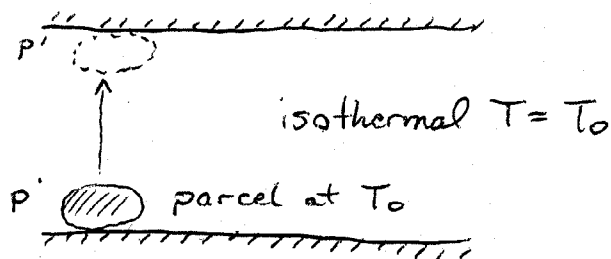Convective transport of heat is a central theme in geodynamics. It determines the evolution of the planet and powers most of the geological processes we observe at the surface. We begin the geodynamics lectures with a discussion of the present-day thermal state of the mantle. We examine the temperature distribution inside the mantle and assess the influence of phase transitions. We also consider the observational constraints on the temperature and identify the need for thermal boundary layers.

Temperature Distribution in the mantle

What is the temperature distribution in a vigorously convecting fluid? Our initial expectation might be that a vigorously stirred fluid is isothermal. This expectation is incorrect and the error springs from an improper extrapolation of kitchen-scale intuition to planetary-scale processes

Isotherm Fluid

isothermal $T = T_0$

parcel at T_0

what happens when the parcel at P is moved to P'?

Suppose that we move the parcel to P' by vigorous stirring (e.g. no time for heat exchange with surroundings)

Question: is the parcel at P' in thermal equilibrium?

Answer: No! the parcel is subject to adiabatic decompression and is at a lower temperature than $T_0 \rightleftharpoons$ not in thermal equilibrium

Conclusion: an isothermal fluid is not established and maintained by vigorous mixing

Adiabatic Temperature Distribution

To pursue the question of an equilibrium temperature distribution we require results from classical thermodynamics. We will make use of "Maxwell relations" (named after James Clerk Maxwell)

1st Law

$$dU = T ds - P dV \qquad \dots\dots (1)$$

where

$T ds =$ addition of heat (s is entropy)

$P dV =$ work (done by system on surronndings)

$dU =$ change in internal energy

interpretation: internal energy is increased by addition of heat or by compressing the system.

The internal energy for a homogeneous (non-reacting system) is a function of two state variables (say x & y)

Then $\quad U = U(x,y)$

$$dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy$$

If we let $\quad U = U(S,V)$

$$\therefore dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV$$

Comparison with 1st Law indicates that

$$T = \left(\frac{\partial U}{\partial S}\right)_V \quad\text{and}\quad P = -\left(\frac{\partial U}{\partial V}\right)_S$$

but

$$\frac{\partial^2 U}{\partial S \partial V} = \frac{\partial^2 U}{\partial V \partial S} \qquad \text{(e.g. order doesn't matter)}$$

$$\therefore \boxed{\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V} \qquad \text{Maxwell relation \# 1}$$

We can define other functions (besides U)

$\quad H = U + PV \qquad\qquad$ Enthalpy
$\quad F = U - TS \qquad\qquad$ Helmholtz free energy
$\quad G = U - TS + PV \qquad$ Gibbs free energy

The utility of these functions emerges from their differential form

For example

$$dH = dU + PdV + VdP$$
$$= TdS - PdV + PdV + VdP \quad (\text{by } 1^{st} \text{ Law})$$
$$= TdS + VdP$$

If we view $H = H(S, P)$

$$\therefore dH = \left(\frac{\partial H}{\partial S}\right)_P dS + \left(\frac{\partial H}{\partial P}\right)_S dP$$

This means that

$$T = \left(\frac{\partial H}{\partial S}\right)_P \quad \text{and} \quad V = \left(\frac{\partial H}{\partial P}\right)_S$$

Since

$$\frac{\partial^2 H}{\partial P \partial S} = \frac{\partial^2 H}{\partial S \partial P} \implies \boxed{\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P}$$

Maxwell relation # 2

Application

We are interested in the temperature change with temperature at constant entropy

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P = \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial S}\right)_P$$

By definition $\left(\frac{\partial V}{\partial T}\right)_P = \alpha V$ where α is the coefficient of thermal expansion.

