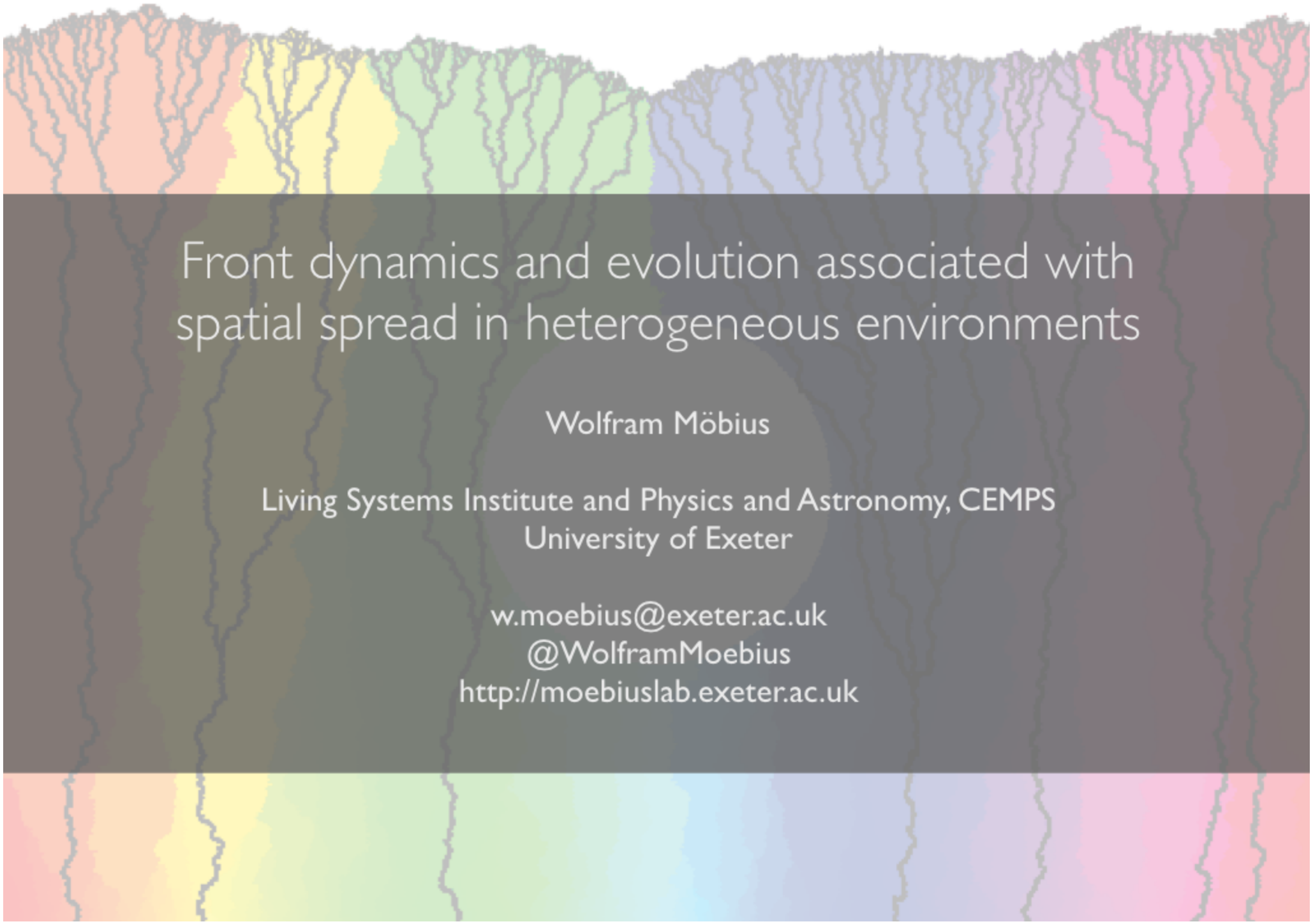


Front dynamics and evolution associated with spatial spread in heterogeneous environments

Wolfram Möbius

Living Systems Institute and Physics and Astronomy, CEMPS
University of Exeter

w.moebius@exeter.ac.uk
@WolframMoebius
http://moebiuslab.exeter.ac.uk

Range expansions - a process occurring on all scales

[Wikipedia]

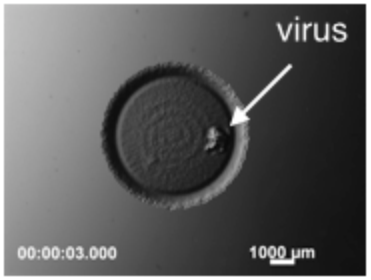
Migration out of Africa

What is humans' evolutionary history?

Invasive alien species

What is effect on ecosystems?
How to control invasions?

[Wikipedia]



Part of life at microbial scale

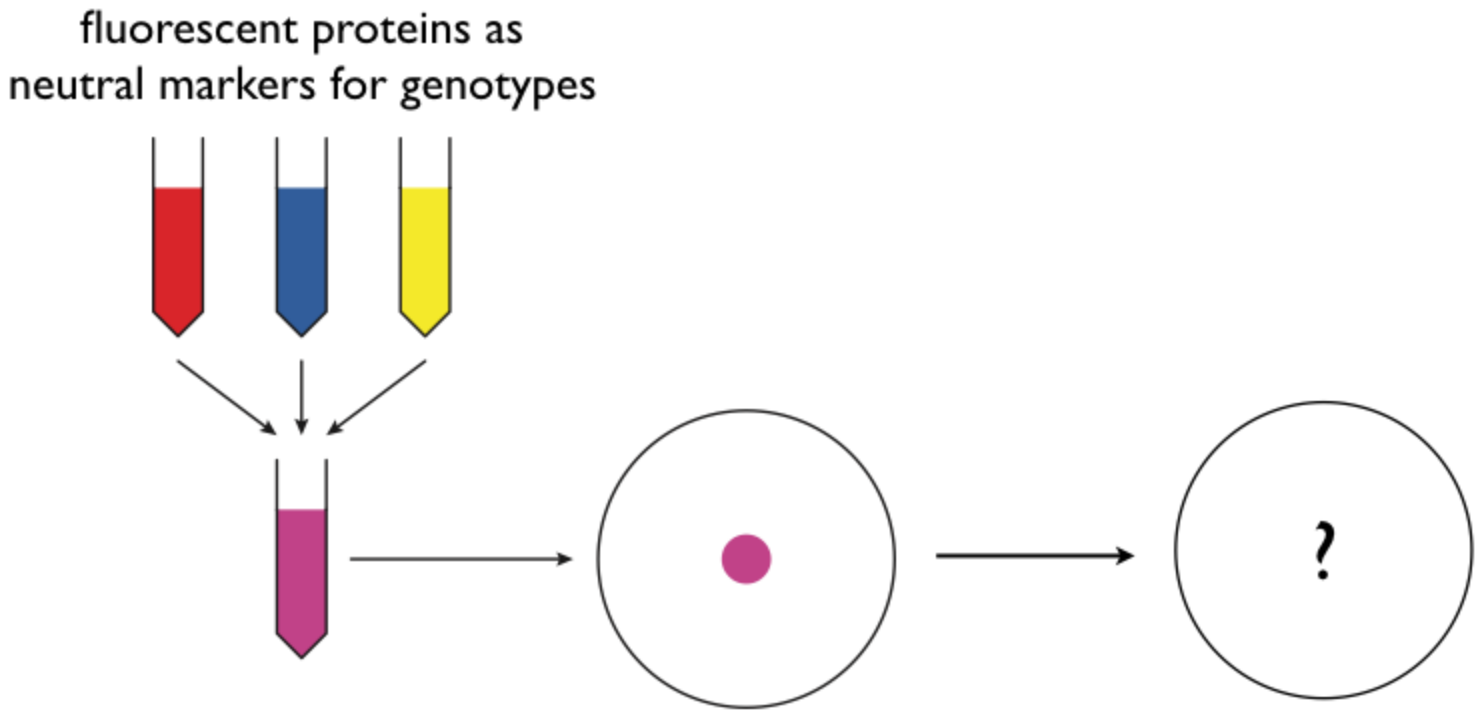
How do expansions shape microbial world?

large distances,
long times

small distances,
short times

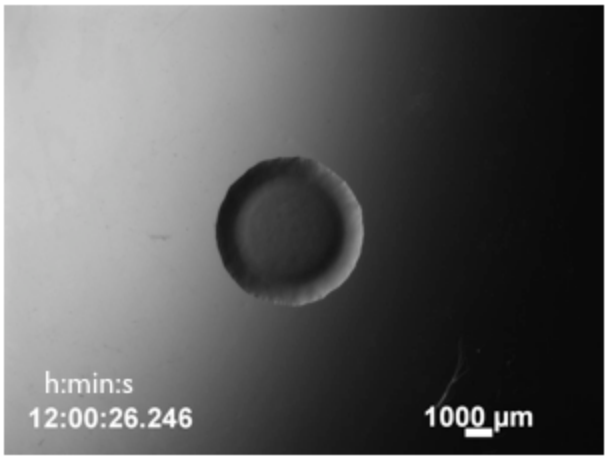
Example: Population expansion in two dimensions

Hallatschek et al., 2007: uniform expansion (invasion of agar plate) with standing neutral diversity



Example: Population expansion in two dimensions

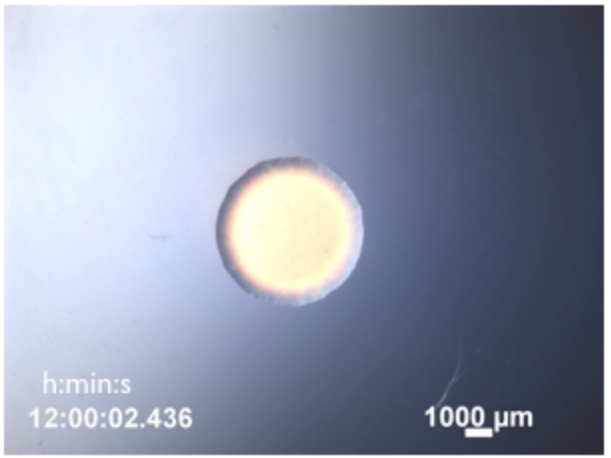
population dynamics



spatial distribution
of population

time evolution of population density

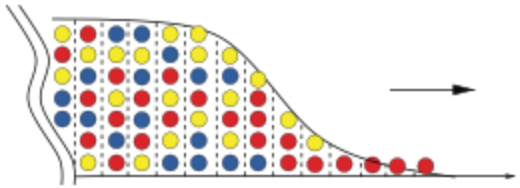
population genetics



genetic composition /
genetic diversity

time evolution of allele frequencies

surface → demixing



[adapted from Hallatschek & Nelson, 2008]

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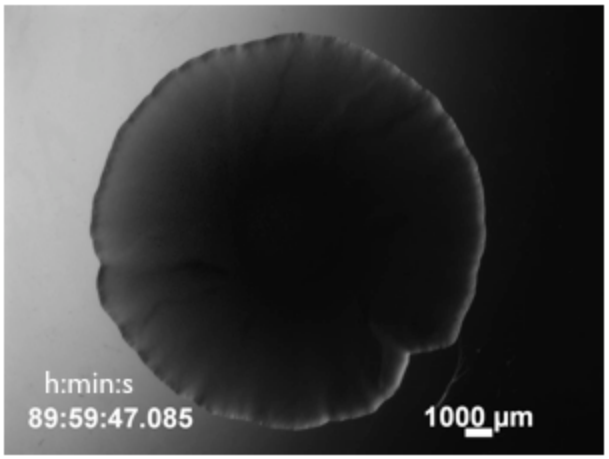

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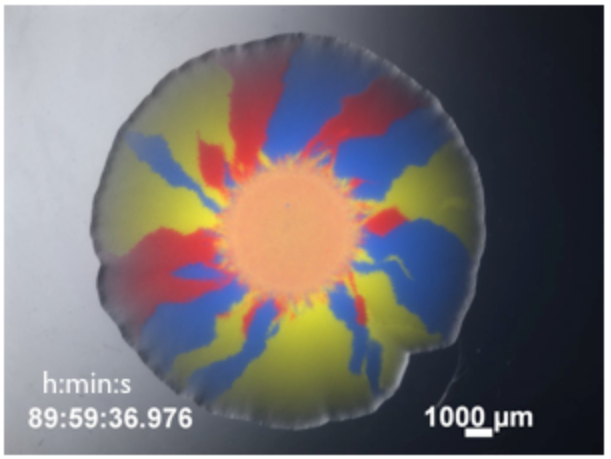

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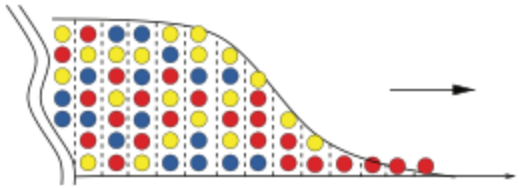

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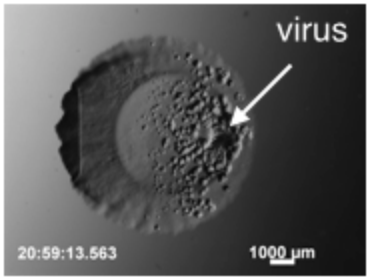


Part of life at microbial scale

How do expansions shape microbial world?

large distances,
long times

small distances,
short times

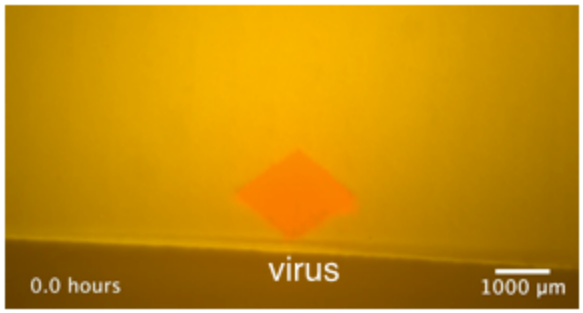
How does the environment (and the expansion process)
shape the ecological and evolutionary dynamics?

Approach: Studying highly simplified scenarios quantitatively

How does the environment shape ecological and evolutionary processes?

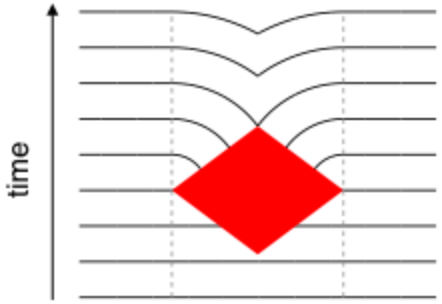
E.g., what happens if a population front encounters a single region that cannot be invaded?

experiments

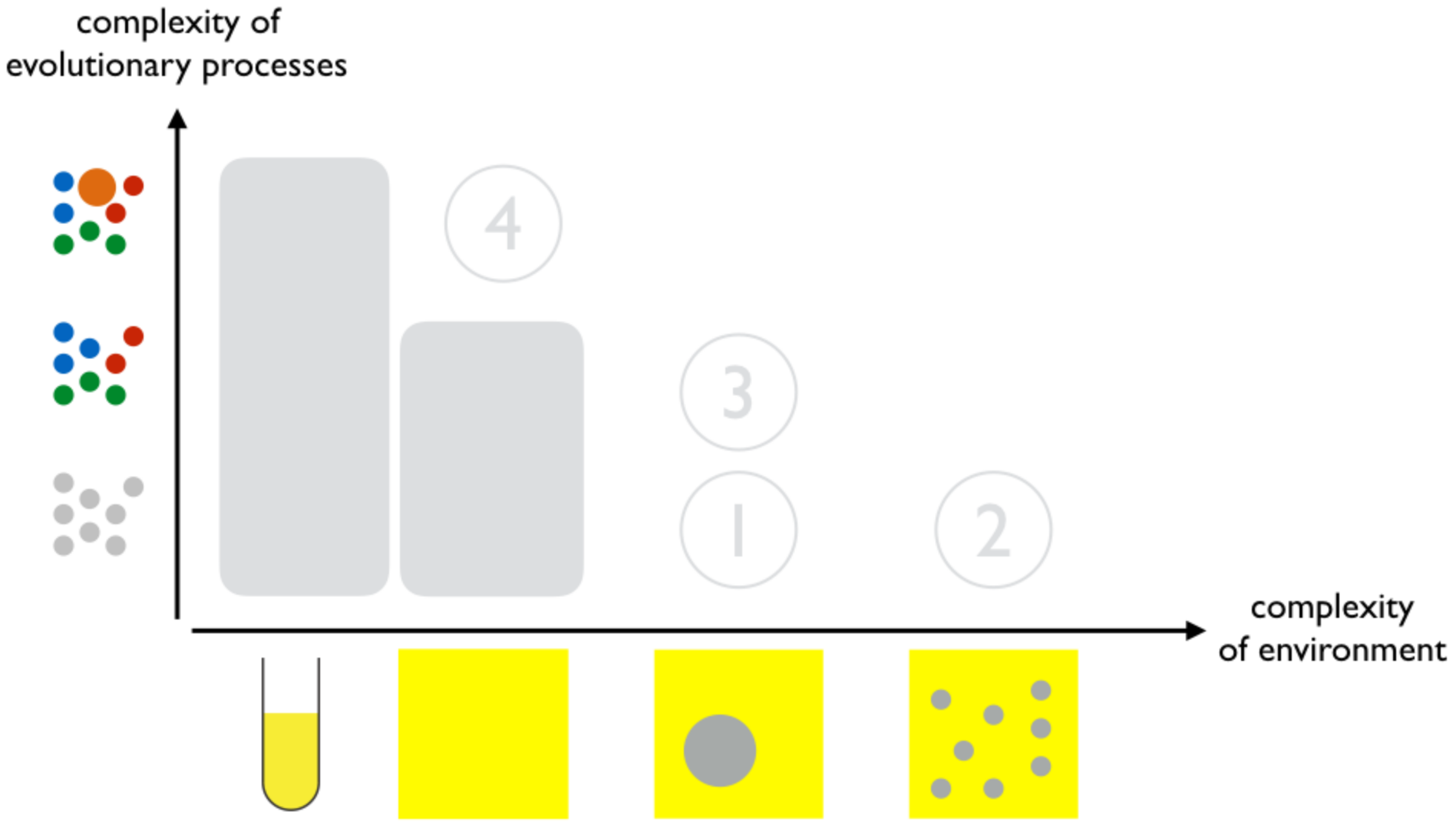


simulations

'pen and paper' mathematics & physics



Front dynamics and evolution associated with spatial spread in heterogeneous environments



