Topological classification of quasi-periodically driven quantum systems

arXiv: 1808.07884

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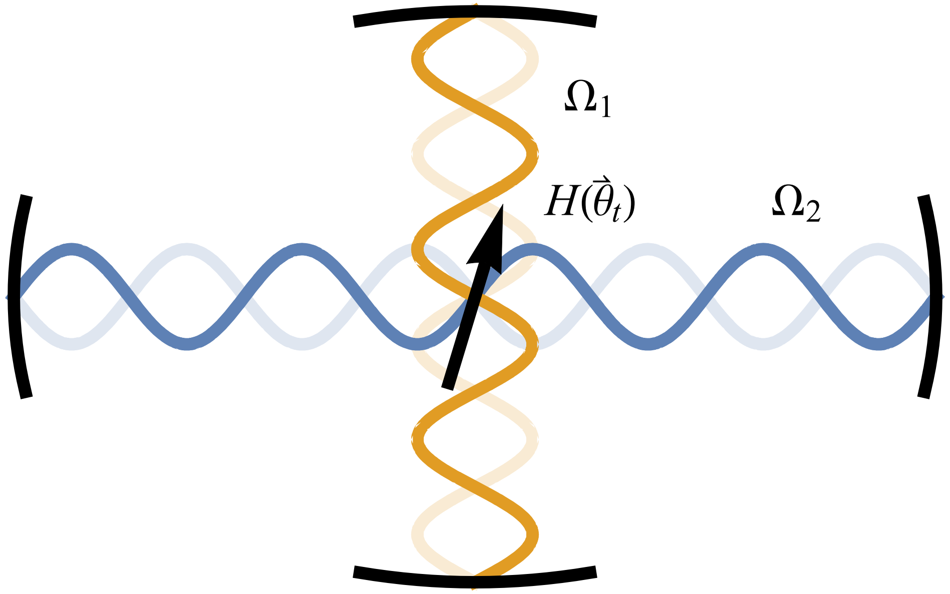
BOSTON UNIVERSITY

Quasi-periodically driven systems

What is quasi-periodic driving?

- Two incommensurate classical drives
- Quasiperiodicity
  - Repetitive structure
  - Sharp Fourier peaks
  - No time-translational symmetry
  - No Bloch-Floquet theorem



$$H(\Omega_1 t + \theta_{01}, \Omega_2 t + \theta_{02})$$

Quasi-periodically driven ‘qudit’

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Quasi-periodically driven systems

What is quasi-periodic driving?

- Two incommensurate classical drives
- Quasiperiodicity
  - Repetitive structure
  - Sharp Fourier peaks
  - No time-translational symmetry
  - No Bloch-Floquet theorem

$$H = \vec{B}(t) \cdot \vec{\sigma}$$

$$\vec{B}(t) = \begin{pmatrix} \sin(\Omega_1 t + \theta_{01}) \\ \sin(\Omega_2 t + \theta_{02}) \\ m - \cos(\Omega_1 t + \theta_{01}) - \cos(\Omega_2 t + \theta_{02}) \end{pmatrix}$$

Quasi-periodically driven 'qudit'

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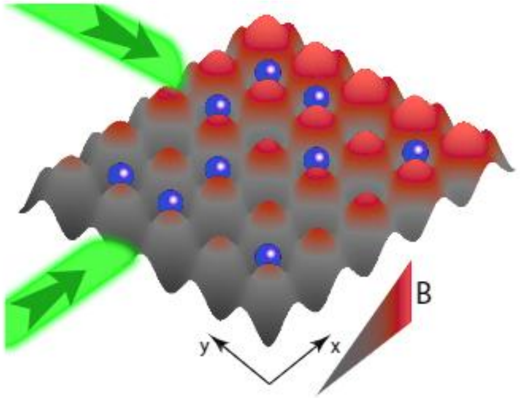
Motivation

Why look at quasi-periodically drive quantum systems?

- Driven quantum systems
  - Response functions
  - Hamiltonian engineering

$$x(t) = \int_{-\infty}^{t} \mathrm{d}t' \, \chi(t - t') h(t')$$



Bukov et al. AIP, 64, 2, 139-226

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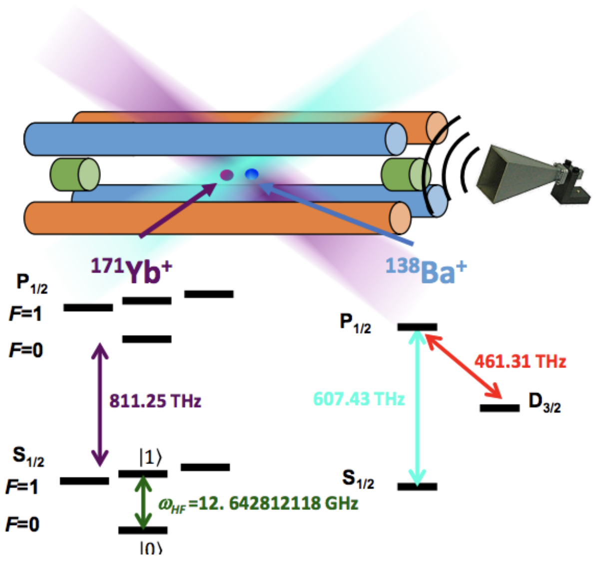
Motivation

Why look at quasi-periodically drive quantum systems?

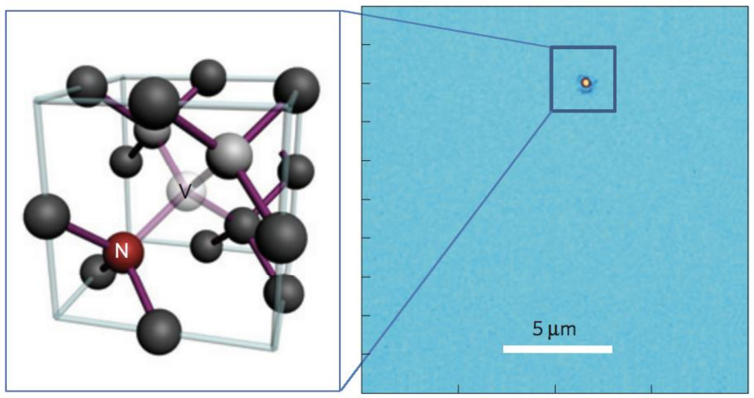
- Driven quantum systems
  - Response functions
  - Hamiltonian engineering
  - Long time dynamics

Trapped ions



Wang et al.
Nat. Photonics 11, 646-650 (2017)

N-V centres



Balasubramanian et al.
Nat. Mater. 8, 383–387 (2009)

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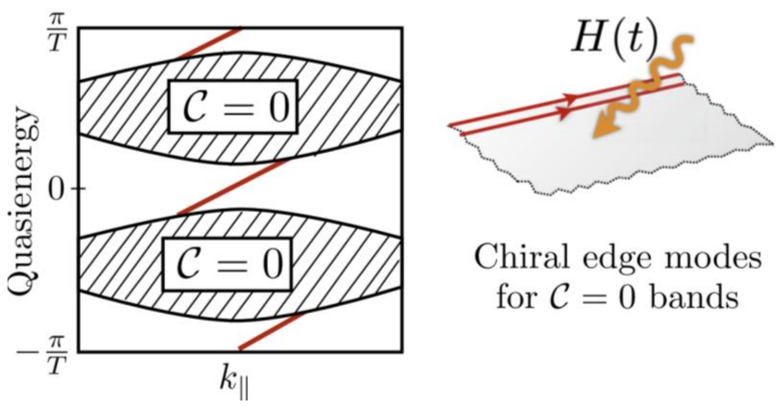
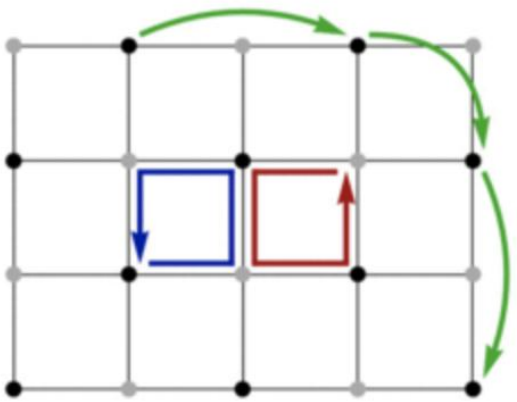
Motivation

Why look at quasi-periodically drive quantum systems?

- Driven quantum systems
  - Response functions
  - Hamiltonian engineering
  - Long time dynamics
  - Floquet Topological Order



Rudner et al. PRX **3**, 031005



Harper and Roy, PRL **118**, 115301

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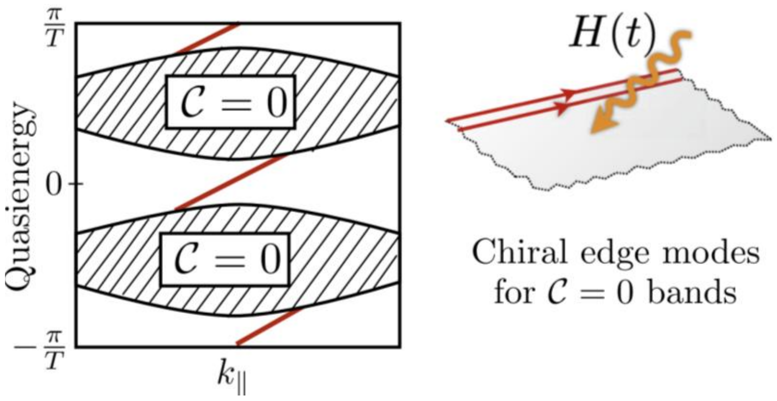
Motivation

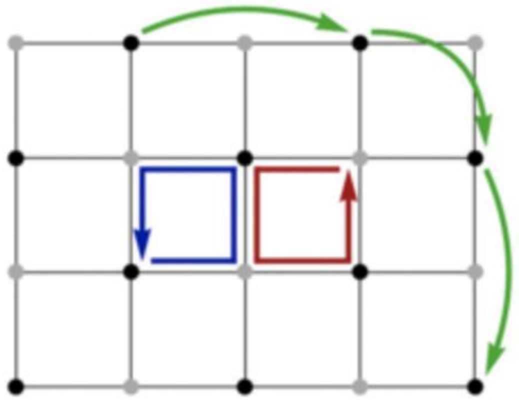
Why look at quasi-periodically drive quantum systems?

