Amontons' law in single-asperity contacts: Pros and cons

(an attempted literature review)

No universally accepted viewpoint ⇒ No universal behavior

Which characteristics or materials properties favor: $F \sim L$?

Which characteristics or materials properties favor: $F \sim A$?

Rob Carpick, UW-Madison
Martin Müser, U. Western Ontario

Generic theoretical analysis

assume constitutive law: $\tau = \tau_0 + \alpha\sigma + \ldots$

$$L = \int_{\text{real contact}} d^2r\, \sigma(r)$$

$$F = \int_{\text{real contact}} d^2r\, \{\tau_0 + \alpha\sigma(r)\} = \tau_0 A + \alpha L$$

insert your favorite model for contact mechanics:

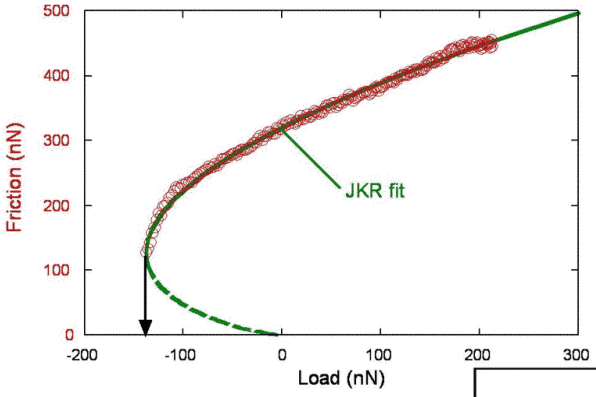
- Hertz: $A \sim L^{2/3}$ ⇒ $\tau_0 L^{2/3}$ dominates αL for "small" L
- other contact models show even stronger non-linearity at $L=0$

"pure Amontons": - τ_0 is negligible (e.g. small adhesion)
- interface is locally flat (e.g. large R_c)

"pure area friction": - α is negligible
- R_c is small

Single Asperity Friction: examples

Platinum-coated tip, Mica sample, in UHV



$$F_f = \tau \cdot A$$

friction | contact area

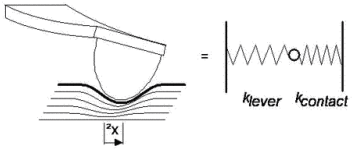
shear strength

Carpick et al., J. Vac. Sci. Technol. B **14** 1289 (1996)
Carpick et al., Langmuir **12** 3334 (1996)

materials	shear strength τ(MPa)	τ/G_{eff}	adhesion energy γ(mJ/m^2)	contact radius at pull-off, number of unit cells
Pt/mica: max.	910	1/25	404	8.6 nm, 2000 unit cells
SiN_x/ mica	52	1/430	24	5.3 nm, 740 unit cells
WC/ H-diamond (111)	240	1/1600	10	1.1 @ zero load, 140 unit cells

Indirect measure of contact area: Lateral Stiffness

$$k_{contact} = 8 \cdot G^* \cdot a_c$$



k=contact stiffness, G=shear modulus* a_c = contact radius
FROM: K.L. Johnson, Contact Mechanics

Carpick et al, JOM, 2004

AFM: SiN_x on NaCl, in UHV

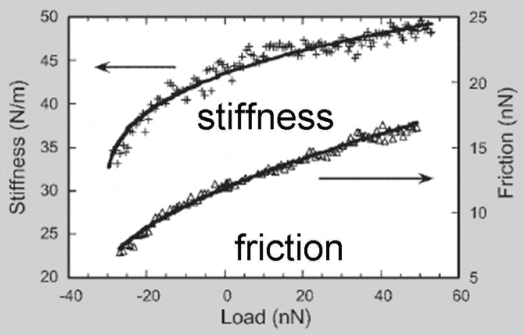


Figure 3. Crosses: lateral stiffness (k_{xx}) vs. load data for an SiN_x tip on mica in UHV. Triangles: F_f vs. load. Solid lines: fits of the JKR model.

