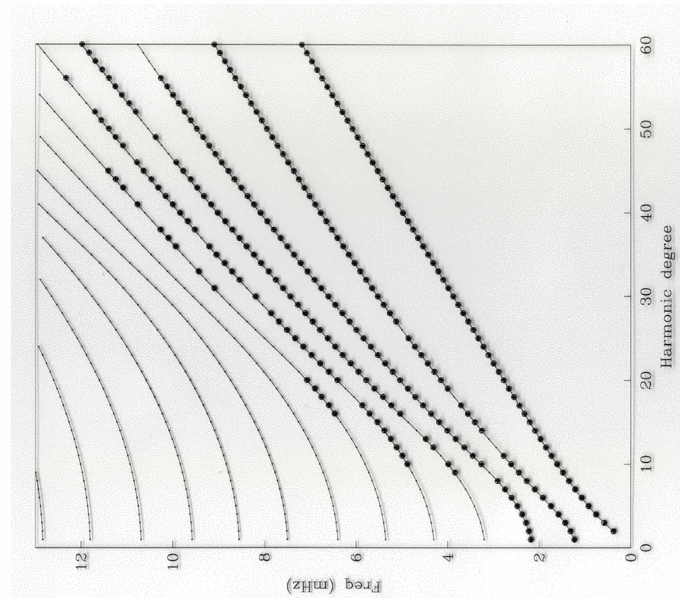
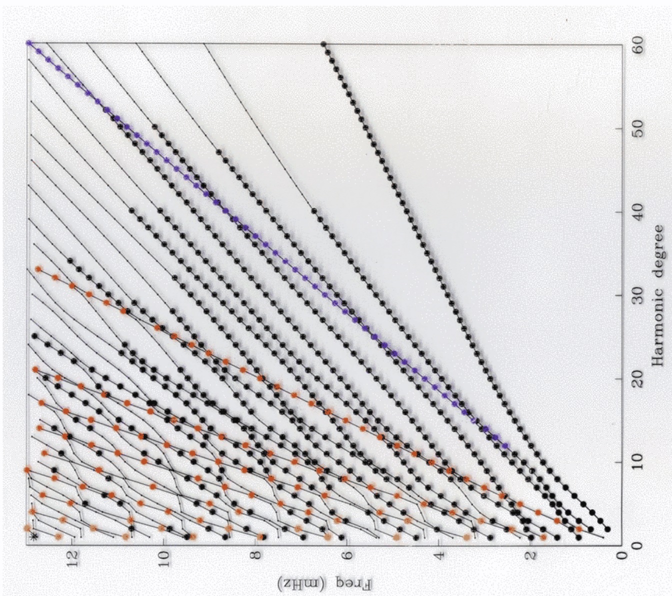
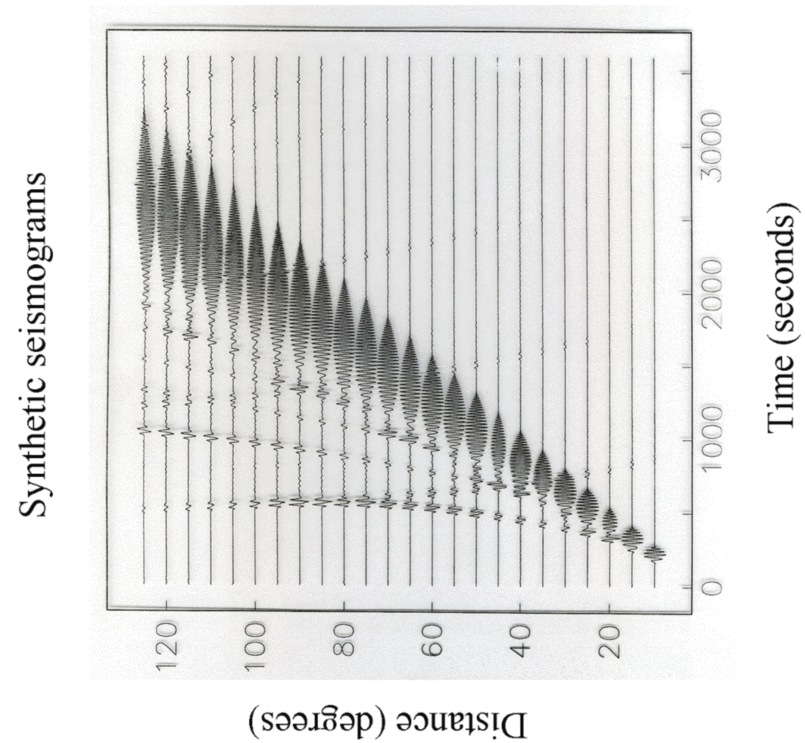
Surface waves

Guy Masters

Cider 2006




Harmonic degree
Freq (mHz)


Synthetic seismograms
Time (seconds)
Distance (degrees)

Standing waves and travelling waves

- Seismogram as a mode sum:

$$s(t) = \sum_k A_k \cos(\omega_k t + \phi_k) e^{-\alpha_k t}$$

In epicentral coordinates, A_k includes both the source excitation and the geometrical mode behavior as a function of epicentral distance, Δ. This latter term is proportional to a spherical harmonic Y_l^m. For surface waves, l is large and m is small, then

$$Y_l^m \simeq \frac{1}{\pi\sqrt{\sin\Delta}} \cos[(l + \tfrac{1}{2})\Delta - \frac{\pi}{4} + \frac{m\pi}{2}] e^{im\phi}$$

Jeans formula is $ka = l + \frac{1}{2}$ so with arc distance given by $x = a\Delta$,

$$Y_l^m \simeq \frac{1}{\pi\sqrt{\sin\Delta}} \cos[kx - \frac{\pi}{4} + \frac{m\pi}{2}] e^{im\phi}$$

Seismogram of a single mode becomes proportional to a plane wave:

$$s(t) \propto e^{(i\omega t - kx)}$$

Phase and group velocity

In general $k = k(\omega)$ (dispersion). Mode sum becomes:

$$s(x,t) = \int B(\omega) e^{i(\omega t - k(\omega)x)}\, d\omega \quad c(\omega) = \frac{\omega}{k(\omega)}$$

$B(\omega)$ due to source – slowly varying. Consider phase, f,

$$f = \omega t - k(\omega)x$$

Main contribution to integral when phase is stationary ($df/d\omega = 0$). If ω_s is the point where the phase is stationary occurs, we have:

$$t - \frac{dk}{d\omega}(\omega_s)x = 0$$

$$U = \frac{x}{t} = \frac{d\omega}{dk}$$

The energy associated with a particular frequency group centered on ω_s travels with the "group velocity", $U(\omega_s)$.

Phase or group velocity?

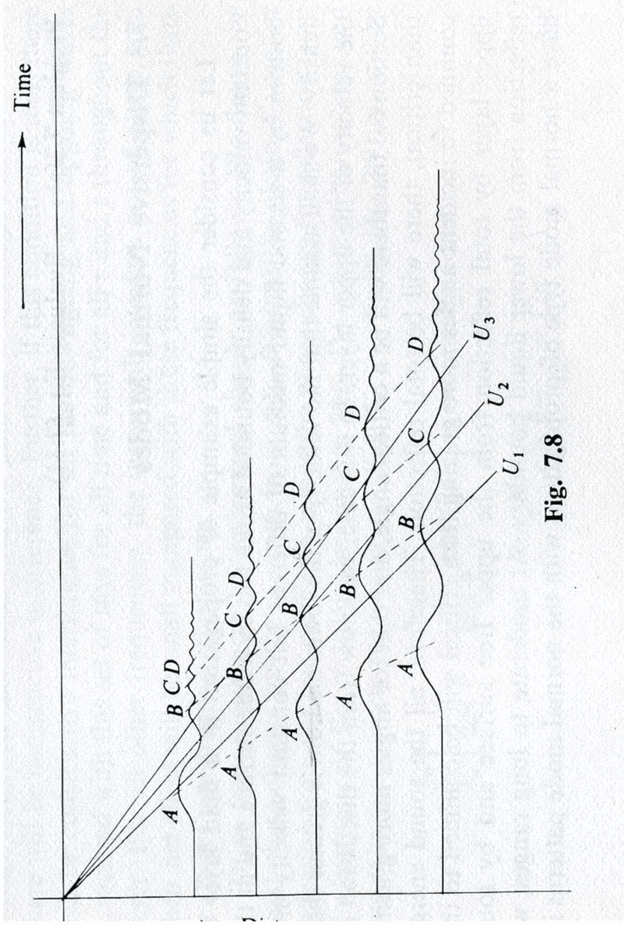


Fig. 7.8

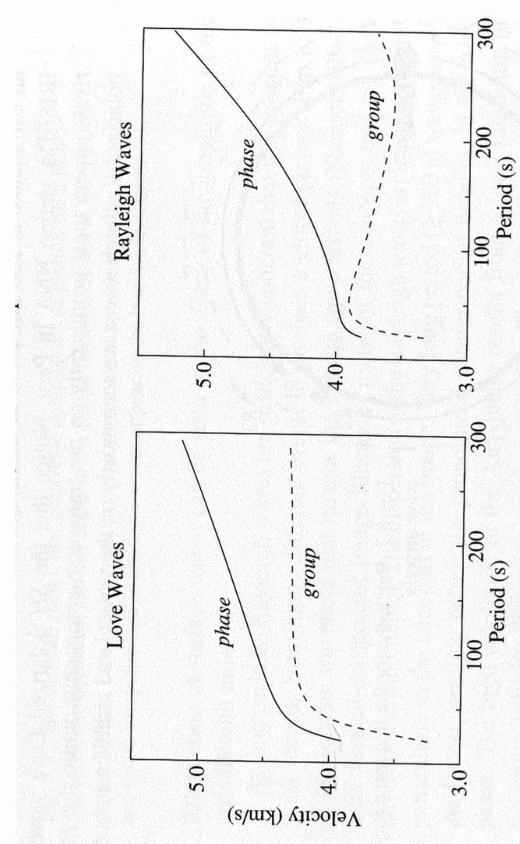


Fig. 8.6. Fundamental Love and Rayleigh dispersion curves computed from the isotropic PREM model (courtesy of Gabi Laske).

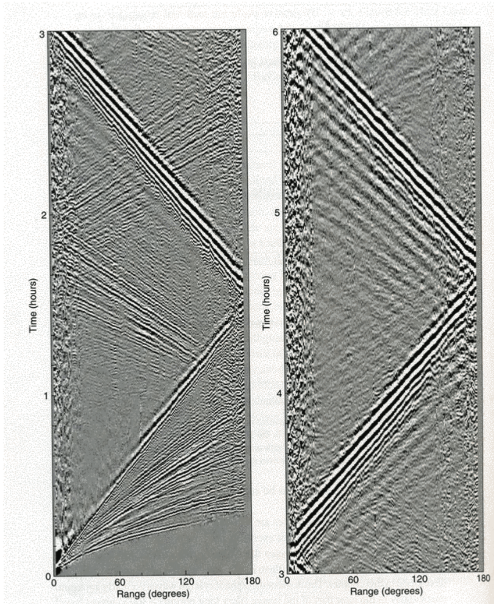


Fig. 8.9. An image of Earth's long-period seismic response on vertical component seismographs as a function of time and distance to an earthquake. Positive amplitudes are shown as black, and negative amplitudes are shown as white. The Rayleigh wave arrivals $R1$ and $R2$ are visible in the left panel showing the first 3 hours of data, whereas $R3$ and $R4$ are seen on the right panel.