- Distinct classifications only in extended spatial dimensions

- QP systems – "synthetic" spatial dimensions



Rudner et al. PRX **3**, 031005



Harper and Roy, PRL **118**, 115301

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Motivation

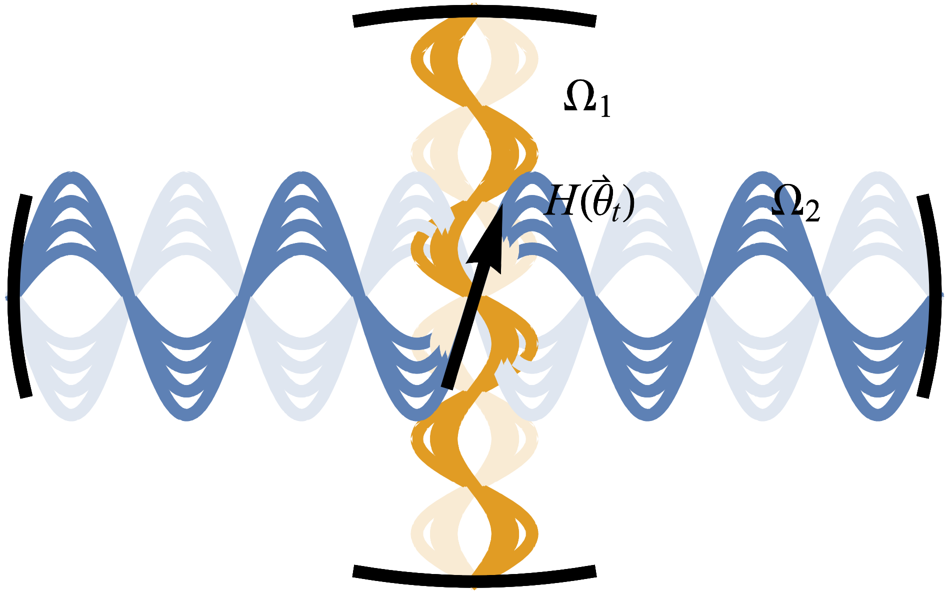
Why look at quasi-periodically drive quantum systems?

- What happens beyond Floquet?

- Martin, Refael, and Halperin showed energy pumping of topological origin. [PRX 7, 041008]

$$|\psi(t)\rangle = \sum_{\epsilon} \alpha_\epsilon e^{-i\epsilon t} |\phi^\epsilon(\Omega t + \theta_0)\rangle.$$

Bloch-Floquet theorem



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Questions

- What is the full classification of generic QP driven systems?
- What physics does this classification control?
- More exotic physics with different H? Or more levels? Or fast driving?
- What is the stability of these classifications?

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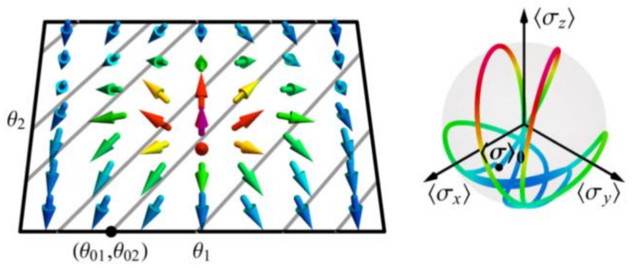
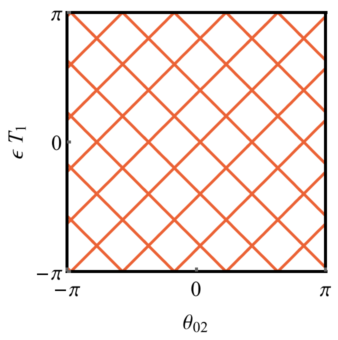
Main message

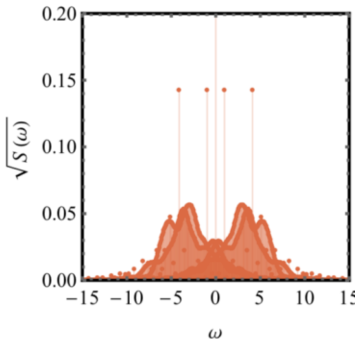
If you only remember one (three) thing(s)…

① Topology → ② Bandstructure → ③ Dynamics

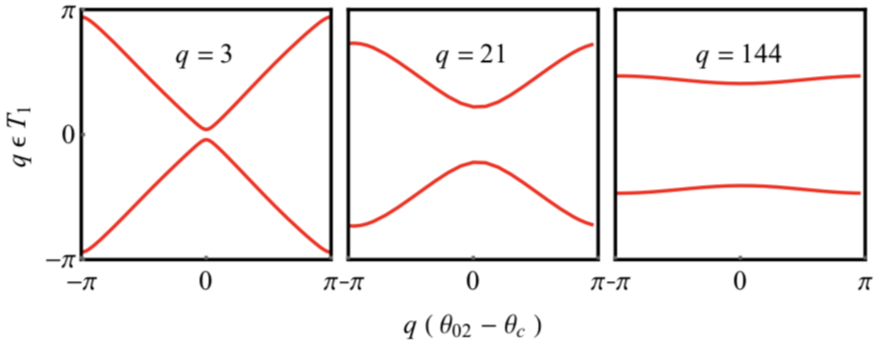
$C \in \mathbb{Z}$







Stability and realisation



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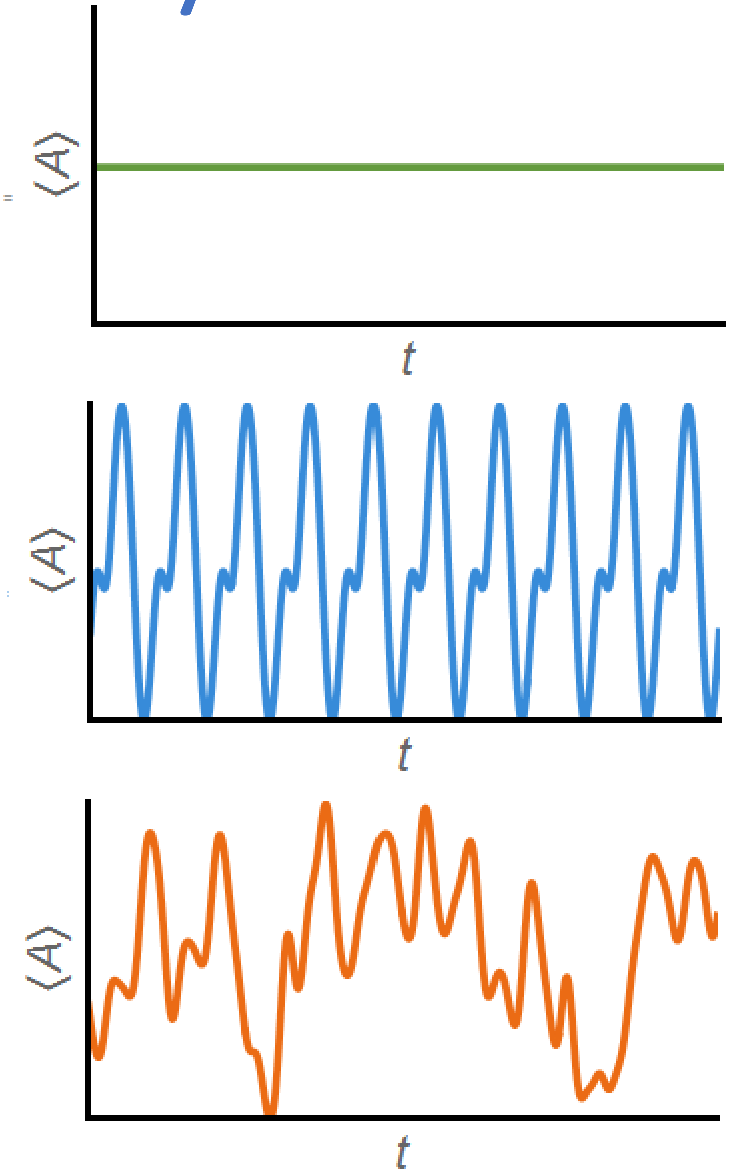
Why is two tones different?



Static system

$$|\psi(t)\rangle = \mathrm{e}^{-i\epsilon t}|\phi^{\epsilon}\rangle$$

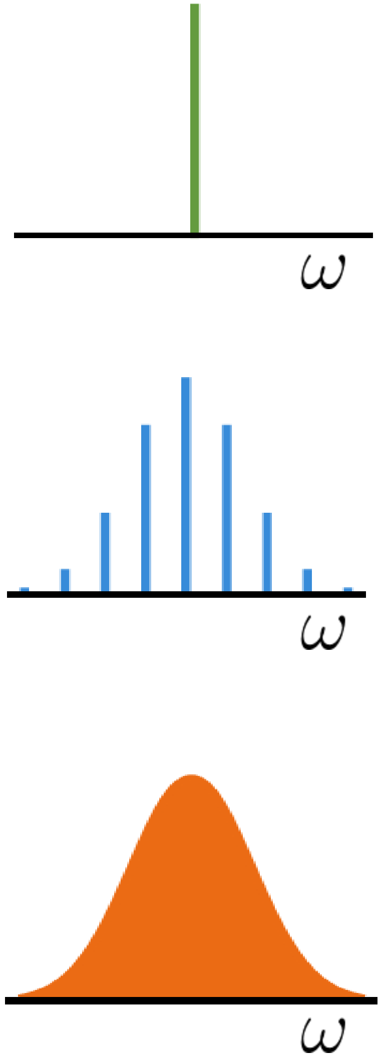
Periodic Hamiltonian

$$|\psi(t)\rangle = \mathrm{e}^{-i\epsilon t}\sum_{n}\mathrm{e}^{-in\Omega t}|\phi^{\epsilon}_{n}\rangle$$

Quasiperiodic Hamiltonian

$$|\psi(t)\rangle = \mathrm{e}^{-i\epsilon t}\sum_{\vec{n}}\mathrm{e}^{-i\vec{n}\cdot\vec{\Omega}t}|\phi^{\epsilon}_{\vec{n}}\rangle$$

Fourier spectra



Synthetic dimensions: The frequency lattice

A lattice model in synthetic dimensions

Fourier representation of the Hamiltonian

$$H(\Omega_1 t + \theta_{01}, \Omega_2 t + \theta_{02}) = \sum_{\vec{n}} H_{\vec{n}} \mathrm{e}^{-i\vec{n}\cdot(\vec{\Omega}t+\vec{\theta}_0)}.$$

Substitute both into the time-dependent Schrödinger equation $i\partial_t|\psi\rangle = H|\psi\rangle$

$$\epsilon|\tilde{\phi}^{\epsilon}_{\vec{n}}\rangle = \sum_{\vec{m}} \left(H_{\vec{n}-\vec{m}} \mathrm{e}^{-i(\vec{n}-\vec{m})\cdot\vec{\theta}_0} - \vec{n}\cdot\vec{\Omega}\delta_{\vec{n}\vec{m}} \right) |\tilde{\phi}^{\epsilon}_{\vec{m}}\rangle$$