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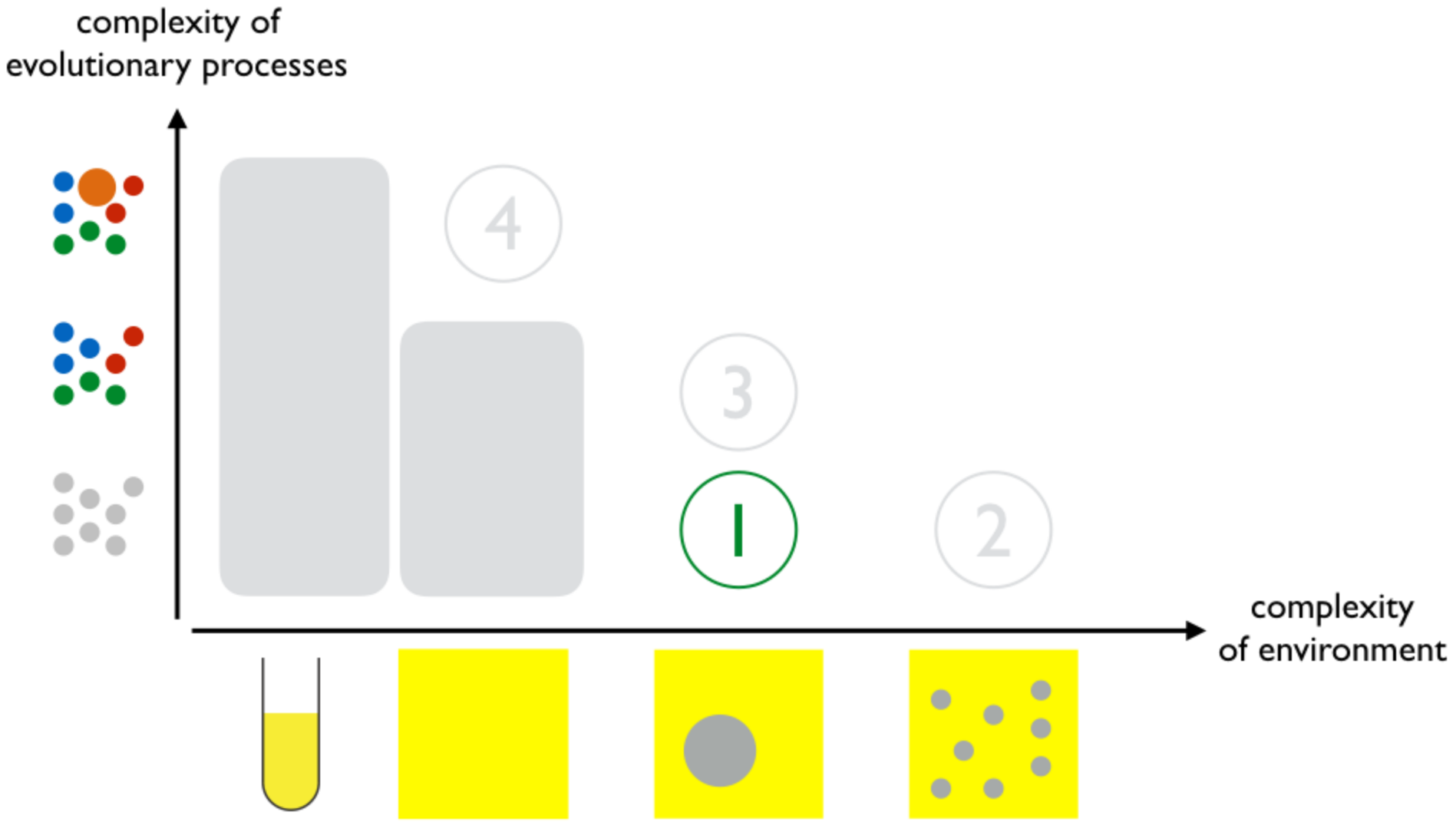
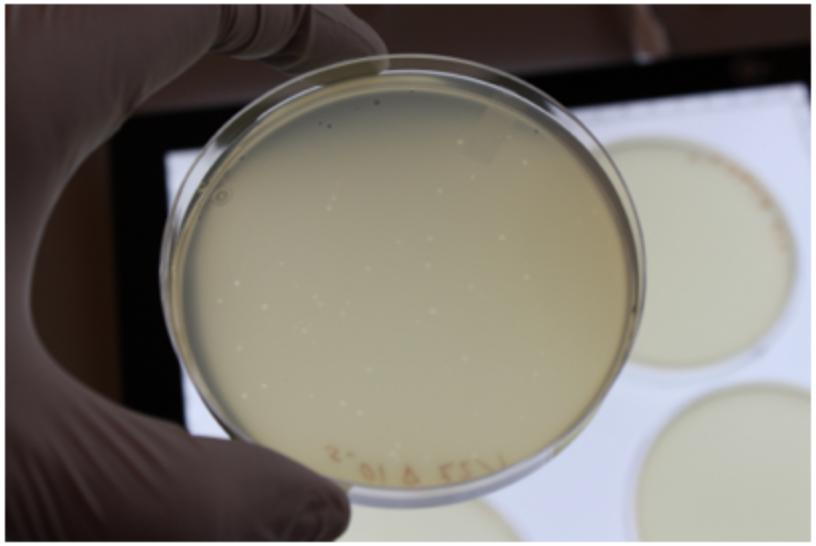


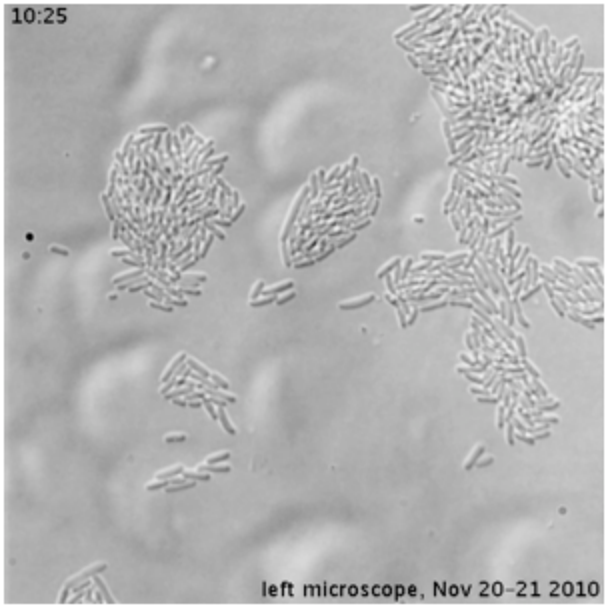
Phage on E. coli lawn - bench-scale model system

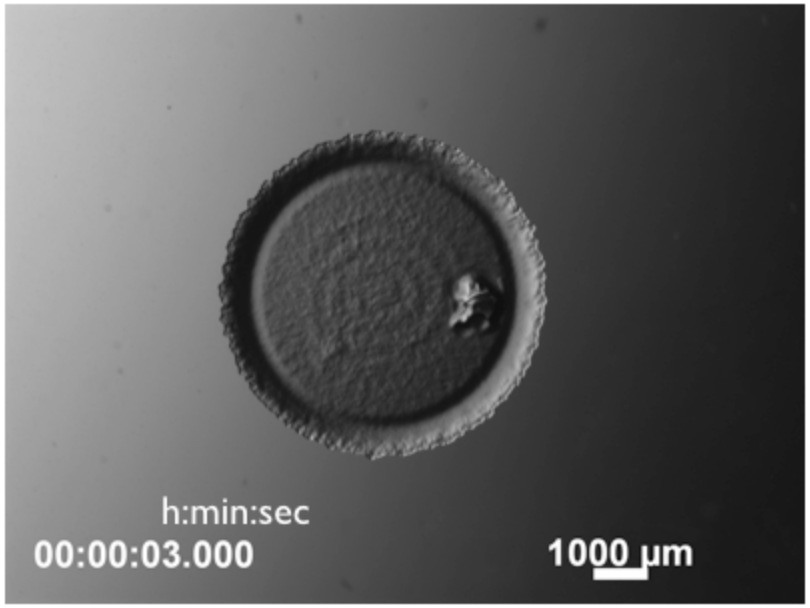
1 phage + 1 bacterium

↓

'burst size' * phage







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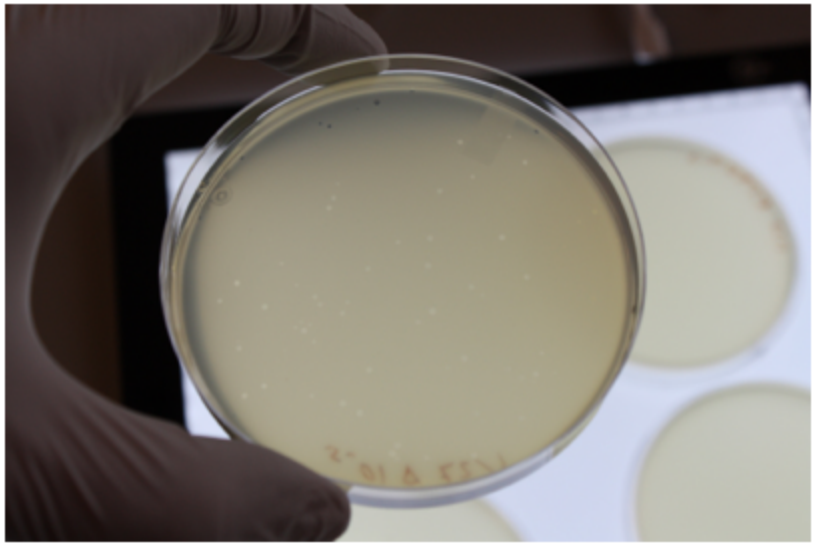
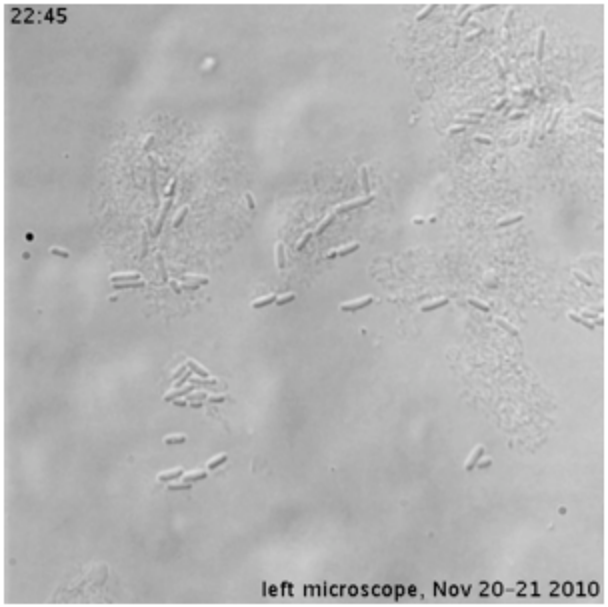
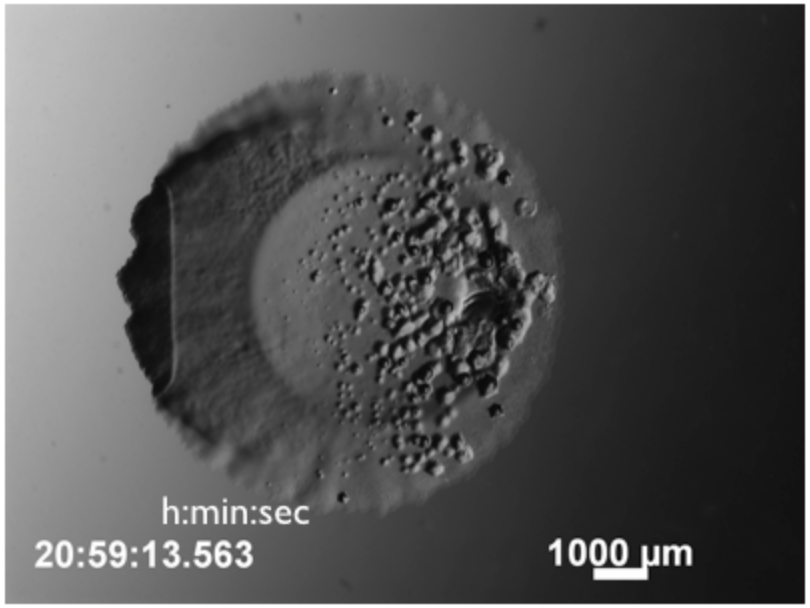






Population spread in complex environments

expanding population

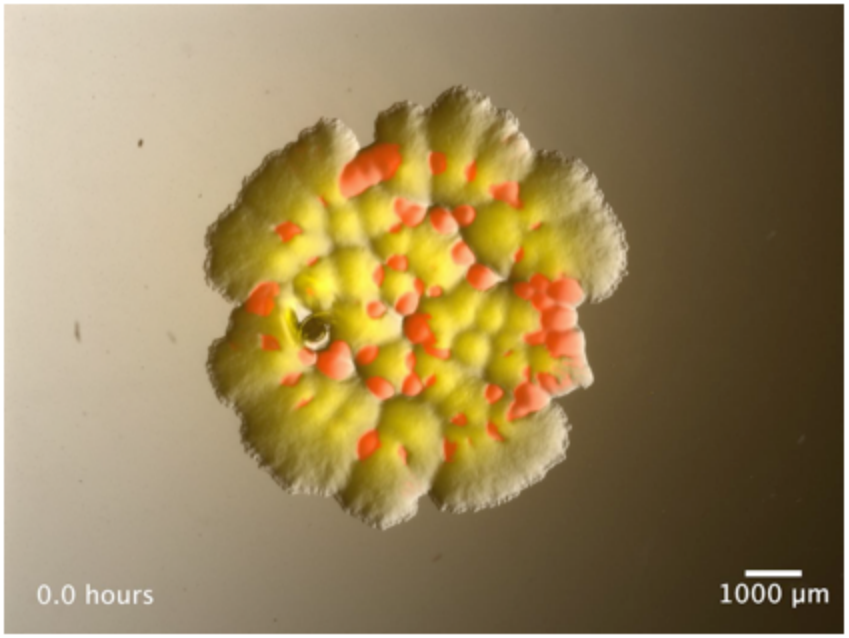
favorable environment

unfavorable environment

bacteriophage T7

WT *E. coli*

E. coli ΔwaaC



- What is **shape** of perturbed front?
- Effect of **two-dimensionality**?
- What **characterises front shape**?
- What is **effect on front speed**?

Population spread in complex environments

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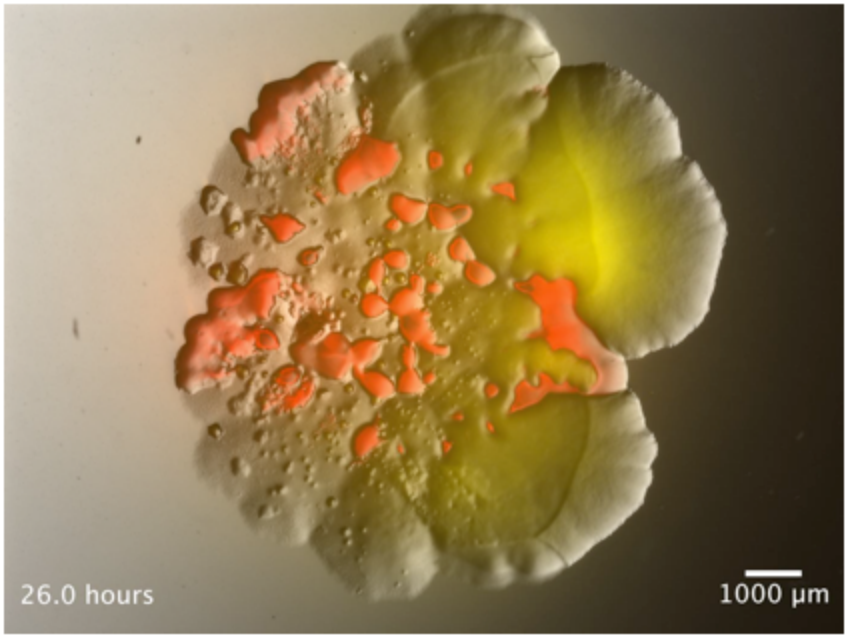

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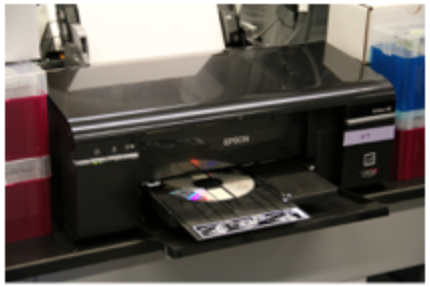
Inkjet printer technique to create artificial environments

designed environment

(re)fillable cartridges
with *E. coli*

black cartridge: resistant bacteria
yellow cartridge: susceptible bacteria
other cartridges: Millipore water



consumer inkjet printer with agar pad on CD tray

based on Cohen et al., PLoS ONE, 2009
with significant modifications

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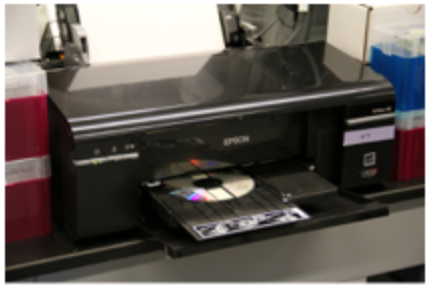

consumer inkjet printer with agar pad on CD tray

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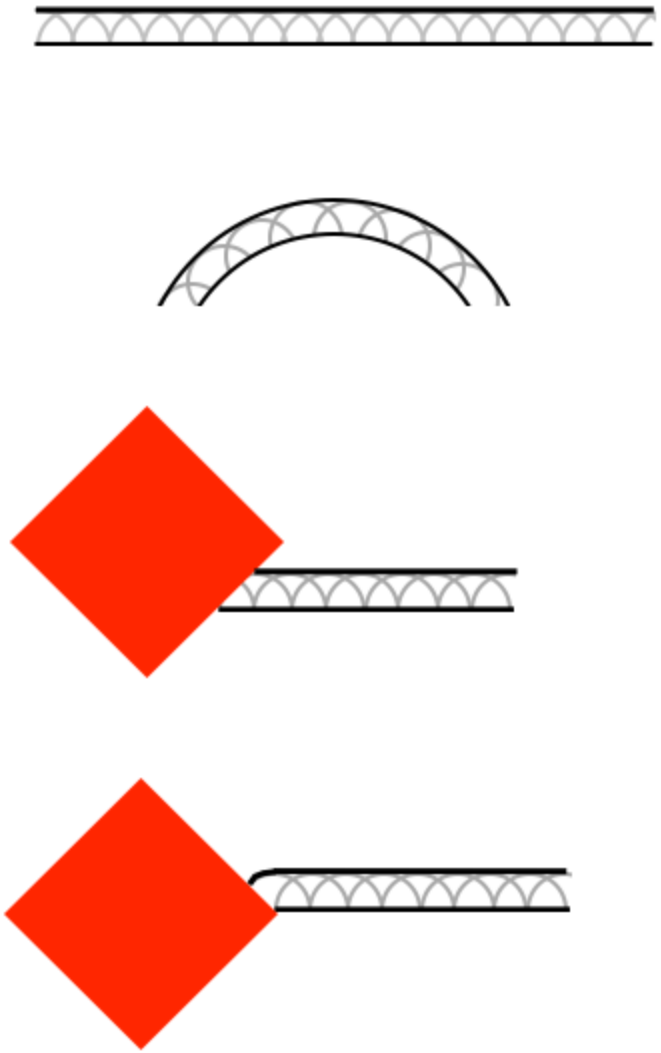
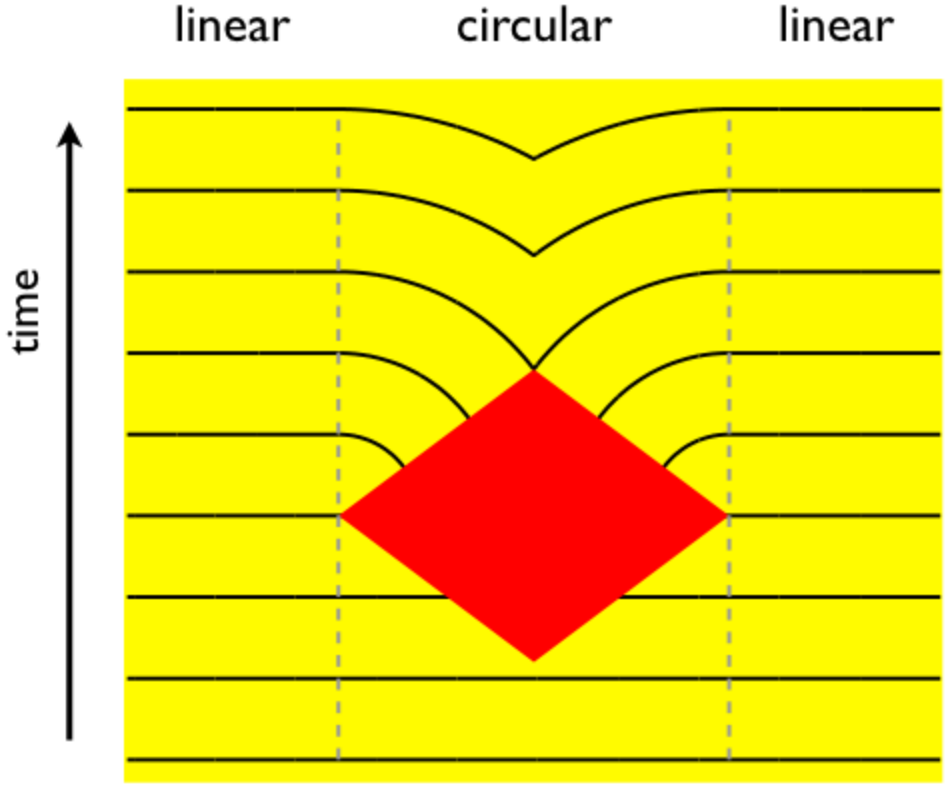
Constant speed model - inspired by Huygens' principle

