Geodynamics

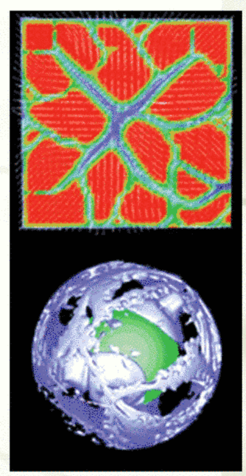
Day	Lecturer	Lectures
1	LHK	Introduction
1	CLB	Dynamical Observations of the Earth
3	PVK	Present-day mantle convection: slabs, plumes and hotspots
4	PVK	Tutorial 1: Mantle convection modeling
5	CLB	Instantaneous flow models: Theory and applications
5	CLB	Tutorial 2: Geoid modeling
8	LHK	Heat budget, thermal evolution models
8	LHK/PVK	Tutorial 3: Thermal evolution modeling
10	PVK	Long-term evolution of the mantle, looking back in
12	LHK	Thermochemical Convection
12	LHK/PVK	Tutorial 4: Chemical Geodynamics

Governing Equations

Momentum- $\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v}) = -\nabla \mathbf{p} + \nabla \cdot \mathbf{T} + \mathbf{f}$

Energy - $\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T + H$

Mass - $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$

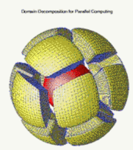


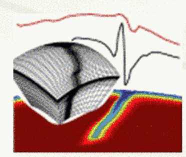
[*Tackley*, 1999]

Non-linear
What is right Constitutive Relation? $\sigma = -p\mathbf{I} + 2\eta\dot{\varepsilon}$

How to solve?

- Numerical methods for PDE's
  - Finite Difference, Spectral, Finite element, Finite Volume, etc.
  - Flexibility
    - Grids (geometry, adaptability)
    - Resolution
    - Material property contrasts
  - Speed!
- Regional vs. Global
  - Boundary conditions
  - Resolution, Speed
  - Nature of problem





- Inputs
  - Material properties (from mineral physics)
    - α, κ, ρ
      - as a function of (P,T,X)
    - Rheology (viscosity, but not only)
      - As a function $(P,T,X,\sigma,\dot{\varepsilon})$
    - P dependence requires compressibility
  - Energy sources (from geochemistry, and ...)
    - Rate of internal heating
    - Basal heating (heat flow coming out of the core)
  - Chemical Composition (from geochemistry in a broad sense)

Simplifications

-Infinite Prandtl # fluid: i.e. Inertial forces are not important
-Fluid is Incompressible, Newtonian
-Properties Homogeneous

$$\nabla p - \eta \nabla^2 \mathbf{v} = \mathbf{f}$$

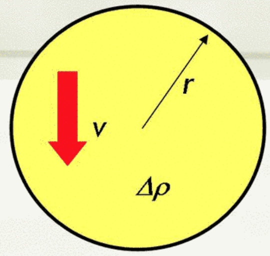
$$\mathbf{f} = \delta\rho\mathbf{g}$$

$$\delta\rho = \alpha\rho_o \Delta T$$

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T = \kappa\nabla^2 T + H$$

But suppose you know $\delta\rho$?

Falling Sphere



Buoyancy

$$F_B = g\Delta m = \frac{4\pi r^3 \Delta\rho g}{3}$$

Resistance

$$F_R = A\sigma_R = 4\pi r^2 \eta \dot{\varepsilon} \approx -4\pi r^2 \eta \frac{v}{r} = -4\pi r \eta v$$

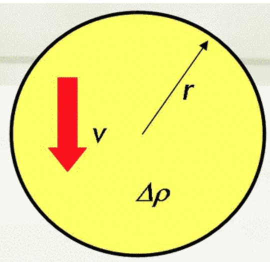
Force Balance: $F_B + F_R = 0$

$$v \approx \frac{1}{3}\frac{r^2 \Delta\rho g}{\eta}$$

Actual coefficient varies between 1/3 (inviscid) and 2/9 (solid)

Falling Sphere

$$v \approx \frac{1}{3}\frac{r^2 \Delta\rho g}{\eta}$$



Object	r	Δρ (kg/m^3)	η (Pa s)	v
Plume head	500 km	30	10^{22}	80 km/Ma
Xenolith	10 cm	200	100	90 km /week

Solve Stokes in a self-gravitating Earth

$$\nabla p - \eta \nabla^2 \mathbf{v} = \delta\rho \mathbf{g}$$

$$\mathbf{g} = -\nabla \Phi$$

$$\nabla^2 \Phi = 4\pi G \delta\rho$$

Take each variable: $[v_r, v_\theta, v_\varphi, \tau_{rr}, \tau_{r\theta}, \tau_{r\varphi}, \delta p, \delta\rho]$

Expand using spherical harmonics, scalar:

$$\delta\rho = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \delta\rho_l^m(r) Y_l^m(\theta, \phi)$$

and vector fields:

$$\mathbf{v}(r,\theta,\phi) = \hat{\mathbf{r}} U(r,\theta,\phi) + \nabla_1 V(r,\theta,\phi) - \hat{\mathbf{r}} \times (\nabla_1 W(r,\theta,\phi))$$

[Alterman et al., 1959; Takeuchi and Hasegawa, 1965; Kaula, 1975; Hager and O'Connell, 1979; 1981]

Aside: Spherical Harmonics

The spherical harmonics are the angular portion of the solution to Laplace's equation in spherical coordinates

$$\nabla^2 \Phi = 0$$

$$Y_l^m(\theta,\phi) = (-1)^m \left[\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}\right]^{\frac{1}{2}} P_l^m(\cos\theta) e^{im\phi}$$

$$\nabla_1^2 Y_l^m = -l(l+1) Y_l^m \qquad \frac{\partial}{\partial \phi} Y_l^m = im Y_l^m$$

Spherical Harmonics

Continuing…..

$$v_r = a_\ell^m(r)Y_\ell^m$$
$$v_\theta = b_\ell^m(r)Y_{\ell,\theta}^m + c_\ell^m(r)Y_{\ell,\varphi}^m$$
$$v_\varphi = b_\ell^m(r)Y_{\ell,\varphi}^m - c_\ell^m(r)Y_{\ell,\theta}^m$$

$$Y_{\ell,\varphi}^m = \frac{1}{\sin\theta}\frac{\partial Y_\ell^m}{\partial\varphi}$$
$$Y_{\ell,\theta}^m = \frac{\partial Y_\ell^m}{\partial\theta}$$

[So for example the continuity (mass conservation) equation]

$$a_\ell^{\prime m} = \frac{-2a_\ell^m}{r} + \frac{\ell(\ell+1)b_\ell^m}{r}$$

…. Substitute the expansions for velocity, stress, with those for pressure and density… We end with 6 coupled ODEs

…..We could solve NUMERICALLY… but if we perform a simple variable substitution for each spherical harmonic coefficient

….. We get two nicely defined differential equations

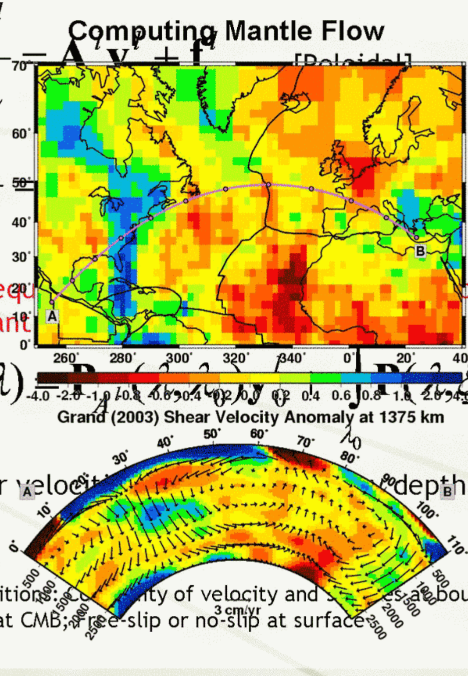
[See Hager and O'Connell, 1979 and 1981 or better yet, ask Guy for his lecture notes!!!]

Propagator Matrices

$$\frac{d\mathbf{y}^l}{d\lambda}$$
$$\frac{d\mathbf{t}^l}{d\lambda}$$
$$= \ln(r/R_E)$$

These eq… …r matrices
[See Gant…

$$\mathbf{y}(\lambda) = \ldots \xi)\mathbf{f}^l(\xi)d\xi$$



We can now solve for velo… …depth once P is known!

