Dynamo Theory
KITP, Santa Barbara
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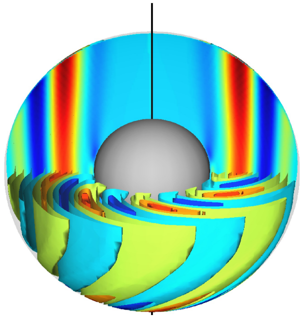

Bistability and hysteresis of dipolar dynamos generated by chaotic convection in rotating spherical shells

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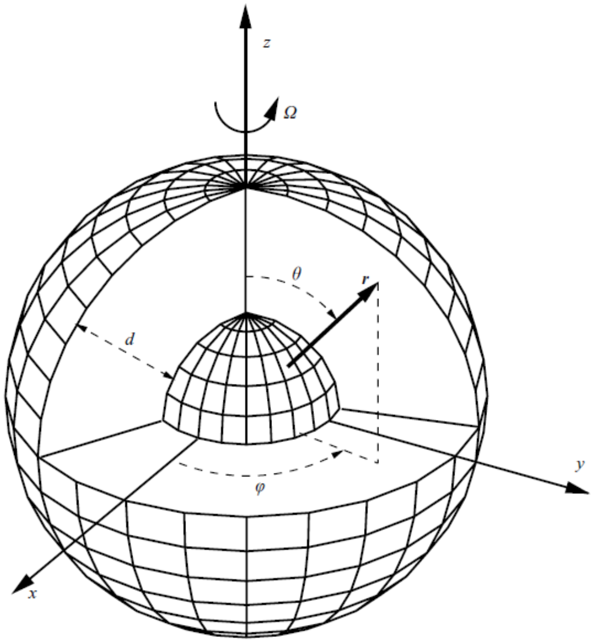
Convective spherical shell dynamos



Simitev & Busse, JFM ,2004

Model equations & parameters

Boussinesq approximation

$$\nabla \cdot \boldsymbol{u} = 0, \quad \nabla \cdot \boldsymbol{B} = 0,$$

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} =$$

$$-\nabla \pi - \tau \boldsymbol{k} \times \boldsymbol{u} + \Theta \boldsymbol{r} + \nabla^2 \boldsymbol{u} + \boldsymbol{B} \cdot \nabla \boldsymbol{B},$$

$$P \left(\partial_t \Theta + \boldsymbol{u} \cdot \nabla \Theta \right) = R\, \boldsymbol{r} \cdot \boldsymbol{u} + \nabla^2 \Theta,$$

$$P_m (\partial_t \boldsymbol{B} + \boldsymbol{u} \cdot \nabla \boldsymbol{B}) = P_m\, \boldsymbol{B} \cdot \nabla \boldsymbol{u} + \nabla^2 \boldsymbol{B}.$$

$$R = \frac{\alpha \gamma \beta d^6}{\nu \kappa}, \quad \tau = \frac{2 \Omega d^2}{\nu}, \quad P = \frac{\nu}{\kappa}, \quad P_m = \frac{\nu}{\lambda}$$

Basic state & scaling

$$T_S = T_0 - \beta d^2 r^2 / 2$$

$$\boldsymbol{g} = -d \gamma \boldsymbol{r}$$

Length scale:	d
Time scale:	d^2/ν
Temp. scale:	$\nu^2/\gamma \alpha d^4$
Magn. flux density:	$\nu (\mu \varrho)^{1/2}/d$

Boundary Conditions

$$\boldsymbol{r} \cdot \boldsymbol{u} = \boldsymbol{r} \cdot \nabla \boldsymbol{r} \times \boldsymbol{u}/r^2 = 0,$$