Measuring phase

Measuring phase

The phase of the Fourier transformed seismogram is just kx (with a source term) but we need to be careful how we measure this to avoid bias. Measure relative phase between data and synthetic or measure phase difference between orbits. Transfer function approach.
Single spectral estimate:

$$S_{obs}(\omega) = T(\omega) S_{syn}(\omega)$$

$$T(\omega) = \frac{S_s^*(\omega) S_o(\omega)}{S_s^*(\omega) S_s(\omega)} = \frac{C_{12}(\omega)}{C_{11}(\omega)}.$$

Better to use a multi-taper estimate. Minimize

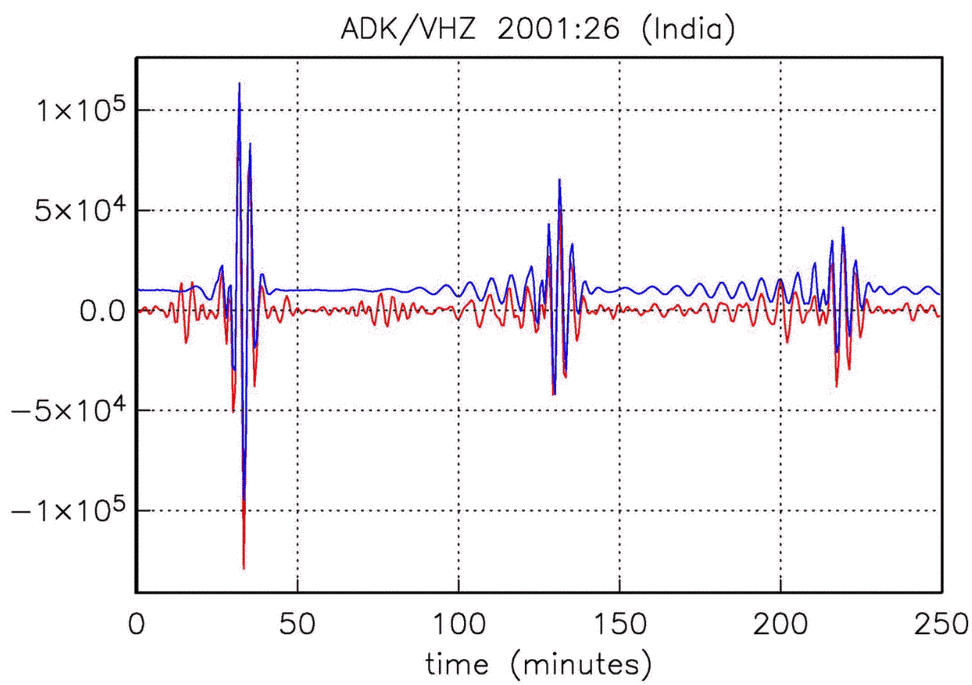
$$\|\mathbf{S}_o(\omega) - T \cdot \mathbf{S}_s(\omega)\|^2$$

Solution is:

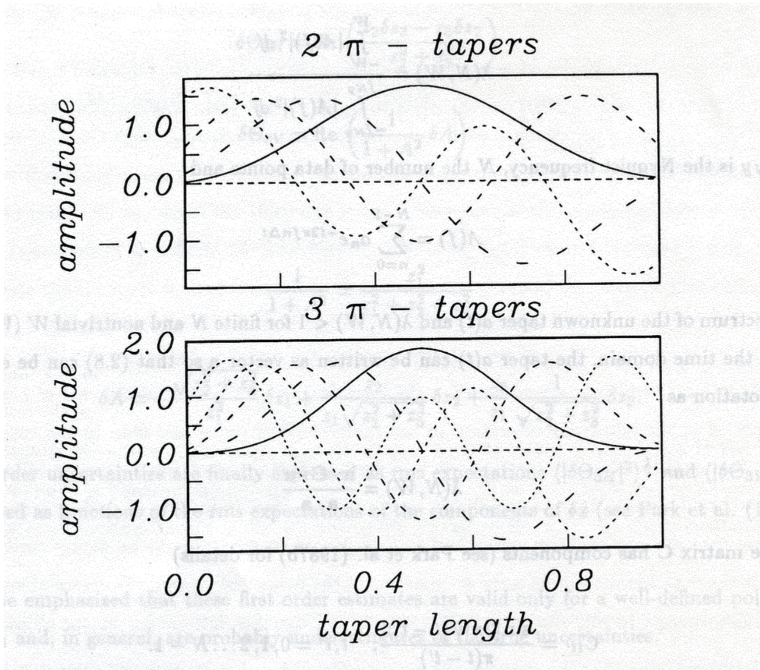
$$T(\omega) = \frac{\mathbf{S}_o(\omega) \cdot \mathbf{S}_s^*(\omega)}{\mathbf{S}_s(\omega) \cdot \mathbf{S}_s^*(\omega)}.$$

T related to perturbation in (complex) wavenumber:

$$T(\omega) = e^{-\delta\bar{\gamma}(\omega)x} e^{-i\delta\bar{k}(\omega)x}$$



Red = data, blue = fundamental mode synthetic



Phase and group velocity maps

Phase and group velocity maps

- Ray theory (great circle propagation):

$$e^{ikx} \Longrightarrow e^{i\bar{k}x} \qquad \text{where} \qquad \bar{k} = \frac{1}{x}\int_0^x k(x')dx'.$$

where $k = k(\omega, \theta, \phi)$ or

$$c(\omega, \theta, \phi) = c_0(\omega) + \delta c(\omega, \theta, \phi)$$

In terms of measured phase:

$$\delta\Psi = \delta\bar{k}a\Delta = -\frac{\omega a}{c_0}\int_0^\Delta \frac{\delta c(\theta,\phi)}{c_0}\, d\gamma$$

- Ray theory for group velocity

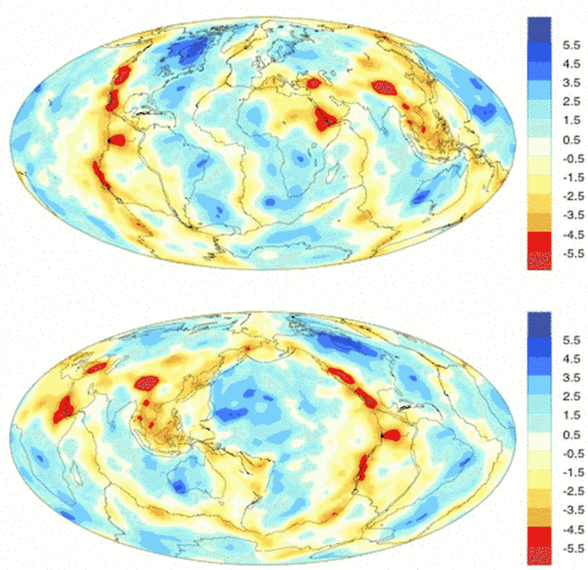
$$t_g = a\int_0^\Delta \frac{1}{U} d\gamma$$

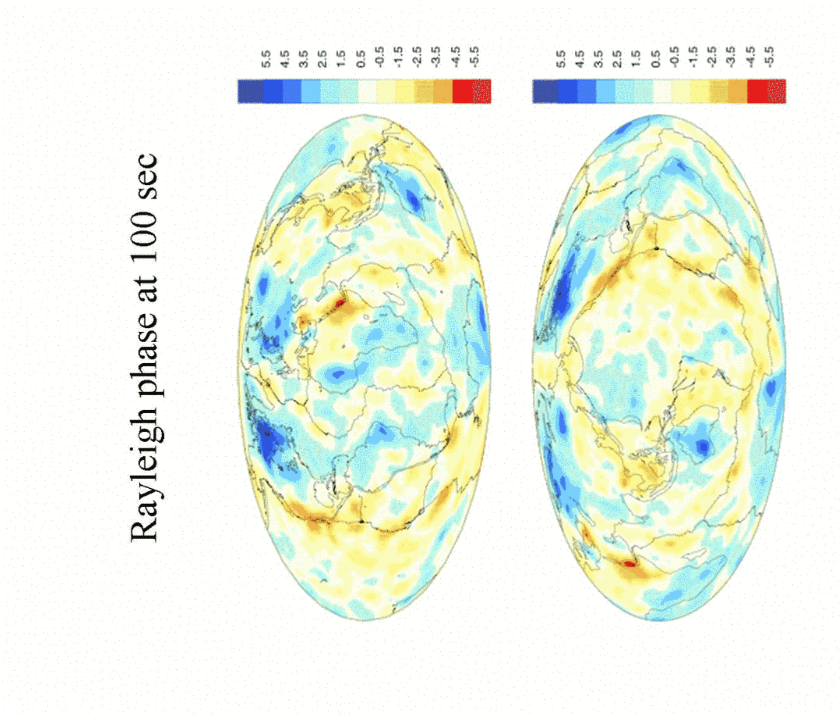
Perturbation from reference group arrival time:

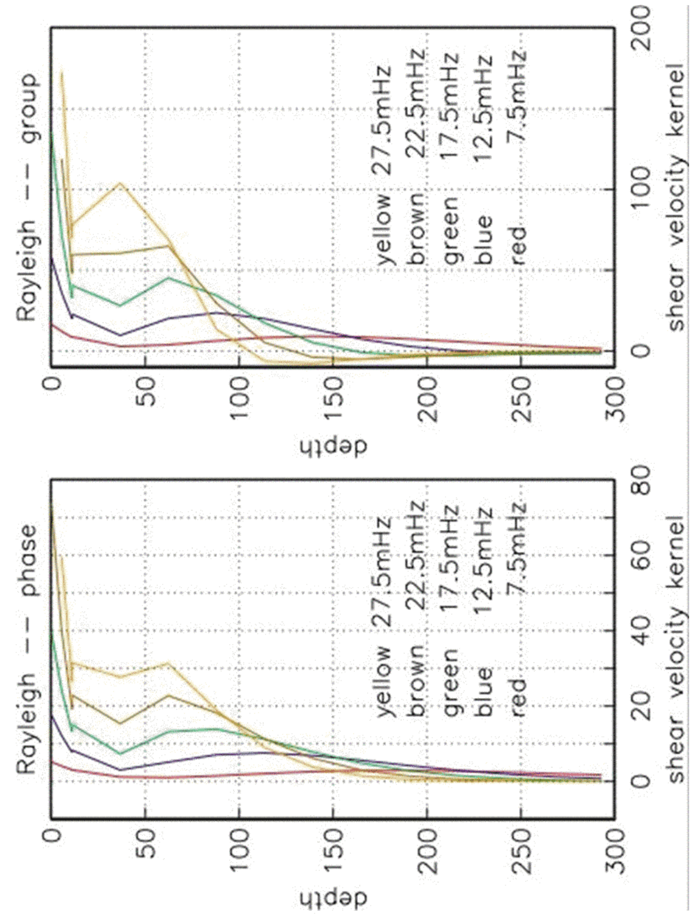
$$\delta t_g = -\frac{a}{U_0}\int_0^\Delta \frac{\delta U(\theta,\phi)}{U_0}\, d\gamma$$

