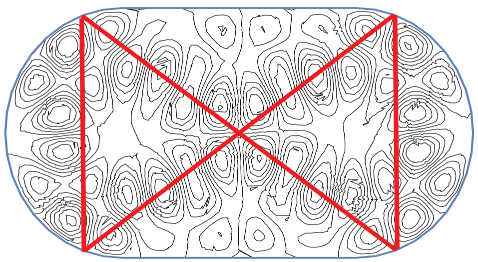
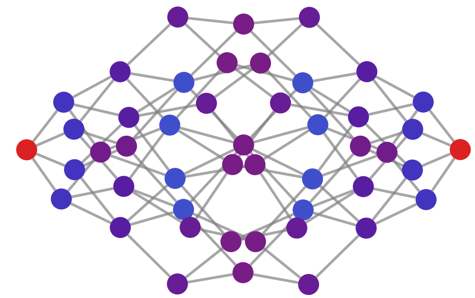
Weak ergodicity breaking from quantum many-body scars



[Maksym Serbyn](#)
IST Austria



in collaboration with
C. Turner, A. Michailidis, D. Abanin, Z. Papic

Nature Physics 14, 745–749 (2018)
arXiv:1806.10933

IST AUSTRIA

KITP, 2018

Kavli Institute for Theoretical Physics
University of California, Santa Barbara

Does an isolated system reach thermal equilibrium?

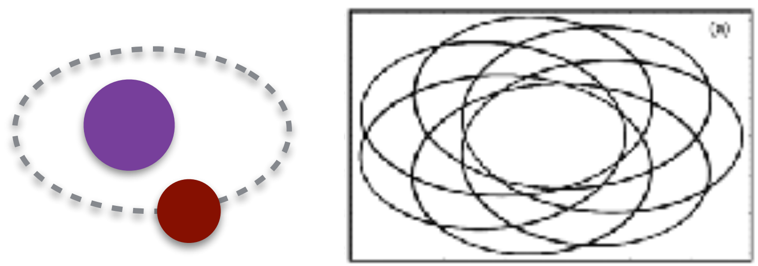
Classical

Yes:
Ergodic systems

chaos → ergodicity

No:
Integrable systems

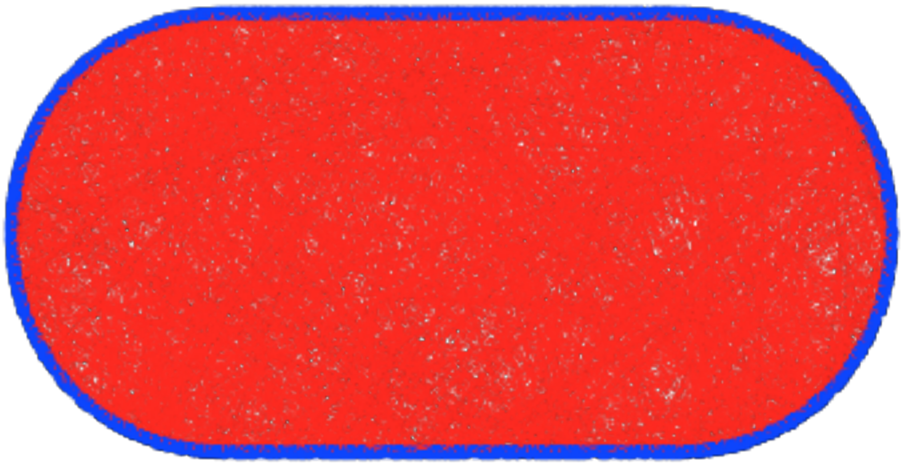
stable to weak perturbations
[Kolmogorov-Arnold-Moser theorem]



Quantum

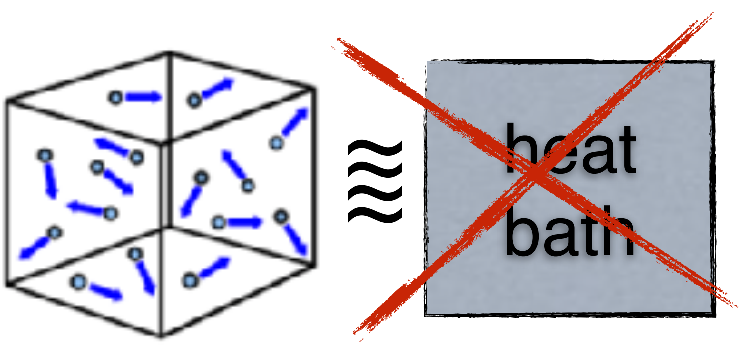
Chaos as a route to statistical equilibrium

- Systems "forget" initial conditions; explore all configurations:



- Generic systems are chaotic

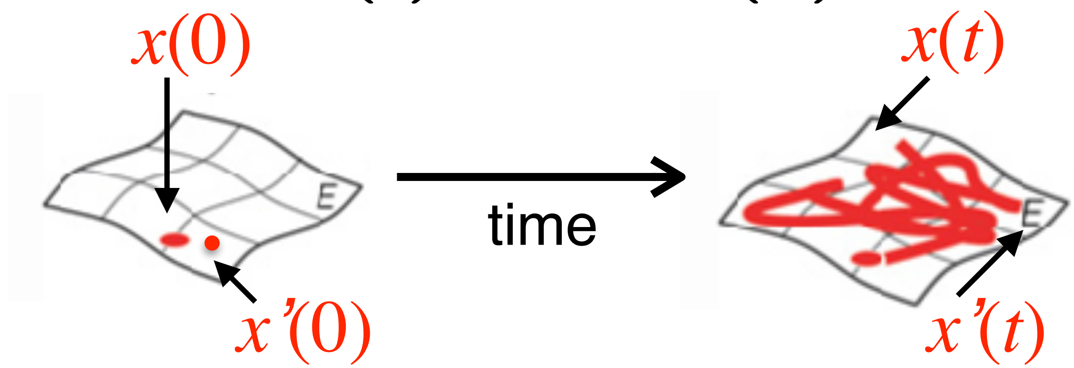
  Even isolated classical systems establish temperature



Not all systems are equally chaotic

Alexandr Lyapunov (1857-1918)

$$\delta x(t) \propto e^{\lambda t}\delta x(0)$$



Solar system

Lyapunov time $t=1/\lambda \sim 10^6$ years

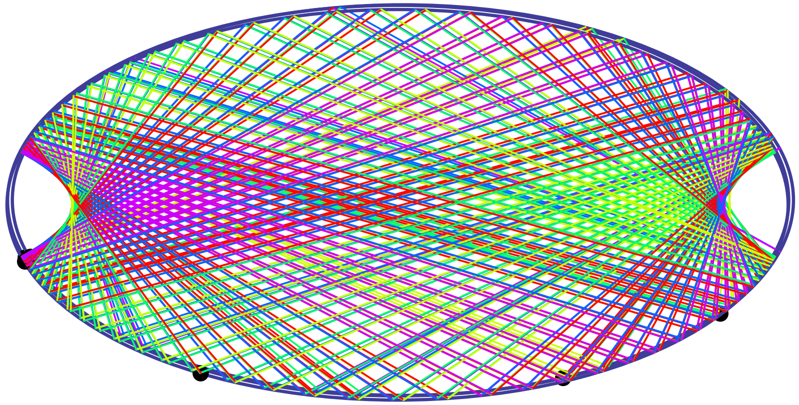
$\delta x(0)=10^{-42}$ m →

δx(500 mil. years)= 150 m

Large $\lambda \neq$ fast equilibration

Escaping chaos: integrability

- Additional conservation laws/symmetries:



- Dynamics is constrained to tori; full phase space is not explored
- KAM theorem: non-resonant tori survive perturbations

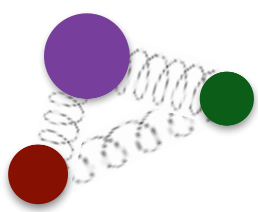
[Kolmogorov 1954; Arnold 1963; Moser 1962]

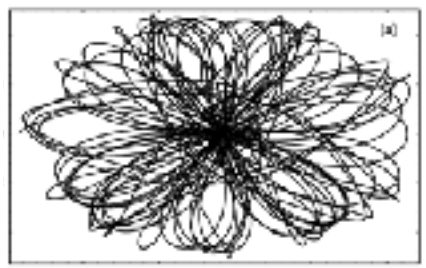
Does an isolated system reach thermal equilibrium?

	Yes:	**No:**
	Ergodic systems	**Integrable systems**

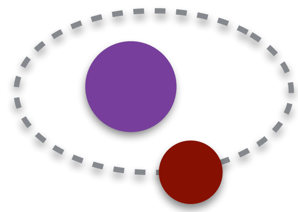
Classical

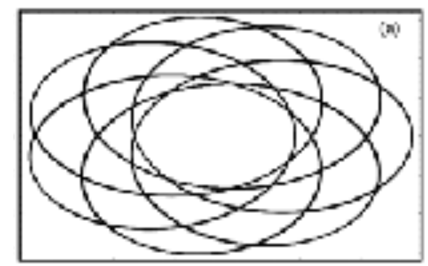
chaos → ergodicity





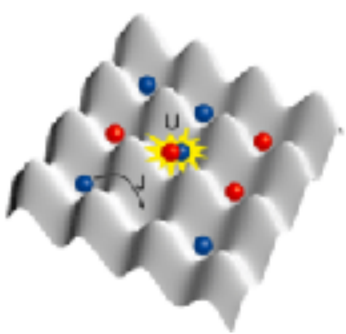
stable to weak perturbations
[Kolmogorov-Arnold-Moser theorem]

