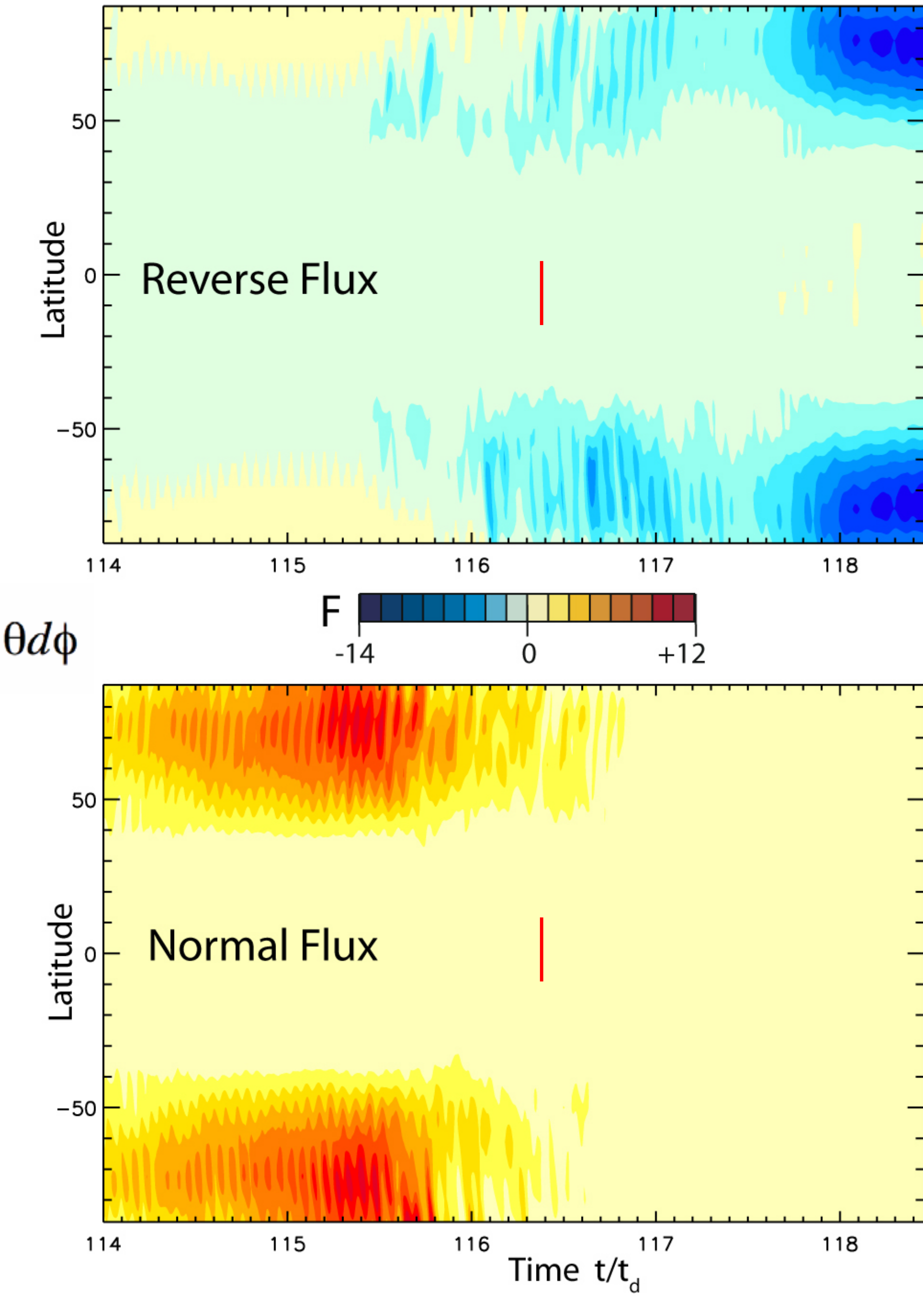
B_r (cmb)