Boundary Conditions: C… …ity of velocity and … at boundary interfaces
Use: Free-slip at CMB; free-slip or no-slip at surface

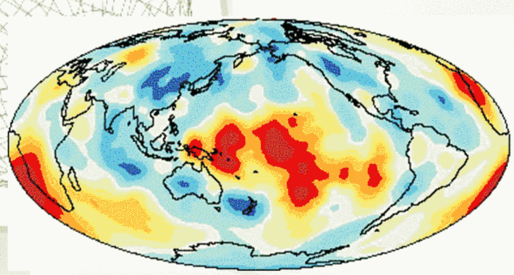
Advantages and Disadvantages

-Spectral Solutions VERY FAST! (You'll see)
-Can change radial viscosity profile (explore effects of viscosity structure)
-Spherical Shell
-Predict observables (Geoid, Topography, Plate Motions, Flow (Anisotropy))
-Can explore compressibility (less than 10% effect at long wavelengths)

-LACK OF RHEOLOGICAL COMPLEXITY
  Lateral viscosity variations
  Plate boundary rheology
-MUST ASSUME A DENSITY HETEROGENEITY
-No TIME DEPENDENCE

So now what?

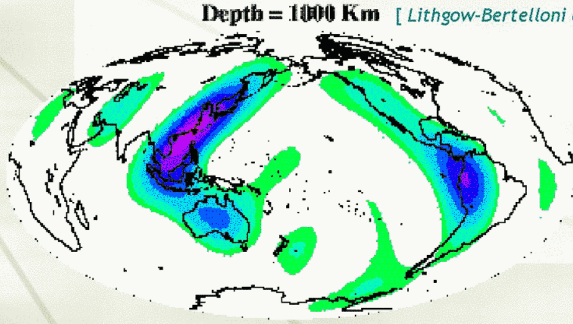
Seismic Tomography- Convert velocity to density---- BUT HOW?



Mantle Density Heterogeneity Model

[*Masters and Bolton*]

Based on Geologic Information-Plate Motion History

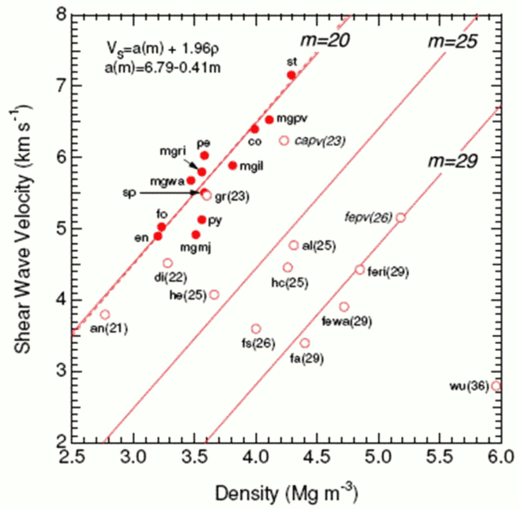


[*Lithgow-Bertelloni and Richards*, 1998]

Velocity-Density Scaling

Birch's law

$\delta v = a \delta \rho$



Factors=0.1-0.5 g s/km cm^3

Density-Velocity Scaling

$$R_{\rho/S} = \left(\frac{\delta \ln \rho}{\delta \ln V_S}\right)_{Depth}$$

If due to lateral T variations:

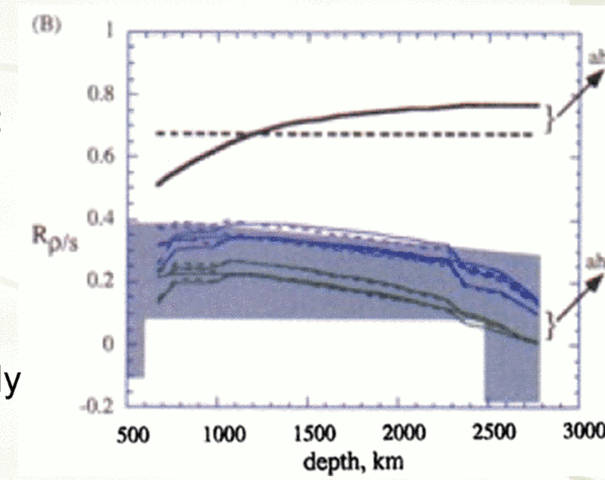
$$R_{\rho/S} = \left(\frac{\partial \ln \rho}{\partial T}\right)_P \left(\frac{\partial \ln V_S}{\partial T}\right)_P^{-1}$$

Attenuation (anelasticity) decreases value significantly

Claim: Cannot be negative

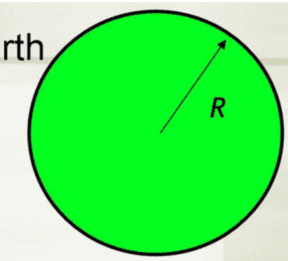
Not so! (phase transformations)

$$\left(\frac{\partial \ln \rho}{\partial T}\right)_P = \left(\frac{\partial \ln \rho}{\partial T}\right)_{P,\mathbf{n}} + \left(\frac{\partial \ln \rho}{\partial \mathbf{n}}\right)_{P,T} \left(\frac{\partial \mathbf{n}}{\partial T}\right)_P$$

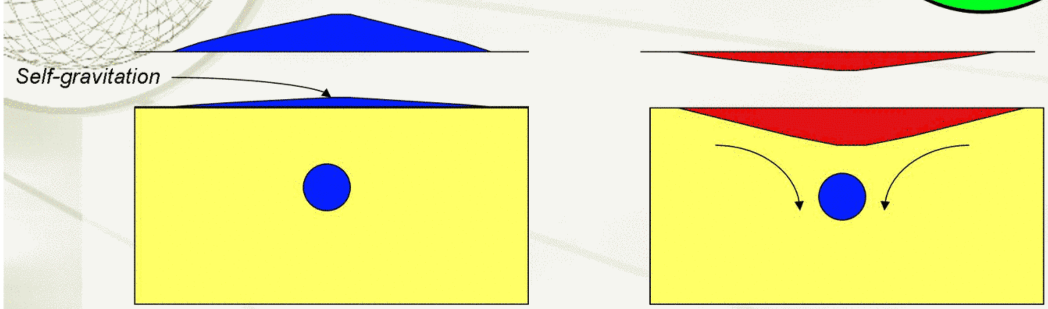


Karato and Karki (2001)

Geoid Anomaly



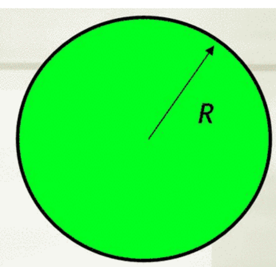
$$\Delta N(\theta,\varphi) \sim \int_0^R \Delta\rho(r,\theta,\phi) r dr$$



Deflection of upper surface represents a mass deficit that opposes mass excess of sphere. Amount of deflection depends on viscosity structure.

Downward deflection of core-mantle boundary also a mass deficit

Geoid Anomaly



$$\delta\rho(r,\theta,\phi) = \sum_{l,m} D_{lm}(r) Y_{lm}(\theta,\phi)$$

$$\delta N_{lm}(\theta,\phi) = \int_0^R G(r,l) D_{lm}(r) dr$$

Formally separate structure (D_{lm}) and dynamics (G)

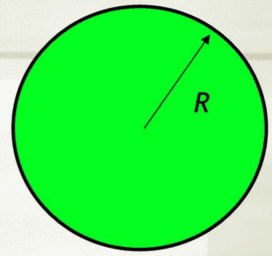
Green's function or Kernel $G(r,l)$ is the geoid anomaly due to a unit density anomaly of wavelength l at depth r

G depends on viscosity structure. Compute separately.