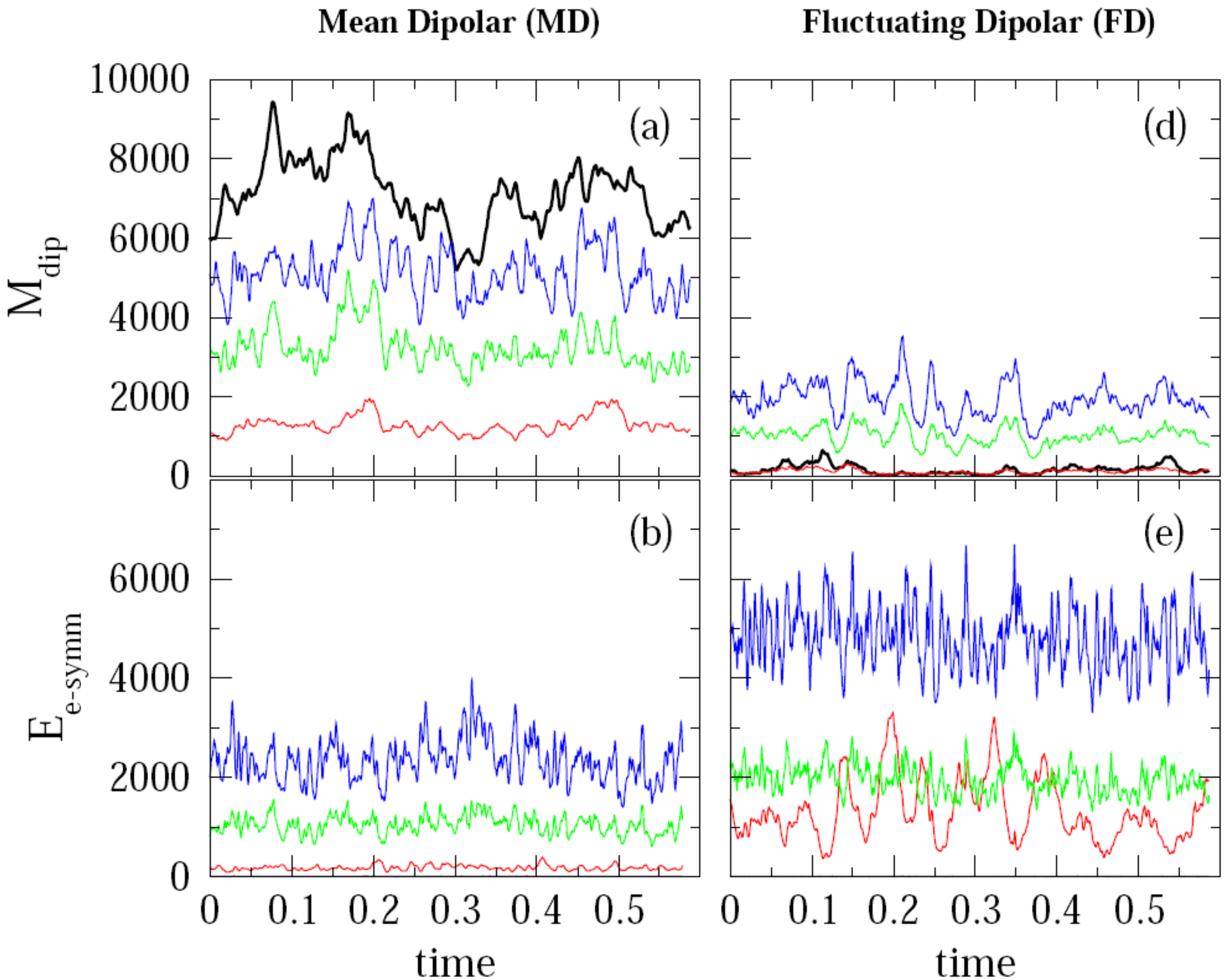
$$\hat{\boldsymbol{e}}_{\boldsymbol{r}} \cdot \boldsymbol{B}_{\text{int}} = \hat{\boldsymbol{e}}_{\boldsymbol{r}} \cdot \boldsymbol{B}_{\text{ext}},$$

$$\hat{\boldsymbol{e}}_{\boldsymbol{r}} \times \boldsymbol{B}_{\text{int}} = \hat{\boldsymbol{e}}_{\boldsymbol{r}} \times \boldsymbol{B}_{\text{ext}},$$

$$\Theta = 0, \text{ at } r = r_i \equiv 2/3 \text{ and } r_o \equiv 5/3$$

Two types of dipolar dynamos generated by chaotic convection

Energy densities



- ***Fully chaotic*** (***large-scale turbulent***) *regime.*
- ***Two chaotic attractors*** *for the same parameter values.*
- *Essential qualitative difference: contribution of the* ***mean poloidal dipolar energy***