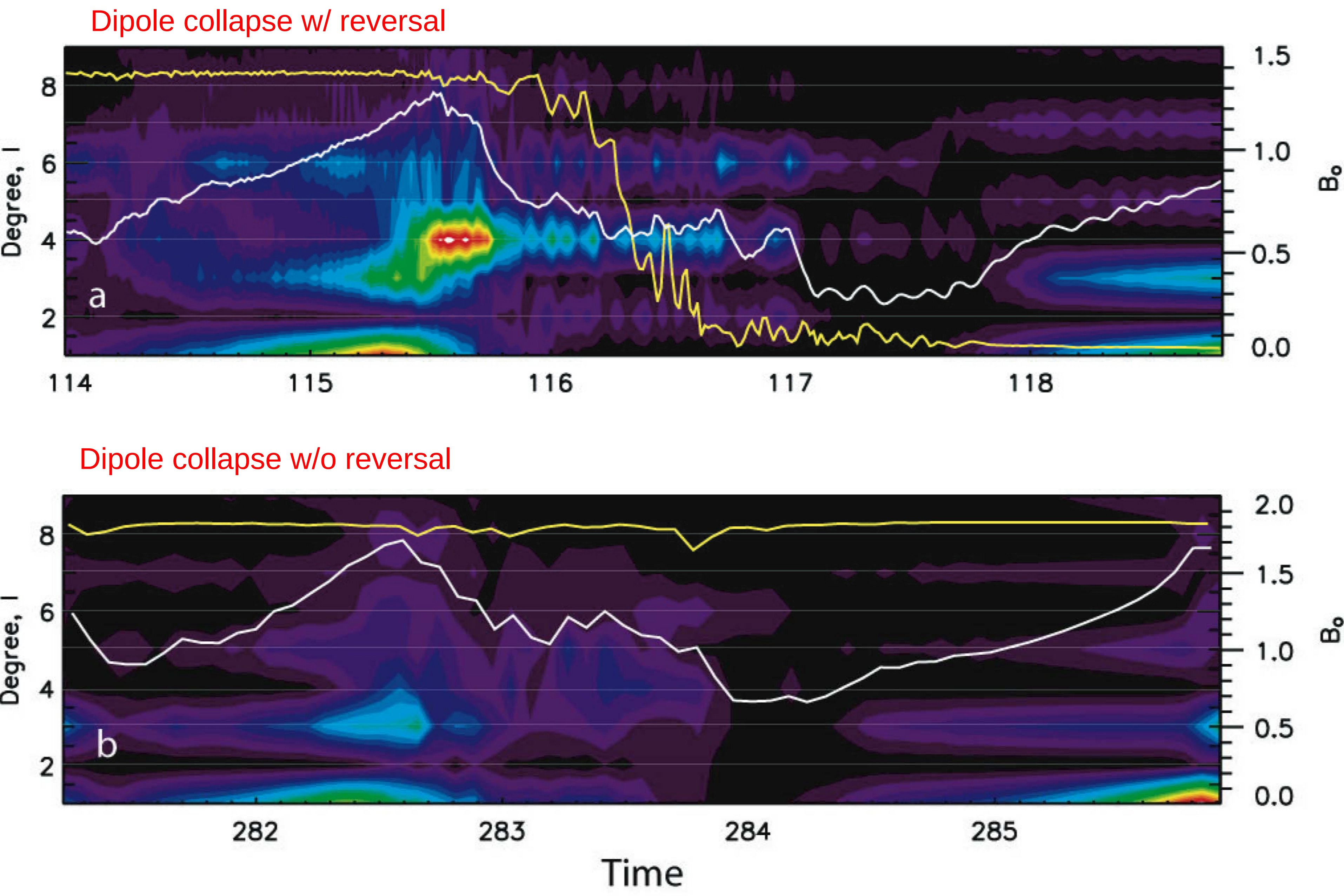
before reversal

after

$$F(\theta,t) = \int_0^{2\pi} B_r(r=r_o)\cos\theta \, d\phi$$



Lowes Spectra on the cmb versus Time



Summary

↸ Buoyancy flux saturation:
- o dipole moment $M \sim (Fd)^{1/3}$ [limited applicability]
- o convective velocity $U \sim (d/\Omega)^{1/5} F^{2/5}$

↸ Low frequency variability:
- o KE-ME tradeoff saturation
- o f^{-2} - $f^{-7/3}$ frequency spectra
- o dipole collapse events

↸ Polarity reversals:
- o follow dipole collapse
- o reversed flux precursors
- o energy spectrum cascade