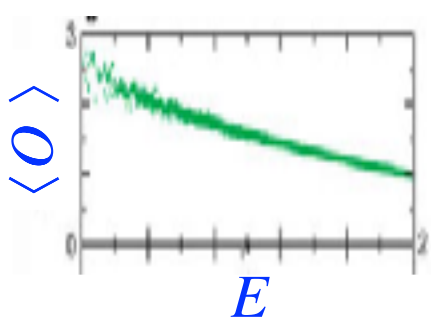




Quantum

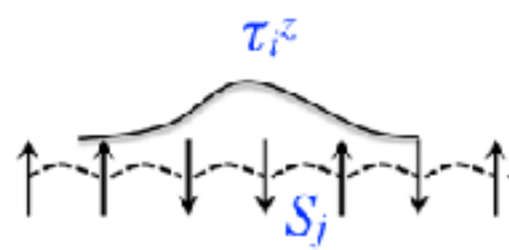
Thermalizing phases





MBL phases

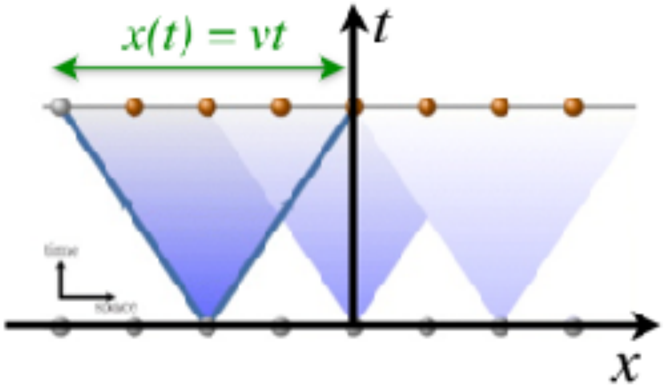
emergent integrability

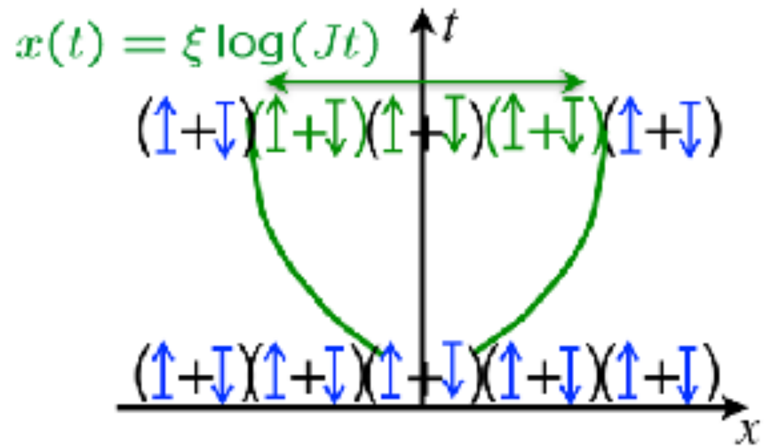


Ergodic phase vs MBL phase

$S_{\mathrm{ent}} \propto vt$

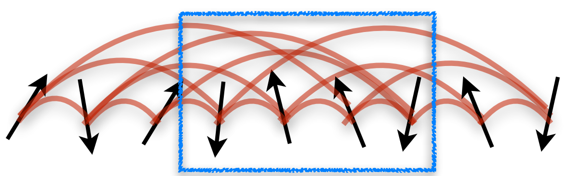
$S_{\mathrm{ent}} \propto \xi \log Jt$





Q: Intermediate behavior possible?

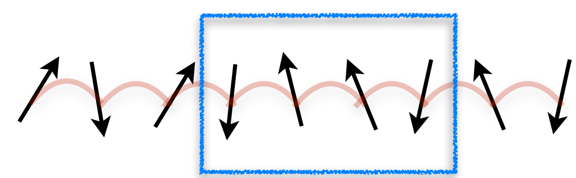
$S_{\mathrm{ent}}(A) \propto \mathrm{vol}(A)$



Eigenstate Thermalization Hypothesis

[Srednicki'94]
[Rigol,Dunjko,Olshanii'08]

$S_{\mathrm{ent}}(A) \propto \mathrm{area}(A)$

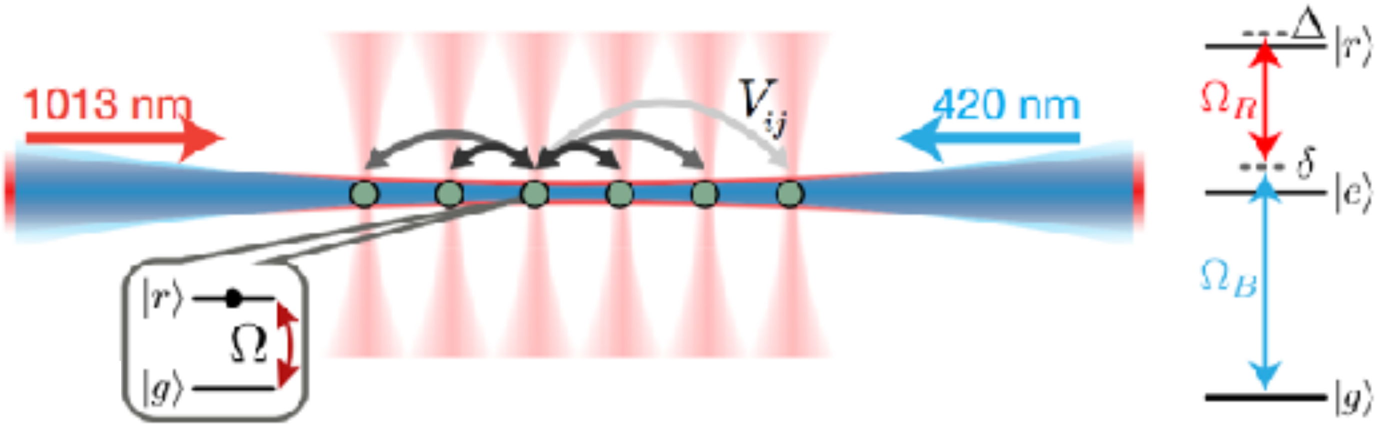


Quasi-Local Integrals of Motion

[see arXiv:1804.11065 for a review]

Experiments on Rydberg atoms array

Atom-by-atom assembly of Rydberg chain



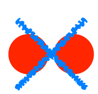
[Bernien et al, Nature 2017, arXiv:1707.04344]

Effective description: two states per atom:

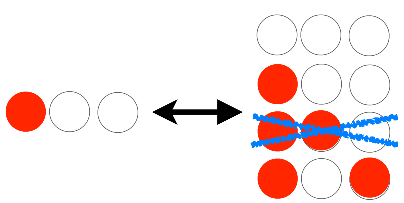
● excited (Rydberg) state

○ ground state

Rydberg blockade

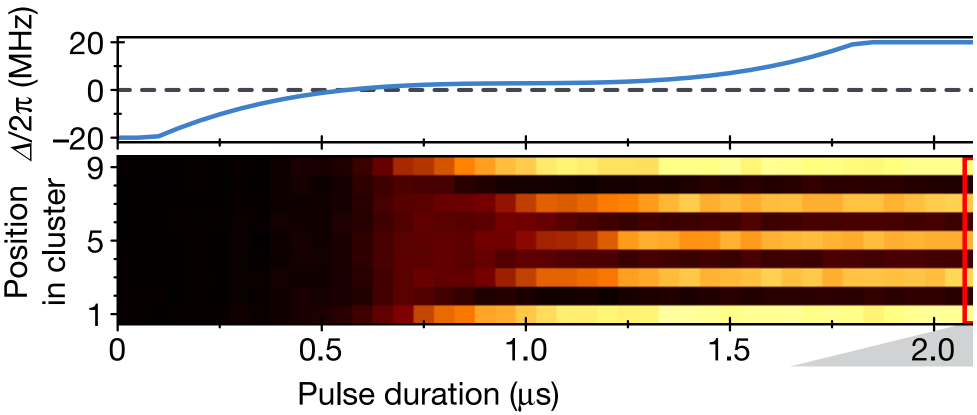


Dynamical constraint



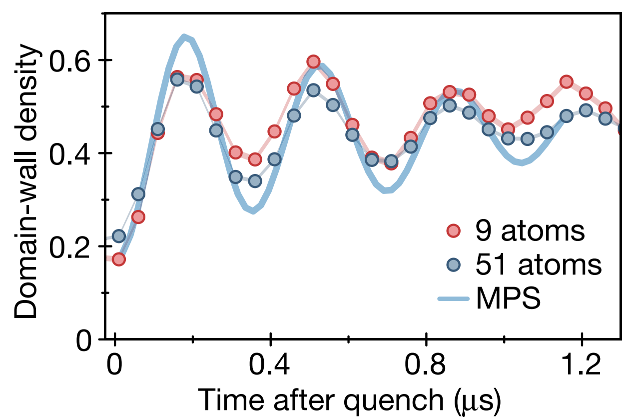
Experimental puzzle: long-time oscillations

Preparation of state



Observable:

$$O = \frac{1}{L} \sum_i P_i^\circ P_{i+1}^\circ$$



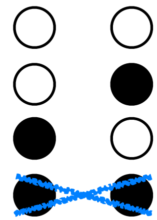
Néel: long-time oscillations

Other initial states: rapid relaxation

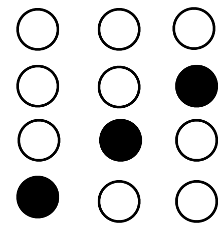
[Bernien et al, Nature 2017, arXiv:1707.04344]

PXP model as a graph

Hilbert space: $\mathcal{D}_2 = 3$



$\mathcal{D}_3 = 4$



$$\mathcal{D}_L = F_{L-1} + F_{L+1}$$

sum of Fibonacci #

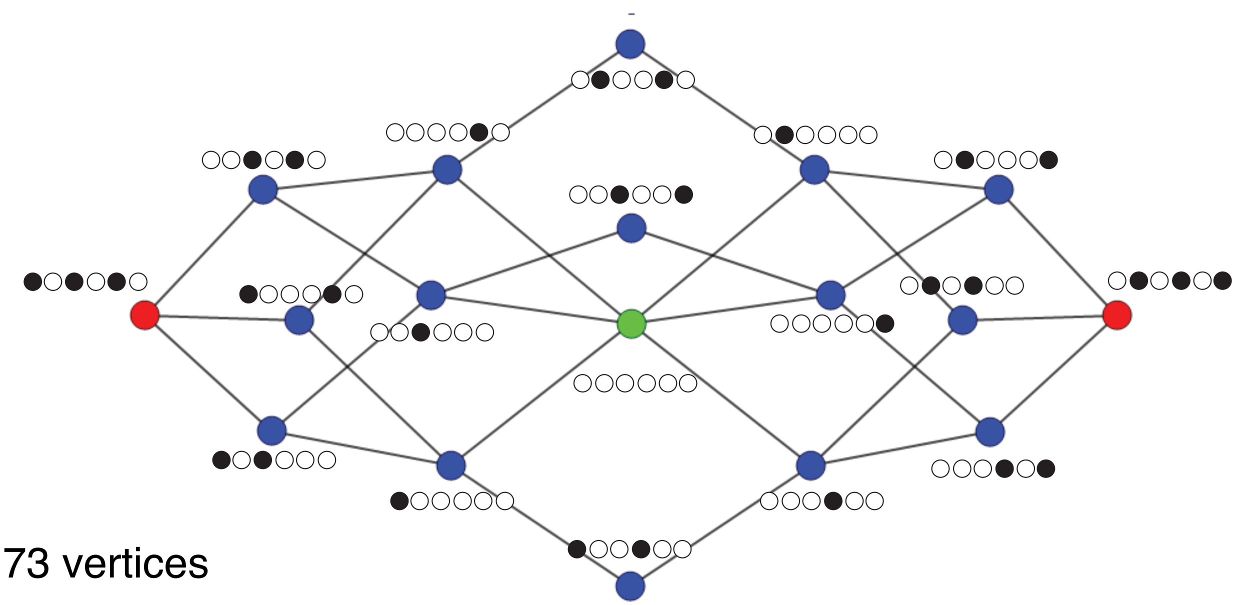
no tensor product structure!

