KITP QDyn Workshop
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Santa Barbara, USA

Institute for Theoretical Physics University of Cologne

Absence of Criticality in Open Floquet Systems — How to Renormalize an Ensemble under Periodic Drive

arxiv:1807.02146

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€€€ :

Deutsche Forschungsgemeinschaft

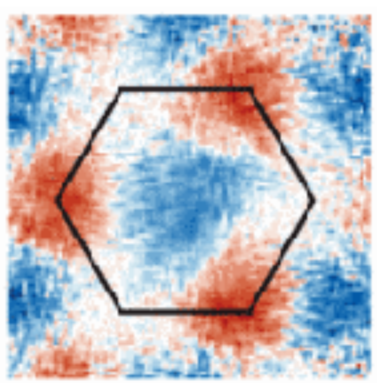
European Research Council

in collaboration with
Steven Mathey (Cologne)

Periodically Driven Quantum Systems

- Realizations in different platforms

Engineering artificial gauge fields

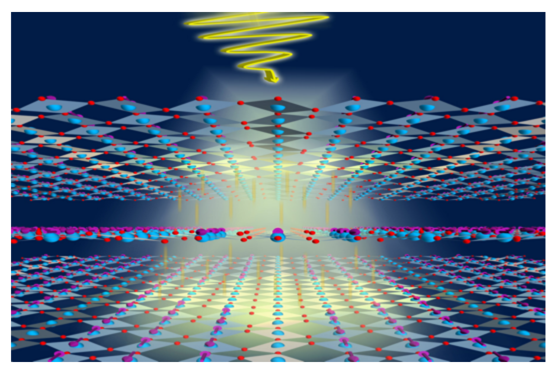


Ultracold atoms

Fläschner et al. Science (2016)
Eckardt, RMP (2017)

driven, closed
often non-interacting

Light induced superconductivity

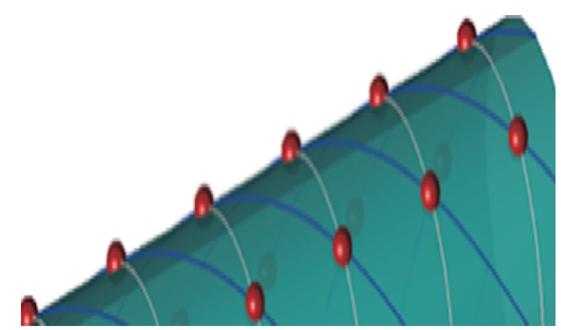


Solid state

Fausti et al. Science (2011)
Mitrano et al. Nature (2016)

driven, open
interacting

Quantum time crystals
Wilczek, PRL (2012)

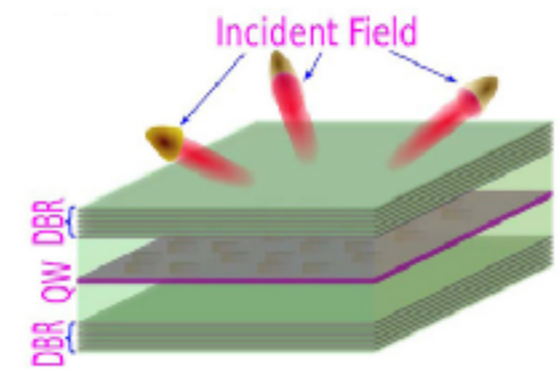


Trapped ions, NV centers

Zhang et al. Nature (2017)
Choi et al. Nature (2017)

driven, closed
short times

Floquet topological insulators



Polaritons

Karzig et al. PRX (2015)
Nalitov et al. PRL (2015)
Ge, Broer, Liew PRB (2018)

driven, open
interacting

➡ challenge: avoid indefinite heating in interacting, driven systems
➡ way out: coupling to a bath (eg. phonons, photons)

Genske, Rosch, PRA (2015)
Seetharam et al., PRX (2015)

Periodically Driven Quantum Systems

- Realizations in different platforms

Engineering artificial | Light induced | Quantum time crystals | Floquet topological

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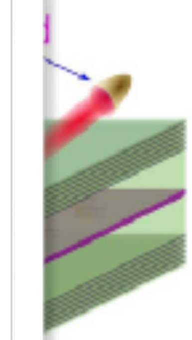


Fläs

(2015)
(2015)
B (2018)

en

Questions:

What is the nature of phase transitions
in periodically driven, open quantum systems (3D)?

How to renormalize an open Floquet system?

(1D closed disordered Ising: Berdanier, Kolodrubetz, Parameswaran, Vasseur, arXiv:1803.00019, arXiv:1807.09767)

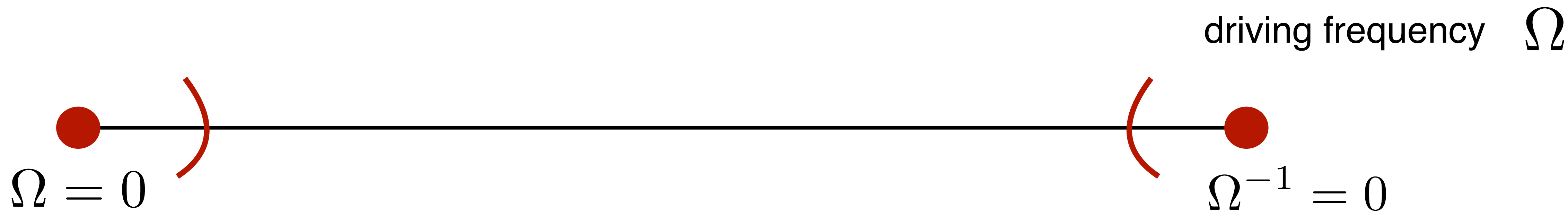
often non-interacting | interacting | short times | interacting

➡ challenge: avoid indefinite heating in interacting, driven systems
➡ way out: coupling to a bath (eg. phonons, photons)

Genske, Rosch, PRA (2015)
Seetharam et al., PRX (2015)

Periodically Driven Open Quantum Systems

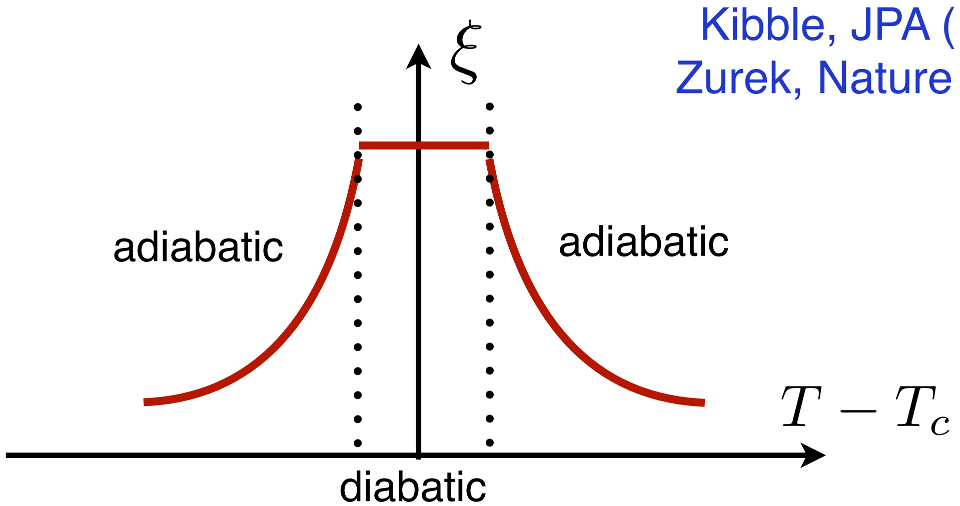
- What is known



Equilibrium limit ⟶ Continuous symmetry breaking phase transitions ⟵ Lindblad limit

Slowly driven regime: Kibble-Zurek mechanism

Kibble, JPA (1976)
Zurek, Nature (1985)



- ξ saturates for $\nu_\tau \sim (T - T_c)$
- production of defects

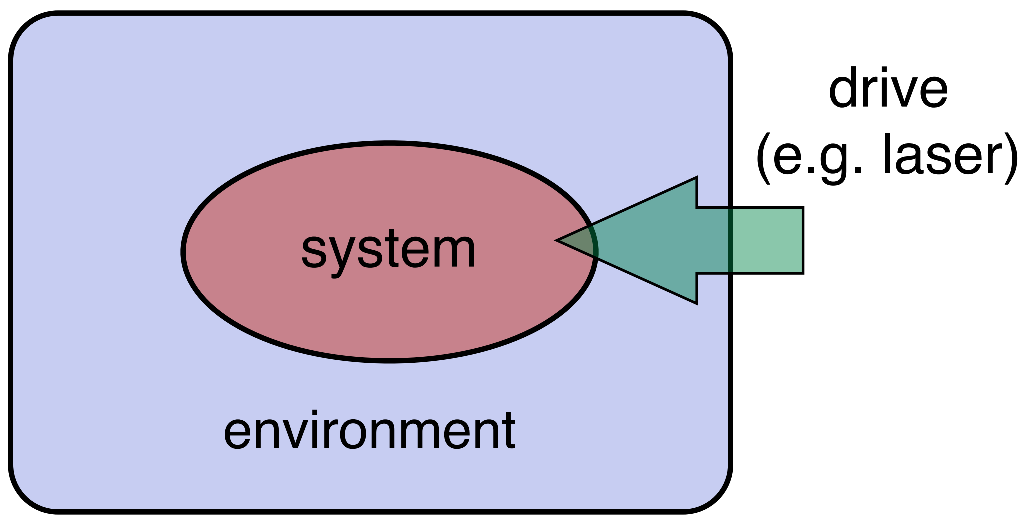
driving rate

➡ Second order phase transition 'masked' by slow drive
➡ Exponents accessible
➡ Universality class not modified

Rapidly driven regime: unknown

- Plan:
  - Description of driven open quantum systems
  - Modified criticality in the infinitely rapidly driven limit
  - Absence of criticality at rapid, finite drive

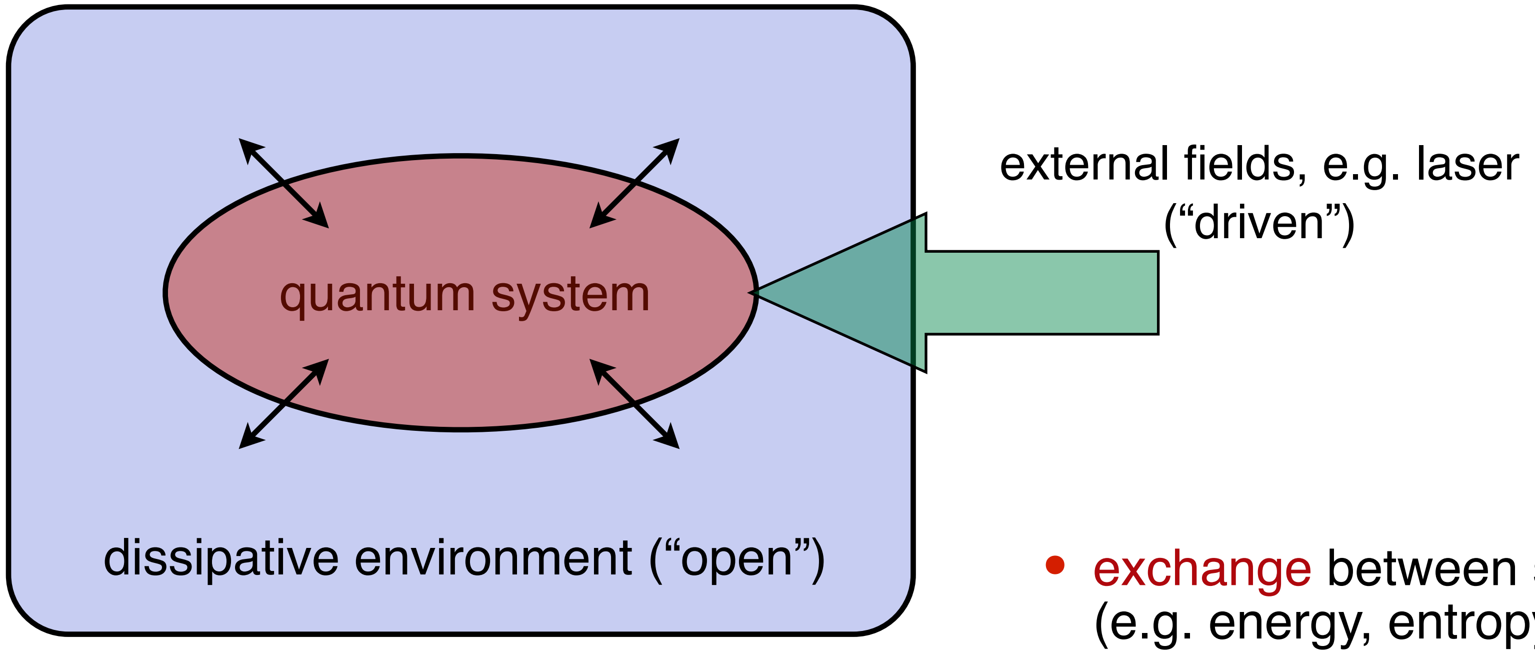
Equilibrium vs. Non-equilibrium



$$\partial_t \rho = -i[H, \rho] + \mathcal{D}[\rho] \quad \Longleftrightarrow \quad \mathrm{e}^{\mathrm{i}\Gamma[\Phi]} = \int \mathcal{D}\delta\Phi \mathrm{e}^{\mathrm{i}S_M[\Phi+\delta\Phi]} \Longleftrightarrow \quad \partial_k \Gamma_k = \frac{\mathrm{i}}{2} \mathrm{Tr}\left[\left(\Gamma_k^{(2)} + R_k\right)^{-1} \partial_k R_k\right]$$

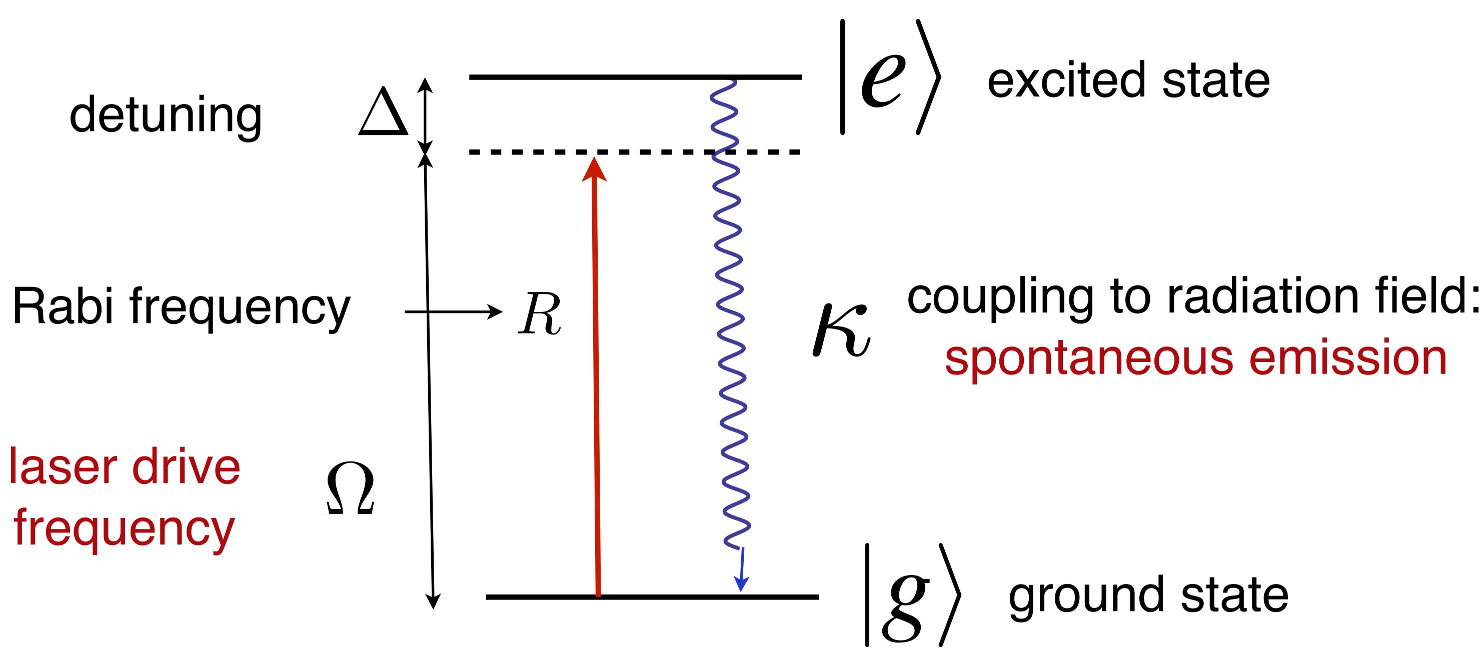
Lindblad limit of driven, open quantum systems

- Quantum Optics: periodically driven and open quantum systems



- exchange between system and bath (e.g. energy, entropy, particle number)

- example: laser driven atom coupled to the radiation field (two-level system)



- simple fact: drive essential to access upper level

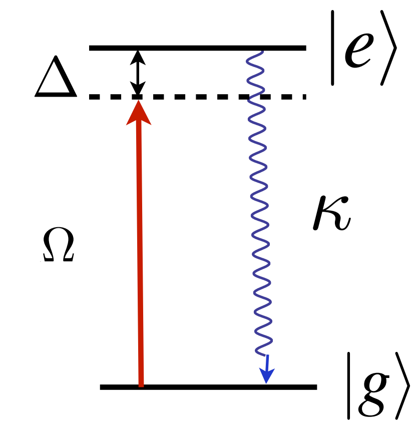
- Implications:

> - no guarantee for detailed balance
> - no obedience of the second law of thermodynamics (state purification)

Lindblad limit

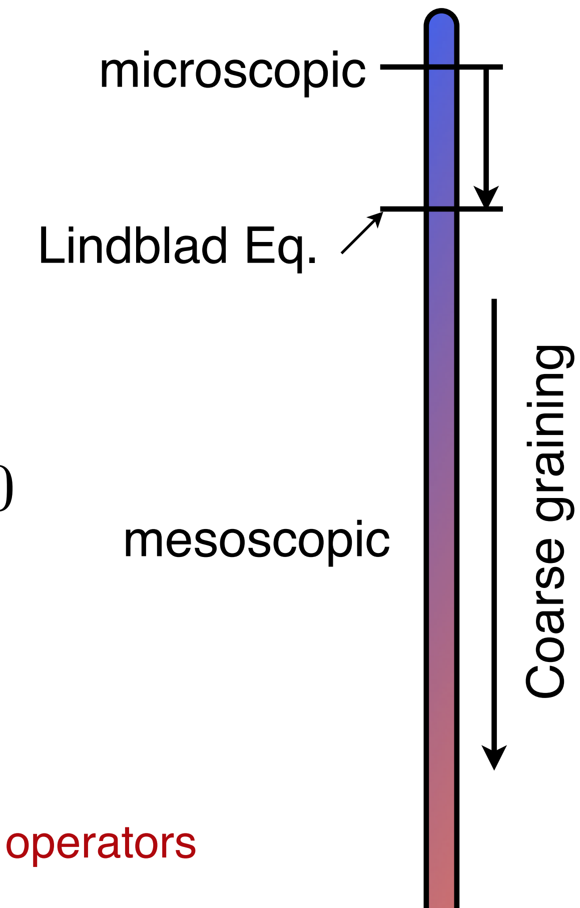
- microscopically system-bath setting

$Re^{i\Omega t}|e\rangle\langle g| + h.c.$



$$\partial_t \rho_{\text{tot}} = -i[H + H_{\text{SB}} + H_{\text{B}}, \rho_{\text{tot}}]$$

continuum of harmonic oscillators



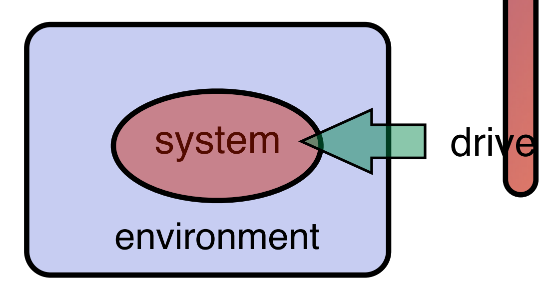
- typical regime: $\frac{\Delta}{\Omega} \sim 10^{-4} \ll 1$