We only need to evaluate $\left(\frac{\partial T}{\partial S}\right)_P$

We know that $dH = TdS$ (at constant P)

$\uparrow$ heat added

$= C_p \, dT$ (at constant P)

$\therefore \; T dS = C_p \, dT$ (all at constant P)

$\therefore \; \dfrac{dT}{dS} = \dfrac{T}{C_p}$

This means that

$$\left(\dfrac{dT}{dP}\right)_S = \dfrac{\alpha V T}{C_p} = \dfrac{\alpha T}{\rho C_p} \qquad *$$

The mantle is nearly hydrostatic (deviations are small)

$$\dfrac{dP}{dr} = -\rho g$$

$\underset{\nwarrow}{} $ acceleration due to gravity

$\therefore \; \dfrac{dT}{dr} = -\dfrac{\alpha g T}{C_p}$ adiabatic gradient

example

$\alpha = 2 \times 10^{-5} \; K^{-1}$
$C_p = 1000 \; J \, kg^{-1} \, K^{-1}$
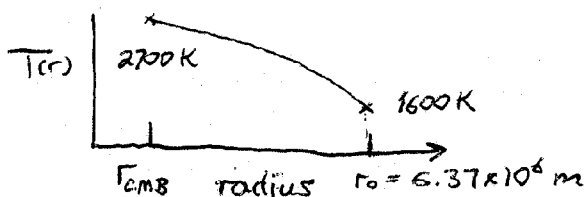$T \sim 2000 \; K$
$g = 9.8 \; m \, s^{-2}$

$\dfrac{dT}{dr} \sim 0.4 \, K/km$

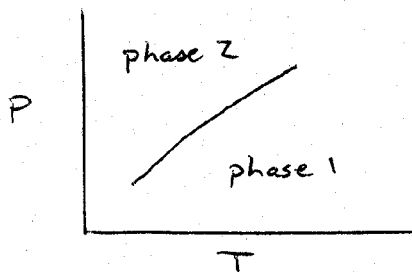Typical temperature gradient in the crust $\sim 25 \, K/km$

Integrate for $T(r) = T(r_0) \, e^{\alpha g (r_0 - r)/C_p}$ when α, C_p and g are constant

$T(r)$ — 2700 K ... × 1600 K

r_{CMB} radius $r_0 = 6.37 \times 10^6 \, m$

$r_{CMB} = 3.48 \times 10^6 \, m$

Complications

1. Phase transitions – mantle minerals undergo a series of transitions to denser phases with increasing P

Clapeyron Slope (see appendix)

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V} = \gamma$$

ΔV: volume change $(\Delta V < 0)$

ΔS: entropy change   (positive or negative)

i) Olivine $\rightarrow$ Spinel $\Delta V < 0$; $\Delta S < 0$; $\gamma > 0$

ii) Spinel $\rightarrow$ Perovskite + Oxides

$\Delta V < 0$; $\Delta S > 0$; $\gamma < 0$
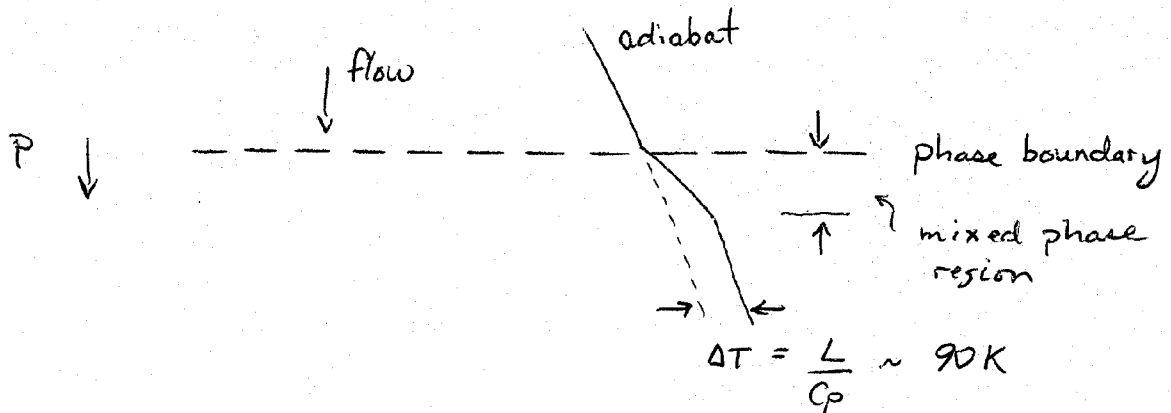
Phase transitions alter adiabat through latent heat