Hamiltonian:

$$H = \sum_i P^{\circ}_{i-1} X_i P^{\circ}_{i+1}$$

[Fendley, Sengupta, Sachdev, PRB'04]
[Fendley, Schoutens, PRL'05]

Hilbert space + Hamiltonian = graph + adjacency matrix



Experiment: L=51

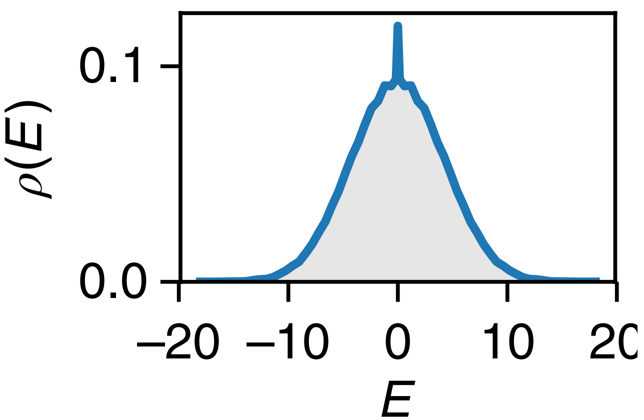
F_{53} =53,316,291,173 vertices

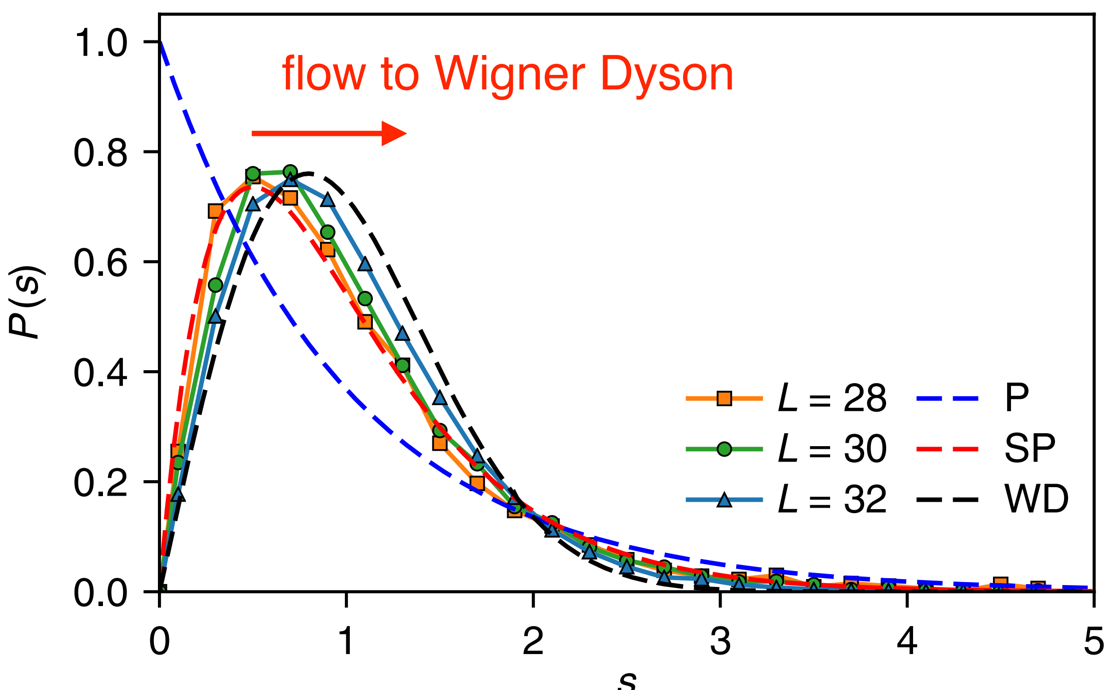
PXP model is non-integrable

Statistics of level spacings

Chaotic/ergodic	Wigner-Dyson	$P(s) = \frac{\pi}{2} s e^{-\frac{\pi}{4}s^2}$
Intermediate/critical	Semi-Poisson	$P(s) = 4 s e^{-2s}$
Integrable/MBL	Poisson	$P(s) = e^{-s}$

Gaussian DOS



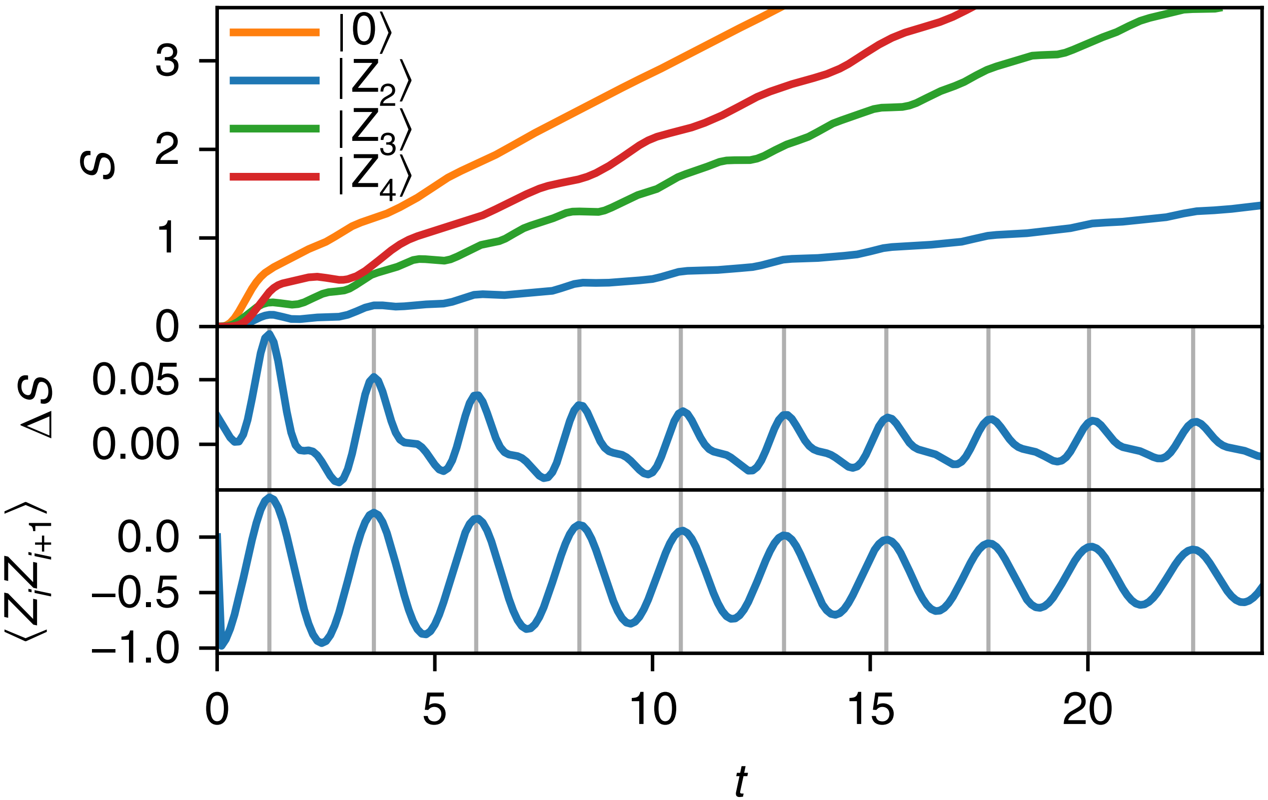


[Turner et al., Nature Physics 2018, arXiv:1711.03528]

Dynamics: ballistic growth of entanglement

Entanglement spreading depends on initial state

$|\mathbb{Z}_2\rangle = $ ●○●○●○ $|\mathbb{Z}_3\rangle = $ ●○○●○○ $|0\rangle = $ ○○○○○○



Long-time oscillations are observed!