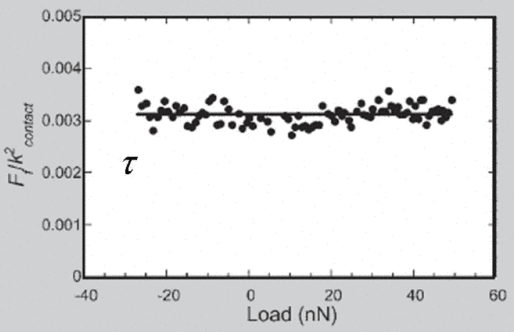
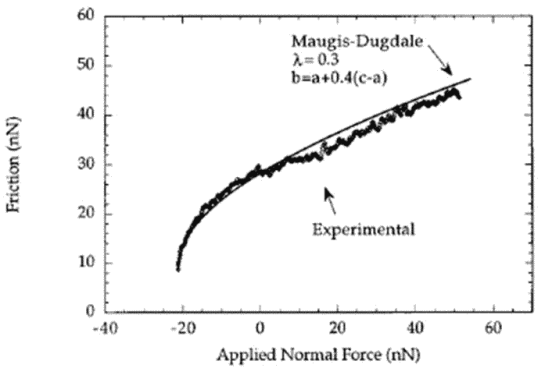
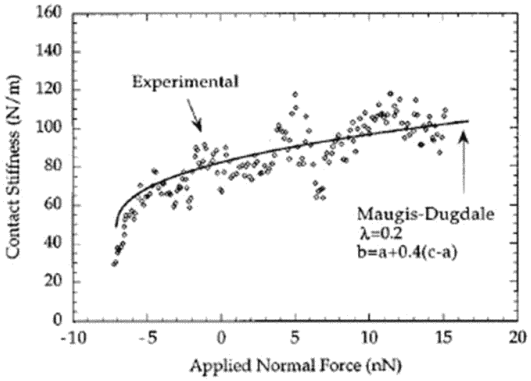


Figure 4. The $F_f / k^2_{contact}$ vs. load, calculated from the stiffness and friction data in Figure 3.





Tip radius (nm)	Static adhesion P_c (nN)	Static adhesion w (J/m^2)	Sliding P_c (nN)	Sliding w (J/m^2)	Sliding τ (N/m^2)	
12	7.0	0.099	4.6	0.065	6.1×10^8	α=0
45	21.9	0.087	21.4	0.084	6.6×10^8	

Lantz et al, PRB, 1997

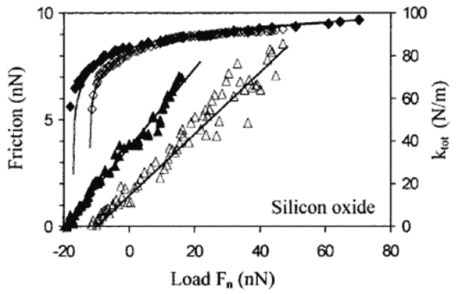


Figure 3. Equivalent lateral contact stiffness k_{tot} (◆, ◇) and friction force (△, ▲) versus normal load in ambient air (filled symbols) and under vacuum (open symbols) for silicon oxide. Data points for k_{tot} are fitted with eq 11 and for friction with a straight line showing that friction is proportional to load.

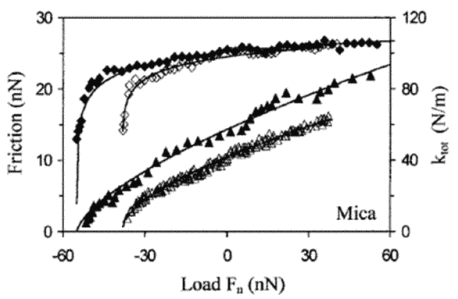


Figure 4. Equivalent lateral contact stiffness k_{tot} (◆, ◇) and friction force (△, ▲) versus normal applied load in ambient air (filled symbols) and under vacuum (open symbols) for mica. Data points for k_{tot} are fitted with eq 11 and for friction with eq 13 showing that friction is proportional to contact area.