Geoid Anomaly

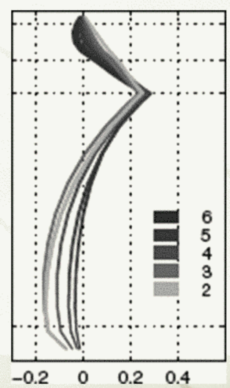
$$\delta\rho(r,\theta,\phi) = \sum_{l,m} D_{lm}(r) Y_{lm}(\theta,\phi)$$

$$\delta N_{lm}(\theta,\phi) = \int_0^R G(r,l) D_{lm}(r)\, dr$$



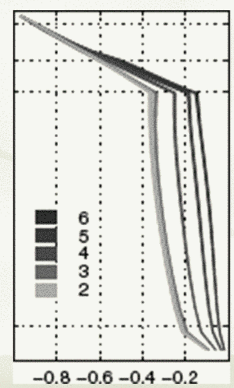
Earth

Geoid Kernel



Standard η structure
LM~50xUM

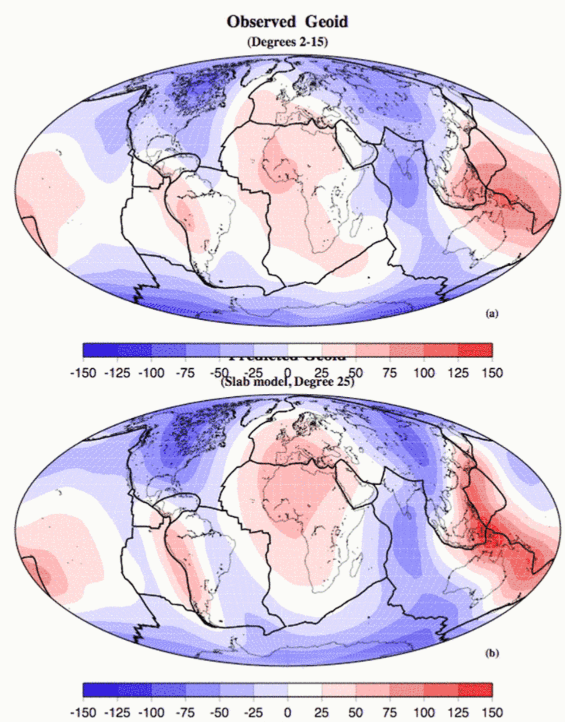
Dynamic topography

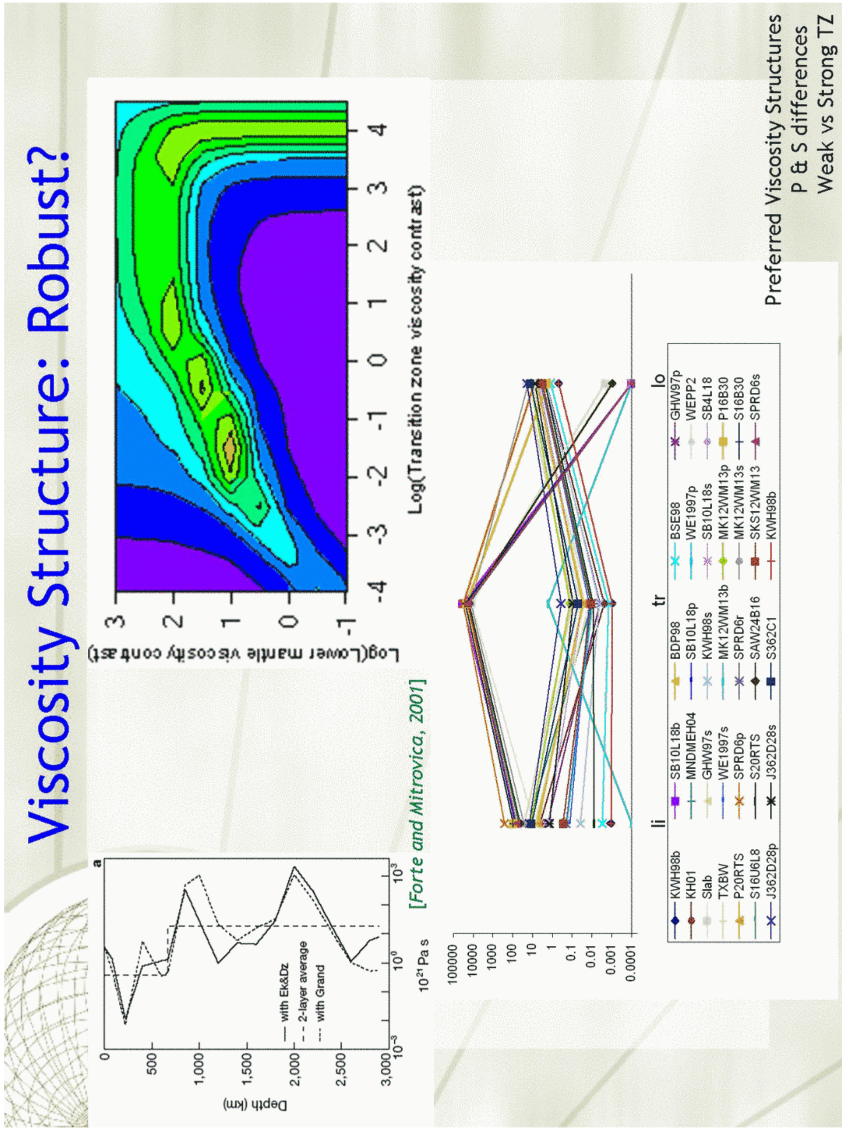


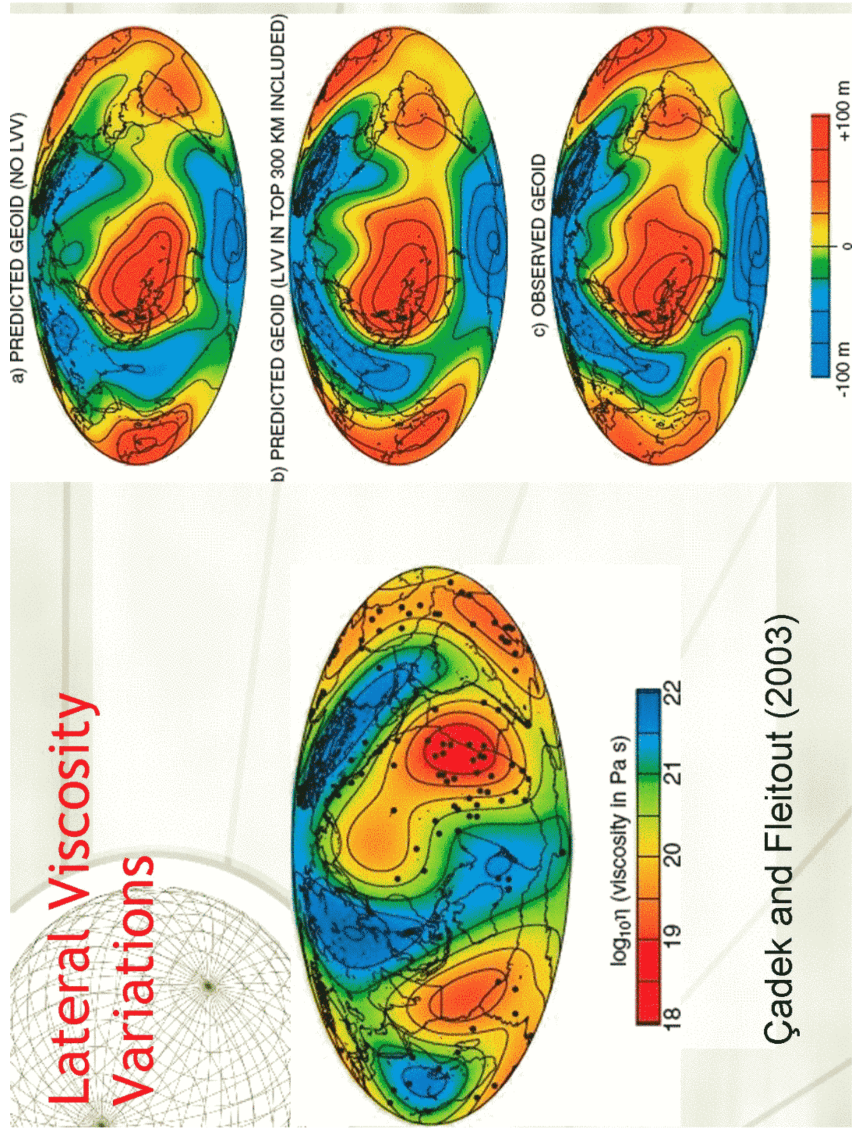
Panasyuk et al. (2000)