(same form as phase velocity inversion)

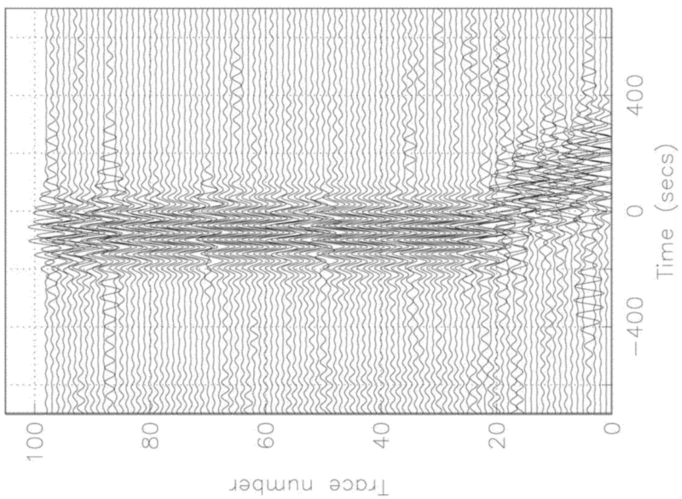
Rayleigh phase at 50 sec



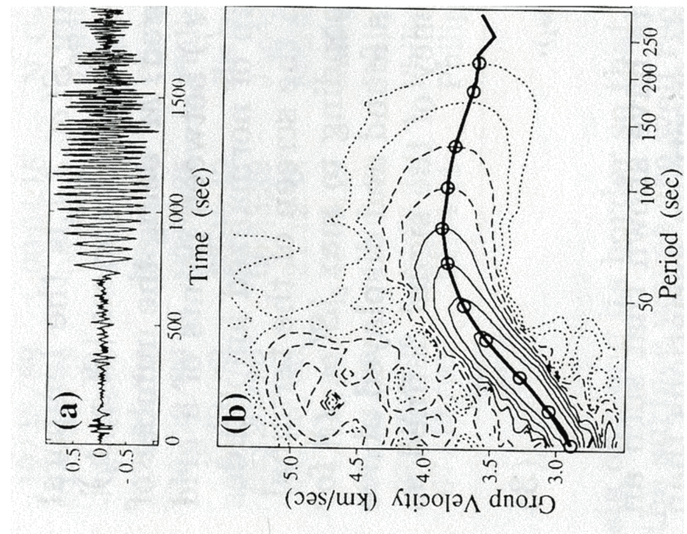

Rayleigh phase at 100 sec
5.5
4.5
3.5
2.5
1.5
0.5
-0.5
-1.5
-2.5
-3.5
-4.5
-5.5


Rayleigh -- group
Rayleigh -- phase
yellow 27.5mHz
brown 22.5mHz
green 17.5mHz
blue 12.5mHz
red 7.5mHz
shear velocity kernel
depth

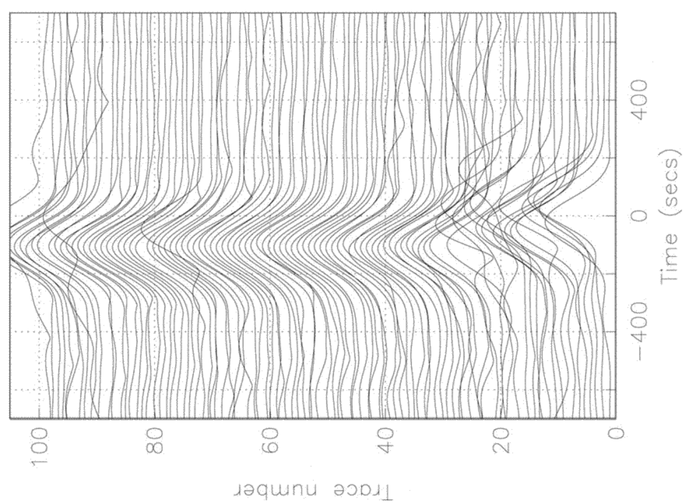
Undispersed 50 s Rayleigh waves



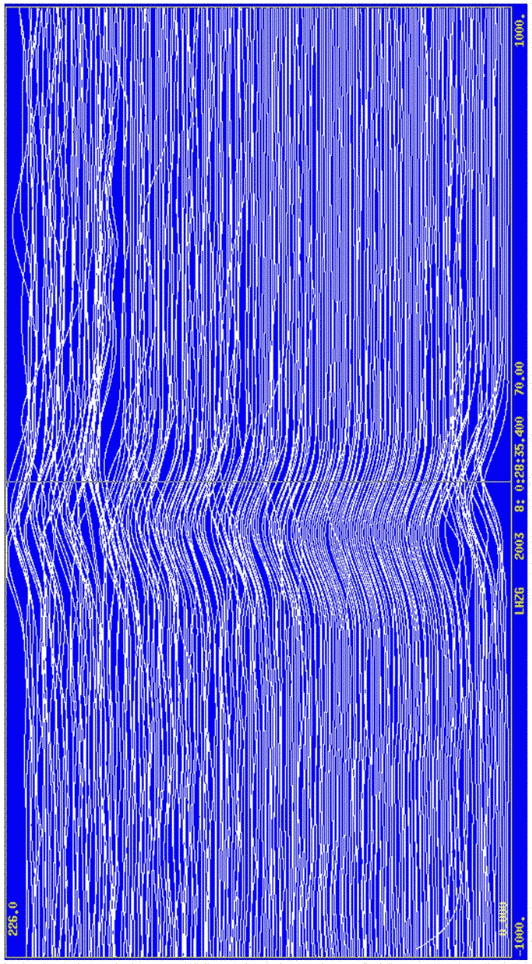
Note large number of cycle shifts possible



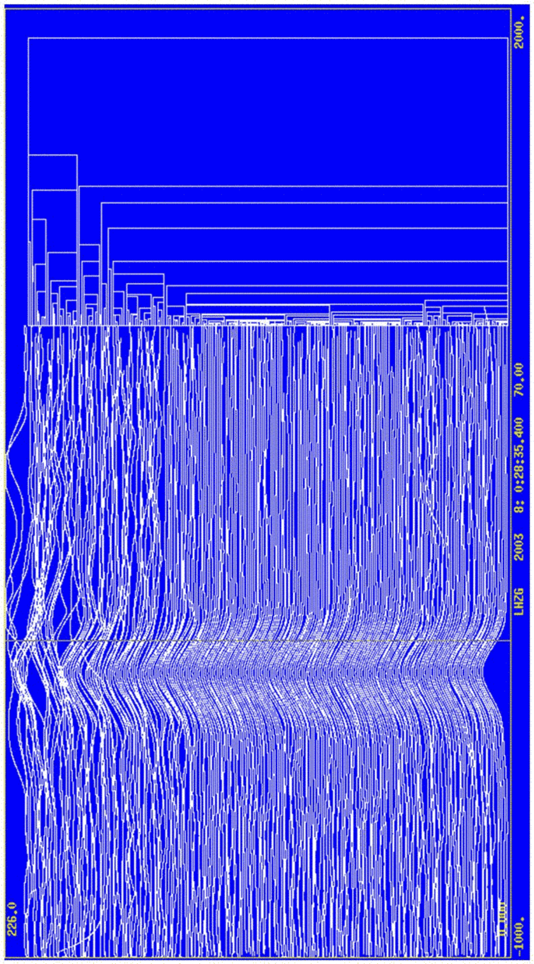
After computation of envelope functions



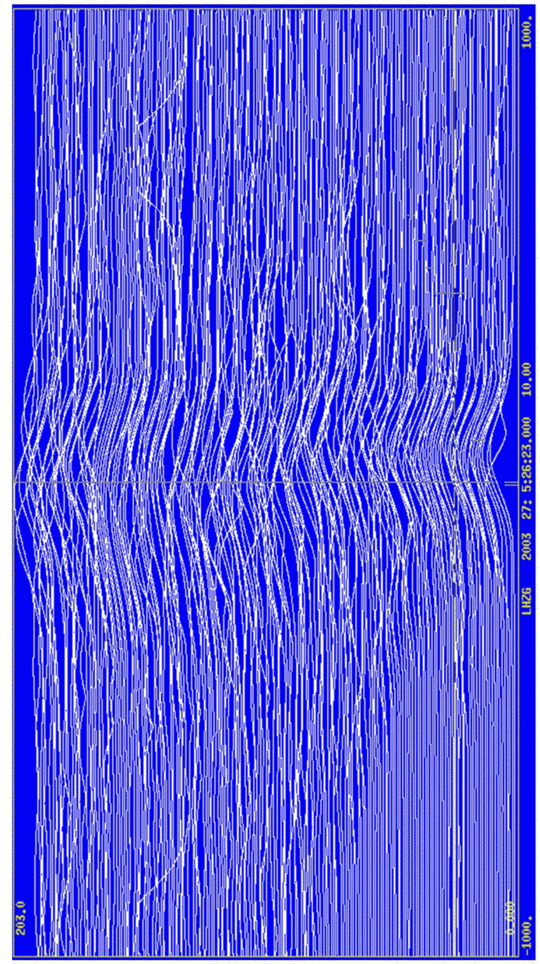
50 sec Rayleigh waves -- a typical example



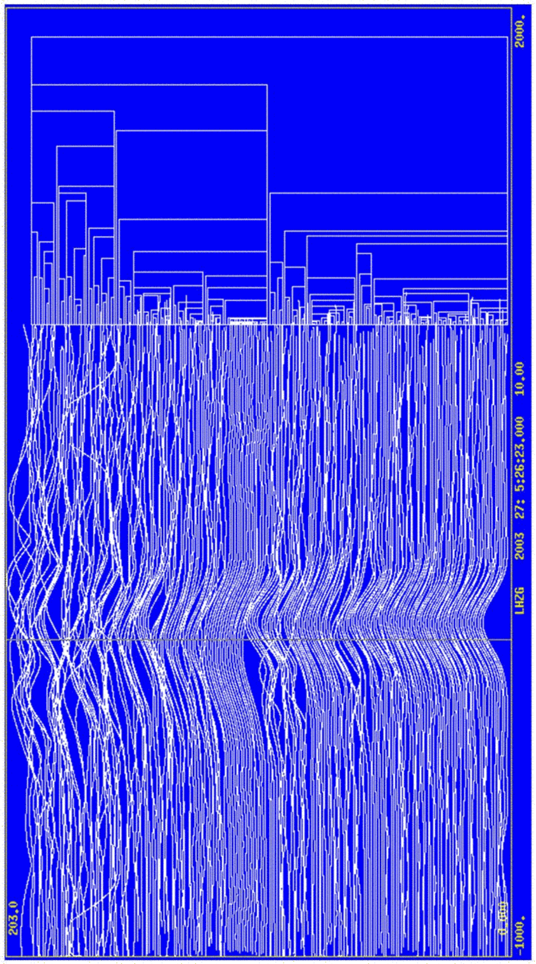
After clustering….