$\vec{n}$

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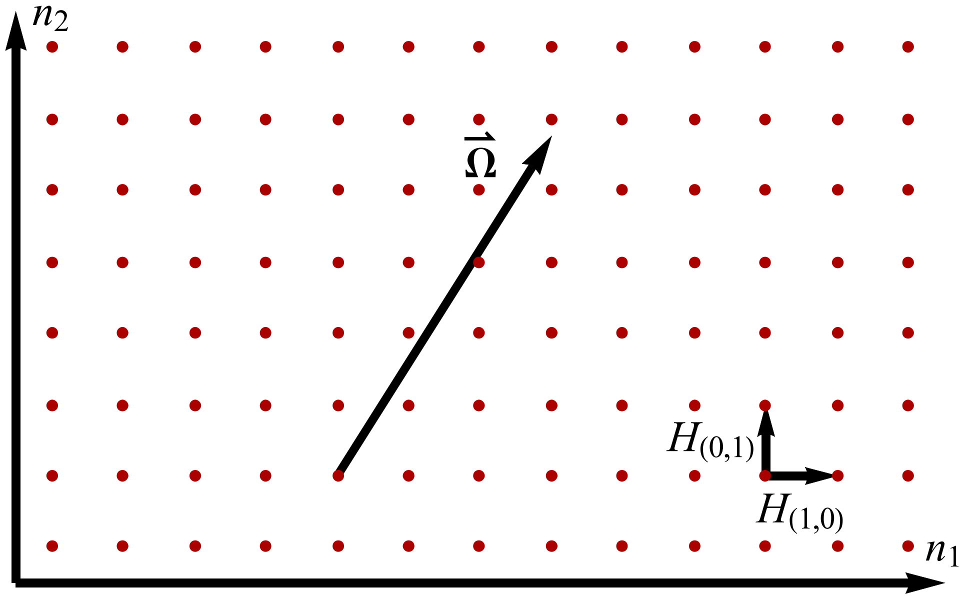
Synthetic dimensions: The frequency lattice

A lattice model in synthetic dimensions

	Time domain	Frequency domain
$H_{\vec{0}}$	Time averaged Hamiltonian	On-site potential
$H_{\vec{m}}$	Fourier component of Hamiltonian	Hop by vector $\vec{m}$
$\lvert\tilde{\phi}^{\epsilon}_{\vec{n}}\rangle$	Fourier component of quasi-energy state	Quasi-energy state projected onto lattice site
$\vec{\Omega}$	Drive frequencies	Electric field
$\vec{\theta}_0$	Initial drive phase angles	Magnetic vector potential



$$\epsilon\lvert\tilde{\phi}^{\epsilon}_{\vec{n}}\rangle = \sum_{\vec{m}} \left(H_{\vec{n}-\vec{m}} e^{-i(\vec{n}-\vec{m})\cdot\vec{\theta}_0} - \vec{n}\cdot\vec{\Omega}\delta_{\vec{n}\vec{m}} \right) \lvert\tilde{\phi}^{\epsilon}_{\vec{m}}\rangle$$

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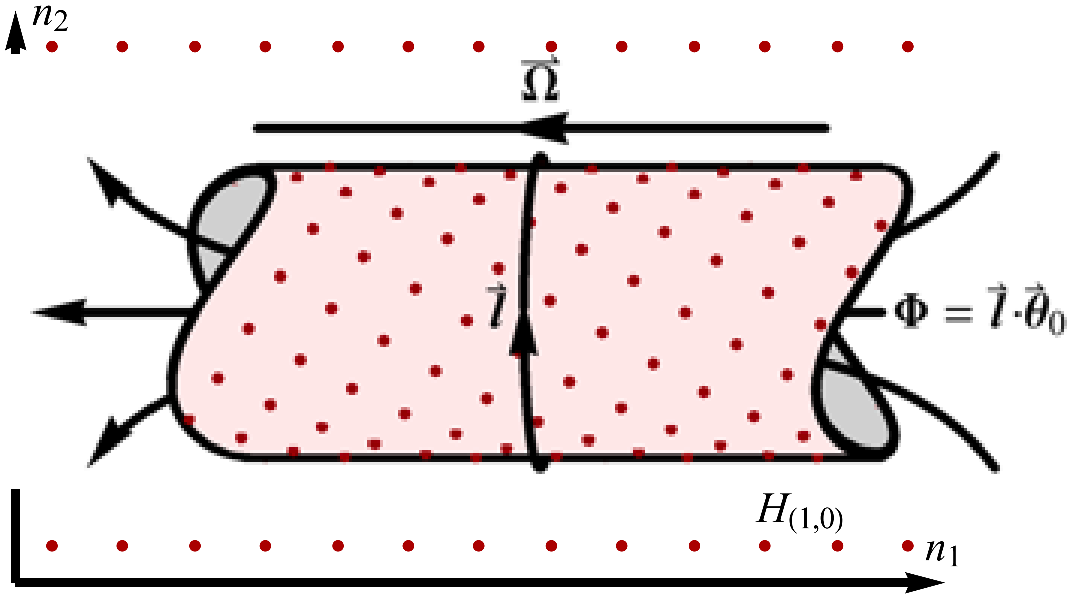
Synthetic dimensions: The frequency lattice

A lattice model in synthetic dimensions

$$\Omega_2/\Omega_1 = p/q$$

$$\Omega_2 q = \Omega_1 p$$

$$\vec{l} = (-p, q)$$



$$\epsilon|\tilde{\phi}^{\epsilon}_{\vec{n}}\rangle = \sum_{\vec{m}} \left(H_{\vec{n}-\vec{m}} e^{-i(\vec{n}-\vec{m})\cdot\vec{\theta}_0} - \vec{n} \cdot \vec{\Omega}\delta_{\vec{n}\vec{m}} \right) |\tilde{\phi}^{\epsilon}_{\vec{m}}\rangle$$

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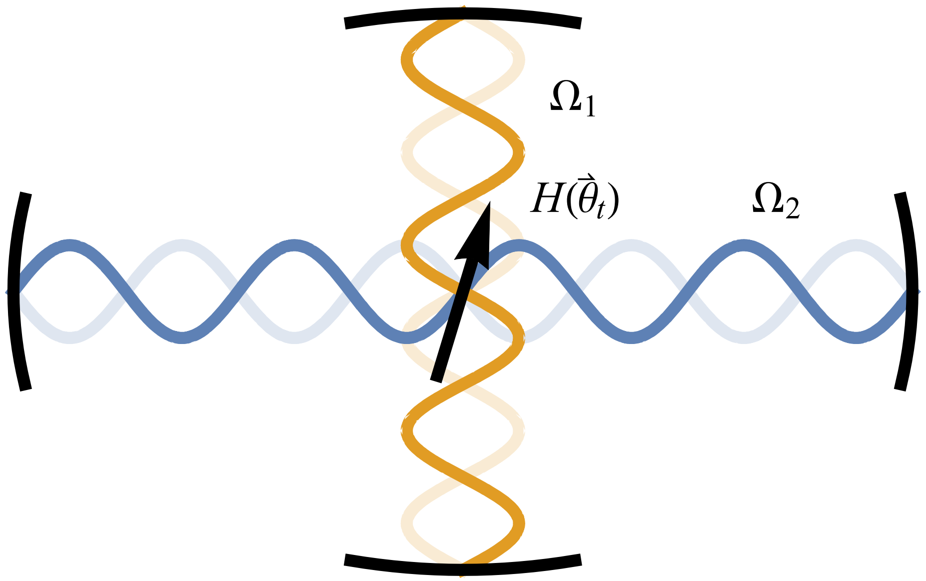
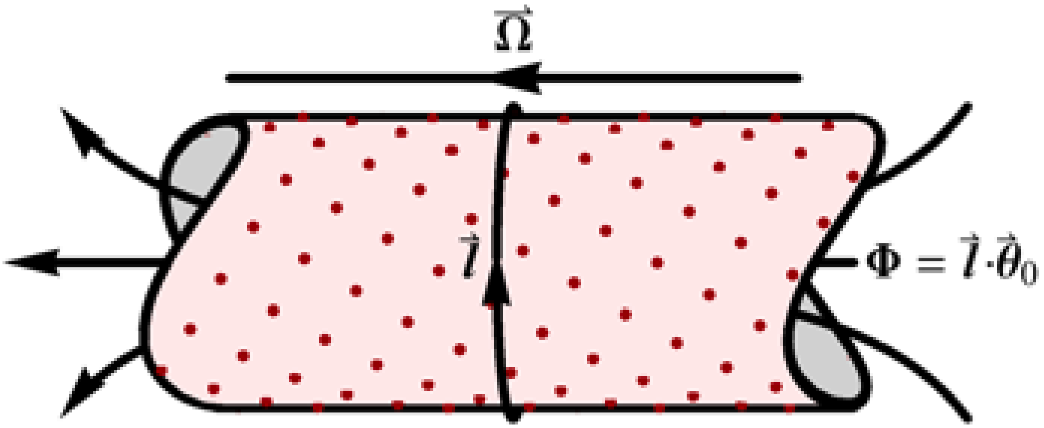
Synthetic dimensions: The frequency lattice

A lattice model in synthetic dimensions



N-level system driven by 2-irrationally related drive tones



N-band translationally invariant hopping model in 2-dimensions

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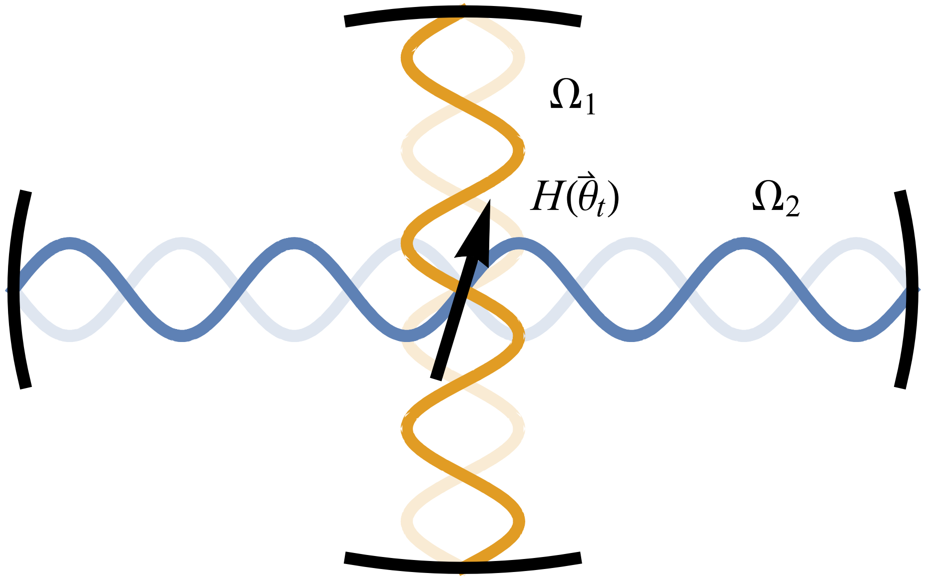
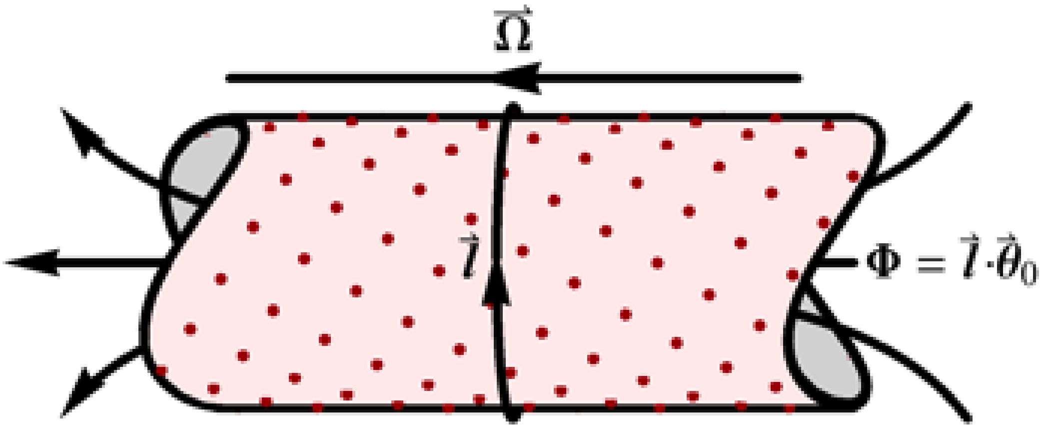
Synthetic dimensions: The frequency lattice