➡ drop corrections in rotating wave approximation (+ Born-Markov): $\Omega^{-1} = 0$

- elimination of bath variables: Lindblad master equation

Lindblad operators

$$\underbrace{\partial_t \rho}_{\text{system}} = \underbrace{-i[H, \rho]}_{\text{coherent evolution}} + \underbrace{\kappa \sum_i (L_i \rho L_i^\dagger - \tfrac{1}{2}\{L_i^\dagger L_i, \rho\})}_{\text{driven-dissipative evolution}}$$

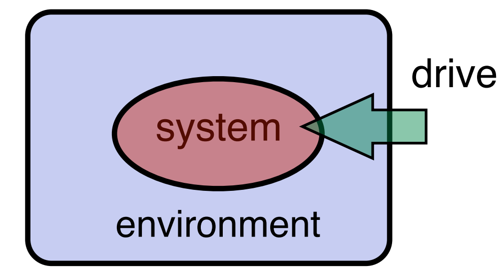


Lindblad limit: interpretation

- Lindblad master equation:

Lindblad operators

$$\partial_t \rho = \underbrace{-i[H,\rho]}_{\text{coherent evolution}} + \underbrace{\kappa \sum_i (L_i \rho L_i^\dagger - \tfrac{1}{2}\{L_i^\dagger L_i, \rho\})}_{\text{driven-dissipative evolution}}$$



- interpretation: rewrite

$$\partial_t \rho = -i(H - i\tfrac{\kappa}{2}\sum_i L_i^\dagger L_i)\rho + h.c. + \kappa \sum_i L_i \rho L_i^\dagger$$

energy decay (dissipation)

ensures probability conservation (fluctuation)

$$\text{"}E - i\Gamma\text{"}$$

$$\partial_t \mathrm{tr}\rho = 0$$

“What is non-equilibrium about it”

- Field theory representation: Keldysh functional integral for stationary states

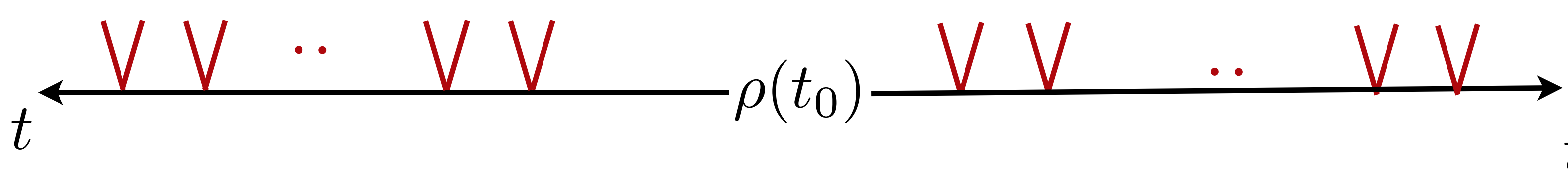
$$U(t,t_0) = e^{-iH(t-t_0)}$$

  - Schrödinger equation: evolving a state vector

$$i\partial_t|\psi\rangle(t) = H|\psi\rangle(t) \quad \Rightarrow |\psi\rangle(t) = U(t,t_0)|\psi\rangle(t_0)$$

  - Heisenberg-von Neumann equation: evolving a state (density) matrix

$$\partial_t\rho(t) = -i[H,\rho(t)] \quad \Rightarrow \rho(t) = U(t,t_0)\rho(t_0)U^\dagger(t,t_0)$$

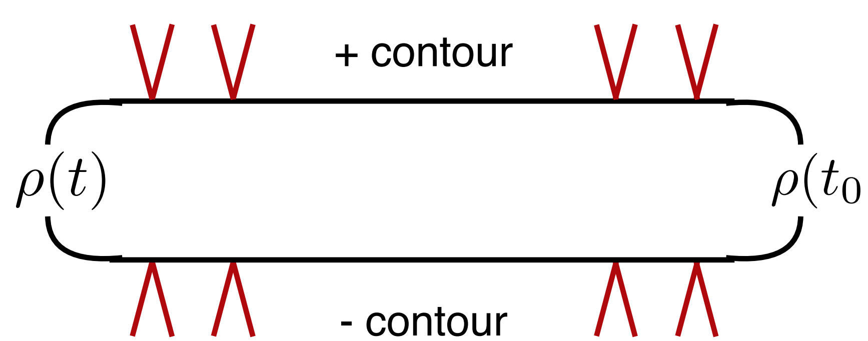


  - Same is true for the Lindblad master equation:

$$\partial_t\rho = -i[H,\rho] + \kappa\sum_i L_i\rho L_i^\dagger - \tfrac{1}{2}\{L_i^\dagger L_i,\rho\} \equiv \mathcal{L}[\rho] \qquad \Rightarrow \rho(t) = e^{\mathcal{L}(t-t_0)}\rho(t_0)$$

  - Keldysh partition function

$$Z = \mathrm{tr}\rho(t\to\infty) = 1$$



Keldysh functional integral

- quantum master equation: $\partial_t \rho = -i[H, \rho] + \mathcal{D}[\rho]$

$$= -i(H\rho - \rho H) + \kappa \sum_i (L_i \rho L_i^\dagger - \tfrac{1}{2} L_i^\dagger L_i \rho - \tfrac{1}{2} \rho L_i^\dagger L_i)$$

- equivalent Keldysh functional integral:

$$Z = \int \mathcal{D}(\Phi_+, \Phi_-) \mathrm{e}^{\mathrm{i}(S_M[\Phi_+, \Phi_-]} \qquad \Phi_\pm = \begin{pmatrix} \phi_\pm \\ \phi_\pm^* \end{pmatrix}$$

$$S_M[\Phi_+, \Phi_-] = \int dt (\phi_+^* i\partial_t \phi_+ - \phi_-^* i\partial_t \phi_- - i\mathcal{L}[\Phi_+, \Phi_-])$$

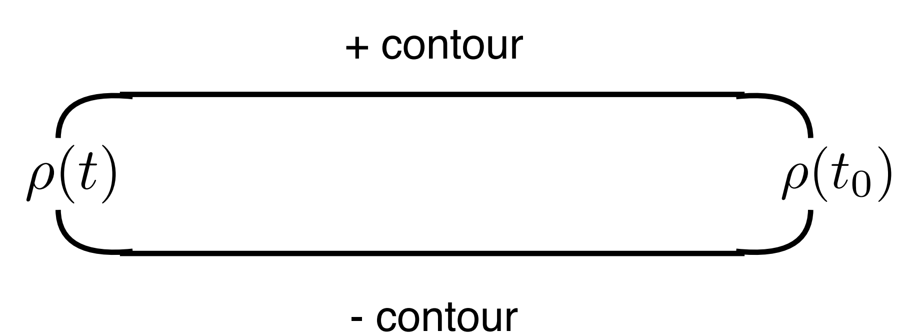
$$\boxed{\mathcal{L}[\Phi_+, \Phi_-] = -i\,(H_+ - H_-) - \kappa \sum_i \left(L_{i,+} L_{i,-}^\dagger - \tfrac{1}{2} L_{i,+}^\dagger L_{i,+} - \tfrac{1}{2} L_{i,-}^\dagger L_{i,-} \right)}$$

$$H_\pm = H(\Phi_\pm) \text{ etc.}$$

➡ recognize Lindblad structure

➡ simple translation table (for normal ordered Liouvillian)

- operator right of density matrix -> - contour
- operator left of density matrix -> + contour



“What is non-equilibrium about it?”

- quantum master equation: $$\underbrace{\partial_t \rho = -i[H,\rho]}_{\Longrightarrow\ S_H} + \underbrace{\mathcal{D}[\rho]}_{\Longrightarrow\ S_{\mathcal{D}}}$$

- Keldysh partition function

$$Z = \int \mathcal{D}(\Phi_+,\Phi_-) e^{\mathrm{i}(S_H[\Phi_+,\Phi_-]+S_{\mathcal{D}}[\Phi_+,\Phi_-])}$$

- equilibrium dynamics generated by a time-independent Hamiltonian alone (global [S+B] energy conservation)

$$S_{\mathcal{D}} = 0$$

Sieberer, Chiocchetta, Täuber, Gambassi, SD PRB (2015)
Aron, Biroli, Cugliandolo, SciPost (2018)
classical limit: Janssen (1976)

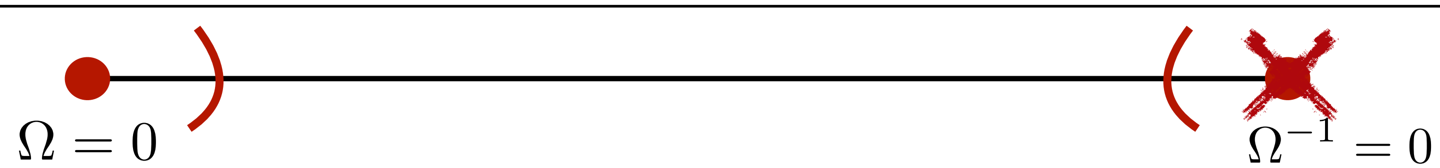
➡ symmetry of Keldysh action under transformation

$$\mathcal{T}_\beta \Phi_\pm(t,\mathbf{x}) = \Phi^*_\pm(-t \pm \mathrm{i}\beta/2, \mathbf{x}) \qquad \Phi_\pm = \begin{pmatrix} \phi_\pm \\ \phi^*_\pm \end{pmatrix}$$

- associated “Ward identities” are equilibrium quantum Fluctuation-Dissipation relations to arbitrary order
- compact functional formulation of KMS boundary condition

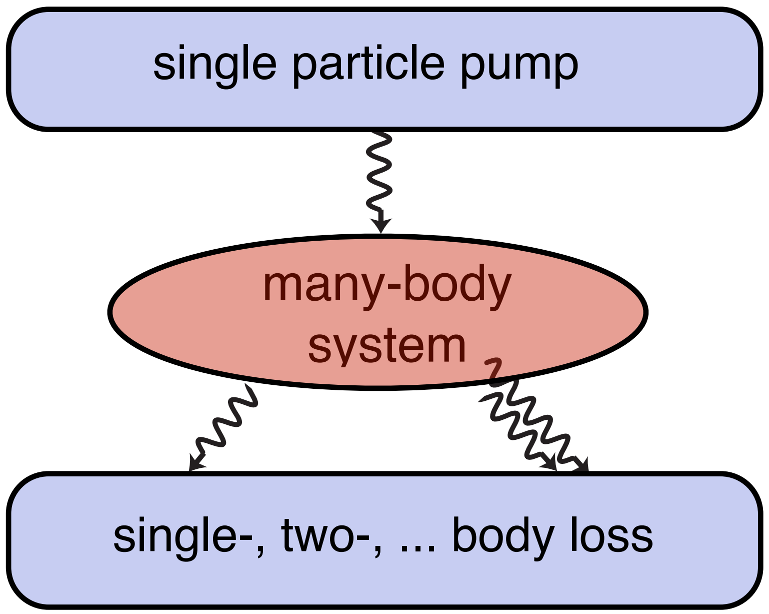
➡ the Liouville operator (or $S_{\mathcal{D}}$) violates this symmetry explicitly (memory of microscopic periodic drive)

➡ consequences of the absence of this symmetry on criticality?



Many-Body Model

- generic microscopic many-body model:



$$\partial_t \rho = -i[H,\rho] + \mathcal{D}[\rho] \equiv \mathcal{L}[\rho]$$

$$H = \int_{\mathbf{x}} \hat{\phi}^\dagger_{\mathbf{x}} \left(\frac{\triangle}{2M} - \mu\right) \hat{\phi}_{\mathbf{x}} + \frac{\lambda}{2} (\hat{\phi}^\dagger_{\mathbf{x}} \hat{\phi}_{\mathbf{x}})^2$$

$$\mathcal{D}[\rho] = \gamma_p \int_{\mathbf{x}} [\hat{\phi}^\dagger_{\mathbf{x}}\, \rho\, \hat{\phi}_{\mathbf{x}} - \tfrac{1}{2}\{\hat{\phi}_{\mathbf{x}}\hat{\phi}^\dagger_{\mathbf{x}}, \rho\}] \quad + \quad \gamma_l \int_{\mathbf{x}} [\hat{\phi}_{\mathbf{x}}\, \rho\, \hat{\phi}^\dagger_{\mathbf{x}} - \tfrac{1}{2}\{\hat{\phi}^\dagger_{\mathbf{x}}\hat{\phi}_{\mathbf{x}}, \rho\}] \quad +$$

single particle pump

single particle loss

$$\kappa \int_{\mathbf{x}} [\hat{\phi}^2_{\mathbf{x}}\, \rho\, \hat{\phi}^{\dagger\,2}_{\mathbf{x}} - \tfrac{1}{2}\{\hat{\phi}^{\dagger\,2}_{\mathbf{x}}\hat{\phi}^2_{\mathbf{x}}, \rho\}]$$

two particle loss

- U(1) phase rotation symmetry

$$\hat{\phi}_{\mathbf{x}} \to e^{i\theta} \hat{\phi}_{\mathbf{x}}$$

- practical evaluation:

closed system Keldysh:
Gasenzer, Pawlowski, PLB 08;
Berges, Hoffmeister, Nucl. Phys. B, 09

open system Keldysh review
Sieberer, Buchhold, SD, ROPP
(2016)

Many-Body Master Equation → Keldysh functional integral → Keldysh Functional Renormalization Group

Wetterich, 93

$$\partial_t \rho = -i[H,\rho] + \mathcal{D}[\rho]$$

$$\mathrm{e}^{\mathrm{i}\Gamma[\Phi]} = \int \mathcal{D}\delta\Phi\, \mathrm{e}^{\mathrm{i}S_M[\Phi+\delta\Phi]}$$

$$\partial_k \Gamma_k = \frac{\mathrm{i}}{2} \mathrm{Tr} \left[\left(\Gamma^{(2)}_k + R_k \right)^{-1} \partial_k R_k \right]$$

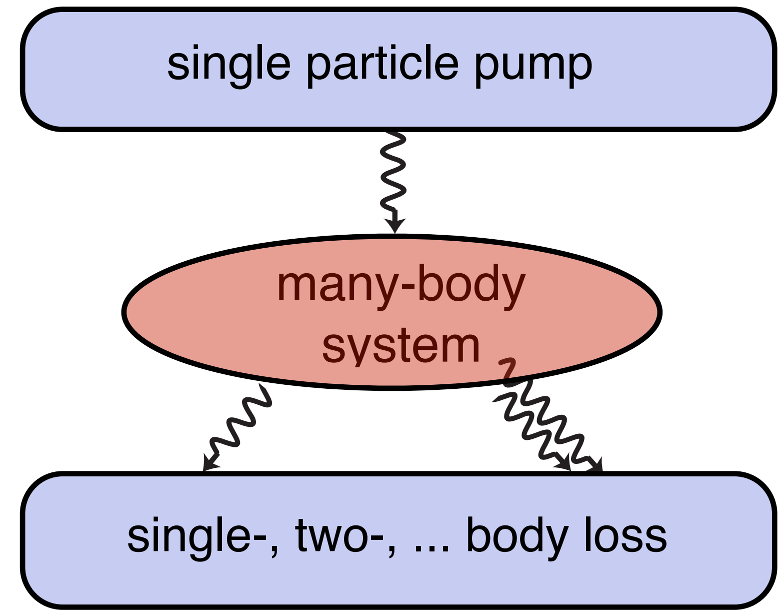
Non-Eq. ϕ^4 Theory: Phase Transition

- 3D: mean field approximation
  - study field expectation value

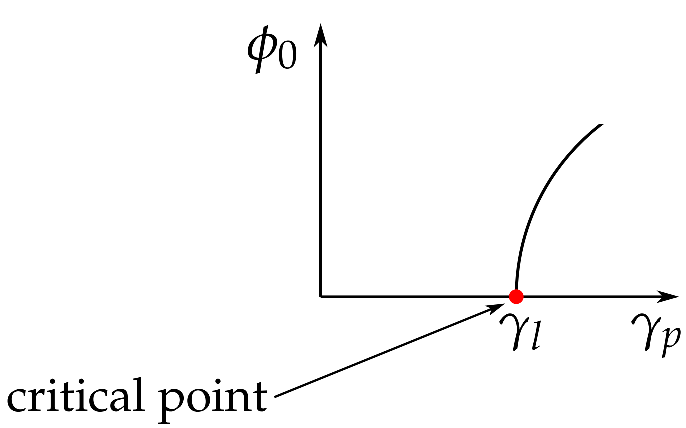
$$\langle\hat{\phi}_{\mathbf{x}}\rangle(t) = \mathrm{tr}[\hat{\phi}_{\mathbf{x}}\rho(t)]$$

  - mean field: factorize correlation functions
  - consider spatially homogeneous configuration

$$\langle\hat{\phi}_{\mathbf{x}}\rangle(t) \equiv \phi(t)$$

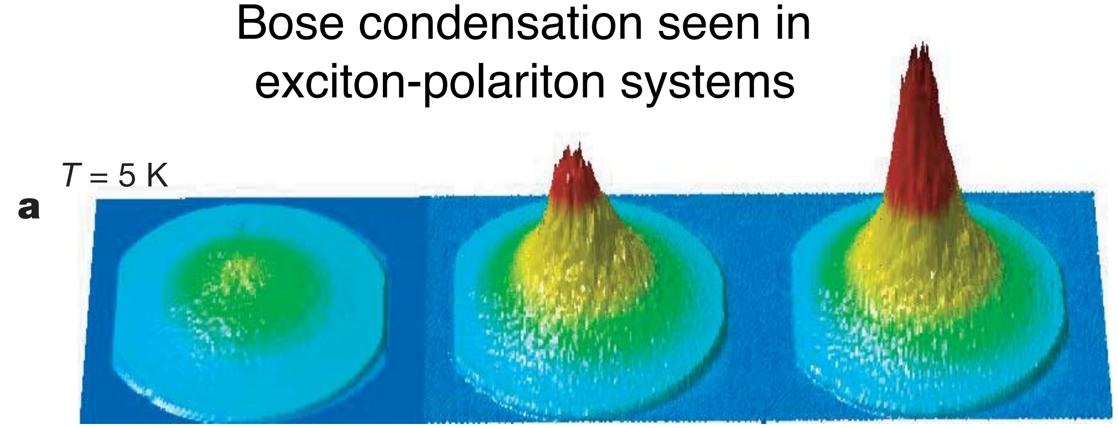


$$\boxed{\partial_t\phi = \left[i\mu - (\gamma_l - \gamma_p) + (-i\lambda - \kappa)\,|\phi|^2\right]\phi}$$