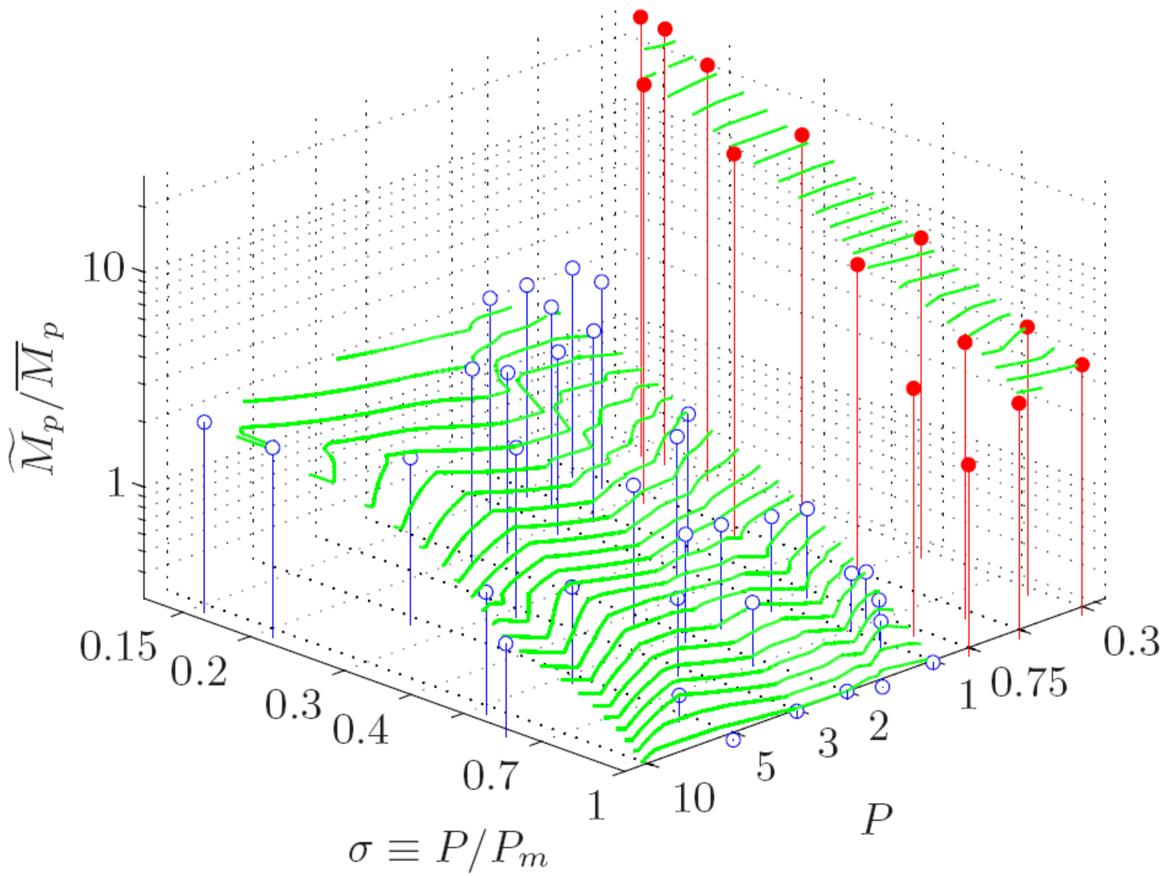
	(ab)	(de)
Rm	133.6	196.5
Mdip/Mtot	0.803	0.527

black.......***mean poloidal***
green......***fluctuating poloidal***
red.........***mean toroidal***
blue.........***fluctuating toroidal***

$R = 3.5 \cdot 10^6,\ \tau = 3 \cdot 10^4,\ P = 0.75 \text{ and } P_m = 1.5$

Regions and transition

Ratio of ***fluctuating to mean poloidal magn energy***



$$R = 3.5 \cdot 10^6, \ \tau = 3 \cdot 10^4$$

Two types of dipolar dynamos

Mean Dipolar (MD)

$$\widetilde{M}_p < \overline{M}_p$$

Fluctuating Dipolar (FD)