A lattice model in synthetic dimensions



N-level system driven by ***D***-irrationally related drive tones



N-band translationally invariant hopping model in ***D***-dimensions

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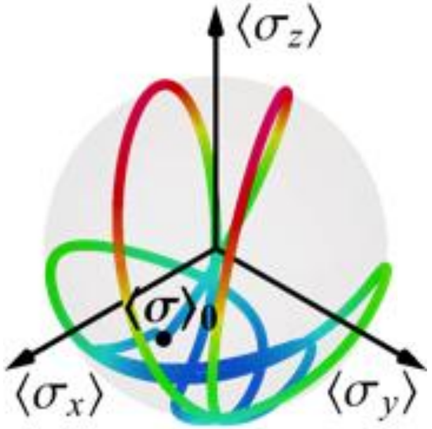
Topological Classification

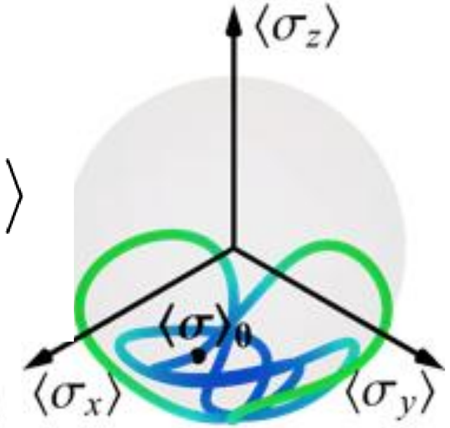
Of quasi-energy-states

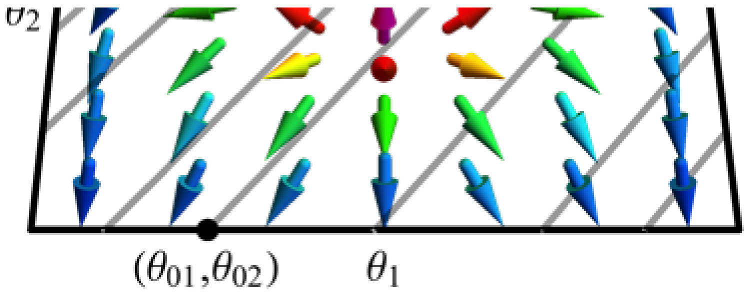
$$|\phi^{\epsilon}(\vec{\Omega}t+\vec{\theta}_0)\rangle = \sum_{\vec{n}} \mathrm{e}^{-i\vec{n}\cdot\vec{\Omega}t}|\tilde{\phi}^{\epsilon}_{\vec{n}}\rangle$$

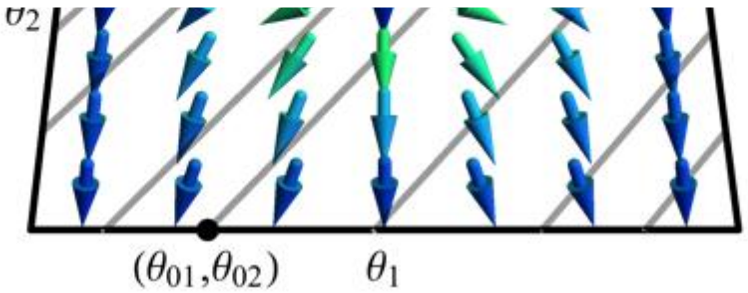
$$C = \frac{1}{2\pi \mathrm{i}}\int_{\mathrm{FZ}} \mathrm{d}^2\theta \left[\langle \partial_{\theta_2}\phi^{\epsilon}|\partial_{\theta_1}\phi^{\epsilon}\rangle - \langle \partial_{\theta_1}\phi^{\epsilon}|\partial_{\theta_2}\phi^{\epsilon}\rangle\right]$$



$$\epsilon|\tilde{\phi}^{\epsilon}_{\vec{n}}\rangle = \sum_{\vec{m}}\left(H_{\vec{n}-\vec{m}}\mathrm{e}^{-i(\vec{n}-\vec{m})\cdot\vec{\theta}_0} - \vec{n}\cdot\vec{\Omega}\delta_{\vec{n}\vec{m}}\right)|\tilde{\phi}^{\epsilon}_{\vec{m}}\rangle$$







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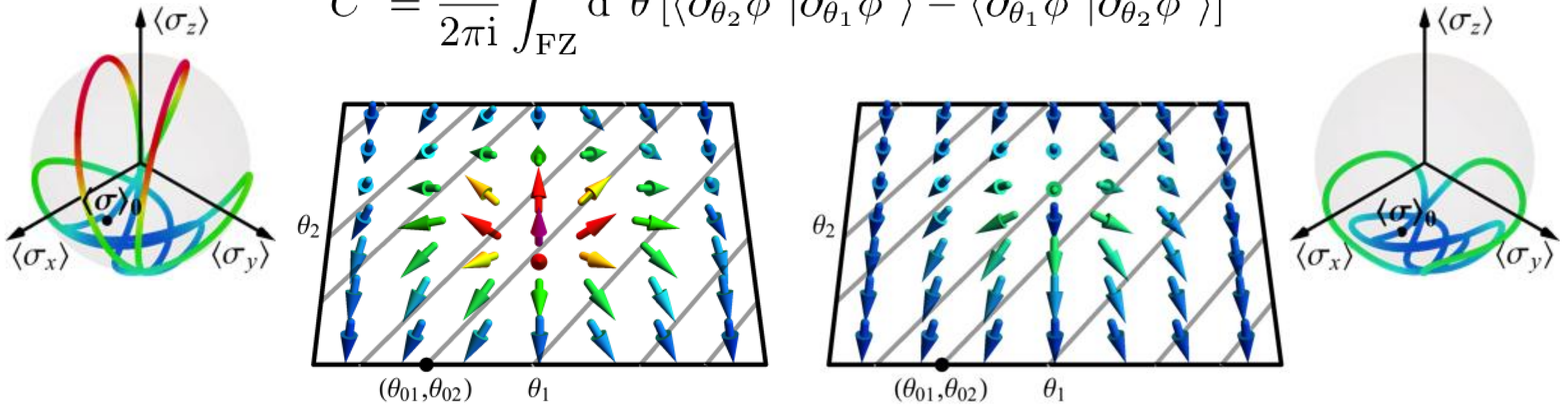
Topological Classification

Of quasi-energy-states

What physics is controlled by this classification?

$$C = \frac{1}{2\pi \mathrm{i}} \int_{\mathrm{FZ}} \mathrm{d}^2\theta \, [\langle \partial_{\theta_2} \phi^\epsilon | \partial_{\theta_1} \phi^\epsilon \rangle - \langle \partial_{\theta_1} \phi^\epsilon | \partial_{\theta_2} \phi^\epsilon \rangle]$$



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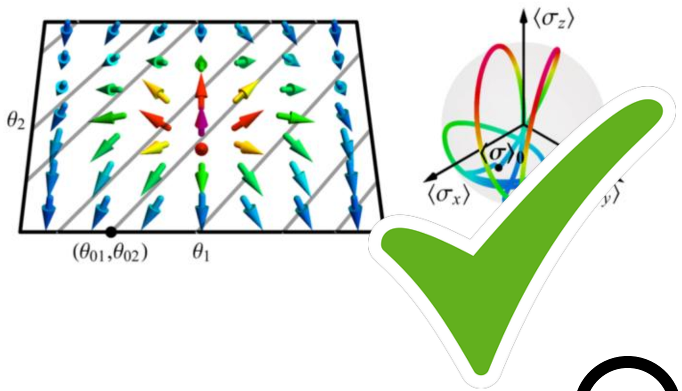
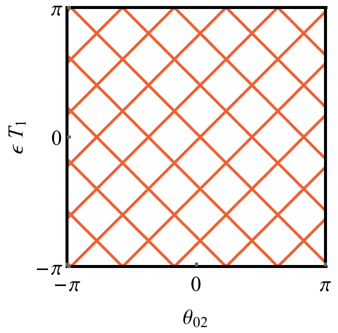
Main message

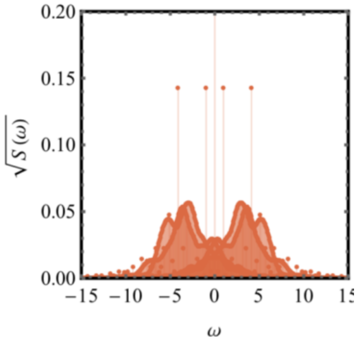
If you only remember one (three) thing(s)…