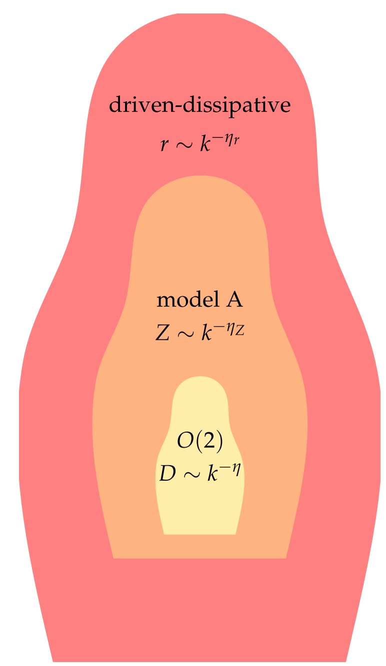
Bose condensation seen in
exciton-polariton systems

Kasprzak et al.,
Nature 2006

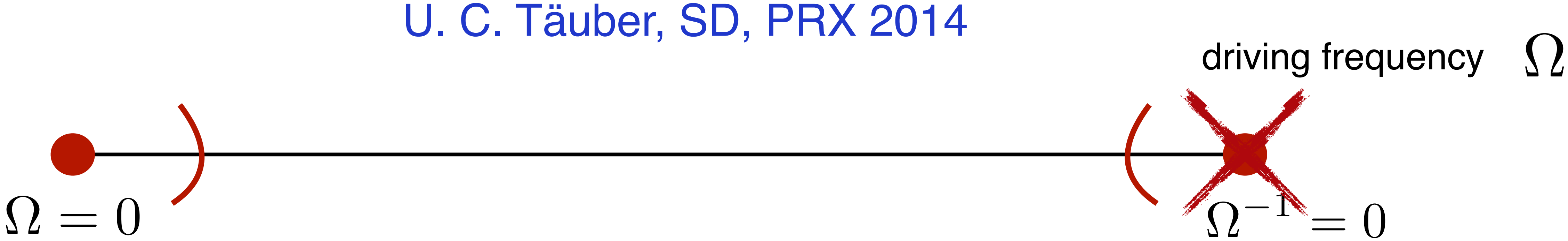


➡ Suggests continuous symmetry breaking phase transition in infinitely rapidly driven limit (many-body laser threshold)

Driven Open Criticality:
Infinitely rapidly driven limit



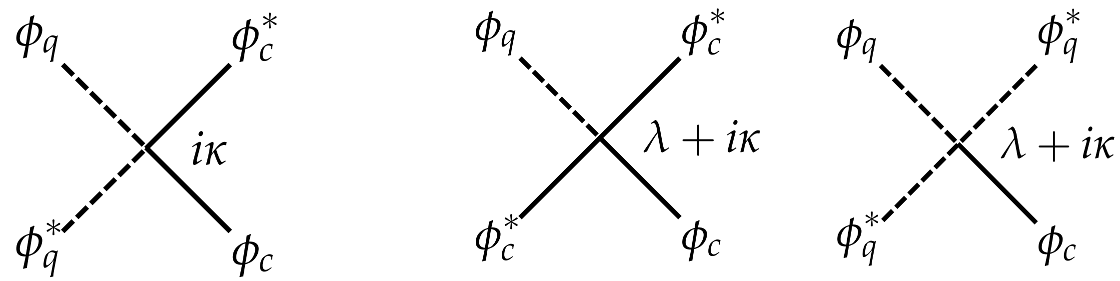
L. Sieberer, S. Huber, E. Altman, SD,
PRL 2013; PRB 2014
U. C. Täuber, SD, PRX 2014

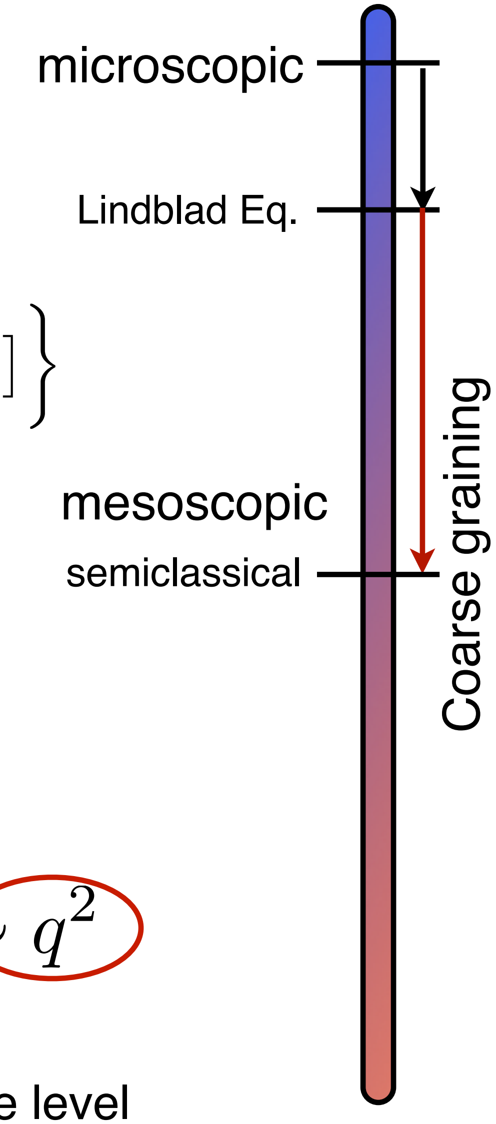


Semiclassical limit: power counting

- Quantum master equation —> Keldysh functional integral, with action

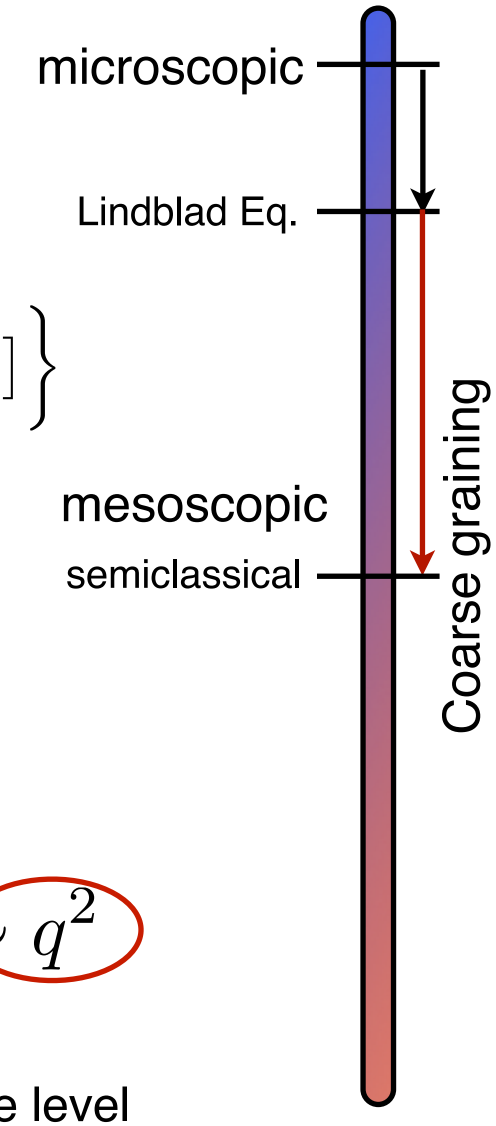
$$\mathcal{S} = \int_{t,\mathbf{x}} \left\{ (\phi_c^*, \phi_q^*) \begin{pmatrix} 0 & P^A \\ P^R & P^K \end{pmatrix} \begin{pmatrix} \phi_c \\ \phi_q \end{pmatrix} + 2i\kappa \phi_c^* \phi_c \phi_q^* \phi_q - \tfrac{1}{2} \left[(\lambda + i\kappa) \left(\phi_c^{*2} \phi_c \phi_q + \phi_q^{*2} \phi_c \phi_q \right) + c.c. \right] \right\}$$





- Gaussian sector close to a critical point:
  - retarded/advanced $P^R(\omega, \mathbf{q}) = Z\omega - (A + iD)\mathbf{q}^2 \overbrace{- \mu + i(\gamma_l - \gamma_p)/2}^{\to 0} \sim q^2$
  - Keldysh component $P^K = i(\gamma_l + \gamma_p) \sim q^0$ finite Markovian noise level
- Canonical field dimensions: $[\phi_c] = \dfrac{d-2}{2} < [\phi_q] = \dfrac{d+2}{2}$

Semiclassical limit: power counting



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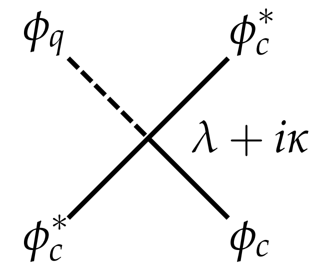

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➡ Equivalent to MSRJD functional integral

➡ Equivalence to phenomenological semiclassical Langevin equations (phase coherence preserved)

Wouters and Carusotto PRL (2006);
Szymanska, Keeling, Littlewood PRL (2004)

➡ Non-equilibrium analog of classical criticality

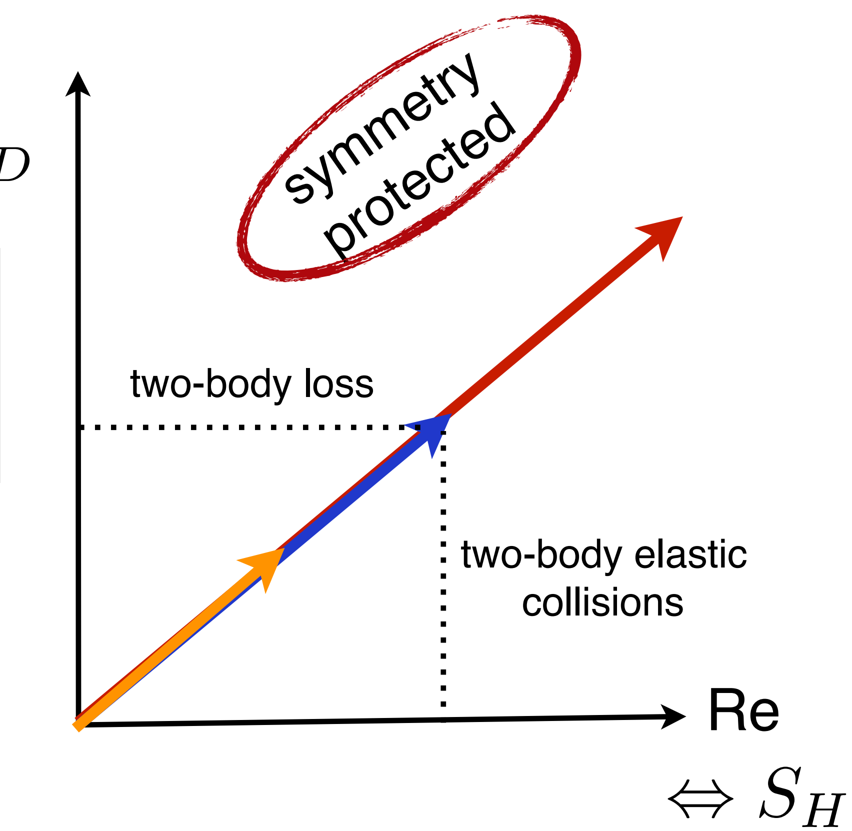
➡ Possible to evade: non-equilibrium analog of quantum criticality (dark state engineering)

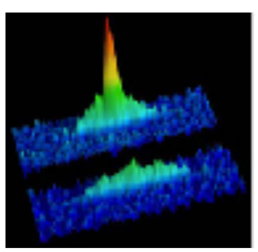
Marino, SD PRL (2016)

“What is non-equilibrium about it”, cont’d

- implication of equilibrium symmetry in semiclassical limit
- reversible and irreversible contributions to action: $S = S_H + S_D$

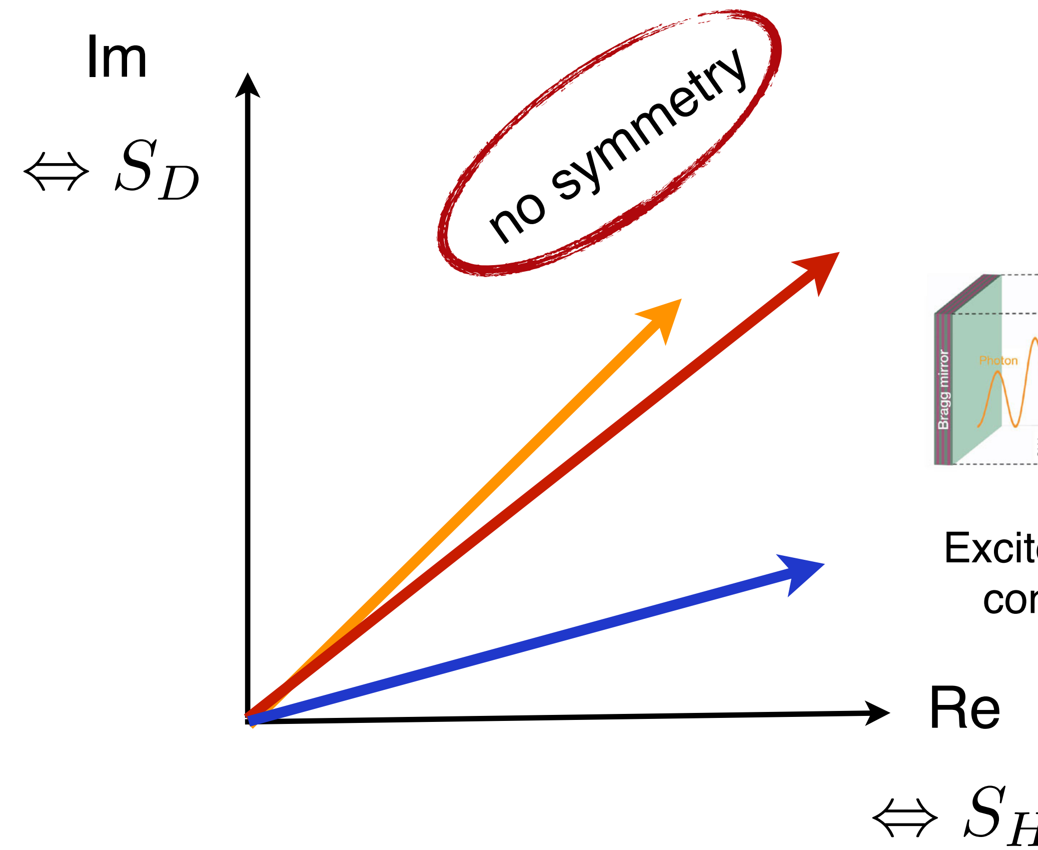
equilibrium dynamics

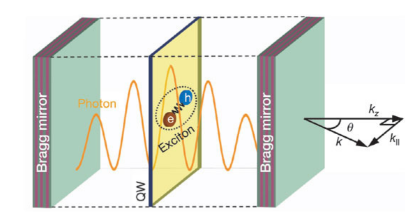




atomic Bose condensates

non-equilibrium dynamics

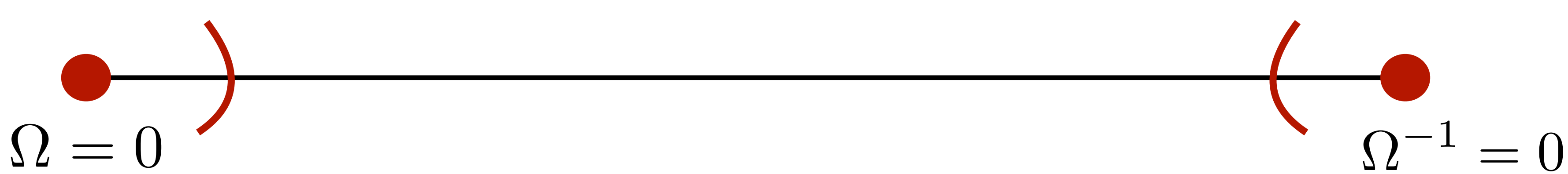




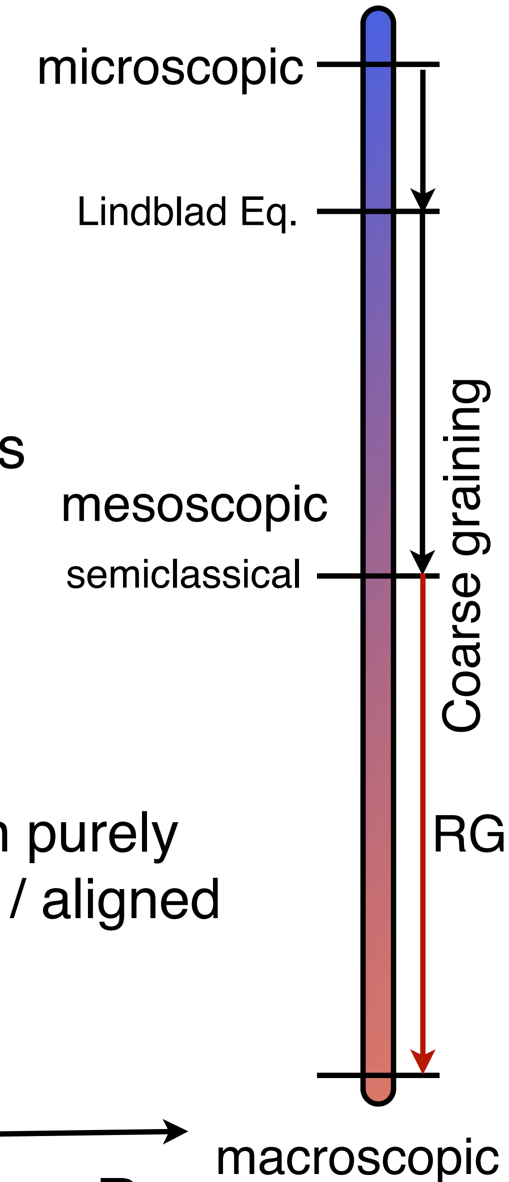
Exciton-Polariton condensates

- coherent and dissipative dynamics may occur simultaneously
- but they are not independent

