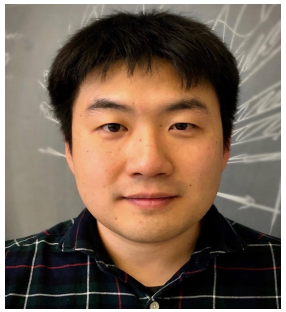
# Observable quantum information transitions with and without measurements

Ehud Altman – UC Berkeley

Based on work and related collaborations with:

Yimu Bao

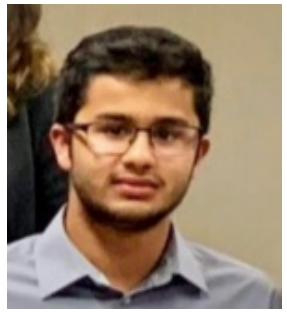
Zack Weinstein



Soonwon Choi

Sam Garratt

Anasuya Lyons



Rohith Sajith

QSA @LBNL

QLCI @Berkeley

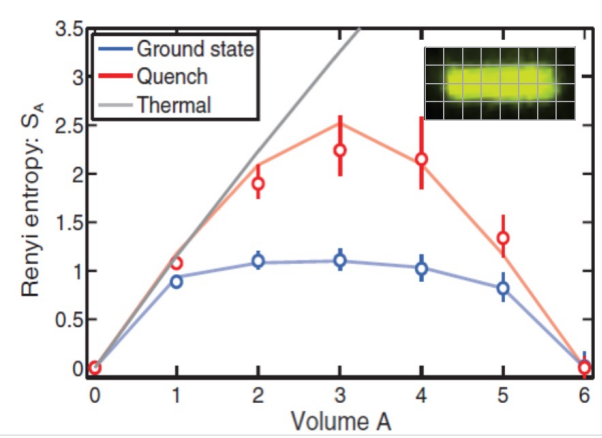
Shane Kelly

Jamir Marino

# New platforms offer access to previously hidden aspects of many body systems

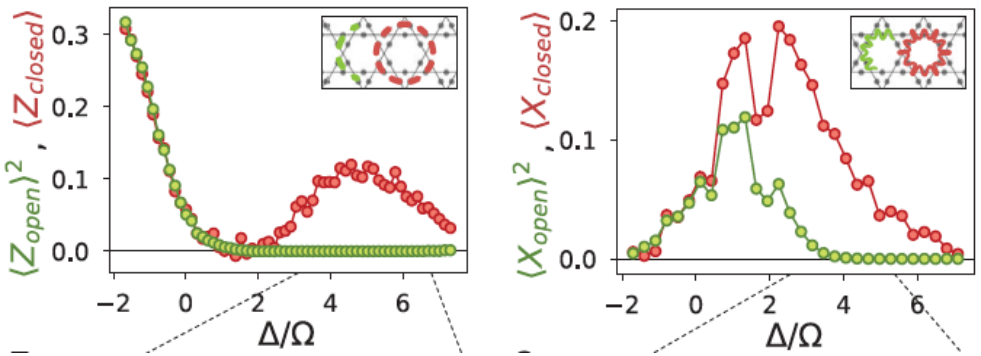
**Entanglement entropy:**

Greiner Group (2018)



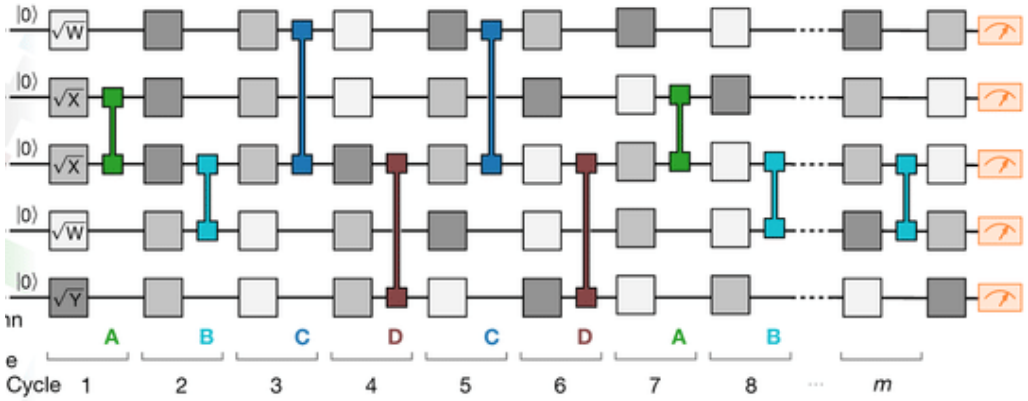

**String correlators (topo. order):**

Lukin, Greiner, Vuletic groups (2020)

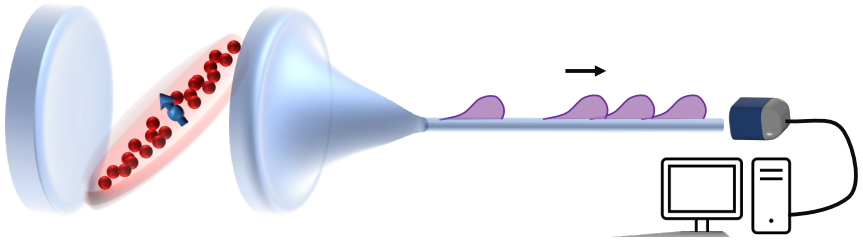



**Quantum measurement and scrambling:**

Google team (Nature 2019)



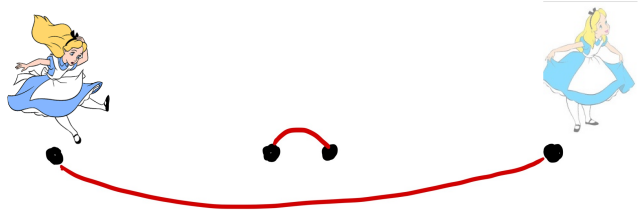

Continuous **measurement**:



# Measurement in quantum mechanics

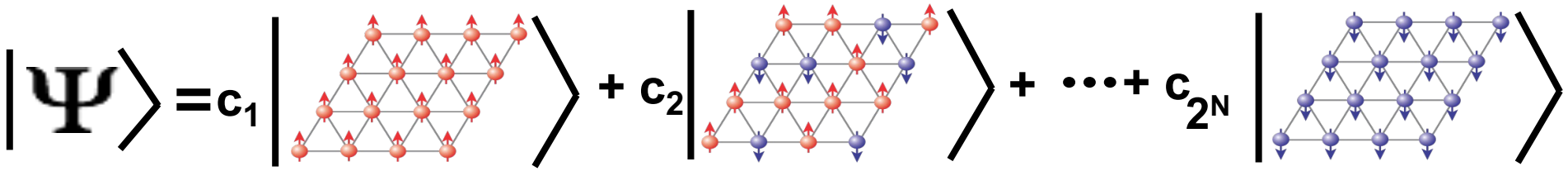
$$|\psi\rangle \mapsto \frac{\hat{P}_\mu|\psi\rangle}{\sqrt{\langle\psi|\hat{P}_\mu|\psi\rangle}}$$


- Measurements can destroy quantum correlations
- But can also create larger scale correlations



**How do these effects manifest in many-body systems?**

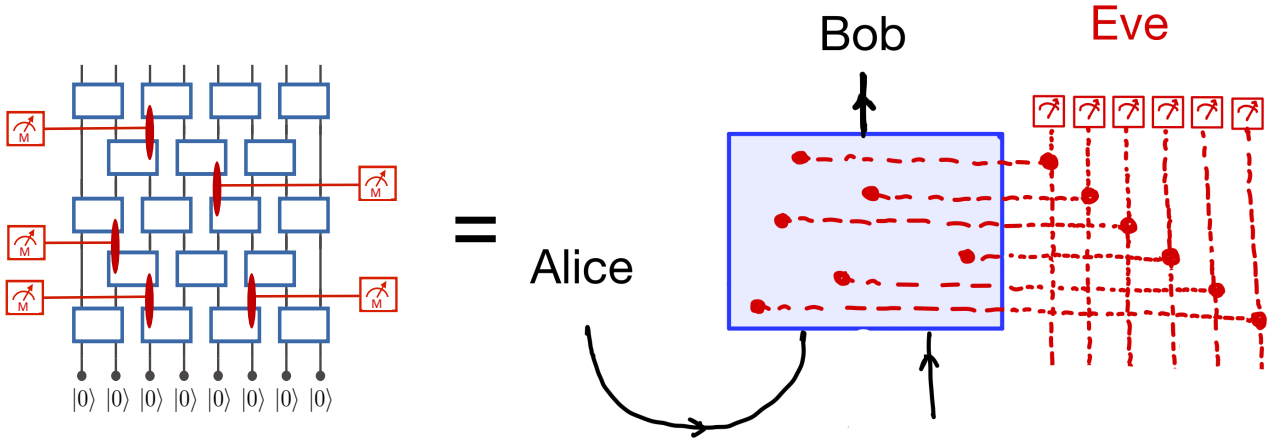
**New phenomena from partial measurement of correlated states?**



# This talk

1. A brief intro to measurement induced phase transitions. Hard to observe.

   Is Alice encoded in the output despite Eve?




2. An observable information transition:

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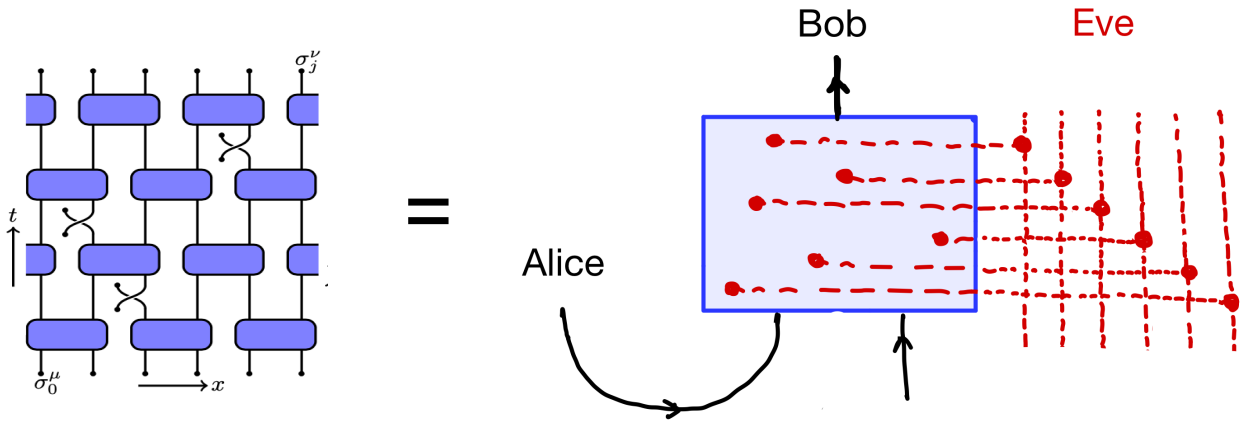
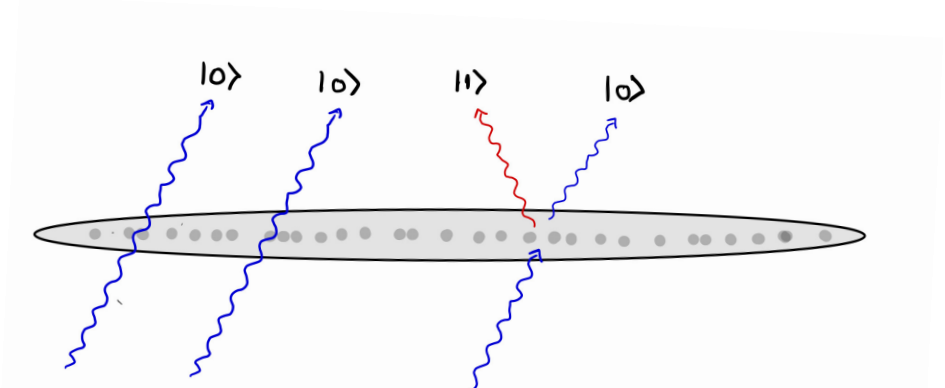