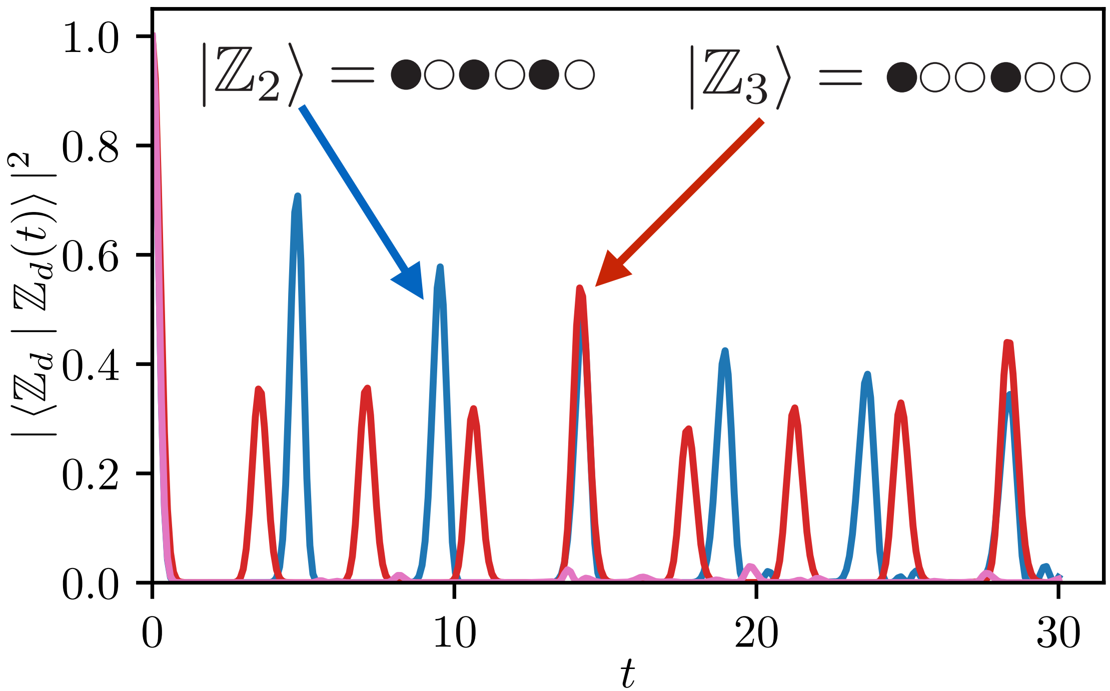
[Turner et al., Nature Physics 2018, arXiv:1711.03528]

Dynamics: revivals of many-body fidelity

Probability to return to Néel/Z_3 state:

$$\mathcal{F} = |\langle Z_2 | e^{-iHt} | Z_2 \rangle|^2$$

L=24 atoms; full Hilbert space:103,682

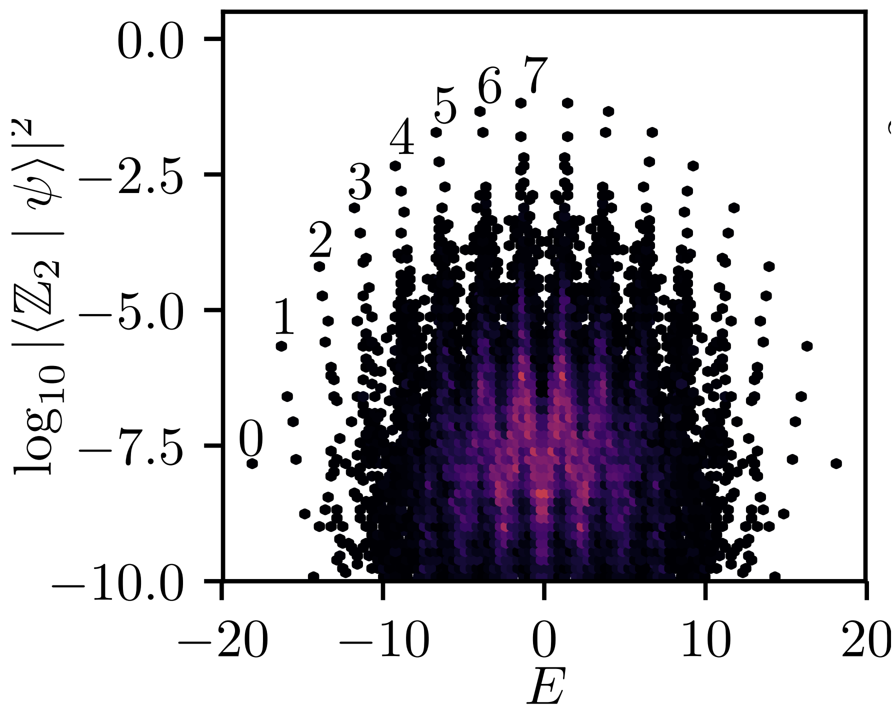


Origin of periodic revivals?

Z_2 special band of eigenstates

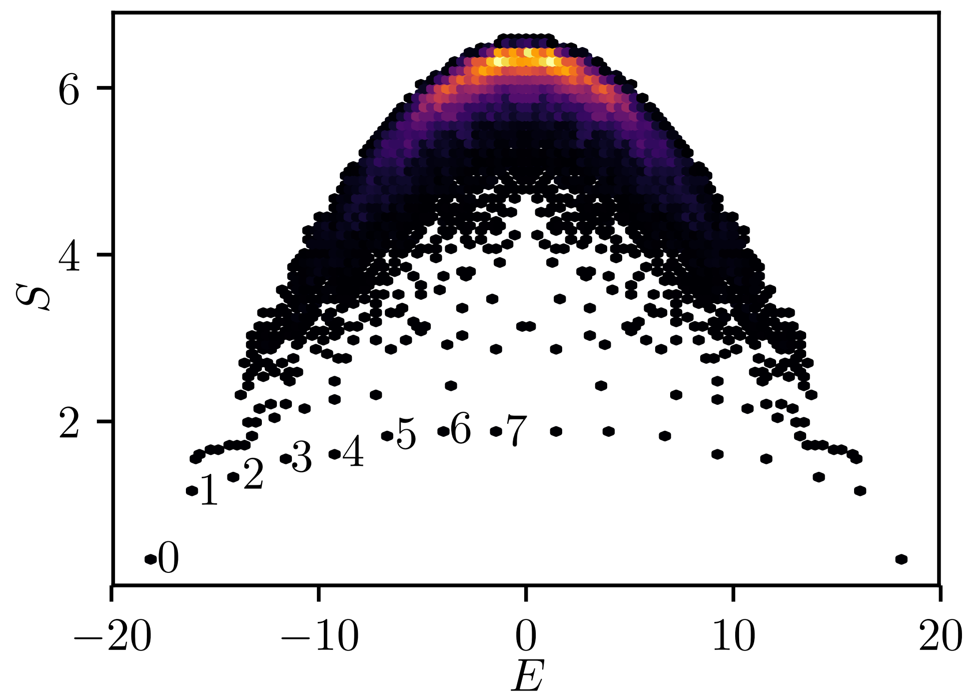
enhanced overlaps with

$|\mathbb{Z}_2\rangle = $ ●○●○●○



Vs. random eigenstates in ergodic systems

anomalously low entanglement entropy



Vs. volume law S_{ent} in ergodic systems area-law in MBL systems

How to understand these special states?

Breakdown of thermalization by special eigenstates

Eigenstate Thermalization Hypothesis:

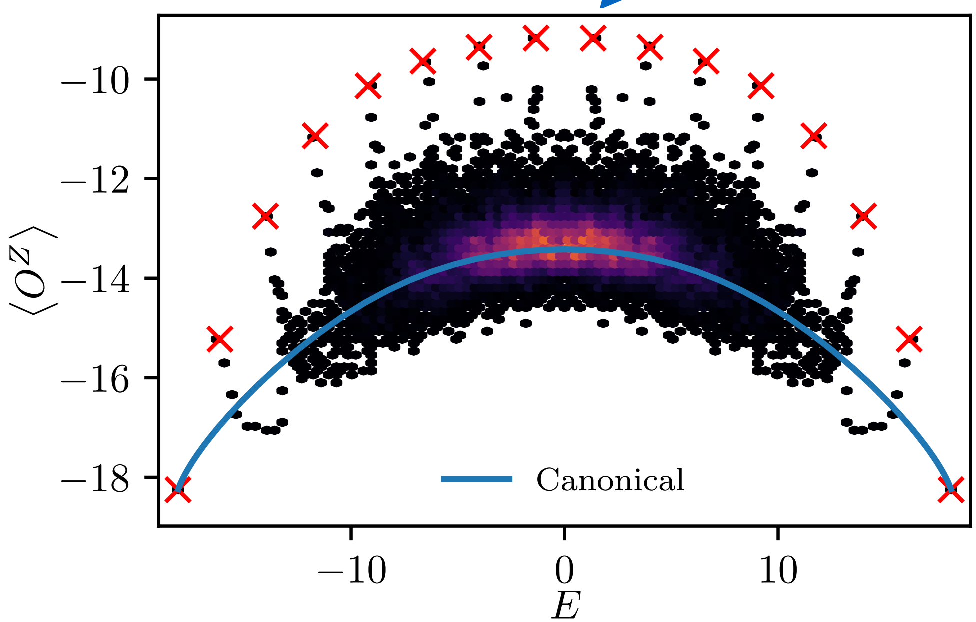
volume-law entanglement

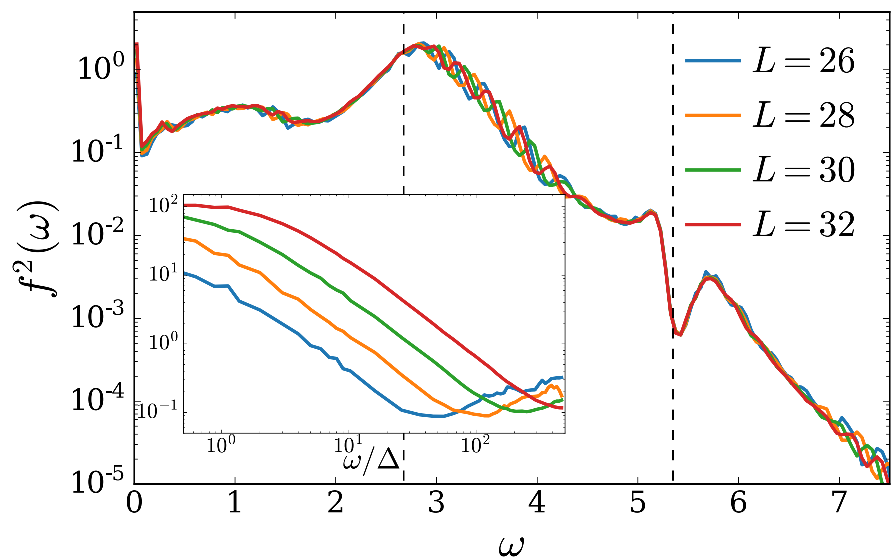
small fluctuation of local observables

small off-diagonal matrix elements

$$O_{\alpha\beta} = \mathcal{O}(E)\delta_{\alpha\beta} + e^{-S(E)/2} f(E,\omega) R_{\alpha\beta}$$

[Turner et al., arXiv:1806.10933]