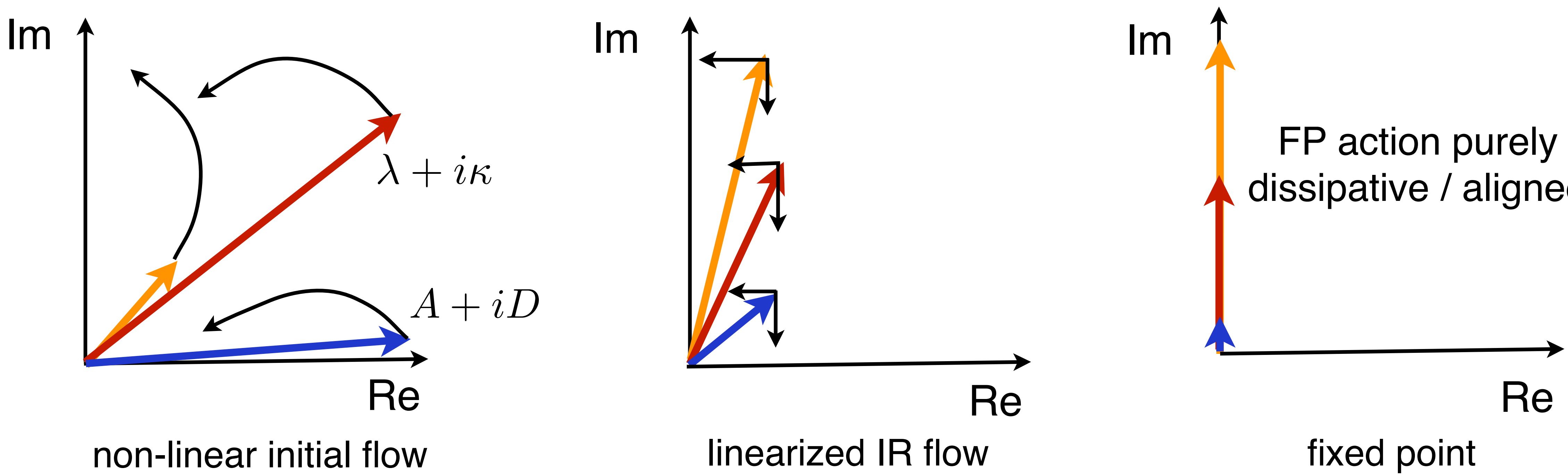
- coherent and dissipative dynamics do occur simultaneously
- they result from different dynamical resources



Schematic RG flow



- How much information on breaking of detailed balance is lost at criticality?
- Flow in the complex plane of couplings: (Functional) RG for driven open systems



- initial values: $\Gamma_{k\approx\Lambda_0} \approx S$
- universal domain encoding universality class

➡ decoherence

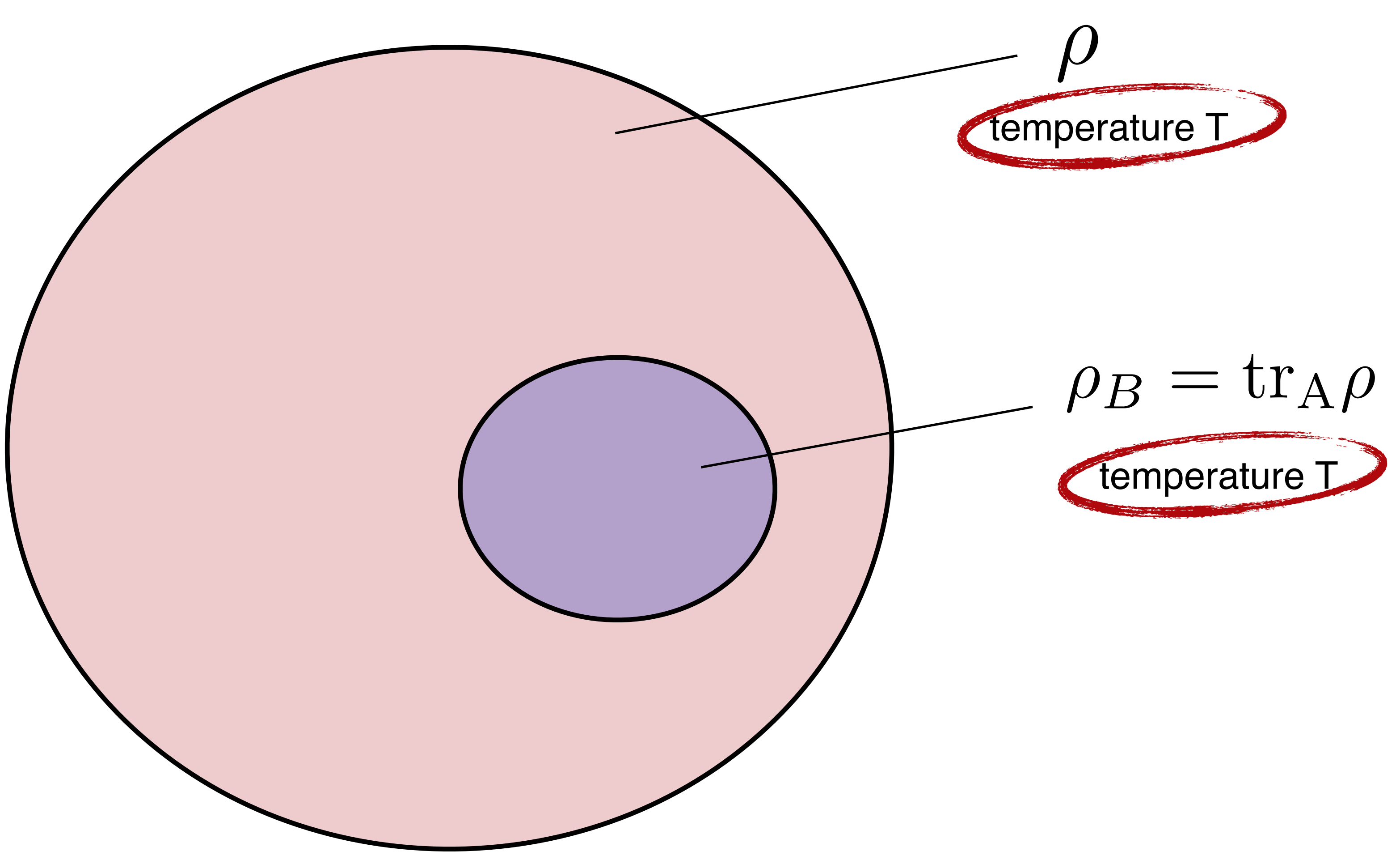
➡ asymptotic thermalization

L. Sieberer, S. Huber, E. Altman, SD, PRL 2013; PRB 2014
U. C. Täuber, SD, PRX 2014

Asymptotic Low-Frequency Thermalization

- global thermal equilibrium: all subparts in equilibrium with each other

  <=> Temperature is invariant under the partition



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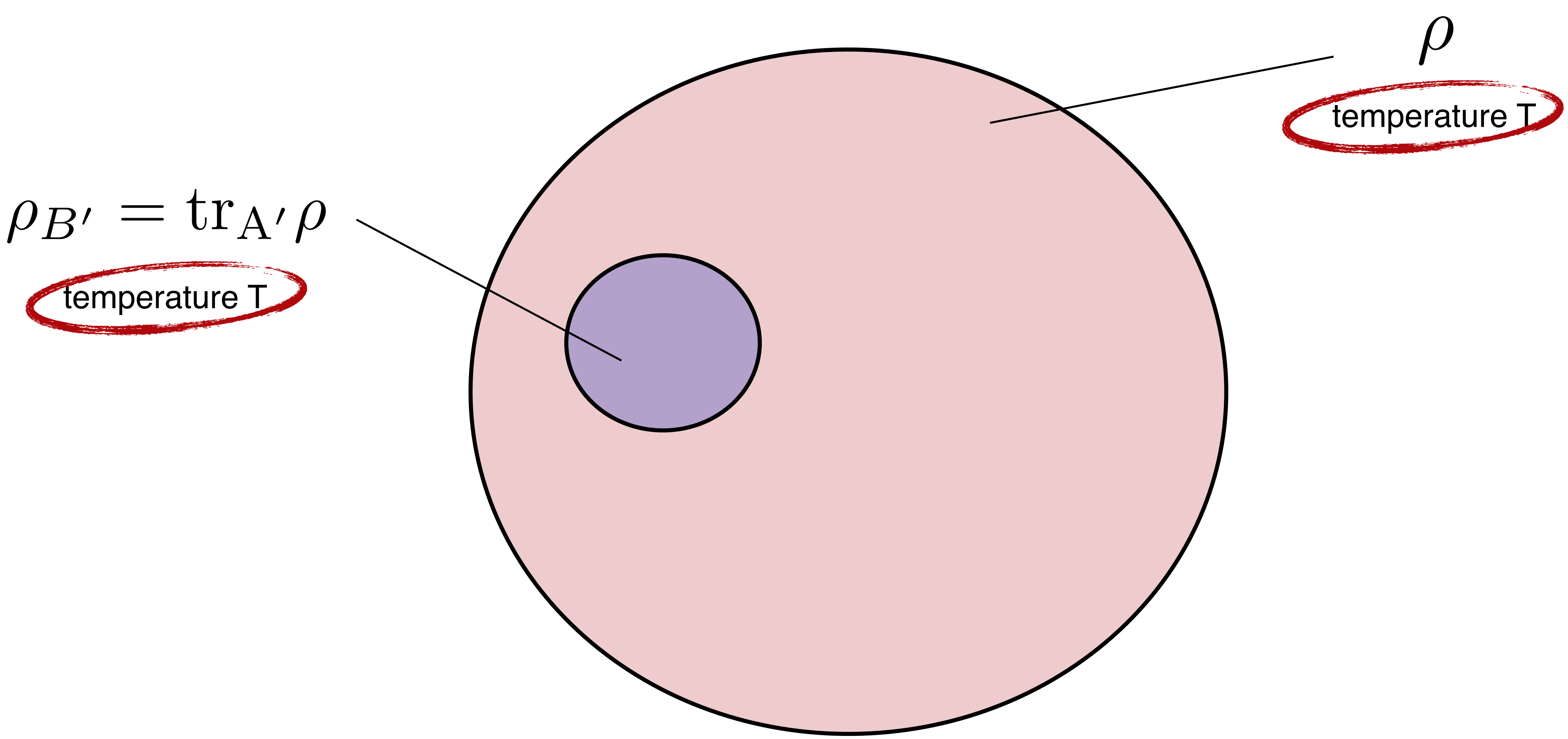
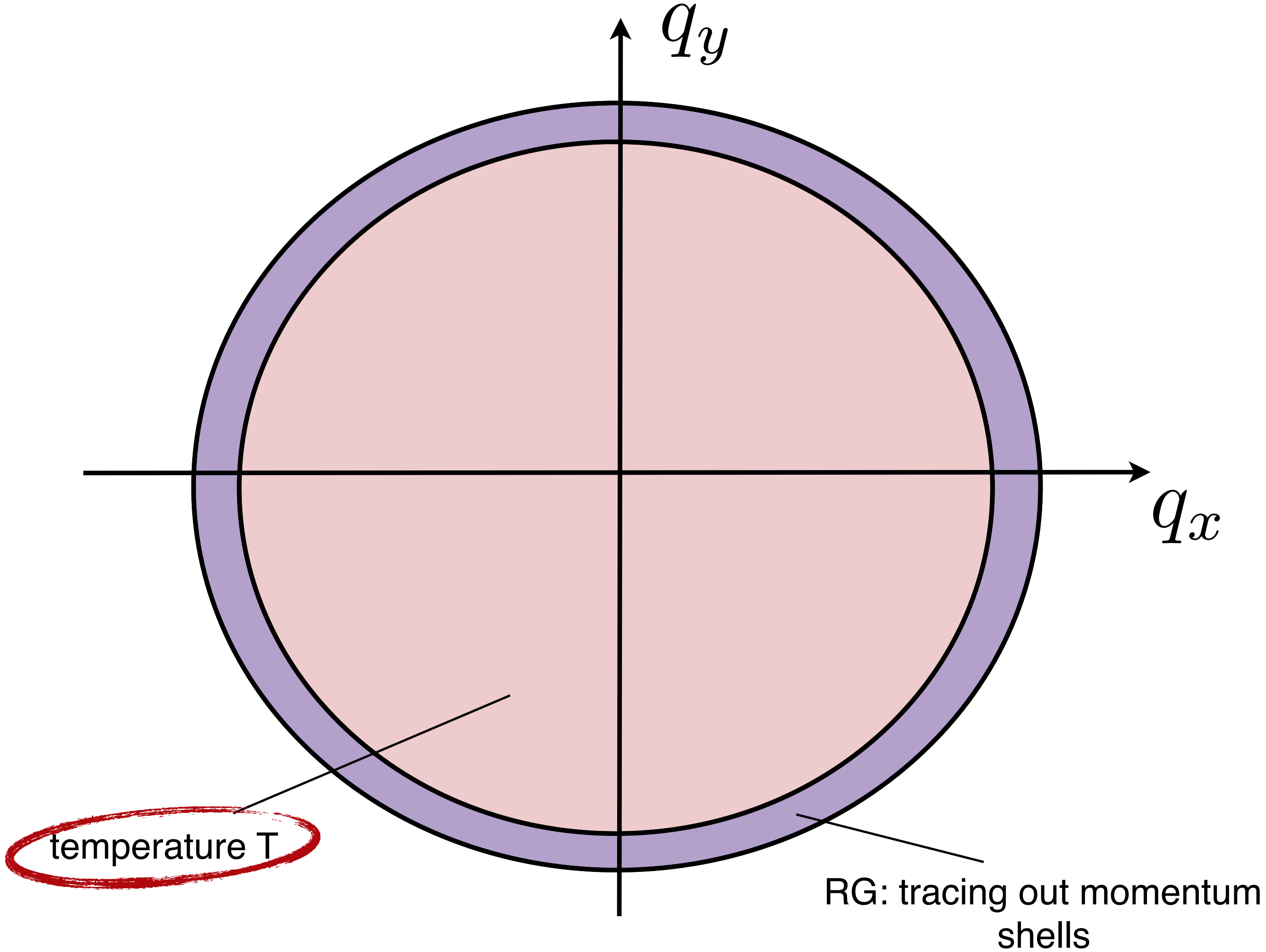


Asymptotic Low-Frequency Thermalization

- global thermal equilibrium: all subparts in equilibrium with each other

  <=> Temperature is invariant under the partition

RG: <=> Temperature is scale invariant



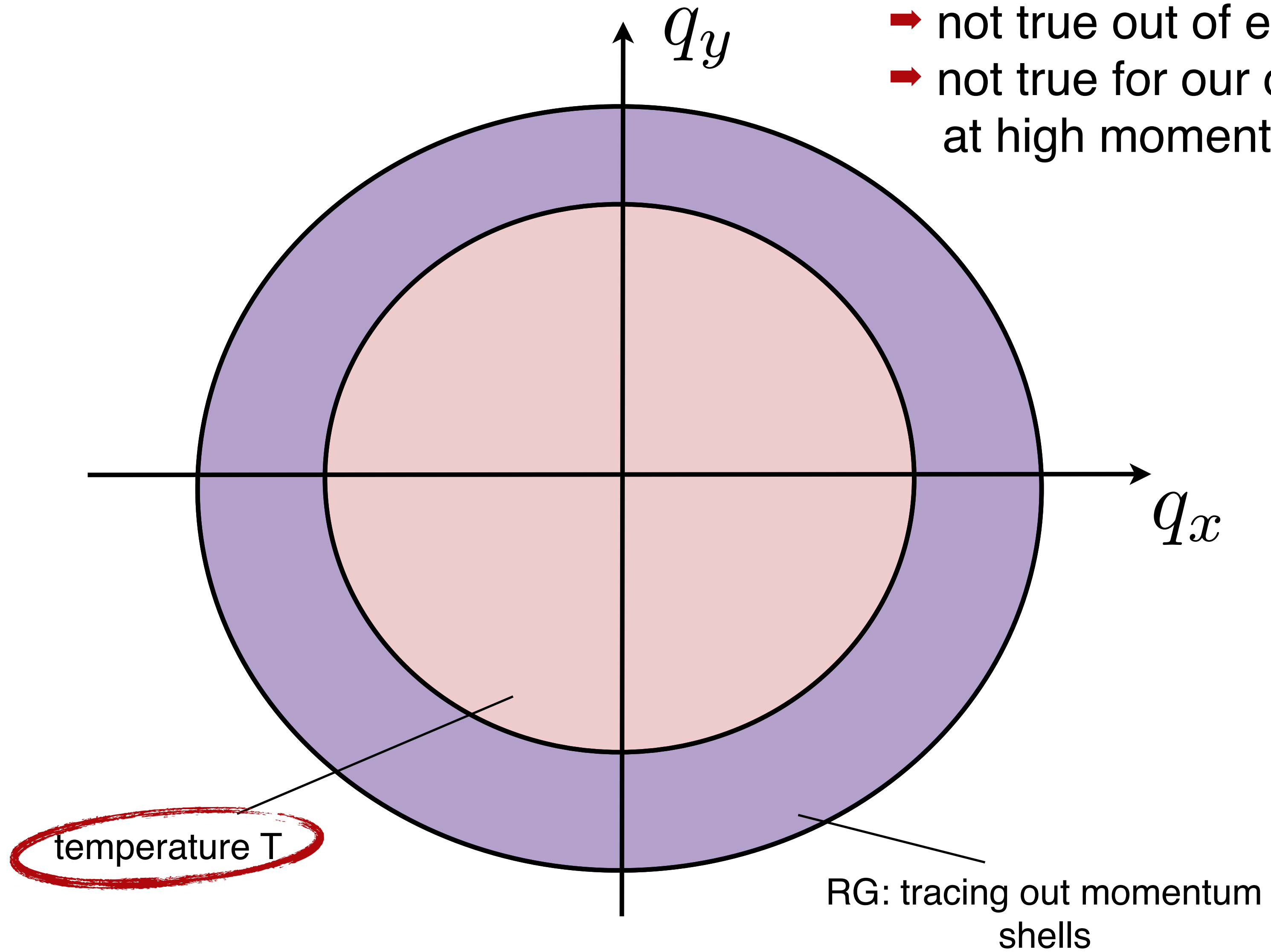
Asymptotic Low-Frequency Thermalization

- global thermal equilibrium: all subparts in equilibrium with each other

  <=> Temperature is invariant under the partition

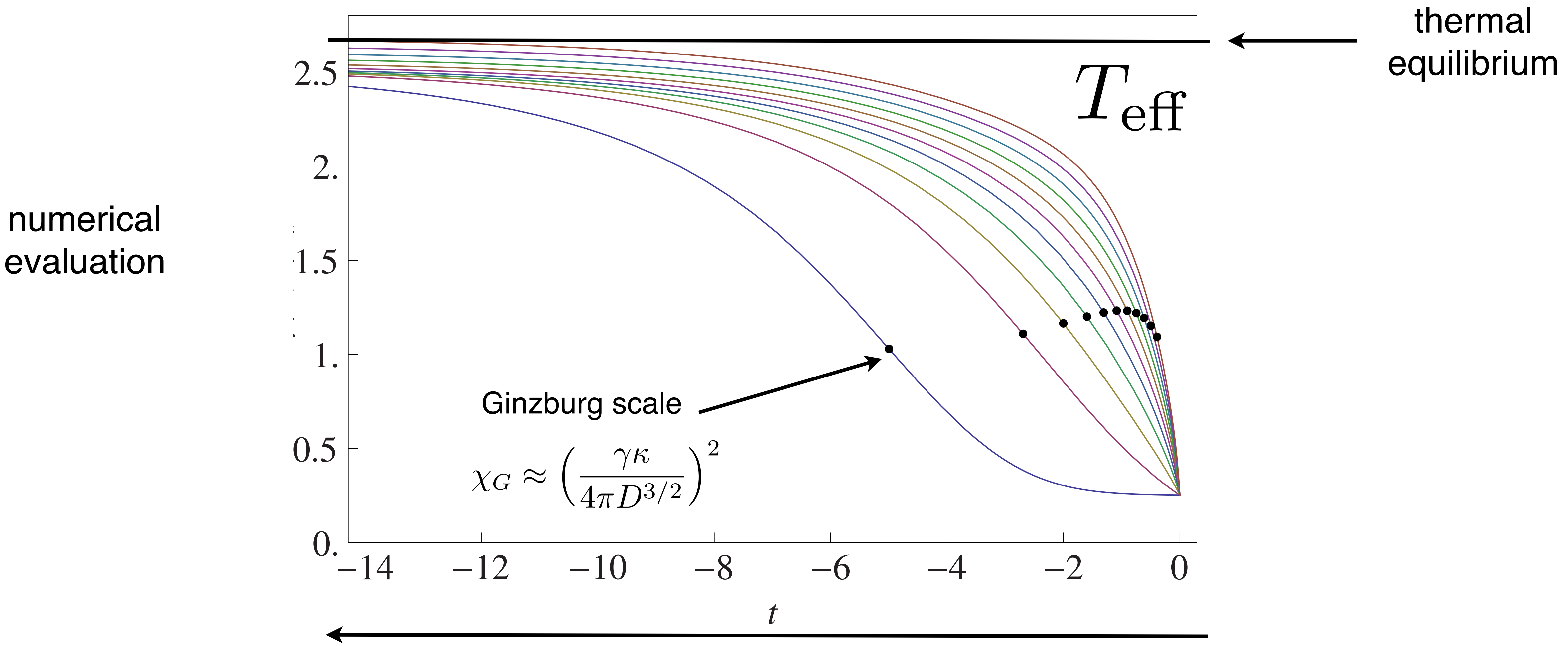
RG: <=> Temperature is scale invariant

➡ not true out of equilibrium
➡ not true for our driven-dissipative system at high momenta