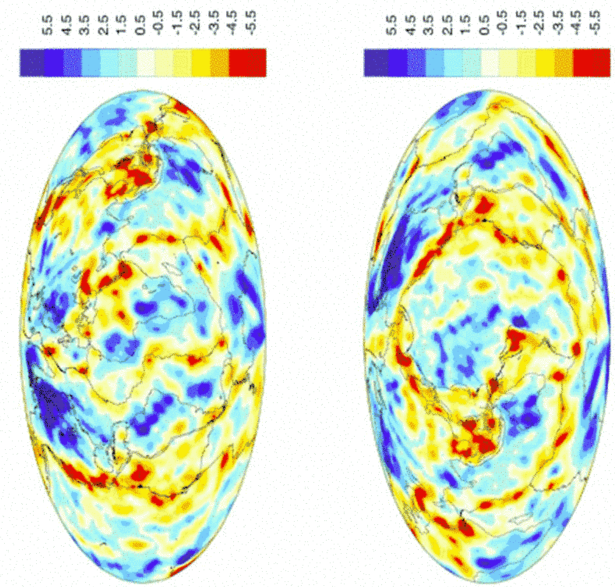
A more complicated example

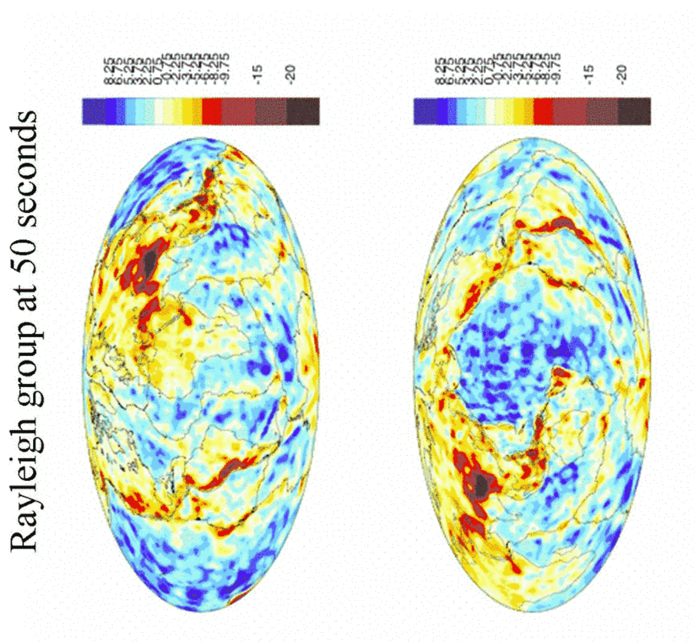


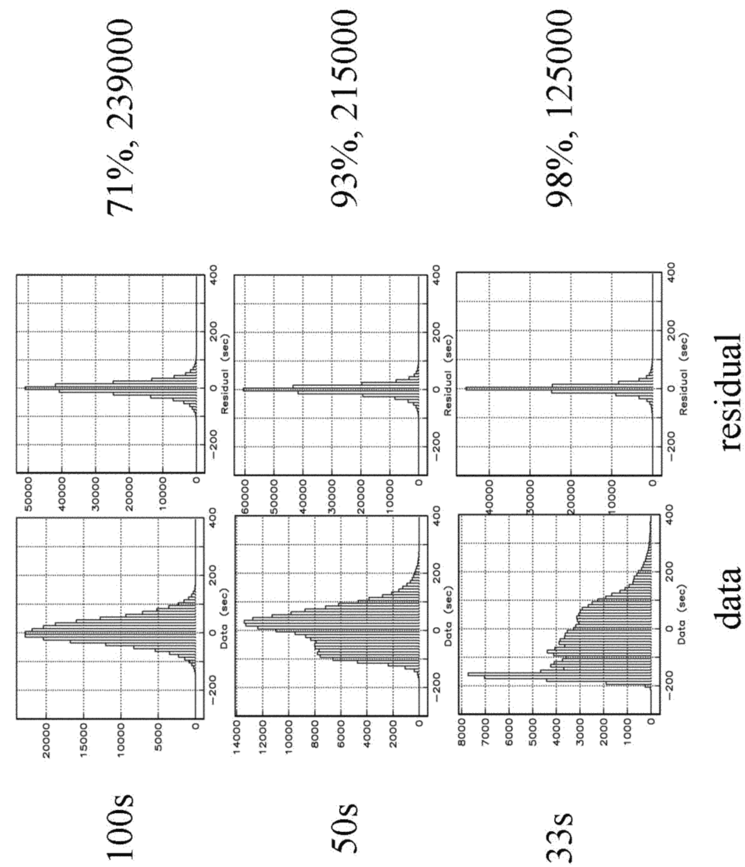
After clustering….

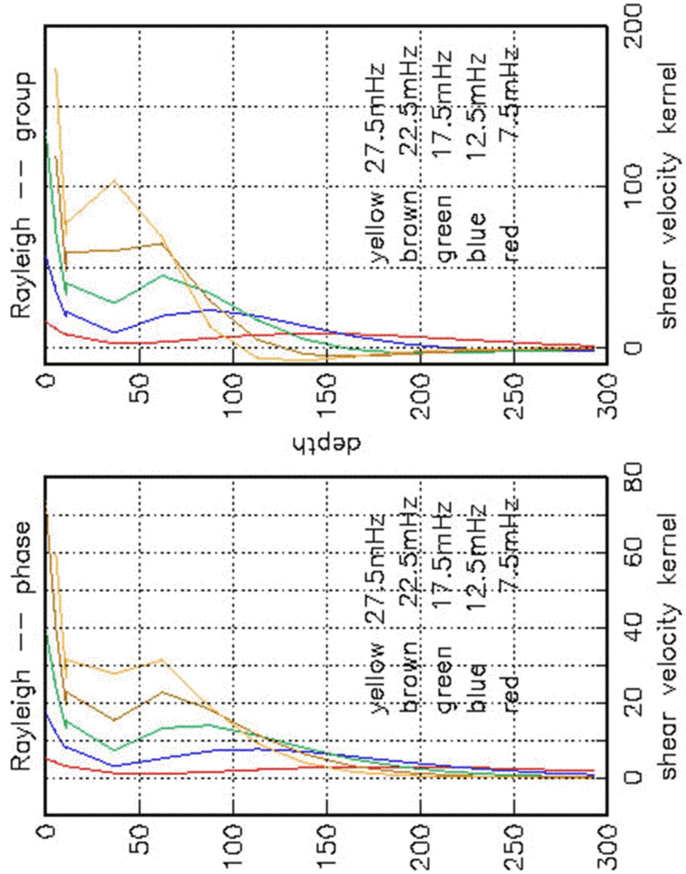


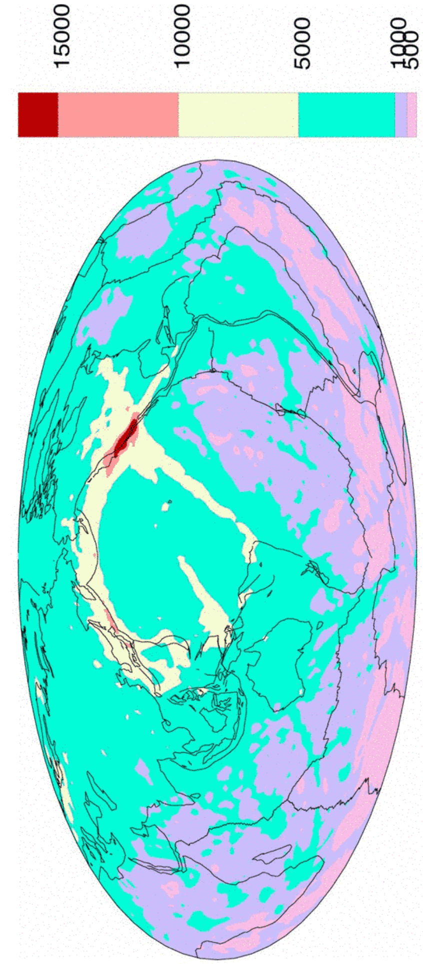
Rayleigh group at 100 seconds




Rayleigh group at 50 seconds
8.25
6.75
5.25
3.75
2.25
0.75
-0.75
-2.25
-3.75
-5.25
-6.75
-8.25
-9.75
-15
-20

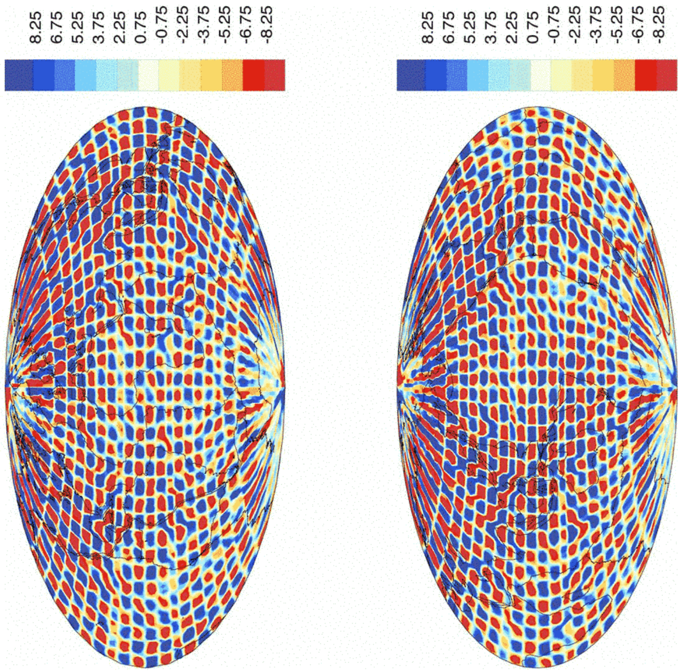

71%, 239000
93%, 215000
98%, 125000
100s
50s
33s
data
residual
Data (sec)
Residual (sec)



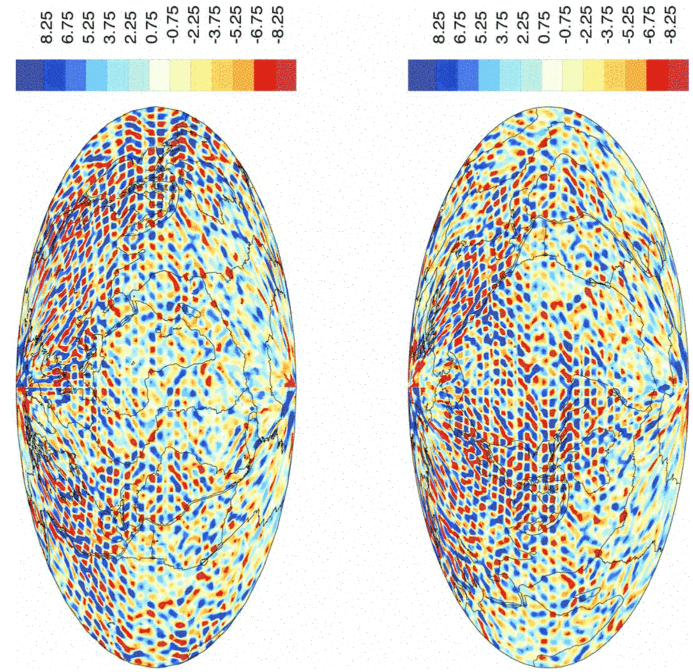


Hit count, 2 degree blocks

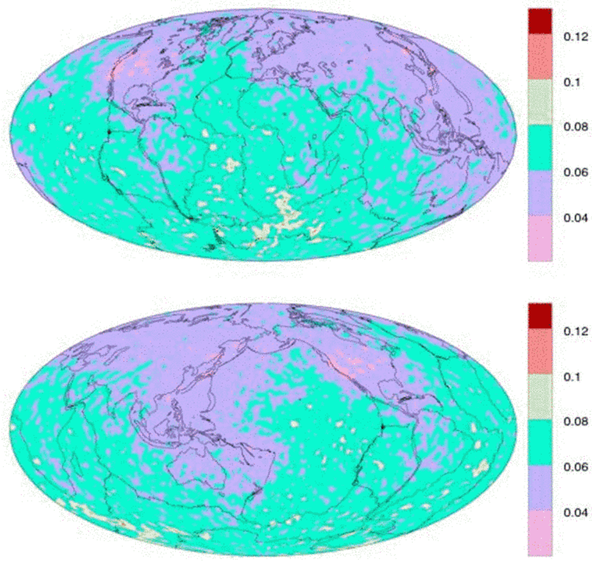
Resolution of 1000km structure is global