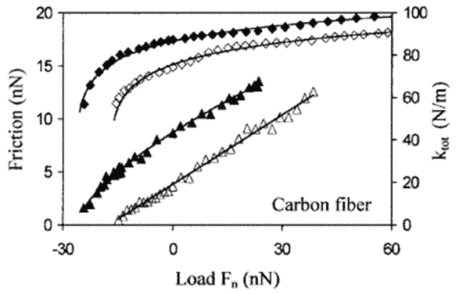


Figure 5. Equivalent lateral contact stiffness k_{tot} (◆, ◇) and friction force (△, ▲) versus normal applied load in ambient air (filled symbols) and under vacuum (open symbols) for carbon fiber. Data points for k_{tot} are fitted with eq 11 and for friction with eq 13 in ambient air showing that friction is proportional to contact area and under vacuum with a straight line showing that it is proportional to load.

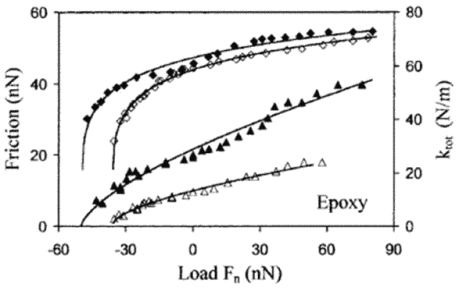


Figure 6. Equivalent lateral contact stiffness k_{tot} (◆, ◇) and friction force (△, ▲) force versus normal applied load in ambient air (filled symbols) and under vacuum (open symbols) for epoxy. Data points for k_{tot} are fitted with eq 11 and for friction with eq 13 showing that friction is proportional to contact area.

Piétrement and Troyon, Langmuir, 2001

	silicon oxide		mica	
	air	vacuum	air	vacuum
λ			0.01	0.18
α	0.15	0.14	0	0
τ_0 (MPa)	60	0.1	450	290

	carbon fiber		epoxy	
	air	vacuum	air	vacuum
λ	0.01		0.08	0.15
α	0	0.19	0	0
τ_0 (MPa)	300	33	270	113

Piétrement and Troyon, Langmuir, 2001

TRENDS to be checked

$F \sim A$ favored by

- small R_c
- low yield strength / extreme p's $A \sim L$ (at micro scale)
- strong adhesion
- complex surfaces (?)
- absence of bound-lubes

$F \sim L$ favored by

- flat interfaces
- weak adhesion
- simple surfaces (?)
- presence of bound-lubes

Tools

	$R_{curvature}$	$R_{contact}$	σ	local roughness
SFA	1mm	0.1mm	10MPa	?
AFM	30nm	5nm	1GPa	> atomic
MD	∞ [20nm]	10nm	0-10GPa	small [large]

Dry friction

$\tau = \tau_0 + \alpha\sigma + \ldots$

small α — constant or small τ_0

- AFM
  - "generic" AFM data [Schwarz et al., PRB (1997)] — NaCl grain (?) on NaCl [Socoliuc et al., PRL (2004)]
  - $\tau_0 \approx 3$GPa (?) [? tips on different C compounds] — MoS_2 flakes on MoS_2 [Miura et al., EPL (2002)]
- Simulation — often consistent with: constant or small τ_0
  - sometimes apparently consistent with: $F \sim A$

    however $\tau_0 = \tau_0(R_c)$ Muser, EPL (2001), Luan et al., Nature (2005)
  - independent shear stresses in *incommensurate* systems

    Al_2O_3-Al_2O_3 interface [Zhang, PRB (2005)]

    directed bonds can break superlubricity

    $\tau_0 \approx 0.1$GPa (similar to adsorbed hydrocarbons)

Lubricated friction

$\tau = \tau_0 + \alpha\sigma + \ldots$

(valid as constitutive law for generic bound-lubes)

- SFA "area friction" and "load friction": SAMS [McGuiggan, J. Adh. (2004)]

  KCl electrolyte solution
  [Berman et al., Tribol. Lett. (1998)]
  Small friction coefficient: 0.07

- Simulation (flat interfaces)

  CH_3-SAMS
  0.09GPa+0.03p

  CF_3-SAMS
  0.05GPa+0.12p

  Park et al., Langmuir (2004)

(curved interfaces) Samailov (?) and Persson see $F \sim A$ (poster in Porquerolles)
friction of droplet-like structure