propagation in normal direction

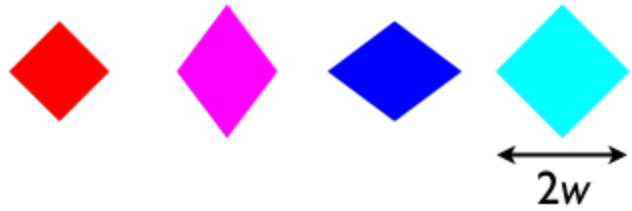




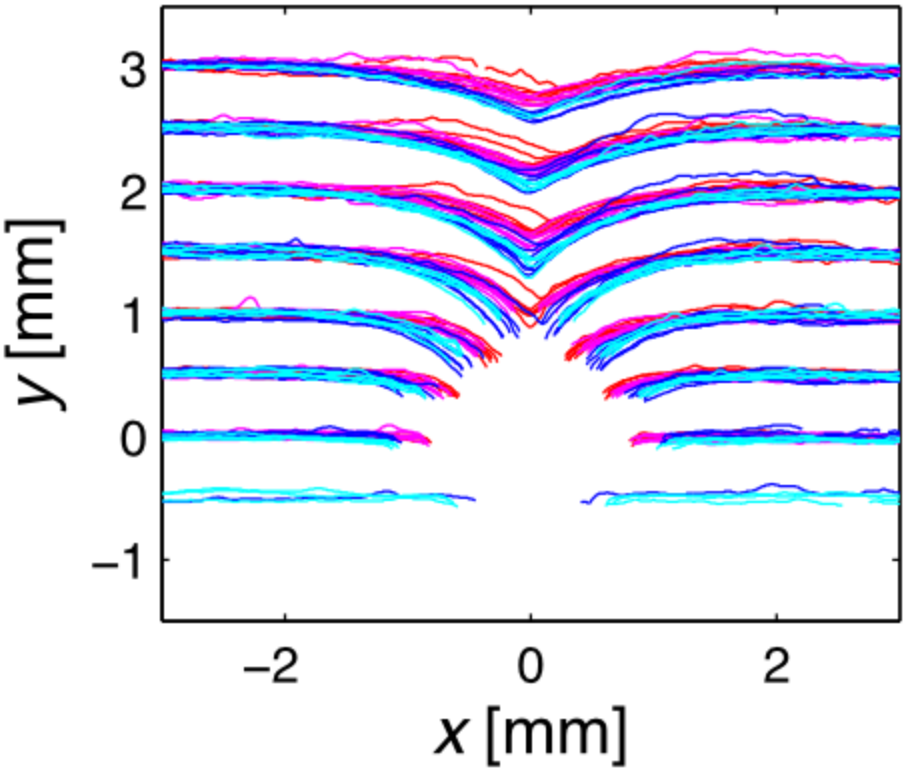
rhombus is convenient example,
but approach generalizable!

Testing predictions of constant speed model

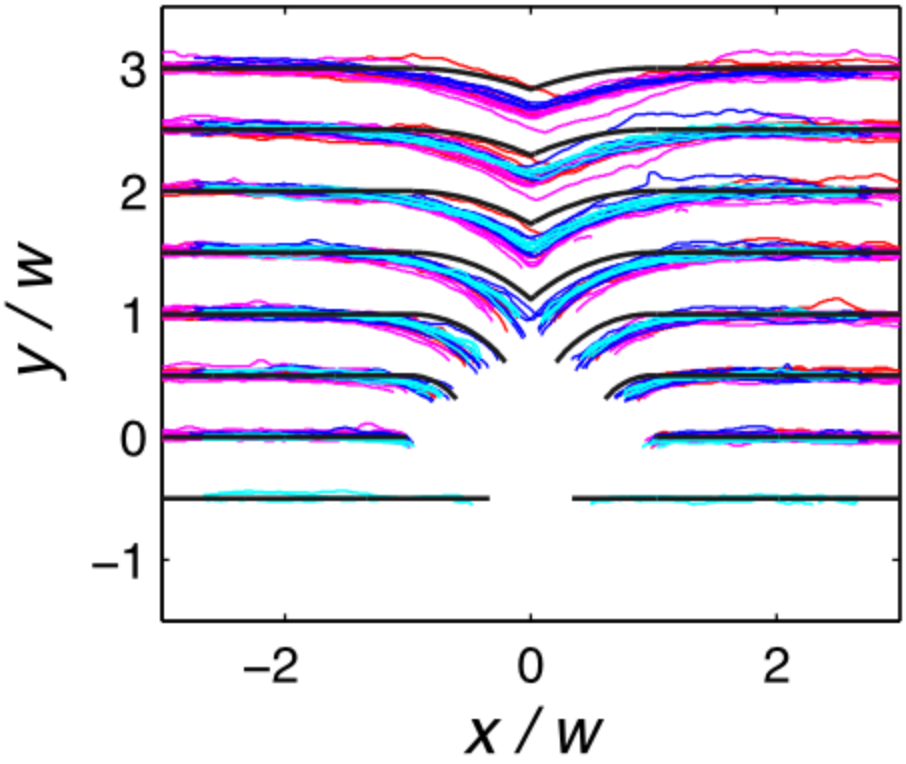
four different shapes:



unscaled data



scaled data & model



Toward a more microscopic model

- one strategy: life cycle of bacteriophage T7 + diffusion of phage

$$V + B \rightleftharpoons VB \rightarrow y \cdot V$$

→ 3 reaction rates, 1 burst size
1 diffusion coefficient
1 initial bacterial density

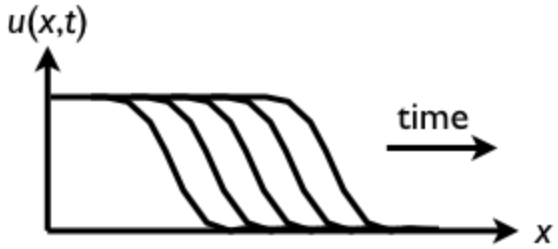
along the lines of
[Yin and McCaskill, 1992]

- simplified model: logistic growth of phage population + diffusion of phage

→ FKPP equation for (dimensionless) population density $u(x,t)$
(Fisher-Kolmogorov-Petrovsky-Piscunov)

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + ku(1-u)$$

diffusion logistic growth



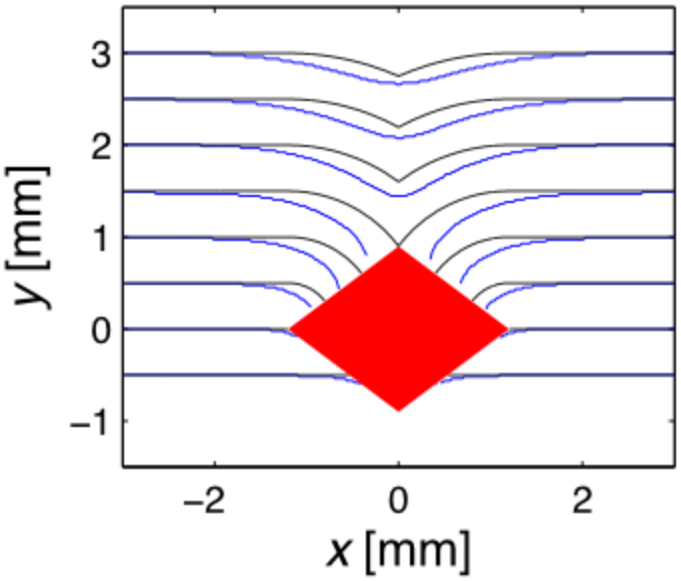
$$v = 2\sqrt{Dk} \qquad \xi = \sqrt{\frac{D}{k}}$$

Reaction-diffusion description

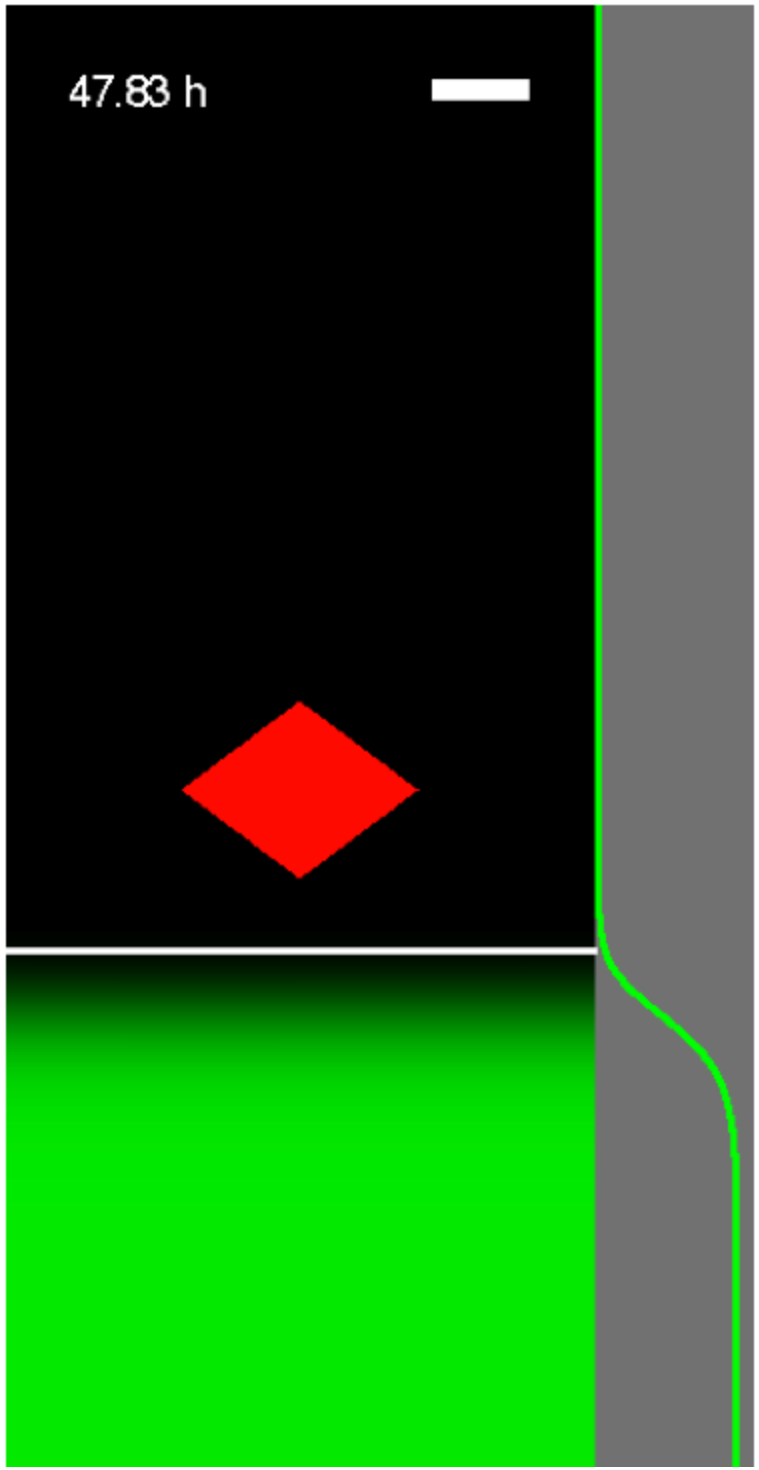
FKPP equation for population density $u(x,t)$

use $v = 2\sqrt{Dk}$ to set k outside obstacle
$k = 0$ inside obstacle
→ numerics without parameter fitting

semi-quantitative agreement with experiments



reaction-diffusion model
constant speed model



Reaction-diffusion description

two reasons for lag:

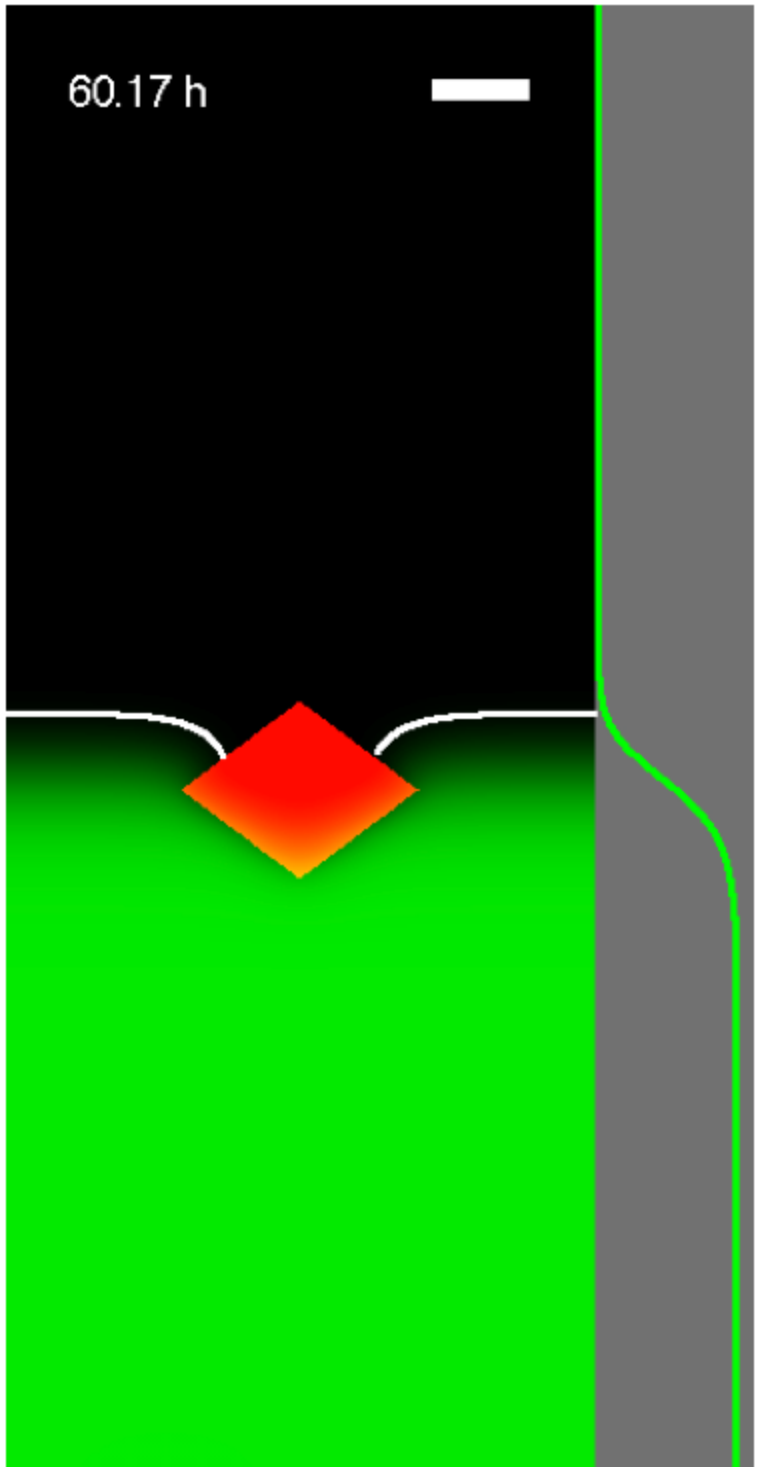
$$\xi = \sqrt{\frac{D}{k}}$$

boundary layer

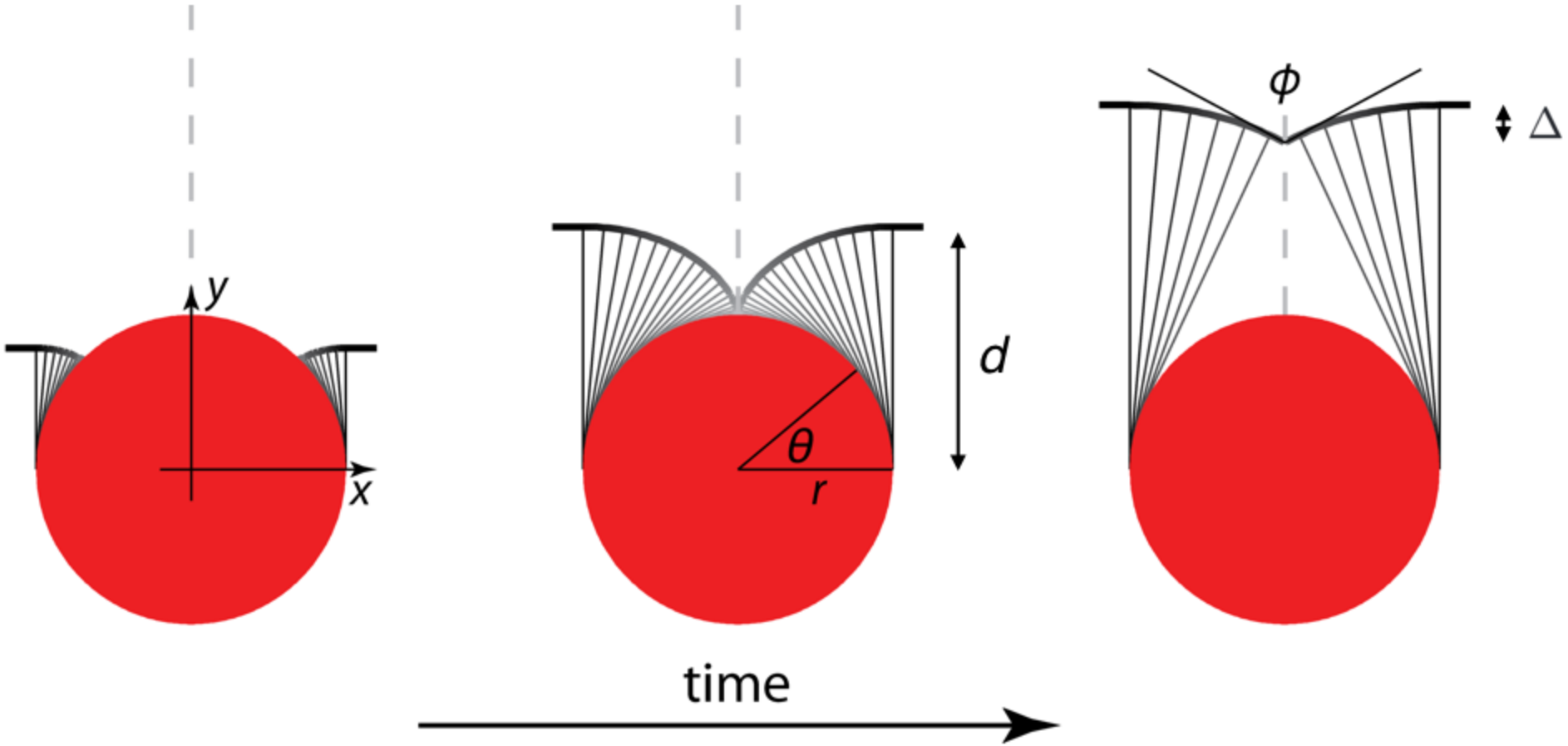
large front curvature

relative importance should become small for large obstacles

constant speed model good for large enough rhombus-shaped obstacles

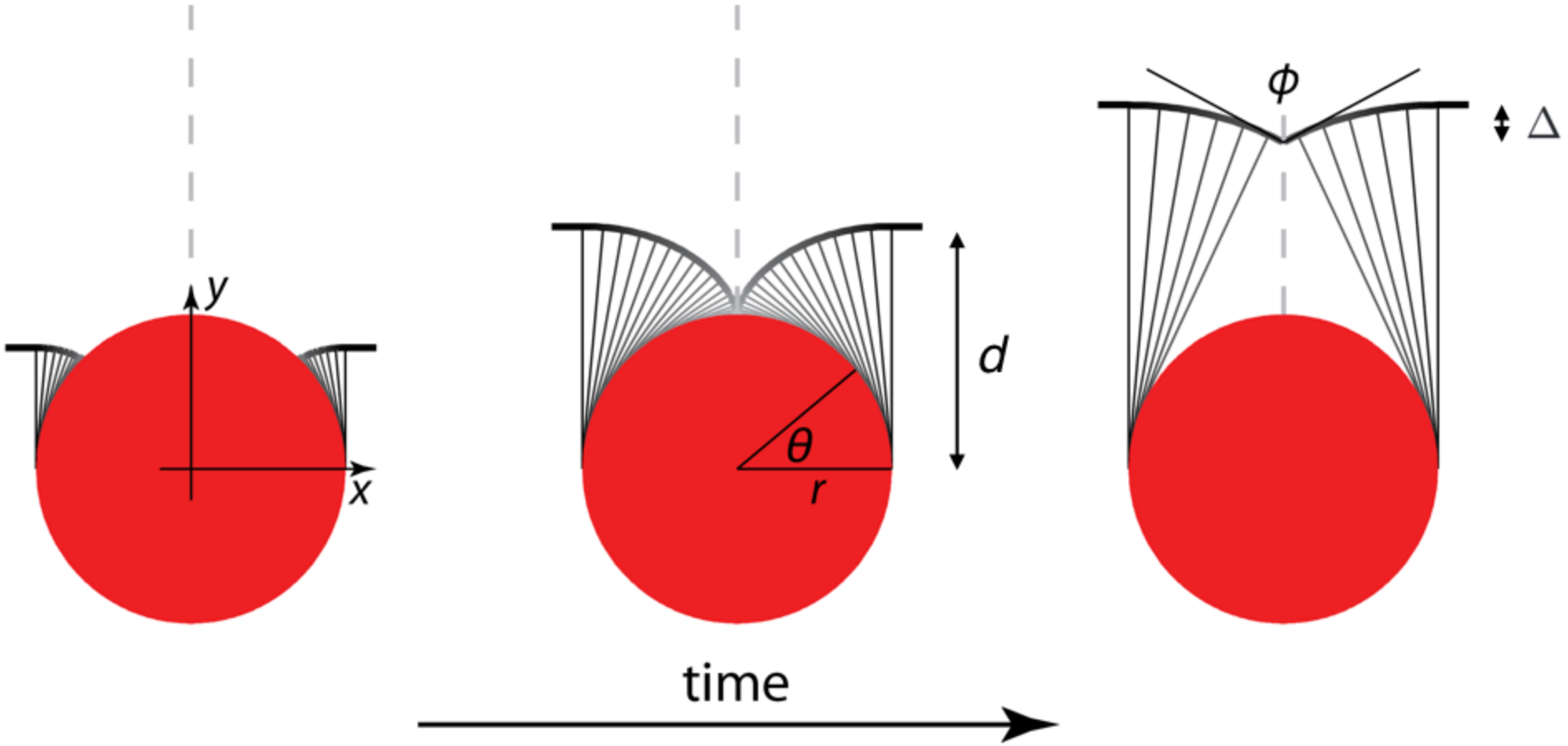


Beyond rhombus-shaped obstacles



$$x(\theta) = r\cos\theta - (d - \theta r)\sin\theta$$
$$y(\theta) = r\sin\theta + (d - \theta r)\cos\theta$$

Beyond rhombus-shaped obstacles

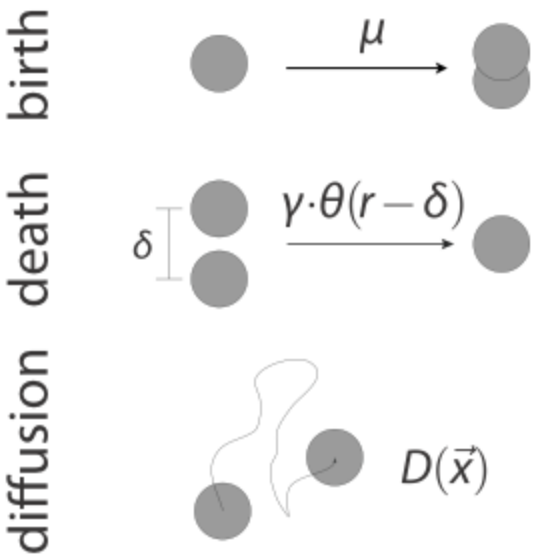


width/radius, not shape,
of obstacle matters

$$\phi \approx \pi - \frac{2r}{d}$$

$$\Delta \approx \frac{r^2}{2d}$$

Particle-based simulation



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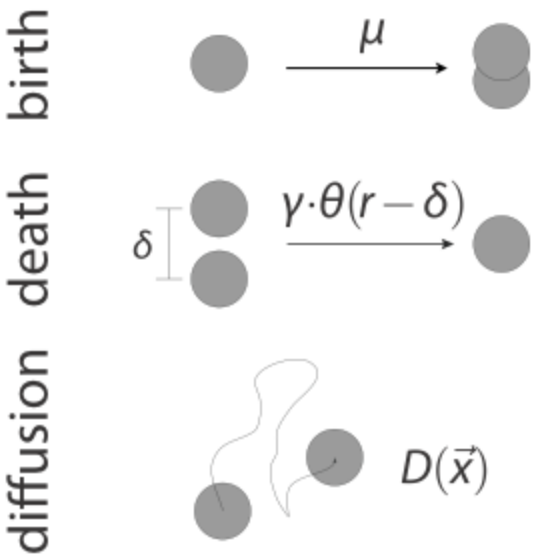
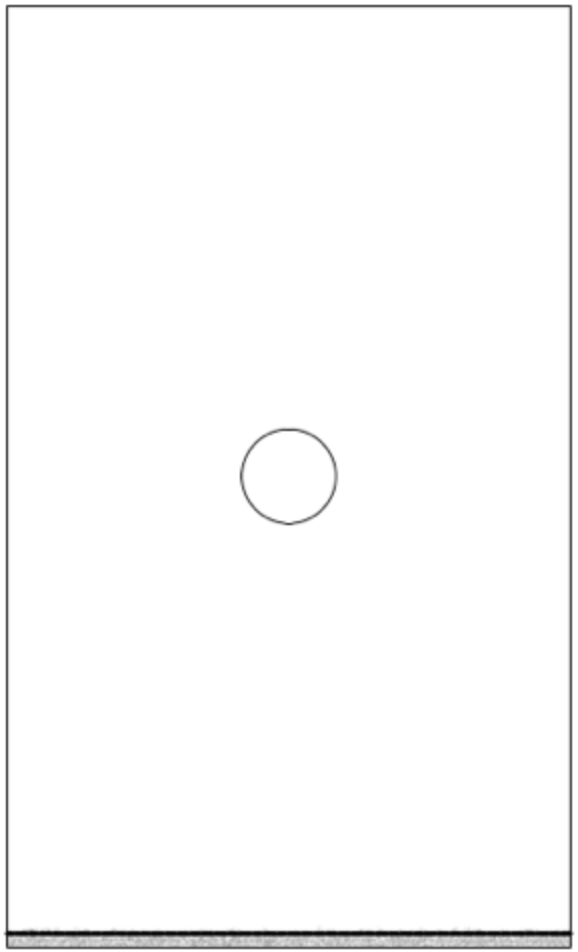


Region that does not support (invading) population, a.k.a. obstacle



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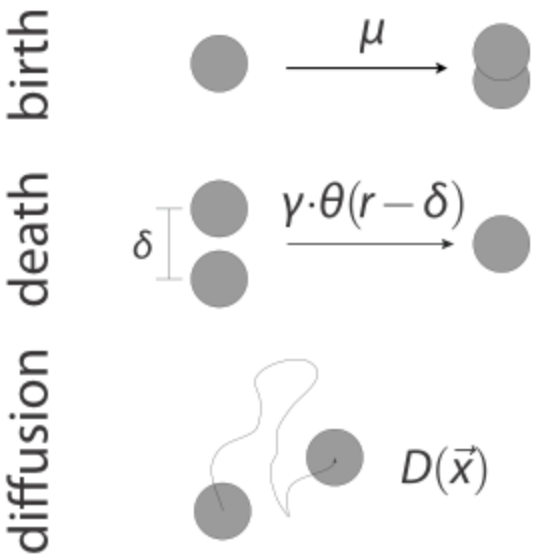

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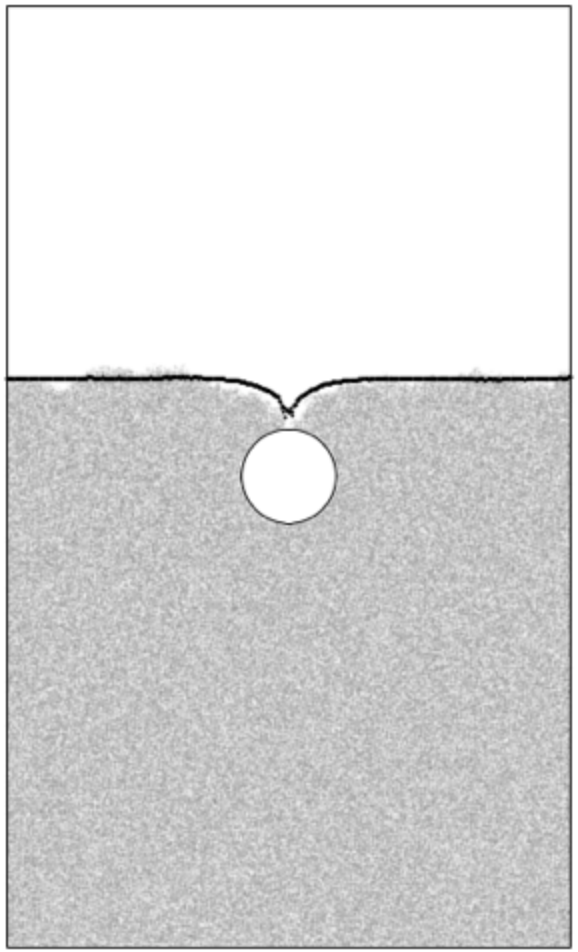

Particle-based simulation compared to geometrical optics prediction

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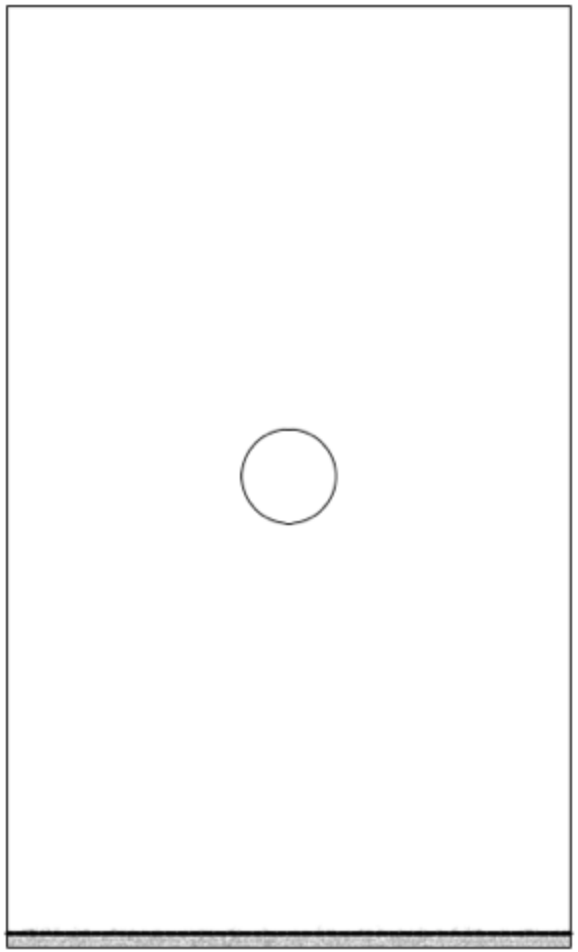

average front

constant speed

far-distance solution

'scattering' from the corners
can guide intuition for obstacles
with complex shape

Particle-based simulation compared to geometrical optics prediction

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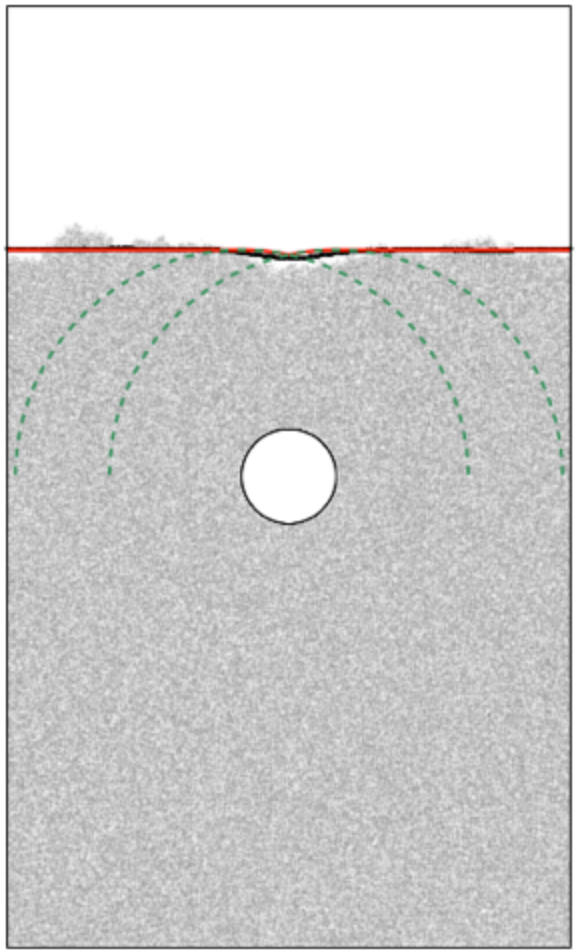

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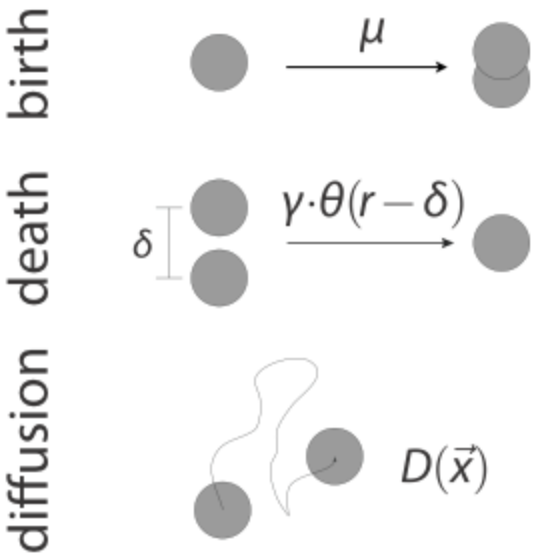
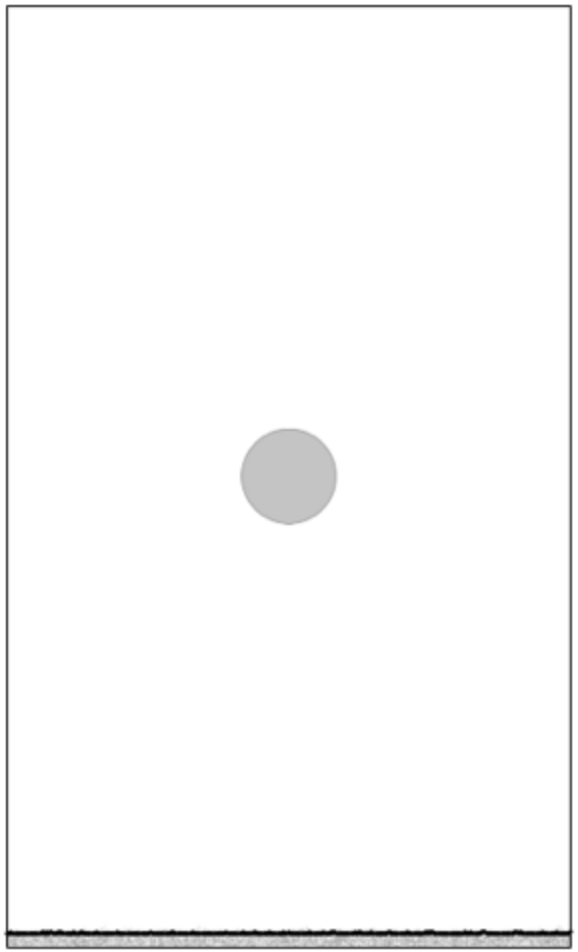
Complement of obstacles - hotspots



Region within which population expands faster, a.k.a. hotspot



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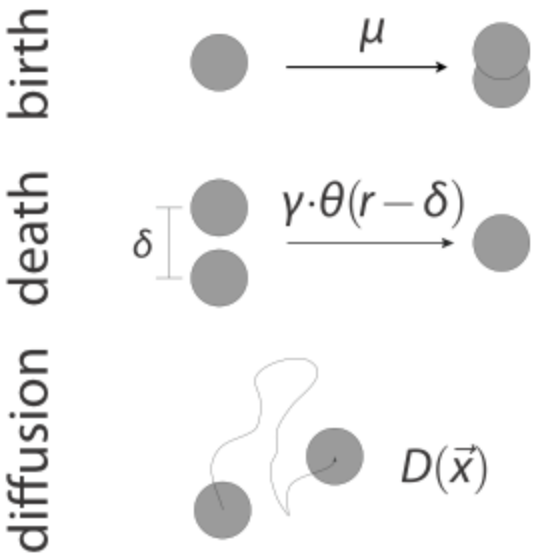