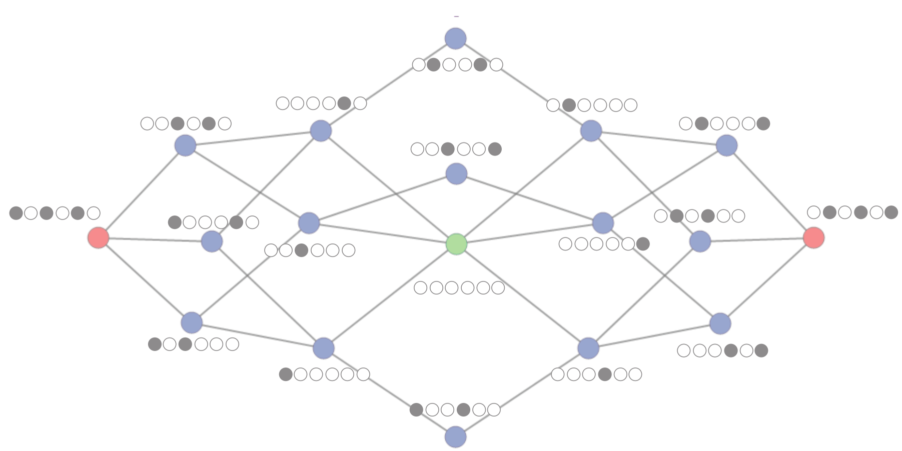




How to understand these special states?

$$H^+ = \sum_{i\in \text{ even}} P_{i-1}\sigma_i^+ P_{i+1} + \sum_{i\in \text{ odd}} P_{i-1}\sigma_i^- P_{i+1}$$

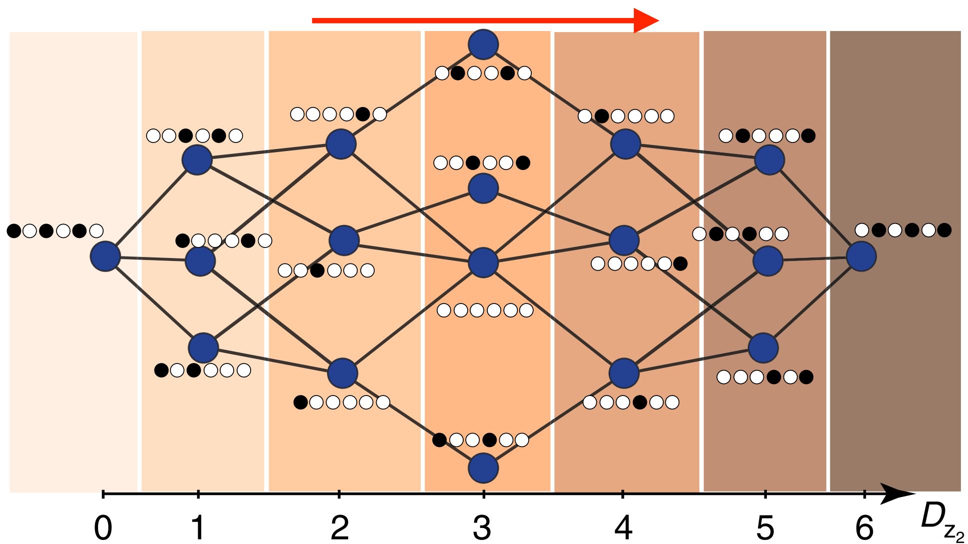
Forward scattering approximation



Constructing the special band of eigenstates

$$H = H_+ + H_- = \text{forward} + \text{backward}$$

$$H^+ = \sum_{i\in \text{ even}} P_{i-1}\sigma_i^+ P_{i+1} + \sum_{i\in \text{ odd}} P_{i-1}\sigma_i^- P_{i+1}$$



$$\left[H^+, H^-\right] = H^z = \sum_{i\in \text{ even}} P_{i-1} Z_i P_{i+1} - \sum_{i\in\text{odd}} P_{i-1} Z_i P_{i+1}$$

Incomplete spin algebra:

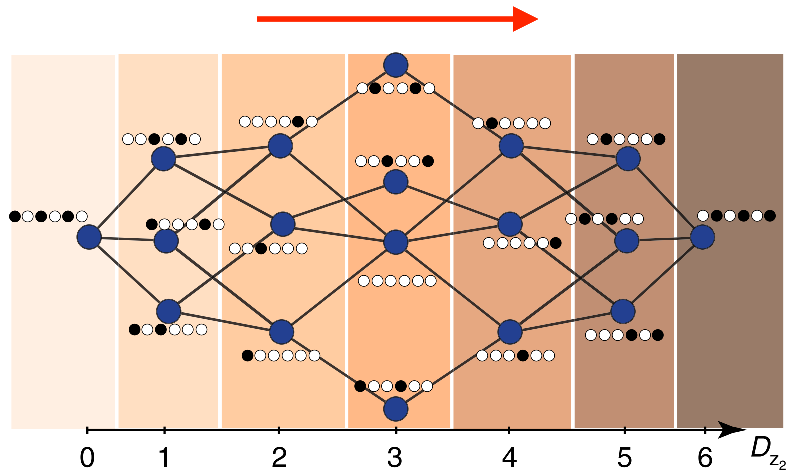
$$\left[H^z, H^+\right] = 2H^+ - \sum_{\text{odd}} P_{i-1} S_i^+ P_{i+1} (P_{i-2} + P_{i+2}) - \sum_{\text{even}} \text{h.c.}$$

corrections

Forward scattering approximation

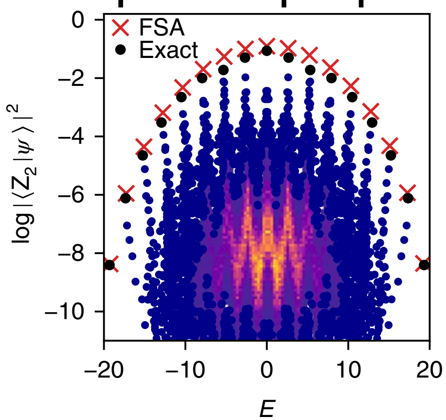
Forward scattering H^+ to construct basis:

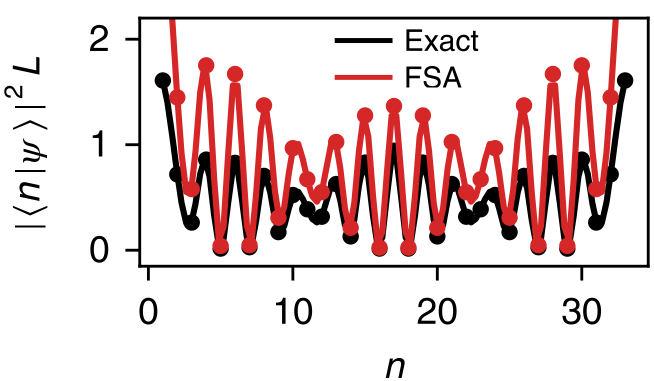
$$|\mathbb{Z}_2\rangle, \; H^+|\mathbb{Z}_2\rangle, \; [H^+]^2|\mathbb{Z}_2\rangle, \; \ldots, \; [H^+]^n|\mathbb{Z}_2\rangle$$



Projected Hamiltonian = tridiagonal matrix

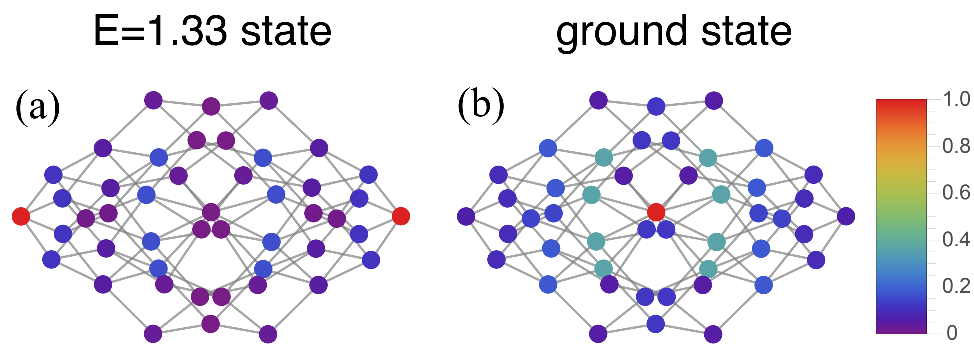
Captures properties of highly excited special states