TRENDS to be checked

$F \sim A$ favored by	$F \sim L$ favored by
• small R_c	• flat interfaces
• low yield strength / extreme p's	$A \sim L$ (at micro scale)
• strong adhesion	• weak adhesion
• complex surfaces (?)	• simple surfaces (?)
• absence of bound-lubes	• presence of bound-lubes

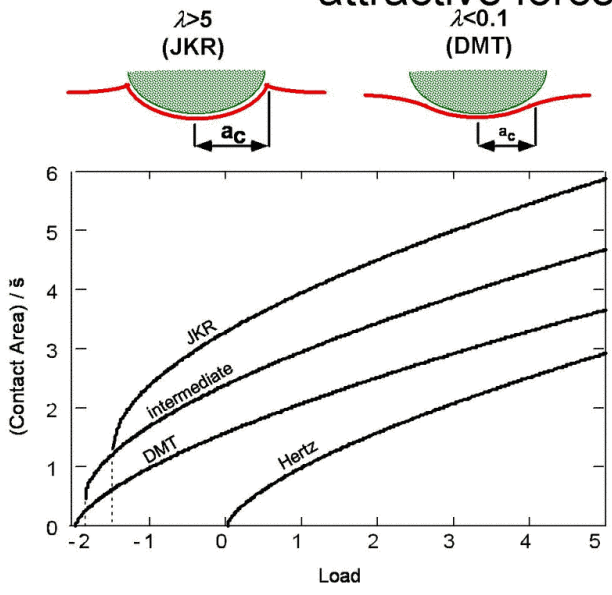
Table 1. A Comparison of Shear Strengths and Adhesion Energy Measurements by Various Techniques for Certain Interfaces

materials in contact	environment	shear strength (Pa)	adhesion energy (J/m²)	ref
		AFM Results		
SiN tip/mica	air, ~55% RH	6.8×10^8	determined by capillary formation	125
Si tip/$C_{60\ islands}$	air, ~55% RH	$\tau = \tau_0 + \alpha P$: $\tau_0 \approx 3.3 \times 10^9$, $\alpha \approx 0$	determined by capillary formation	174
Si tip/GeS		$\tau = \tau_0 + \alpha P$: $\tau_0 \approx 1.2 \times 10^9$, $\alpha \approx 1.1$; or $\tau_0 = 0$, $\alpha \approx 1.5$		
Si tip/NaCl: terraces	dry N_2	9.6×10^7 (JKR) 1.45×10^8 (extended JKR)	0.027–0.035[a]	173
steps		1.42×10^8 (JKR) 4.78×10^8 (extended JKR)		
Pt-coated tip/mica	UHV	9.1×10^8 max, 2.7×10^8 min	0.404 max, 0.019 min	172
C_{60}/NaCl	UHV	$(0.5–1) \times 10^5$		188
MoO_3/MoS_2	dry N_2	1.1×10^6		189
Cd–arachidate/Cd–arachidate	ambient	$(1.0 \pm 0.2) \times 10^6$		190
		SFA Results		
mica/mica	dry air	2.5×10^7		178
mica/mica	dry air/N_2		0.123–0.157[b]	191
fatty acid monolayers–$(C_{n-1}H_{2n-1})$-COOH between Mica sheets	ambient	$\tau = \tau_0 + \alpha P$		179
$n = 14$[1](myristic acid)		$n = 14$: $\tau_0 = 6 \times 10^5$, $\alpha = 0.034$		
$n = 18$ (steric acid)		$n = 18$: $\tau_0 = 6 \times 10^5$, $\alpha = 0.038$		
$n = 22$[1](behenic acid)		$n = 22$: $\tau_0 = 22 \times 10^5$, $\alpha = 0.048$		
		QCM Results[c]		
Kr monolayer/Au(111)	UHV	5×10^{-1}		136
		Macroscopic Results		
SiO–Cu-bonded interface	*d*	$(0.56–1.67) \times 10^9$		192
glass/stearic acid/Al	ambient	$\tau = \tau_0 + \alpha P$: $\tau_0 = 5.8 \times 10^6$, $\alpha = 0.068$		193
MoS_2-sputtered films on hard substrates	dry air	$\tau = \tau_0 + \alpha P$: $\tau_0 = 2.48 \pm 0.5 \times 10^7$, $\alpha = 0.001 \pm 0.001$		194