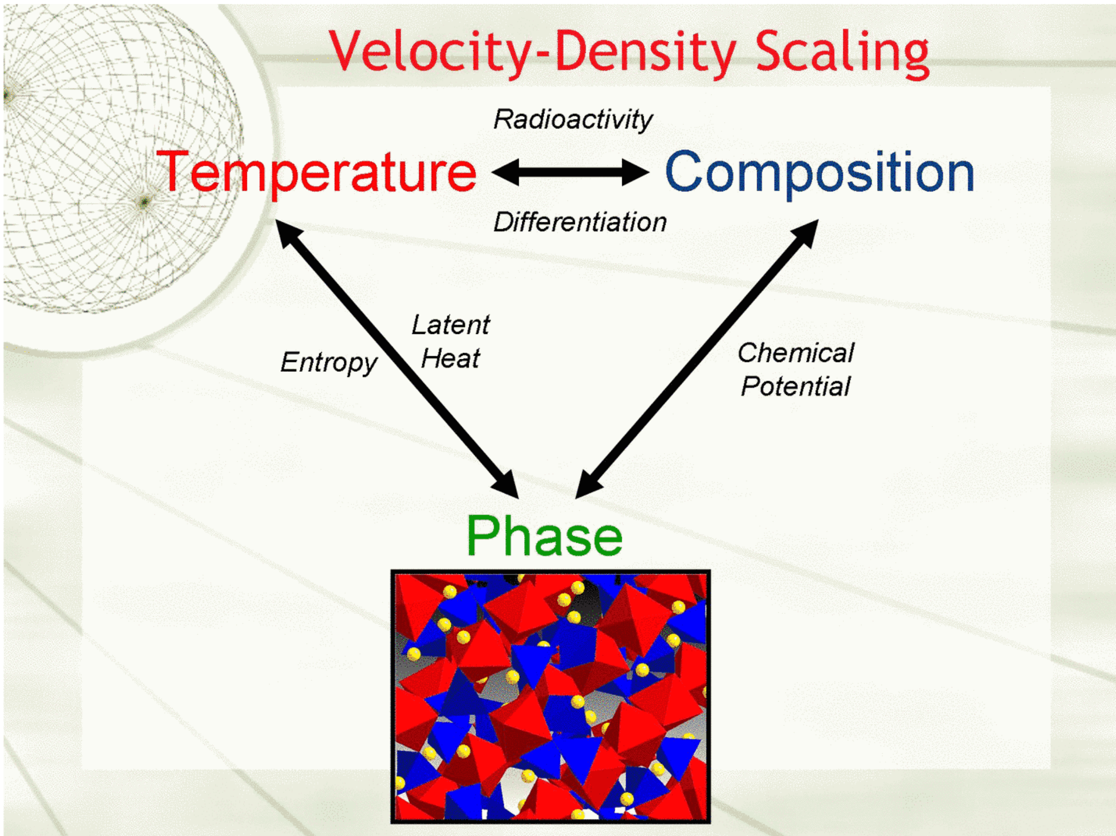
Predicted Geoid

Best fitting viscosity structure
Lithosphere-10 * UM
Lower Mantle-50 * UM




Viscosity Structure: Robust?
Log(Transition zone viscosity contrast)
Log(Lower mantle viscosity contrast)
[Forte and Mitrovica, 2001]
Depth (km)
with Ek&Dz
2-layer average
with Grand
Preferred Viscosity Structures
P & S differences
Weak vs Strong TZ


Lateral Viscosity Variations
Çadek and Fleitout (2003)
$\log_{10}\eta$ (viscosity in Pa s)
a) PREDICTED GEOID (NO LVV)
b) PREDICTED GEOID (LVV IN TOP 300 KM INCLUDED)
c) OBSERVED GEOID
-100 m
0
+100 m



Density-Velocity Scaling

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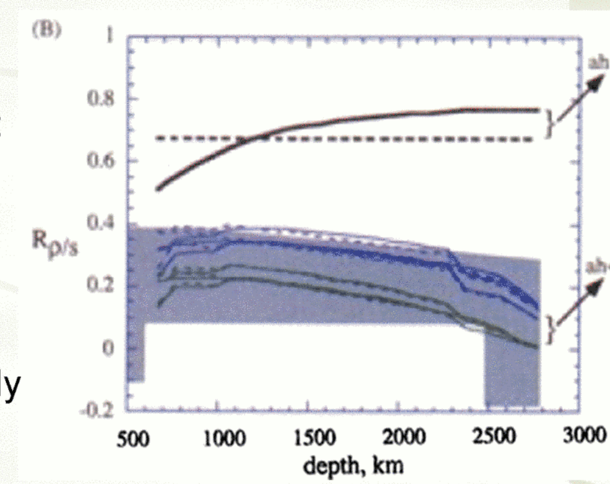
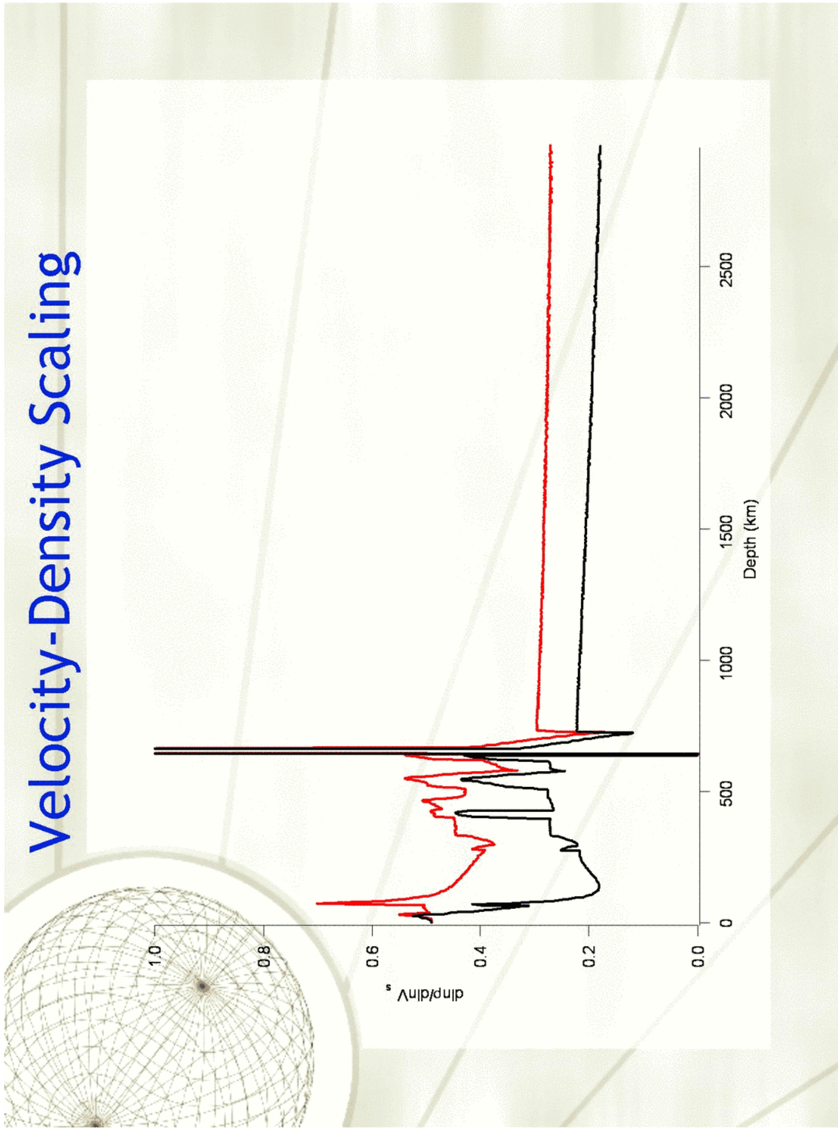
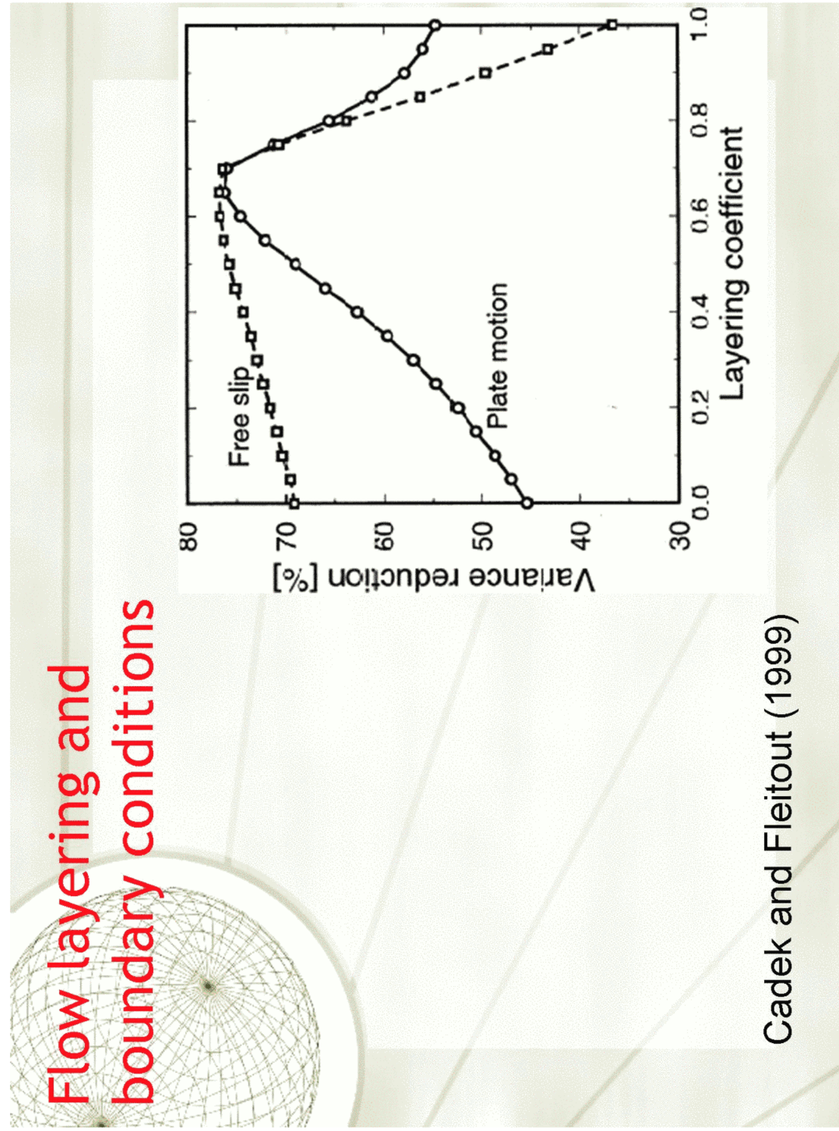
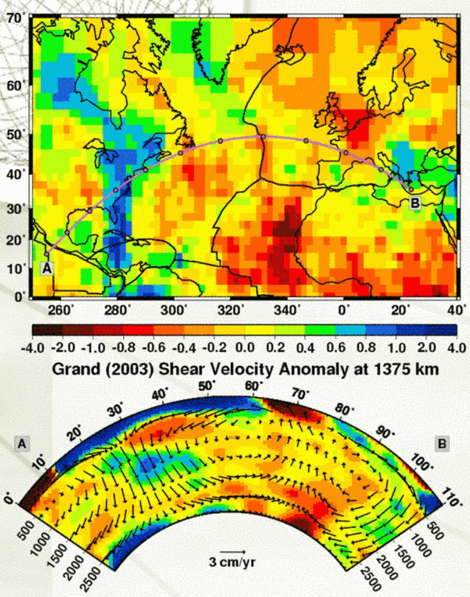