Asymptotic Low-Frequency Thermalization

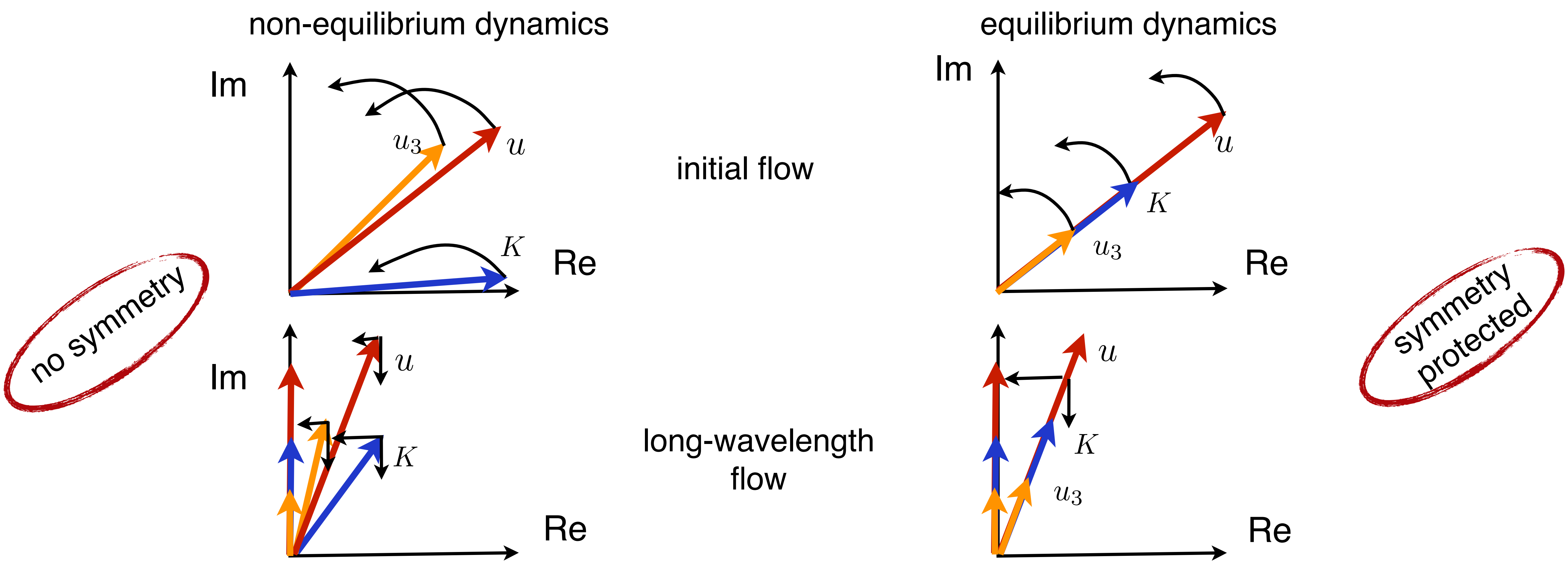
- emergent scale invariant effective temperature in the universal low-momentum regime: asymptotic thermalization



flow to long wavelength/small momenta, constant in equilibrium

New universality: Equilibrium vs. non-equilibrium fine structure

- RG approach to fixed point



- lowest eigenvalue

$$\eta_R \approx -0.143$$

- eigenvalue of flow speed

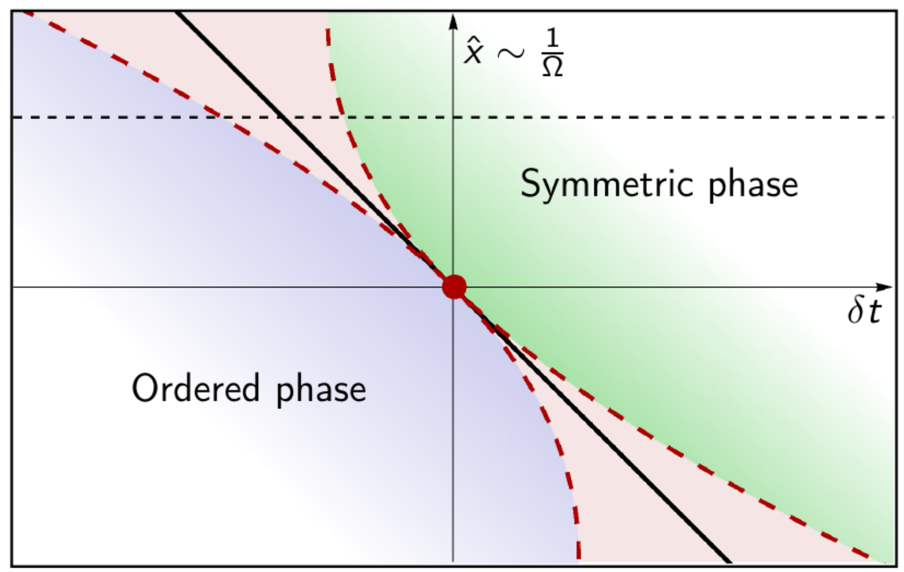
$$\eta_r \approx -0.101$$

➡ identifies new independent critical exponent, measuring universal decoherence

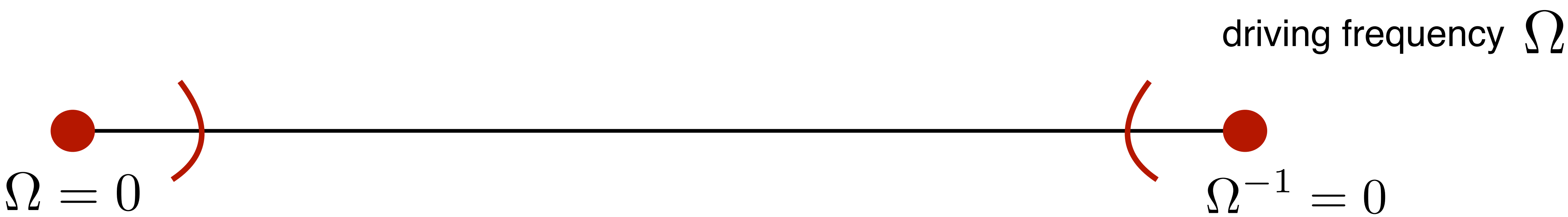
➡ equilibrium and driven systems in different dyn. universality classes

➡ physical reason: independence of coherent and dissipative dynamics

Periodically Driven Open Criticality: Rapidly Driven Regime



S. Mathey, SD, arxiv:1807.02146

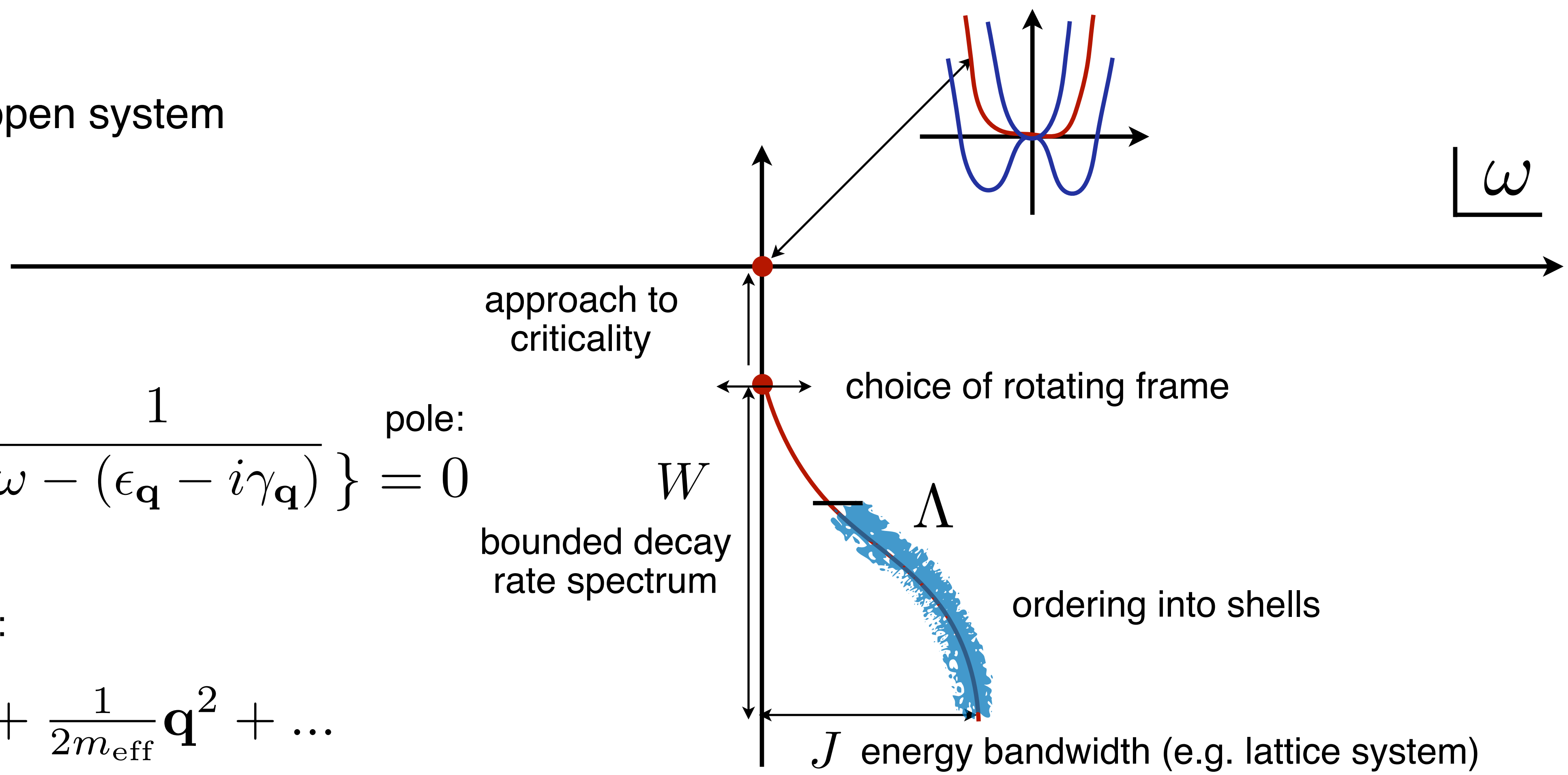


Basic Physical Picture

- First guess: fast driving scale Ω => no effect on long wavelength critical properties
- But: energy not conserved, defined only $\mathrm{mod}(\Omega)$: ‘high’ and ‘low’ energies not well defined

  (can exchange energy quanta $n \cdot \hbar\Omega$ with drive)
- Pole structure of single particle Green’s function (dynamic susceptibility) with bounded spectrum

undriven, open system



$$G^R(\omega, \mathbf{q}) = \frac{1}{\omega - (\epsilon_{\mathbf{q}} - i\gamma_{\mathbf{q}})} \quad \text{pole: } \} = 0$$

small momenta:

$$\epsilon_{\mathbf{q}} = \omega_0 + \frac{1}{2m_{\mathrm{eff}}}\mathbf{q}^2 + \ldots$$

$$\gamma_{\mathbf{q}} = \gamma_0 + D\mathbf{q}^2 + \ldots$$

$\rightarrow 0$ at critical point

Basic Physical Picture

- First guess: fast driving scale Ω => no effect on long wavelength critical properties
- But: energy not conserved, defined only $\mathrm{mod}(\Omega)$: 'high' and 'low' energies not well defined

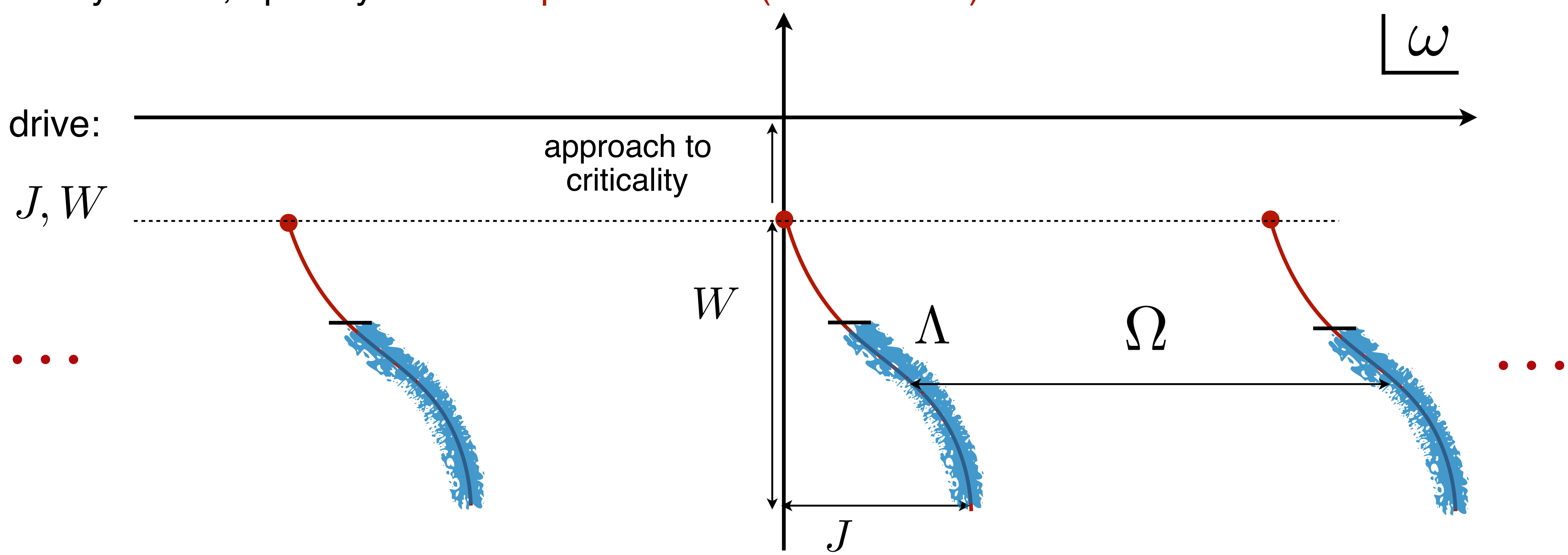
  (can exchange energy quanta $n \cdot \hbar\Omega$ with drive)

- Pole structure of single particle retarded Green's function with bounded energy / decay rate spectrum

periodically driven, open system: Floquet theorem (linear PDEs)

rapid drive:

$\Omega > J, W$

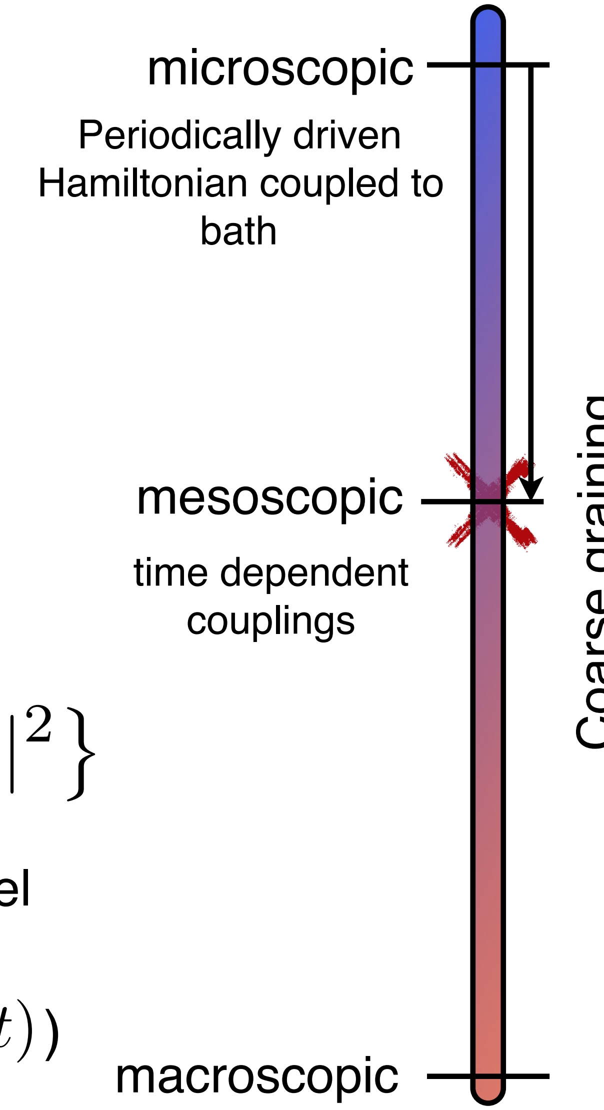


➡ Periodic drive leads to massive degeneracy of near critical poles

➡ Need to treat on equal footing, but RG possible within each strip

➡ Effect on criticality not obvious

Setup of the problem: Mesoscopic action



- microscopically: periodically driven Hamiltonian on a lattice (e.g. hopping, interaction), coupled to bath

