Quantum Time Crystals and Floquet SPTs: A summary

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Collaborators: S. Sondhi (Princeton), V. Khemani (Harvard), Lukin Group (Harvard).

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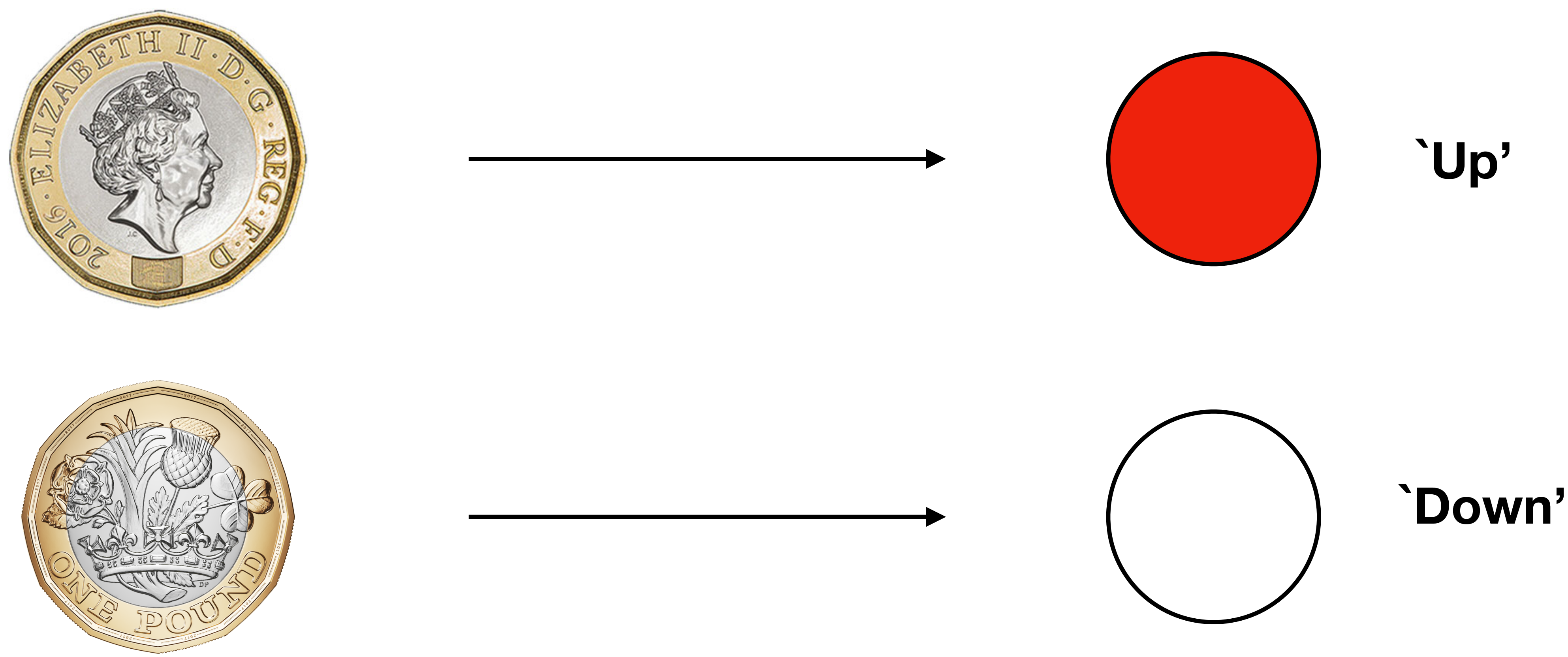
Phys. Rev. B 96, 115127 (2017)
Experiment! Nature 543, 221 (2017)