1 Topology → 2 Bandstructure → 3 Dynamics

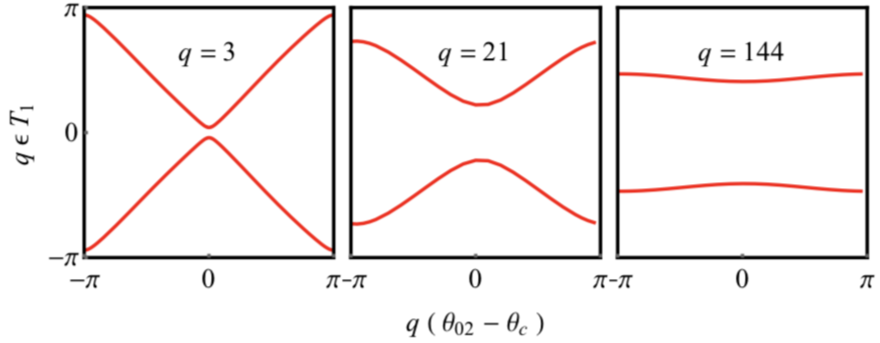
$C \in \mathbb{Z}$







4 Stability and realisation



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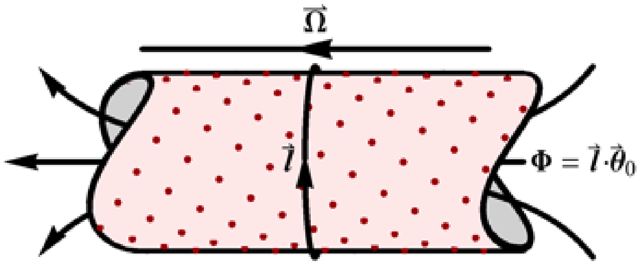
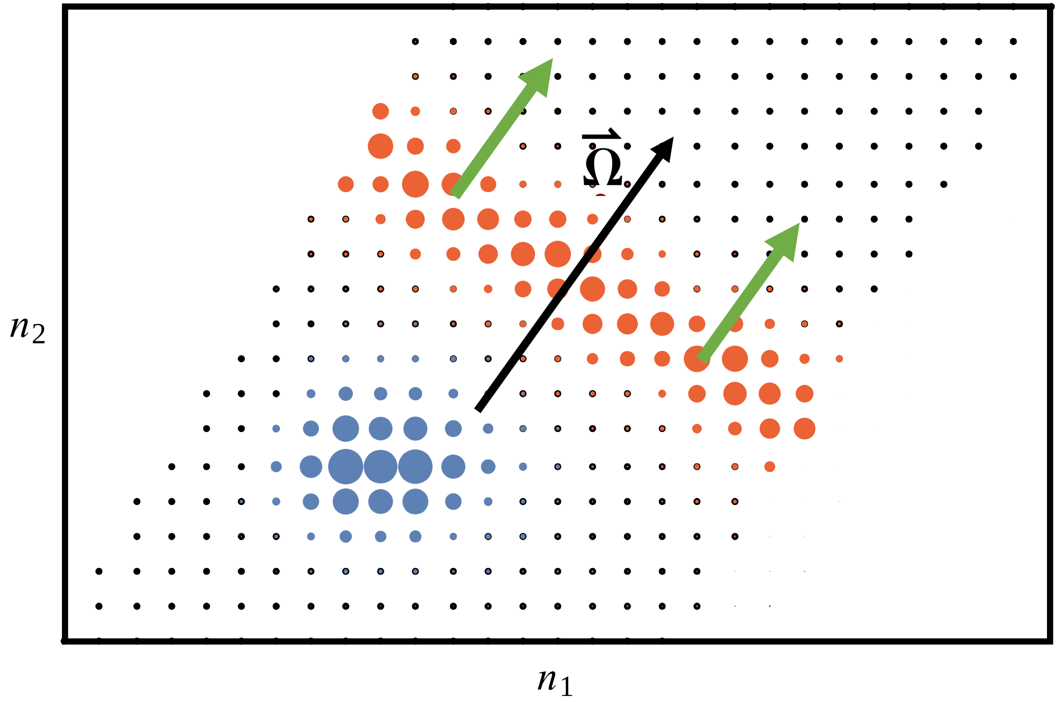
Band-structure

Variation of Quasi-energy with boundary conditions

- Confined to quasi-1D strip
- States can localise or delocalise





$$\epsilon|\tilde{\phi}^{\epsilon}_{\vec{n}}(\vec{\theta}_0)\rangle = \sum_{\vec{m}} \left(H_{\vec{n}-\vec{m}} \mathrm{e}^{-i(\vec{n}-\vec{m})\cdot\vec{\theta}_0} - \vec{n}\cdot\vec{\Omega}\delta_{\vec{n}\vec{m}} \right) |\tilde{\phi}^{\epsilon}_{\vec{m}}(\vec{\theta}_0)\rangle$$

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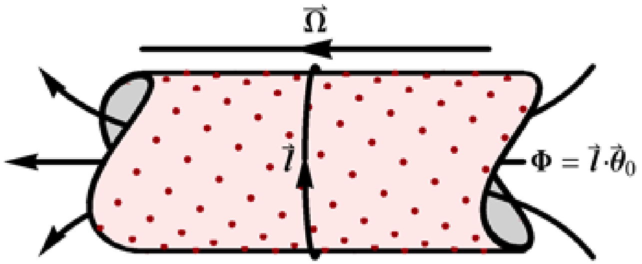
Band-structure

Variation of Quasi-energy with boundary conditions

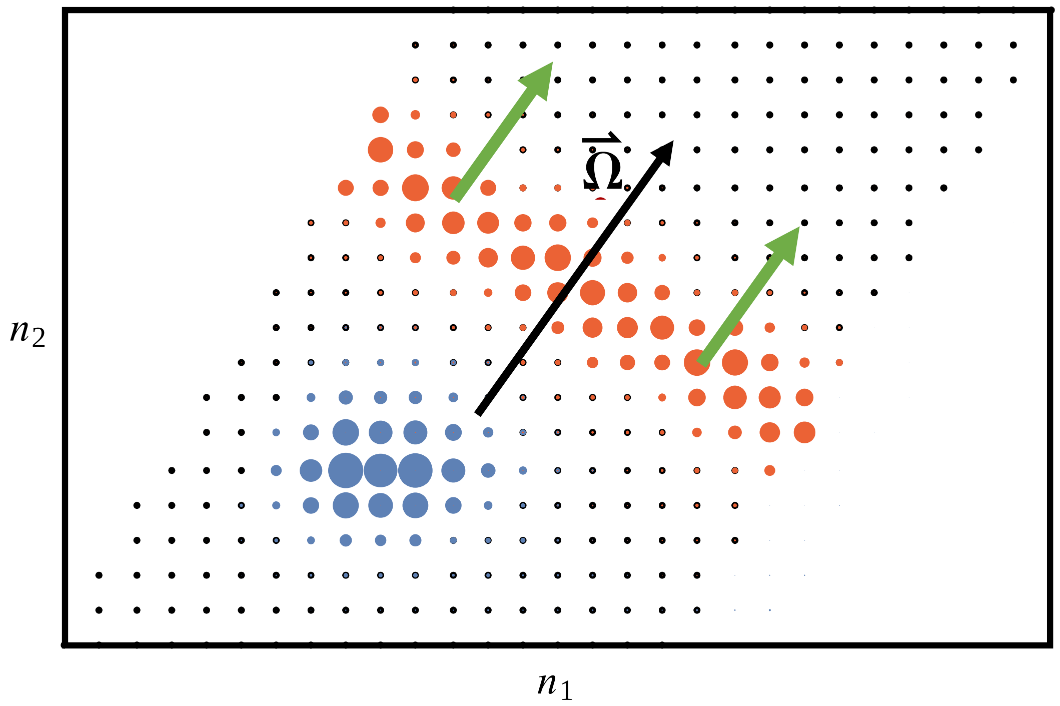
- The topological states carry hall current

- Faradays Law: varying flux does work on currents



- Work done on the current is converted into electric potential

varying boundary conditions $\vec{\theta}_0$



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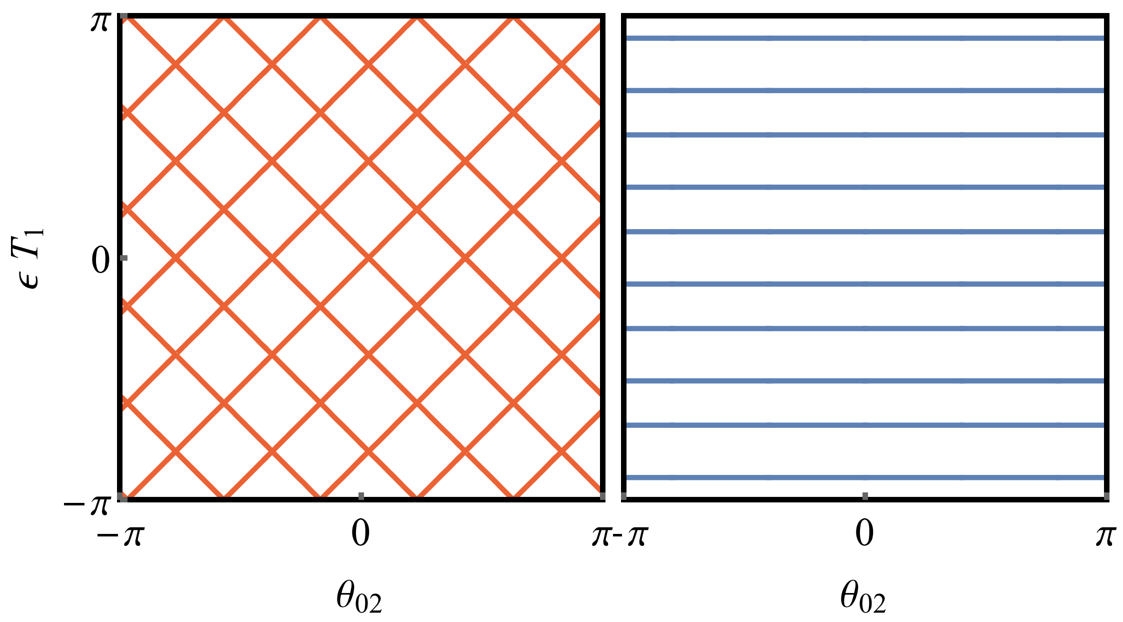
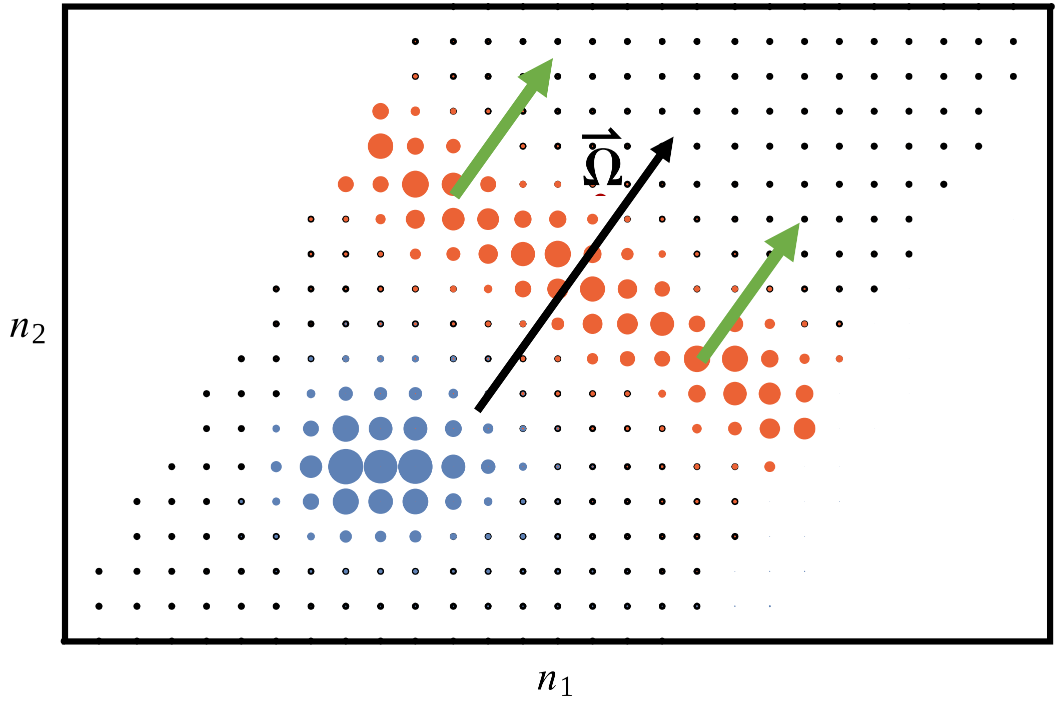
Band-structure

Variation of Quasi-energy with boundary conditions



varying boundary conditions $\vec{\theta}_0$



$$\nabla_{\vec{\theta}_0}\epsilon_j(\vec{\theta}_0) = \frac{C_j}{2\pi}(-\Omega_2, \Omega_1)$$

$$\epsilon|\tilde{\phi}^{\epsilon}_{\vec{n}}(\vec{\theta}_0)\rangle = \sum_{\vec{m}} \left(H_{\vec{n}-\vec{m}} \mathrm{e}^{-i(\vec{n}-\vec{m})\cdot\vec{\theta}_0} - \vec{n}\cdot\vec{\Omega}\delta_{\vec{n}\vec{m}} \right) |\tilde{\phi}^{\epsilon}_{\vec{m}}(\vec{\theta}_0)\rangle$$

