Observable quantum information transitions with and without measurements

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Based on work and related collaborations with:

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New platforms offer access to previously hidden aspects of many body systems

**Entanglement entropy:**
Greiner Group (2018)

**Quantum measurement and scrambling:**
Google team (Nature 2019)

**String correlators (topo. order):**
Lukin, Greiner, Vuletic groups (2020)

**Continuous measurement:**
Measurement in quantum mechanics

\[ |\psi\rangle \mapsto \frac{\hat{P}_\mu |\psi\rangle}{\sqrt{\langle \psi |\hat{P}_\mu |\psi\rangle}} \]

- Measurements can destroy quantum correlations
- But can also create larger scale correlations

How do these effects manifest in many-body systems?

New phenomena from partial measurement of correlated states?

\[ |\Psi\rangle = c_1 |\Psi_1\rangle + c_2 |\Psi_2\rangle + \cdots + c_{2^N} |\Psi_{2^N}\rangle \]
This talk

1. A brief intro to measurement induced phase transitions. Hard to observe.
   Is Alice encoded in the output despite Eve?

2. An observable information transition:
   Can Eve decode Alice from her part of the wavefunction?

3. How do measurements affect critical correlations in quantum ground states
Monitored system ≠ open system

1. System coupled to a bath
   • Describes usual decoherence processes.
   • System gets entangled with environment d.o.f.
   • A pure initial state generally becomes mixed.
   • Density matrix evolves by the Lindblad equation (for a Markovian bath)

2. Monitored system
   • A pure initial state remains pure.
   • Time evolution described as an ensemble of pure state trajectories
   • Observer must make use of the gained information (post-selection or feedback)
   • If measurements are discarded then the observer is no different from a bath.
Measurement induced phase transition in hybrid quantum circuits

Random unitary gates

$$|\psi\rangle \rightarrow \prod_x U_{x,t} |\psi\rangle$$

Single qubit measurements applied with probability $p$:

$$|\psi\rangle \rightarrow \frac{\hat{P}_\mu |\psi\rangle}{\sqrt{\langle \psi | \hat{P}_\mu |\psi\rangle}}$$

Project on outcome $\mu$ with probability $\langle \hat{P}_\mu \rangle$
Entanglement entropy for a given instance of measurement outcomes

$$\tilde{m} = \{m_1, m_2, m_3, \ldots\}$$

Entanglement entropy for a given instance of measurement outcomes

$$\langle S_A \rangle_u = \langle \sum_{\tilde{m}} p_{\tilde{m}} S_{A, \tilde{m}} \rangle_u$$

Ensemble averaged entanglement entropy of a sub-system after a long time \( t \approx L \rightarrow \infty \):
Numerical evidence for a phase transition

Skinner, Ruhman, Nahum PRX 2019; Li, Chen, Fisher PRB 2018, PRB 2019

\[ \langle S_A \rangle_u = \left\langle \sum_{\hat{m}} p_{\hat{m}} S_{A,\hat{m}} \right\rangle_u \]

Entanglement entropy for a given instance of measurement outcomes

[Li, Chen and Fisher PRB 2019]:

Measure with probability p
Mapping to an effective classical “spin model”

\[ U \otimes U^* \otimes U \otimes U^* = \]

\[ \sum_m p_m \text{tr}(\rho_A^2) \sim \]

Entanglement entropy = free energy of a domain wall

Volume law phase = ferromagnet

Area law phase = paramagnet

\[ S^{(2)}(A) = F^{(2)}_{dw} - F^{(2)}_0 \]

\[ \Delta F^{(2)}_{dw} \sim L \]

\[ \Delta F^{(2)}_{dw} \sim \text{const} \]

Bao, Choi, EA PRB 2020; Jian, You, Vasseur, Ludwig PRB 2020
(Based on earlier work on unitary circuits by Nahum, Vijay Haah PRX 2018)
Understanding in terms of quantum information encoding

Choi, Bao, Qi and EA, PRL 2020; Bao, Choi and EA PRB 2020; Gullans and Huse PRX 2020

Phase transition in the quantum channel capacity:

\[ I_c(Alice \rightarrow Bob) \]

From the perspective of the observer Eve:

transition in the information Eve can extract on the initial state

\[ \mathcal{F}(Eve|Alice) \]  (Fisher information)

Observing the transition is fundamentally hard because it requires post-selecting on an extensive number of measurement outcomes.
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Scrambling phase transition in a random circuit

[Zack Weinstein, Shane Kelly, Jamir Marino, EA (Unpublished)]

Scrambling circuit:

Operator growth:

Operator growth measured by the OTOC:

\[ C_r(t) = \langle [X_0(t), Y_r]^2 \rangle \]

Operator size:

\[ n(t) = \sum_r C_r(t) \]
Scrambling phase transition in a random circuit

Scrambling circuit:

Operator growth measured by the OTOC:

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Unitary circuit coupled to ancillas

With probability $p$ apply a swap with ancilla:

$$C_r(t) = \langle [X_0(t), Y_r]^2 \rangle$$

The swaps kill branches of the tree leading to a directed-percolation transition in the OTOC at a critical value $p=p_c$. 