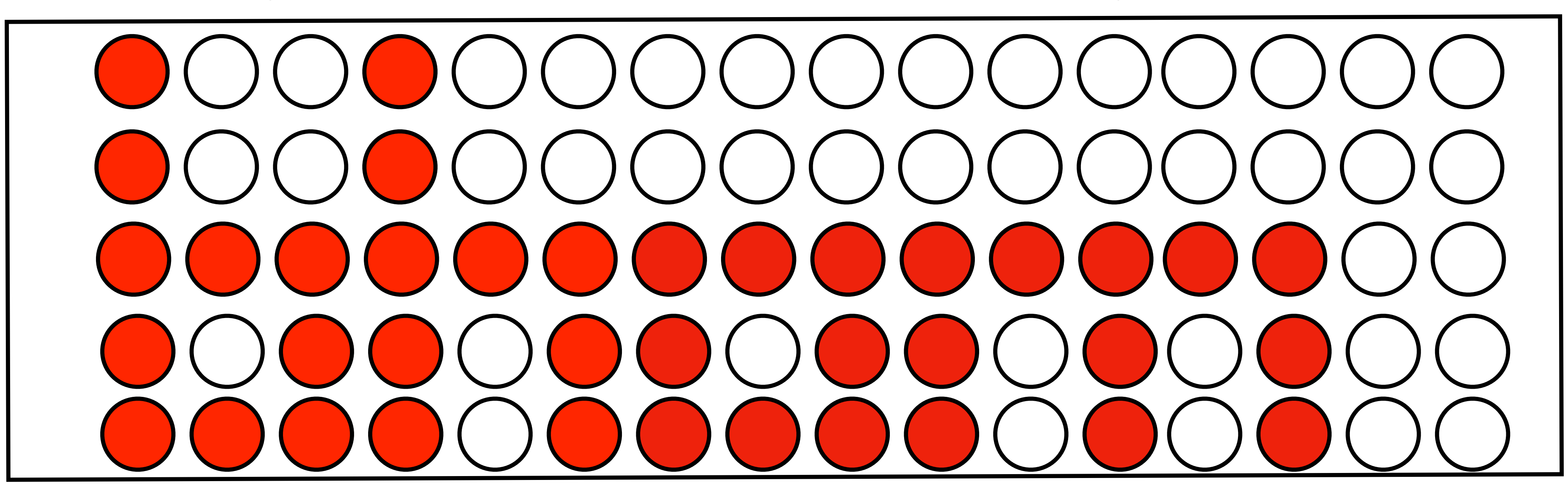
Preamble

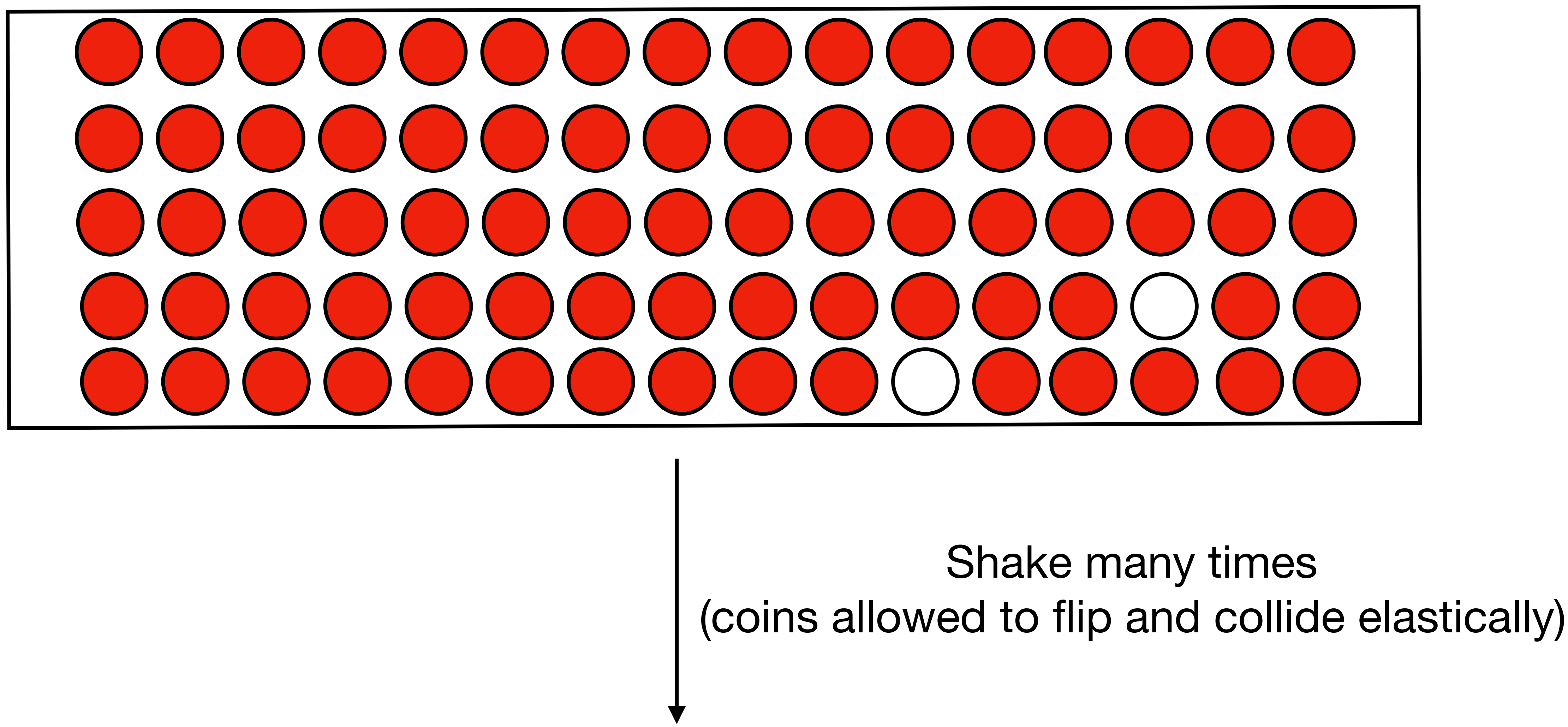
Frivolous thought experiment

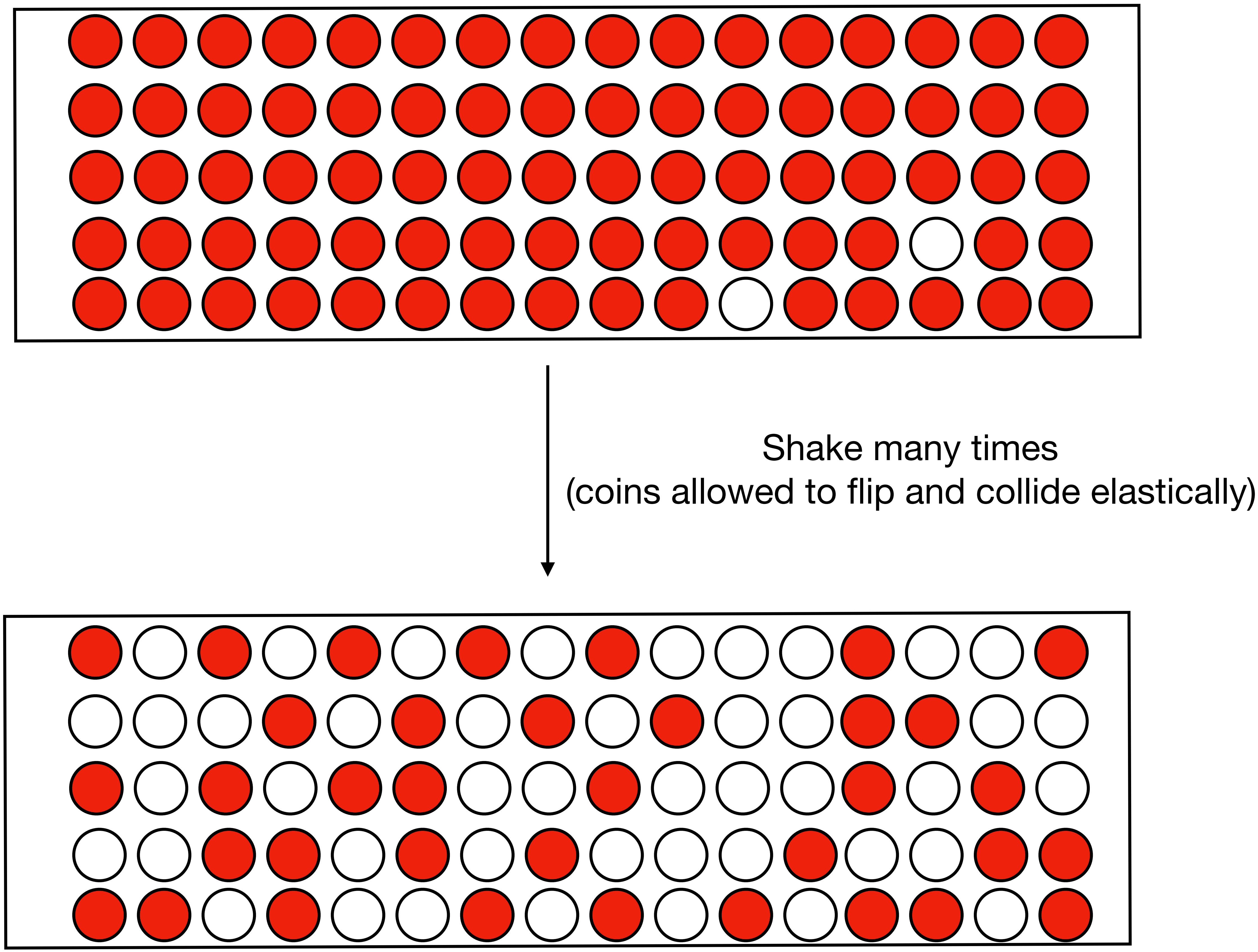


Fill a box with many such coins

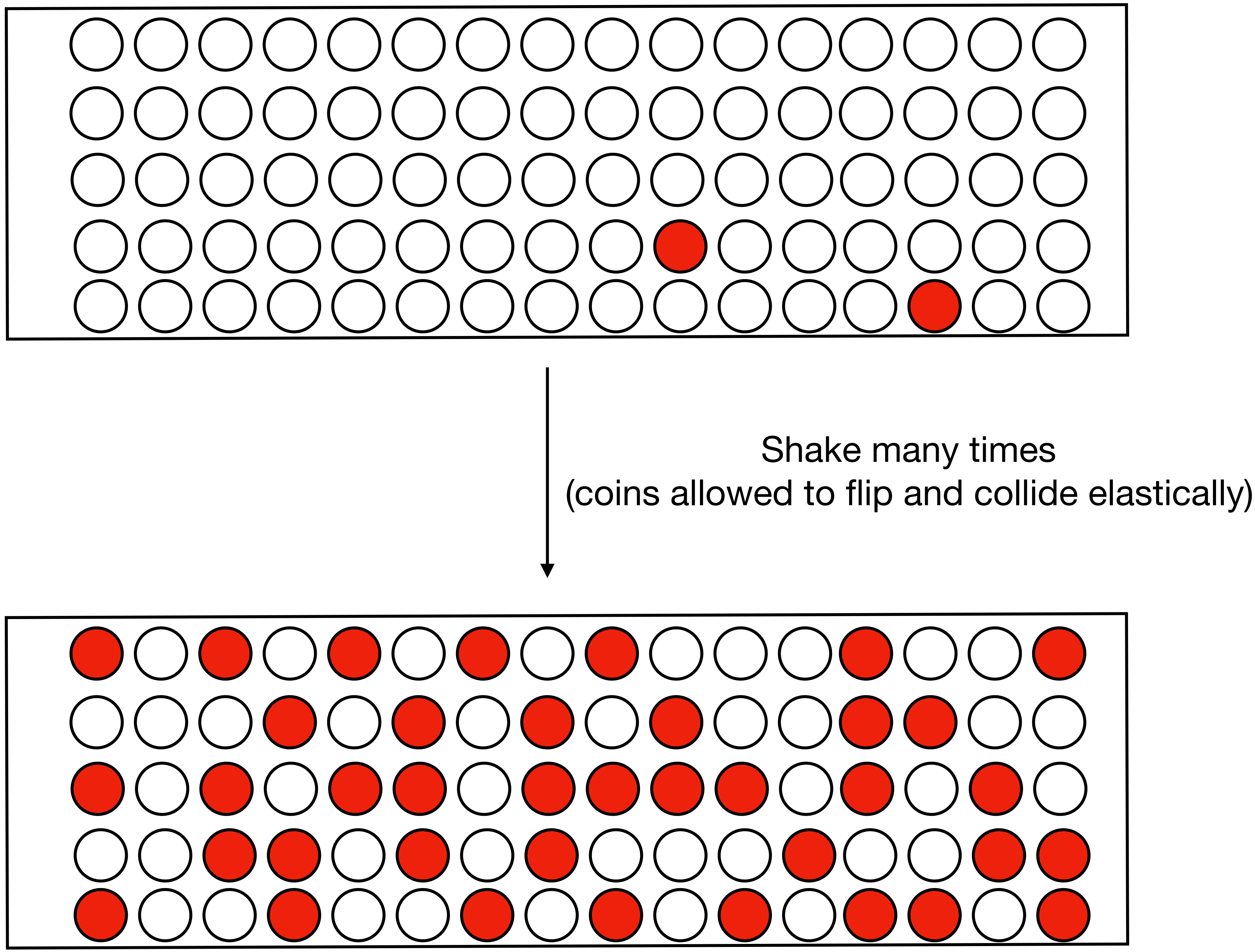
Fill a box with many such coins




Shake many times
(coins allowed to flip and collide elastically)

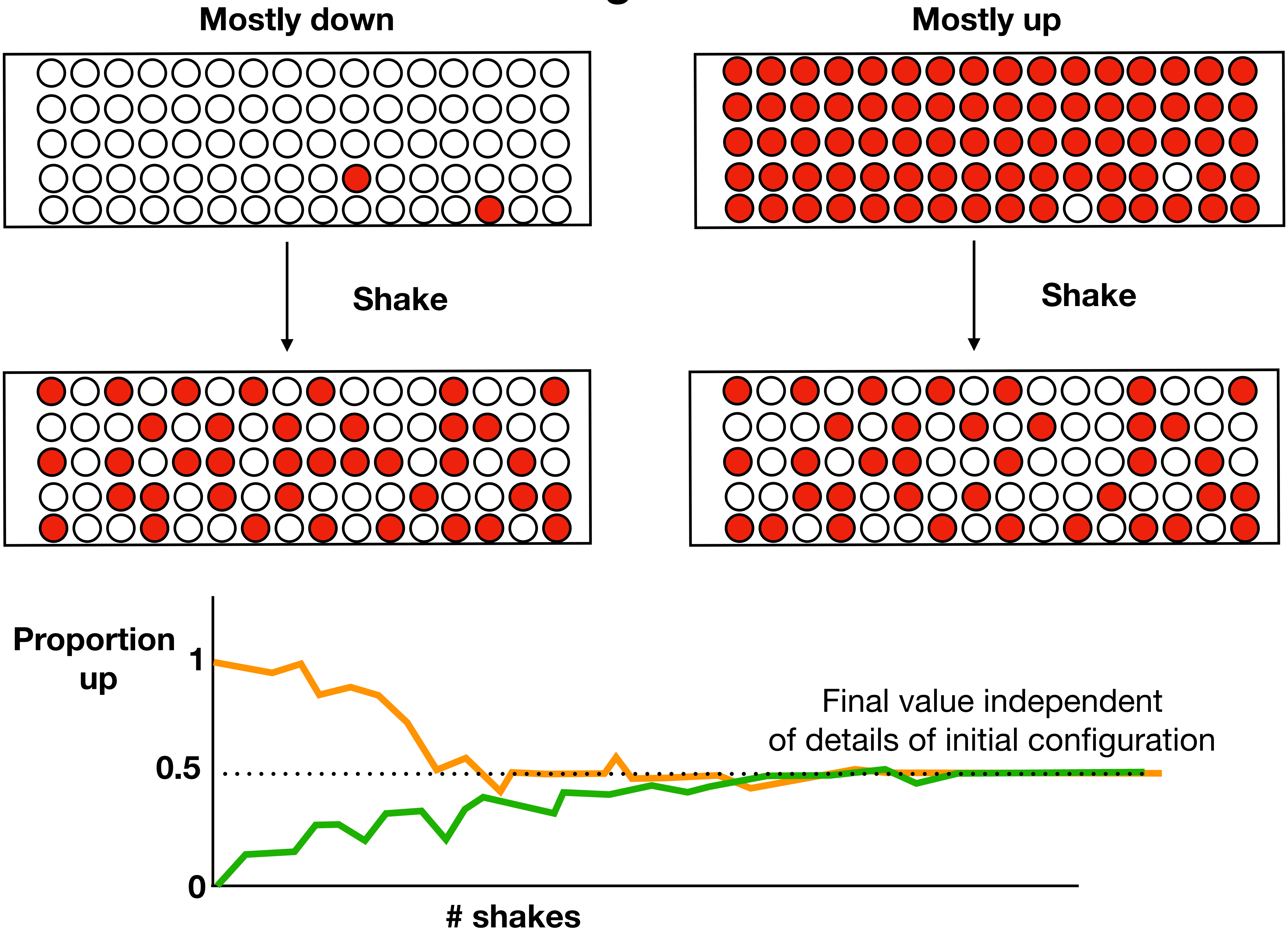


To good approximation, half of the coins now point up, and the other half point down



To good approximation, half of the coins now point up, and the other half point down

Ergodic


Mostly down
Mostly up
Shake
Shake
Proportion up
1
0.5
0
shakes
Final value independent of details of initial configuration

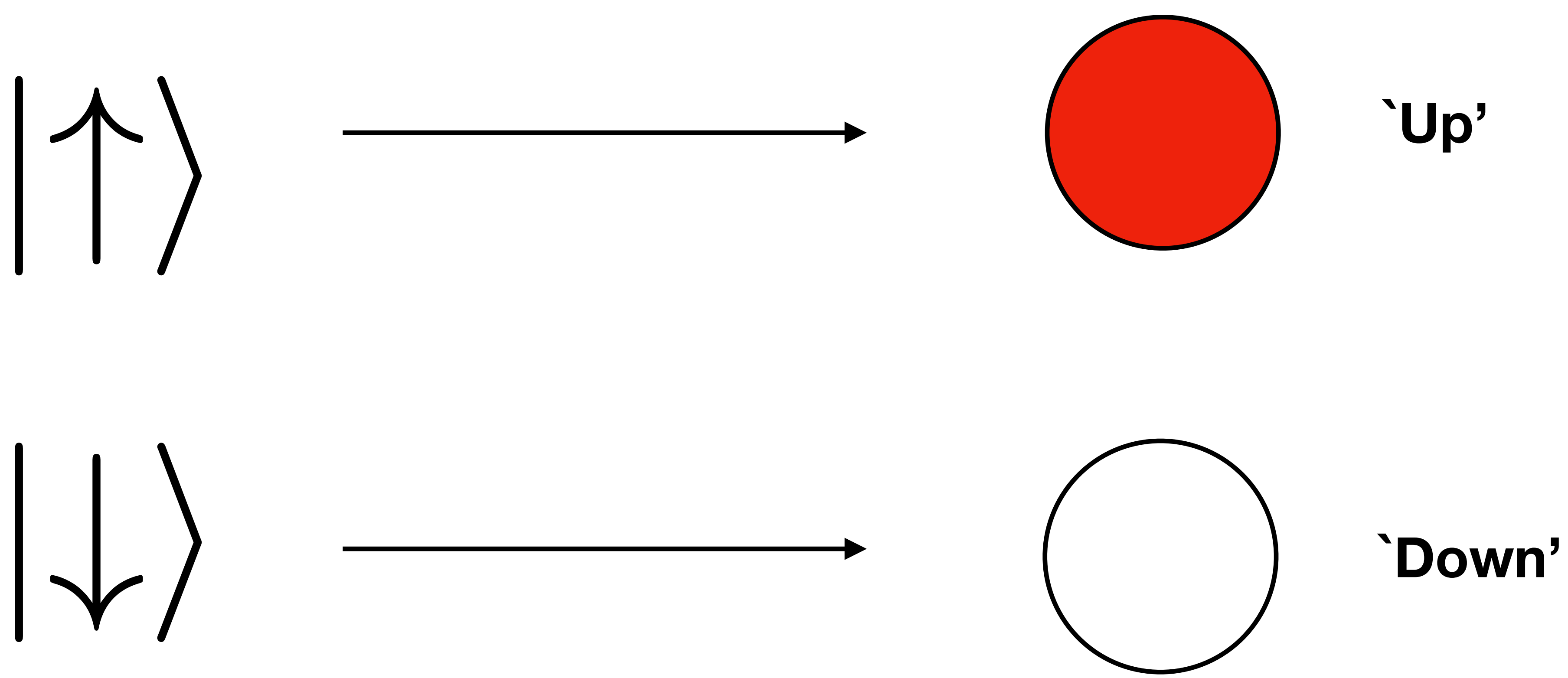
Lesson

- Conjecture: generic + **isolated** (i.e., no dissipation) + periodically driven + many body classical systems are ergodic — local observables eventually forget about the system's initial conditions.
- The same story was expected to hold for analogous driven quantum systems. However…
- It appears that isolated **quantum** systems can sometimes robustly non-ergodic.
- There are now a few examples (MBL, MBL Floquet SPTs, **time crystals**).