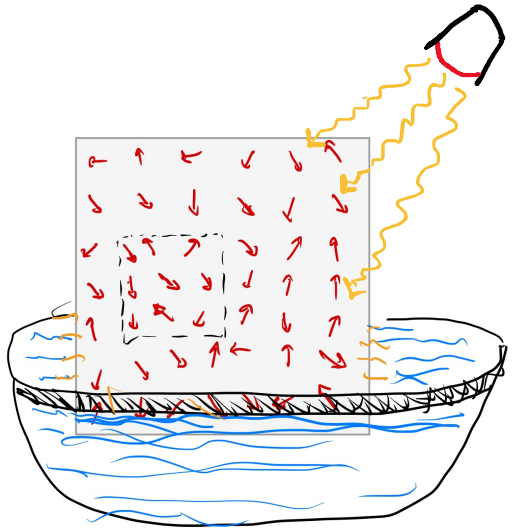

3. How do measurements affect critical correlations in quantum ground states



# Monitored system ≠ open system

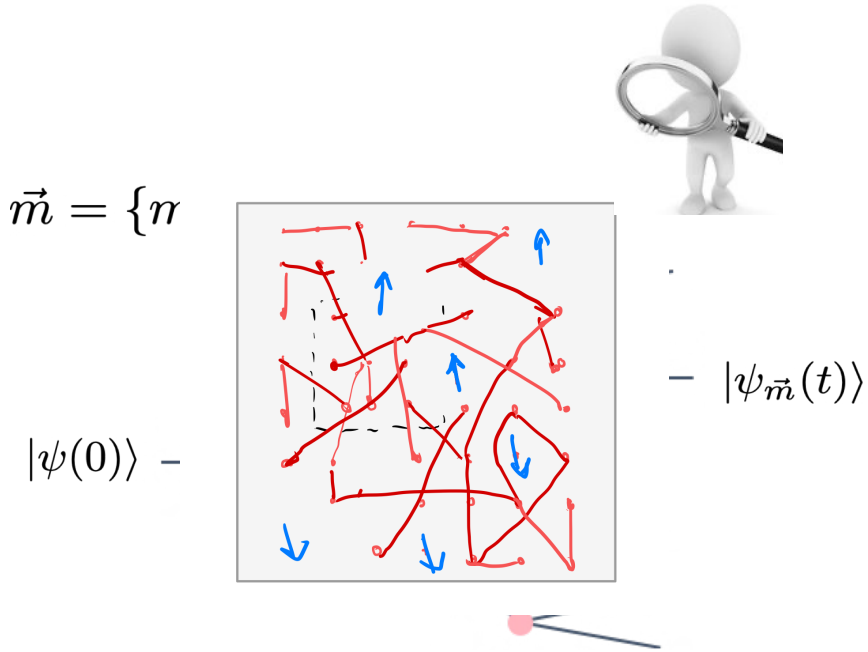
## 1. System coupled to a bath

- Describes usual decoherence processes.
- System gets entangled with environment d.o.f.
- A pure initial state generally becomes mixed.
- Density matrix evolves by the Lindblad equation (for a Markovian bath)



## 2. Monitored system

- A pure initial state remains pure.
- Time evolution described as an ensemble of pure state trajectories
- Observer must make use of the gained information (post-selection or feedback)
- If measurements are discarded then the observer is no different from a bath.



# Measurement induced phase transition in hybrid quantum circuits

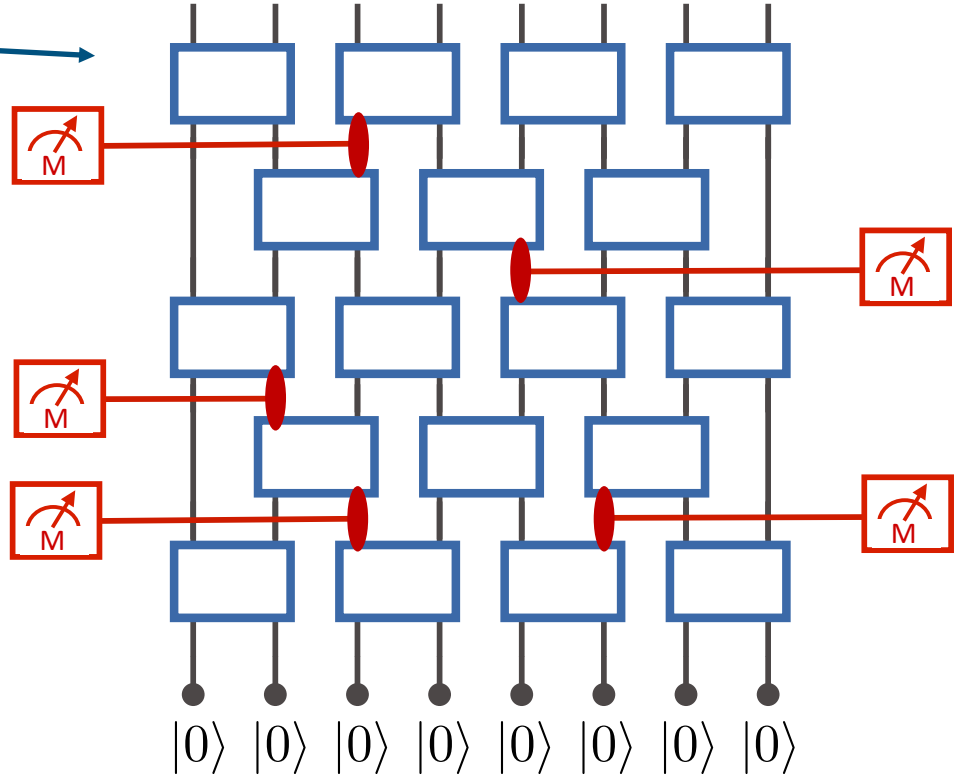
Random unitary gates

$$|\psi\rangle \to \prod_x U_{x,t}|\psi\rangle$$

Single qubit measurements applied with probability p:

$$|\psi\rangle \mapsto \frac{\hat{P}_\mu|\psi\rangle}{\sqrt{\langle\psi|\hat{P}_\mu|\psi\rangle}}$$

Project on outcome $\mu$ with probability $\langle\hat{P}_\mu\rangle$



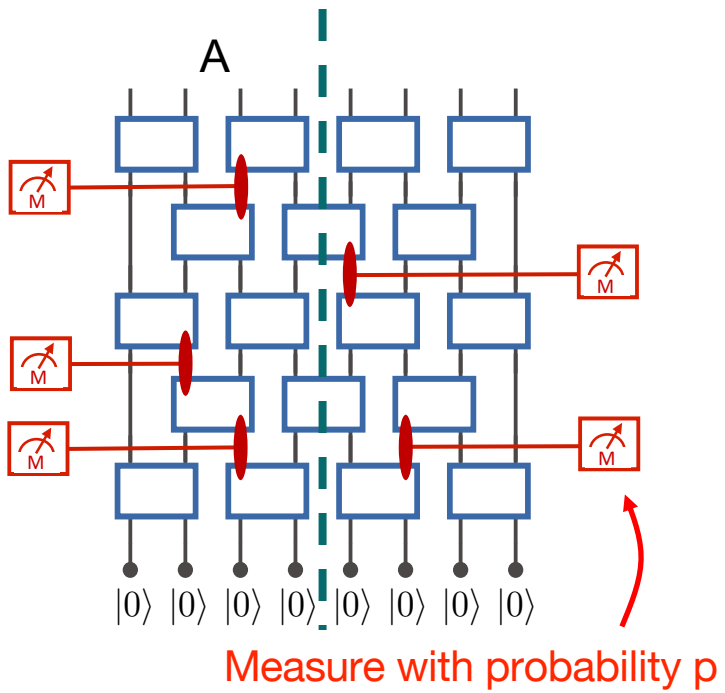
# Entanglement in hybrid quantum circuits

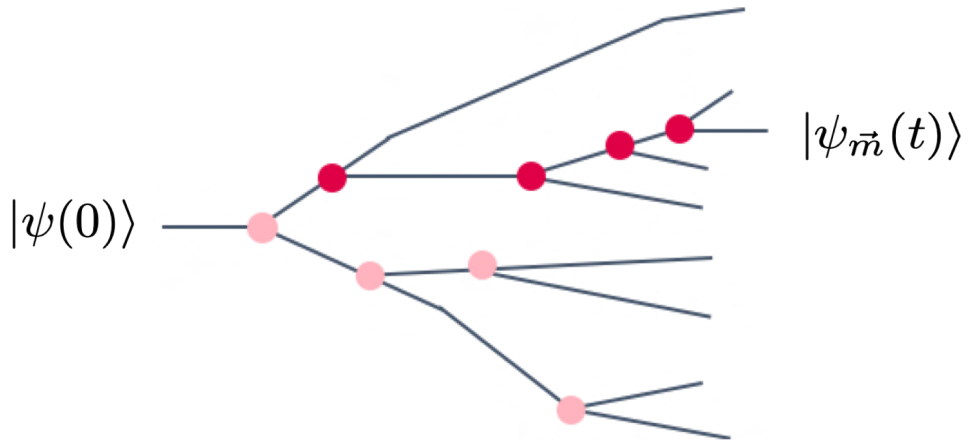
Ensemble averaged entanglement entropy of a sub-system after a long time ( $t \sim L \to \infty$ ):

$$\langle S_A \rangle_{\mathcal{U}} = \langle \sum_{\vec{m}} p_{\vec{m}} S_{A,\vec{m}} \rangle_{\mathcal{U}}$$

Entanglement entropy for a given instance of measurement outcomes

$$\vec{m} = \{m_1, m_2, m_3, \ldots\}$$



# Numerical evidence for a phase transition

Skinner, Ruhman, Nahum PRX 2019;
Li, Chen, Fisher PRB 2018, PRB 2019

$$\langle S_A \rangle_{\mathcal{U}} = \langle \sum_{\vec{m}} p_{\vec{m}} S_{A,\vec{m}} \rangle_{\mathcal{U}}$$

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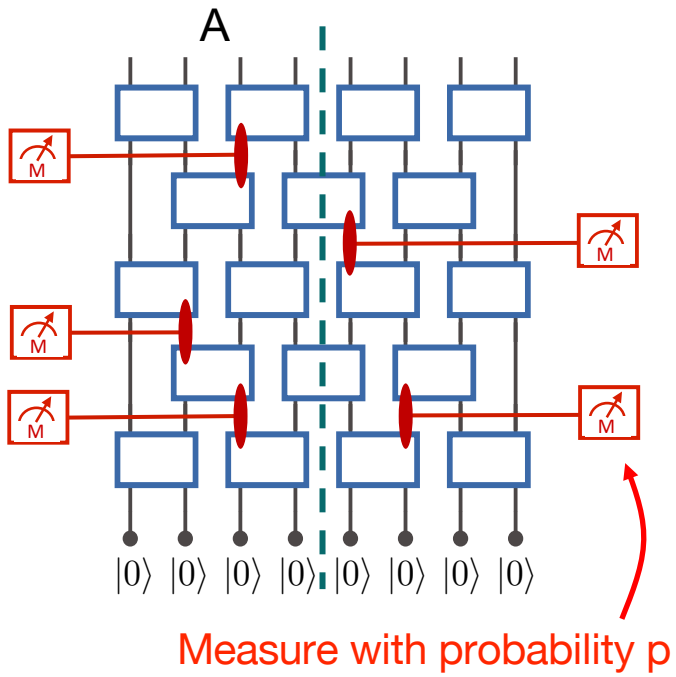
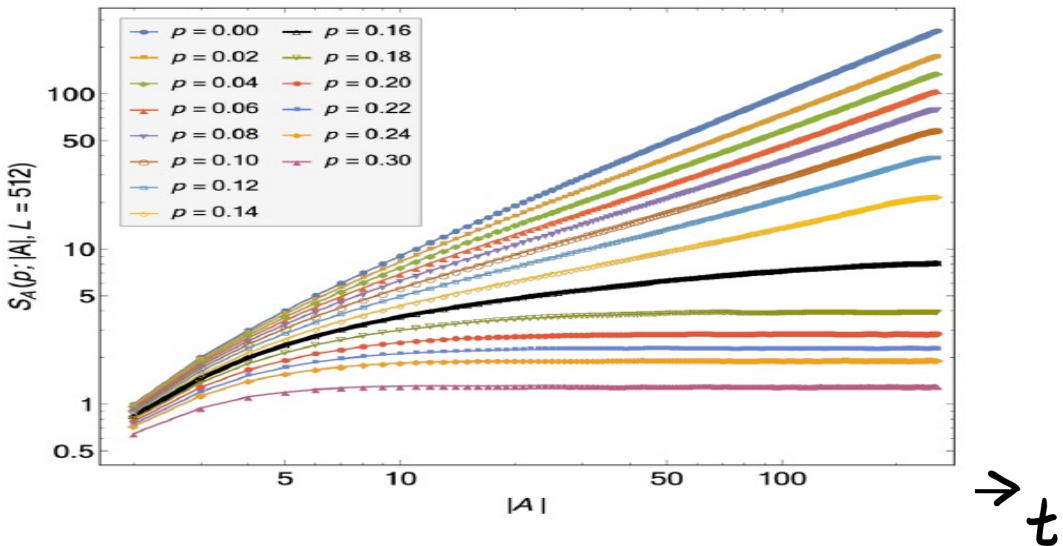
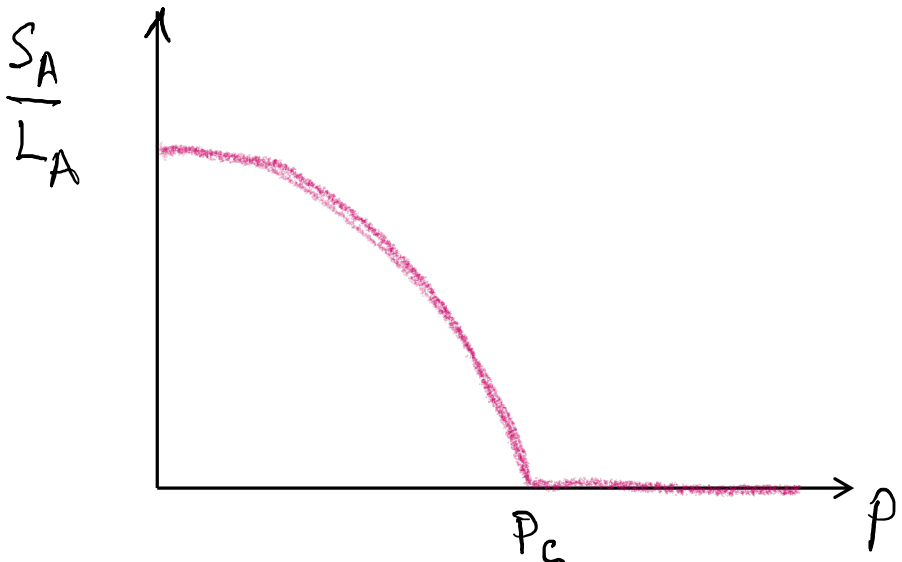