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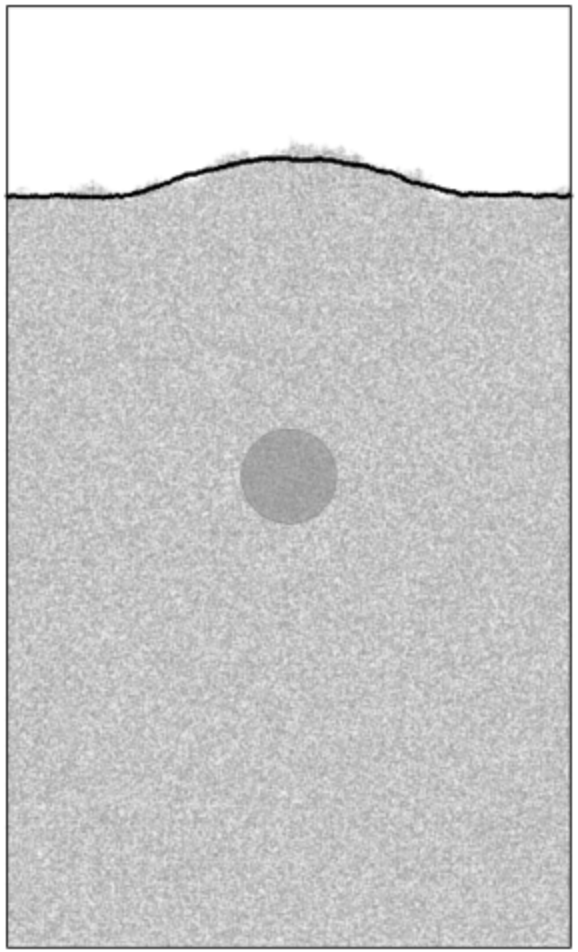
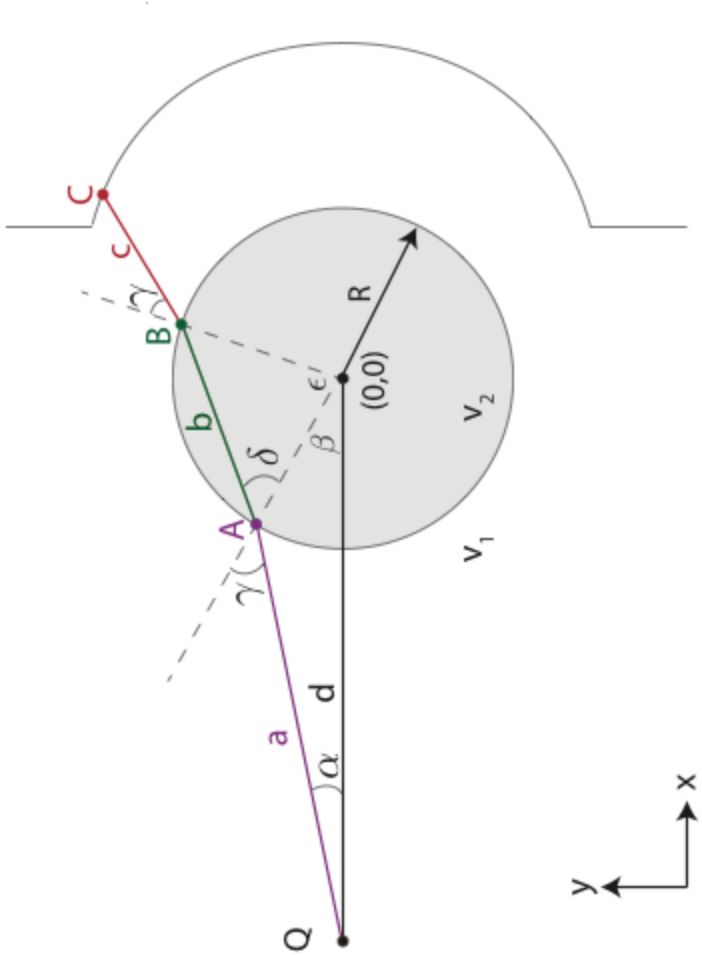
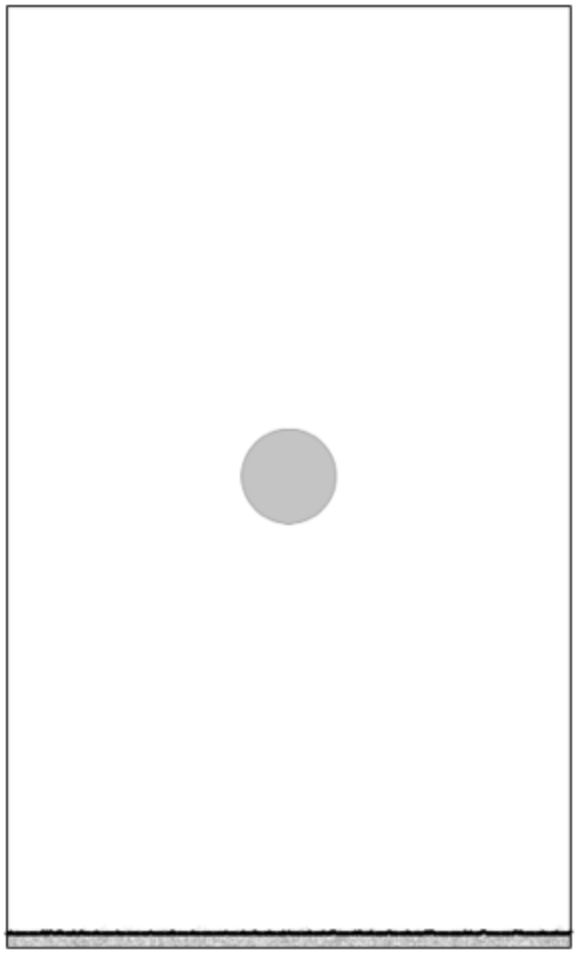
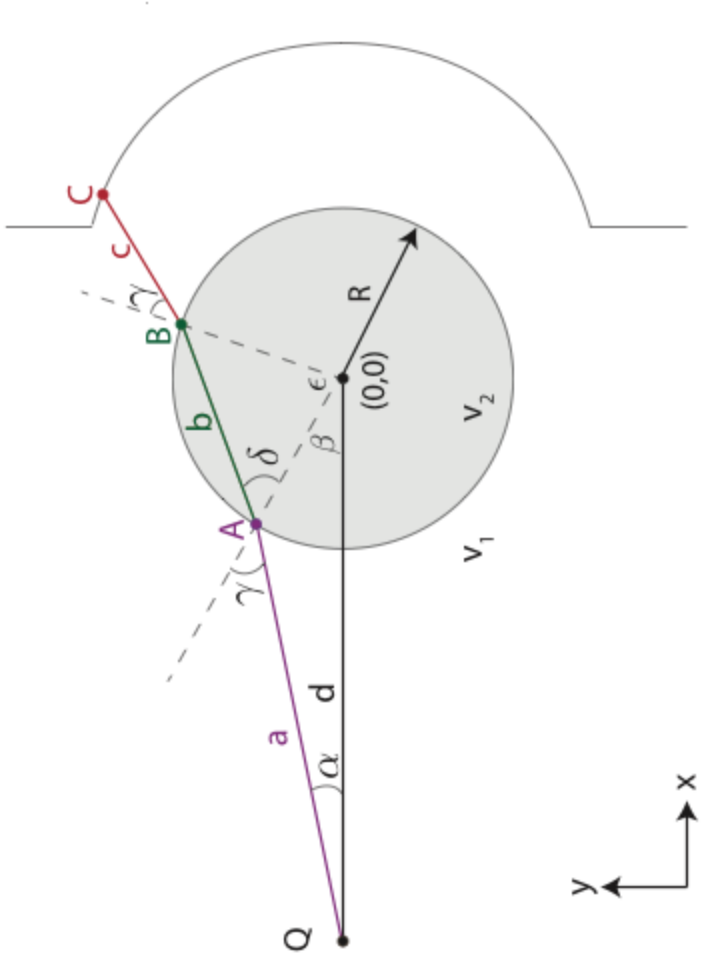


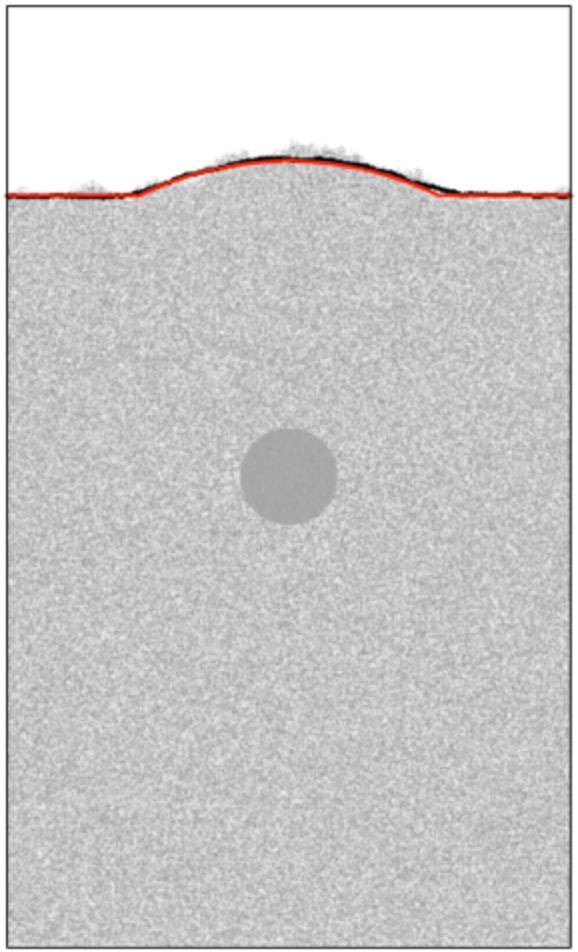
Complement of obstacles - hotspots - and geometrical optics





Complement of obstacles - hotspots - and geometrical optics





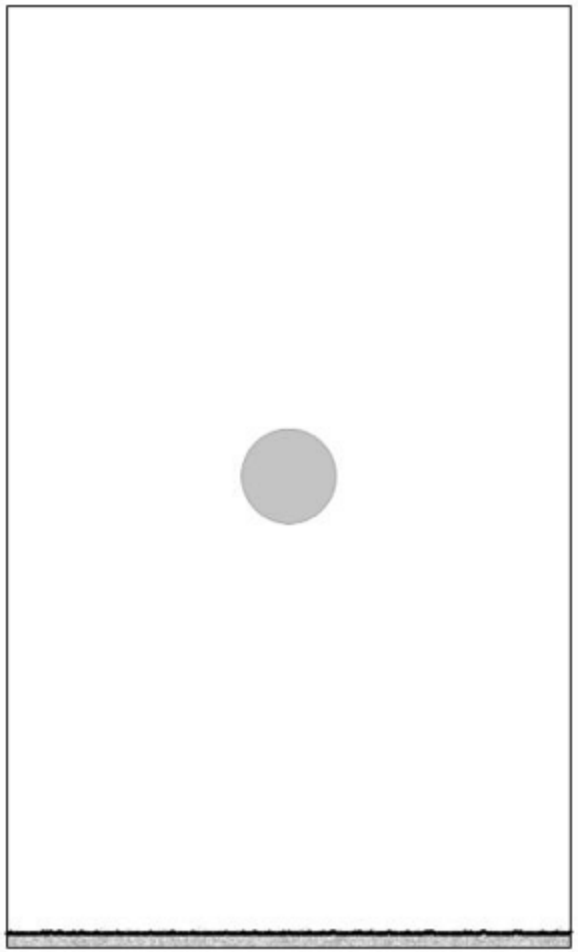
Complement of obstacles - hotspots - and geometrical optics

average front

geometrical optic

far-distance solut

finite-sized perturbation,
expands behind hotspot,
constant ‘speed-up’



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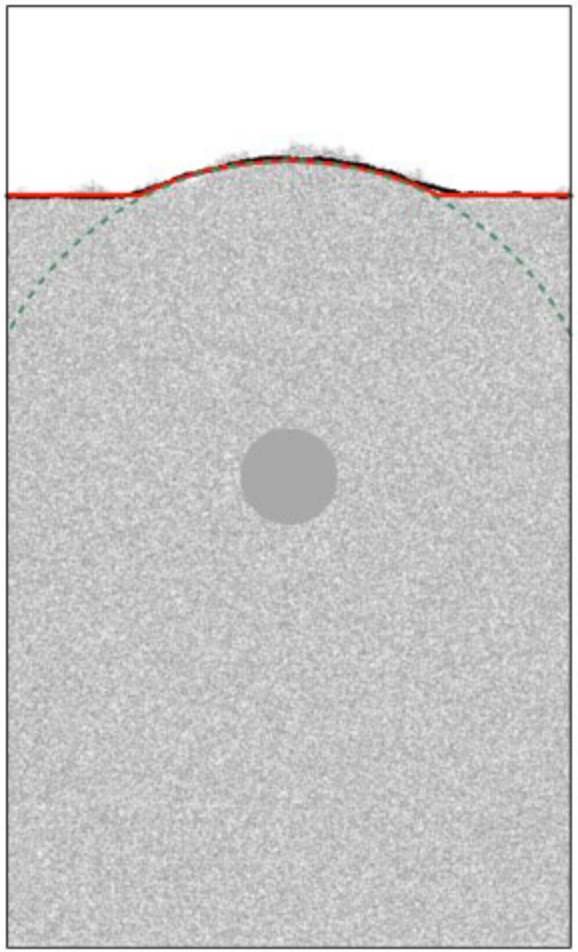
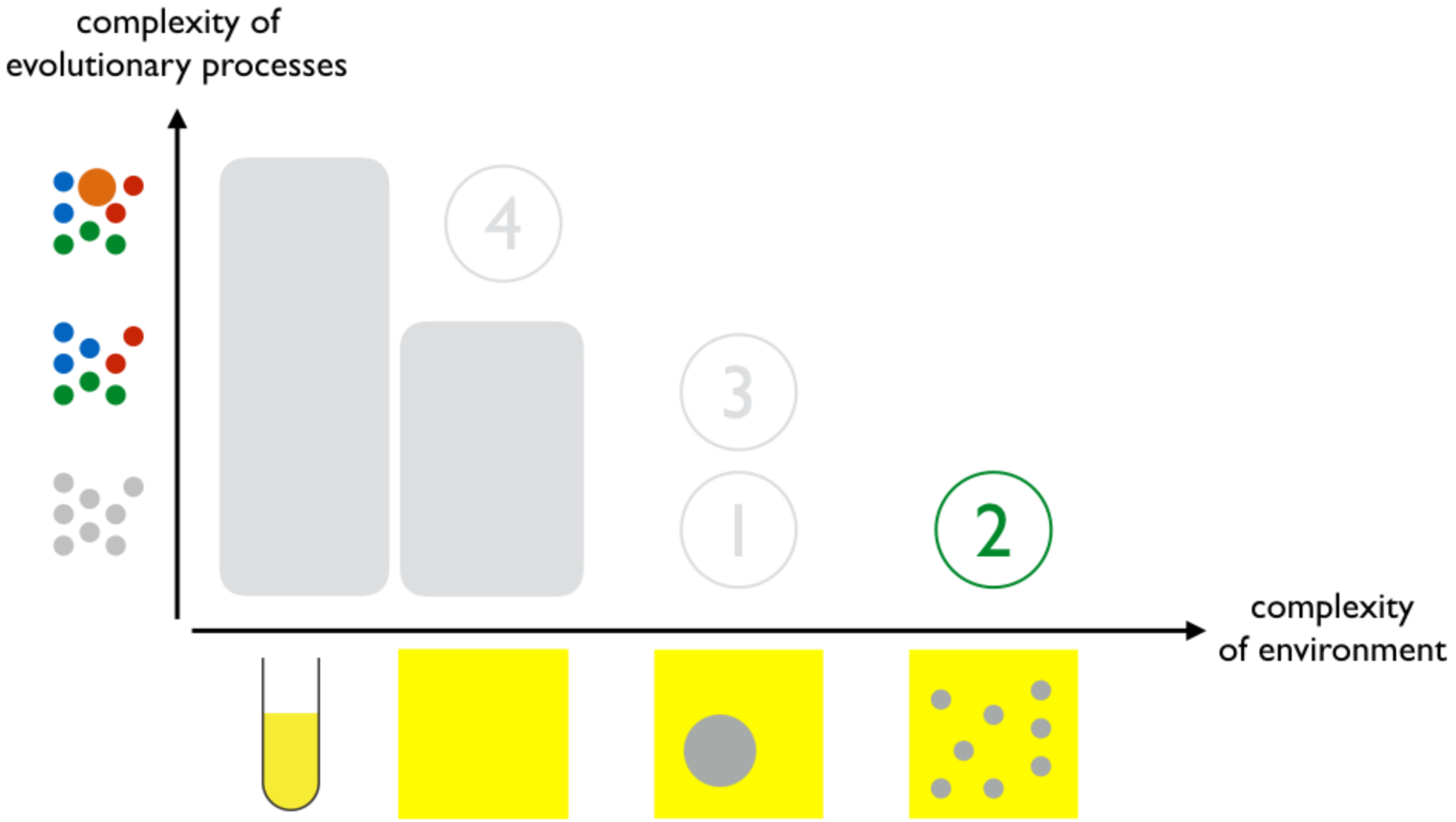


- think forest fires, not lava flows for population waves!
- Huygens' principle good prediction on large scales
- obstacles:
  - transient perturbation of front, limited to width of obstacle
  - universal front shape far away, determined by width, 'scattering from sides of obstacle'
- (circular) hotspots:
  - permanent perturbation of front, expanding outwards
  - radial wave originating from hotspot describes front far away

with David Nelson and Andrew Murray
Möbius, Murray, Nelson PLoS Comp Biol, 2015

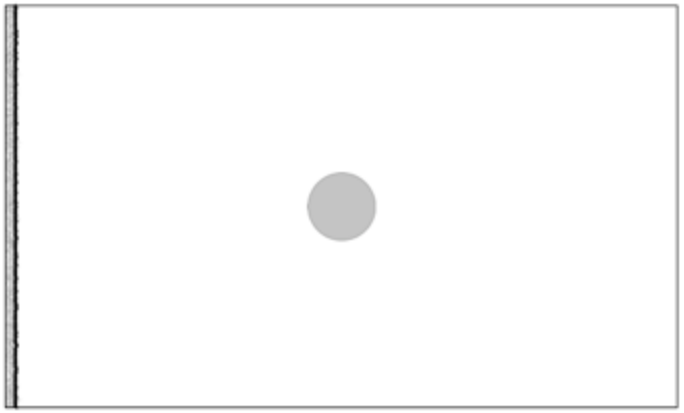
with Kim Alards, Francesca Tesser, David Nelson, Roberto Benzi, Federico Toschi

Front dynamics and evolution associated with spatial spread in heterogeneous environments

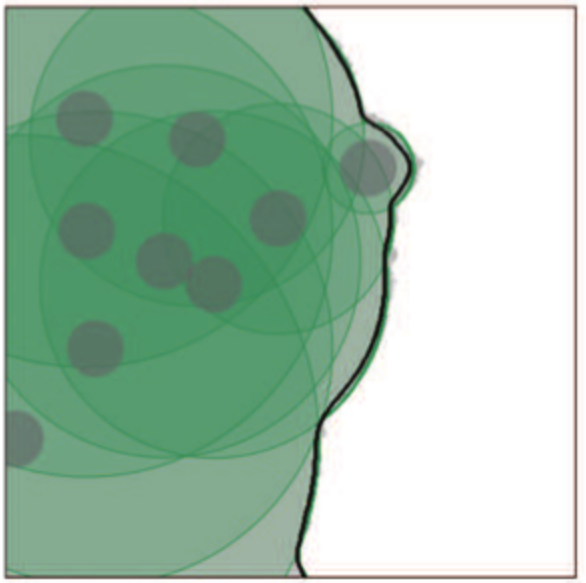


Many hotspots - scattering description

front far away described by instantaneous scattering / acceleration event



event-based numerics



applicable to dilute
(and very dense) systems

Least-time solution - obtained using Fast Marching Method

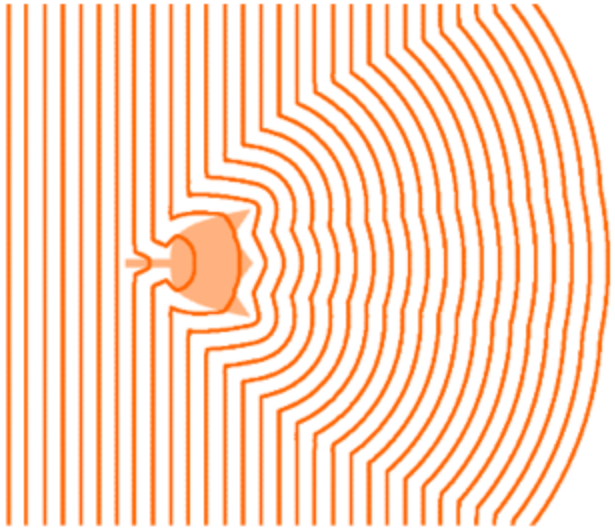
local speed

arrival time

$v(\vec{x}) \rightarrow T(\vec{x}) \rightarrow$ fronts as iso-arrival-time lines

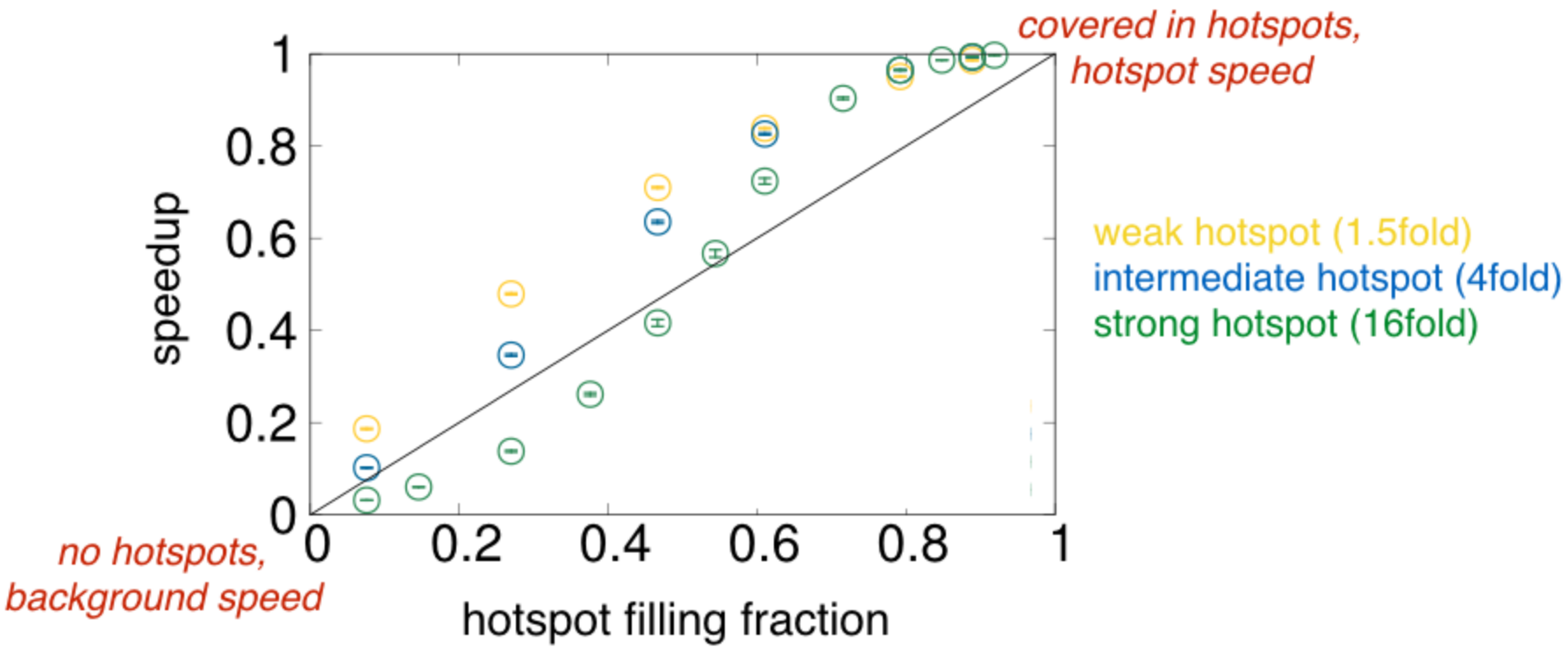
$$\left|\nabla T(\vec{x})\right| = \frac{1}{v(\vec{x})}$$

numerically solving **Eikonal equation** using **Fast Marching Method**
(related to Dijkstra's algorithm for finding shortest path on graph)