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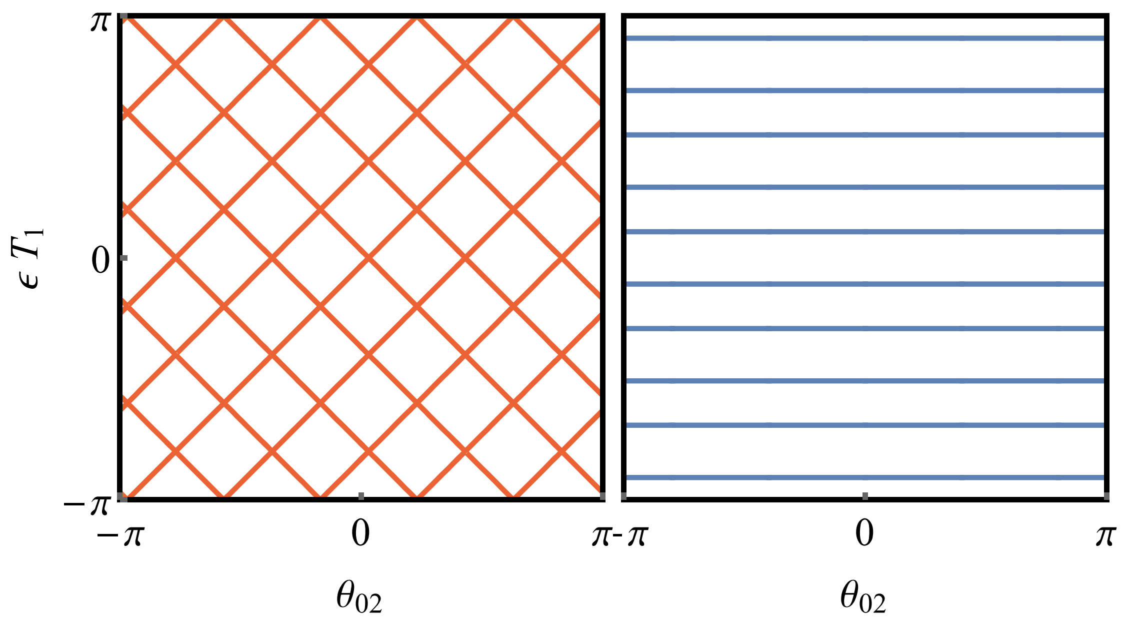
Band-structure

Variation of Quasi-energy with boundary conditions



$$\nabla_{\vec{\theta}_0}\epsilon_j(\vec{\theta}_0) = \frac{C_j}{2\pi}(-\Omega_2, \Omega_1)$$

Distinct from usual analysis:

Topology and bandstructure are fixed by each other

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Main message

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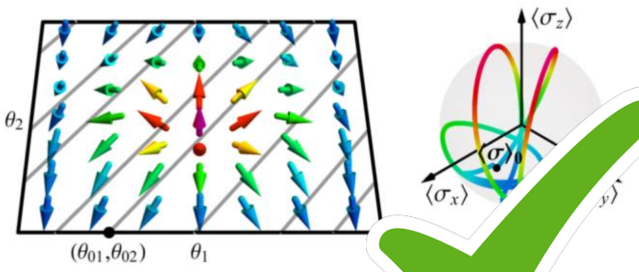
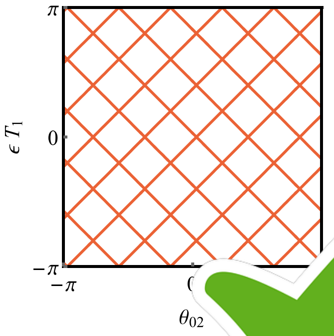
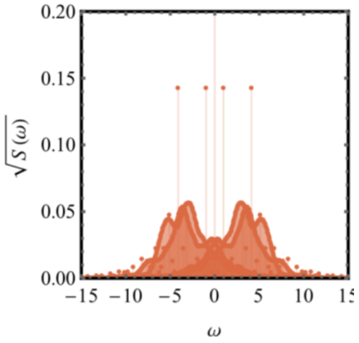
$C \in \mathbb{Z}$







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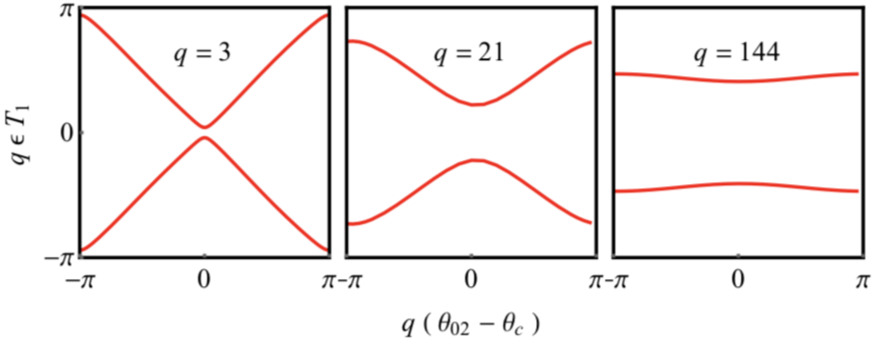

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Dynamical consequences

Three signatures of topology in real time dynamics

$$\boxed{\nabla_{\vec{\theta}_0} \epsilon_j(\vec{\theta}_0) = \frac{C_j}{2\pi}(-\Omega_2, \Omega_1)}$$

1. Pumping

2. Sensitivity to initial conditions

3. Behaviour of observable expectation values

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1: Pumping

Three signatures of topology in real time dynamics

- Hall current flows on the frequency lattice

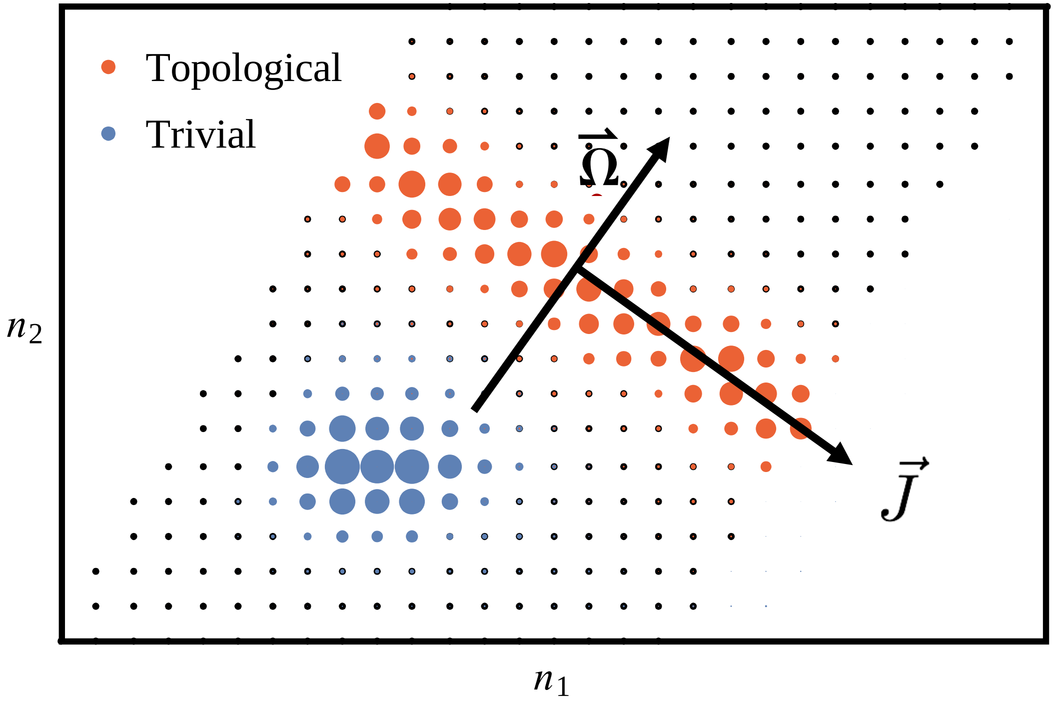
$$\vec{J} = \frac{C_j}{2\pi}(\Omega_2, -\Omega_1)$$

$$\frac{dE_1}{dt} = \Omega_1 J_1 = \frac{C_j \Omega_1 \Omega_2}{2\pi}$$

- This current moves photons from one drive to the other

$$\frac{dE_1}{dt} = \Omega_1 \frac{\partial \epsilon}{\partial \theta_{01}}$$

- Pumping seen in the model of Martin et al, PRX 7, 041008



$$\epsilon|\tilde{\phi}^{\epsilon}_{\vec{n}}(\vec{\theta}_0)\rangle = \sum_{\vec{m}} \left(H_{\vec{n}-\vec{m}} e^{-i(\vec{n}-\vec{m})\cdot\vec{\theta}_0} - \vec{n}\cdot\vec{\Omega}\delta_{\vec{n}\vec{m}} \right) |\tilde{\phi}^{\epsilon}_{\vec{m}}(\vec{\theta}_0)\rangle$$

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1: Pumping

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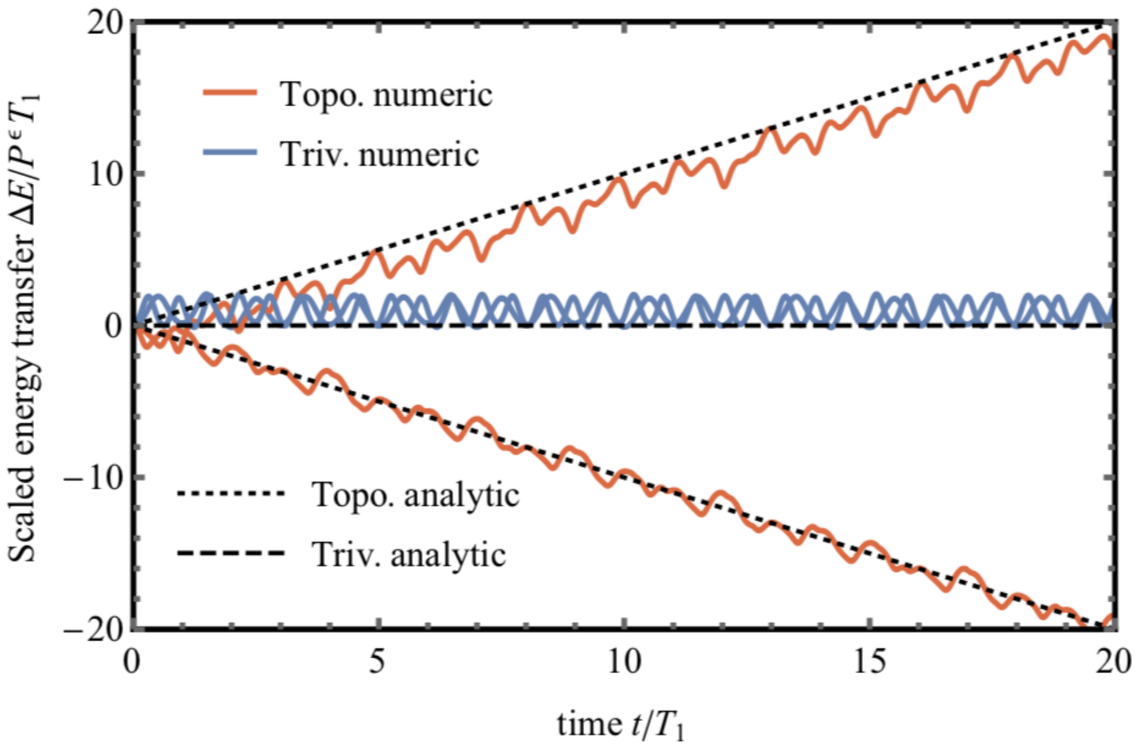