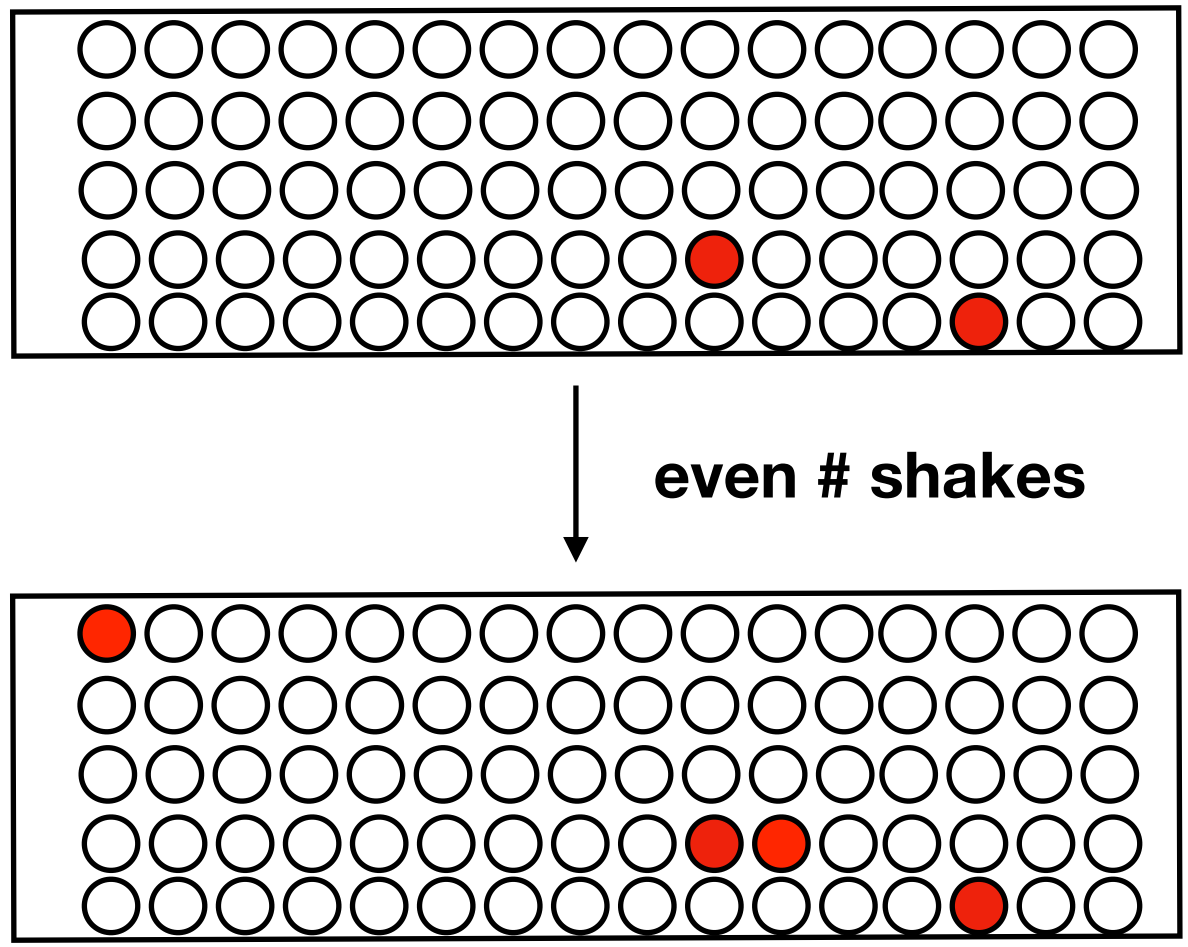
Quantum Time Crystals

Quantum Time Crystals

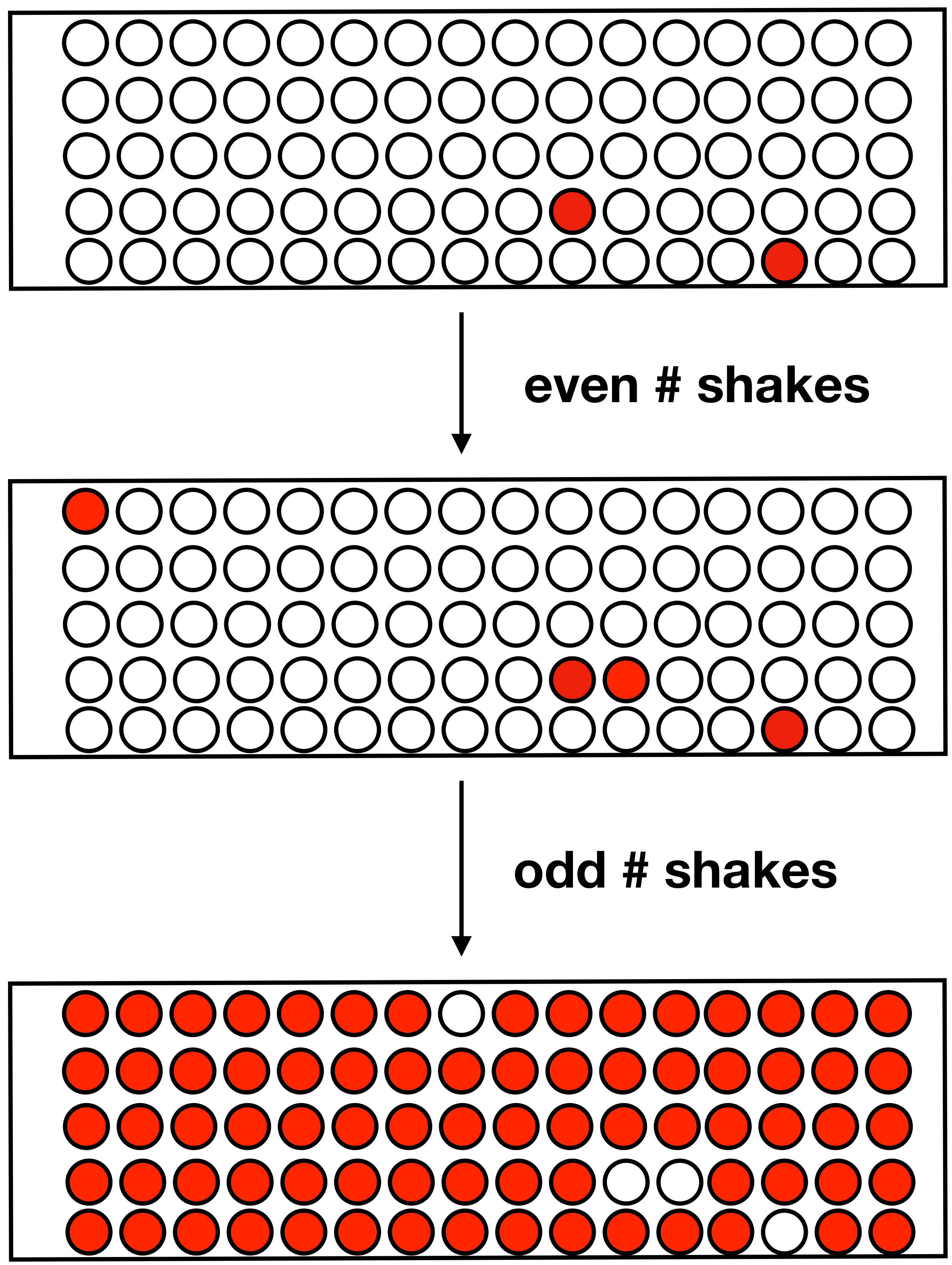
Instead of coins we have spin 1/2 degrees of freedom.