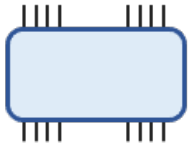

[Li, Chen and Fisher PRB 2019]:



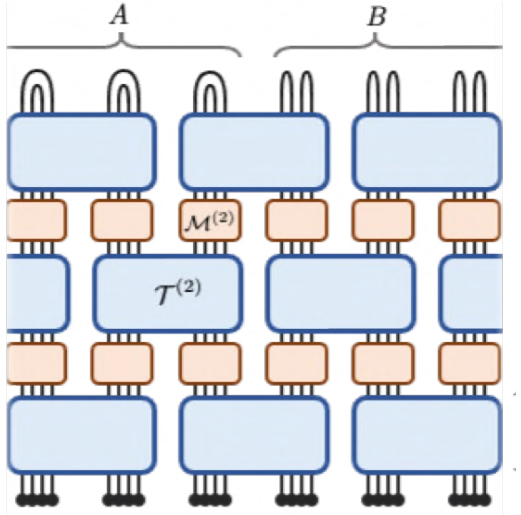



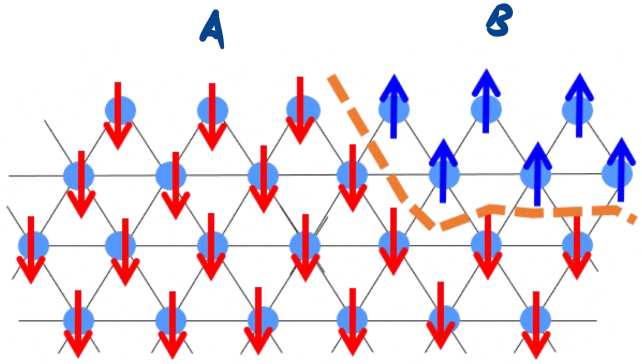
# Mapping to an effective classical "spin model"

$$\overline{U \otimes U^* \otimes U \otimes U^*} =$$



$$\overline{\sum_m p_m \mathrm{tr}(\rho_A^2)} \sim$$




**Entanglement entropy = free energy of a domain wall** $S^{(2)}(A) = F_{dw}^{(2)} - F_0^{(2)}$

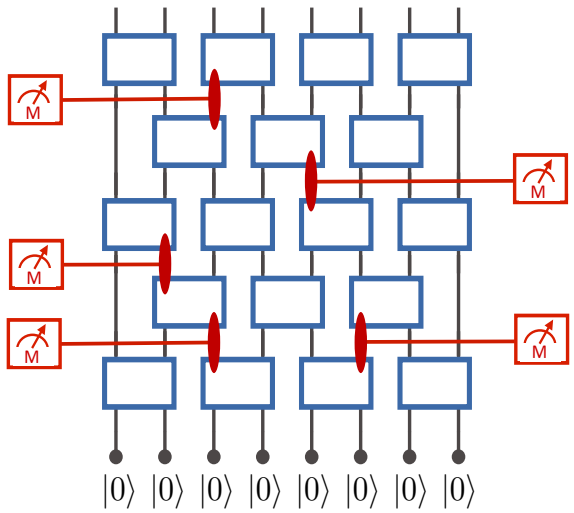
Volume law phase = ferromagnet $\Delta F_{dw}^{(2)} \sim L$

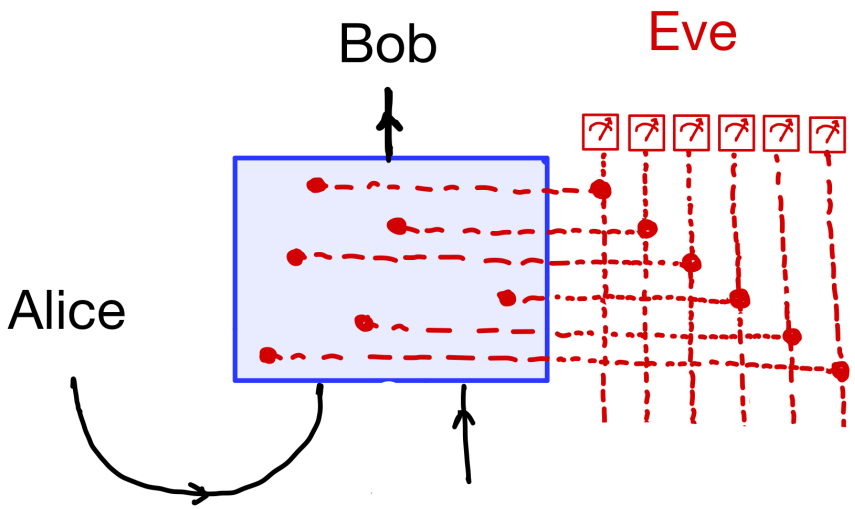
Area law phase = paramagnet $\Delta F_{dw}^{(2)} \sim \mathrm{const}$

Bao,Choi,EA PRB 2020; Jian,You,Vasseur,Ludwig PRB 2020
(Based on earlier work on unitary circuits by Nahum, Vijay Haah PRX 2018)

# Understanding in terms of quantum information encoding

Choi, Bao, Qi and EA, PRL 2020; Bao, Choi and EA PRB 2020; Gullans and Huse PRX 2020




Phase transition in the quantum channel capacity: $I_c(\text{Alice} \to \text{Bob})$

From the perspective of the observer Eve:
transition in the information Eve can extract on the initial state $\mathcal{F}(\text{Eve}|\text{Alice})$ (Fisher information)

Observing the transition is fundamentally hard because it requires post-selecting on an extensive number of measurement outcomes.

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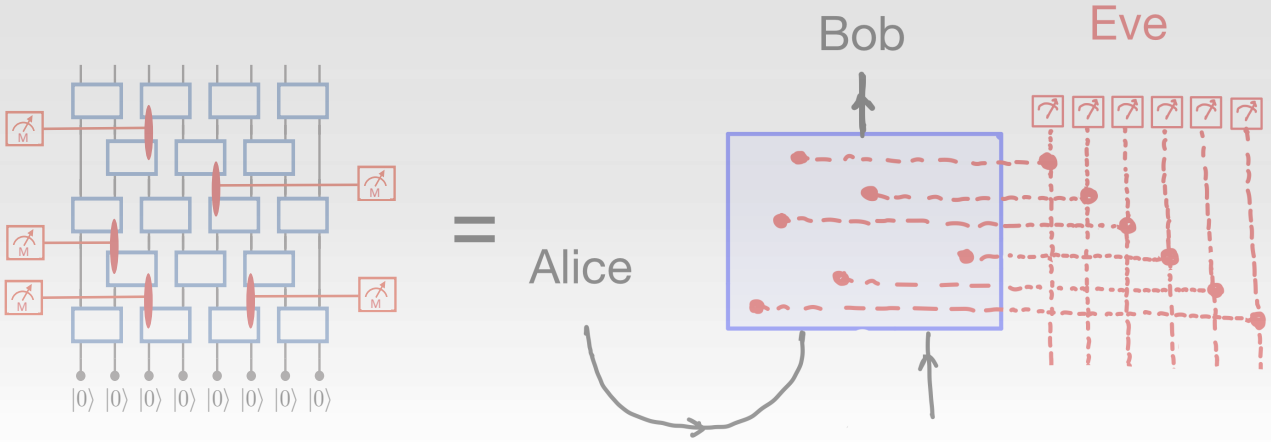


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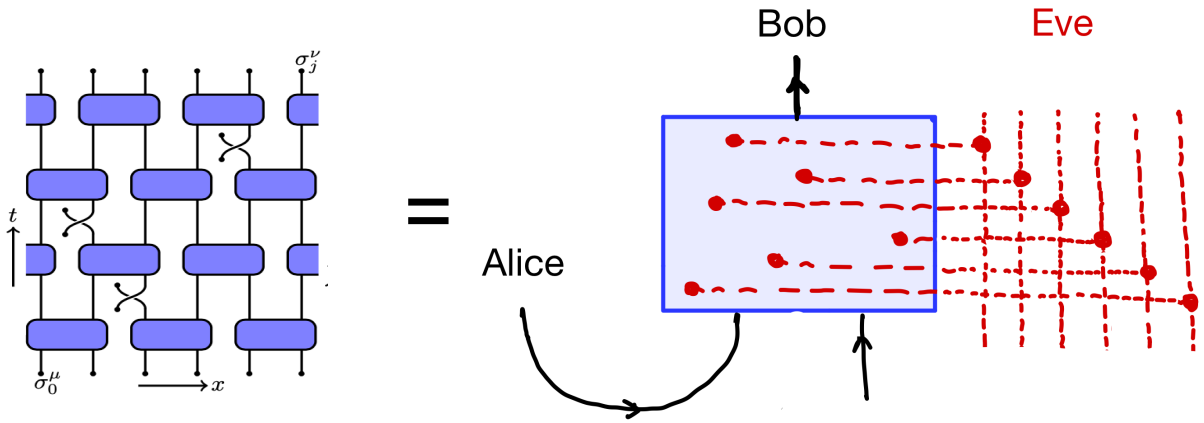


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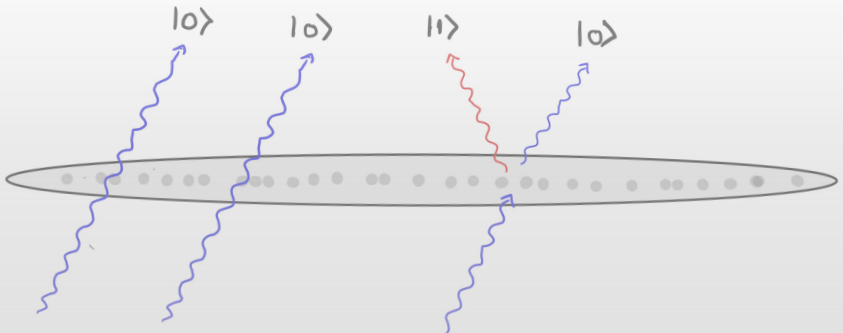