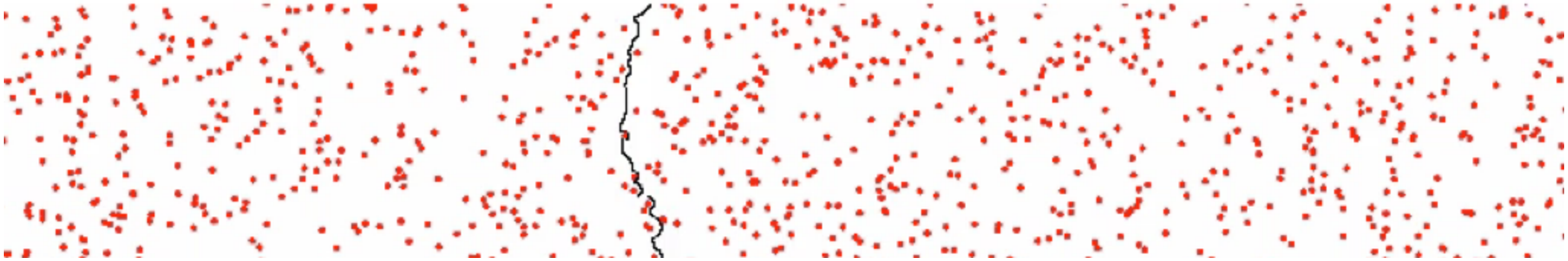


'first application'

Effective front speed in presence of many hotspots



- always faster than 1D analogue
- low density → local ‘protrusions’ lead to speed-up



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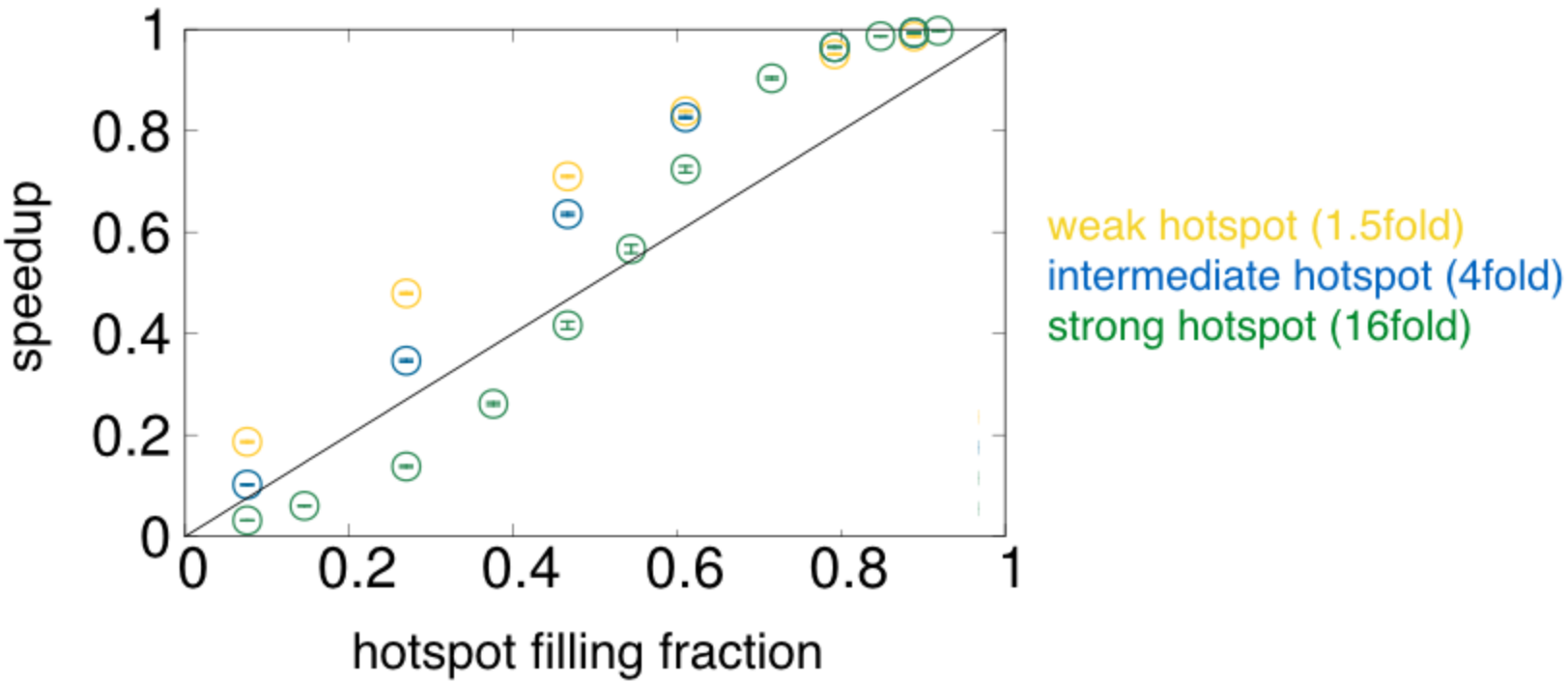
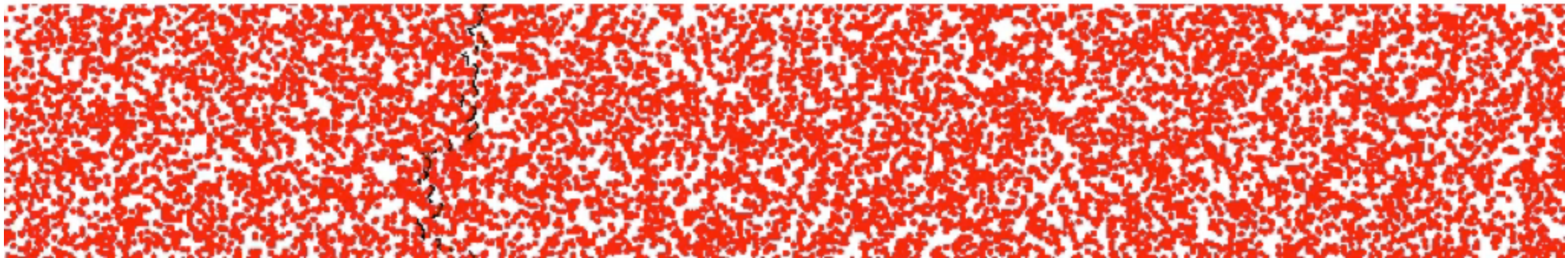
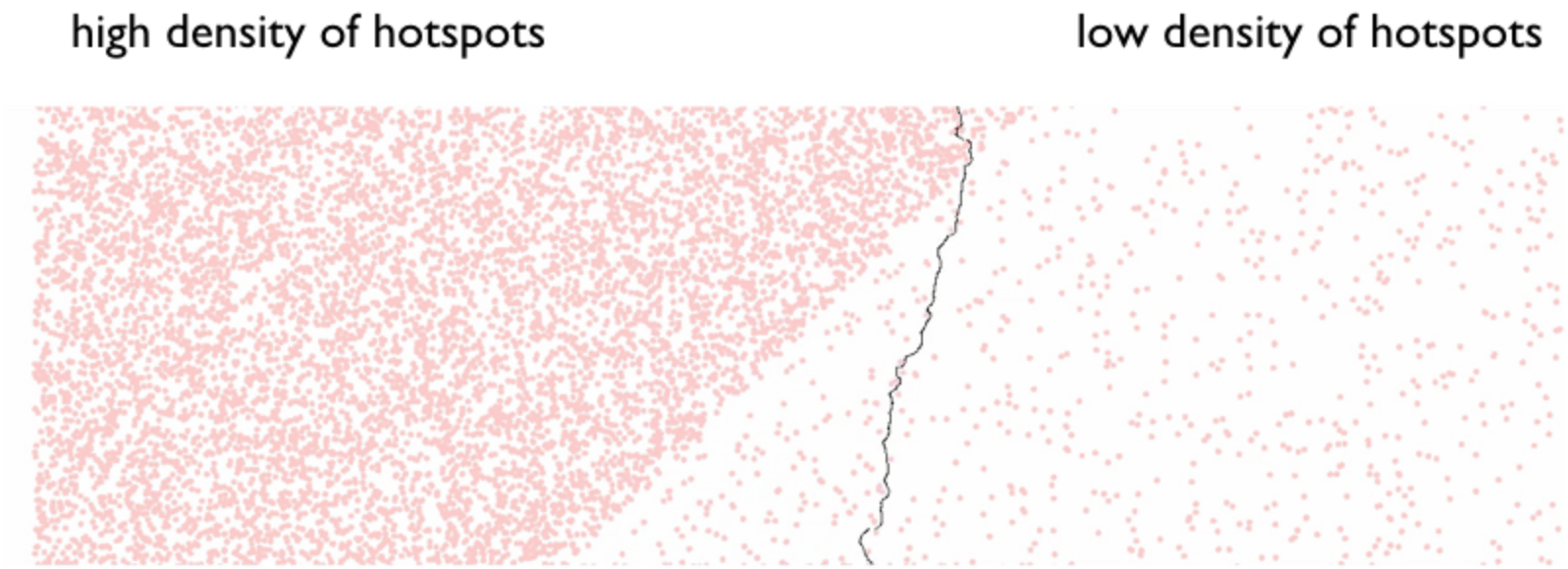


- always faster than 1D analogue
- large density → front follows shortest path through maze



Coarsening - a meta-environment?

Coarsening - a meta-environment?



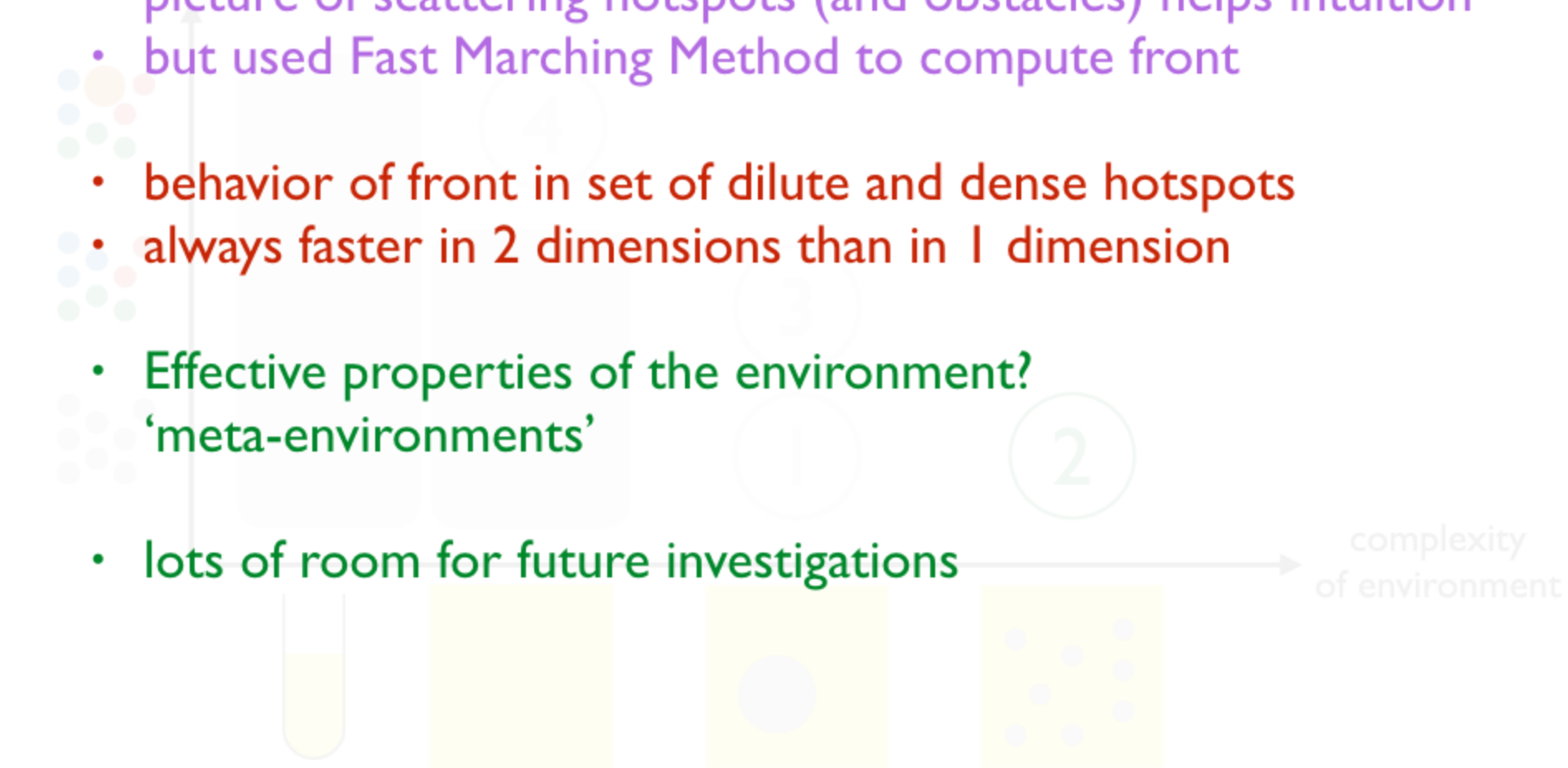
Front dynamics and evolution associated with spatial spread in heterogeneous environments

- picture of scattering hotspots (and obstacles) helps intuition
- but used Fast Marching Method to compute front

- behavior of front in set of dilute and dense hotspots
- always faster in 2 dimensions than in 1 dimension

- Effective properties of the environment? 'meta-environments'

- lots of room for future investigations



with Kim Alards, Francesca Tesser, David Nelson, Roberto Benzi, Federico Toschi

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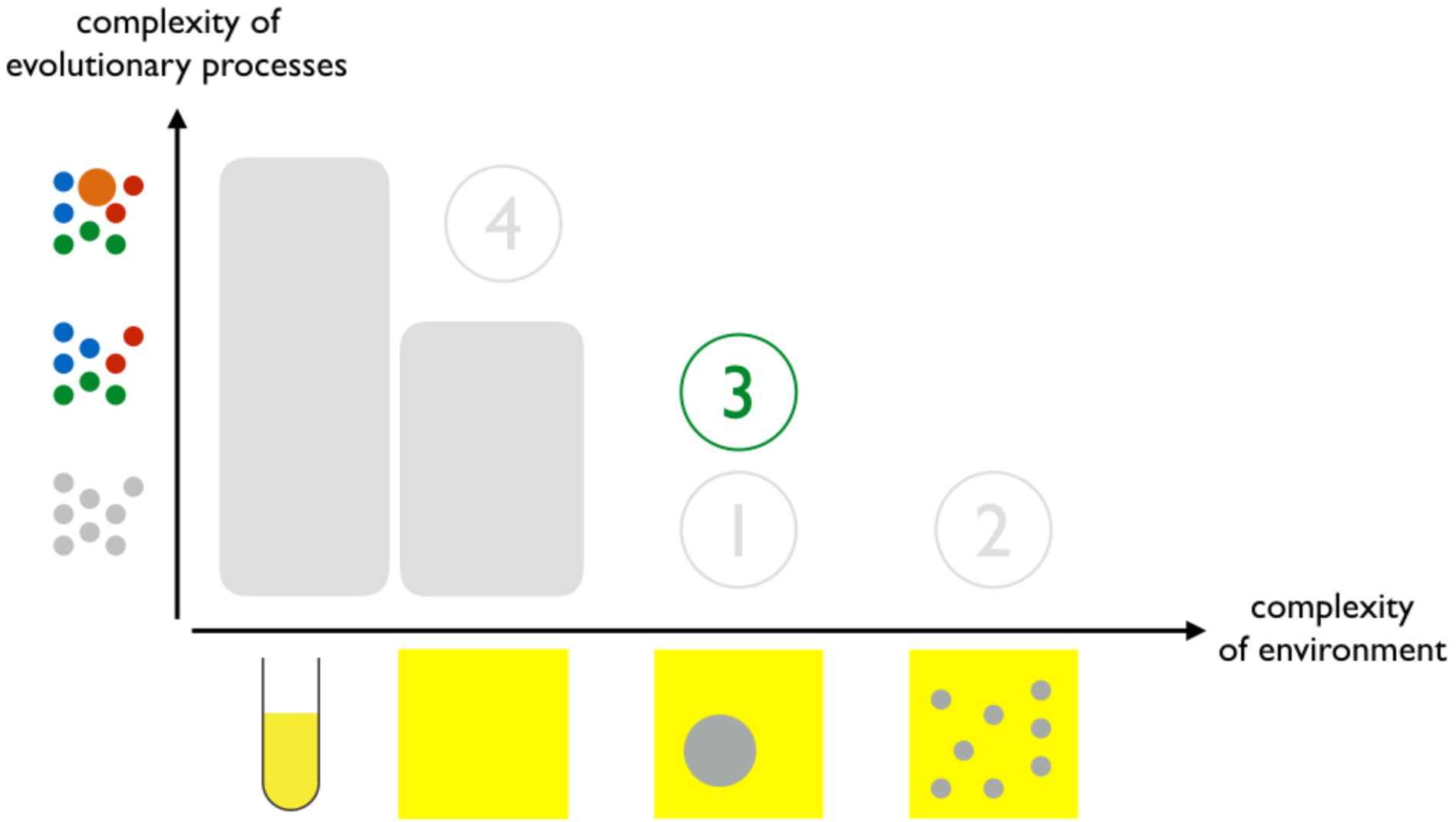
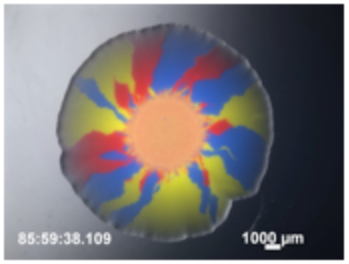
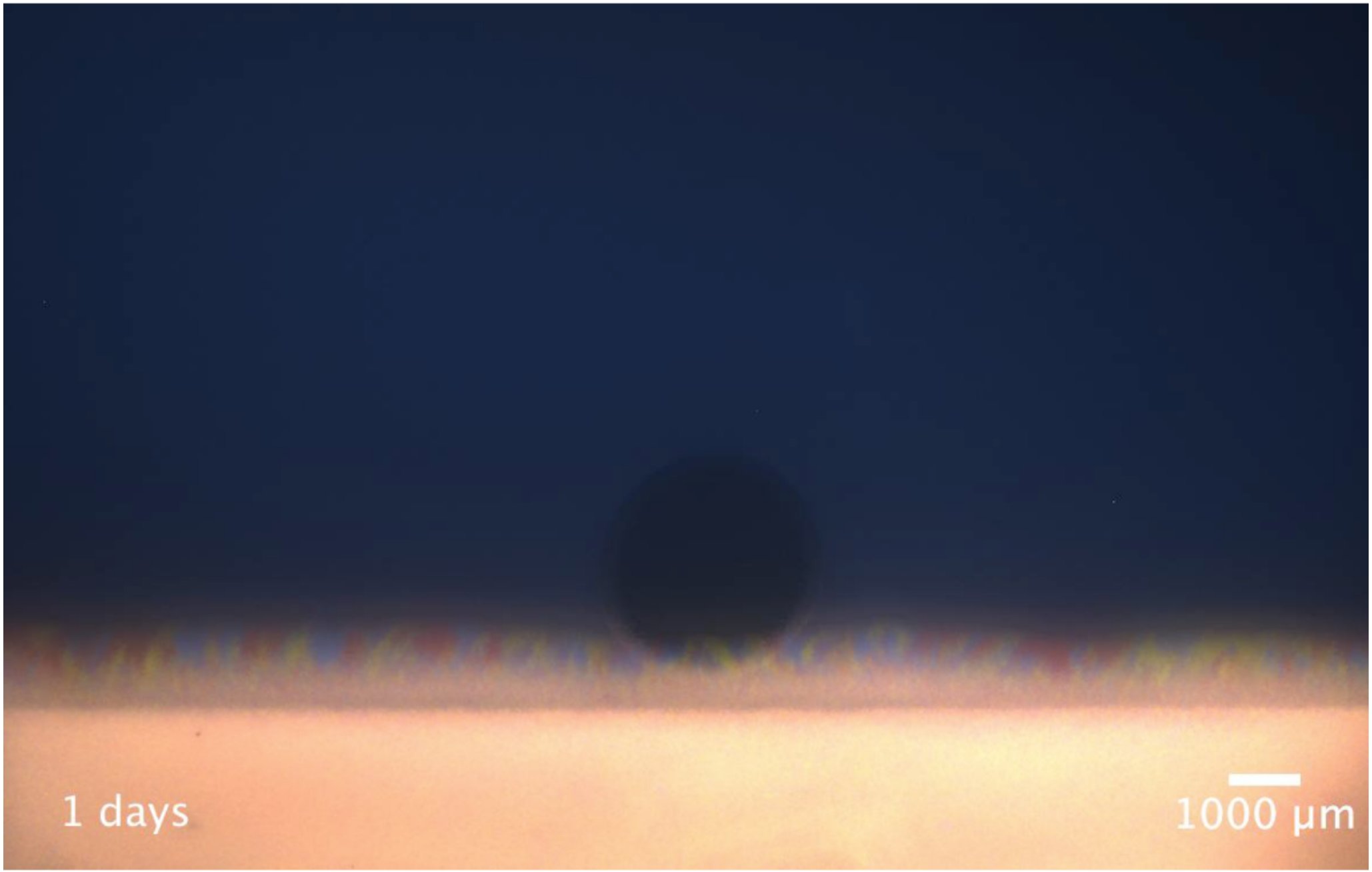