$$L = T \Delta S$$

i) Olivine $\rightarrow$ Spinel is exothermic
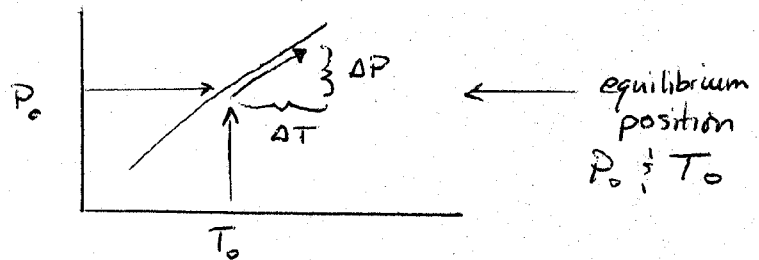
(heat given off because entropy of spinel is lower)

ii) Spinel $\rightarrow$ Perovskite is endothermic

(heat is absorbed to raise the entropy of perovskite + oxides)

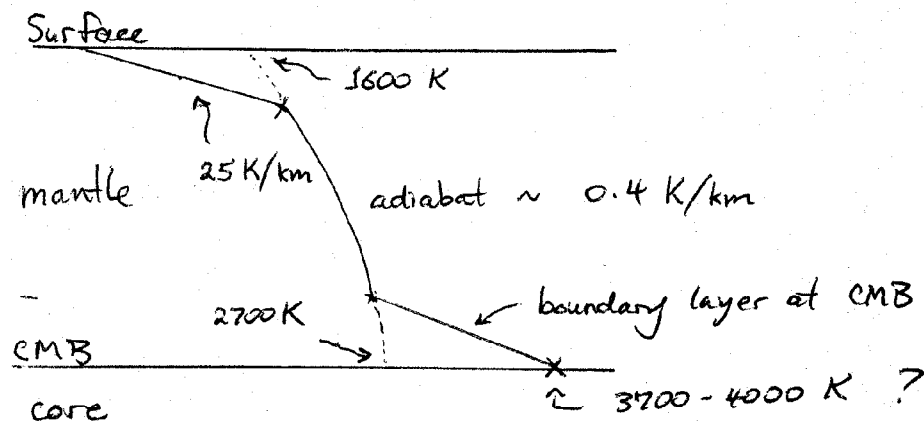How does latent heat alter temperature profile?

exothermic case

$$\Delta T = \frac{L}{Cp} \sim 90 K$$

temperature profile is controlled by Clapeyron slope in mixed phase region

for endothermic case $\Delta T \sim \frac{L}{Cp} \sim 70 K$

2. Boundary Layers — temperature profile deviates from an adiabat anywhere the mantle is not well-mixed. This usually happens near boundaries where convective heat flow vanishes

3. Effect of Radioactive Heating $\dot{Q}_R$ (W/kg)

_____

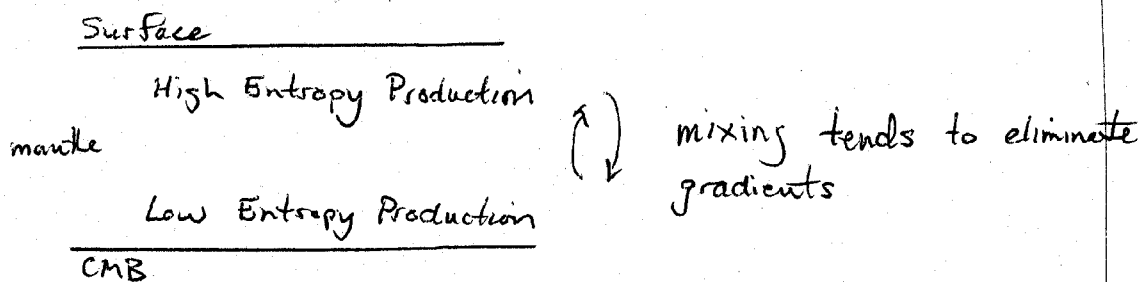Well mixed $S = $ constant
$\dot{Q}_R = $ constant