# Scrambling phase transition in a random circuit

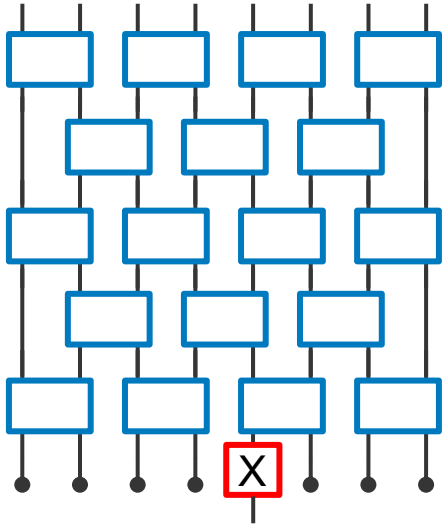
[Zack Weinstein, Shane Kelly, Jamir Marino, EA (Unpublished)]

Zack Weinstein

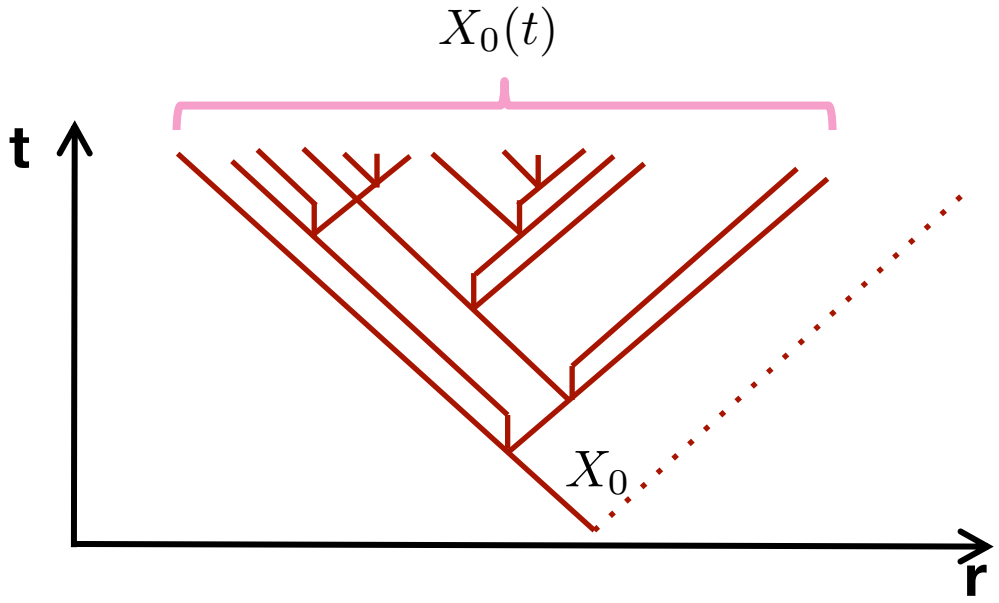
Shane Kelly

Jamir Marino

Scrambling circuit:




Operator growth:

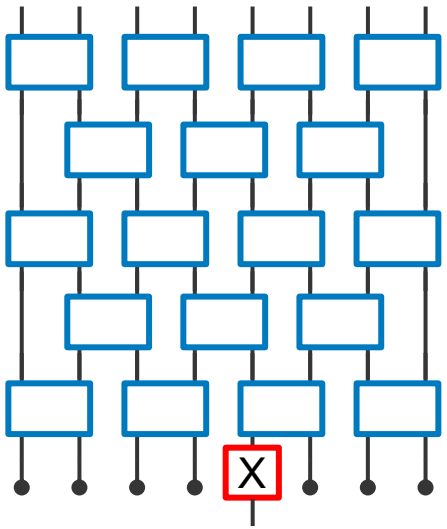


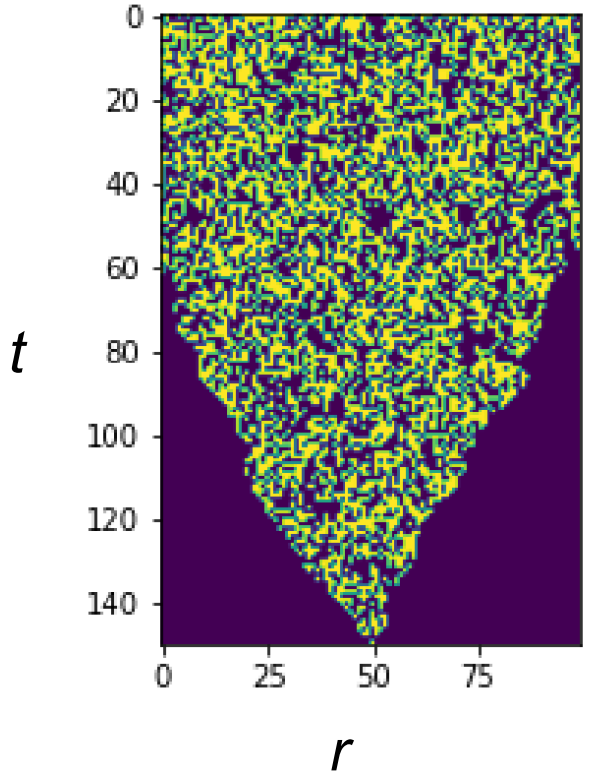

Operator growth measured by the OTOC: $C_r(t) = \langle [X_0(t), Y_r]^2 \rangle$

Operator size: $n(t) = \sum_r C_r(t)$

# Scrambling phase transition in a random circuit

Scrambling circuit:




Operator growth measured by the OTOC: $C_r(t) = \langle [X_0(t), Y_r]^2 \rangle$

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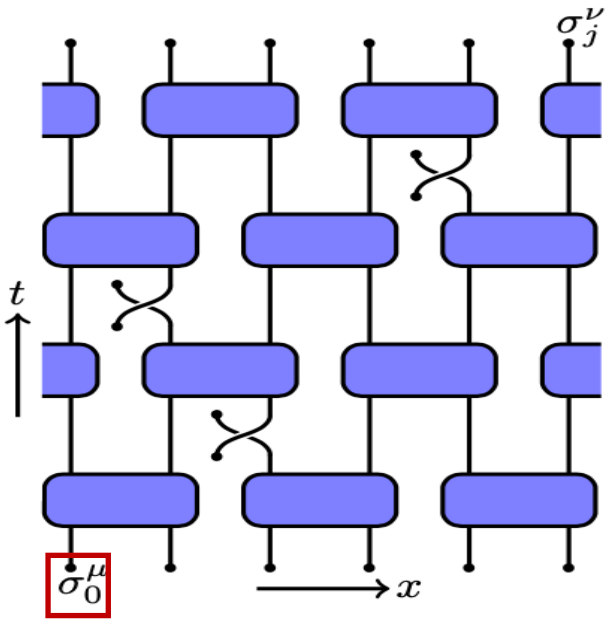
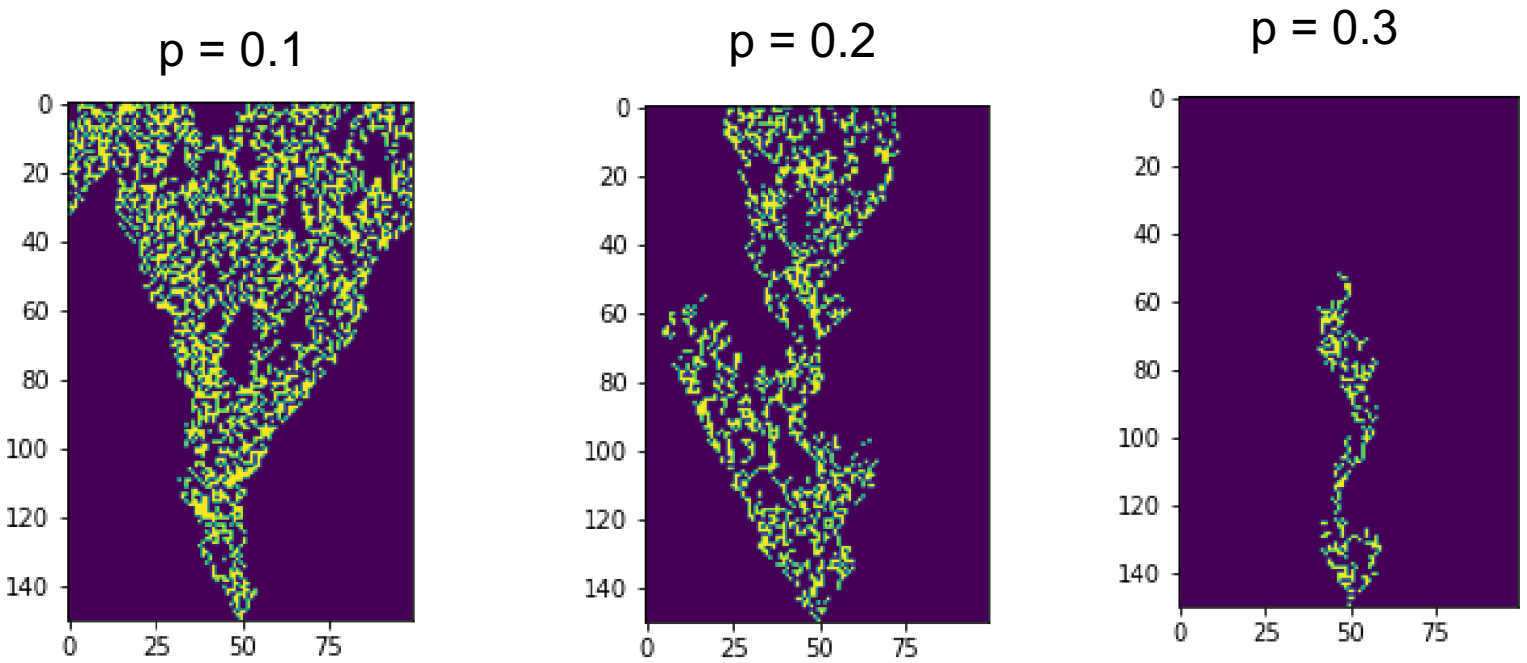
# Unitary circuit coupled to ancillas

With probability p
apply a swap with ancilla:



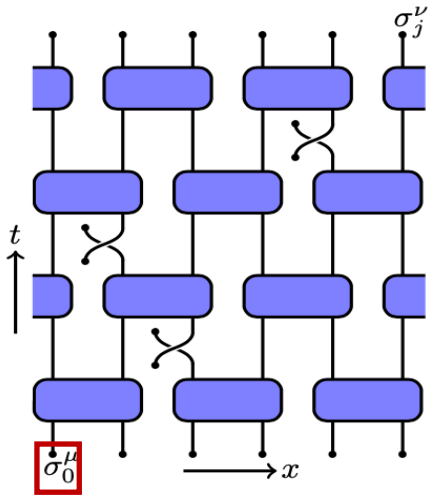

$$C_r(t) = \langle [X_0(t), Y_r]^2 \rangle$$

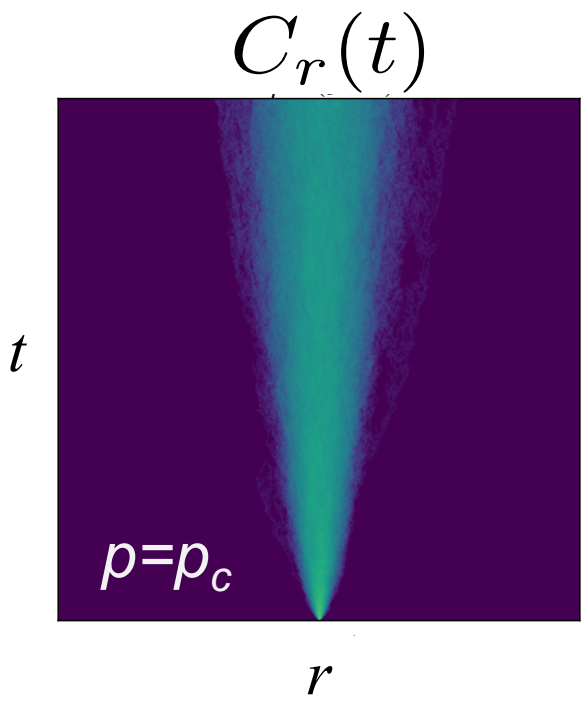


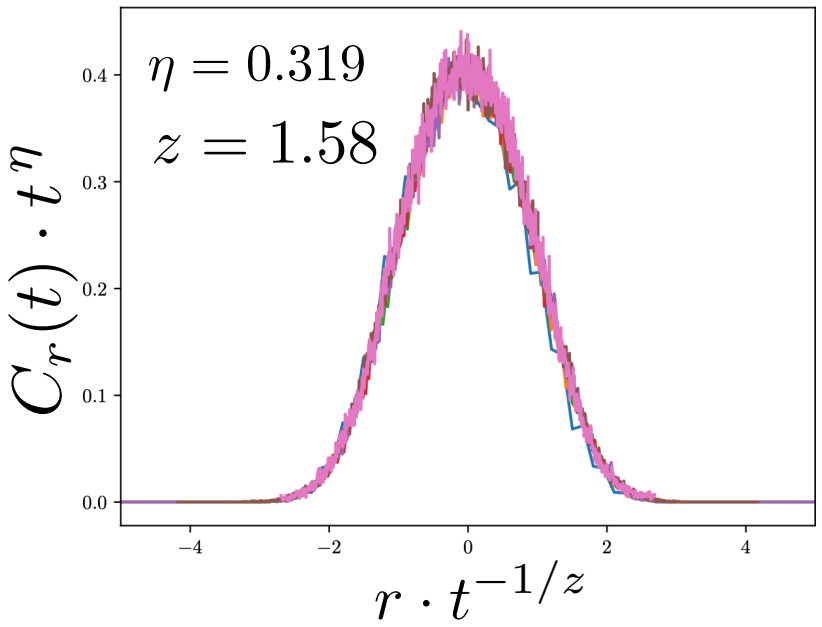

The swaps kill branches of the tree leading to a directed-percolation transition in the OTOC at a critical value $p=p_c$

# Unitary circuit coupled to ancillas

Collapse of the critical operator density profile at different times using directed percolation exponents:

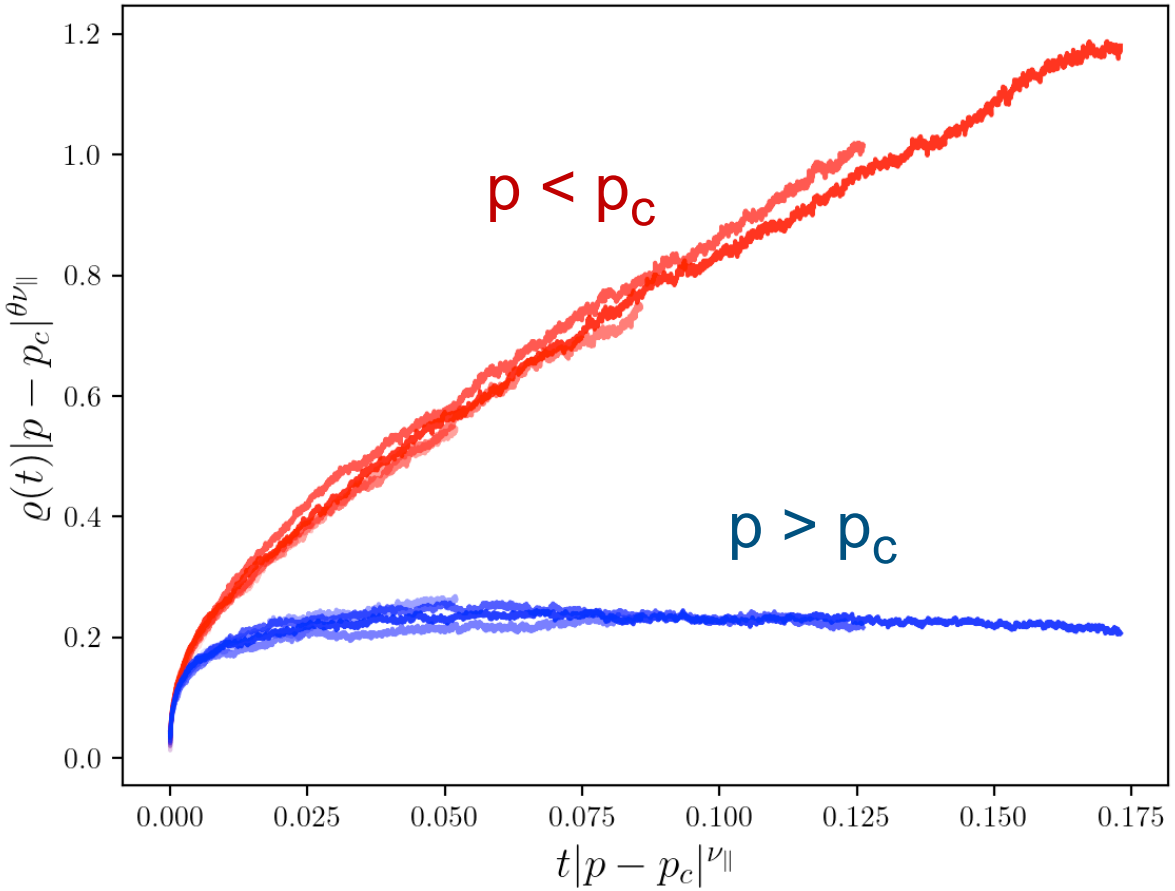


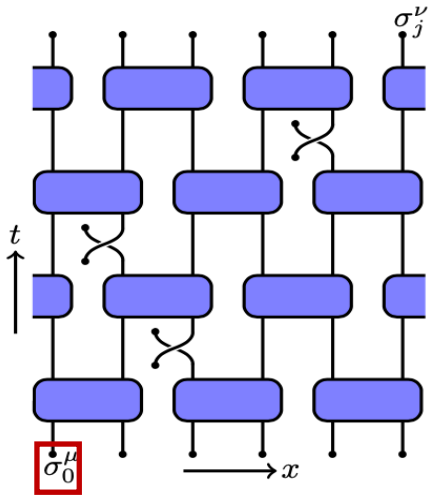



# Unitary circuit coupled to ancillas

Data collapse of the operator density versus time off criticality ($\rho(t)$ for different values of p):

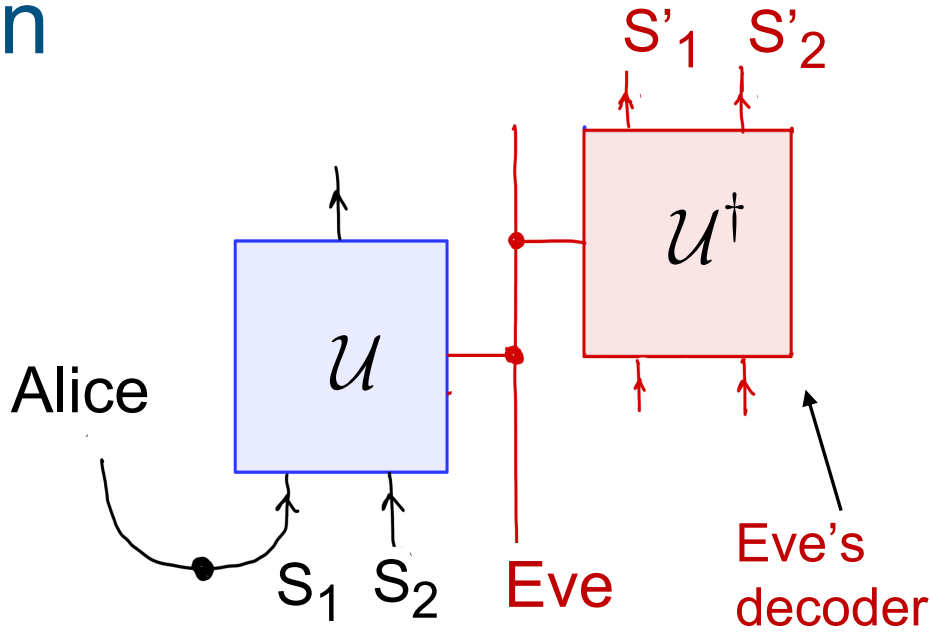


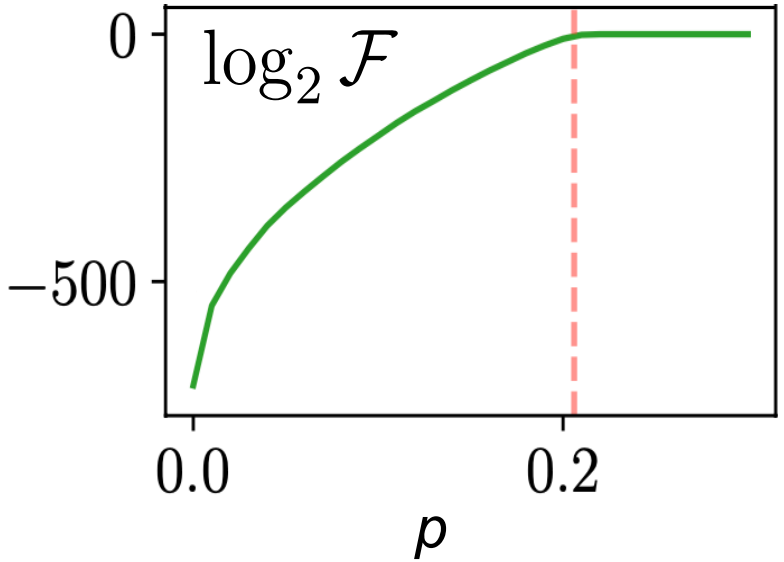



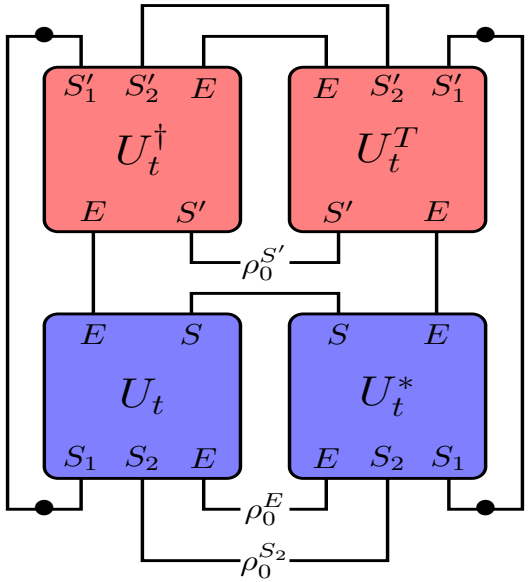
# Decoding transition

For $p>p_c$ Eve can decode Alice with perfect fidelity using this scheme:

$$\mathcal{F} = \mathrm{tr}\left(|\Psi_{AS_1'}\rangle\langle\Psi_{AS_1'}|\mathcal{U}^\dagger_{S'E}\mathcal{U}_{SE}\rho_{ASES'}\mathcal{U}^\dagger_{SE}\mathcal{U}_{S'E}\right)$$

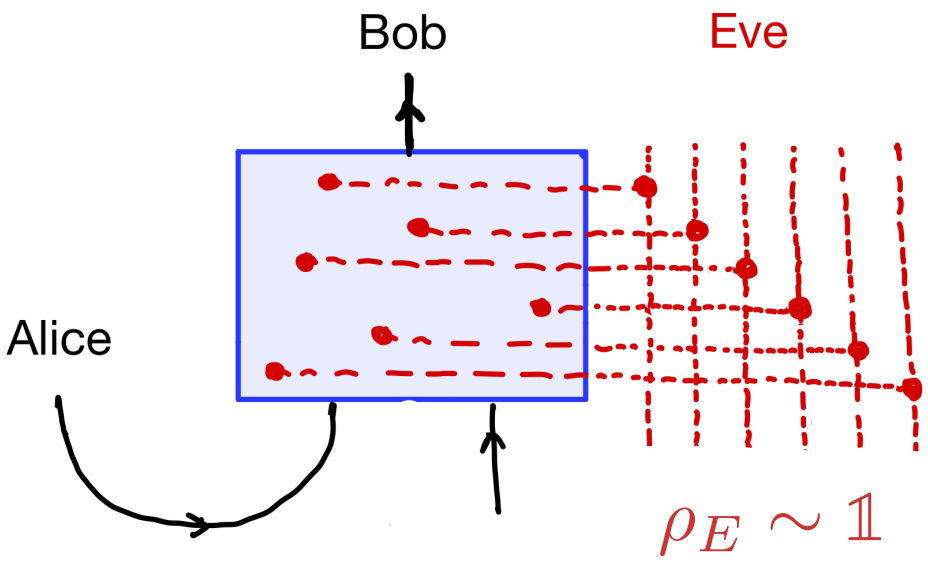


# Transition in the quantum channel capacity

$$I_c(\text{Alice} : \text{Eve}) = S_E - S_{AE}$$




Only if eve has zero knowledge of her initial state. Otherwise, the quantum channel capacity always saturates to the maximum value.