Neutral genetic diversity and one obstacle



prediction: genetic structure shaped by obstacle and front shape, complementary to 'spatial bottlenecks'

Neutral genetic diversity and one obstacle



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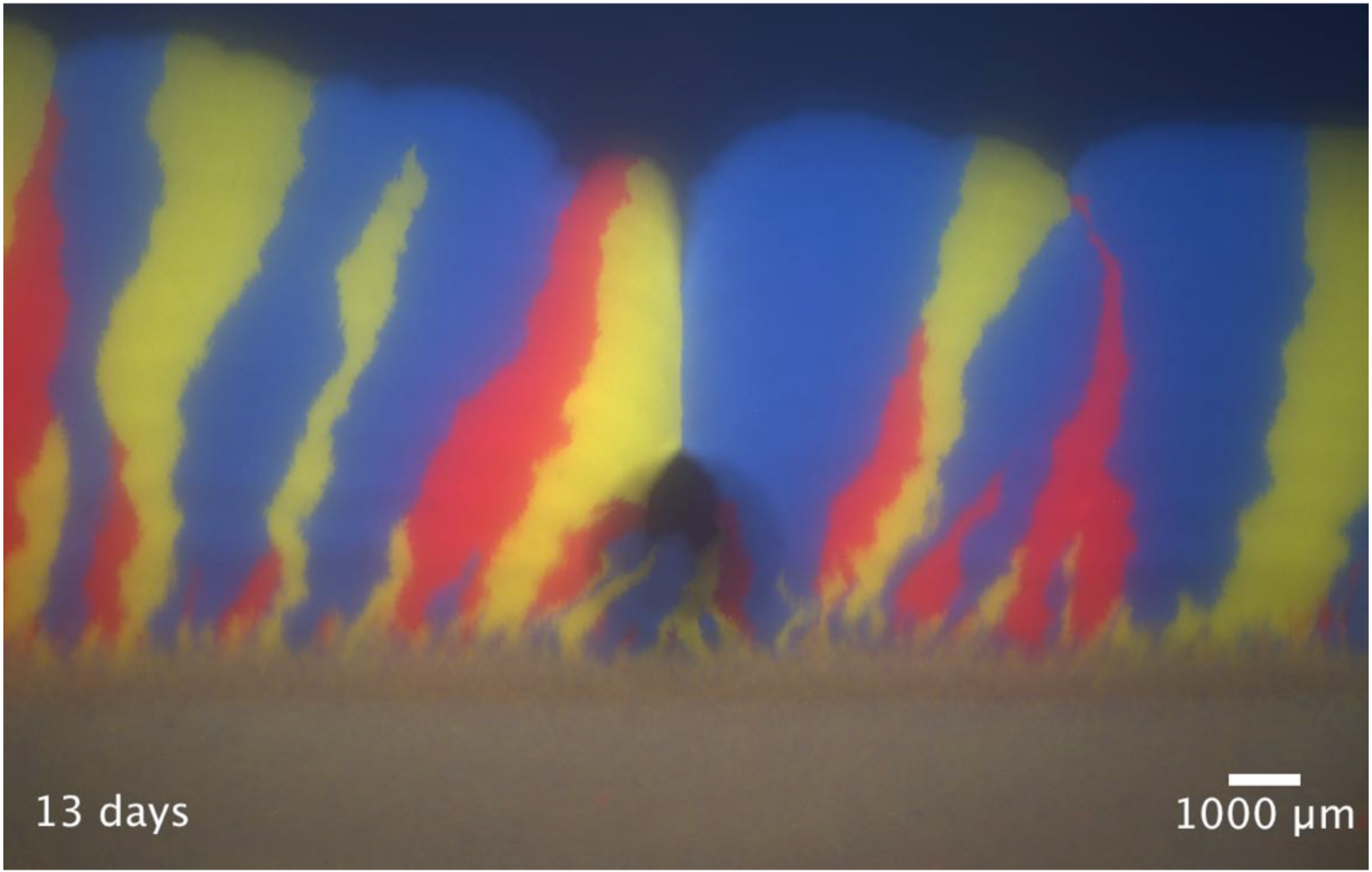

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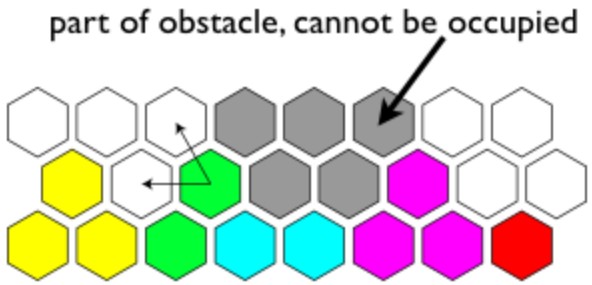
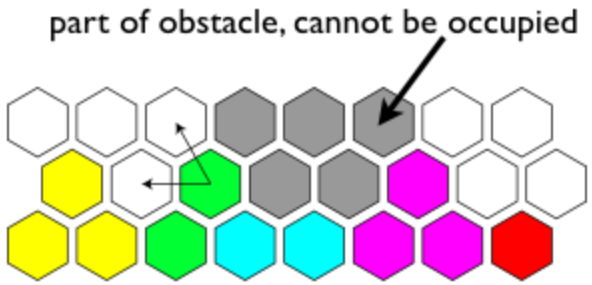


stochastic simulation of propagating front with large number of genotypes using variant of Eden model [Korolev et al., RMP, 2010]

‘geometry-enhanced genetic drift’

Neutral genetic diversity and one obstacle



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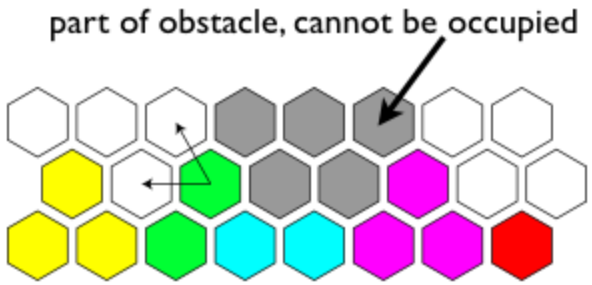

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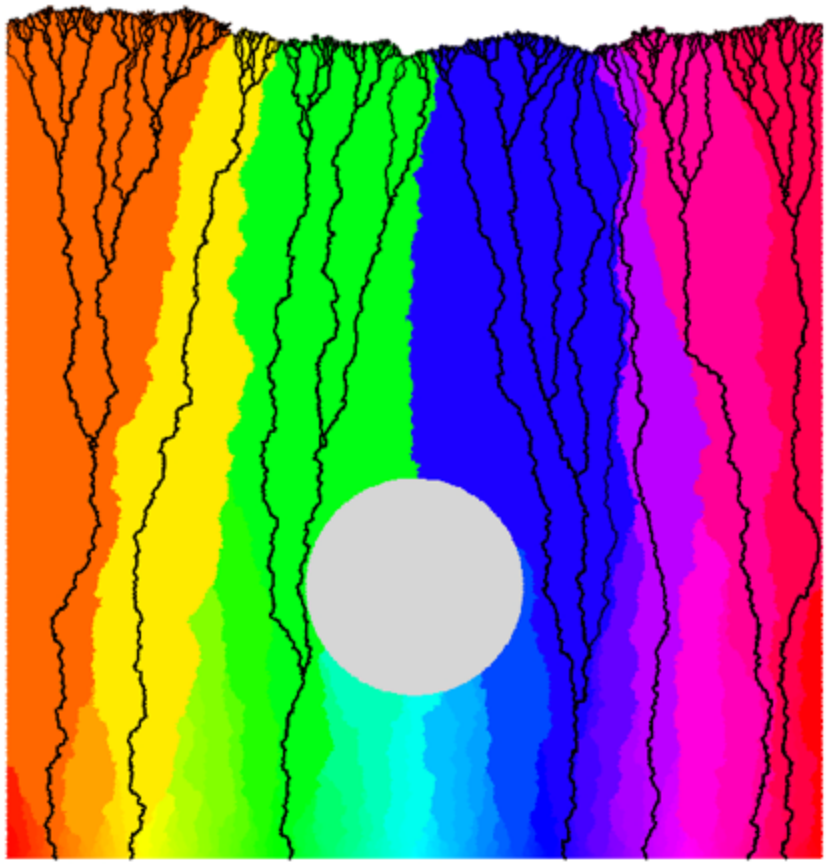

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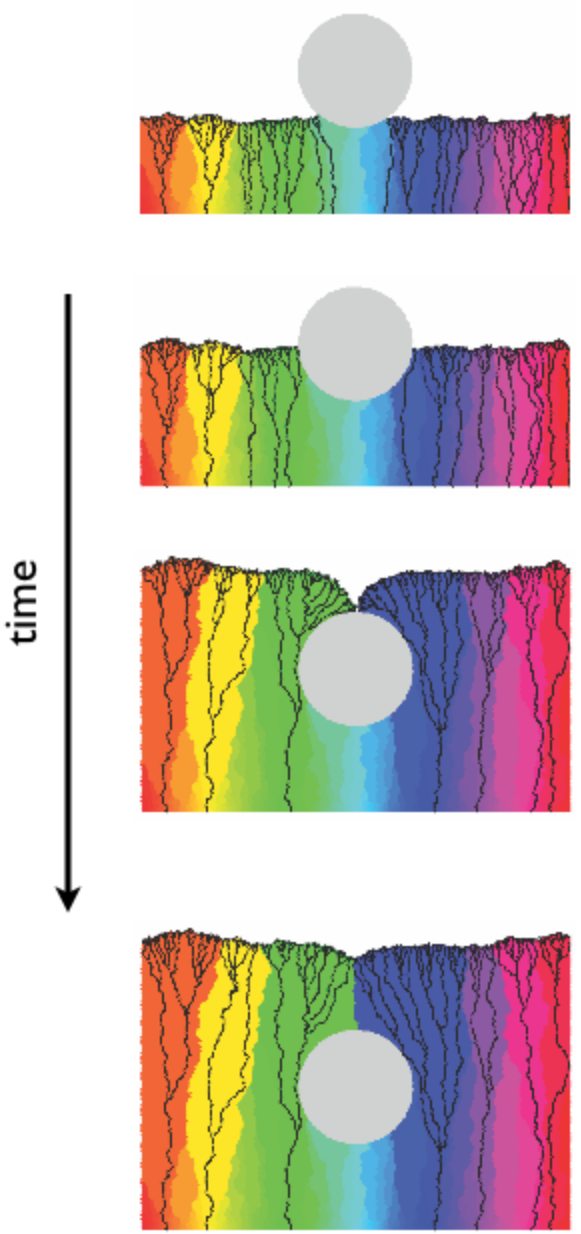


description in terms of sector boundaries

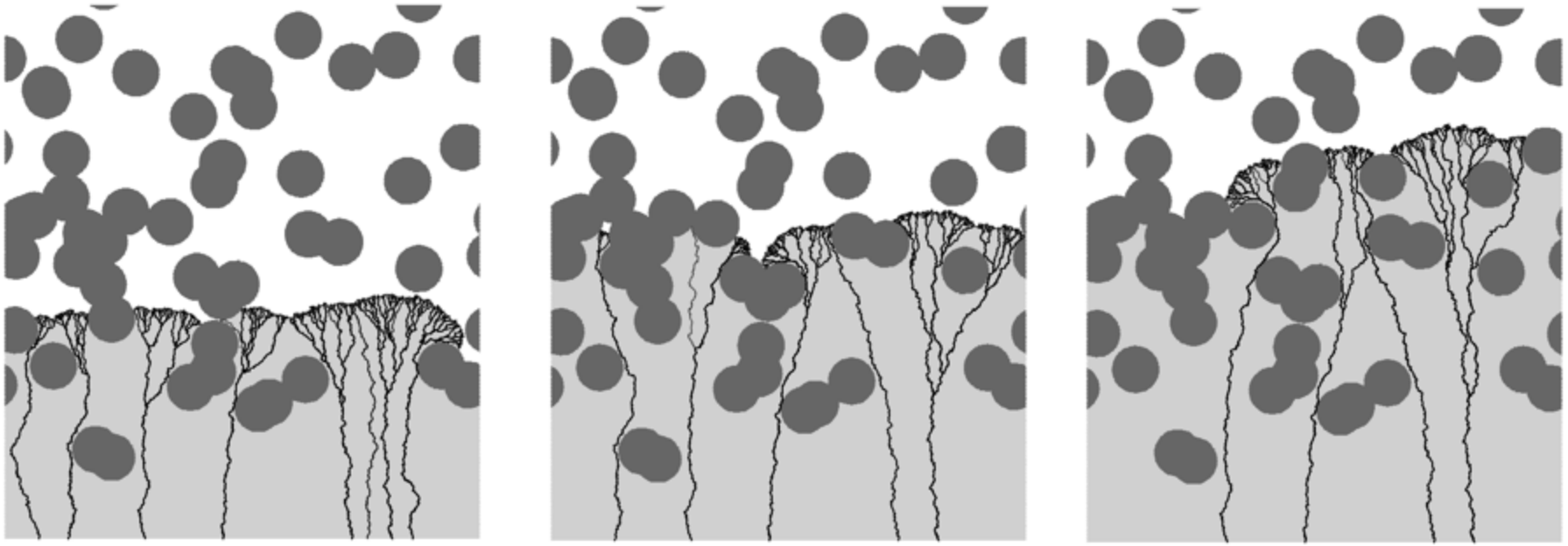
random walkers along front

description in terms of lineages

paths of least time with fluctuations

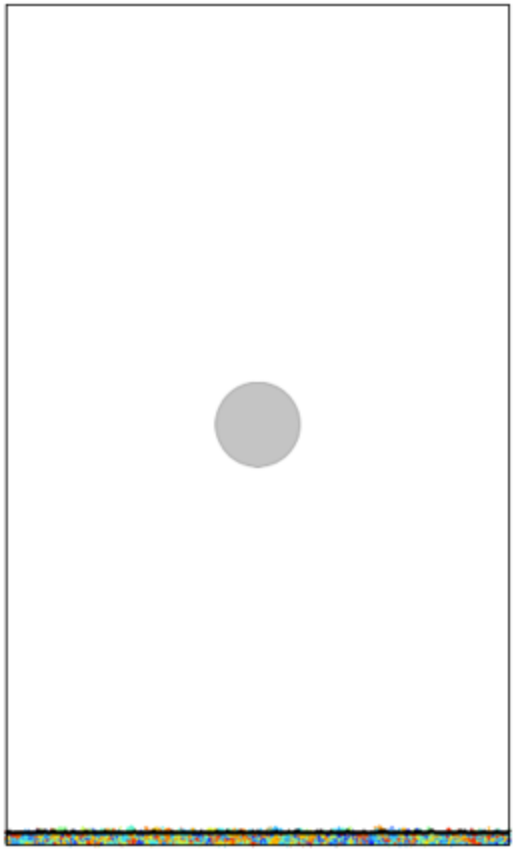
Lineages in the presence of many obstacles

can be understood as paths of least time + fluctuations?



time

‘Geometry-enhanced genetic drift’ for a single hotspot



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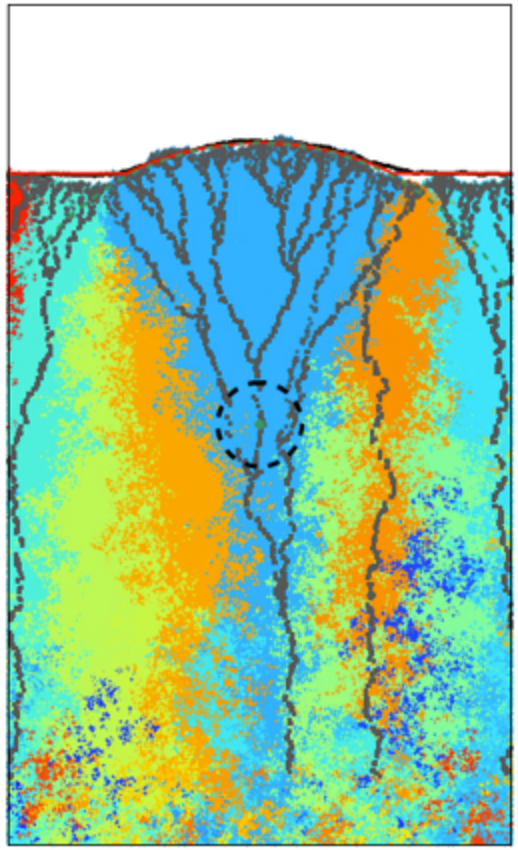