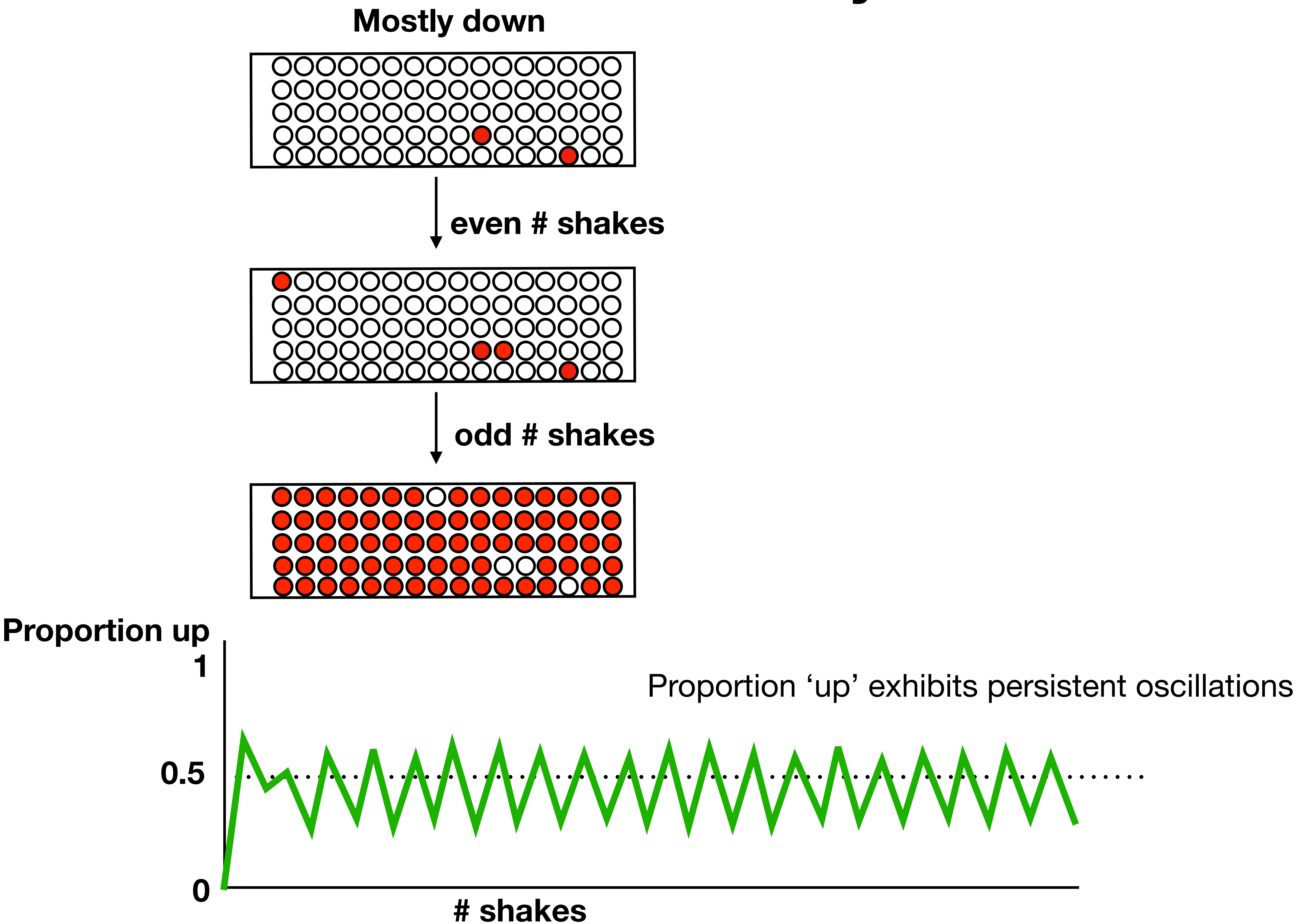
Quantum Time crystal



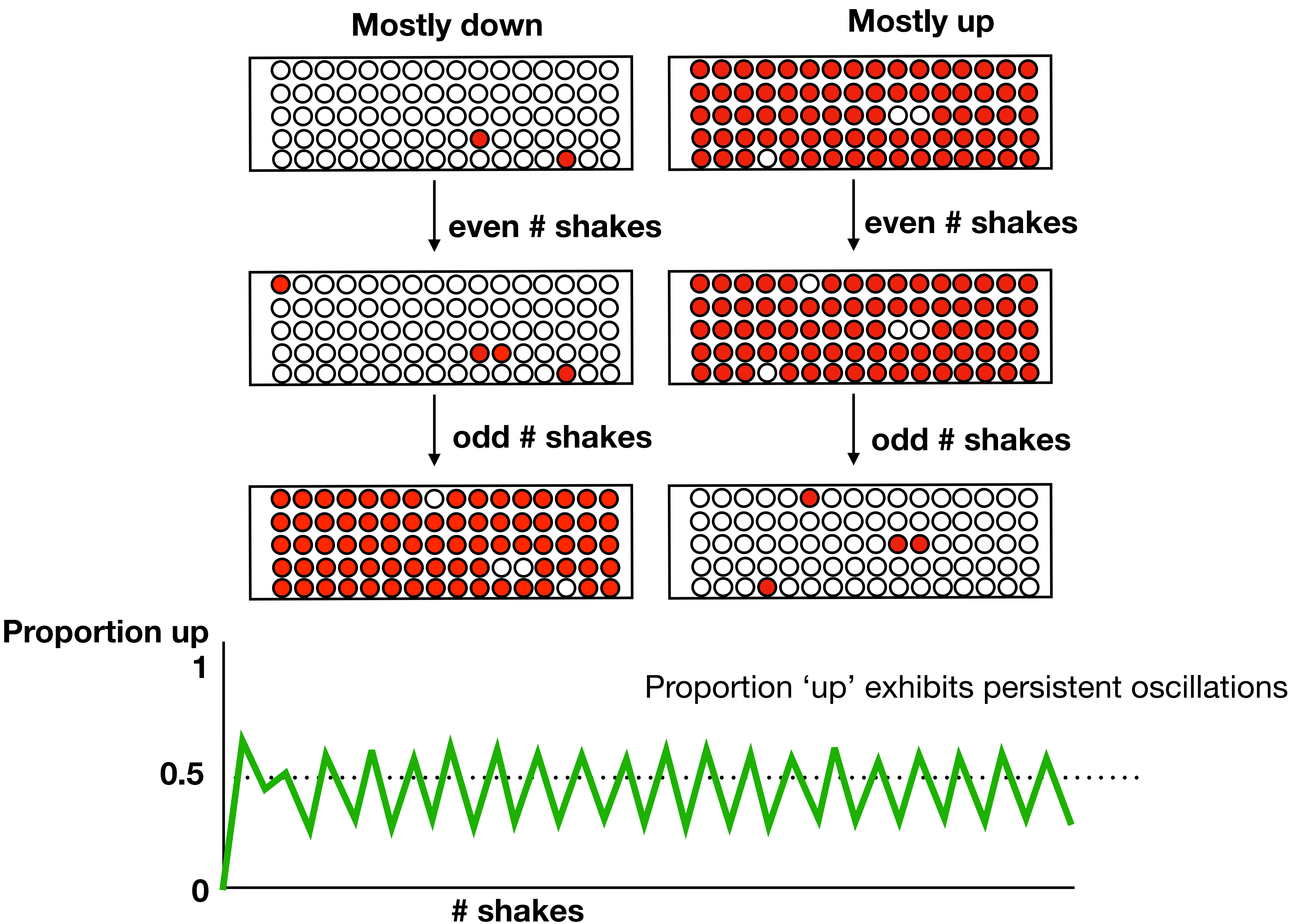
Quantum Time crystal



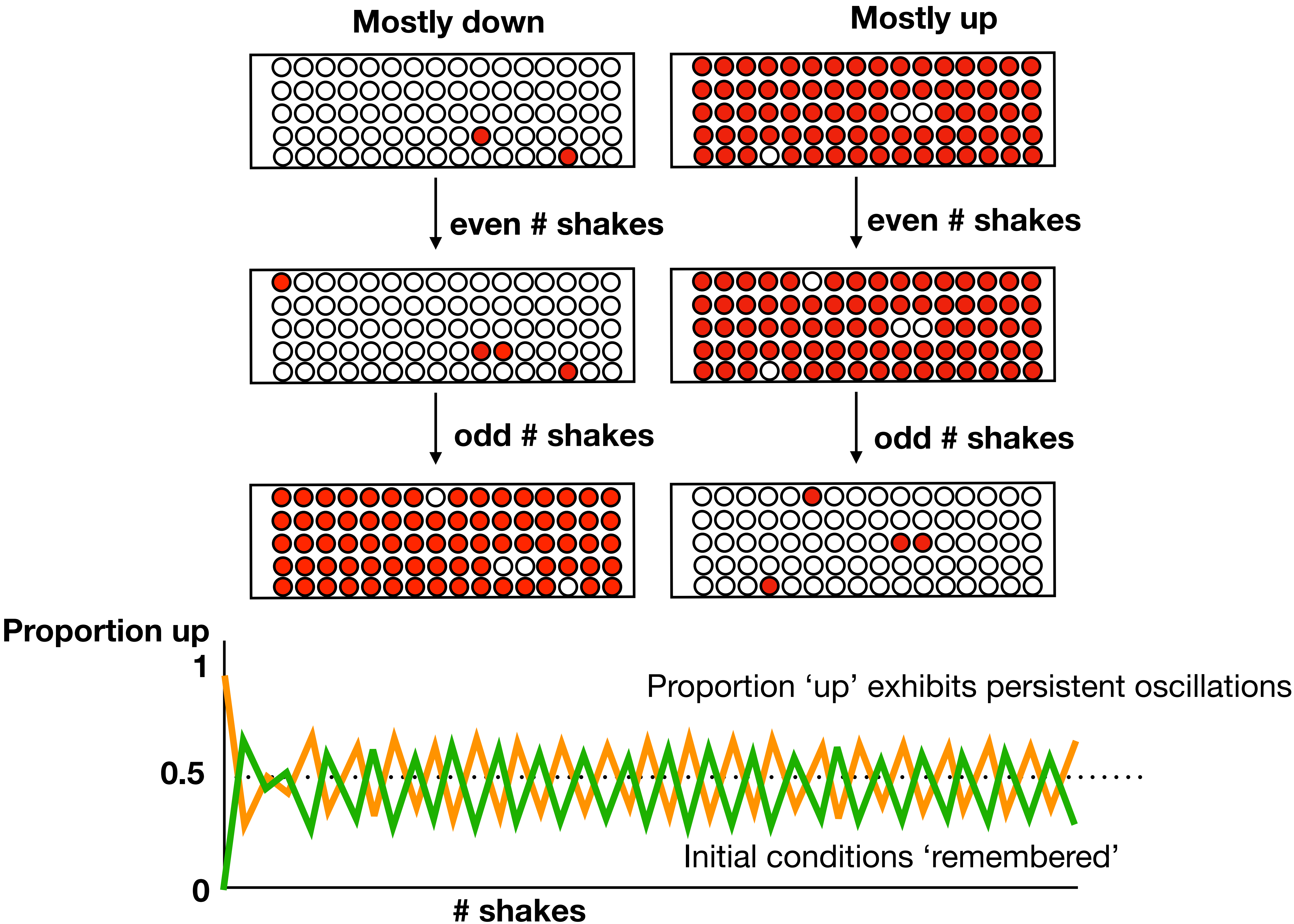
Quantum Time crystal

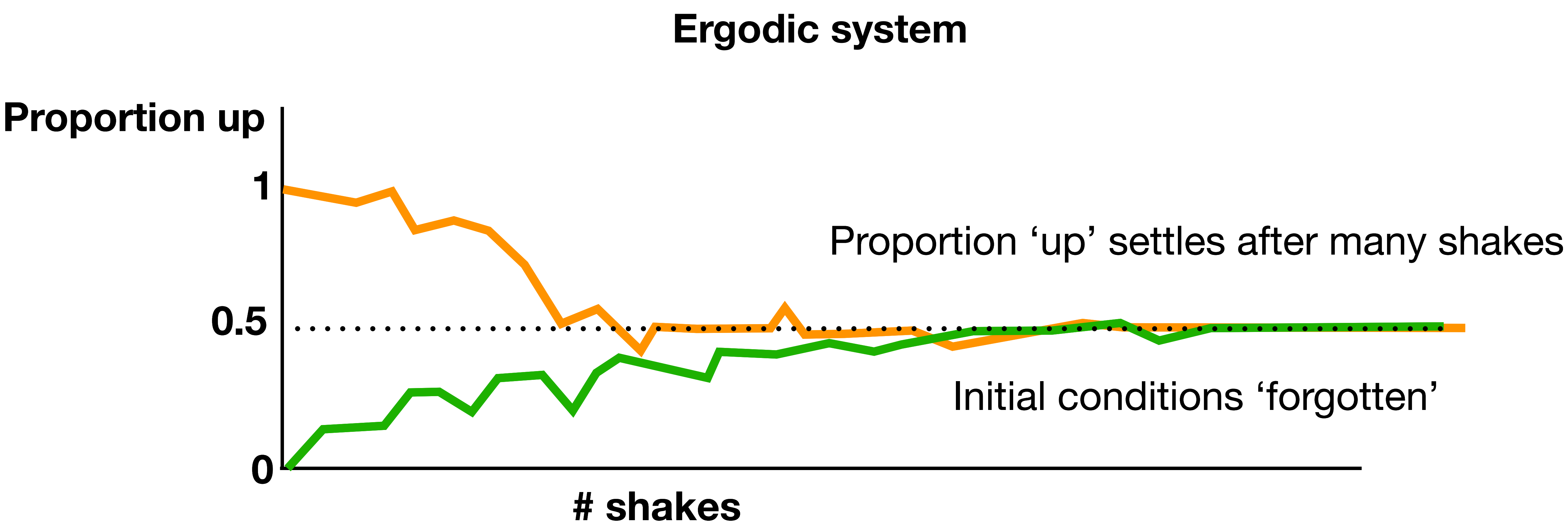


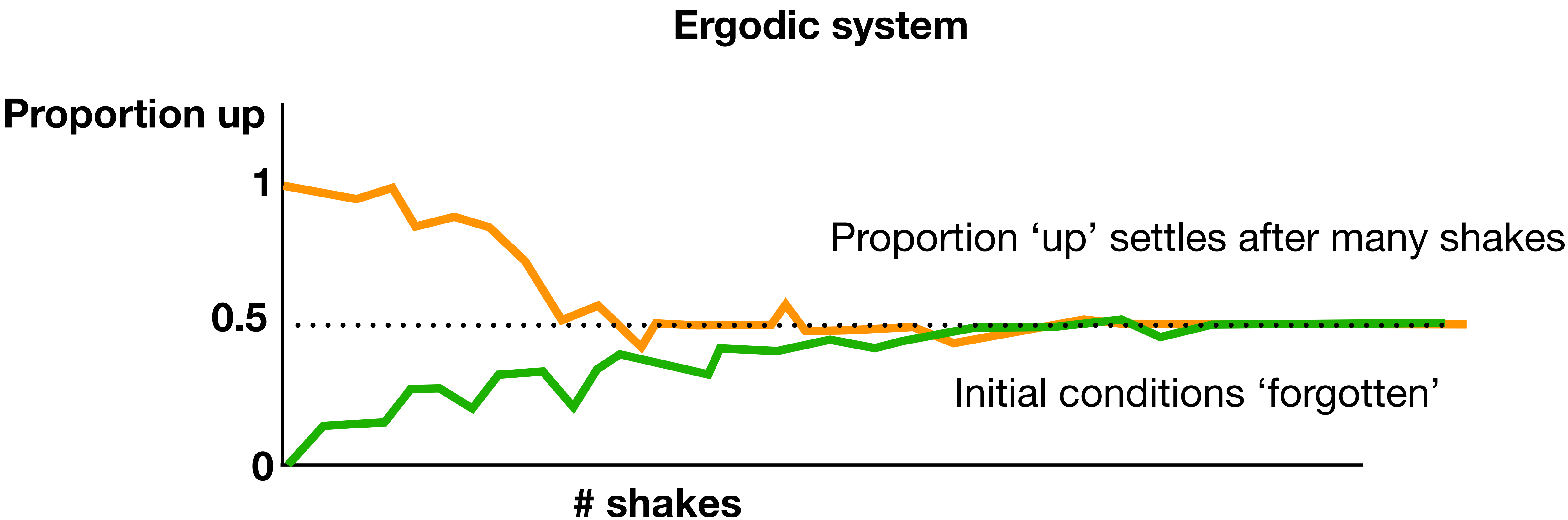
Quantum Time crystal



Quantum Time crystal




Ergodic system
Proportion up
1
0.5
0
shakes
Proportion 'up' settles after many shakes
Initial conditions 'forgotten'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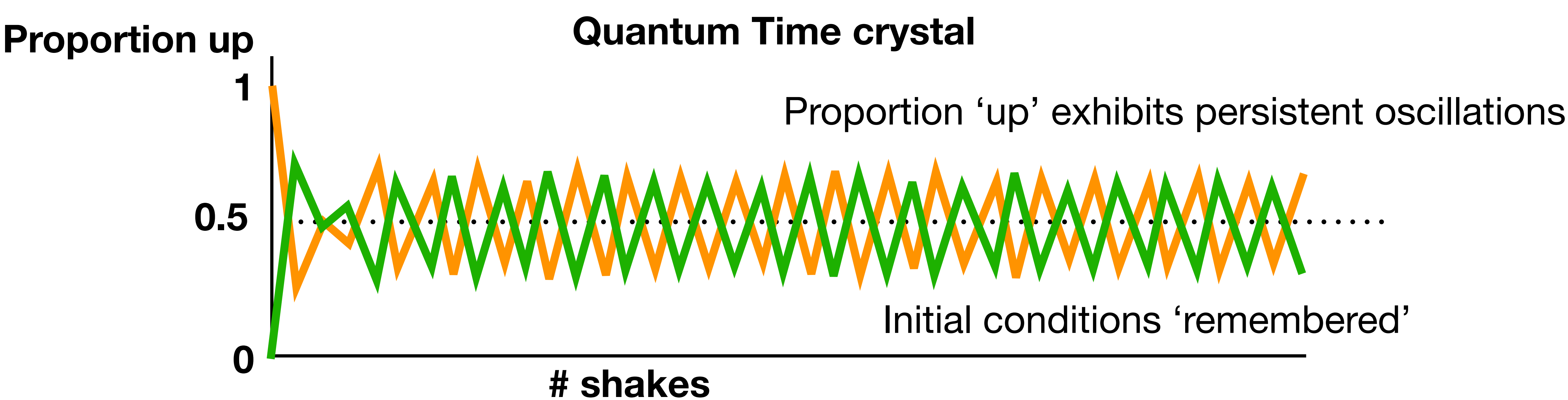
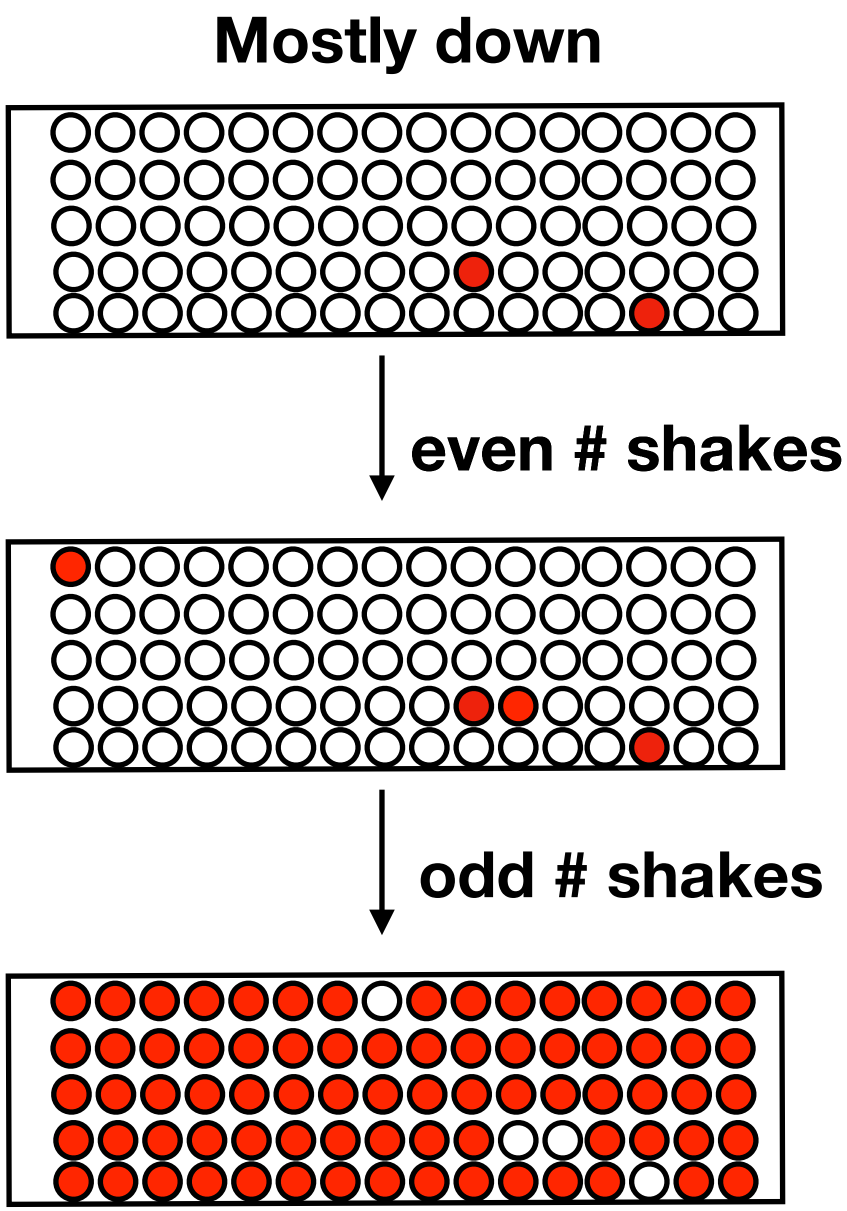
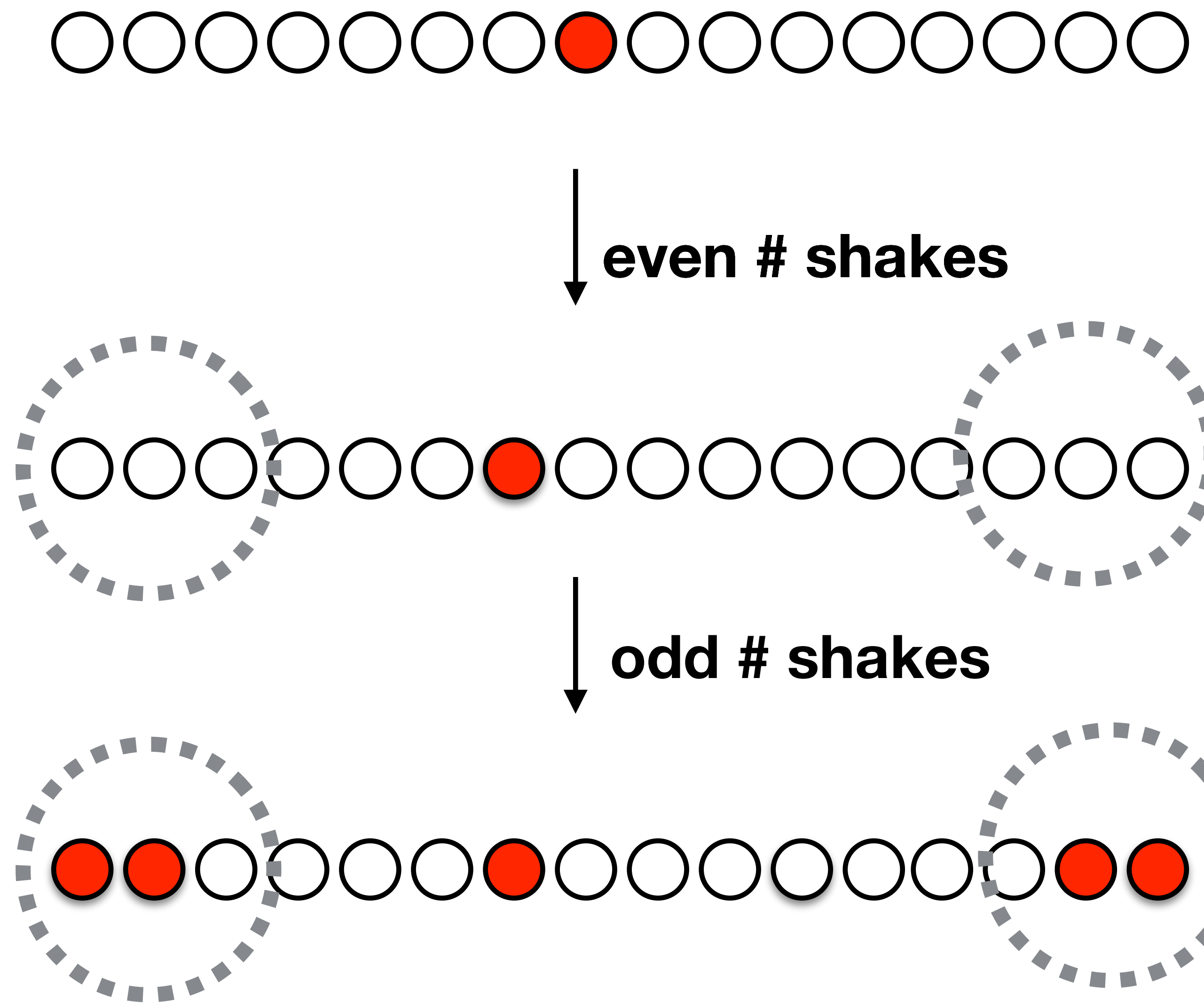

Quantum Time crystal
Proportion up