Karato and Karki (2001)


Velocity-Density Scaling
dlnρ/dlnV$_s$
0.0
0.2
0.4
0.6
0.8
1.0
0
500
1000
1500
2000
2500
Depth (km)


Flow layering and boundary conditions
Variance reduction [%]
30
40
50
60
70
80
0.0
0.2
0.4
0.6
0.8
1.0
Layering coefficient
Free slip
Plate motion
Cadek and Fleitout (1999)

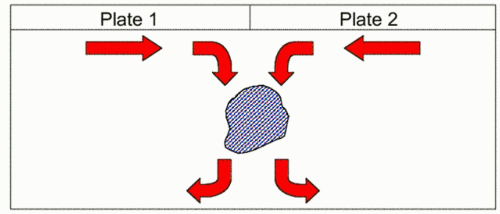
Computing Mantle Flow



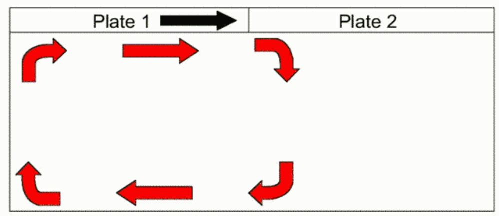
Predicting Plate Motions

Torque Balance

Driving Torques $\vec{Q}_i$



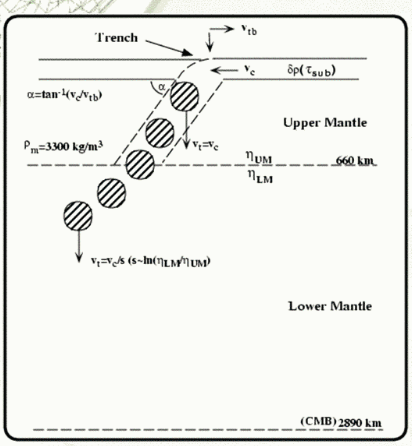
Resisting Torques $\mathbf{M}_{ij}\vec{\omega}_j$



$$\vec{\omega}_j = \mathbf{M}_{ij}^{-1}\vec{Q}_i$$

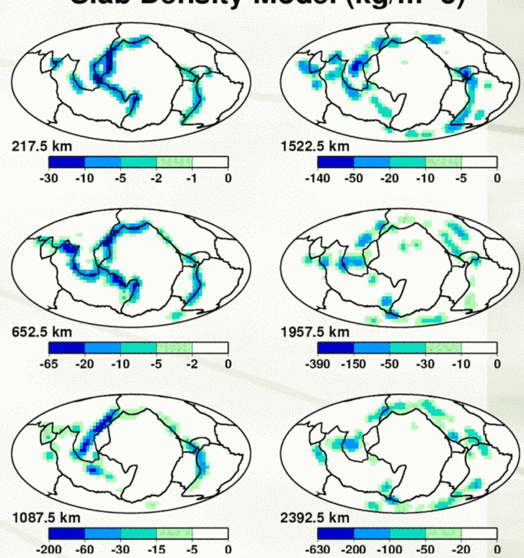
Estimating the Locations and Densities of Mantle Slabs

Drop plates into the mantle at regions of convergence



After *Lithgow-Bertelloni and Richards* [1998]

Slab Density Model (kg/m^3)