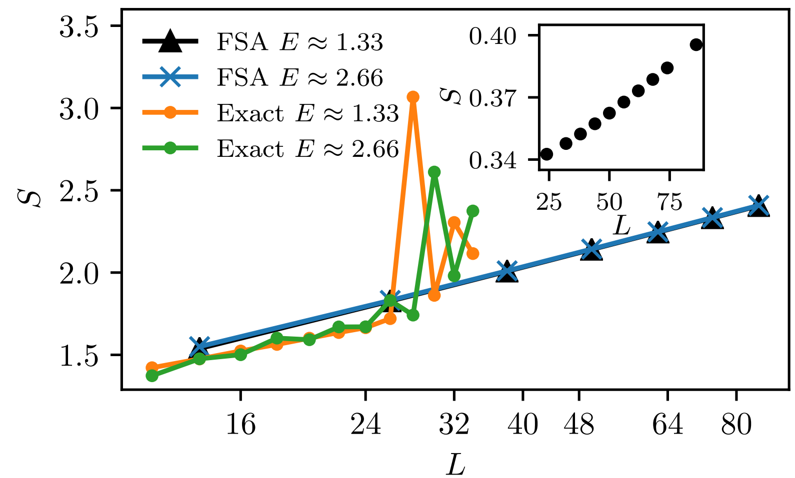
Structure of special eigenstates

Concentration on parts of Hilbert space

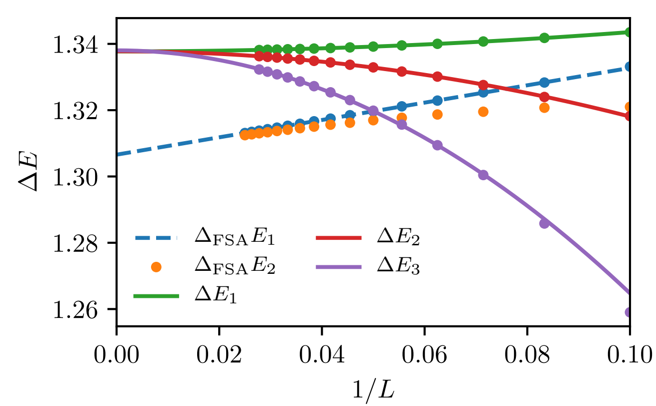


Low entanglement

$S_{\mathrm{ent}}(A) \propto \log L_A$

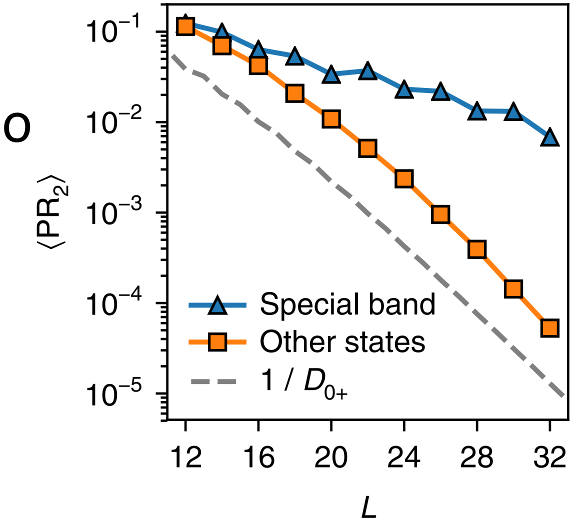


Constant ΔE
+ $O(1/L^2)$ corrections



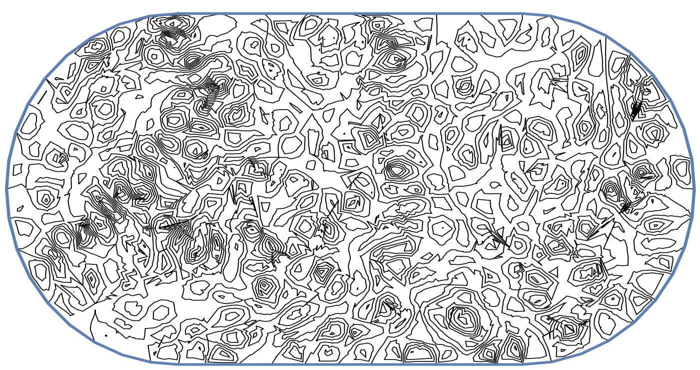
Special eigenstates as quantum many-body scars

- Concentration, low entanglement, participation ratio
- Constant energy separation
- Remaining eigenstates are “conventional”

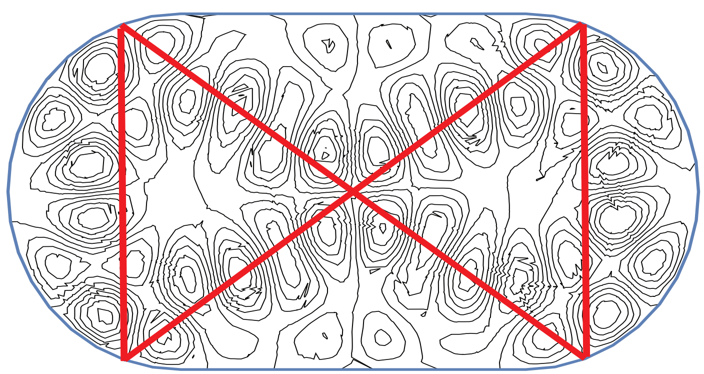


Quantum scars in single-particle chaos

typical eigenstates

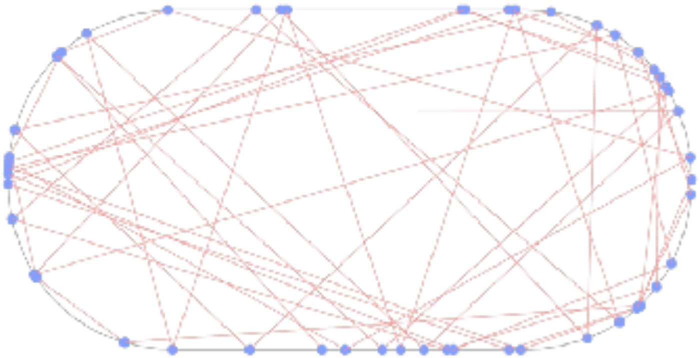


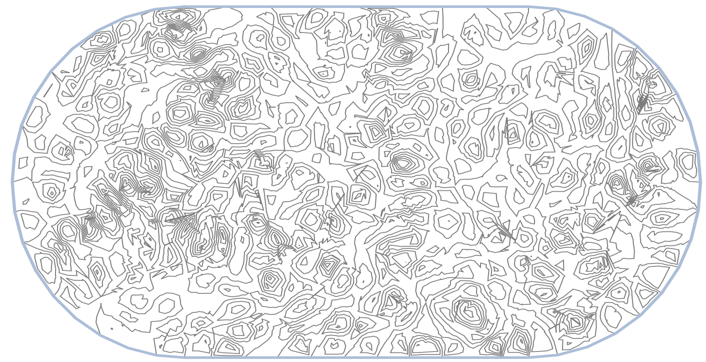
quantum scarred eigenstates



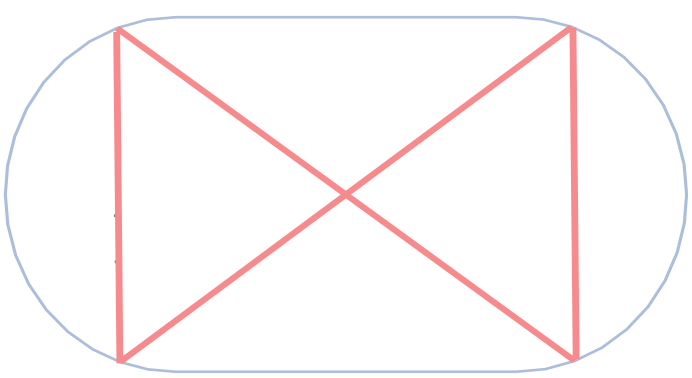
[Heller, PRL’84]

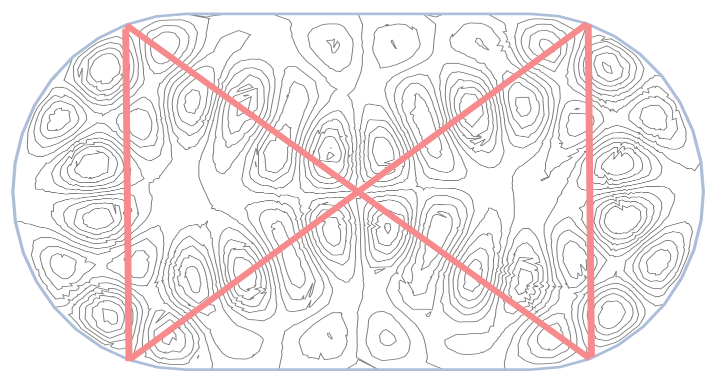
Unstable **classical** orbits influence **quantum** eigenstates





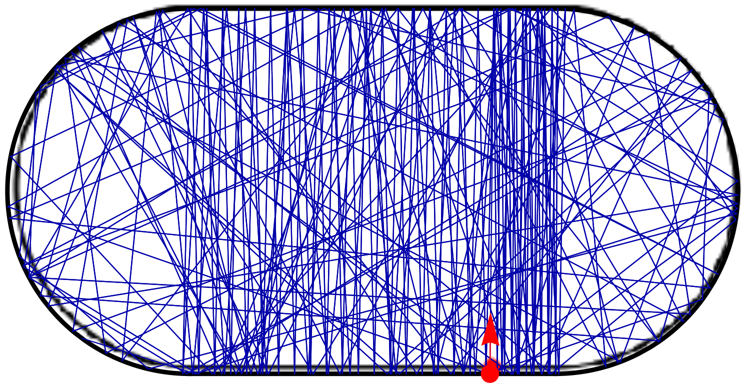
Quantum many-body scars

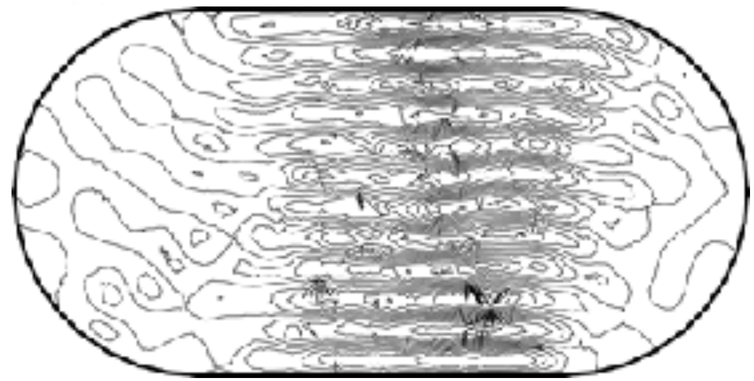




Properties of single-particle quantum scars

I. Different trajectories with small Lyapunov exponent $\lambda_L T \ll 1$



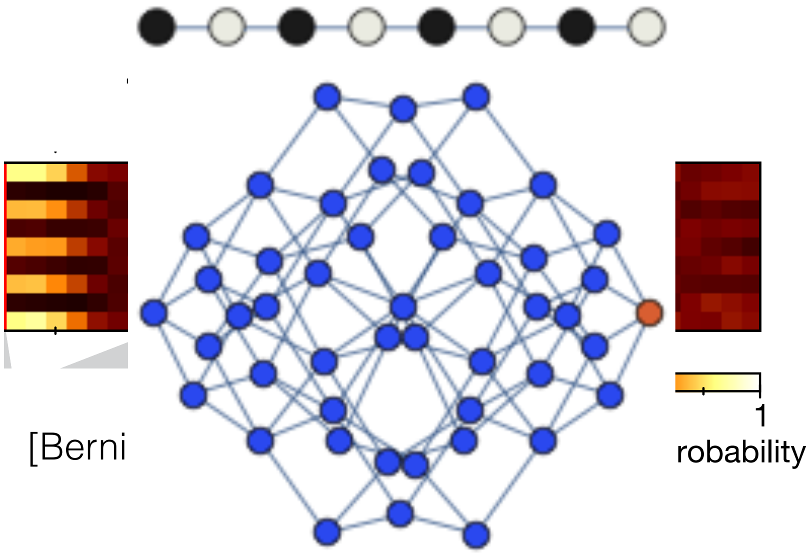


Small Lyapunov exponent → stronger scarring

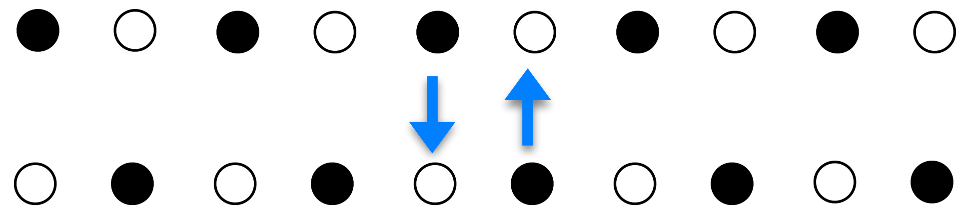
II. Stability to perturbations when periodic orbit is not destroyed

Do these properties hold in many-body system?