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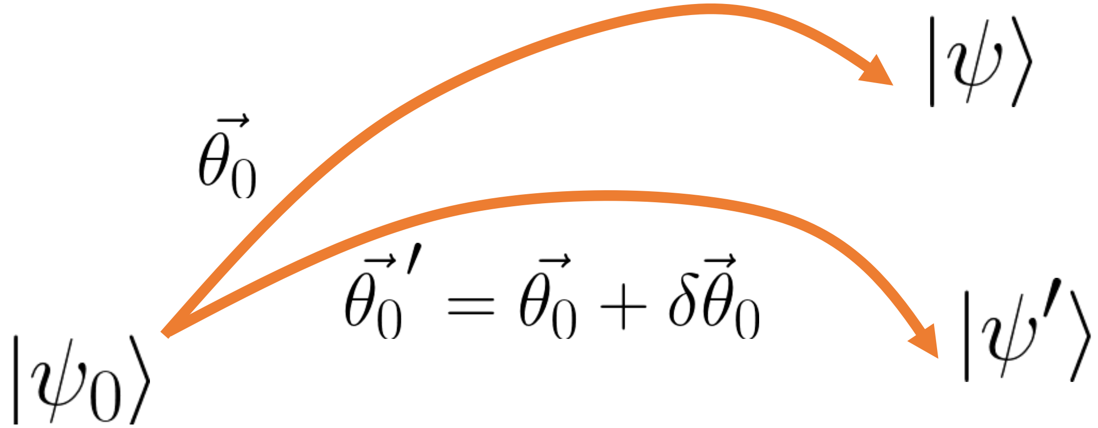
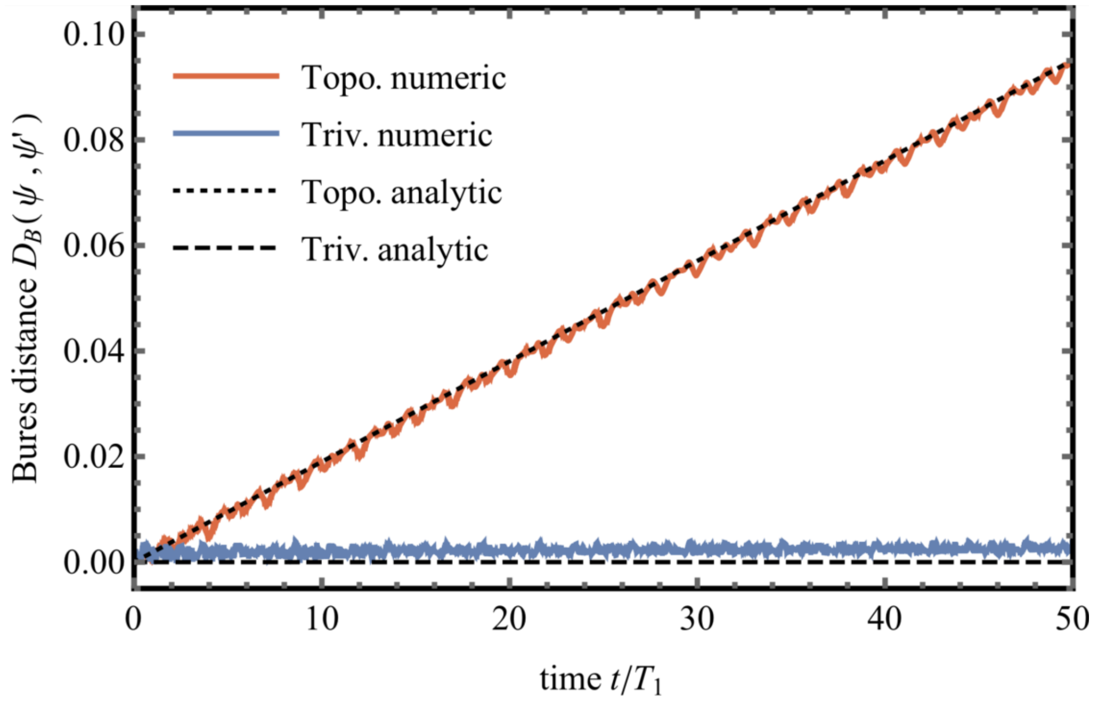
2: Sensitivity to initial conditions

Three signatures of topology in real time dynamics

$$\nabla_{\vec{\theta}_0} \epsilon_j(\vec{\theta}_0) = \frac{C_j}{2\pi}(-\Omega_2, \Omega_1)$$

$$|\psi(t)\rangle = \sum_{\epsilon} \alpha_\epsilon \mathrm{e}^{-\mathrm{i}\epsilon(\theta_{01},\theta_{02})t} |\phi^\epsilon(\Omega_1 t + \theta_{01}, \Omega_2 t + \theta_{02})\rangle$$





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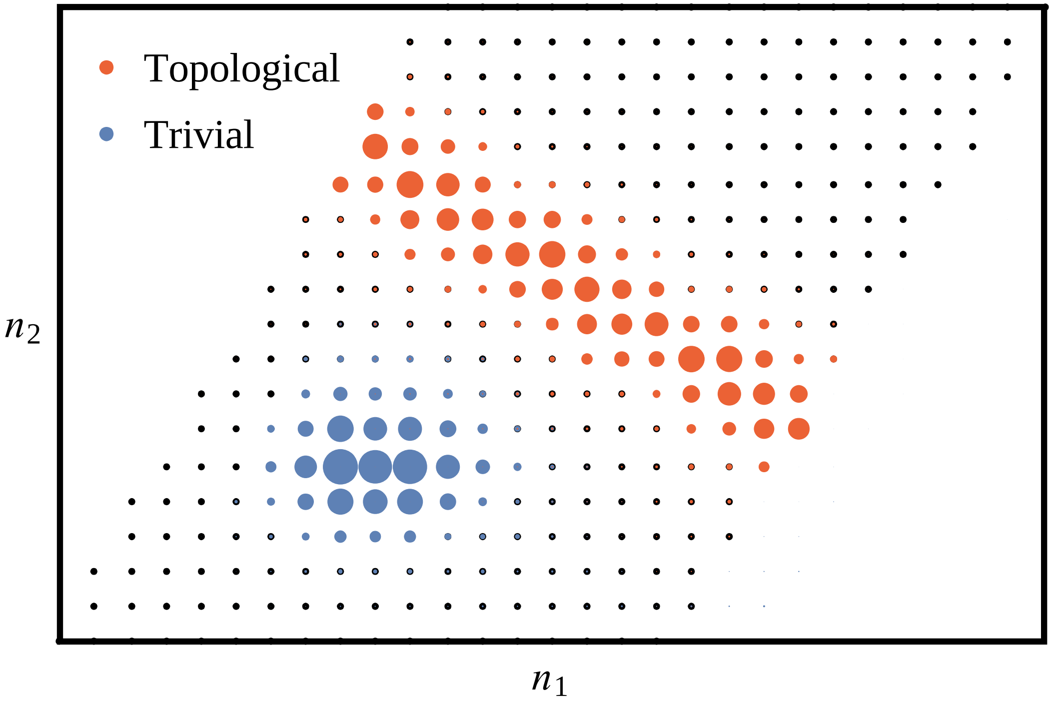
3: Behaviour of observables

Three signatures of topology in real time dynamics

- Topo quasi-energy states have infinitely many large Fourier components

- Trivial quasi-energy states have finitely many.



arXiv: 1808.07884

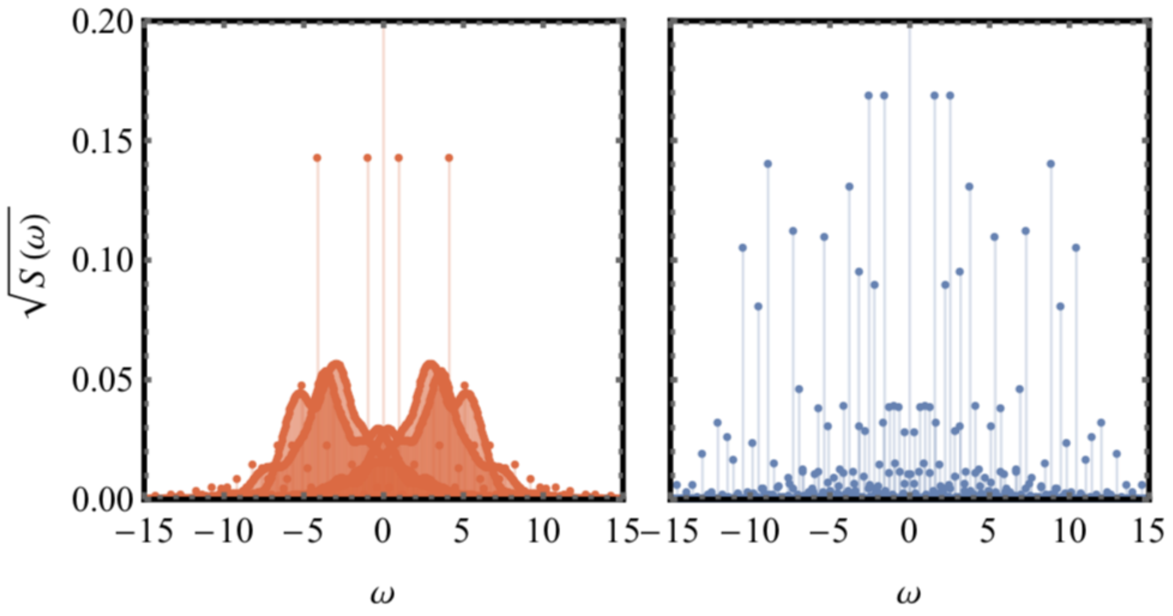
3: Behaviour of observables

Three signatures of topology in real time dynamics

- Topo quasi-energy states have infinitely many large Fourier components
- Trivial quasi-energy states have finitely many.
- Topo observables are not QP, they have no repetitive structure, Fourier spectra are dense

$$A(t) = \langle \psi | \hat{A} | \psi \rangle$$

$$S(\omega) = |A(\omega)|^2$$



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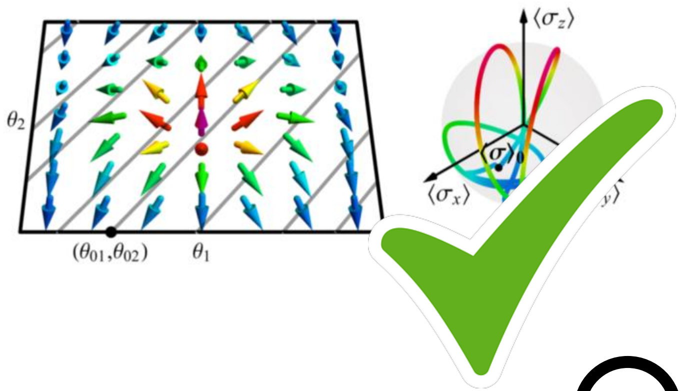
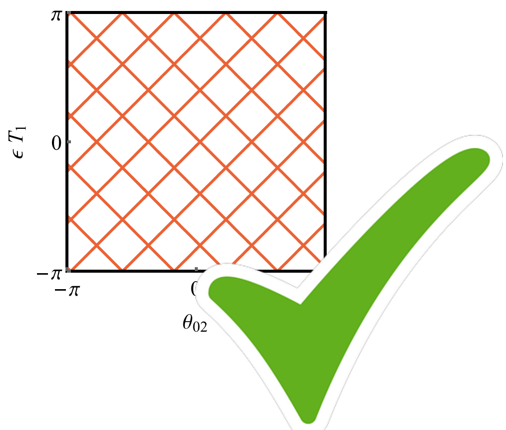
Main message