1
0.5
0
Proportion 'up' exhibits persistent oscillations
Initial conditions 'remembered'
shakes

Quantum Floquet Time crystal



Floquet SPT



Response frequency = shaking frequency/2

Today

- Present new classes of out-of-equilibrium quantum phases of matter: (1) Time crystals, and (2) Floquet symmetry protected topological phases.

- What I mean by ‘equilibrium’, ‘phase of matter’ and ‘out-of-equilibrium phase of matter’?

- A summary of more recent progress in the field.

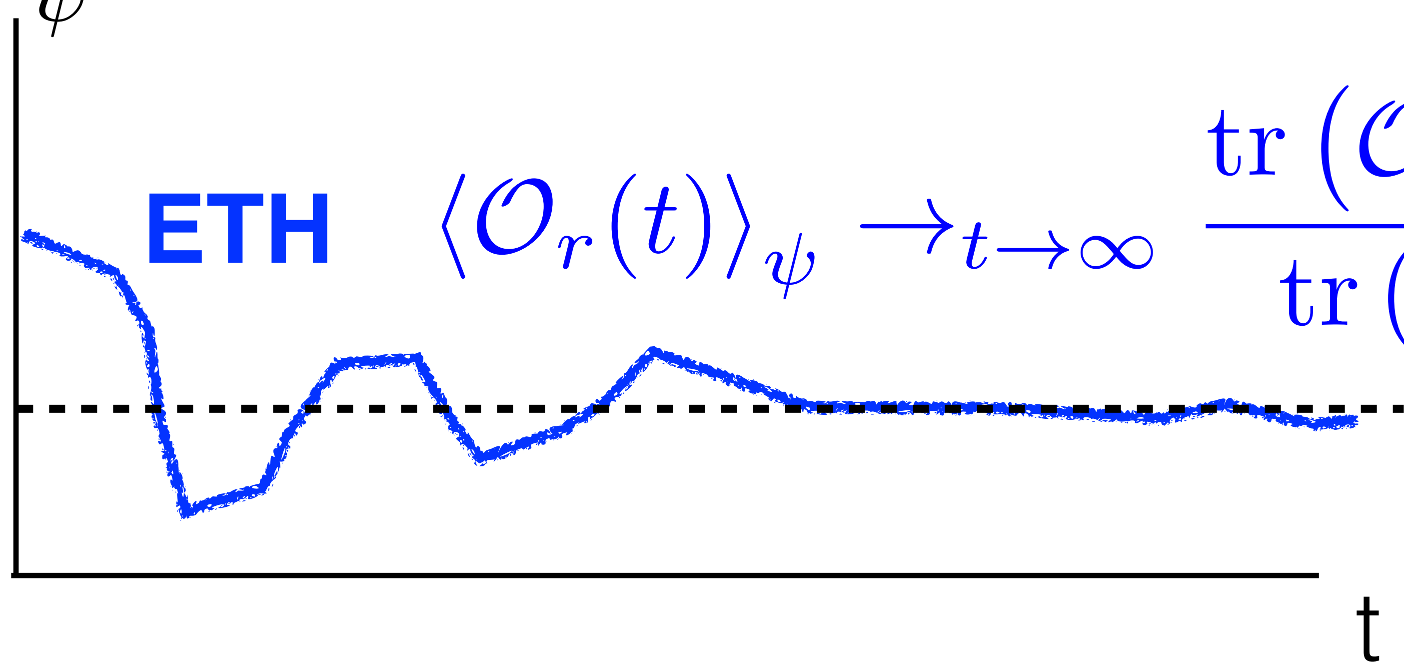
Equilibrating Quantum systems

- Old (incomplete) lore: Generic (interacting) many quantum systems thermalize/come to equilibrium.