$p = 0.1$  

$p = 0.2$  

$p = 0.3$
Unitary circuit coupled to ancillas

Collapse of the critical operator density profile at different times using directed percolation exponents:

\[ C_r(t) \cdot t^{\eta} \]

\[ \eta = 0.319 \]

\[ z = 1.58 \]
Unitary circuit coupled to ancillas

Data collapse of the operator density versus time off criticality ($\rho(t)$ for different values of $p$):

$p < p_c$

$p > p_c$
For $p > p_c$ Eve can decode Alice with perfect fidelity using this scheme:

$$
\mathcal{F} = \text{tr} \left( |\Psi_{AS_1'}\rangle \langle \Psi_{AS_1'}| \mathcal{U}_{S'E}^{\dagger} \mathcal{U}_{SE}^{\dagger} \rho_{ASES'} \mathcal{U}_{SE} \mathcal{U}_{S'E} \right)
$$
Transition in the quantum channel capacity

\[ I_c(Alice : Eve) = S_E - S_{AE} \]

Only if eve has zero knowledge of her initial state. Otherwise, the quantum channel capacity always saturates to the maximum value.
How do measurements affect quantum critical correlations?


Given a quantum state measure a subset of the qubits, then evaluate the correlations in the resulting state. **There is no dynamics!**

Example: detection of particles in a 1d quantum liquid
One dimensional quantum liquids

- Universal long wavelength description: Luttinger liquid

Dual descriptions in terms of phase or density fluctuations

\[ S = \frac{K}{2\pi} \int dx d\tau \left[ \dot{\theta}^2 + (\nabla \theta)^2 \right] \]

\[ S = \frac{1}{2\pi K} \int dx d\tau \left[ \dot{\phi}^2 + (\nabla \phi)^2 \right] \]

\[ \delta n(x) \approx -\frac{1}{\pi} \nabla \phi(x) + \frac{1}{\pi} \cos\left[2\pi \rho_0 x - 2\phi(x)\right] \]

K=1 corresponds to non interacting fermions or hard-core bosons.
K<1: fermions with repulsive interactions or bosons with longer range repulsion
One dimensional quantum liquids

- Quantum critical states with continuously tunable exponents.

Realized with ultracold atoms in optical lattices:

Density correlations:

\[
\langle \Psi_{gs} | \delta n(x) \delta n(0) | \Psi_{gs} \rangle \sim c_1 \left( \frac{1}{x} \right)^2 + c_2 \cos(2\pi \rho_0 x) \left( \frac{1}{x} \right)^{2K} 
\]

\[
\langle \nabla \phi(x) \nabla \phi(0) \rangle 
\]

\[
\langle e^{i(2\phi(x) - 2\phi(0))} \rangle 
\]

Phase correlations:

\[
\langle \Psi_{gs} | \psi(x) \psi(0) | \Psi_{gs} \rangle \sim \langle e^{i(\theta(x) - \theta(0))} \rangle \sim \left( \frac{1}{x} \right)^{\frac{1}{2K}} 
\]
Quantum Non-Demolition Measurement

- Weak measurement of the density everywhere: probe light interacts weakly with the particles.
- Measure polarizations of photons at different locations.
- Outcome $|1\rangle = $ particle found at this location
  Outcome $|0\rangle = $ “No click”. Particle occupation remains indefinite

How does the partial/weak measurement affect the critical correlations?

$$\langle \Psi_{gs} | P_m n(x)n(0) P_m | \Psi_{gs} \rangle \sim c_1 \left( \frac{1}{x} \right)^2 + c_2 \cos(2\pi \rho_0 x) \left( \frac{1}{x} \right)^{2K}$$

$$\langle \Psi_{gs} | P_m \psi^\dagger(x)\psi(0) P_m | \Psi_{gs} \rangle \sim \left( \frac{1}{x} \right)^{\frac{1}{2K}}$$

Important: there is no dynamics. Perform measurements then evaluate correlations in the output state.
Warmup: post-select on the null measurement outcome (no clicks)

The no click state:

\[ |\Psi_{nc}\rangle = e^{-\int dx v(x)n(x)} |\Psi_{gs}\rangle \]

If the measurement strength \( v(x) \) is oscillating in space at a wavelength commensurate with the particle density, then we can represent it in terms of the long-wavelength fields:

\[ |\Psi_{nc}\rangle = e^{-v \int dx \cos[2\phi(x)]} |\Psi_{gs}\rangle \]
Warmup: post-select on the null measurement outcome (no clicks)