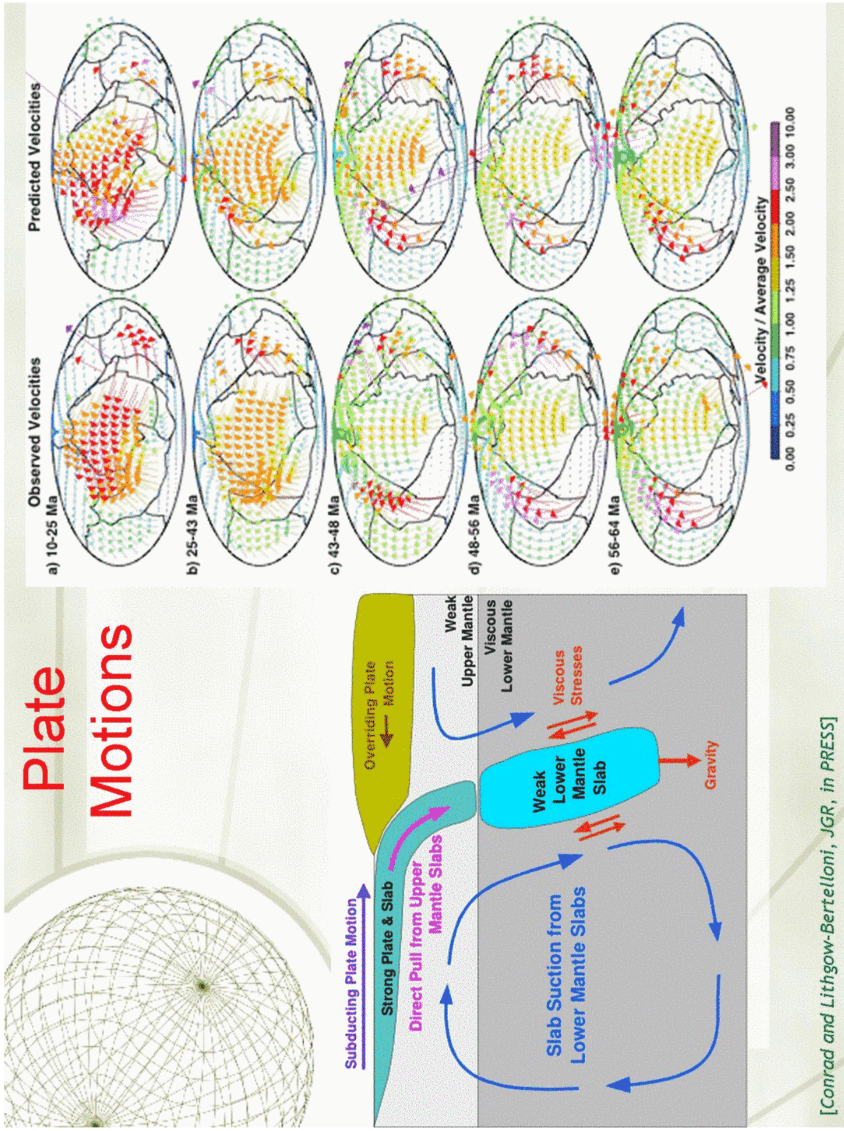
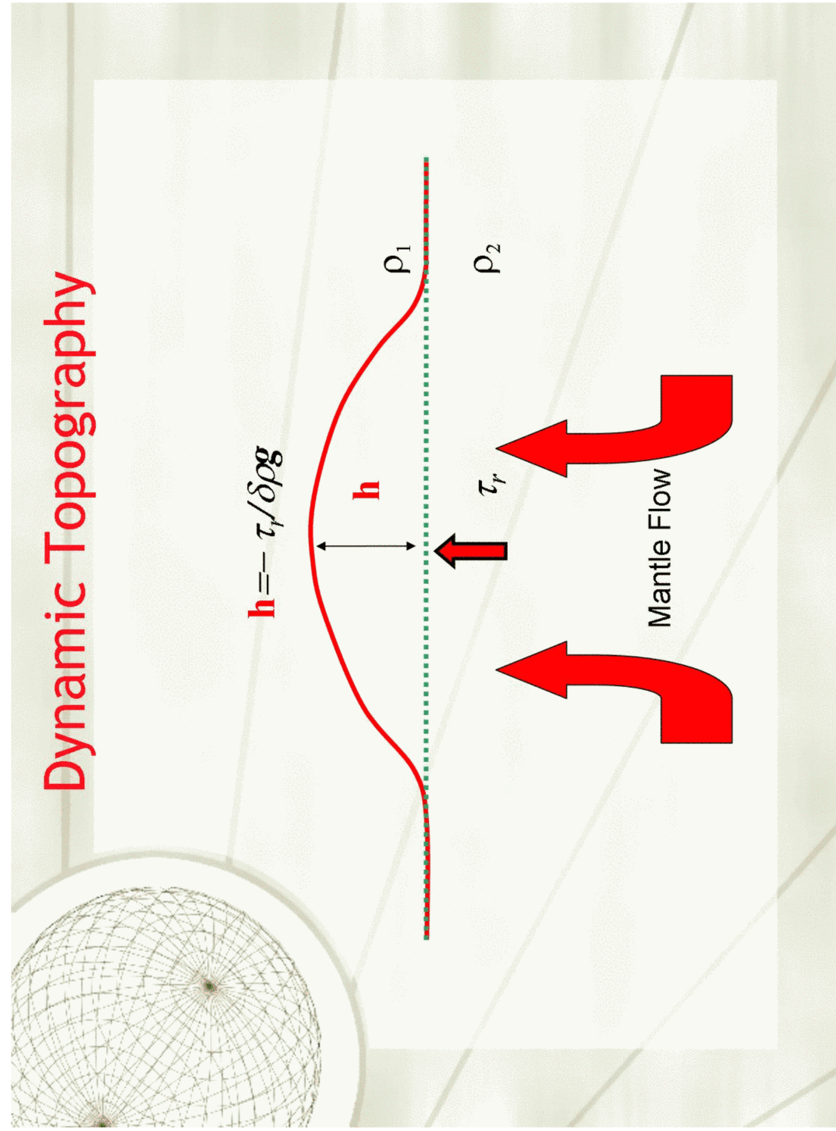
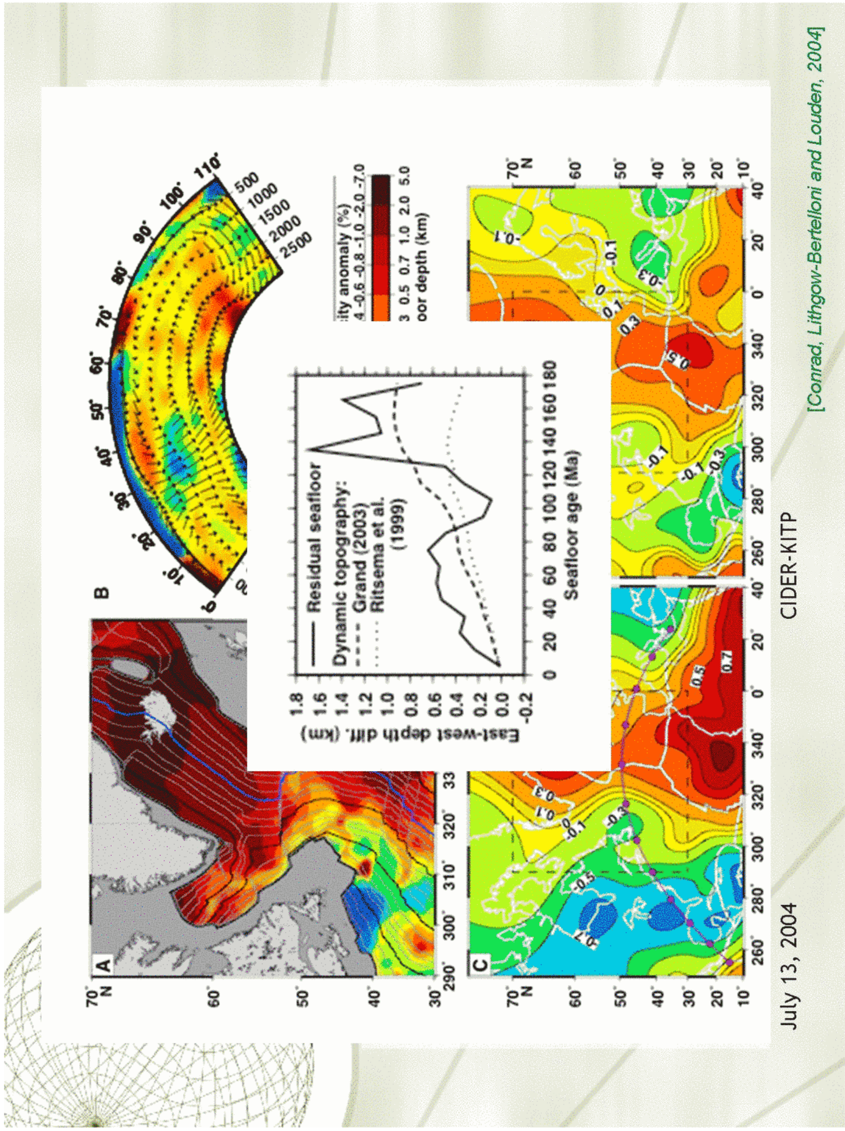
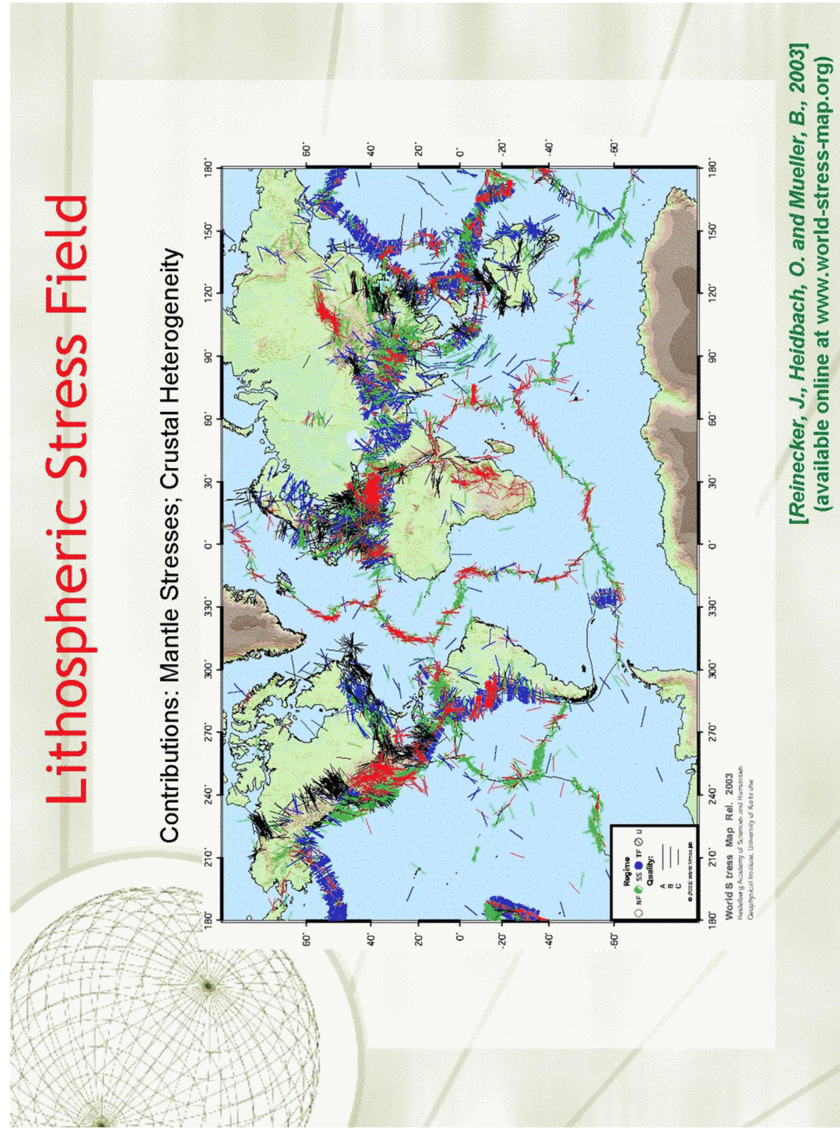
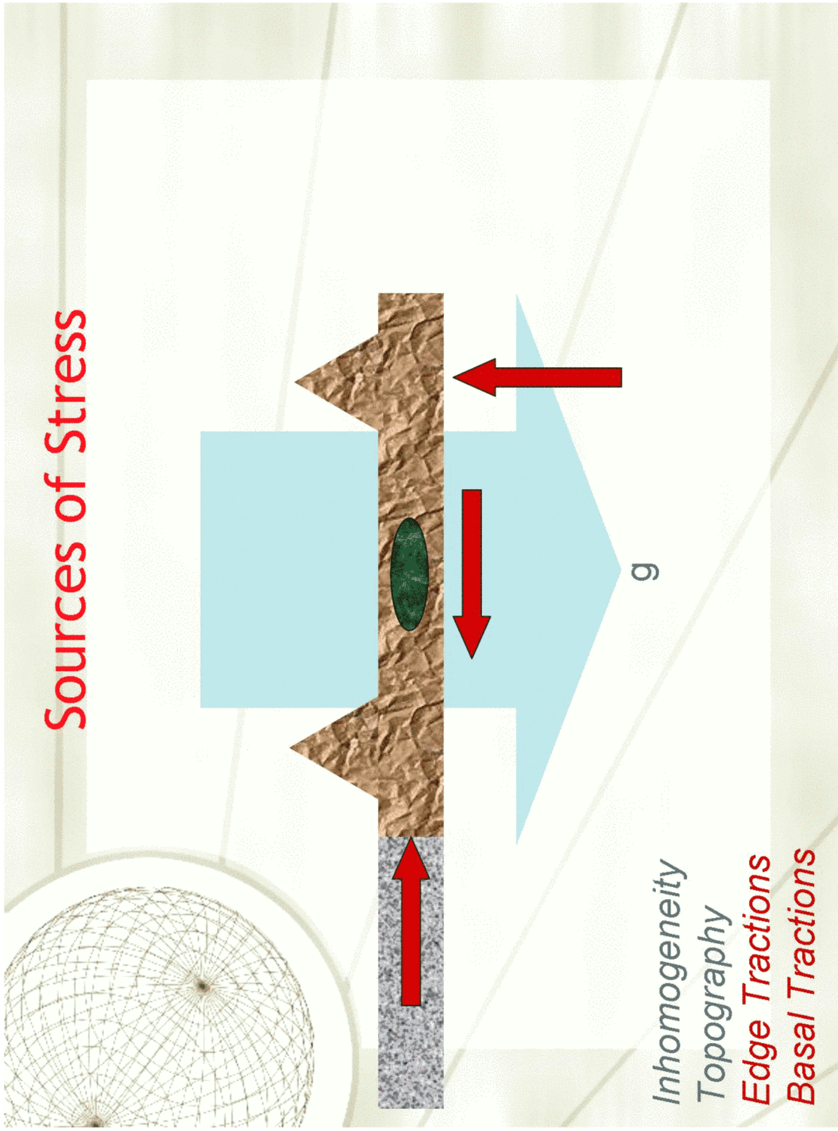