Initial state: $|\psi\rangle$ $\text{Gen. local observable: } \mathcal{O}_r$

$\langle \mathcal{O}_r(t) \rangle_\psi$

ETH $\langle \mathcal{O}_r(t) \rangle_\psi \to_{t\to\infty} \frac{\mathrm{tr}\left(\mathcal{O}_r e^{-\beta_\psi H}\right)}{\mathrm{tr}\left(e^{-\beta_\psi H}\right)}$



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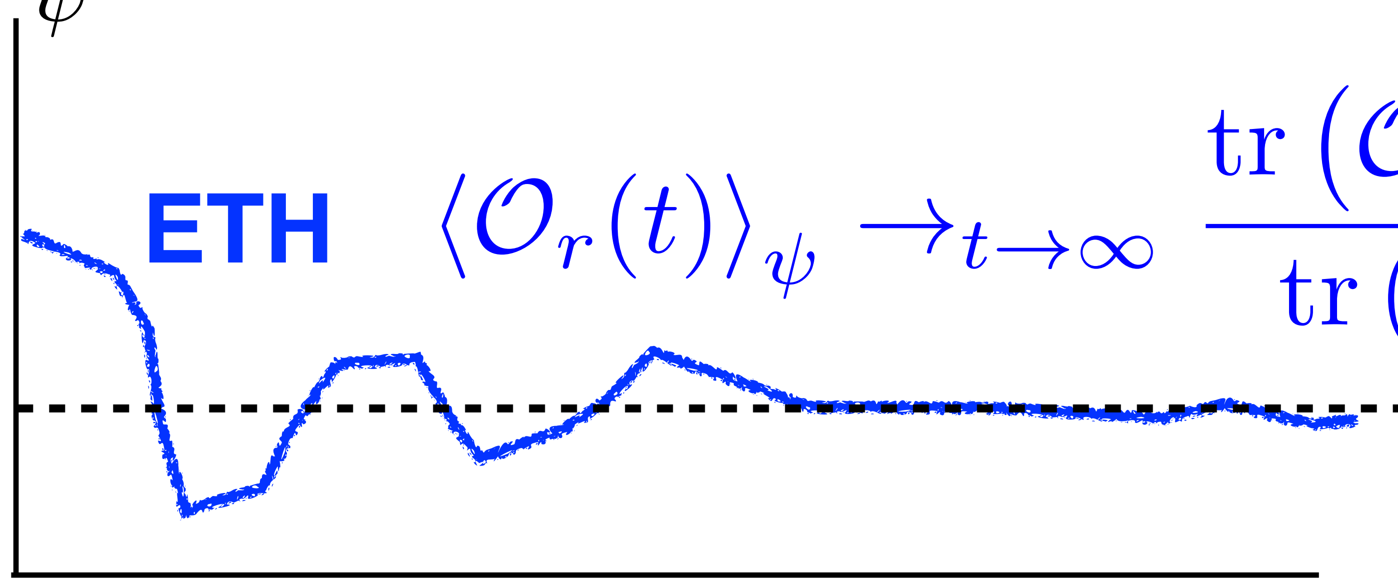


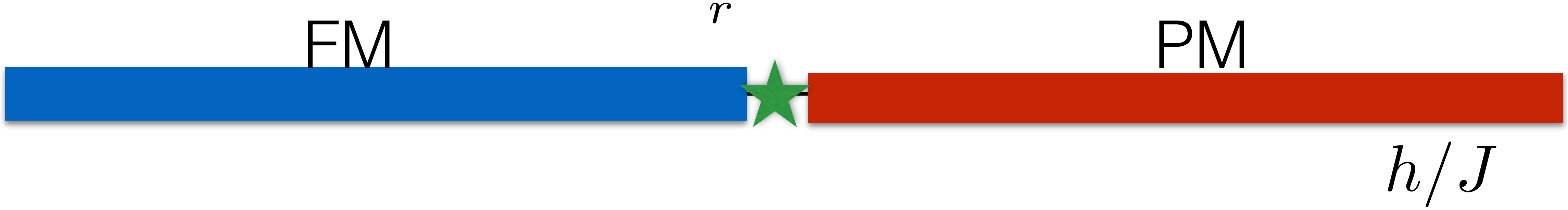
t

$$\langle H \rangle = \mathrm{tr}(e^{-\beta_\psi H} H)/\mathrm{tr}(e^{-\beta_\psi H})$$

Phases of Thermalizing Quantum systems

The Gibbs ensemble tells us about the long time behavior of systems that come to equilibrium. It therefore makes sense to characterize those systems in terms of the long distance correlation behavior of their Gibbs ensembles. This is how we define equilibrium phases of matter.

$$H_{\text{TFIM}} = -\sum_r J Z_r Z_{r+1} + h X_r$$

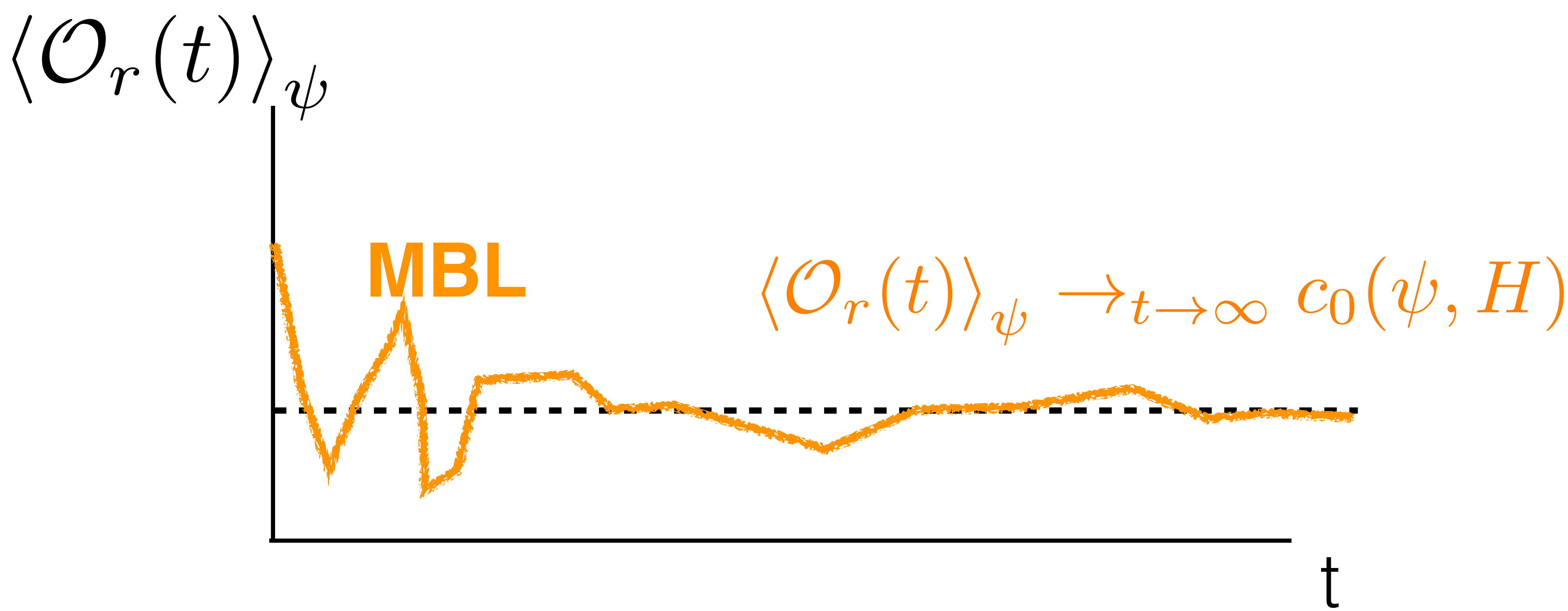


$$\langle Z_r Z_s \rangle_c^{\text{GS}} = O(1) \qquad \langle \mathcal{O}_r \mathcal{O}_s \rangle_c^{\text{GS}} = O(e^{-|r-s|/\xi})$$

MBL

We now know that some isolated quantum systems fail to thermalize. e.g., many body localised MBL systems. At long times, the observables in these phases are NOT determined by a Gibbs ensemble. MBL is a robust phenomenon, stable to sufficiently small perturbation to the underlying Hamiltonian.

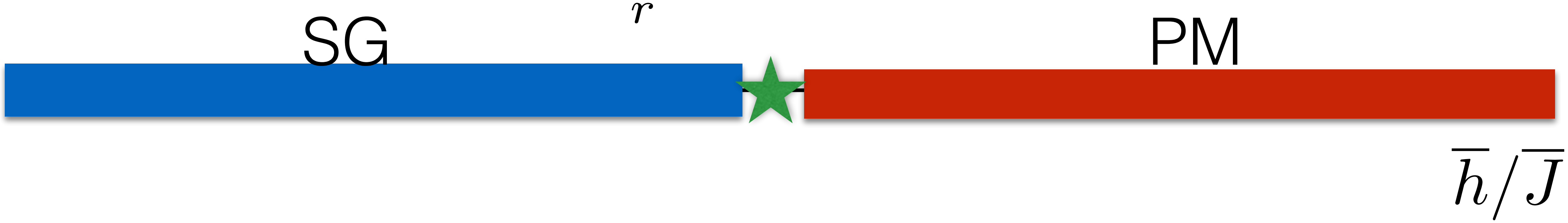
Initial state: $|\psi\rangle$ $\quad$ $\text{Gen. local observable: } \mathcal{O}_r$



MBL phases?

As MBL systems do not settle to a Gibbs ensemble, it does not make sense to characterise these phases by examining the correlation behaviour within their Gibbs ensembles. MBL systems can be thought of more fruitfully as 'eigenstate phases'. Within such a phase, all of the eigenstates of a many body Hamiltonian have a common quantum order (e.g., long range order, topological edge modes). This common quantum order determines the late time behaviour giving rise to **distinctive dynamical signatures**. (Huse et al '13; see also Pekker et al, Vosk et al, Chandran et al '14, Bahri et al '15)

$$H_{\mathrm{MBL}} = -\sum_r J_r Z_r Z_{r+1} + h_r X_r + \ldots$$



$$\langle Z_r(t \to \infty) Z_r(0) \rangle_\psi = O(1) \qquad \langle Z_r(t \to \infty) Z_r(0) \rangle_\psi = 0$$

Phases of matter

Both the equilibrium and MBL problems involve characterising the eigenstates of local time independent Hamiltonians with an aim towards characterizing long time behavior?

What new quantum phenomena/phases can arise for driven systems i.e., systems with time varying Hamiltonians?

!?!

Floquet systems in brief

- Floquet systems: Time periodic Hamiltonians H(t)=H(t+T). (Floquet symmetry or discrete time translation symmetry).

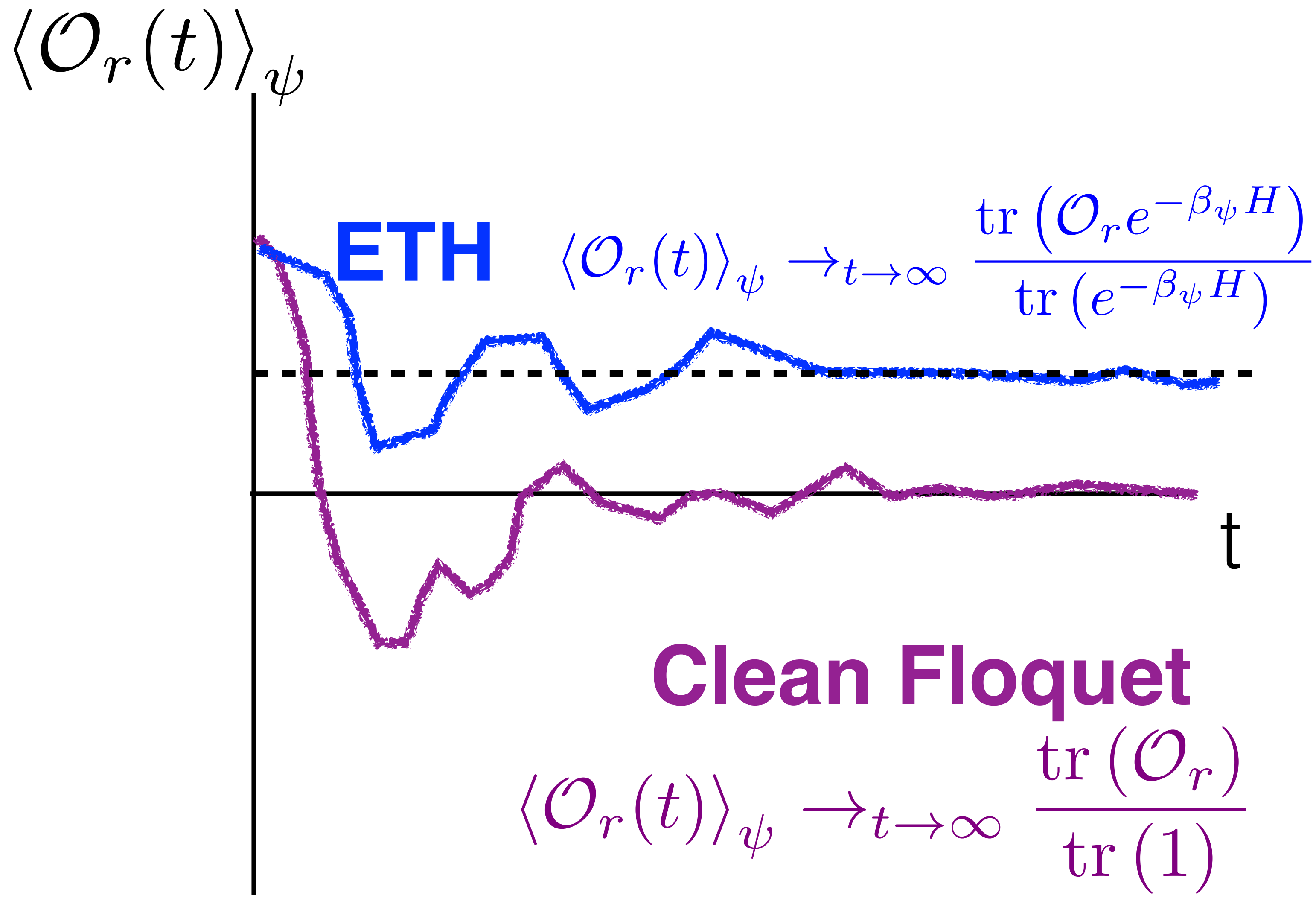
- ‘Floquet phase’ is characterized by the long time behavior of local observables. This in turn is determined by the eigenstate properties of

- $$U_f = \mathcal{T} e^{-i \int_0^T H(t') dt'}$$

What was expected in Floquet systems?