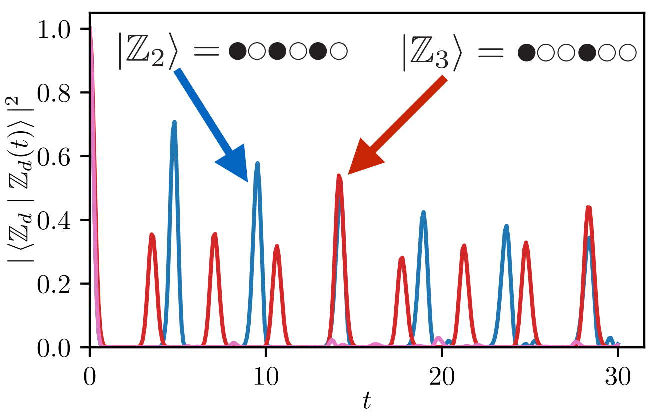
Trajectories and special bands: Z_2 and Z_3



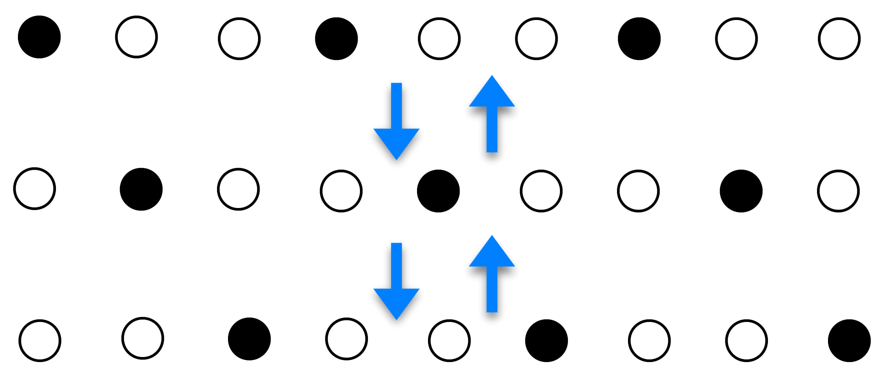
trajectory connecting 2 Néel states



Z_3 special eigenstates

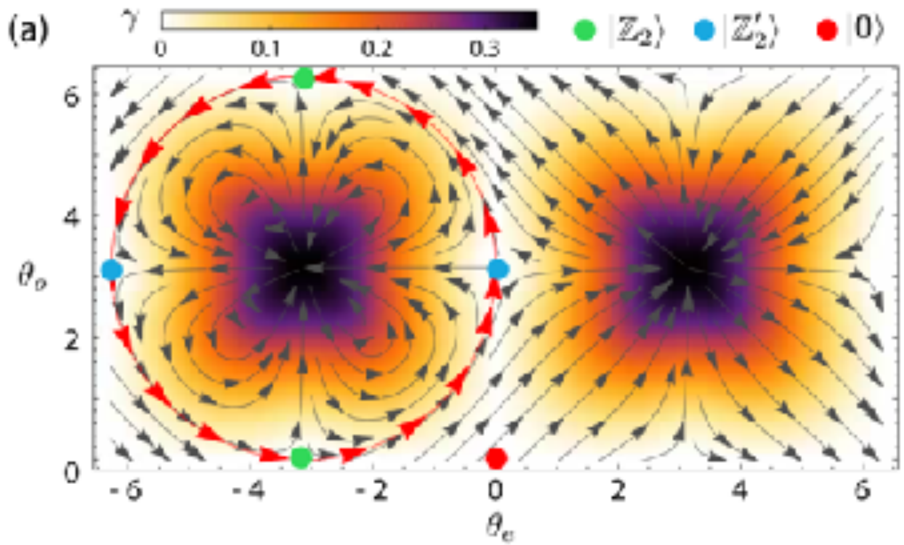


trajectory connecting period-3 CDW



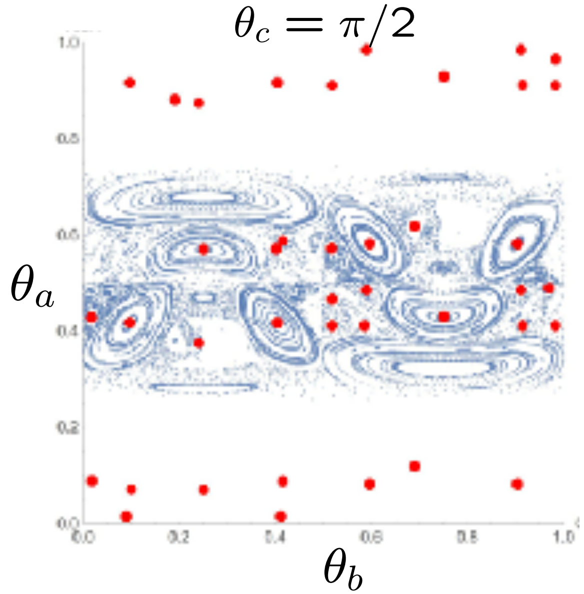
Time dependent variational principle

Z_2 band trajectory with TDVP

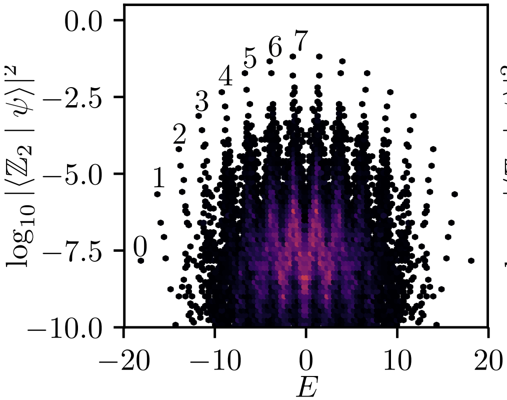


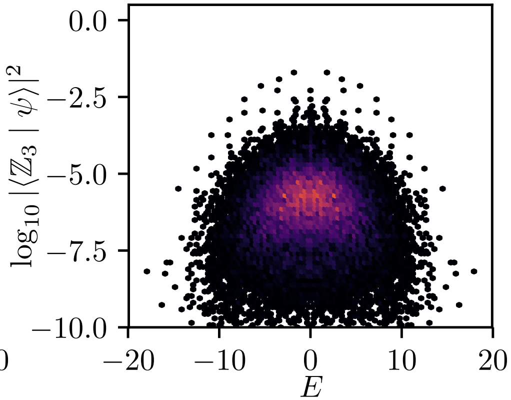
[WW Ho et al, arXiv:1807.01815], see poster!
[Haegeman et al'11]
[Bernien et al, Nature'17]

Z_3 band trajectories



[Michailidis et al, in preparation]





Z_3 trajectory
is more unstable
→ weaker scarring

Effect of perturbations

Rydberg-motivated perturbations:
oscillations are more damped
thermalization is enhanced

[Turner et al., arXiv:1806.10933]

$$\delta H_0 = g_0 \sum_j Q_j,$$

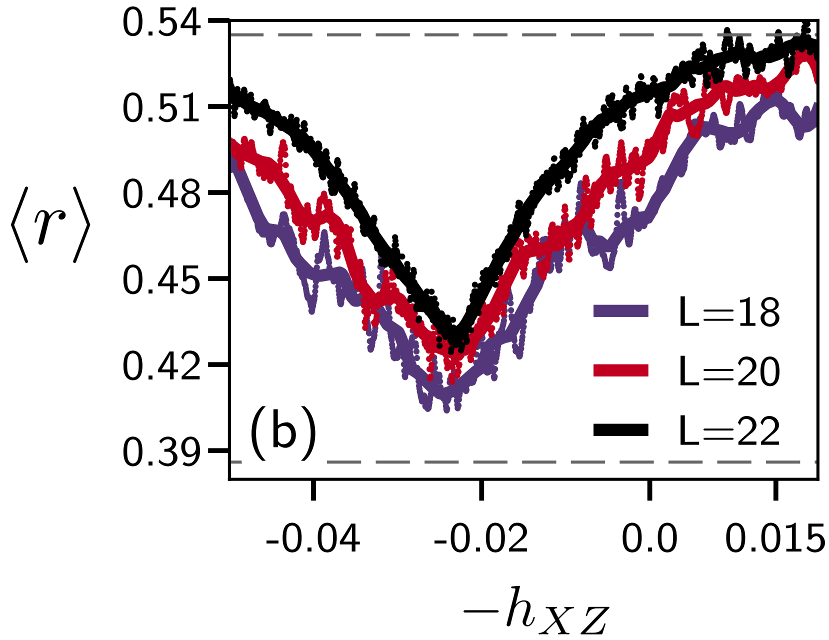
$$\delta H_{\rm nn} = g_{\rm nn} \sum_j P_{j-1}(\sigma_j^+ \sigma_{j+1}^- + \sigma_j^- \sigma_{j+1}^+) P_{j+2}$$

Special perturbation:
improves oscillations
slows down thermalization

[Khemani et al., arXiv:1807.02108]

$$h_{XZ} \approx 0.02$$

$$\delta H = h_{XZ} \sum_i P_{i-1} X_i P_{i+1} (Z_{i-2} + Z_{i+2})$$



Q: are scars caused
by proximate integrability?