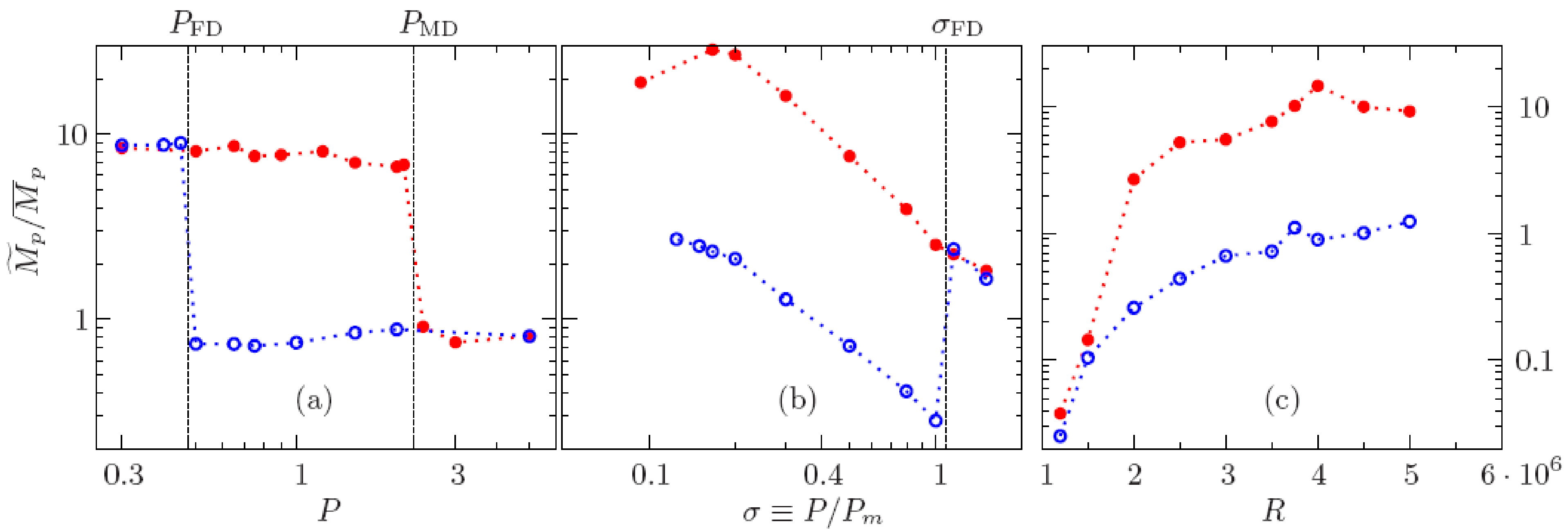
$$\widetilde{M}_p > \overline{M}_p$$

- *MD and FD dynamos correspond to rather* ***different chaotic attractors*** *in a fully chaotioc system*
- *The transition between them is not gradual but is an* ***abrupt jump*** *as a critical parameter value is surpassed.*
- *The nature of the transition is complicated.*

	MD	FD
Mdip/Mtot	(0.62,1)	(0.41,56)

Bistability and hysteresis in the MD <==> FD transition

Bistability and hysteresis *in the ratio of fluctuating poloidal to mean poloidal magn energy*