Correlations in the no click state:

\[
\langle n(x)n(0) \rangle_{\text{nc}} = \lim_{\beta \to \infty} \langle \Psi_{\text{ref}} | e^{-\beta H_{\text{LL}}} e^{-v \int dx \cos(2\phi)} n(x)n(0) e^{-v \int dx \cos(2\phi)} e^{-\beta H_{\text{LL}}} | \Psi_{\text{ref}} \rangle
\]

\[
= \int \mathcal{D}\phi \, e^{-S_{\text{nc}}[\phi]} \, \delta n(x) \delta n(0)
\]
The “no click” action

\[ \langle n(x)n(0) \rangle_{nc} = \int D\phi e^{-S_{nc}} n(x)n(0) \]

\[ S_{nc} = \frac{1}{2\pi K} \int dx d\tau \left[ (\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] - \nu \int dx \cos(2\phi) \]

= Wick rotated impurity problem [Kane and Fisher PRL 1992]
Phase transition tuned by the Luttineger parameter $K$

$$S_{nc} = \frac{1}{2\pi K} \int dx d\tau \left[ (\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] - v \int dx \cos(2\phi)$$

Scaling of the measurements:

$$\frac{dv}{d\ell} = (1 - K) v$$

$K > 1$ Measurements are irrelevant

Long distance correlations unaffected for any finite measurement strength

Phase:

$$\langle e^{i[\hat{\theta}(x) - \hat{\theta}(0)]} \rangle \sim x^{-1/(2K)}$$

Smooth component of the density:

$$\langle \nabla \hat{\phi}(0) \nabla \hat{\phi}(x) \rangle \sim x^{-2}$$
Phase transition tuned by the Luttinger parameter $K$.

$$S_{nc} = \frac{1}{2\pi K} \int dx d\tau \left[ (\partial_\tau \phi)^2 + (\partial_x \phi)^2 \right] - v \int dx \cos(2\phi)$$

Scaling of the measurements:

$$\frac{dv}{d\ell} = (1 - K) v$$

$K < 1$ Measurements are **relevant**

Non perturbative effect on long distance correlations:

$$\langle e^{i[\hat{\theta}(x)-\hat{\theta}(0)]} \rangle \sim x^{-1/(2K)} \rightarrow x^{-1/K}$$

$$\langle \nabla \hat{\phi}(0) \nabla \hat{\phi}(x) \rangle \sim x^{-2} \rightarrow x^{-2/K}$$

Performing local measurements has a highly non-local effect on the quantum state!
To implement the average over measurement outcomes we need to introduce N replicas taking the limit N \to 1 at the end:

\[ s_N[\{\varphi_\alpha\}] = \sum_{\alpha=1}^{N} s[\varphi_\alpha] - \frac{\mu}{2N\pi^2} \int dx \sum_{\alpha<\beta} \cos[2(\varphi_\alpha - \varphi_\beta)] \]

The random measurement outcomes couple the replicas attempting to lock them to each other.

Need to look at quantities that are non-linear in the density matrix, e.g.:

\[ C^{(2)}(r) \equiv \sum_{m} p_m |\langle \psi_m | e^{i(\theta(r)-\theta(0))} | \psi_m \rangle|^2 \]

Perturbative RG reveals a transition at K=1/2 in this case.
How to observe the transition

Nonlinear in $\rho_m \Rightarrow$ unobservable

$$C^{(2)}(r) \equiv \sum_m p_m \left| \langle \psi_m | e^{i(\theta(r)-\theta(0))} | \psi_m \rangle \right|^2$$

Alternative linear quantity:

$$C_w(r) = \sum_m p_m w_m \langle \psi_m | e^{i(\theta(r)-\theta(0))} | \psi_m \rangle$$

The weight $w_m$ utilizes knowledge of the measurement outcomes.

If the correlation is calculable classically then a natural choice for $w_m$ is:

$$w_m = \langle e^{i(\theta(r)-\theta(0))} \rangle_m^{\text{calc}}$$

Observe the transition through the quantum-classical estimator:

$$C_{CQ}(r) = \sum_m p_m \langle e^{i(\theta(r)-\theta(0))} \rangle_m^{\text{calc}} \langle e^{i(\theta(0)-\theta(0))} \rangle_m$$
Summary

- **MIPT = “encoding transition”**
  Hard to observe even with a perfect quantum computer (post-selection problem)

- **Scrambling transition = “decoding transition”**
  Observable using a quantum computer e.g. by measuring OTOC (no post-selection)

- Partial local measurement of a quantum critical state can alter the long-distance critical correlations.
  - Mapping to boundary critical phenomena
  - An observable quantum-classical estimator:

\[
C_{CQ}(r) = \sum_m p_m \langle e^{i(\theta(r) - \theta(0))} \rangle_m \langle e^{i(\theta(r) - \theta(0))} \rangle_m
\]
Outlook

• Decoding transitions in different universality classes?

• Is it possible to show quantum advantage using the quantum classical estimators? Instances for which:

\[ C_{CC}(r) = \sum_{m} p_m \langle e^{i(\theta(r) - \theta(0))} \rangle_m \langle e^{i(\theta(r) - \theta(0))} \rangle_m \]

\[ C_{CQ}(r) = \sum_{m} p_m \langle e^{i(\theta(r) - \theta(0))} \rangle_m \langle e^{i(\theta(r) - \theta(0))} \rangle_m \]

In situations we now understand there is only a polynomial gap between these estimators