Making quantum scars exact

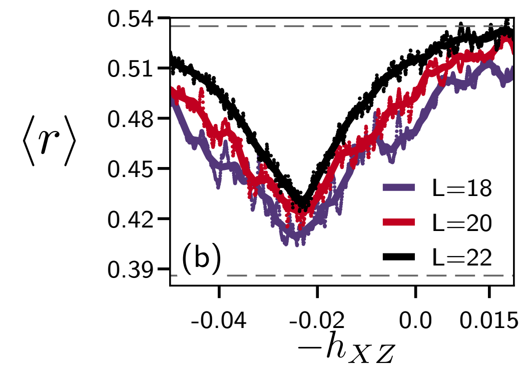
Two **different** "special points":

$$\delta H = h_{XZ} \sum_i P_{i-1} X_i P_{i+1} (Z_{i-2} + Z_{i+2})$$

Max non-ergodicity

$$h_{XZ} \approx 0.02$$

[Khemani et al., arXiv:1807.02108]



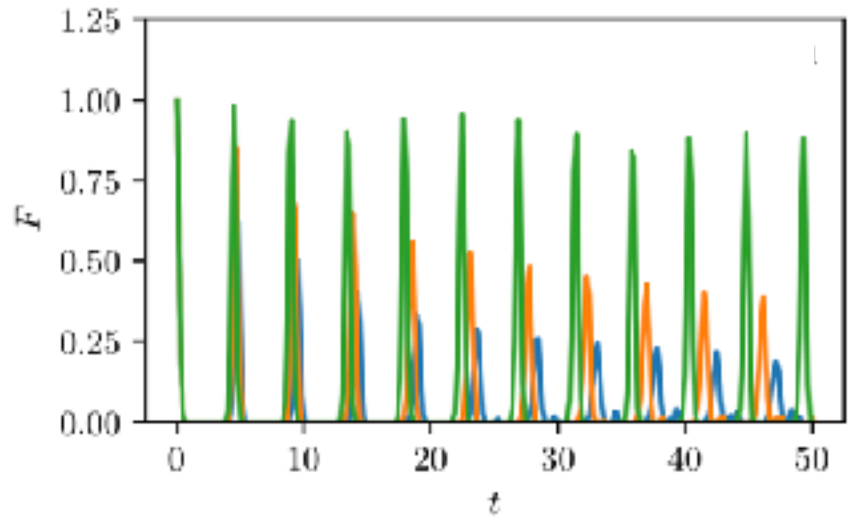
Making scars exact:

$$h_{XZ} \approx 0.055$$

[Choi et al., in preparation]

Longer-range deformation:
Fidelity revives to 1
up 10^{-6} corrections!

L=24, quench from Neel

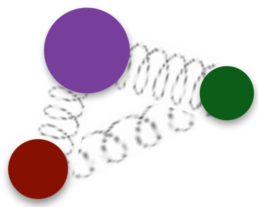


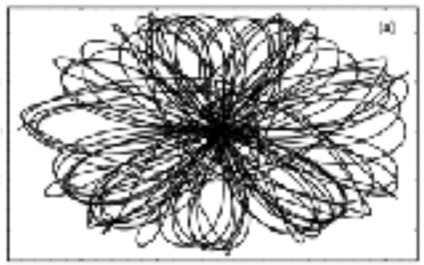
Ergodicity and integrability

Classical

<u>Ergodic systems</u>

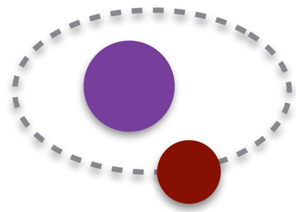
chaos → ergodicity

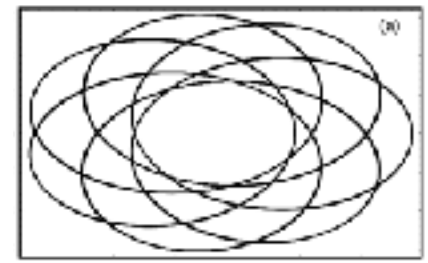




<u>Integrable systems</u>

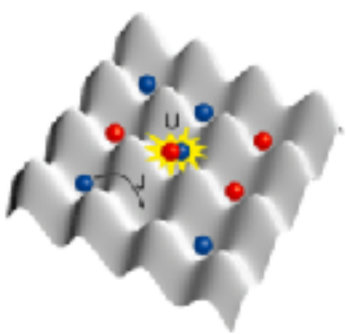
stable to weak perturbations
[Kolmogorov-Arnold-Moser theorem]

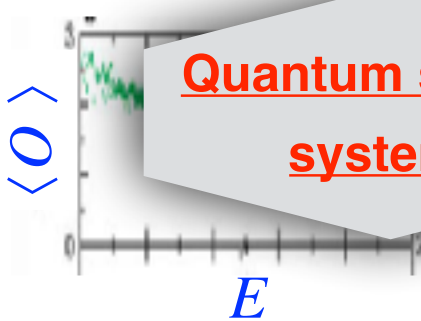




Quantum

Thermalizing phases

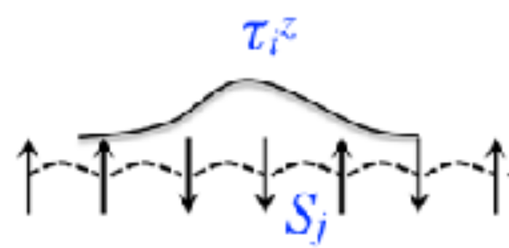




<u>Quantum scarred systems</u>

MBL phases

emergent integrability

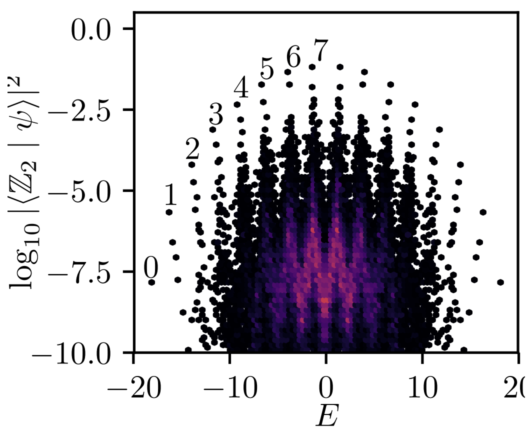


Quantum many-body scars

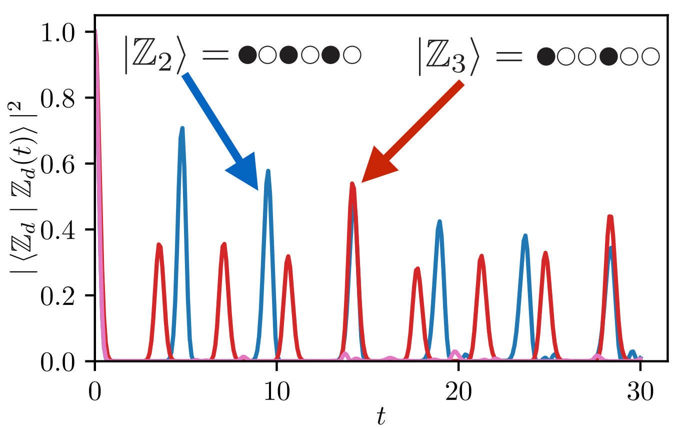
- Weak ergodicity breaking:
  - ✳ special eigenstates: low entanglement, large $Z_{2,3}$ overlaps, no ETH
  - ✳ explains recent experiments; survives deformation; can be improved

- Open questions:
  - ✳ classes of models with quantum scars?
  - ✳ semiclassical limit, meaning/value of Lyapunov exponent?
  - ✳ use scars and/or zero modes to protect quantum information?

[Nat. Phys. 14, 745–749 (2018)]



[arXiv:1806.10933]



Related work:

[Bernien et al, Nature 2017]

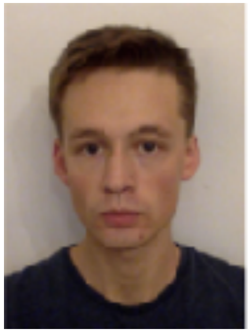
[WW Ho et al., arXiv:1807.01815]

[Khemani et al., arXiv:1807.02108]

Acknowledgments

Collaborations:

Christopher Turner



Alex Michailidis

Dima Abanin

Zlatko Papic

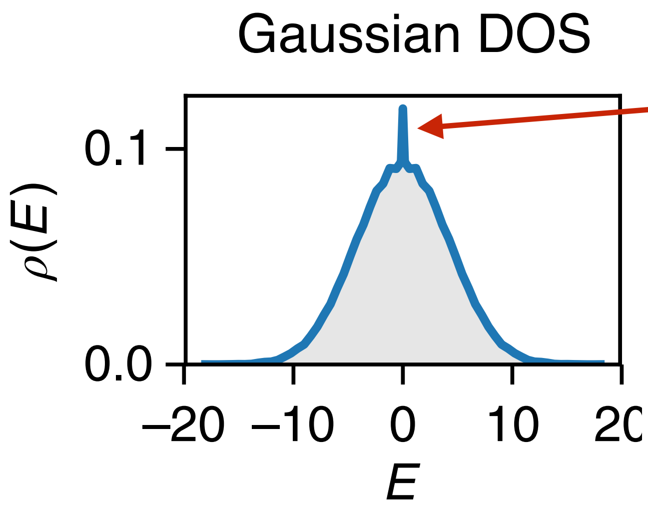
Wen Wei Ho

Soonwon Choi

Hannes Pichler

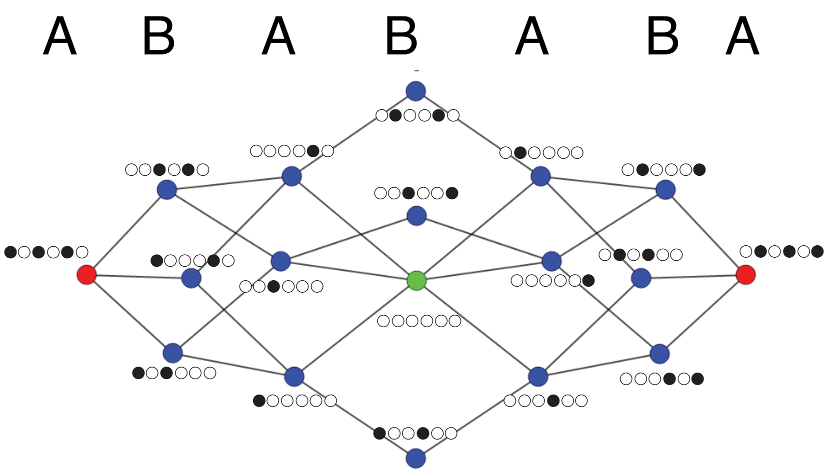
Mikhail Lukin

Degeneracy at E=0



Exponential number of “zero-modes” $\propto \sqrt{\mathcal{D}}$

bipartite graph + inversion symmetry



Inversion = +1 sector

but no such states in I=-1

A - # B ≧ # zero modes

Compatible with scarred eigenstates!

more details in [Turner et al., arXiv:1806.10933]