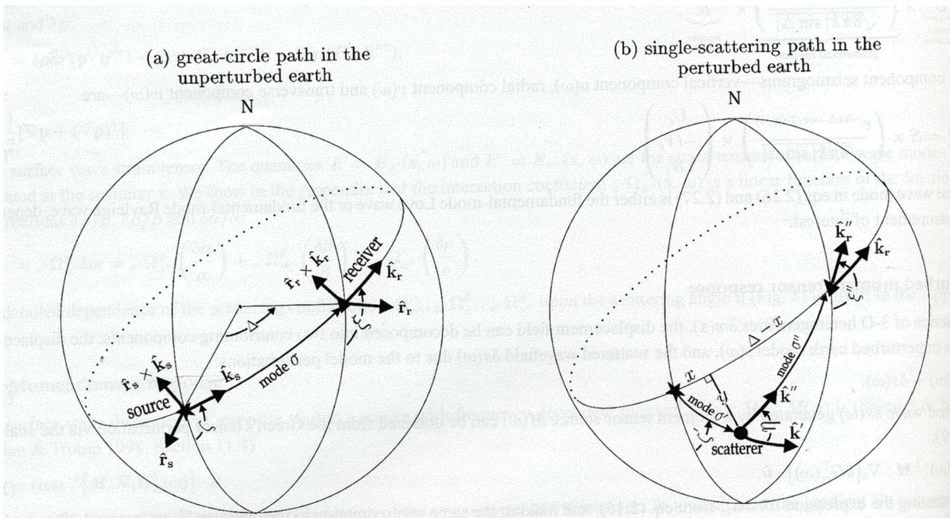
Resolution of finer structure is more variable



Error map for Rayleigh waves at 50 sec



Finite frequency kernels



Finite Frequency Kernels

- Born (single scattering) approximation

$$s(\omega) \rightarrow s(\omega) + \delta s(\omega)$$

$$\delta s(\omega) = \sum_{\sigma'\sigma''} \int \mathcal{S}' \times \left(\frac{e^{-i(k'\Delta' - n'\pi/2 + \pi/4)}}{\sqrt{8\pi k' |\sin\Delta'|}} \right) \times {}_{\sigma'}\Omega_{\sigma''} \times \left(\frac{e^{-i(k''\Delta'' - n''\pi/2 + \pi/4)}}{\sqrt{8\pi k'' |\sin\Delta''|}} \right) \times \mathcal{R}''\, dV$$

Relation to measured transfer function:

$$s(\omega) + \delta s(\omega) = T(\omega) s(\omega)$$

3D kernel:

$$\delta\Psi(\omega) = \mathrm{Im}\left(\frac{\delta s}{s}\right) = \int_V K_\Psi \delta m\, dV$$

K_Ψ is pretty ugly (see notes) but can be converted to a 2D kernel for a phase velocity perturbation with some approximations:

$$K_\Psi = \frac{2k^2 \sin\left[k(\Delta' + \Delta'' - \Delta) + \pi/4\right]}{\sqrt{8\pi k}\,(|\sin\Delta'||\sin\Delta''|/|\sin\Delta|)}$$

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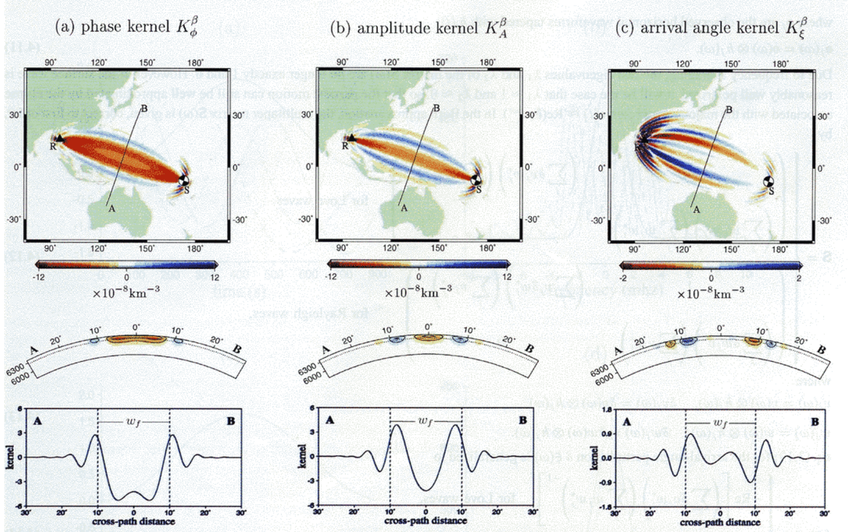
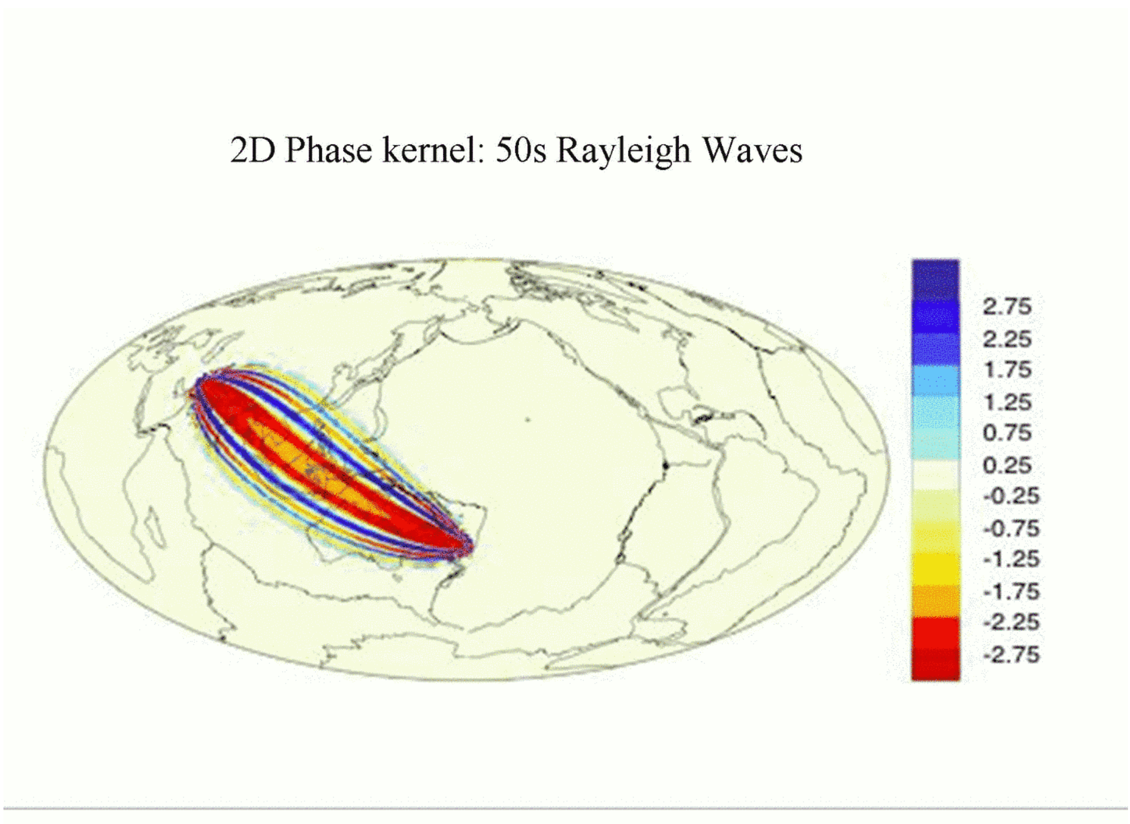
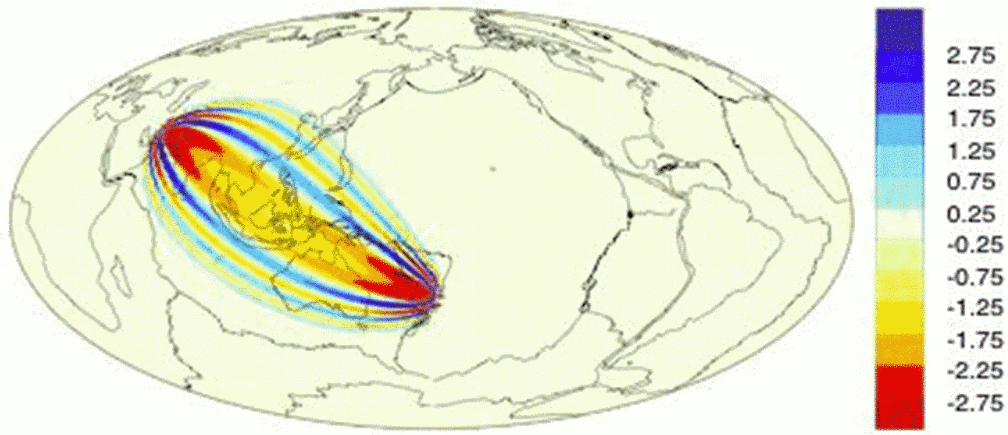


Figure 4. 3-D sensitivity kernels K_ϕ^β, K_A^β, K_ξ^β for a 10 mHz Love wave, excited by a 52 km deep strike-slip seismic source (S). Love wave radiation is maximum in the direction of the source–receiver geometrical ray (see beachball symbol). The epicentral distance to the receiver (R) is $\Delta = 80°$. Sensitivity kernels are for 800 s cosine-taper measurements, with the taper centred at the group arrival time predicted by PREM. Top: Map view of kernels at the depth of approximately greatest sensitivity, 108 km. Middle: Slice view of cross-section AB, dotted lines are plotted at 108 km depth. Bottom: AB cross-section at a depth of 108 km; dashed lines indicate the width of the first Fresnel zone, w_f. Mode-coupling effects have been ignored, i.e. $\sigma_1 = \sigma_2 = \sigma$.