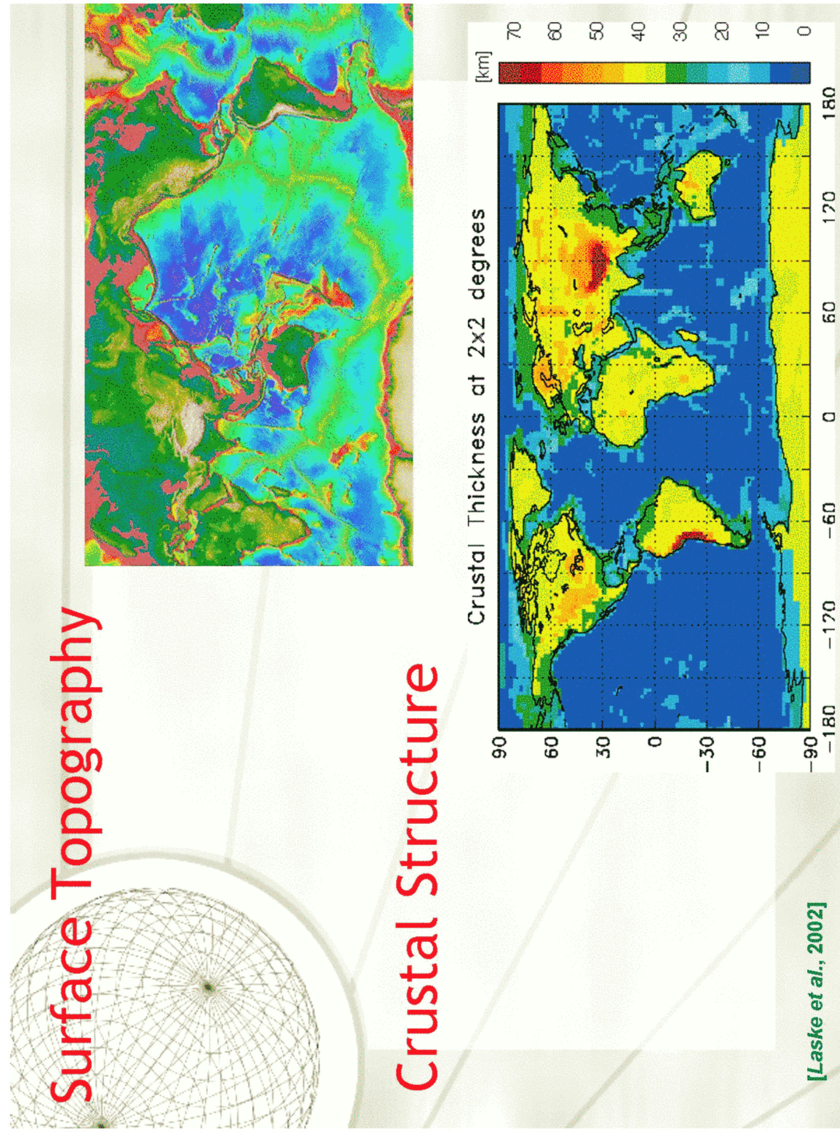
Plate Motions
Observed Velocities
Predicted Velocities
a) 10-25 Ma
b) 25-43 Ma
c) 43-48 Ma
d) 48-56 Ma
e) 56-64 Ma
Velocity / Average Velocity
0.00 0.25 0.50 0.75 1.00 1.25 1.50 2.00 2.50 3.00 10.00
Subducting Plate Motion
Strong Plate & Slab
Direct Pull from Upper Mantle Slabs
Overriding Plate Motion
Weak Upper Mantle
Viscous Lower Mantle
Viscous Stresses
Weak Lower Mantle Slab
Gravity
Slab Suction from Lower Mantle Slabs
[Conrad and Lithgow-Bertelloni, JGR, in PRESS]