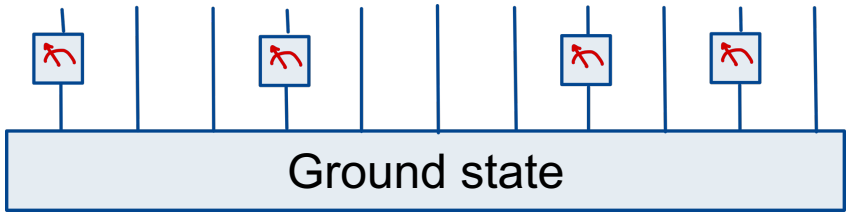
# How do measurements affect quantum critical correlations?

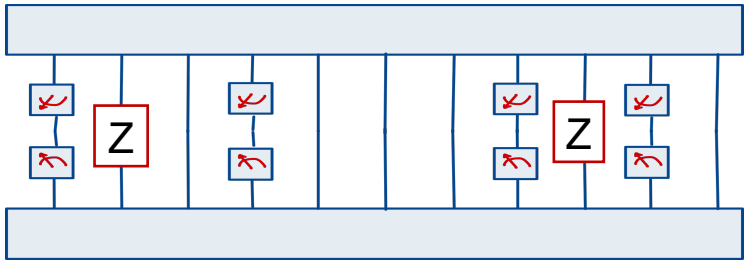
Sam Garratt, Zack Weinstein, EA, arXiv:2207.09476

Sam Garratt

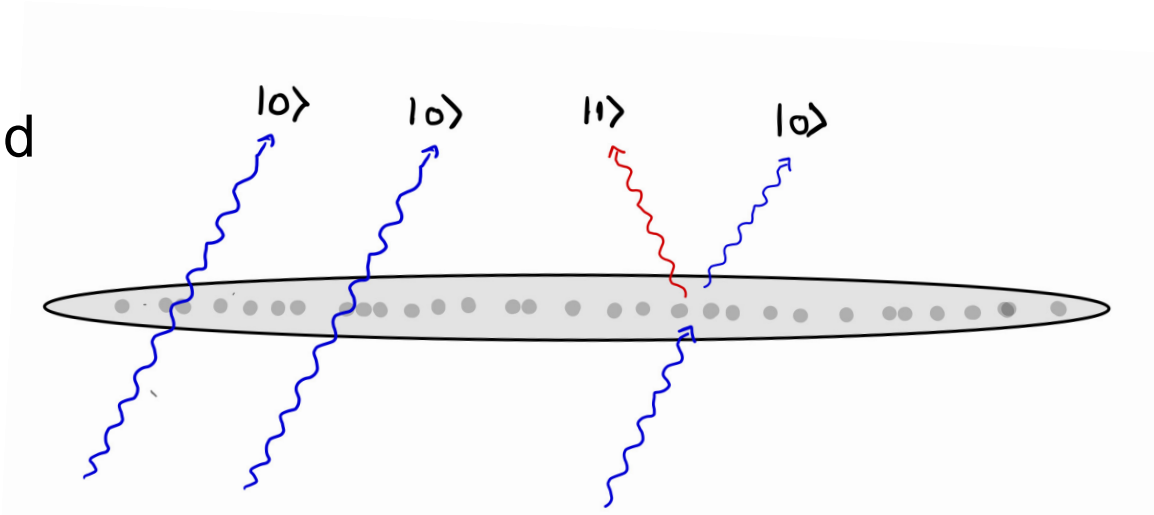
Zack Weinstein

Given a quantum state measure a subset of the qubits, then evaluate the correlations in the resulting state. **There is no dynamics !**




Example: detection of particles in a 1d quantum liquid

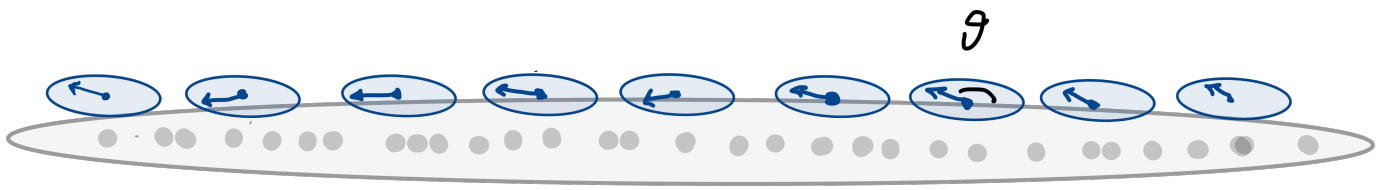


# One dimensional quantum liquids

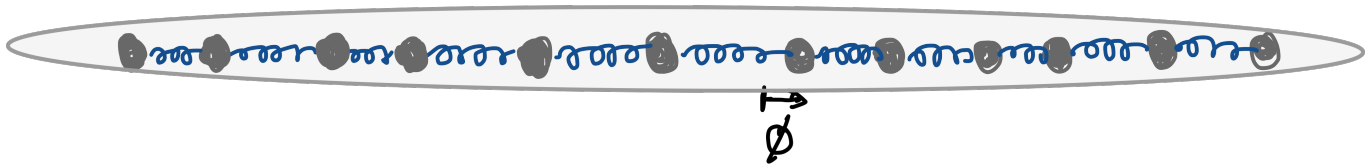
- Universal long wavelength description: Luttinger liquid

  Dual descriptions in terms of phase or density fluctuations

$$S = \frac{K}{2\pi} \int dx d\tau \left[ \dot{\theta}^2 + (\nabla\theta)^2 \right]$$




$$S = \frac{1}{2\pi K} \int dx d\tau \left[ \dot{\phi}^2 + (\nabla\phi)^2 \right]$$



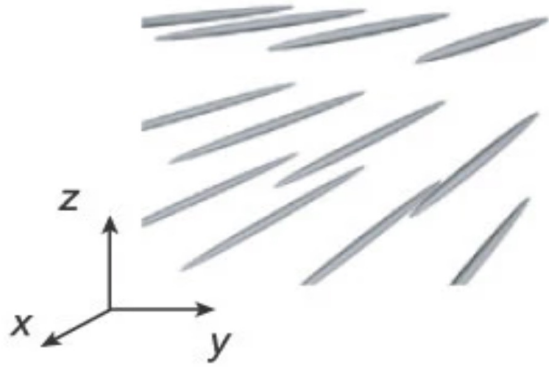

$$\delta n(x) \approx -\frac{1}{\pi}\nabla\phi(x) + \frac{1}{\pi}\cos[2\pi\rho_0 x - 2\phi(x)]$$

K=1 corresponds to non interacting fermions or hard-core bosons.
K<1: fermions with repulsive interactions or bosons with longer range repulsion

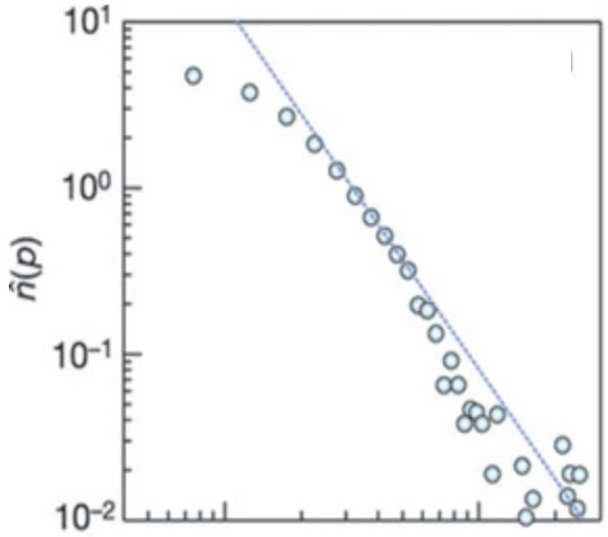
# One dimensional quantum liquids

➡ Quantum critical states with continuously tunable exponents.

Realized with ultracold atoms in optical lattices:




Paredes et. al.
(I. Bloch group) Nature 2004




Density correlations:

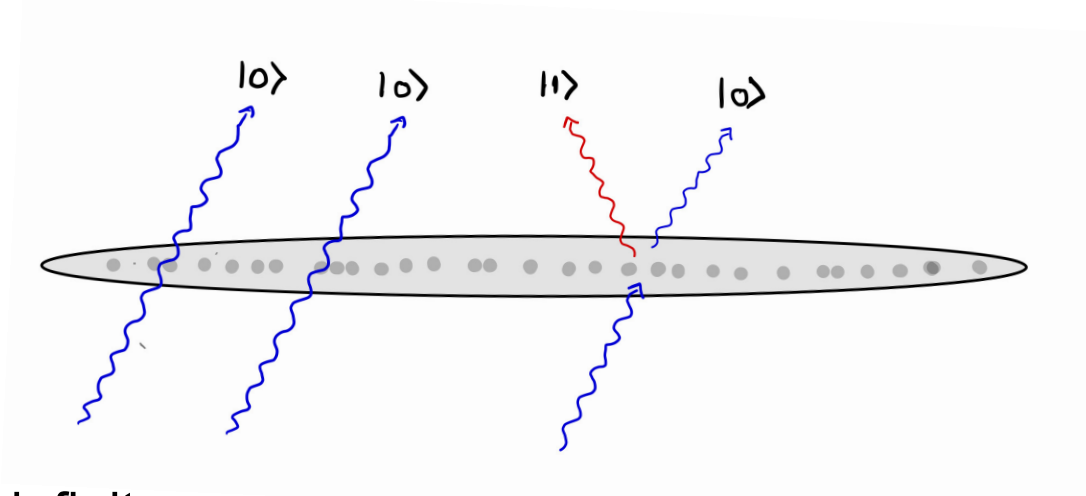
$$\langle \Psi_{\rm gs} | \delta n(x) \delta n(0) | \Psi_{\rm gs} \rangle \sim c_1 \left(\frac{1}{x}\right)^2 + c_2 \cos(2\pi\rho_0 x) \left(\frac{1}{x}\right)^{2K}$$

$\langle \nabla\phi(x) \nabla\phi(0) \rangle$ $\qquad$ $\langle e^{i(2\phi(x) - 2\phi(0))} \rangle$

Phase correlations:

$$\langle \Psi_{\rm gs} | \psi^\dagger(x) \psi(0) | \Psi_{\rm gs} \rangle \sim \langle e^{i(\theta(x) - \theta(0))} \rangle \sim \left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

# Quantum Non-Demolition Measurement




- Weak measurement of the density everywhere: probe light interacts weakly with the particles.
- Measure polarizations of photons at different locations.
- Outcome |1> = particle found at this location
  Outcome |0> = “No click”. Particle occupation remains indefinite