2D Phase kernel: 100s Rayleigh waves



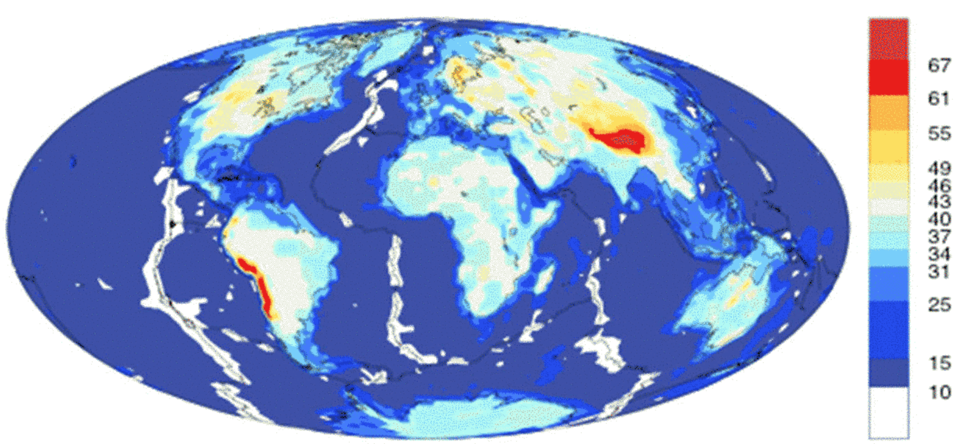
Finite frequency kernels

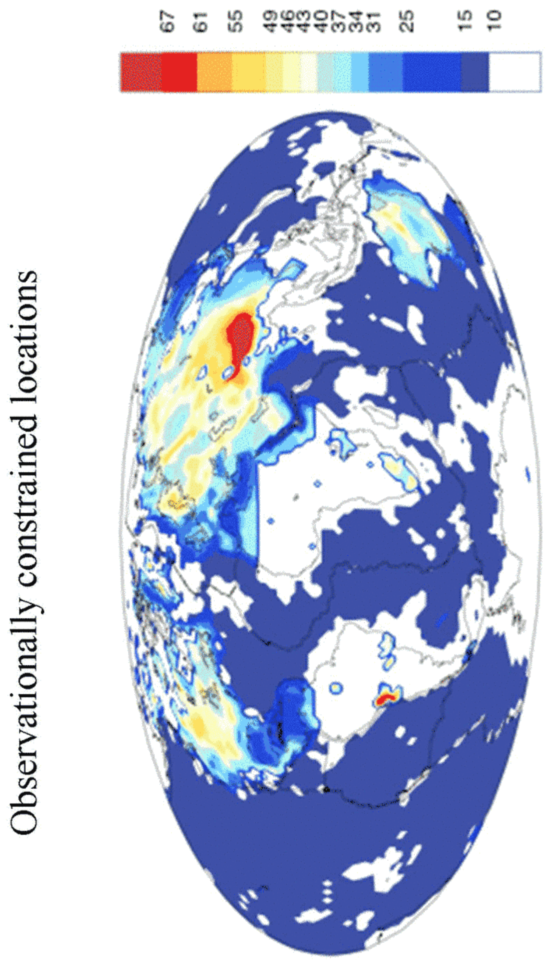
- Expect effect to be greatest at longest periods
- Difficult to make direct comparison with great circle result since different damping is implied
- Comparisons with analyses of SEM synthetics are ambiguous but 2D kernels do not seem effective -- need to use 3d kernels (Zhou et al, 2004)?
- What do you find?

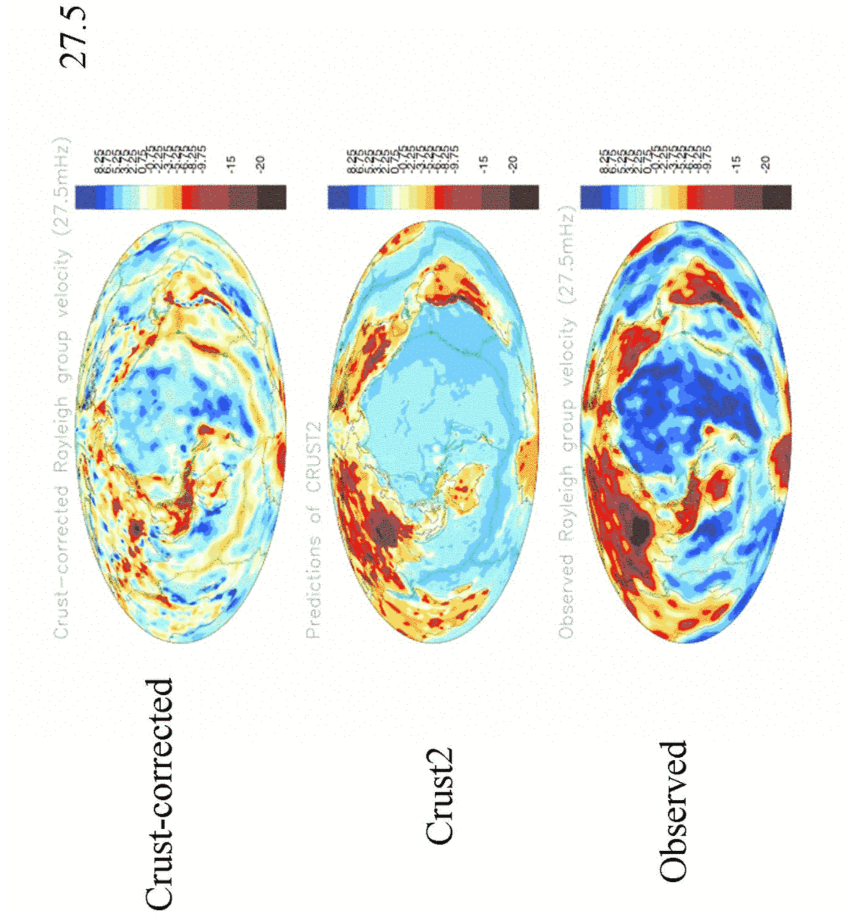
Inversion for crustal thickness

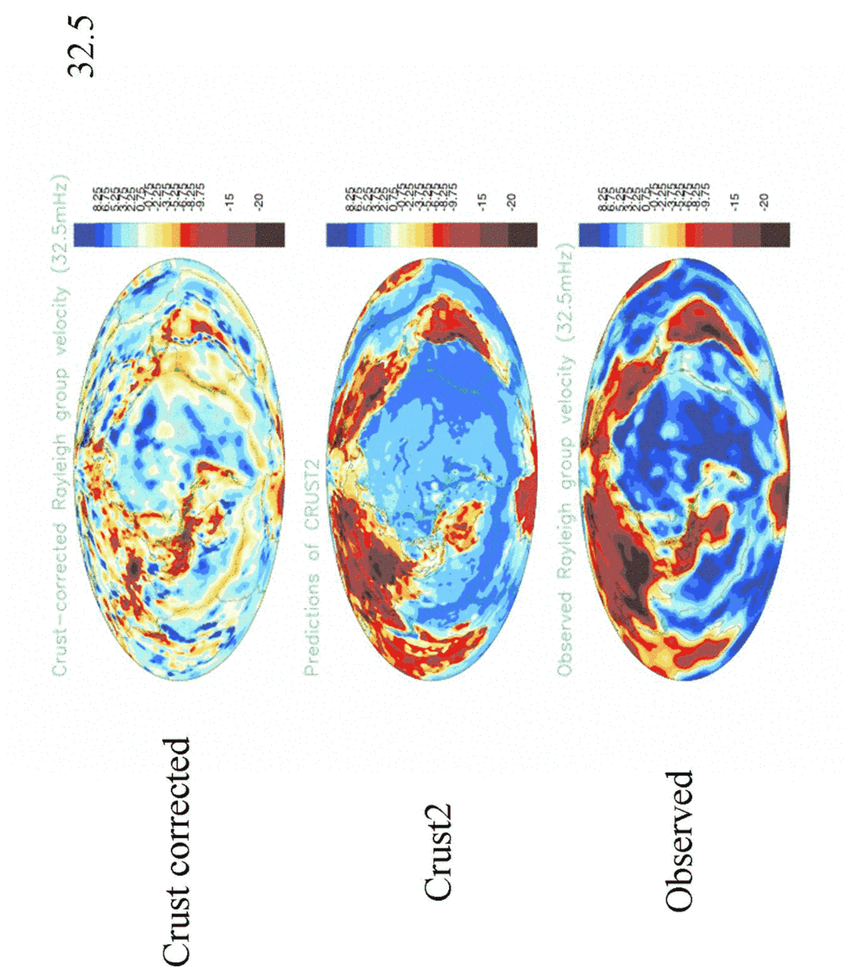
- Correct maps for the CRUST2 model
- Invert corrected maps with additional point constraints from receiver function studies for perturbation to CRUST2 moho depth
- Include only maps for periods shorter than 40 seconds to avoid mantle signal.