- mesoscopic action for finite temperature bath

$$S = \int_{t,\mathbf{x}} \left\{ \phi_q^* \left[i\partial_t - \left(-K\nabla^2 + \mu + \tfrac{g}{2}|\phi_c|^2\right)\right] \phi_c^* + c.c. + 2i\gamma|\phi_q|^2 \right\}$$

generator of dynamics

noise level

- all couplings complex and periodically time dependent (simplicity: just $\mu(t), g(t)$)

- key requirements:
  - drive respects U(1) phase rotation symmetry
  - drive cannot be factored out of generator of dynamics (accidental equilibrium symmetry)

- synchronization to Floquet stationary state

$$\mu(t) = \sum_n \mathrm{e}^{-in\Omega t}\mu_n, \quad g(t) = \sum_n \mathrm{e}^{-in\Omega t} g_n$$

➡ to be found $\{\mu_n, g_n, \ldots\}$

Explicit symmetry breaking and new couplings

- new couplings: symmetry point of view
- explicit symmetry breaking: continuous time translations —> discrete time translations

$$\Phi(t) \to \Phi(t + \Delta t) \qquad \longrightarrow \qquad \Phi(t) \to \Phi(t + n \cdot \tfrac{2\pi}{\Omega})$$

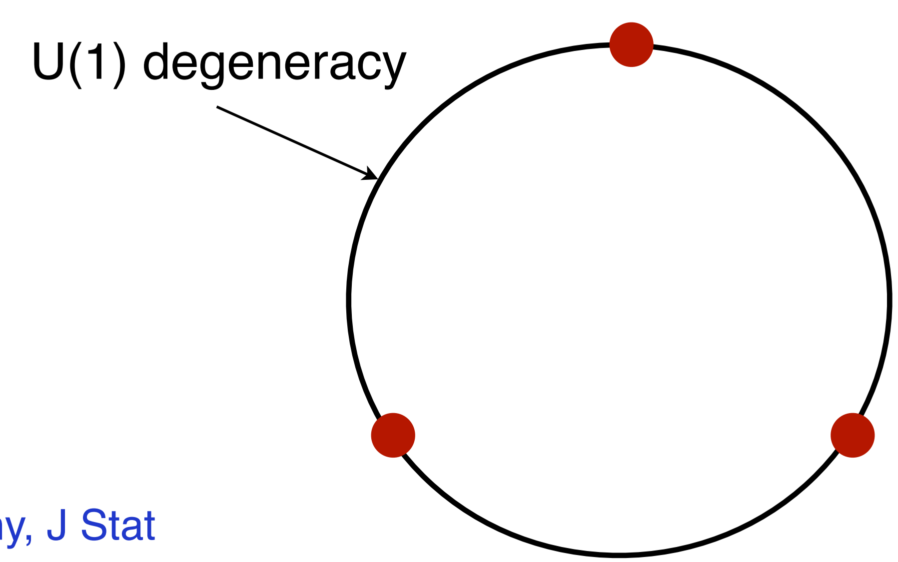
Δt arbitrary $\qquad\qquad$ n integer

- $\{\mu_{n\neq 0}, g_{n\neq 0}, \ldots\}$ ruled out only for
  - undriven problem (continuous time translations)
  - emergent: infinitely rapidly driven limit (averaging out, Lindblad limit)

➡ here: need to classify relevance of new couplings at critical point

- Analogous: external symmetry transformations: continuous rotations —> discrete rotations
  - example: Potts model, $U(1) \simeq O(2) \to Z_3$
  - New correlation length scale around discrete degenerate minima

  ➡ That case: single pole but more relevant couplings: can lead to fluctuation induced first order transition



Golner, PRB (1973)
Manuel Carmona, Pelisetto, Vicari, PRB (2000)

Reviews: Wu, RMP (1982); Aharony, J Stat Phys (2003)

Exact single particle Green's function

- Floquet theorem (time-periodic linear partial differential equations), stationary state:

$$G\left(t+\frac{\tau}{2}, t-\frac{\tau}{2}\right) = \sum_n \mathrm{e}^{-in\Omega t} G_n(\tau) = \sum_n \int_\omega \mathrm{e}^{-i[n\Omega t+\omega\tau]} G_n(\omega)$$

periodic in center-of-mass time t

Wigner Green's functions

- solution:

$$i\partial_t \phi + M(t)\phi = \xi, \quad M(t) = K\mathbf{p}^2 + \mu(t)$$

left general here

$$\Rightarrow G_R(t,t') = -i\theta(t-t')\mathrm{e}^{i\int_{t'}^{t} M(t'')\mathrm{d}t''}$$

$$M_0 = \frac{\Omega}{2\pi}\int_0^{\frac{2\pi}{\Omega}} M(t)\mathrm{d}t = K\mathbf{p}^2 + \mu_0$$

$$\Rightarrow G_{R;n}(\omega) = \frac{\Omega}{2\pi}\int_0^{\frac{2\pi}{\Omega}} dt \mathrm{e}^{in\omega t} \sum_m \frac{J_m\left(2[M(t)-M_0]/\Omega\right)}{\omega + M_0 + \frac{m\Omega}{2}}$$

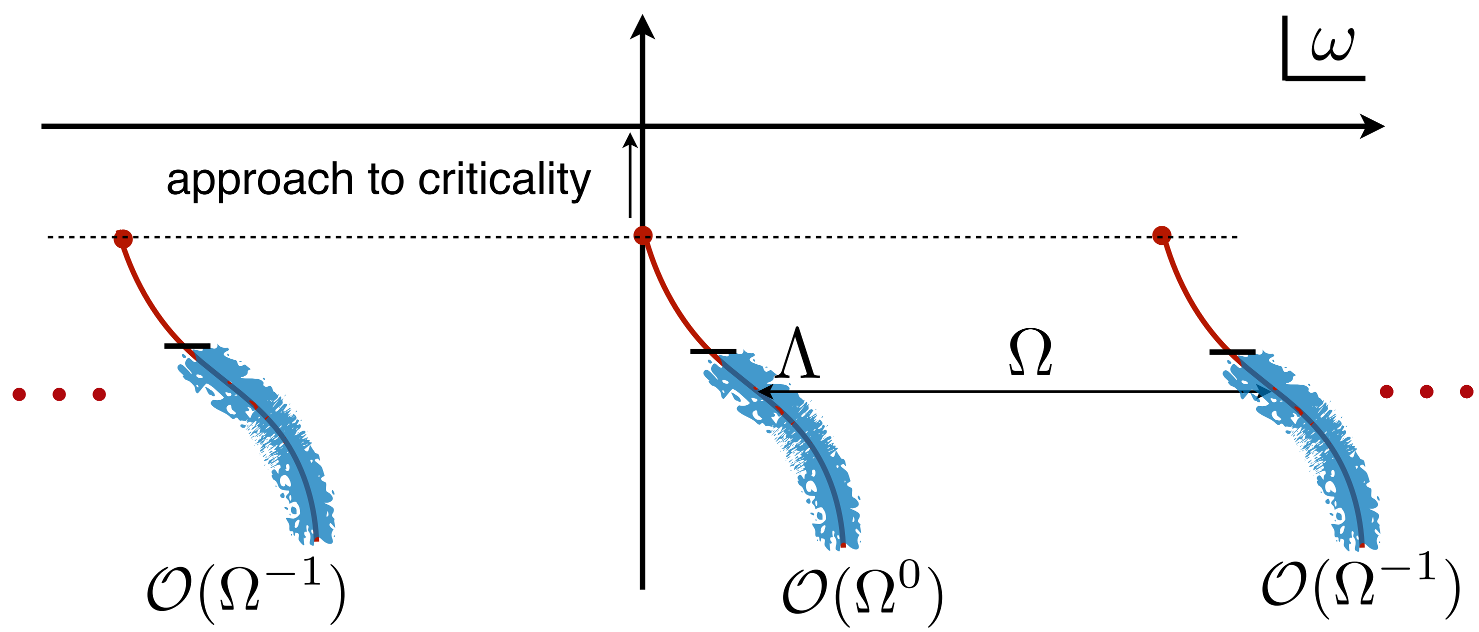
Bessel functions of first kind

Approximate single particle Green's function

- exact Wigner Green's function

$$G_{R;n}(\omega) = \frac{\Omega}{2\pi}\int_0^{\frac{2\pi}{\Omega}} dt e^{in\omega t} \sum_m \frac{J_m\left(2[M(t)-M_0]/\Omega\right)}{\omega + M_0 + \frac{m\Omega}{2}}$$

- key properties:



➡ infinitely many degenerate poles become critical simultaneously

➡ parametric suppression, ordered by distance from central one $\left(\frac{\mu_n}{\Omega}\right)^n$

- Ordering principle: Expansion in inverse drive frequency

$$G_{R;0}(\omega) = \frac{1}{\omega + K\boldsymbol{q}^2 + \mu_0}, \quad G_{R;n\neq 0}(\omega) = \frac{\mu_n}{\left(\frac{n\Omega}{2}\right)^2 - (\omega + K\boldsymbol{q}^2 + \mu_0)^2}$$

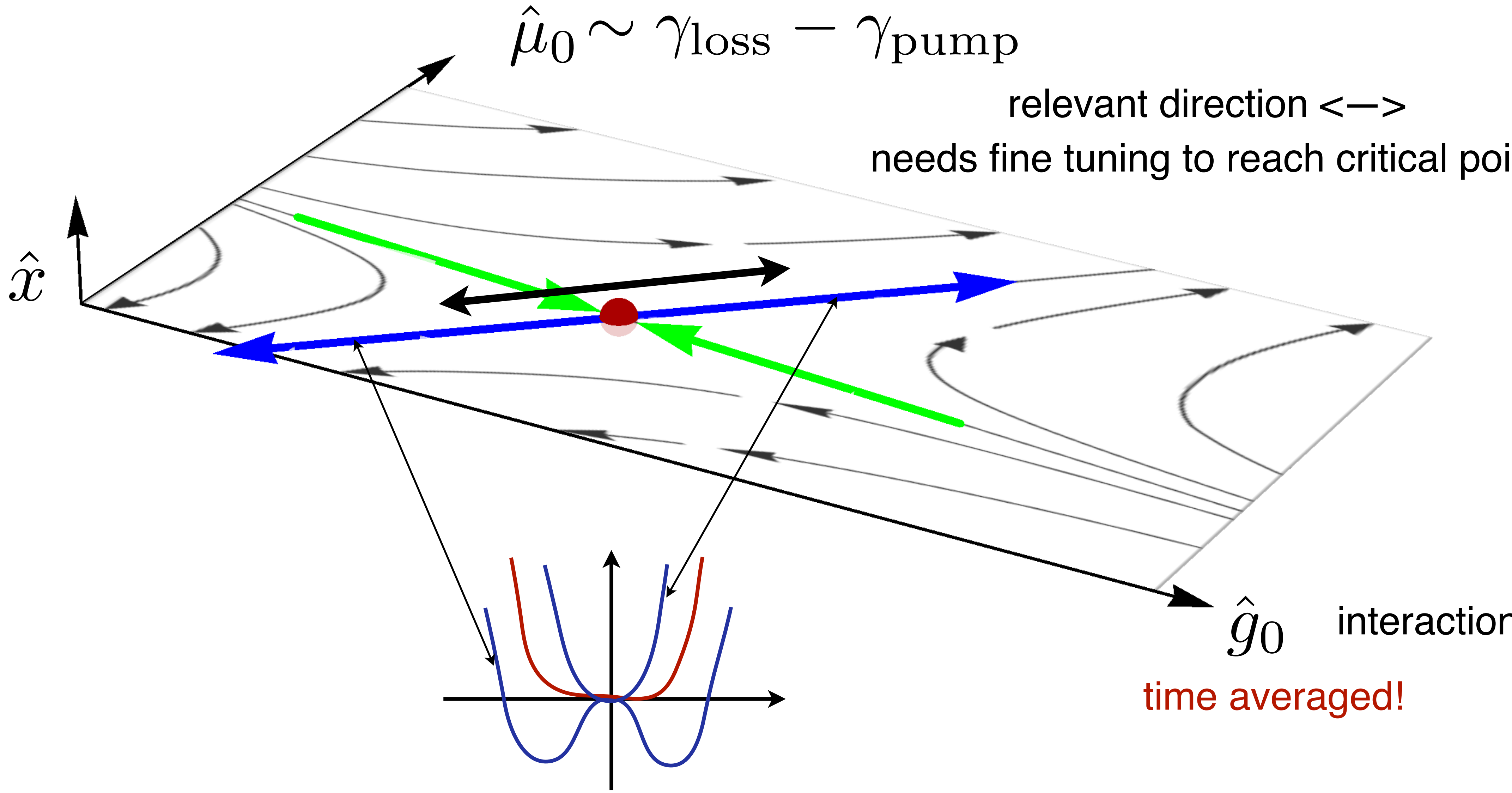
- up to $\mathcal{O}(\Omega^{-1})$: first correction to rotating wave approximation

Renormalization group flow

- fixed point coordinates

$$\begin{pmatrix} \hat{\mu}_0^* \\ \hat{g}_0^* \\ \hat{x}^* \end{pmatrix} \cong \left.\begin{pmatrix} -\epsilon/5 \\ 4\pi^2\epsilon/5 \\ 0 \end{pmatrix}\right\}$$ usual Wilson-Fisher fixed point

time averaged!

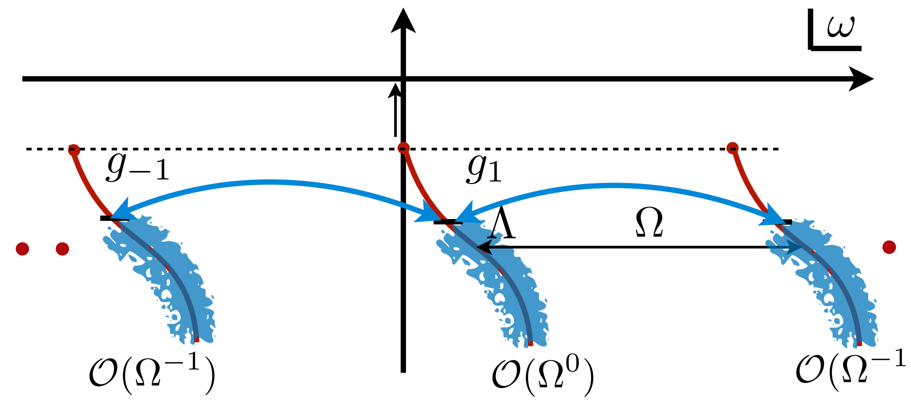


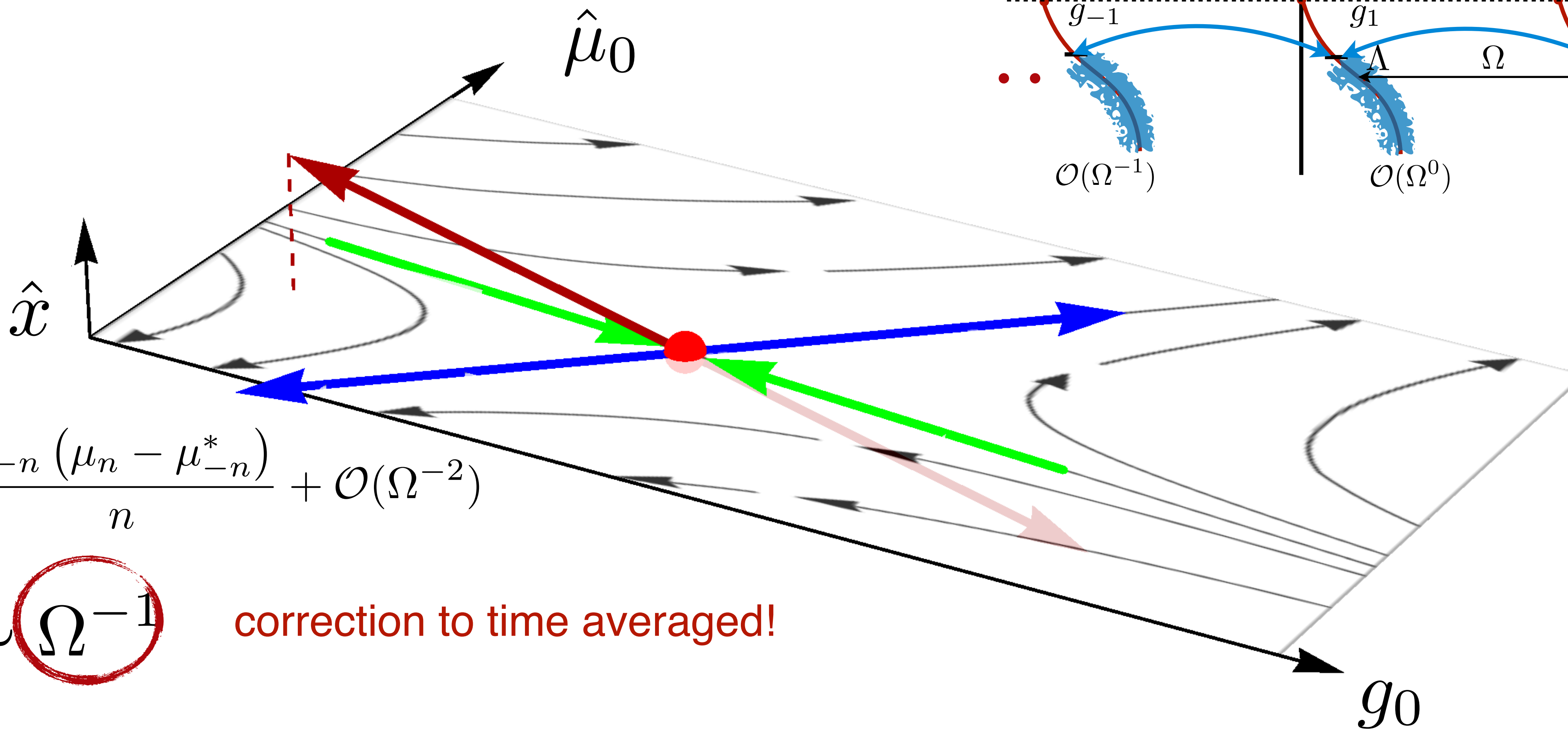
Renormalization group flow

- fixed point coordinates $\epsilon = 4 - d$

$$\begin{pmatrix} \hat{\mu}_0^* \\ \hat{g}_0^* \\ \hat{x}^* \end{pmatrix} \cong \begin{pmatrix} -\epsilon/5 \\ 4\pi^2\epsilon/5 \\ 0 \end{pmatrix} \Bigg\} \text{ usual Wilson-Fisher fixed point}$$