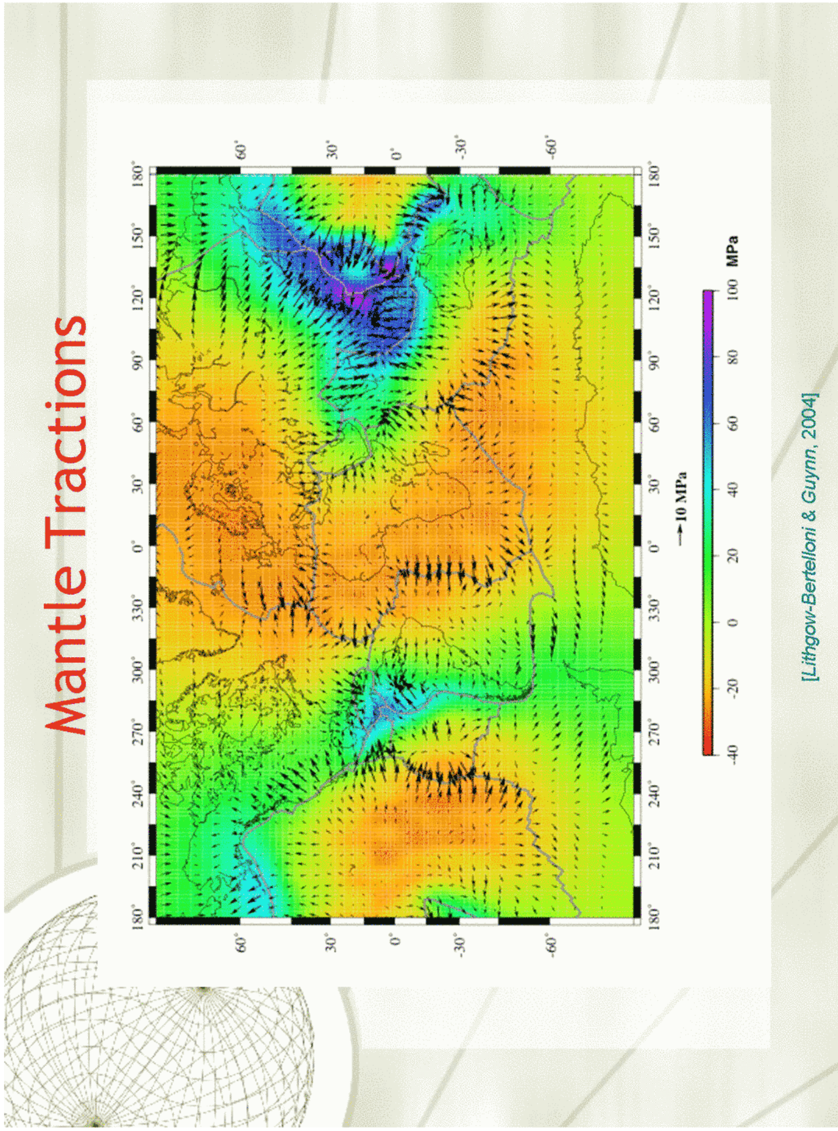

Dynamic Topography
$\mathbf{h} = -\tau_r/\delta\rho g$
h
ρ_1
ρ_2
τ_r
Mantle Flow

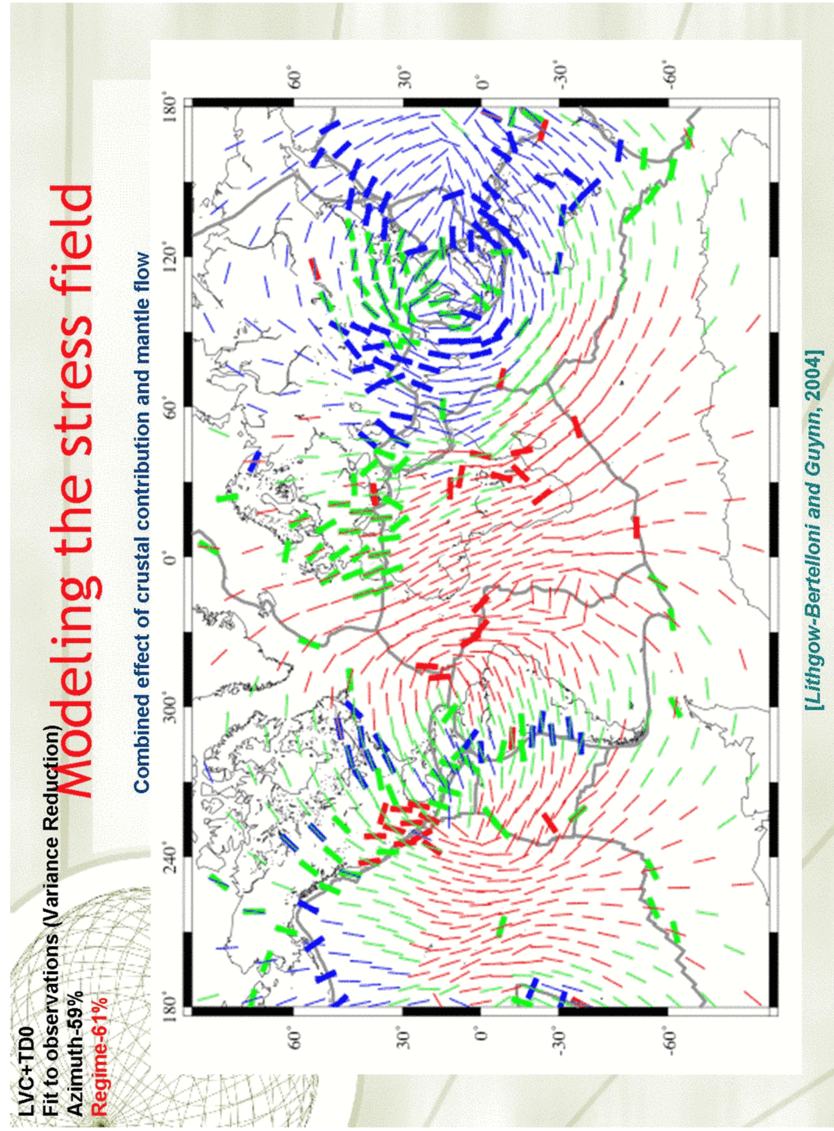

[Conrad, Lithgow-Bertelloni and Louden, 2004]
July 13, 2004
CIDER-KITP
Residual seafloor
Dynamic topography:
Grand (2003)
Ritsema et al. (1999)
Seafloor age (Ma)
East-west depth diff. (km)


Lithospheric Stress Field
Contributions: Mantle Stresses; Crustal Heterogeneity
[Reinecker, J., Heidbach, O. and Mueller, B., 2003]
(available online at www.world-stress-map.org)


Sources of Stress
g
Inhomogeneity
Topography
Edge Tractions
Basal Tractions


Surface Topography
Crustal Structure
Crustal Thickness at 2x2 degrees
[km]
[Laske et al., 2002]


Mantle Tractions
MPa
10 MPa
[Lithgow-Bertelloni & Guynn, 2004]


Modeling the stress field
Combined effect of crustal contribution and mantle flow
LVC+TD0
Fit to observations (Variance Reduction)
Azimuth-59%
Regime-61%
[Lithgow-Bertelloni and Guynn, 2004]