- **Clean interacting** Floquet systems generically thermalise to `infinite temperature'. Local observables exhibit trivial dynamics at long times. (Lazarides & Moessner '15, Abanin et al. '14,'15, D'Alessio & Rigol '15)

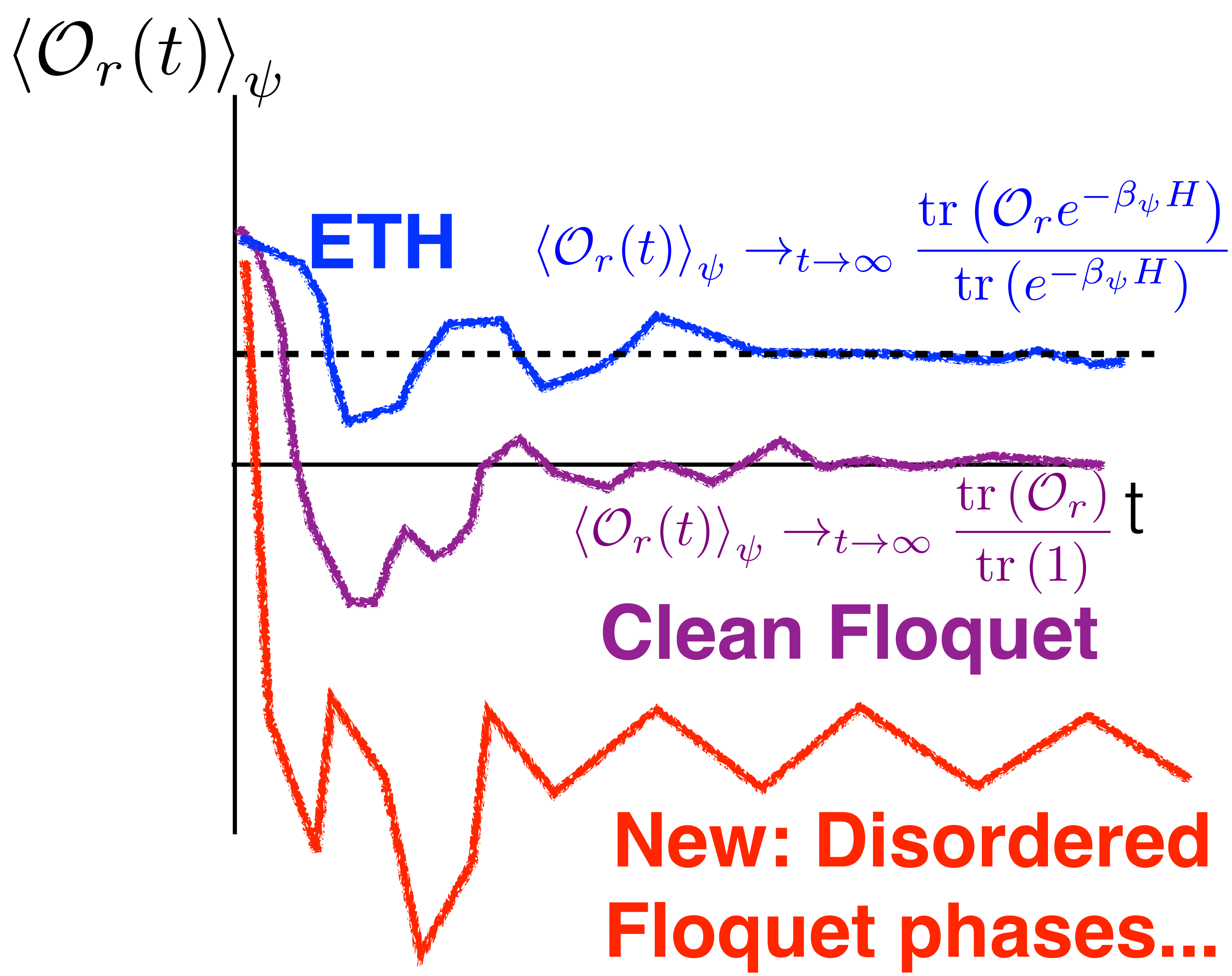
Initial state: $|\psi\rangle$



A surprise!

- In **disordered interacting** Floquet systems, this `trivial' long time behavior can be avoided e.g., observables can show persistent, large, **universal** oscillations at long times e.g., Time crystals and Floquet SPTs.

Initial state: $|\psi\rangle$



What we did

- There is a sharp definition of 'phase of matter' for periodically driven disordered systems encoded in the long time behavior of local observables. (Khemani et al, PRL 116, 250401 (2016))

- Our work consisted of discovering and classifying two classes of Floquet systems.

  1. Symmetry broken Floquet phases, also called time crystals. These phases are robust to any sufficiently small perturbation which preserves the Floquet period. Phase characterized by persistent stroboscopic oscillations in **bulk** observables.(von Keyserlingk + Sondhi PRB **93**, 245146 (2016))

  2. Floquet interacting SPTs. SPT phases protected by Floquet symmetry and some internal symmetry G (e.g., Ising symmetry). Characterized by persistent stroboscopic oscillations in **edge** observables, rather than **bulk** observables..(von Keyserlingk + Sondhi PRB **93**, 245145 (2016))

- In both cases, the new ingredient is that these systems fail to settle at long times (and they spontaneously break $t \rightarrow t+T$ symmetry).

$\mathbb{Z}_2$ Time crystal/πSG in detail

A non-time crystal:

Consider spin-1/2 degrees of freedom in a 1D chain.

It is easy to engineer a time dependent Hamiltonian H(t) with period T, such that.

$$U_{\mathrm{f}} = R^{\hat{x}}_{\pi-\epsilon}$$

$$\langle Z_r(nT)\rangle_\psi = \cos((\pi-\epsilon)n)\,\langle Z_r\rangle_\psi + \sin((\pi-\epsilon)n)\,\langle Y_r\rangle_\psi$$

Bulk observables will precess with frequency ωT=**π**±ϵ. However, this behavior is unstable to small changes to **H(t)→H(t)+V(t)**. For generic clean local V(t), periodic or otherwise, bulk observables are expected to thermalize (to infinite temperature). No late time oscillations.

$$\langle \mathcal{O}_r(t)\rangle_\psi \rightarrow_{t\rightarrow\infty} \frac{\mathrm{tr}\left(\mathcal{O}_r\right)}{\mathrm{tr}\left(1\right)}$$

$\mathbb{Z}_2$ Time crystal/πSG

However, by adding spin-spin interactions and disorder one can to engineer a time dependent Hamiltonian $H_{SG}(t)$ which gives a Floquet evolution exhibiting 2T oscillations. e.g.,

$$H_{\pi\mathrm{SG}}(t) = \begin{cases} \sum_r J_{rs} Z_r Z_s + \sum_r h_r Z_r & 0 < t < t_1 \\ \sum_r X_r & t_1 < t < T = t_1 + \frac{\pi}{2} \end{cases}$$

$$U_{\mathrm{f},\pi\mathrm{SG}} = R_\pi^{\hat{x}} e^{-it_1 \sum_r J_{rs} Z_r Z_s - it_1 \sum_r h_r Z_r}$$

Robustness

$$H_{\pi \mathrm{SG}}(t) = \begin{cases} \sum_r J_{rs} Z_r Z_s + \sum_r h_r Z_r & 0 < t < t_1 \\ \sum_r X_r & t_1 < t < T = t_1 + \frac{\pi}{2} \end{cases}$$

$$U_{\mathrm{f},\pi \mathrm{SG}} = R_{\pi}^{\hat{x}} e^{-it_1 \sum_r J_{rs} Z_r Z_s - it_1 \sum_r h_r Z_r}$$

The resulting long time 2T periodicity is **robust** to sufficiently small changes $H_{SG}(t) \rightarrow H_{SG}(t) + V(t)$ **which preserve the Floquet period**. In particular, for sufficiently small epsilon the following will also yield 2T periodic oscillations at late times.

$$U_{\mathrm{f}} = R_{\pi-\epsilon}^{\hat{x}} e^{-it_1 \sum_r J_{rs} Z_r Z_s - it_1 \sum_r h_r Z_r}$$

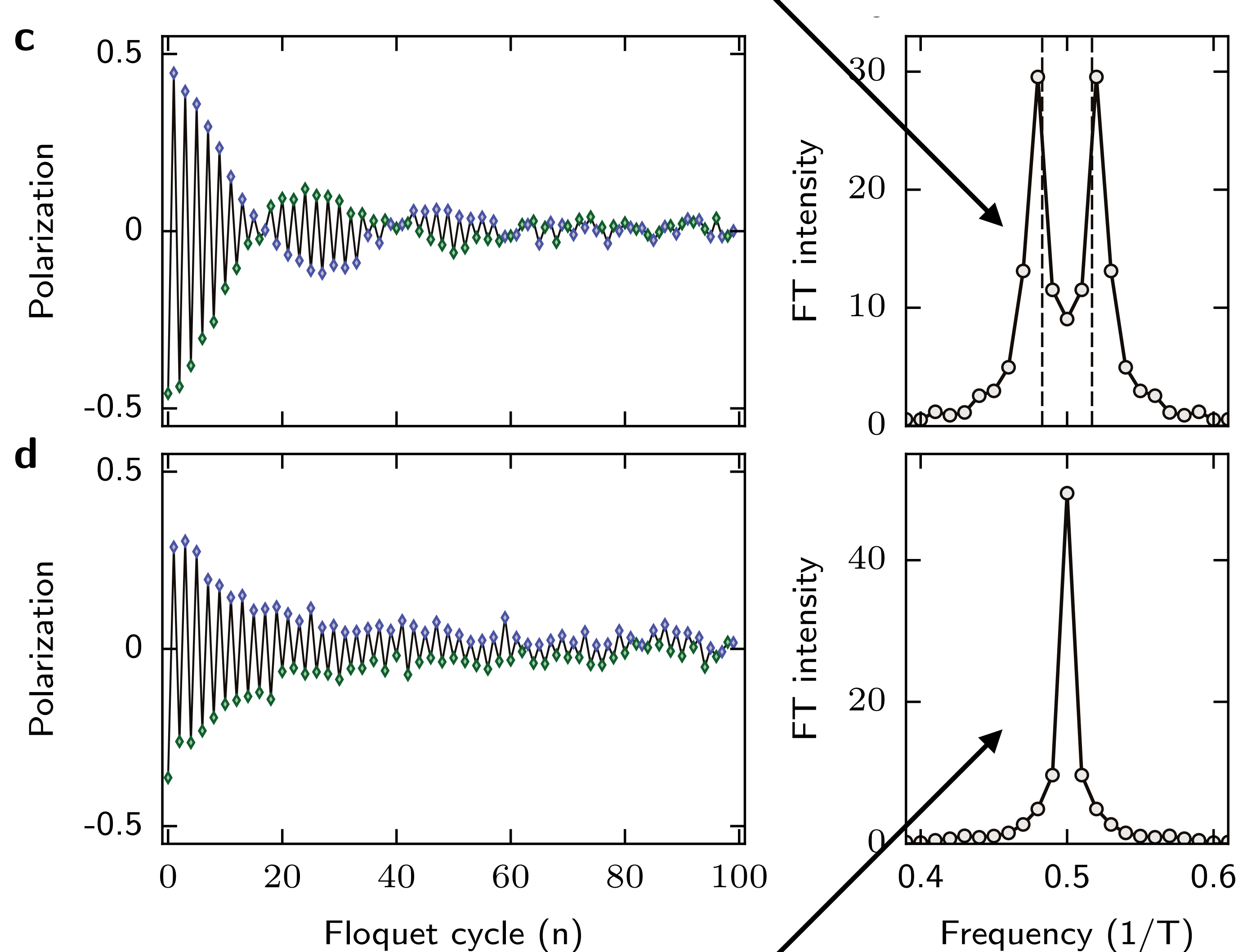
Protocol (simplified)

- Prepare an initial pure product state. Plot discrete fourier transform of the spin magnetization.