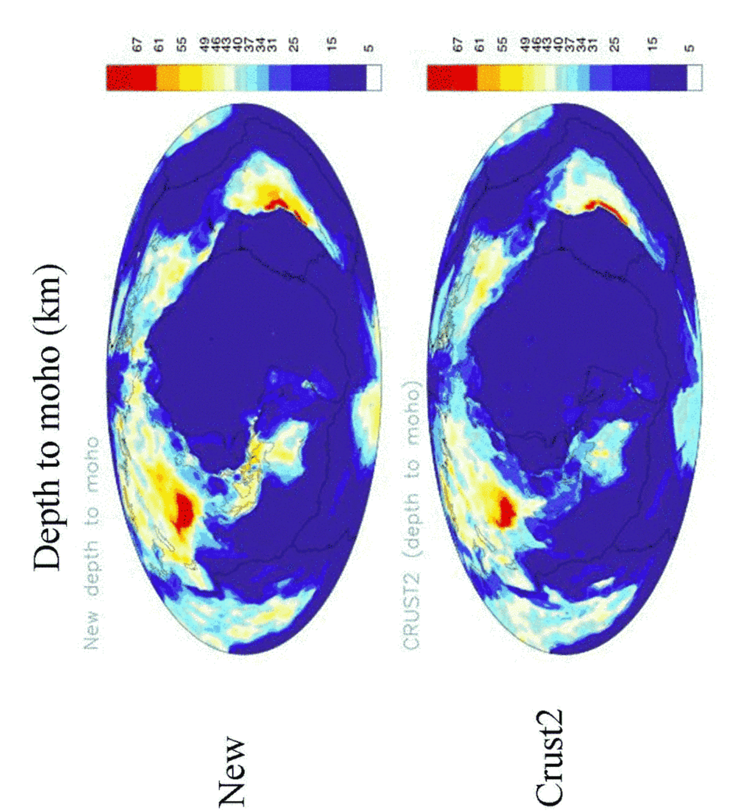
Depth to moho in CRUST2.0

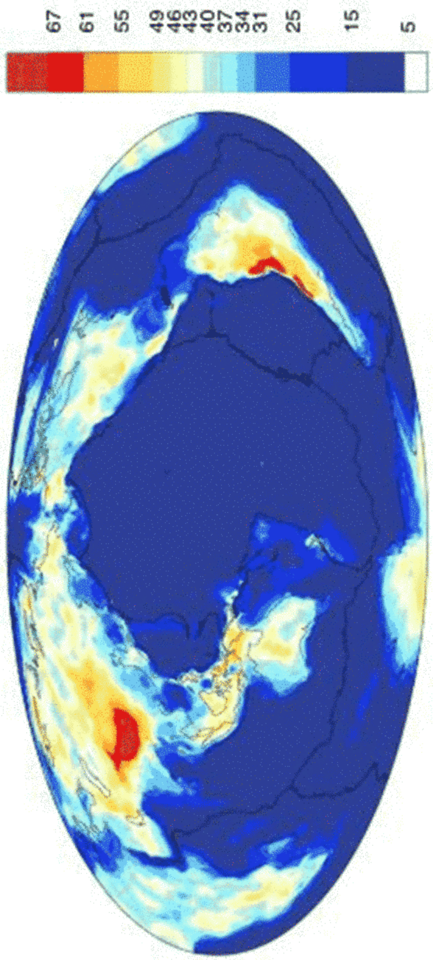


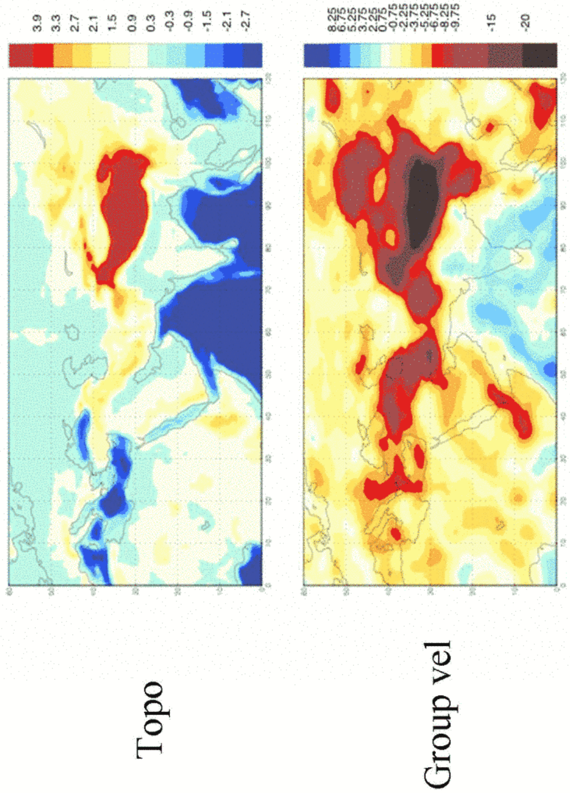

Observationally constrained locations
67
61
55
49
46
43
40
37
34
31
25
15
10

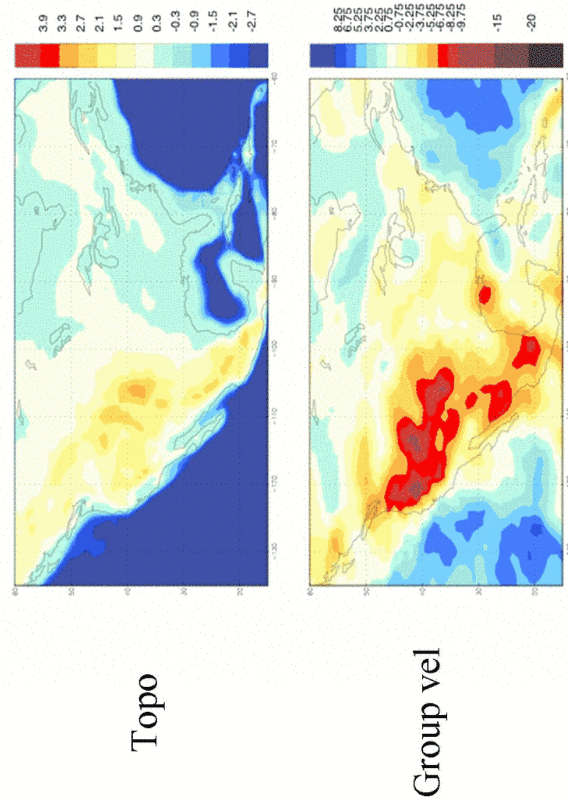

27.5
Crust-corrected Rayleigh group velocity (27.5mHz)
Predictions of CRUST2
Observed Rayleigh group velocity (27.5mHz)
Crust-corrected
Crust2
Observed

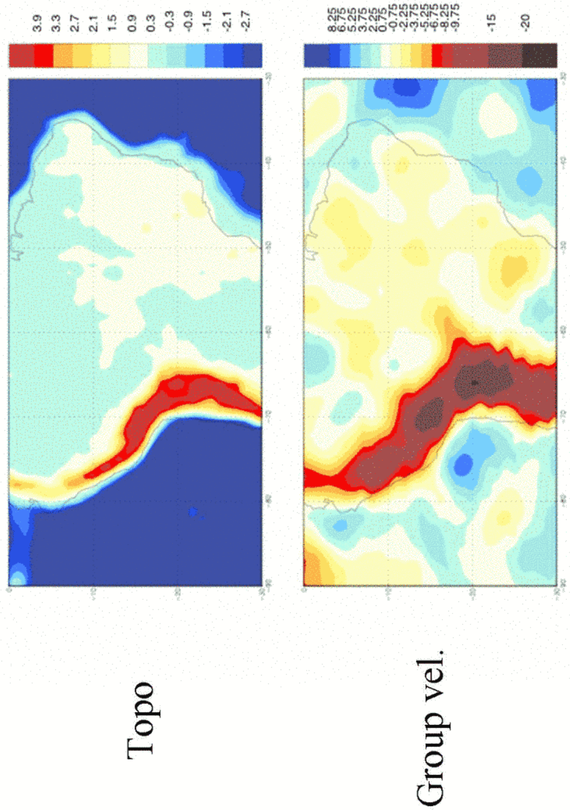

32.5
Crust corrected
Crust2
Observed


Depth to moho (km)
New
Crust2

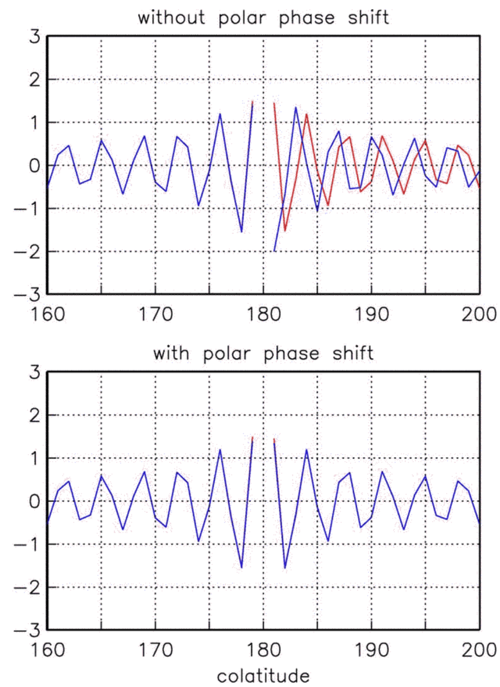

67
61
55
49
46
43
40
37
34
31
25
15
5


3.9
3.3
2.7
2.1
1.5
0.9
0.3
-0.3
-0.9
-1.5
-2.1
-2.7
8.25
6.75
5.25
3.75
2.25
0.75
-0.75
-2.25
-3.75
-5.25
-6.75
-8.25
-9.75
-15
-20
Topo
Group vel


3.9
3.3
2.7
2.1
1.5
0.9
0.3
-0.3
-0.9
-1.5
-2.1
-2.7
8.25
6.75
5.25
3.75
2.25
0.75
-0.75
-2.25
-3.75
-5.25
-6.75
-8.25
-9.75
-15
-20
Topo
Group vel

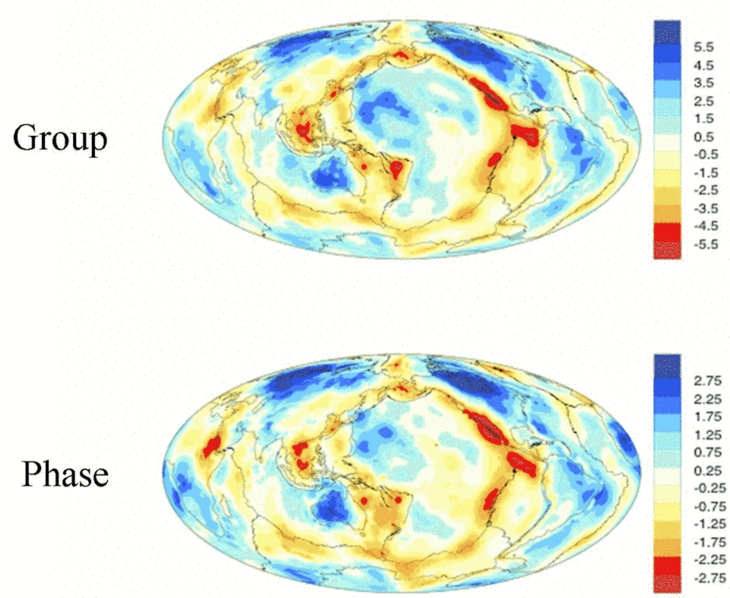

Topo
Group vel.

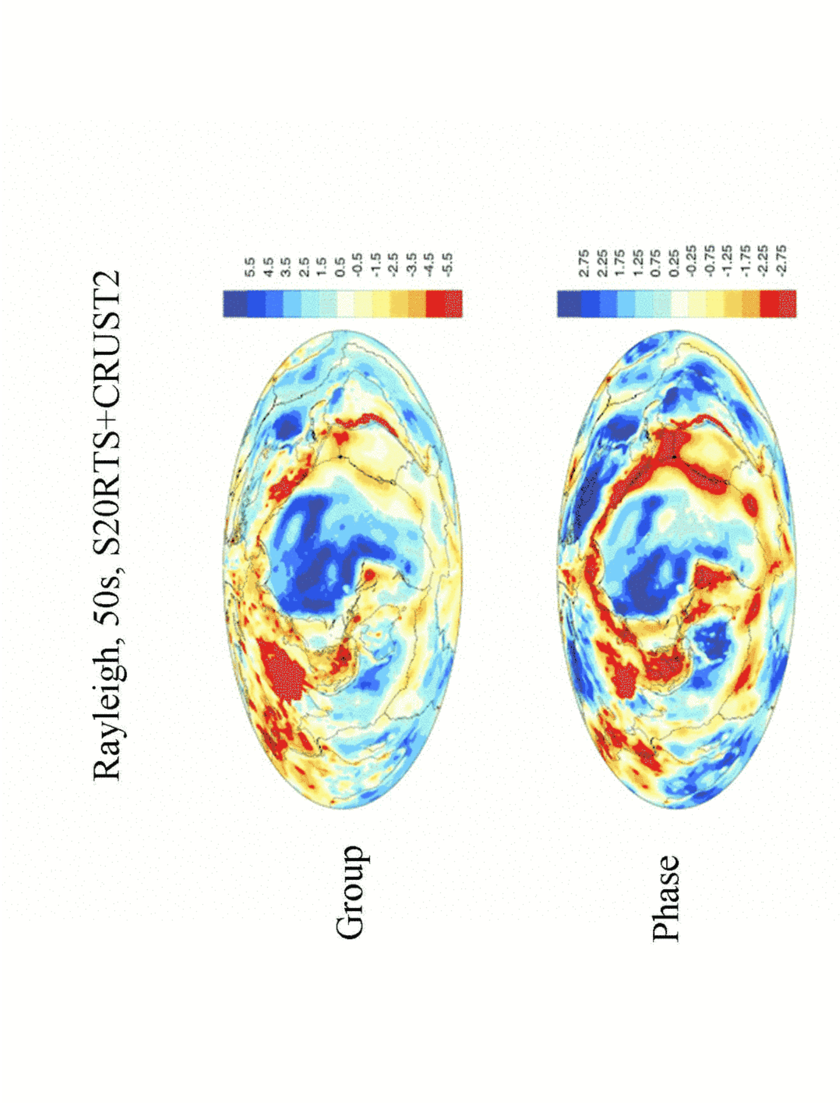
Where do those pole phase shifts come from?