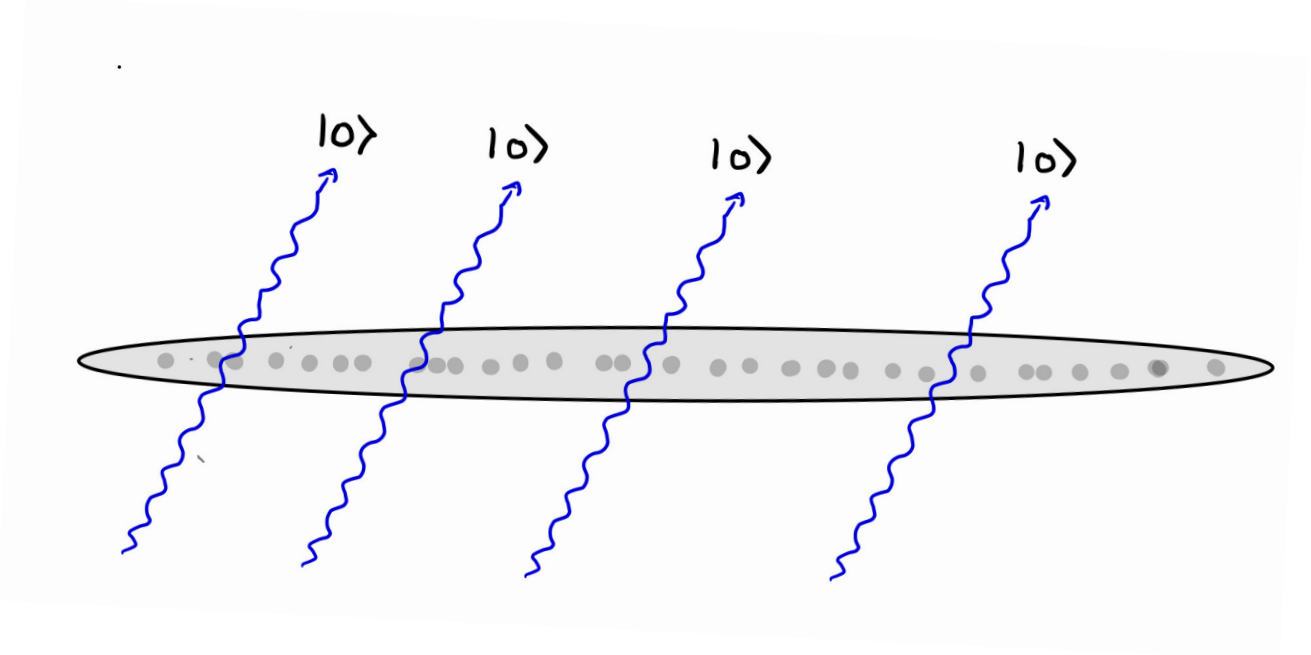
How does the partial/weak measurement affect the critical correlations?

$$\langle \Psi_{\rm gs} | P_m n(x) n(0) P_m | \Psi_{\rm gs} \rangle \overset{?}{\sim} c_1 \left(\frac{1}{x}\right)^2 + c_2 \cos(2\pi\rho_0 x) \left(\frac{1}{x}\right)^{2K}$$

$$\langle \Psi_{\rm gs} | P_m \psi^\dagger(x) \psi(0) P_m | \Psi_{\rm gs} \rangle \overset{?}{\sim} \left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

Important: there is no dynamics. Perform measurements then evaluate correlations in the output state.

# Warmup: post-select on the null measurement outcome (no clicks)

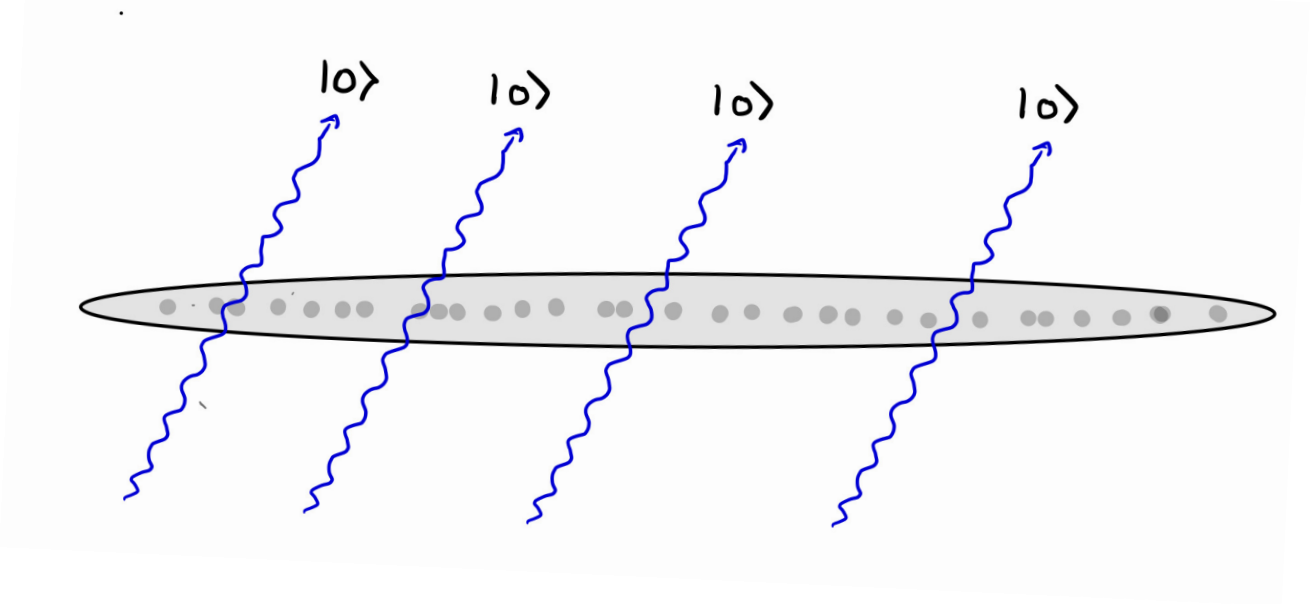



<u>The no click state:</u> $$|\Psi_{\rm nc}\rangle = e^{-\int dx v(x) n(x)} |\Psi_{\rm gs}\rangle$$

If the measurement strength *v*(x) is oscillating in space at a wavelength commensurate with the particle density, then we can represent it in terms of the long-wavelength fields:

$$|\Psi_{\rm nc}\rangle = e^{-v \int dx \cos[2\phi(x)]} |\Psi_{\rm gs}\rangle$$

# Warmup: post-select on the null measurement outcome (no clicks)

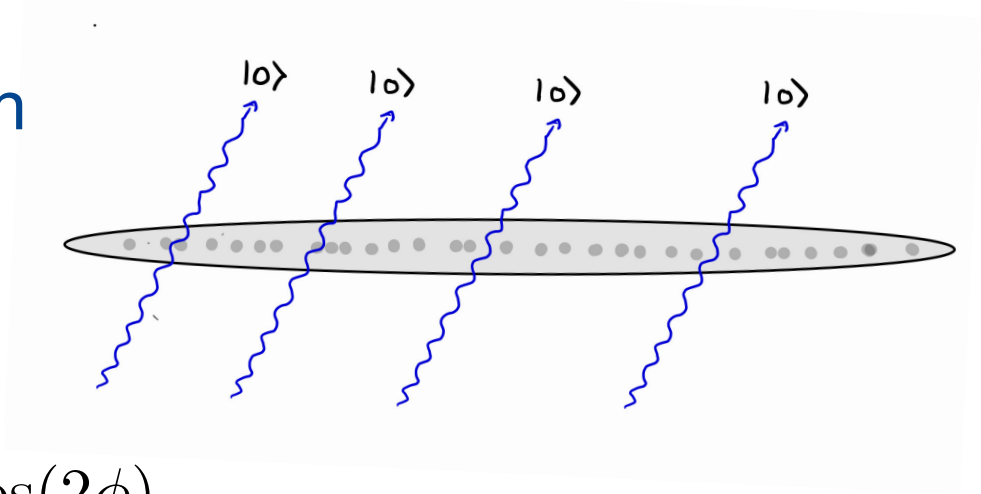



<u>Correlations in the no click state:</u>

$$\langle n(x)n(0)\rangle_{\rm nc} = \lim_{\beta\to\infty} \langle \Psi_{\rm ref}| e^{-\beta H_{\rm LL}} e^{-v\int dx\,\cos(2\phi)} n(x)n(0) e^{-v\int dx\,\cos(2\phi)} e^{-\beta H_{\rm LL}} |\Psi_{\rm ref}\rangle$$

$$= \int \mathcal{D}\phi\, e^{-S_{\rm nc}[\phi]}\, \delta n(x)\delta n(0)$$

# The “no click” action

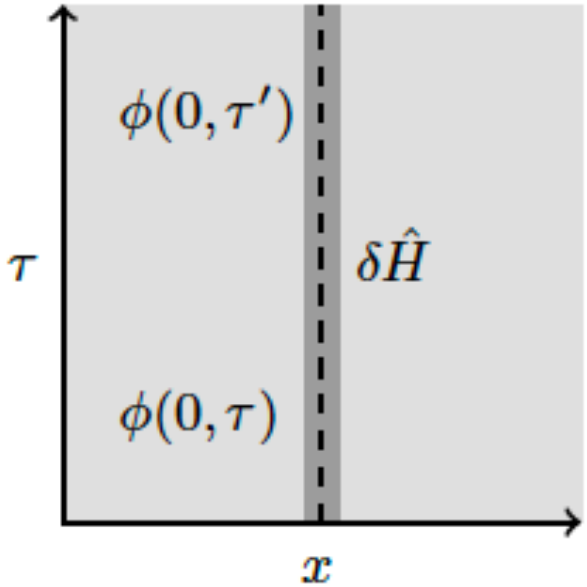



$$\langle n(x)n(0)\rangle_{\rm nc} = \int \mathcal{D}\phi e^{-S_{\rm nc}}\, n(x)n(0)$$

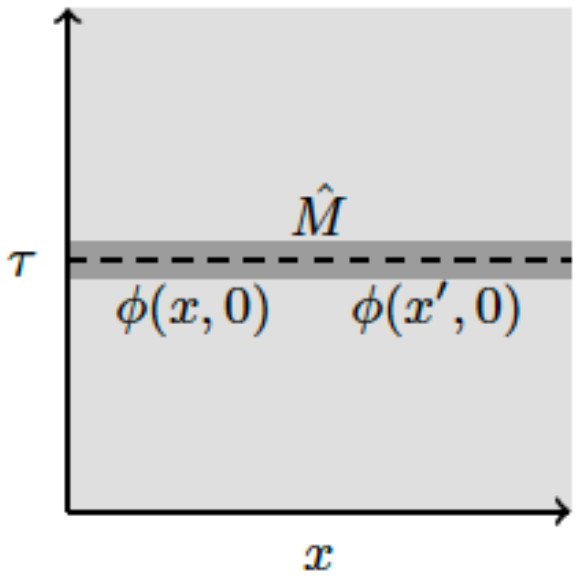
$$S_{\rm nc} = \frac{1}{2\pi K}\int dx d\tau \left[(\partial_\tau\phi)^2 + (\partial_x\phi)^2\right] - v\int dx \cos(2\phi)$$

= Wick rotated impurity problem [Kane and Fisher PRL 1992]

impurity

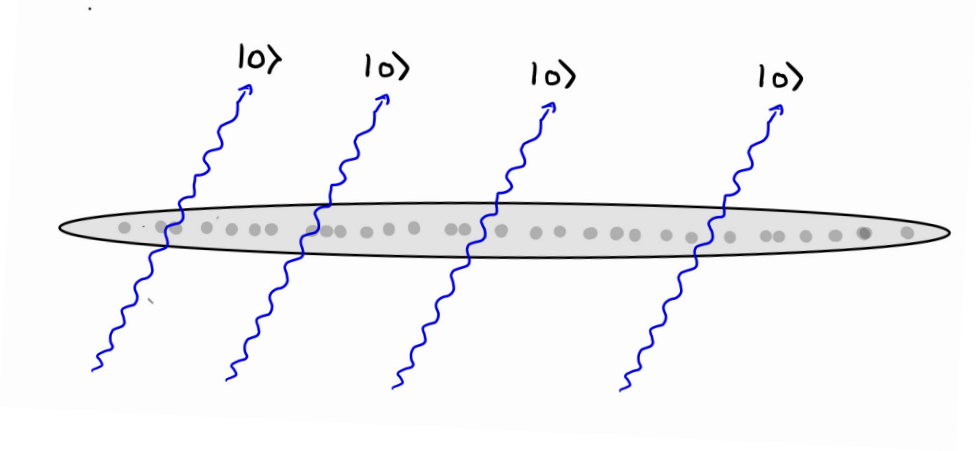



measurements



# Phase transition tuned by the Luttinger parameter K

$$S_{\rm nc} = \frac{1}{2\pi K}\int dx d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2\right] - v\int dx \cos(2\phi)$$