(a) $R = 3.5 \cdot 10^6 \quad P/P_m = 0.5$

(b) $R = 3.5 \cdot 10^6, \; P = 0.75$

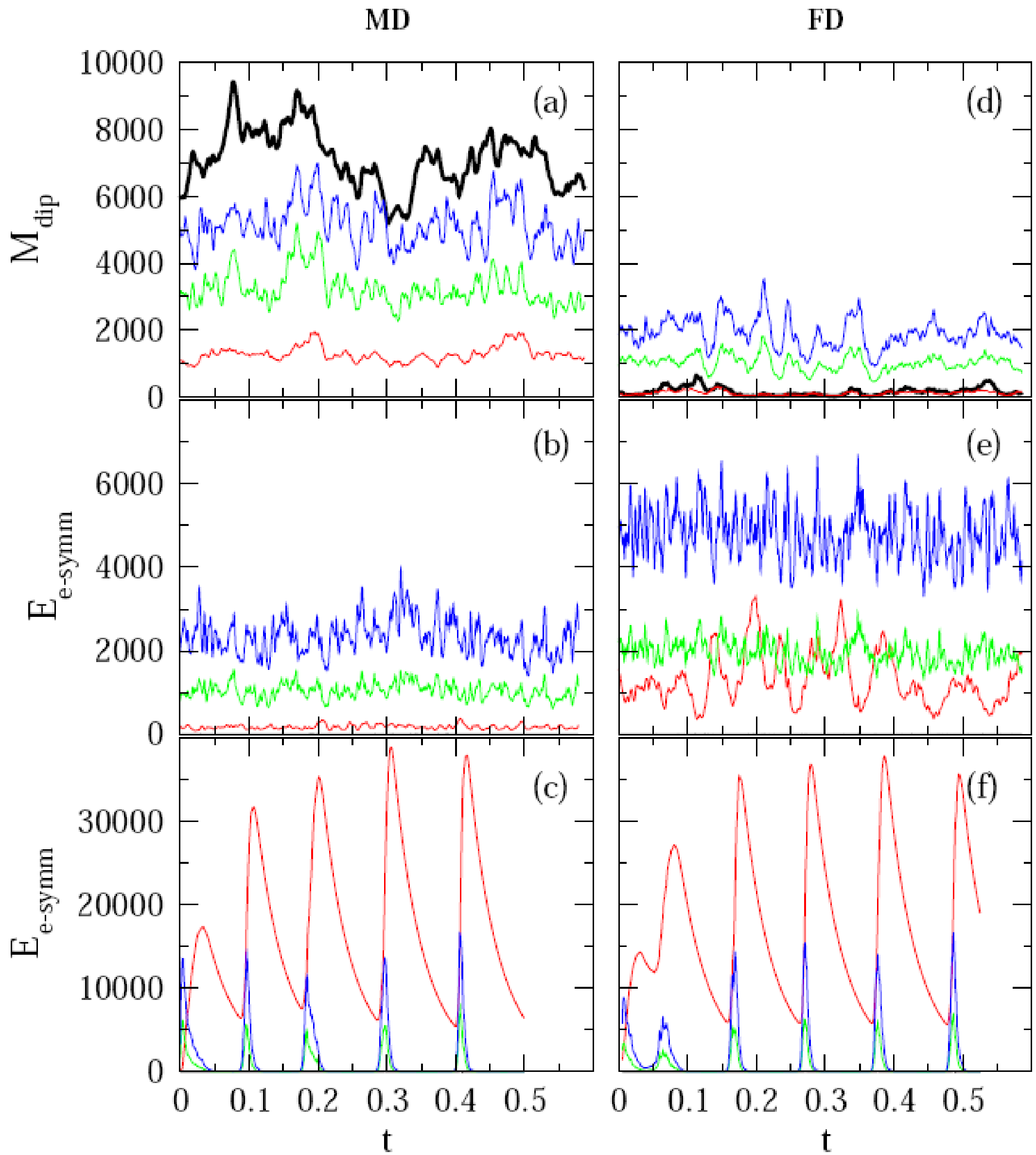
(c) $P = 0.75, \; P_m = 1.5$

in all cases: $\tau = 3 \cdot 10^4$

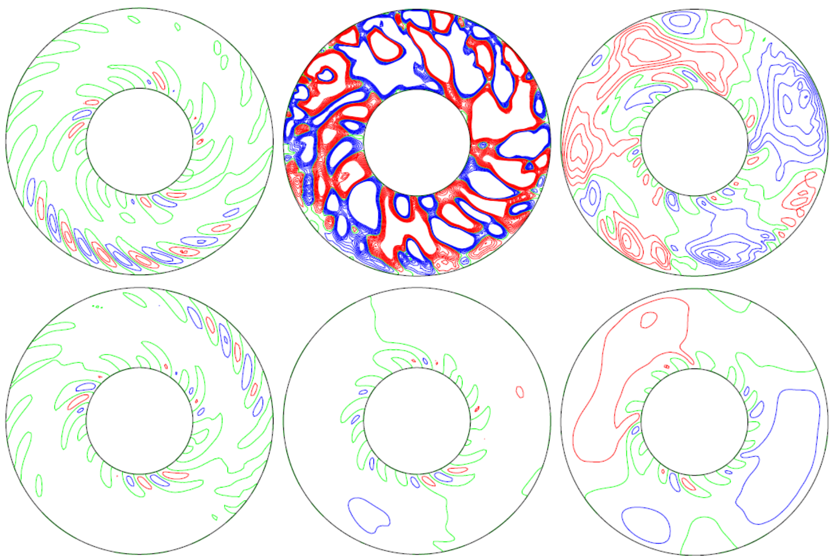
The coexistence is
not an isolated phenomenon
but can be traced with
variation of the parameters.