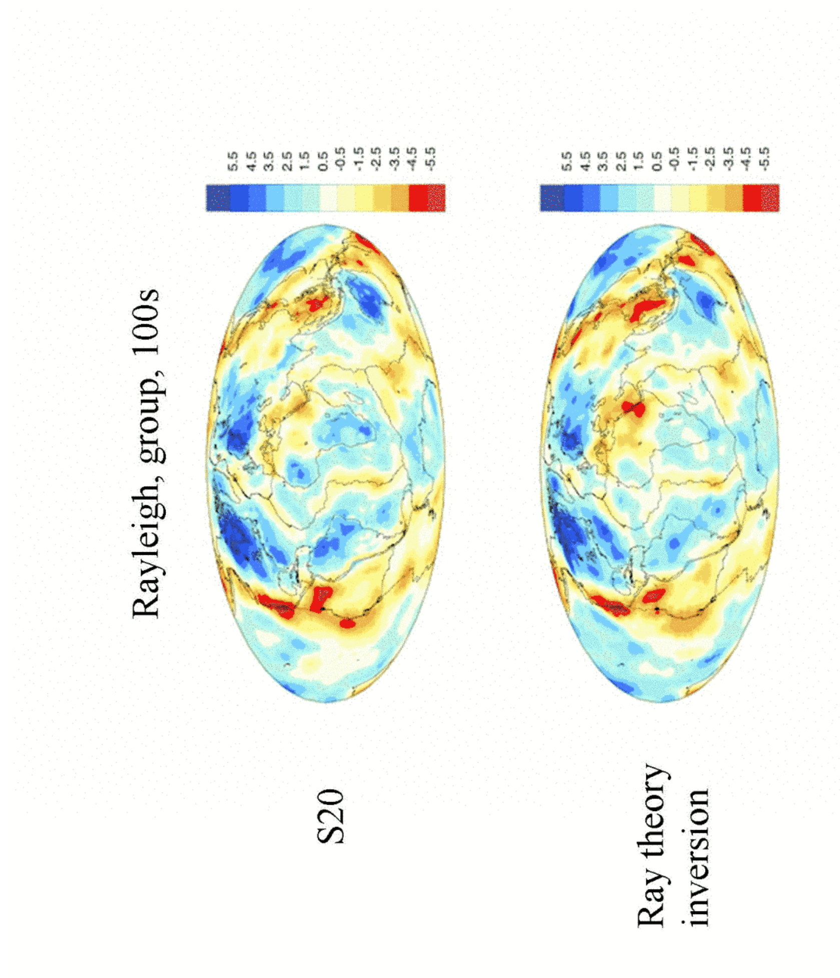


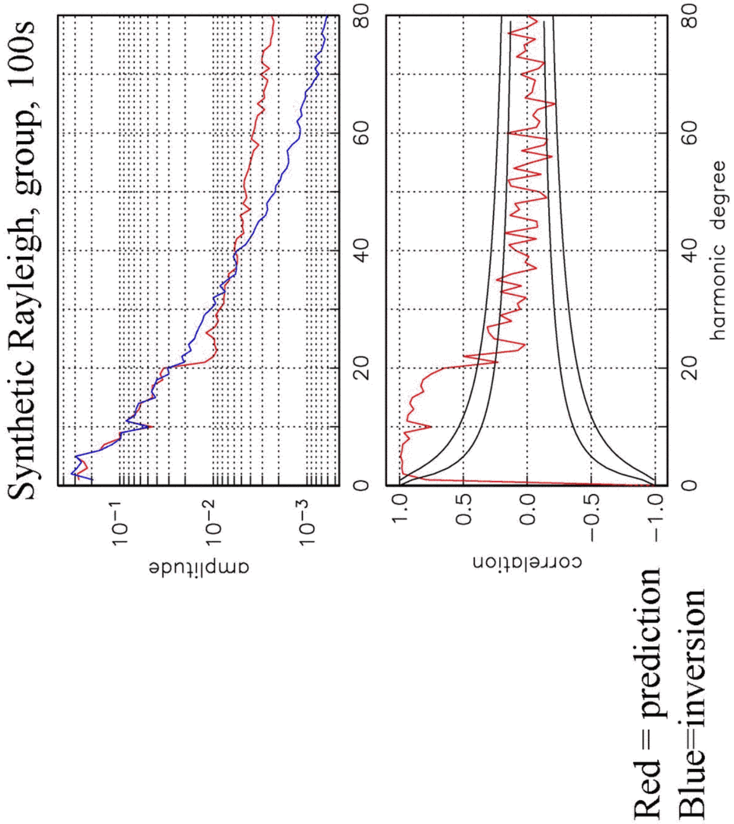
Comparison of Xlm with asymptotic form for l=100

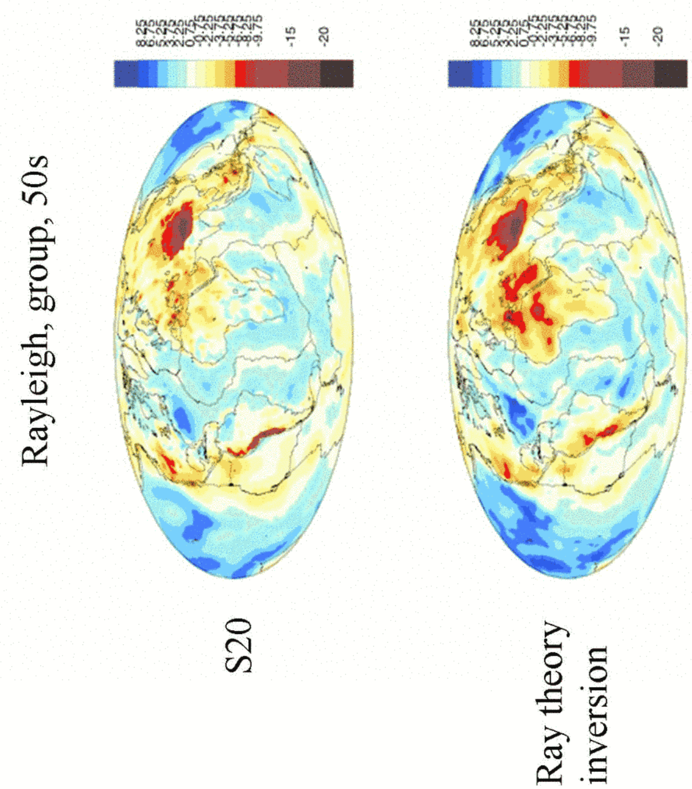
Rayleigh, 100s, s20RTS+CRUST2



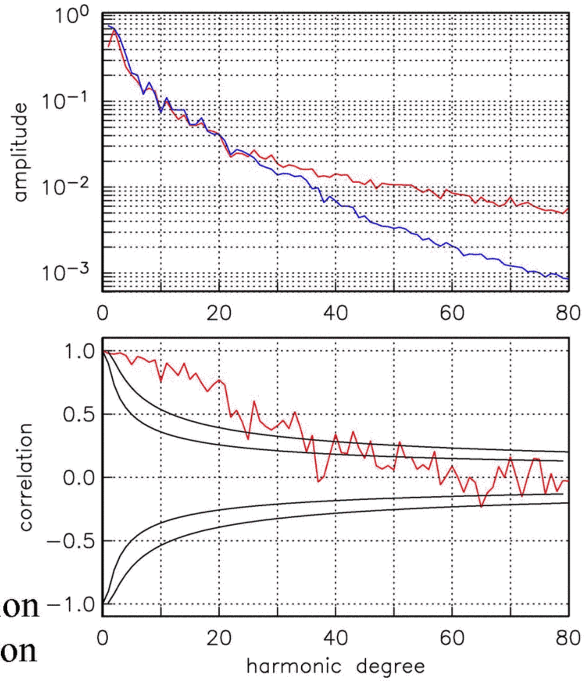

Rayleigh, 50s, S20RTS+CRUST2
Group
Phase
5.5 4.5 3.5 2.5 1.5 0.5 -0.5 -1.5 -2.5 -3.5 -4.5 -5.5
2.75 2.25 1.75 1.25 0.75 0.25 -0.25 -0.75 -1.25 -1.75 -2.25 -2.75


Rayleigh, group, 100s
S20
Ray theory inversion
5.5 4.5 3.5 2.5 1.5 0.5 -0.5 -1.5 -2.5 -3.5 -4.5 -5.5
5.5 4.5 3.5 2.5 1.5 0.5 -0.5 -1.5 -2.5 -3.5 -4.5 -5.5

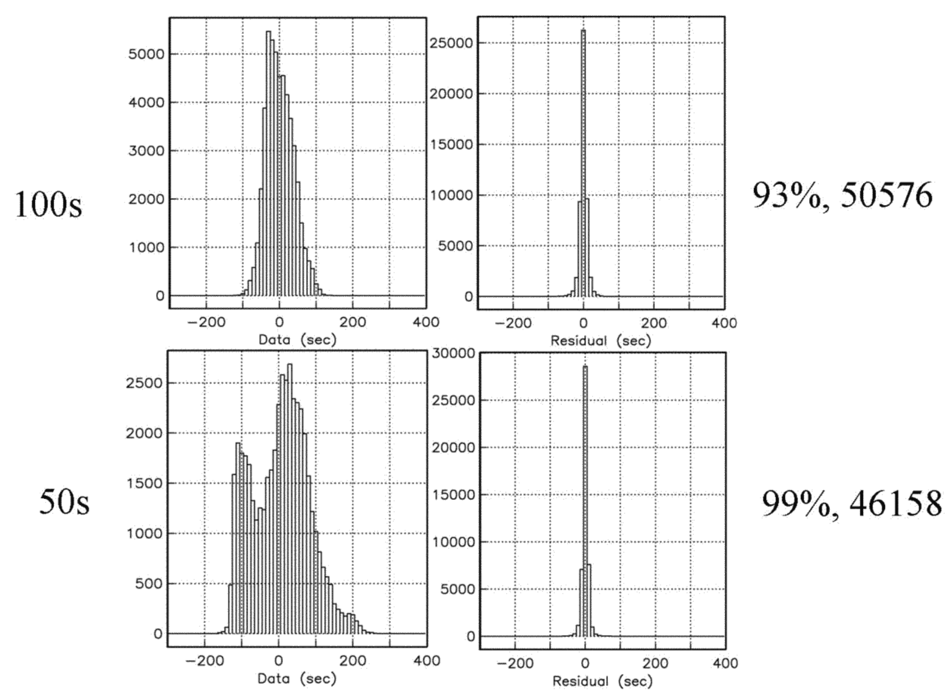

Synthetic Rayleigh, group, 100s
amplitude
correlation
harmonic degree
Red = prediction
Blue=inversion


Rayleigh, group, 50s
S20
Ray theory
inversion

Synthetic Rayleigh, group, 50s



Red = prediction
Blue = inversion



Inversion of SEM data using ray theory