_____

convection homogenizes entropy and the distribution of heat producing elements
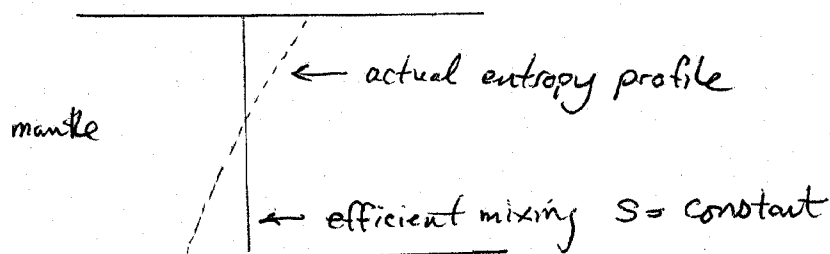
Recall that $dQ = T\,dS$

Entropy Production Associated with $\dot{Q}_R$

$$\dot{S} = \frac{\dot{Q}_R}{T}$$

$\dot{S}$ is not uniformly distributed through the mantle because of $T(r)$

_____
Surface

    High Entropy Production

mantle

    Low Entropy Production
_____
    CMB

↑↓ mixing tends to eliminate gradients

In steady state the entropy flux balances the difference in production

mantle   ← actual entropy profile

    ← efficient mixing $S = $ constant

The well-mixed part of the mantle should be thermally stratified. This is observed in numerical models

Observational Constraints

1. Production of Oceanic Crust: The oceanic crust is produced by melting 10% to 20% of the mantle below a mid-ocean ridge. The thickness and composition of the oceanic crust constrains the potential temperature in the upper mantle (adiabat at $P = 0$). A typical estimate $\sim 1600 K$ [Klein & Langmuir, 1987]

2. Depth of Phase Boundaries: Seismic detection of phase boundaries at 410 km and 660 km determines the temperature when the phase diagram of representative minerals is known
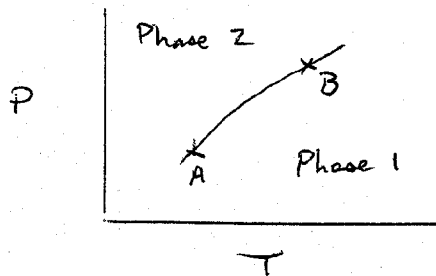
   Olivine $\longrightarrow$ Spinel $T = 1800 K \pm 200K$

   Spinel $\longrightarrow$ Perovskite $T = 2000 K \pm 250 K$

   [Akaogi et al., 1989; Ito & Takahashi, 1989; Katsura & Ito, 1989]

3. Mineral Assemblages of Mantle Xenoliths - phase equilibrium temperature and pressure often used to constrain continental lithosphere

4. Melting Temperature of Iron - constrains temperature at inner-core boundary. Experiments must be extrapolated to high pressure and corrected for impurities. Estimates 5000 to 6000 K

Clapeyron Slope

Phase 2

P

×B

×A   Phase 1

T

Gibbs free energy of two phases must be equal at phase boundary (where both phases are present)

$$\therefore dG = G_2 - G_1 = 0$$

$$G = U - TS + PV$$

$$dG = dU - TdS - SdT + PdV + VdP$$

but 1st law requires

$$dU = TdS - PdV$$

so

$$dG = -SdT + VdP$$

Consider how G in the two phases varies as we move from A to B

$$dG_1 = -S_1 dT + V_1 dP$$
$$dG_2 = -S_2 dT + V_2 dP$$

Equilibrium requires $dG_1 = dG_2$

$$\therefore -S_1 dT + V_1 dP = -S_2 dT + V_2 dP$$

$$\therefore \frac{dP}{dT} = \frac{S_2 - S_1}{V_2 - V_1}$$