$P_{MD} = 2.2$ $\quad P_{FD} = 0.5$
$\sigma_{MD} = 0.07$ $\quad \sigma_{FD} = 1$

The hysteresis is a purely magnetic effect



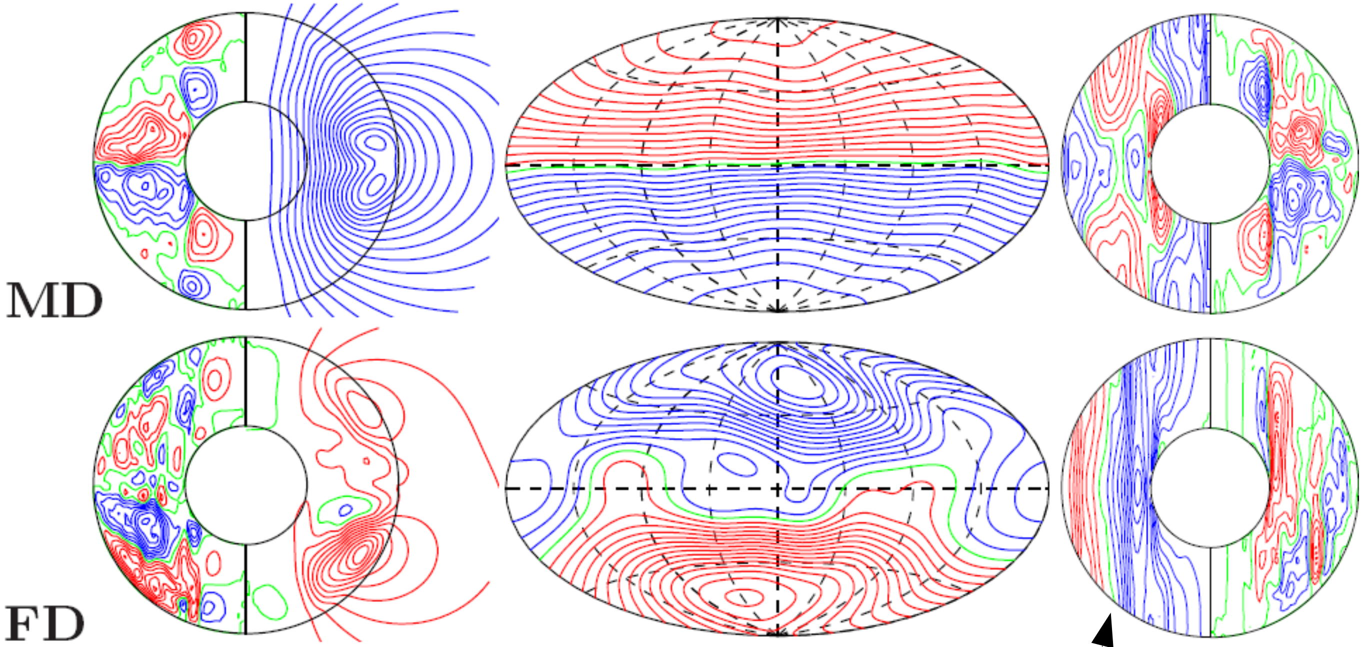
After magnetic field is suppressed both MD and FD dynamos equilibrate to statistically ***identical convective states*** *(period of relaxation oscillations, clockwise)*



Magnetic field is artificially suppressed, i.e. ***non-magnetic convection***

$$R = 3.5 \cdot 10^6, \ \tau = 3 \cdot 10^4 \quad P = 0.75 \text{ and } P_m = 1.5$$

A property comparison of MD and FD dynamos (Spatial structures)



Snapshots of spatial structures

$$R = 3.5 \times 10^6,\ \tau = 3 \times 10^4$$
$$P = 0.75 \text{ and } P_m = 1.5$$

The stronger magnetic field of MD dynamos counteracts differential rotation (diff rotation, meridional streamlines)