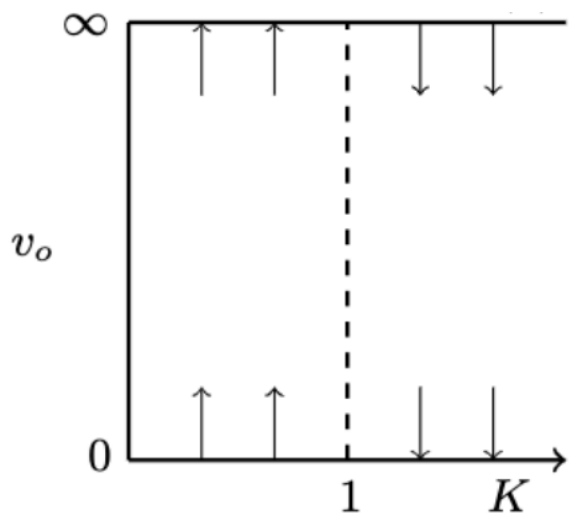

Scaling of the measurements : $\dfrac{dv}{d\ell} = (1-K)\,v$

$K > 1$ Measurements are **irrelevant**

Long distance correlations unaffected for any finite measurement strength

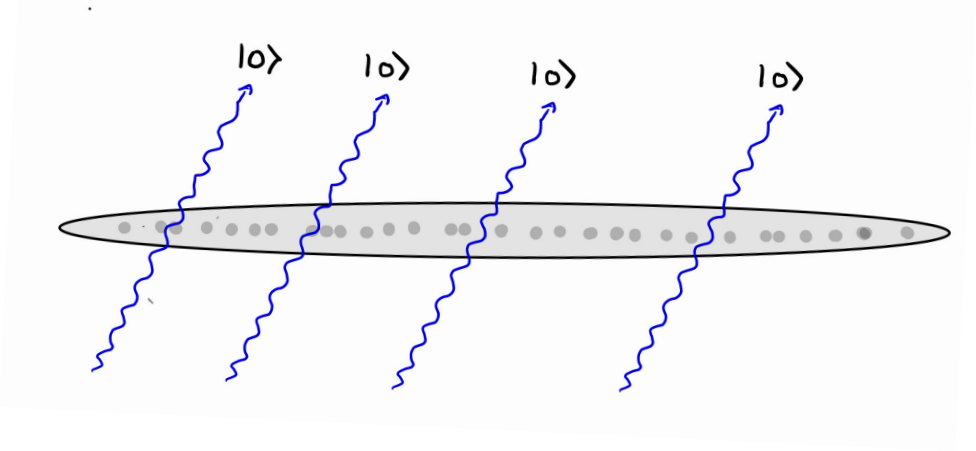
Phase: $\langle e^{i[\hat{\theta}(x)-\hat{\theta}(0)]}\rangle \sim x^{-1/(2K)}$

Smooth component of the density: $\langle \nabla\hat{\phi}(0)\,\nabla\hat{\phi}(x)\rangle \sim x^{-2}$



# Phase transition tuned by the Luttinger parameter K

$$S_{\rm nc} = \frac{1}{2\pi K}\int dx d\tau \left[(\partial_\tau \phi)^2 + (\partial_x \phi)^2\right] - v\int dx \cos(2\phi)$$



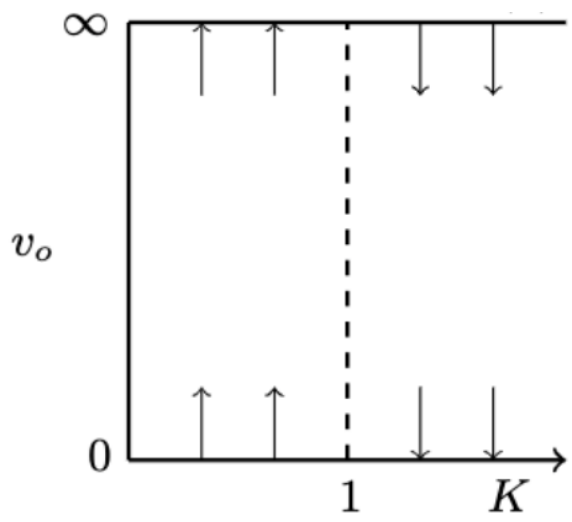

Scaling of the measurements : $\frac{dv}{d\ell} = (1-K)\,v$

$K < 1$ Measurements are **relevant**

Non perturbative effect on long distance correlations:

$$\langle e^{i[\hat\theta(x)-\hat\theta(0)]}\rangle \sim x^{-1/(2K)} \to x^{-1/K}$$

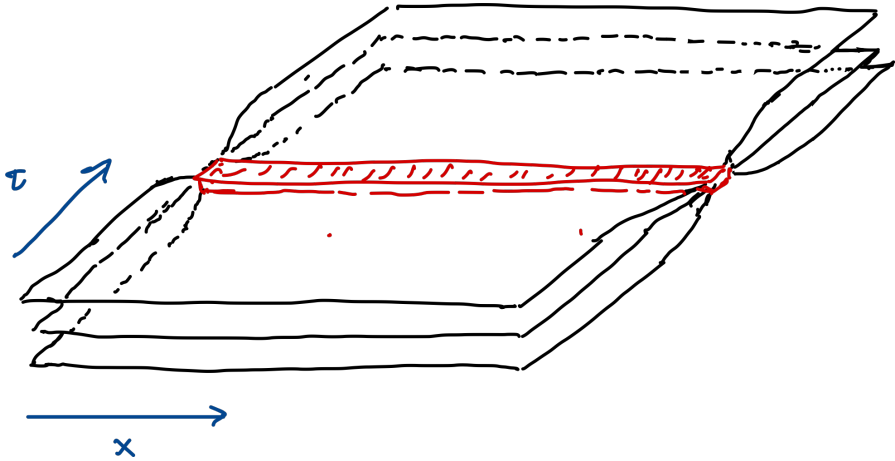
$$\langle \nabla\hat\phi(0)\,\nabla\hat\phi(x)\rangle \sim x^{-2} \to x^{-2/K}$$




Performing local measurements has a highly non-local effect on the quantum state!

# Ensemble of measurement outcomes

To implement the average over measurement outcomes we need to introduce N replicas taking the limit N ->1 at the end:



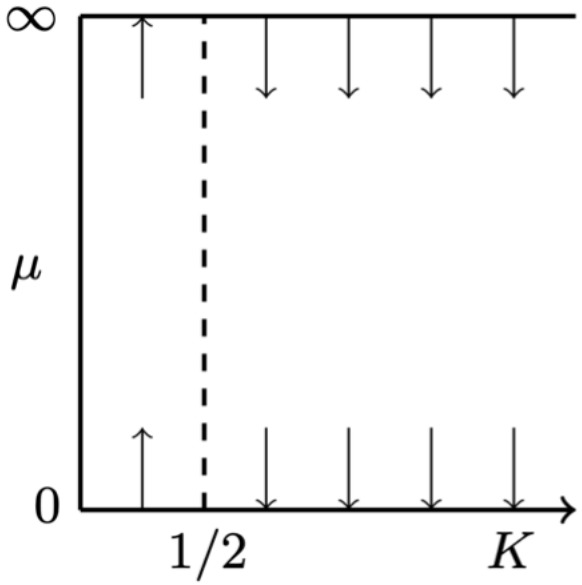

$$s_N[\{\varphi_\alpha\}] = \sum_{\alpha=1}^{N} s[\varphi_\alpha] - \frac{\mu}{2N\pi^2}\int dx \sum_{\alpha<\beta} \cos[2(\varphi_\alpha - \varphi_\beta)]$$

The random measurement outcomes couple the replicas attempting to lock them to each other.

Need to look at quantities that are non-linear in the density matrix, e.g.:

$$C^{(2)}(r) \equiv \sum_m p_m |\langle\psi_m| e^{i(\theta(r)-\theta(0))} |\psi_m\rangle|^2$$

Perturbative RG reveals a transition at K=1/2 in this case.



# How to observe the transition

Nonlinear in $\rho_m$ ➡ unobservable

$$C^{(2)}(r) \equiv \sum_m p_m |\langle \psi_m | e^{i(\theta(r)-\theta(0))} | \psi_m \rangle|^2$$

Alternative linear quantity:

$$C_w(r) = \sum_m p_m w_m \langle \psi_m | e^{i(\theta(r)-\theta(0))} | \psi_m \rangle$$

**The weight $w_m$ utilizes knowledge of the measurement outcomes.**

If the correlation is calculable classically then a natural choice for $w_m$ is :

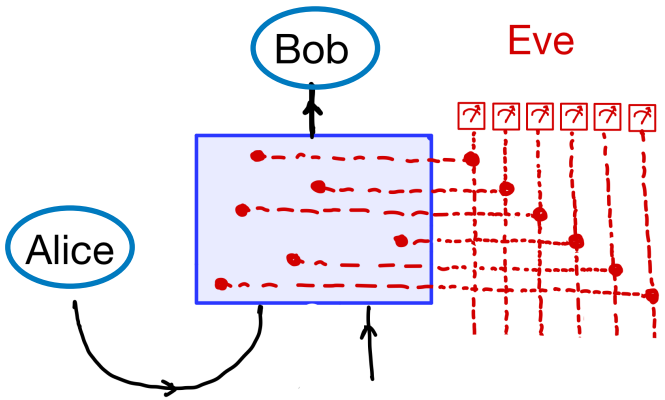
$$w_m = \langle e^{i(\theta(r)-\theta(0))} \rangle_m^{\text{calc}}$$

Observe the transition through the quantum-classical estimator:

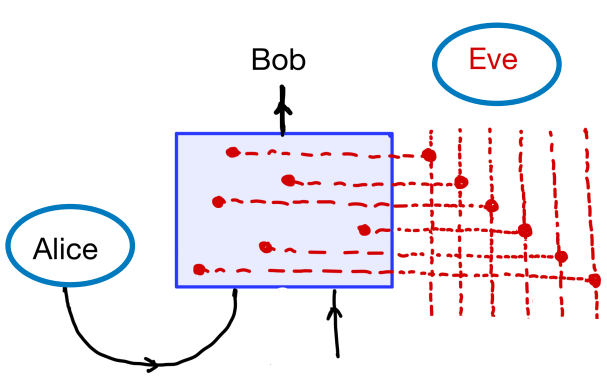
$$C_{CQ}(r) = \sum_m p_m \langle e^{i(\theta(r)-\theta(0))} \rangle_m^{\text{calc}} \langle e^{i(\theta(r)-\theta(0))} \rangle_m$$

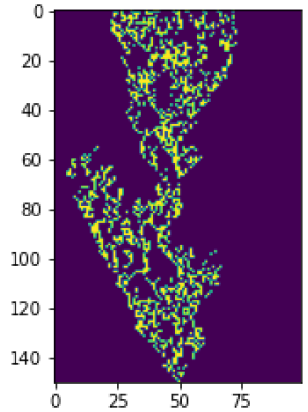
# Summary

- **MIPT = "encoding transition"**
  Hard to observe even with a perfect quantum computer (post-selection problem)



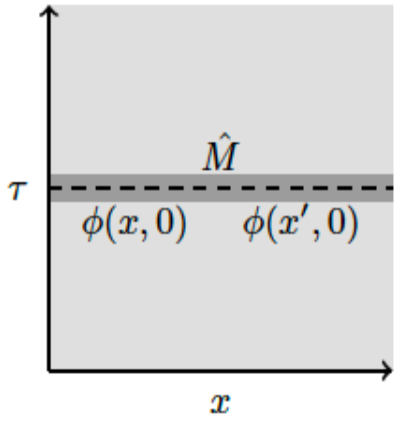

- **Scrambling transition = "decoding transition"**
  Observable using a quantum computer e.g. by measuring OTOC (no post-selection)




- Partial local measurement of a quantum critical state can alter the long-distance critical correlations.
  - Mapping to boundary critical phenomena
  - An observable quantum-classical estimator:

$$C_{CQ}(r) = \sum_m p_m \langle e^{i(\theta(r)-\theta(0))} \rangle_m^{\text{calc}} \langle e^{i(\theta(r)-\theta(0))} \rangle_m$$



# Outlook

- Decoding transitions in different universality classes?

- Is it possible to show quantum advantage using the quantum classical estimators? Instances for which:

$$C_{CC}(r) = \sum_m p_m \langle e^{i(\theta(r)-\theta(0))} \rangle_m^{\text{calc}} \langle e^{i(\theta(r)-\theta(0))} \rangle_m^{\text{calc}}$$ Hard

$$C_{CQ}(r) = \sum_m p_m \langle e^{i(\theta(r)-\theta(0))} \rangle_m^{\text{calc}} \langle e^{i(\theta(r)-\theta(0))} \rangle_m$$ Easy

In situations we now understand there is only a polynomial gap between these estimators