- two relevant directions





$$= \frac{1}{\Omega} \sum_{n=\pm 1} \frac{g_{-n}\left(\mu_n - \mu_{-n}^*\right)}{n} + \mathcal{O}(\Omega^{-2})$$

$$\sim \Omega^{-1}$$

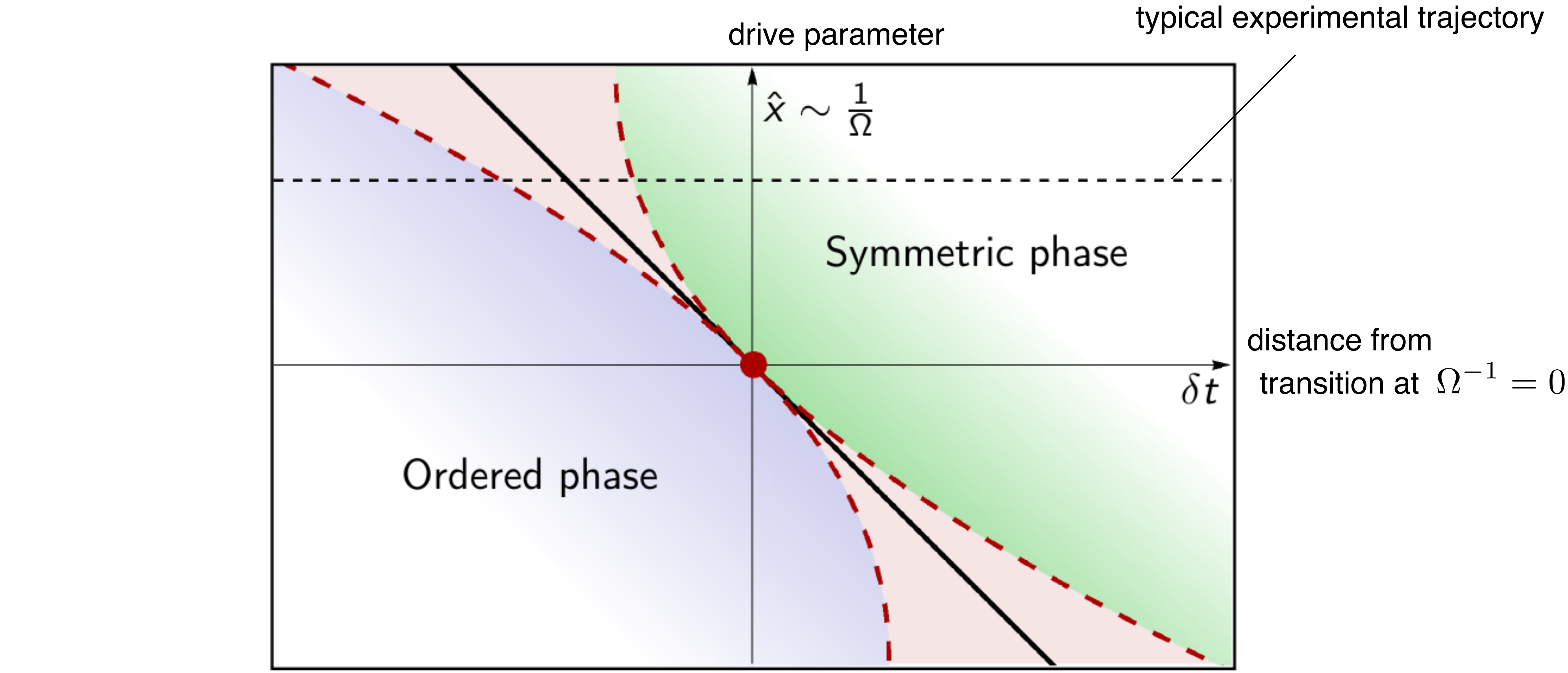
correction to time averaged!

- $\hat{x}$ defines a new independent critical exponent, at $\mathcal{O}(\Omega^{-1}) \times \mathcal{O}(\epsilon)$

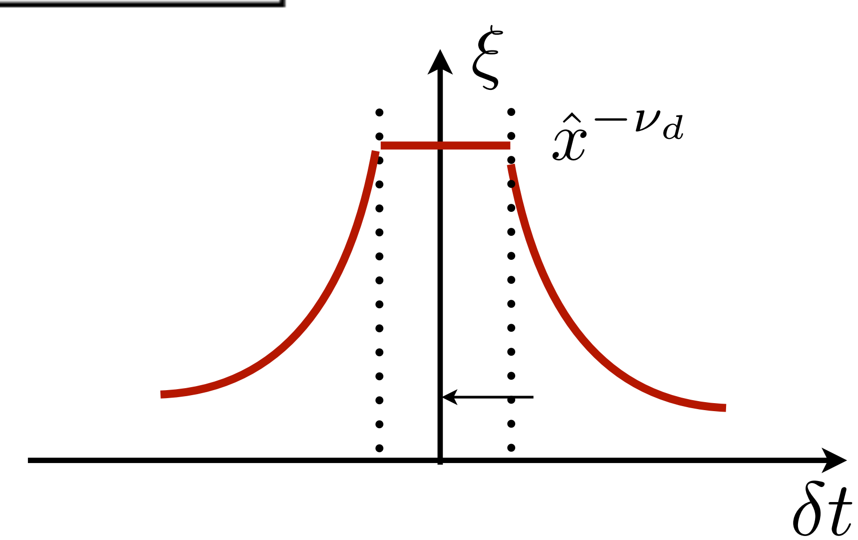
$$\boxed{\nu_d = 1/\epsilon}$$

Phase diagram

- Drive scale turns critical into bicritical point (two relevant directions)

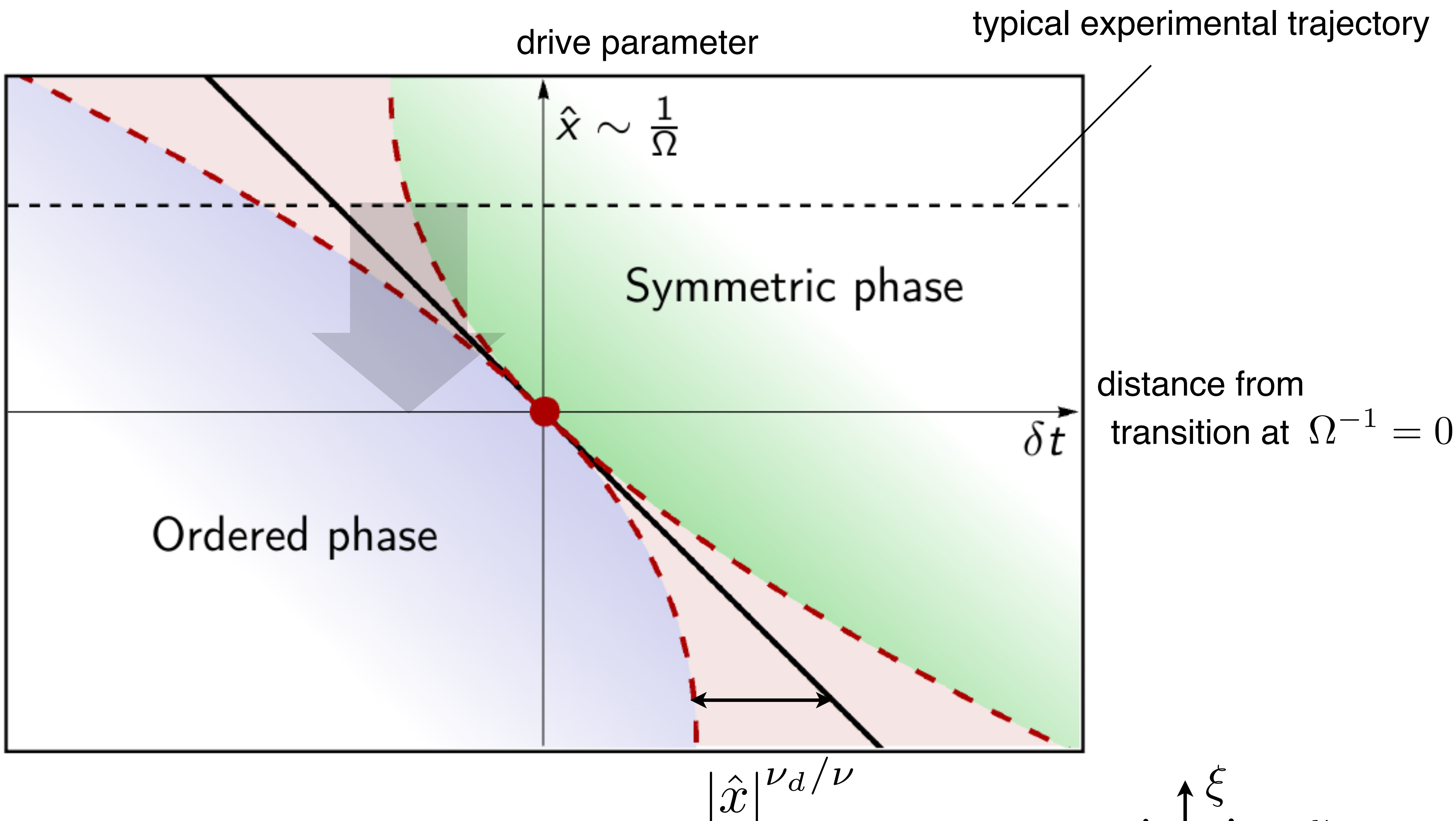


- $\hat{x} = 0$: usual critical physics is visible, e.g. $\xi \sim \delta t^{-\nu}$
- $\hat{x} \neq 0$: correlation length saturates to $\xi \lesssim \hat{x}^{-\nu_d}$
- infinitely rapidly driven smoothly recovered as $\hat{x} \sim \Omega^{-1} \to 0$



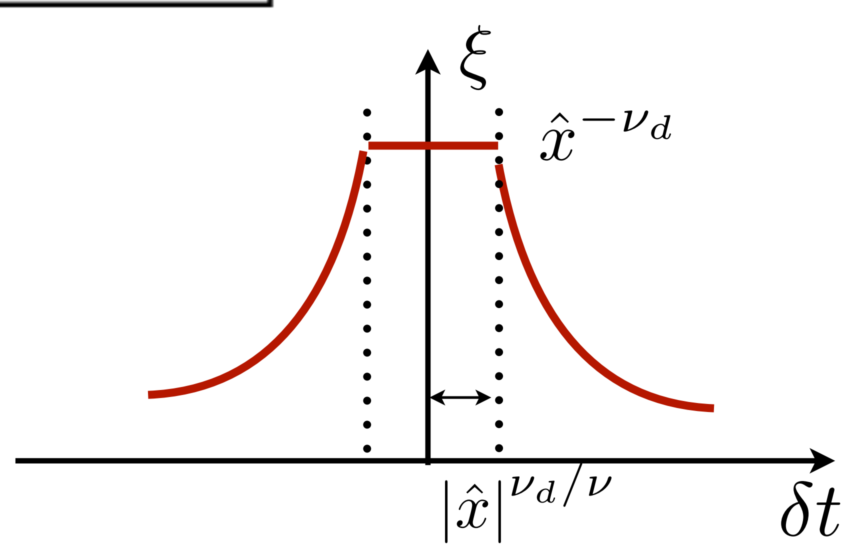
Phase diagram

- Observability



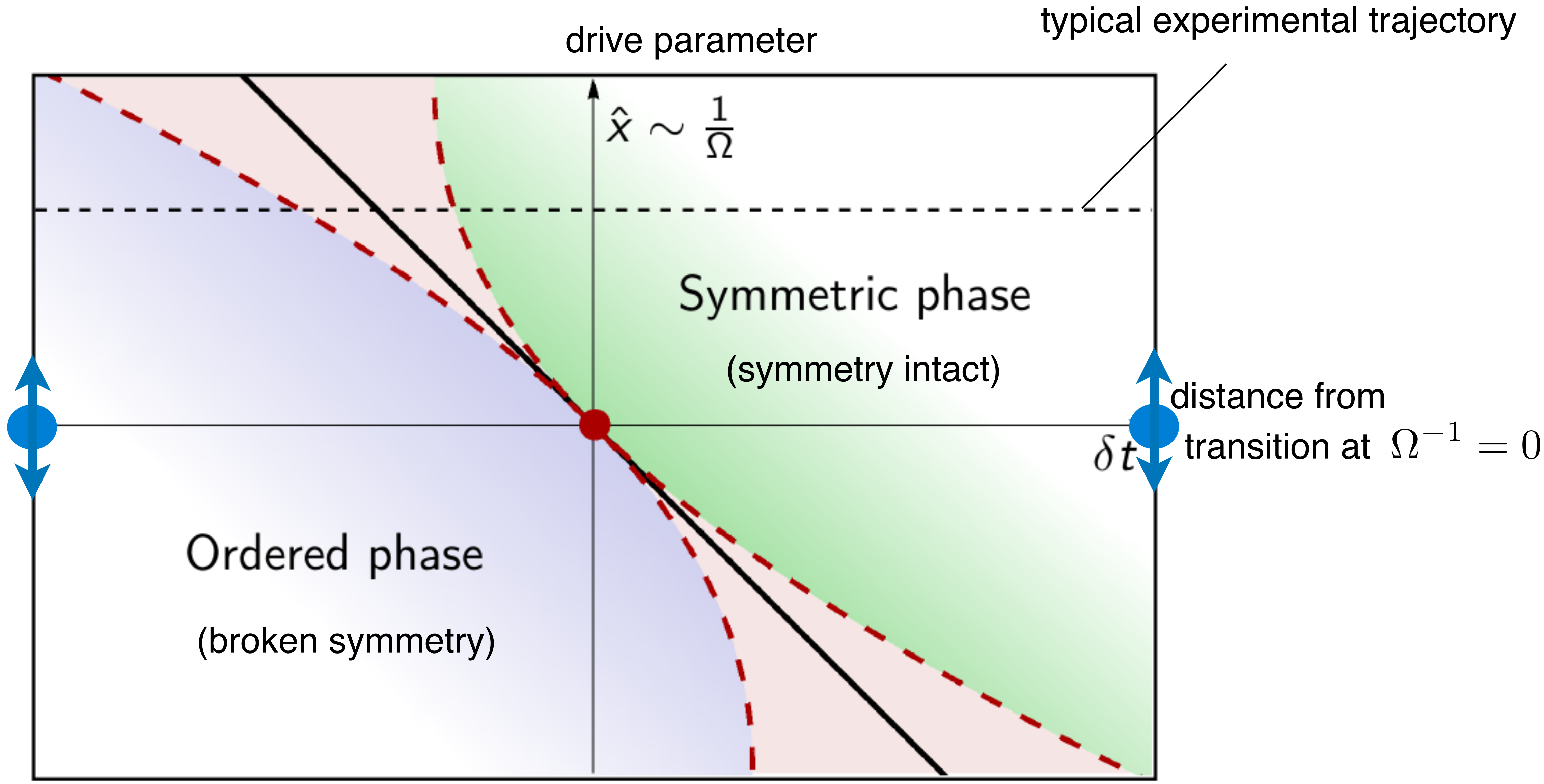
- width of saturation window hosts the new critical exponent

$$|\Delta t| \cong |\hat{x}|^{\nu_d/\nu}$$



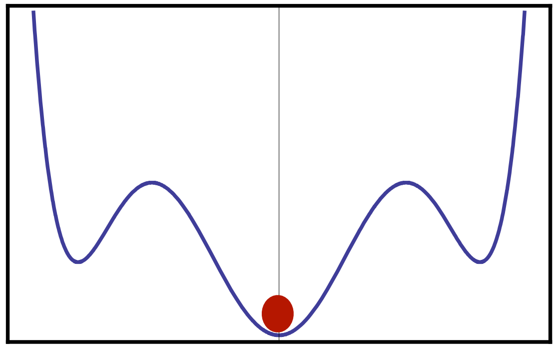
Phase diagram: Nature of Phase Transition

- Fluctuation induced first order phase transition



- Perturbation theory converges despite driving
- Phases macroscopically distinct by symmetry breaking pattern
- Correlation length remains finite despite symmetry breaking transition

➡ Fluctuation induced, weak first order phase transition



analogies: Coleman, Weinberg, PRD (1973); Halperin, Lubensky, Ma, PRL (1974); Fisher, Nelson, PRL (1974)

gauged vector models — superconductors — Goldstone modes

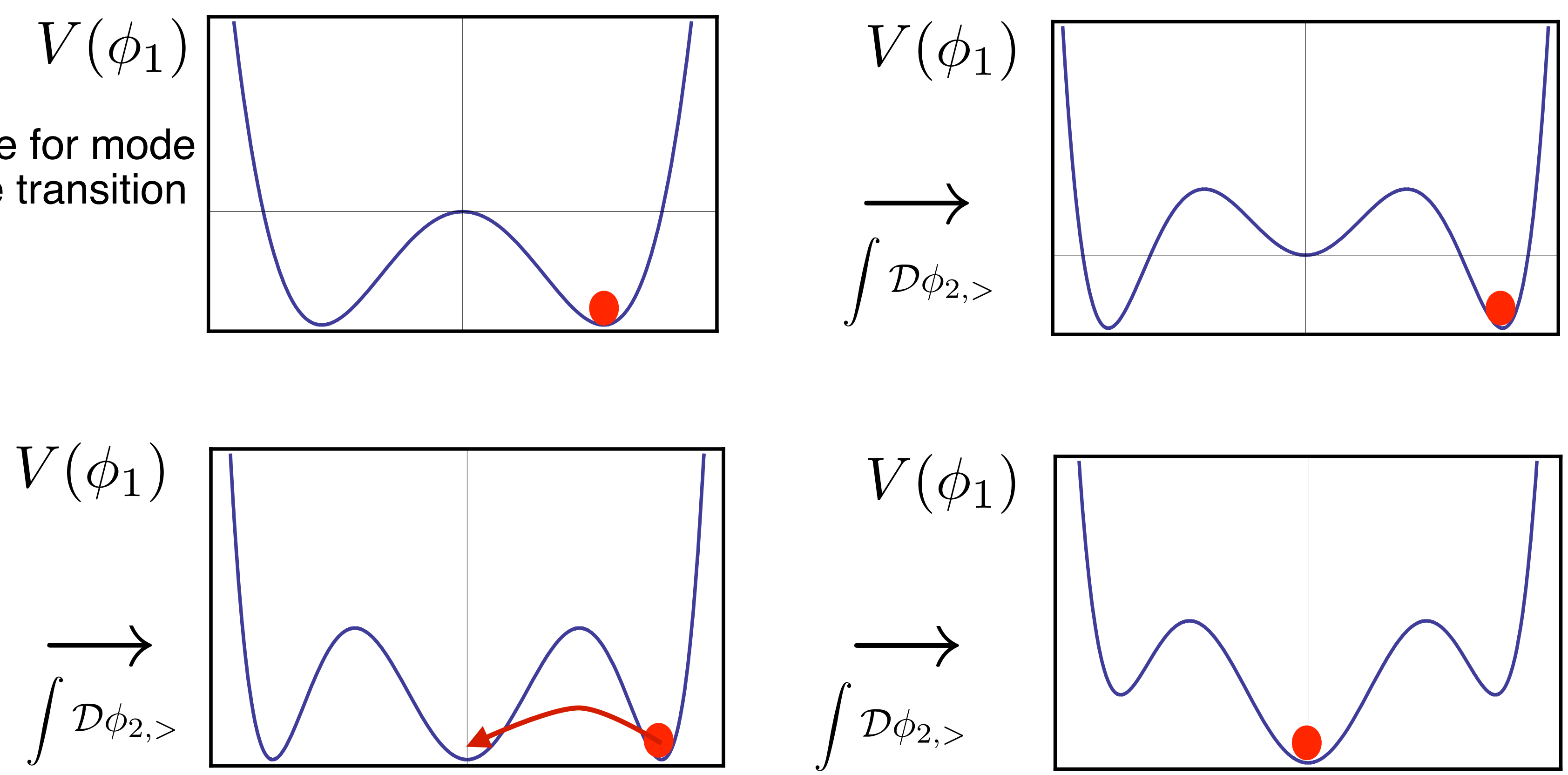
Critical Degeneracy: Coleman-Weinberg phenomenon
Halperin-Lubensky-Ma

- Multiple gapless modes: Critical mode coupled to non-critical gapless modes
  - ➡ Fluctuation induced first order phase transition (w/o explicit symmetry breaking)
  - gauged vector models — Coleman, Weinberg, PRD (1973)
  - superconductors — Halperin, Lubensky, Ma, PRL (1974)
  - Goldstone modes — Fisher, Nelson, PRL (1974)

  and many after!