- Start by just rotating the spins by **π-ϵ every period**. Fix epsilon small (~0.05) and nonzero. The unitary rotates the order parameter by angle **π-ϵ** every period T. **Gives two peaks at ωT=$\pi\pm\epsilon$. Not in putative time crystal phase.**

- Switching on the interactions and disorder, we enter the time crystal phase. **In the thermodynamic limit, get a single robust peak at exactly ωT=π.**

$$U_{\mathrm{f}} = R^{\hat{x}}_{\pi-\epsilon}$$



$$U_{\mathrm{f}} = R^{\hat{x}}_{\pi-\epsilon} e^{-it_1 H_{\mathrm{int}}}$$

Experiments

- At least three experiments on quantum time crystals.
- Lukin Group Harvard: Room temperature, diamond NV^- centre setup. (Choi et al Nature 543, 221 (2017)).
- Monroe Group Maryland: Trapped ion setup (10 effective Ising spins). (Zhang et al Nature 543, 217 (2017))
- Barrett group Yale: ^{31}P nuclear spins in $NH_4H_2PO_4$ (Rovny et al Phys. Rev. Lett. 120, 180603 (2018))
- Roughly the same protocol in all experiments.
- None show *definitive* evidence of the phenomenon, as originally formulated. Although all very interesting and technically impressive experiments.

Experimental issues

1D experiment *possibly* localized in principle, but not in many body limit. (~10 ions)

3D experiments aren't in MBL phase.

Experiments aren't able to probe `the infinitely long time limit'; they are limited by the system-environment coupling.

It is possible, perhaps likely, that something else is being observed in the 3D experiments. Options: '**Critical time crystals**', "**Prethermal time crystal**." (Else et al Phys. Rev. X 7, 011026 (2017)).

Prethermal time crystals?

$$D = \sum J_{rs} Z_r Z_s + \dots$$

$$U_{\mathrm{f}} = R_{x,\pi-\epsilon} e^{-i \sum J_{rs} Z_r Z_s}$$

$$U_{\mathrm{f}}^n \approx R_{x,\pi}^n e^{-inD}$$

- Prethermal time crystals show oscillations which are persistent for a finite but surprisingly $(t_m \sim e^{c/\max(\epsilon,J)})$ long time. Disorder/MBL unnecessary. System eventually heats up.
- If D has a ferromagnetic ordering transition, and initial state is sufficiently cold, then magnetization has stable oscillations for $t<t_m$. (Else et al Phys. Rev. X 7, 011026 (2017), also 1708.01620).
- In Lukin experiment, initial state probably too hot for prethermalization mechanism to hold. Although initial state dependence of those results not systematically studied.

Quantum Time crystals in open systems

- Prethermal time crystals heat up eventually. Perhaps one can absorb the excess energy with a cold external bath. Perhaps can stabilize time crystal `forever'? (Else et al Phys. Rev. X 7, 011026 (2017)).

- **This is a largely unexplored direction. Verify mechanism in concrete model? More generally, how do we define phases and their stability in open driven dissipative systems? Connections to classifications of Lindbladians (Michalakis,Eisert…)?**

What is a critical time crystal?

$$U_f = R^{\hat{x}}_{\pi-\epsilon} e^{-it_1 \sum_r J_{rs} Z_r Z_s - it_1 \sum_r h_r Z_r}$$

- In Harvard system, interactions are naturally disordered and decay as $1/r^3$.
- There is no robust prethermal plateau. Resonance counting arguments indicate magnitude of magnetization initially decays surprisingly slowly, first as a power law in time. Eventually signal decays with finite rate.(1703.04593, Ho, Abanin, Choi, Lukin)
- Consistent with experiment.

Classical time crystals?

- Periodically drive an isolated (i.e., no dissipation) classical system. Expect ergodicity long times — the system heats up and explores all of phase space.

- Ergodicity incompatible with a long time persistent oscillatory behavior described in this talk.

- There is a possibility that classical time crystals exist in driven dissipative systems, with temperature activated decay of TC signal. (Yao et al arXiv:1801.02628)

Conclusion

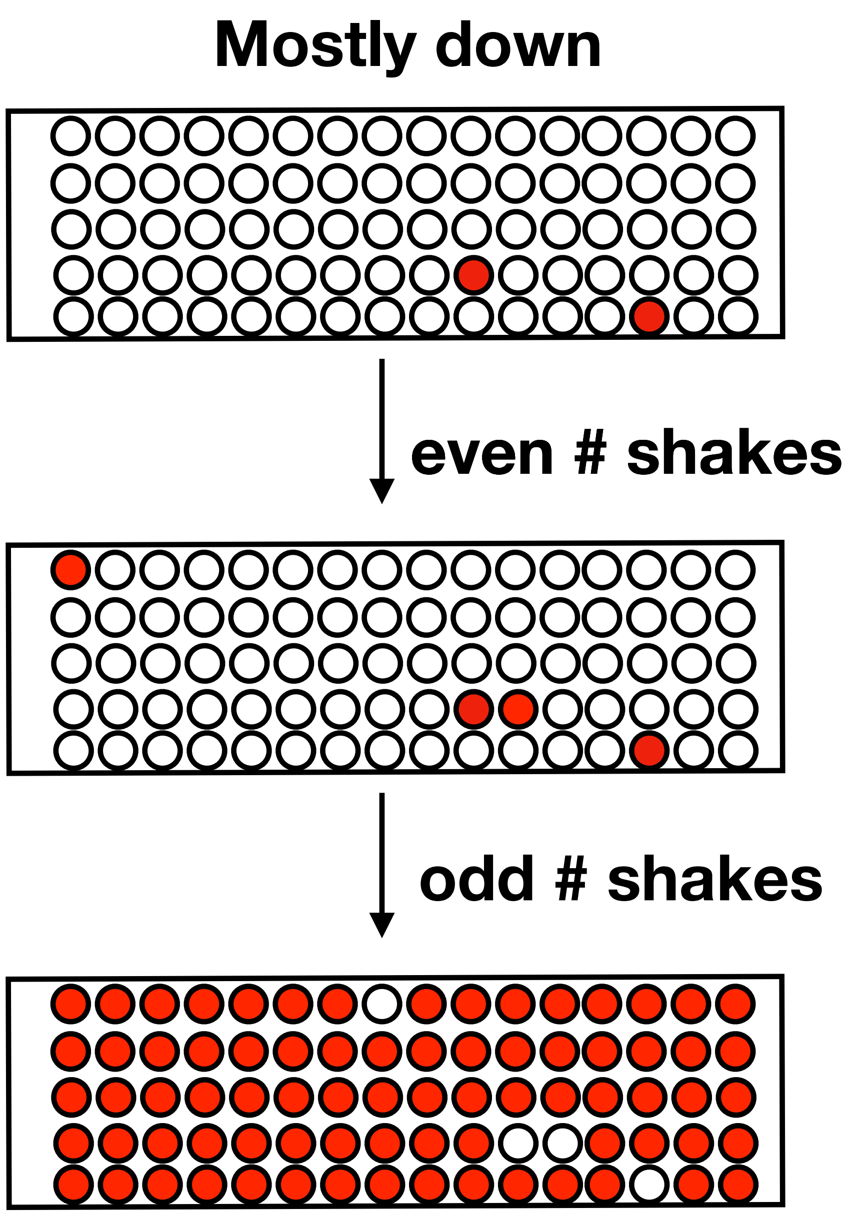
- Classical many body systems: If **Isolated** + **driven** they heat up, and equilibrate. It was thought the same should hold for analogous driven quantum systems.

- (MBL) Time crystals are many body systems which are **Isolated+driven** but: They don't heat up. They show persistent robust oscillations at late times.

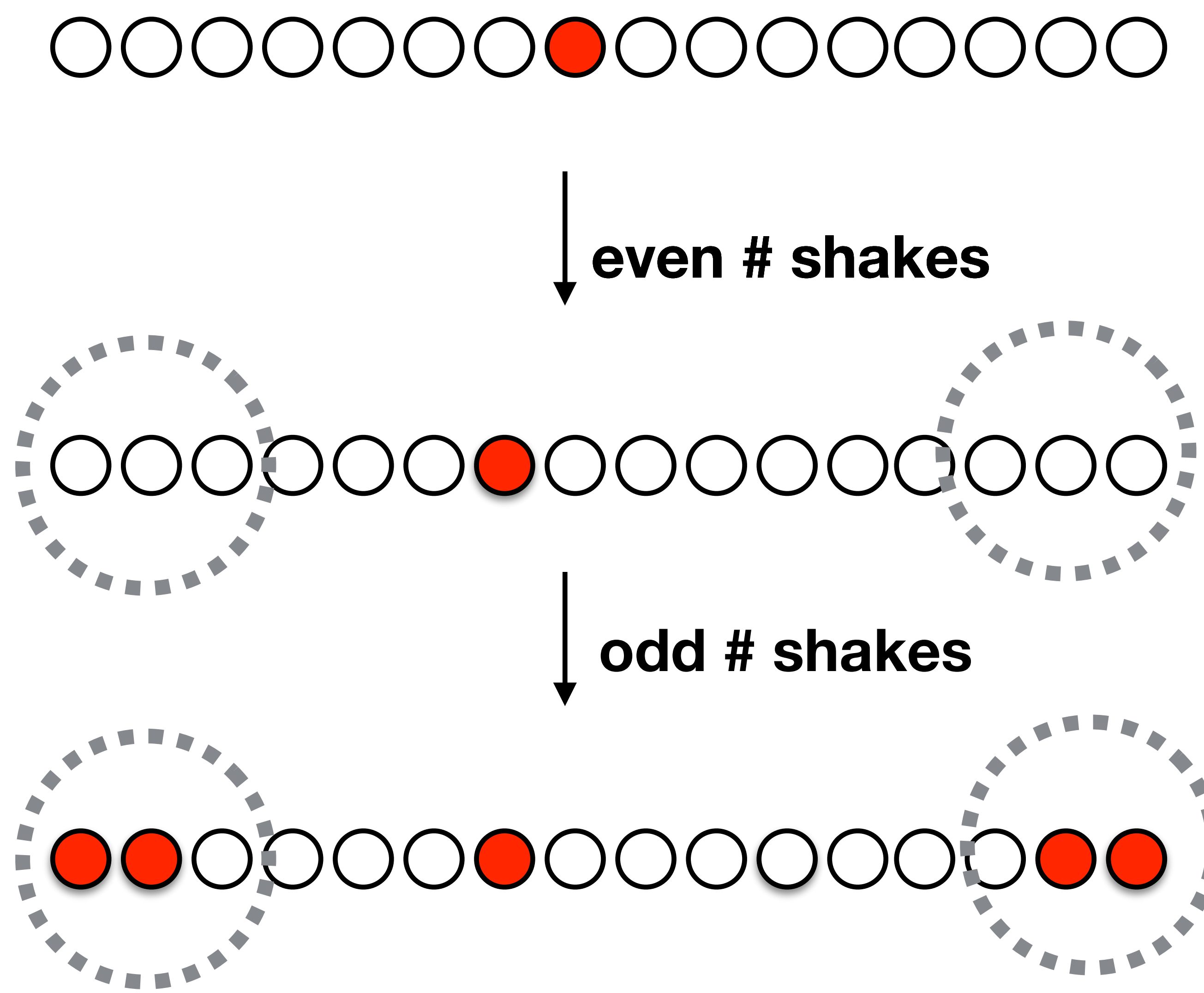
- Experiments: Main limitation is system-environment decoherence.

- Main unanswered questions: Generalize notion of time crystals and their stability to open quantum systems.

Quantum Floquet Time crystal



Floquet SPT



Response frequency = shaking frequency/2

Thanks!

Collaborators: Vedika Khemani (Harvard), Shivaji Sondhi (Princeton).

Support: This work was supported by the Princeton Centre for Theoretical Physics and a Birmingham Fellowship.

Other groups:

Original idea: Wilczek.

See also: Krzysztof Sacha

Station Q: **Dominic Else**, Bauer, Nayak

UCLA: **Fenner Harper**, Rahul Roy.

Berkeley diaspora: **Drew Potter**, **Haruki Watanabe**, Vishwanath

Various prethermalization results: Abanin, **Ho**, Propotov, **Choi**, Lukin, Huveneers, de Roeck. Mori.