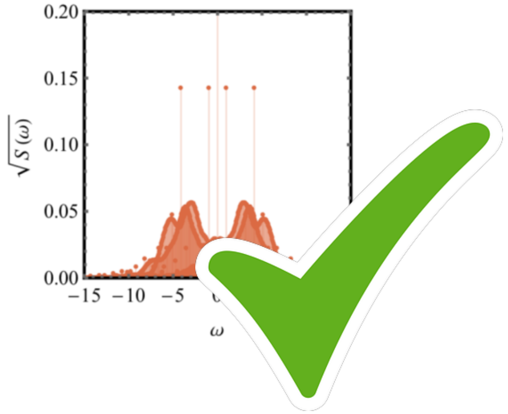
If you only remember one (three) thing(s)...

① Topology → ② Bandstructure → ③ Dynamics

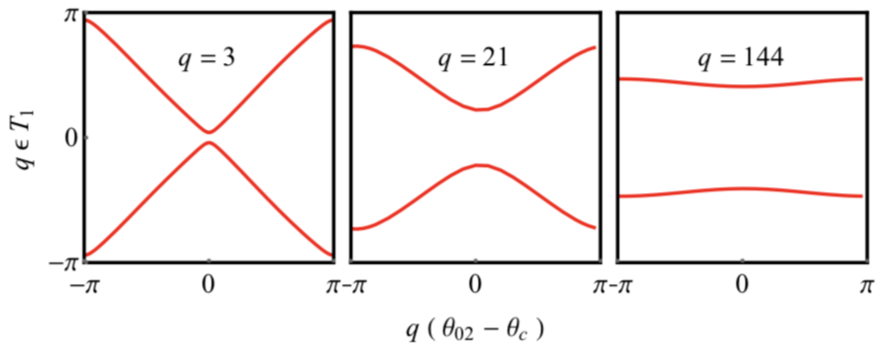
$C \in \mathbb{Z}$







④ Stability and realisation



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Stability of the topological phase

And can we see this in experiment?

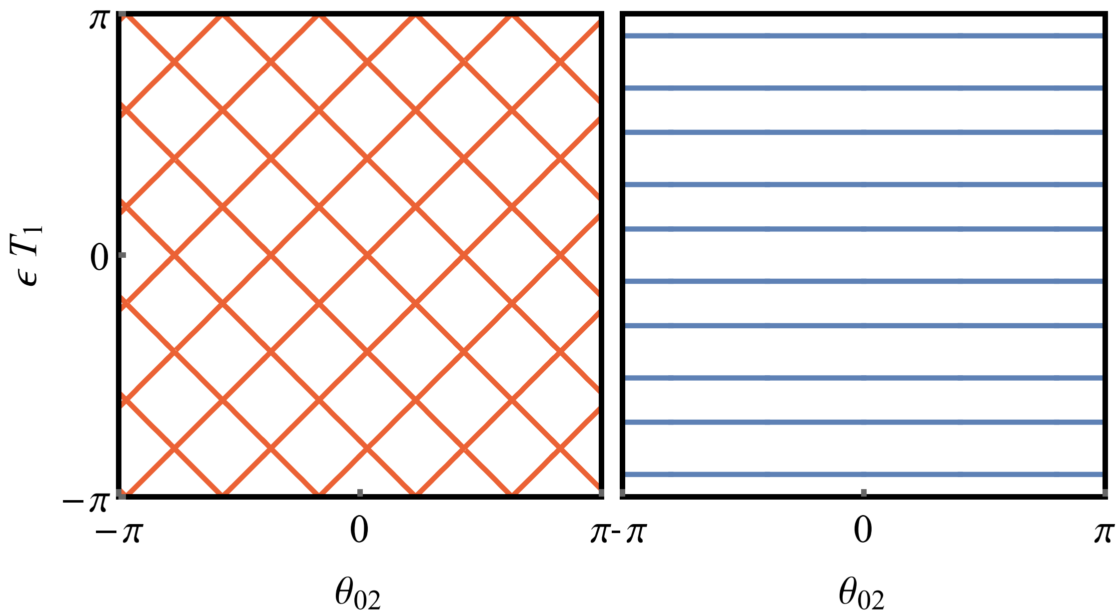
- Topological class stable in the adiabatic limit

$$\sum_j C_j = 0$$

- Landau Zener excitation

$$\log \tau \sim \Omega_1^{-1}, \Omega_2^{-1},$$

- Topological dynamics seen for exponentially long pre-thermal period.



$$\nabla_{\vec{\theta}_0} \epsilon_j(\vec{\theta}_0) = \frac{C_j}{2\pi}(-\Omega_2, \Omega_1)$$

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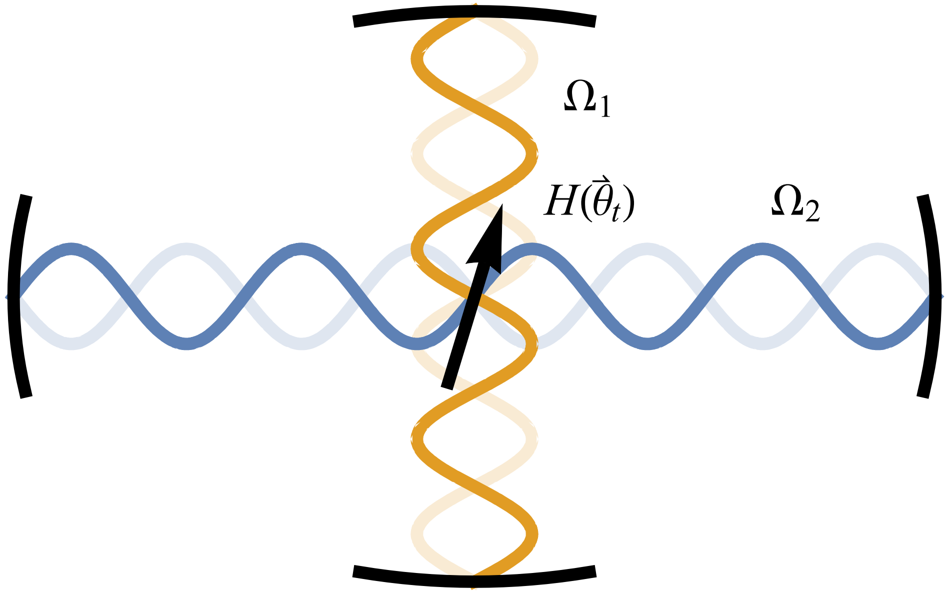
Stability of the topological phase

And can we see this in experiment?

- Tuning with counter-diabatic driving stabilises topological dynamics at finite rate.
- Experimentally accessible
- Generic perturbations lead to trivial dynamics on exponentially long time scales.



$$H_{\mathrm{CD}} = H + V$$

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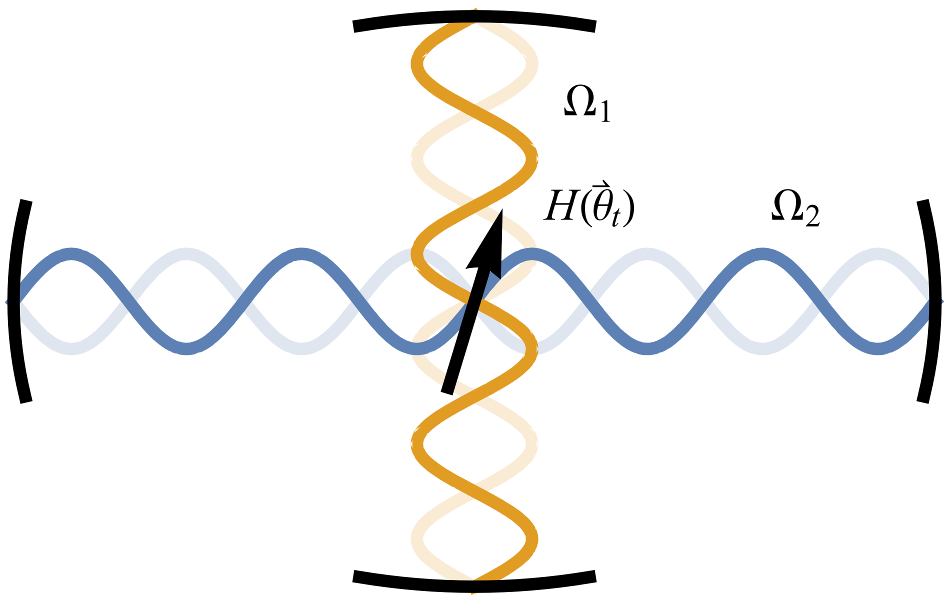
Conclusions

- Map to 'synthetic dimensions'
- Topological classification of states
- Topology controls exp-long lived pre-thermal dynamics
- 3 main dynamical phenomena distinguish these classes
- Phenomenology accessible in experiments, *viz.* Sushkov group



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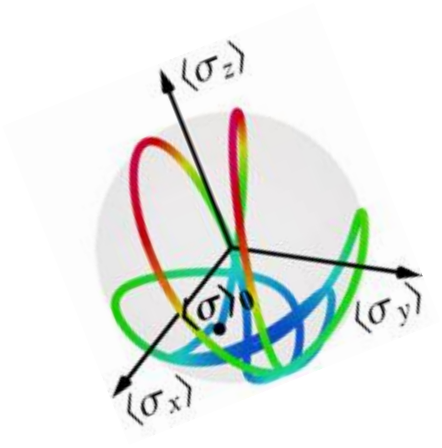
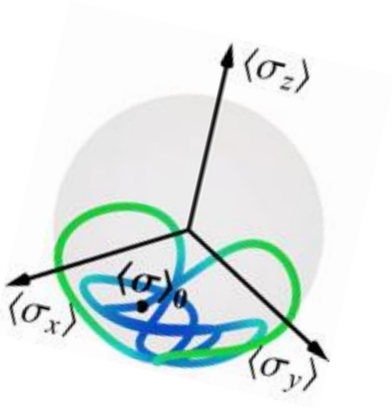


Thank you



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