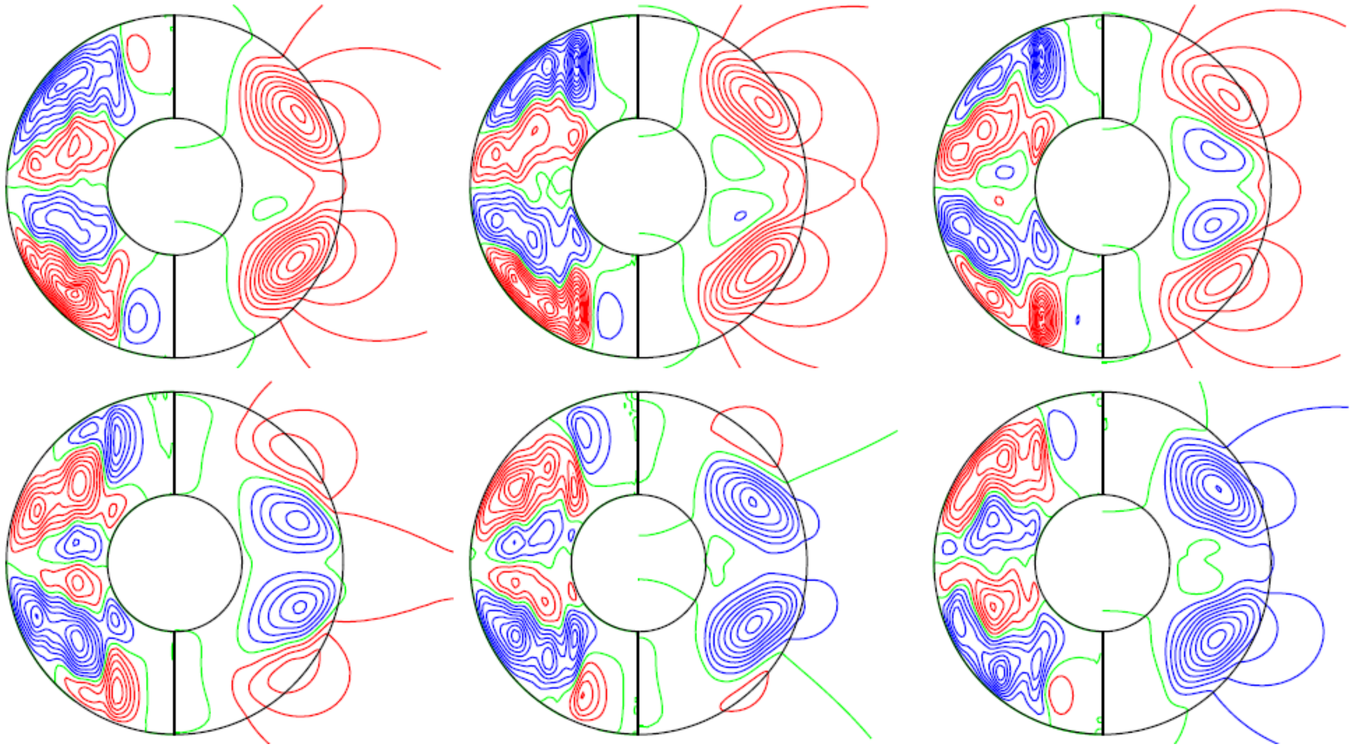
Both MD and FD dynamos appear dipolar from the outside (radial magn field at r=ro+1.3, Earth's surface)

FD dynamos have a somewhat more irregular and small-scale internal structure (Bphi and meridional fieldlines)

A property comparison of MD and FD dynamos (Temporal variations)

- ***Mean Dipolar*** *(MD) dynamos are* ***non-oscillatory.***
- ***Fluctuating Dipolar*** *(FD) dynamos are* ***oscillatory.***

Half-period of oscillation in a FD dynamo (row-by-row)



$$R = 3.5 \cdot 10^6,\ \tau = 3 \cdot 10^4,\ P = 0.75 \text{ and } P_m = 0.65$$

Conclusion

- *Two types of dipolar dynamos can be distinguished:*

    *) *Mean dipolar dynamos (MD)*

    *) *Fluctuating dipolar dynamos (FD)*

- *MD and FD dynamos have rather different properties.*
- *FD dynamos are normally oscillatory. In some cases this may lead to reversals.*
- *The transition between MD and FD dynamos is hysteretic.*
- *Most geodynamo simulations have typical parameters values* ***P in [0.5, 2], Pm in [0.5, 10] and R < 10 Rc which are within the observed hysteresis region****.*

Thank you!