[a] Calculated by present authors based on information in reference. [b] Immediately after cleavage. Adhesion decreases substantially with time due to adsorption of contaminants. [c] QCM measurements are performed at much shorter time scales than AFM and SFA experiments. [d] This experiment measured the ultimate interfacial shear strength for a thin film of silica deposited onto a copper substrate with a stress-induced cracking technique.

from Carpick & Salmeron, Chem. Rev., 1997

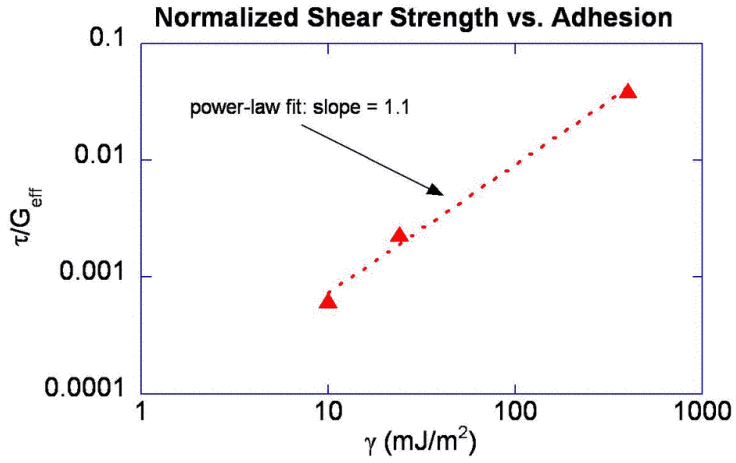
Contact area depends on the range of the attractive forces



$$\lambda = 1.16 \cdot \left(\frac{\gamma^2 \cdot R}{E^{*2} \cdot z_0^{\ 3}} \right)^{1/3}$$

γ=adhesion energy
R=tip radius
E^*=elastic modulus
z_0=equilibrium spacing

The "normalized" shear strength increases monotonically with adhesion



Comparing friction with stiffness measurements gives τ

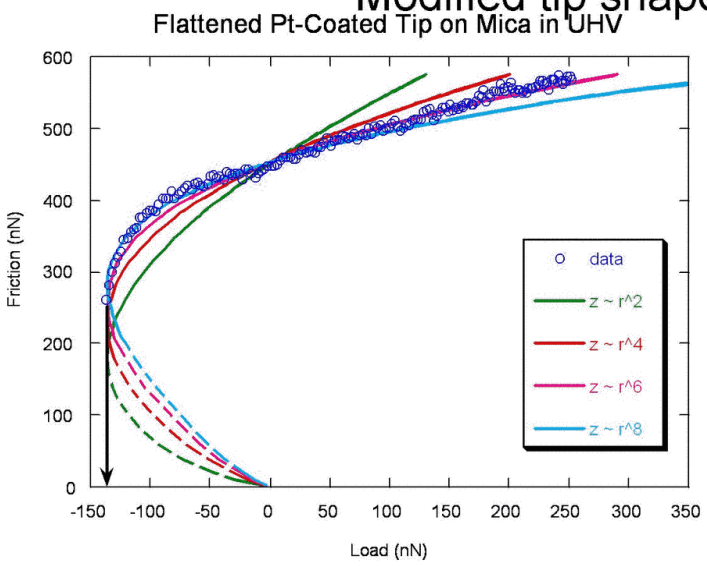
(1) $F_f = \tau \cdot A = \tau \cdot \pi \cdot a^2$

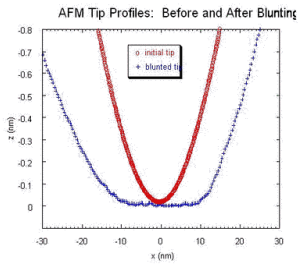
(2) $k_{contact} = 8 \cdot G^* \cdot a$

$\therefore$ plot $F_f / k^2_{contact}$ vs load :

$$\frac{F_f}{k^2_{contact}} = \frac{\tau \cdot \pi}{(8 \cdot G^*)^2} \propto \tau$$

Modified tip shape





tip shape