- Intuition: eliminating the non-critical mode ϕ_2

potential landscape for mode undergoing phase transition



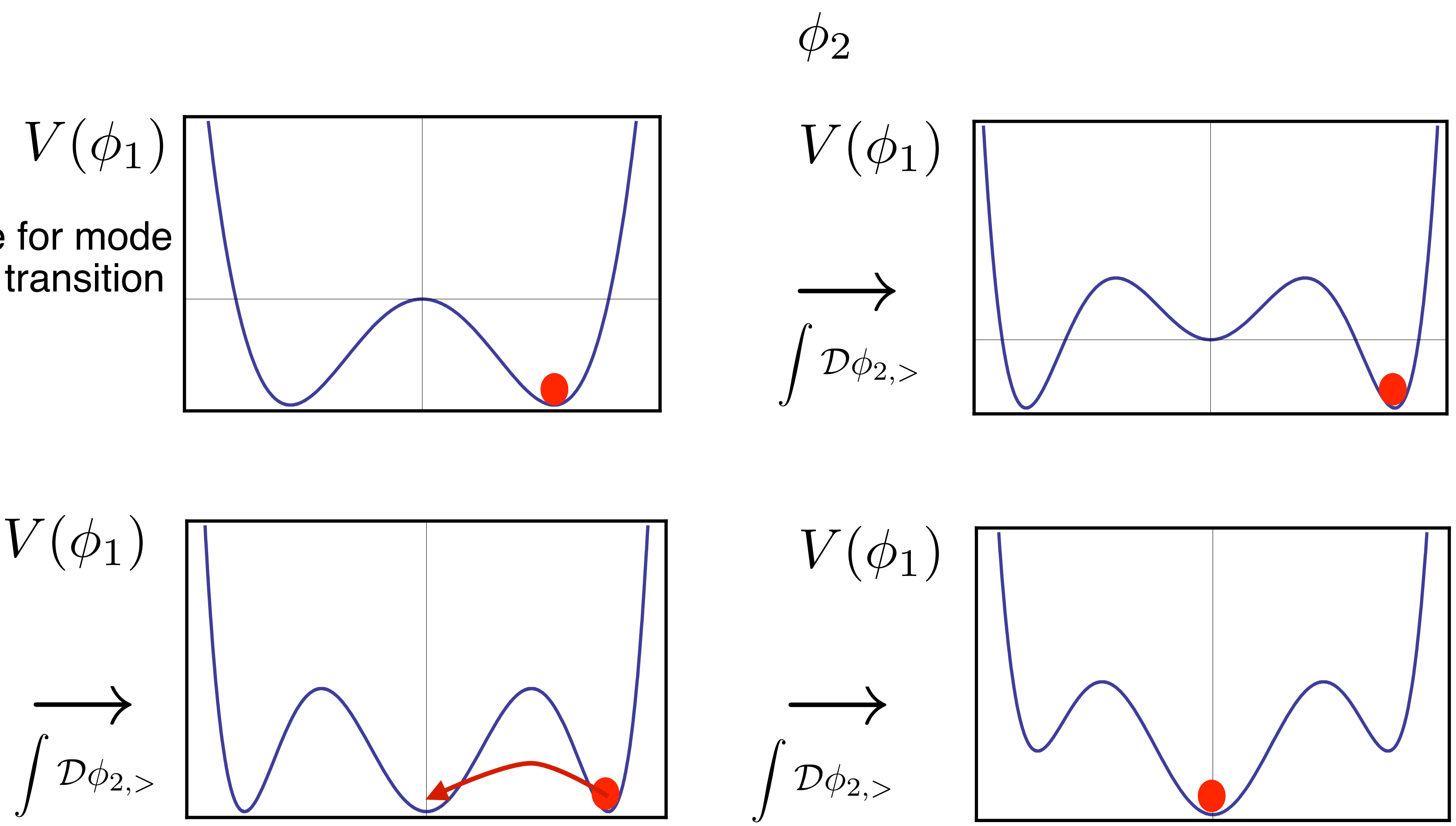
- alternatively: RG picture, runaway flow signals generation of new length scale

Picture: fluctuation induced many-body Kapitza pendulum

Citro et al., AoP (2015); Lerose, Marino, Gambassi, Silva, arxiv:1803.04490

- renormalization of n = 0 sector due to higher FBZs
- 1-loop effective potential for n=0: interaction sign change $g_0^{\text{1-loop}} < 0$
- suggests generation of additional minimum due to fast drive via Kapitza mechanism
- universal: critical degeneracy guaranteed by Floquet theorem

potential landscape for mode undergoing phase transition



- alternatively: RG picture, runaway flow signals generation of new length scale

Outlook: RG formulation of Kibble-Zurek mechanism

- "dual" limit $\Omega \to 0$
- exact Wigner Green's function $G_{R;n}(\omega) = \frac{\Omega}{2\pi}\int_0^{\frac{2\pi}{\Omega}} dt e^{in\omega t} \sum_m \frac{J_m\left(2[M(t) - M_0]/\Omega\right)}{\omega + M_0 + \frac{m\Omega}{2}}$
- expansion in powers of Ω produces derivative expansion of the drive function

$$\mu(t) = \mu + \mu' \cdot t + \ldots \qquad\qquad g(t) = g + g' \cdot t + \ldots$$

- follow the above program:
  - RG equations

$$\partial_t \hat{\mu} = -2\hat{\mu} - \frac{4[2\pi\Omega_d]\hat{g}}{|1+\hat{\mu}|}\left[1 + \frac{\hat{\mu}'}{2(1+\hat{\mu})^2}\right] \qquad \partial_t \hat{\mu}' = -4\hat{\mu}' - \frac{4[2\pi\Omega_d]}{|1+\hat{\mu}|}\left[\hat{g}' - \frac{\hat{g}\hat{\mu}'}{1+\hat{\mu}}\right]$$

$$\partial_t \hat{g} = -(4-d)\hat{g} + \frac{10[2\pi\Omega_d]\hat{g}^2}{|1+\hat{\mu}|\,(1+\hat{\mu})}\left[1 + \frac{3\hat{\mu}'}{4(1+\hat{\mu})^2}\right] \qquad \partial_t \hat{g}' = -(6-d)\hat{g}' + \frac{20[2\pi\Omega_d]\hat{g}}{|1+\hat{\mu}|\,(1+\hat{\mu})}\left[\hat{g}' - \frac{\hat{g}\hat{\mu}'}{1+\hat{\mu}}\right]$$

  - Wilson Fisher fixed point: unmodified

$$\mu^* = -\frac{4-d}{9-d}\,, \qquad g^* = 2^{d-2}\pi^{d/2}\Gamma[d/2]\frac{5(4-d)}{(9-d)\,|9-d|}\,, \qquad \hat{\mu}' = 0\,, \qquad \hat{g}' = 0$$

  - stability matrix: linearize around fixed point

Outlook: RG formulation of Kibble-Zurek mechanism

- "dual" limit $\Omega \to 0$
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- expansion in powers of Ω produces derivative expansion of the drive function

$$\mu(t) = \mu + \mu' \cdot t + \dots \qquad\qquad g(t) = g + g' \cdot t + \dots$$

- follow the above program:
  - structure of stability matrix

$$M = \begin{pmatrix} -2 + \frac{4S_d g^*}{(1+\mu^*)^2} & -\frac{4S_d}{1+\mu^2} & -\frac{2S_d g^*}{(1+\mu^*)^3} & 0 \\ -\frac{20S_d (g^*)^2}{(1+\mu^*)^3} & (d-4) + \frac{20S_d g^*}{(1+\mu^*)^2} & \frac{15S_d (g^*)^2}{2(1+\mu^*)^4} & 0 \\ 0 & 0 & -4 + \frac{4S_d g^*}{(1+\mu^*)^2} & -\frac{4S_d}{1+\mu^2} \\ 0 & 0 & -\frac{20S_d (g^*)^2}{(1+\mu^*)^3} & d - 6 + \frac{20S_d g^*}{(1+\mu^*)^2} \end{pmatrix}$$

$$\equiv \begin{pmatrix} M_1 & X \\ 0 & M_2 \end{pmatrix} \quad \text{with} \quad \boxed{M_2 = -2 + M_1}$$

difference in canonical dimension

identical loop effect!

leading eigenvalues (critical exponents)

$$\boxed{\begin{aligned} y_1 &= 2 - \tfrac{2}{5}\epsilon \\ y_2 &= 2 - y_1 \end{aligned}}$$

➡ additional relevant direction, as in opposite limit

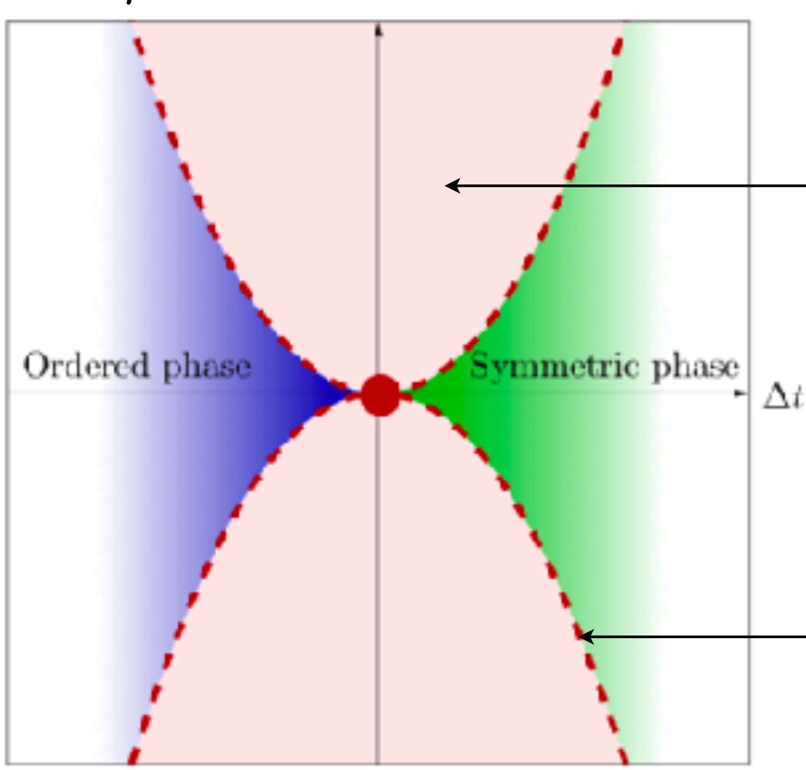
➡ no independent information (at one loop), unlike opposite limit!

Outlook: RG formulation of Kibble-Zurek mechanism

- “dual” limit $\Omega \to 0$
- exact Wigner Green’s function $G_{R;n}(\omega) = \frac{\Omega}{2\pi}\int_0^{\frac{2\pi}{\Omega}} dt e^{in\omega t} \sum_m \frac{J_m\left(2[M(t)-M_0]/\Omega\right)}{\omega + M_0 + \frac{m\Omega}{2}}$
- expansion in powers of Ω produces derivative expansion of the drive function

$$\mu(t) = \mu + \mu' \cdot t + \ldots \qquad g(t) = g + g' \cdot t + \ldots$$

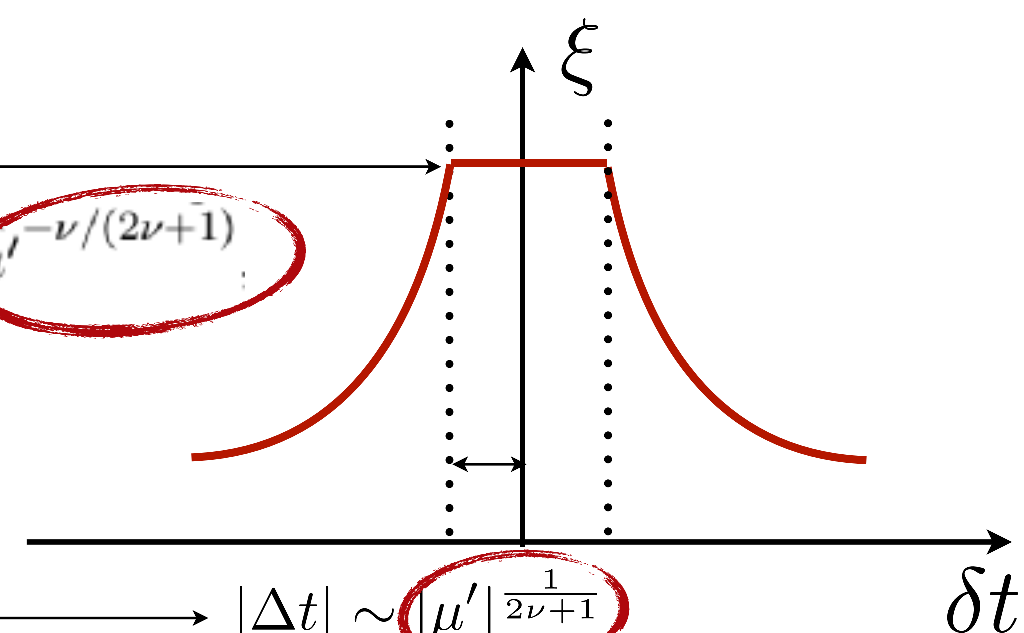
- follow the above program:
  - phase diagram
    $\mu' \sim \Omega$ ramp speed
  - interpretation: Kibble-Zurek



saturation to

$$\xi \sim \hat{\mu}'^{-1/(y+2)} = \mu'^{-\nu/(2\nu+1)}$$

diabatic freeze out at $|\Delta t| \sim |\mu'|^{\frac{1}{2\nu+1}}$



➡ Kibble-Zurek scaling reproduced

Outlook: RG formulation of Kibble-Zurek mechanism

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- expansion in powers of Ω produces derivative expansion of the drive function

$$\mu(t) = \mu + \mu' \cdot t + ... \qquad\qquad g(t) = g + g' \cdot t + ...$$

- open: quantitative test of Kibble-Zurek hypothesis of non-modification of critical exponents (structure of M_1, M_2 at two-loop order)

- guess: structure prevails
  - $\Omega \to 0$: Kibble-Zurek: infrared modification => no change of universal behavior
  - $\Omega^{-1} \to 0$: ultraviolet modification => universal behavior possibly changed

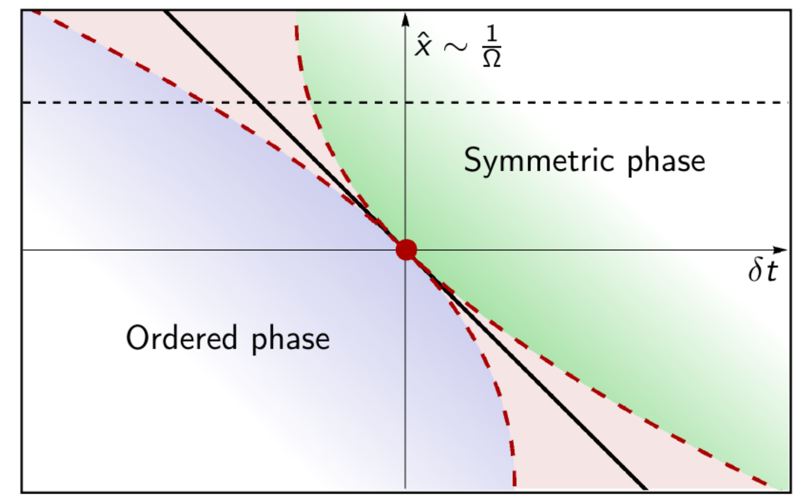
origin of each independent critical exponent must be associated to a UV scale

e.g. $$\langle \phi^*(r)\phi(0)\rangle \sim L^{2-d} \sim a^{-\eta} r^{2-d+\eta}$$

physical length dimension

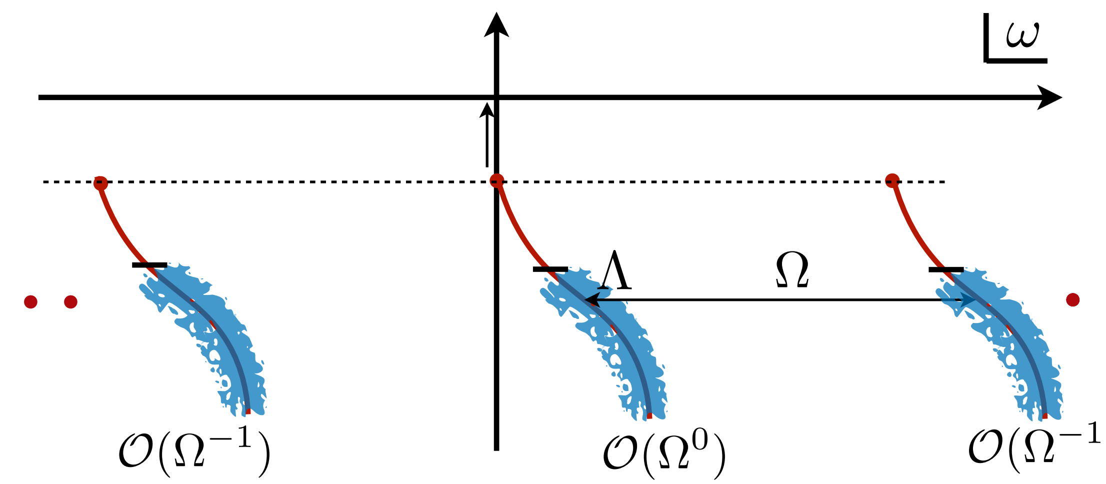
experimentally observed scaling

Conclusions and Outlook



- Absence of criticality in rapidly driven quantum systems above lower critical dimension

- Mechanism
    - Degeneracy of near critical poles => new relevant direction at infinitely rapidly driven fixed point
    - Ordering principle in rapidly driven limit



- completes picture of vicinity of established fixed points

➡ connection between the two limiting regimes?

- applicability in “pre-heating” states of closed Floquet systems (system as its own bath)?