Front dynamics and evolution associated with spatial spread in heterogeneous environments

- front shape helps determine effect on neutral diversity

- 'geometry-enhanced genetic drift',
  an additional layer of 'survival of the luckiest'

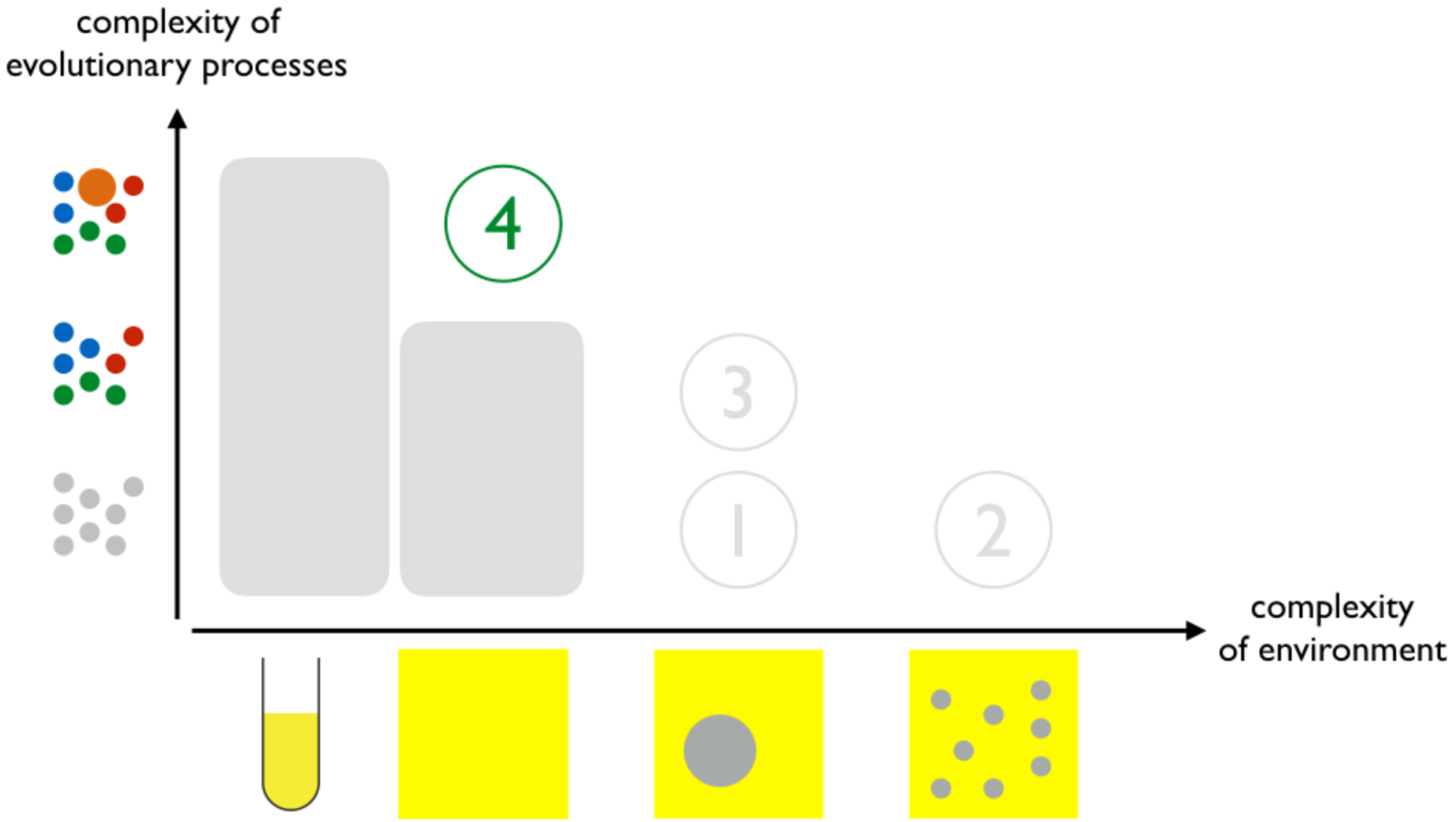
- obstacles similar to, yet different from, spatial bottlenecks
- hotspots result in different effect of 'geometry-enhanced genetic drift'

- What are the consequences for non-neutral evolution?
- How to apply to complex heterogeneous environments?

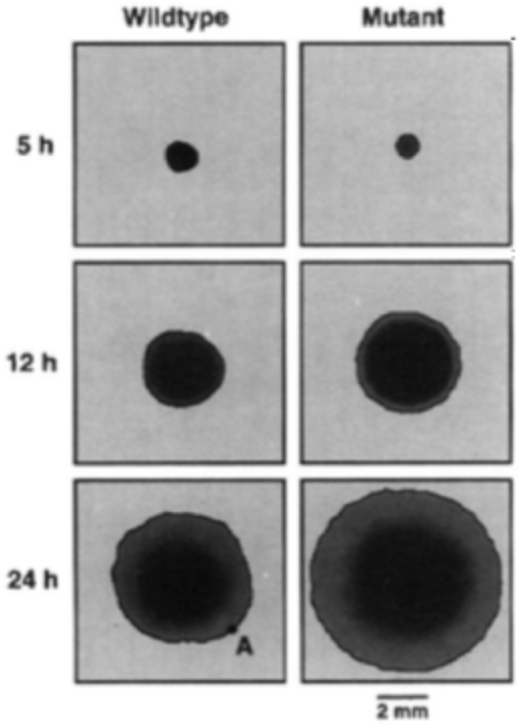
with David Nelson and Andrew Murray
Möbius, Murray, Nelson PLoS Comp Biol, 2015

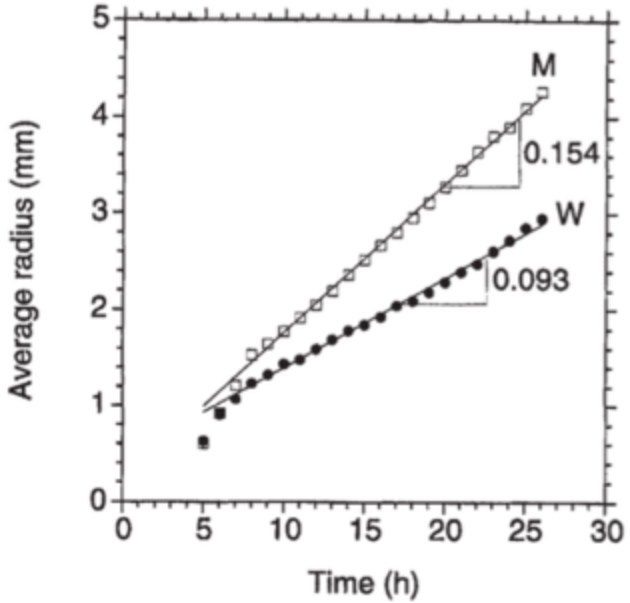
with Kim Alards, Francesca Tesser, David Nelson, Roberto Benzi, Federico Toschi

Front dynamics and evolution associated with spatial spread in heterogeneous environments



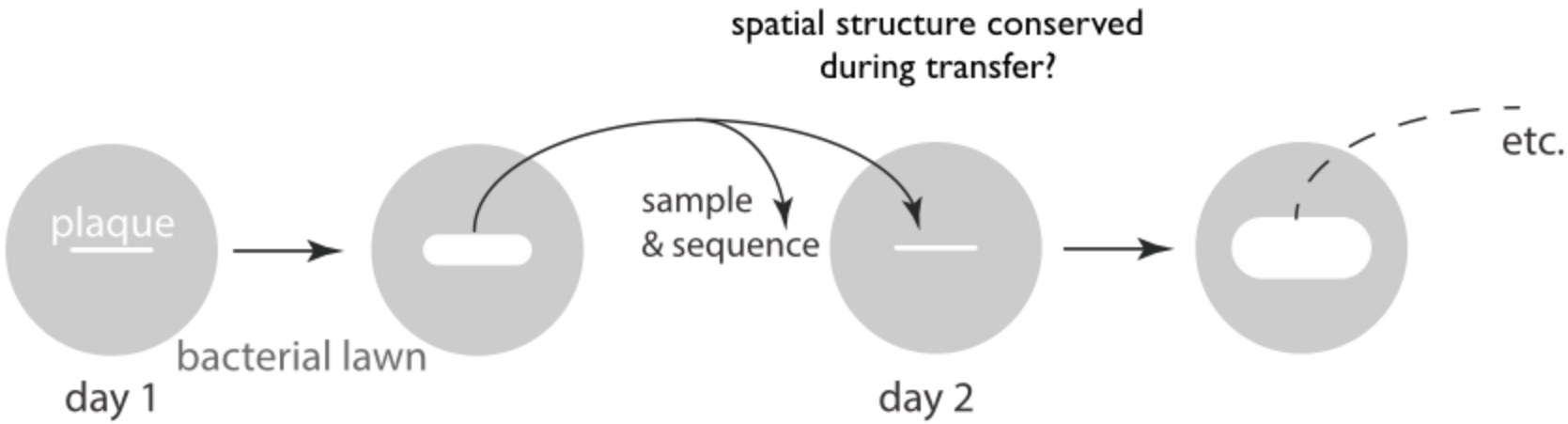
Evolution during a phage range expansion





[Lee & Yin, 1996]

Our evolution experiment



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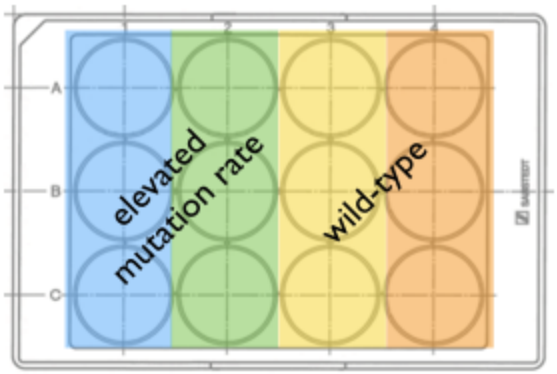

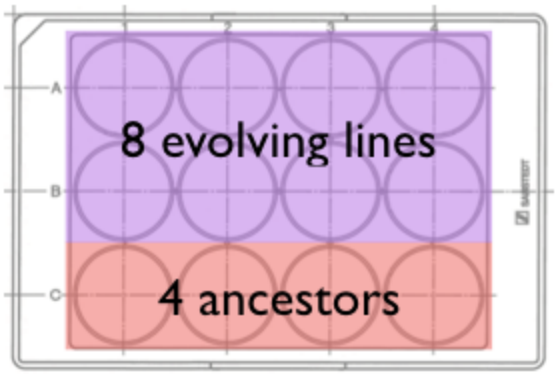

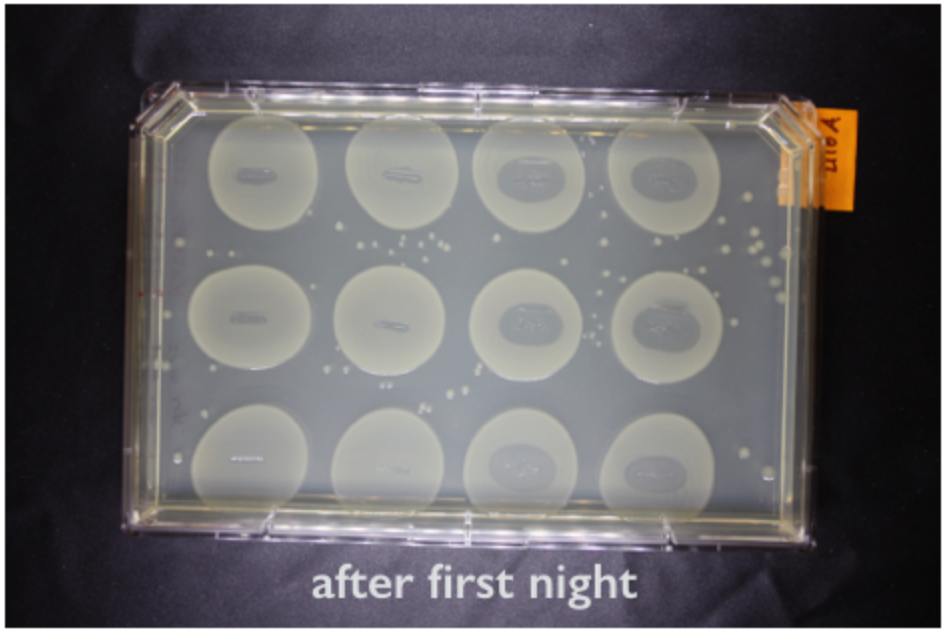

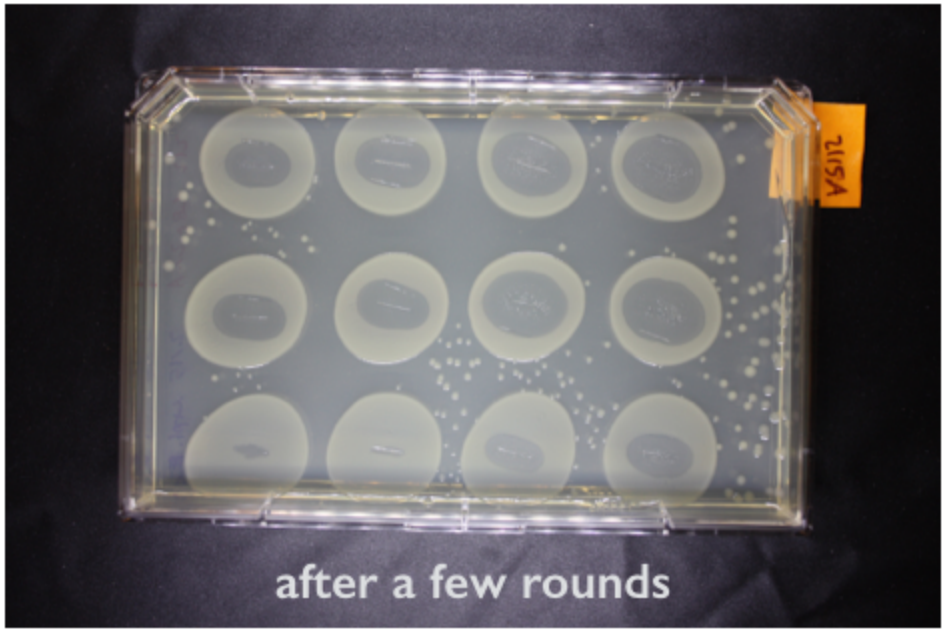

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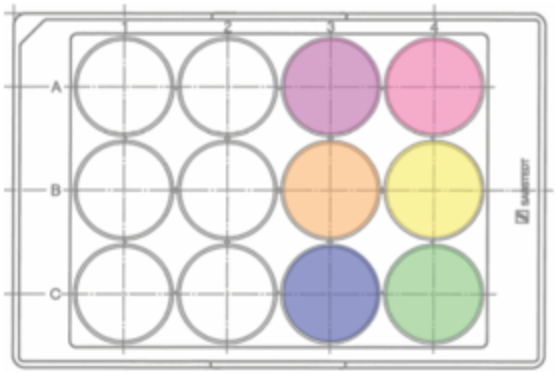
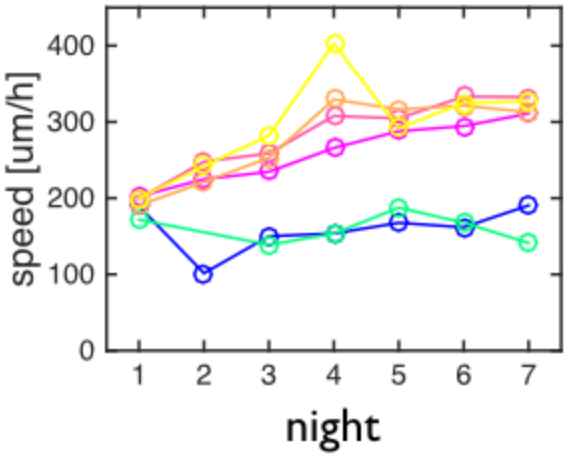


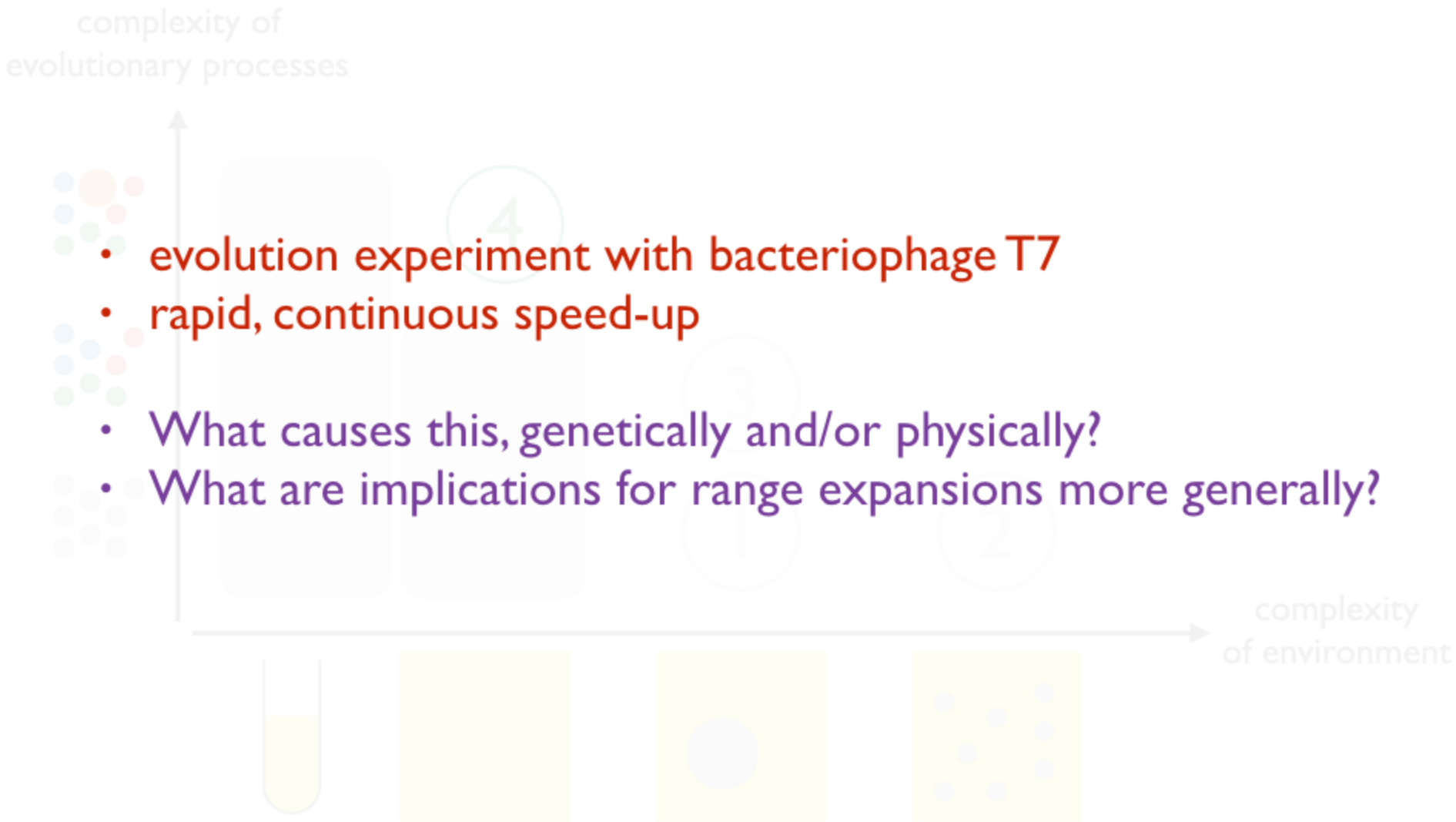


WT evolving strains

WT ancestors

preliminary analysis!

Front dynamics and evolution associated with spatial spread in heterogeneous environments

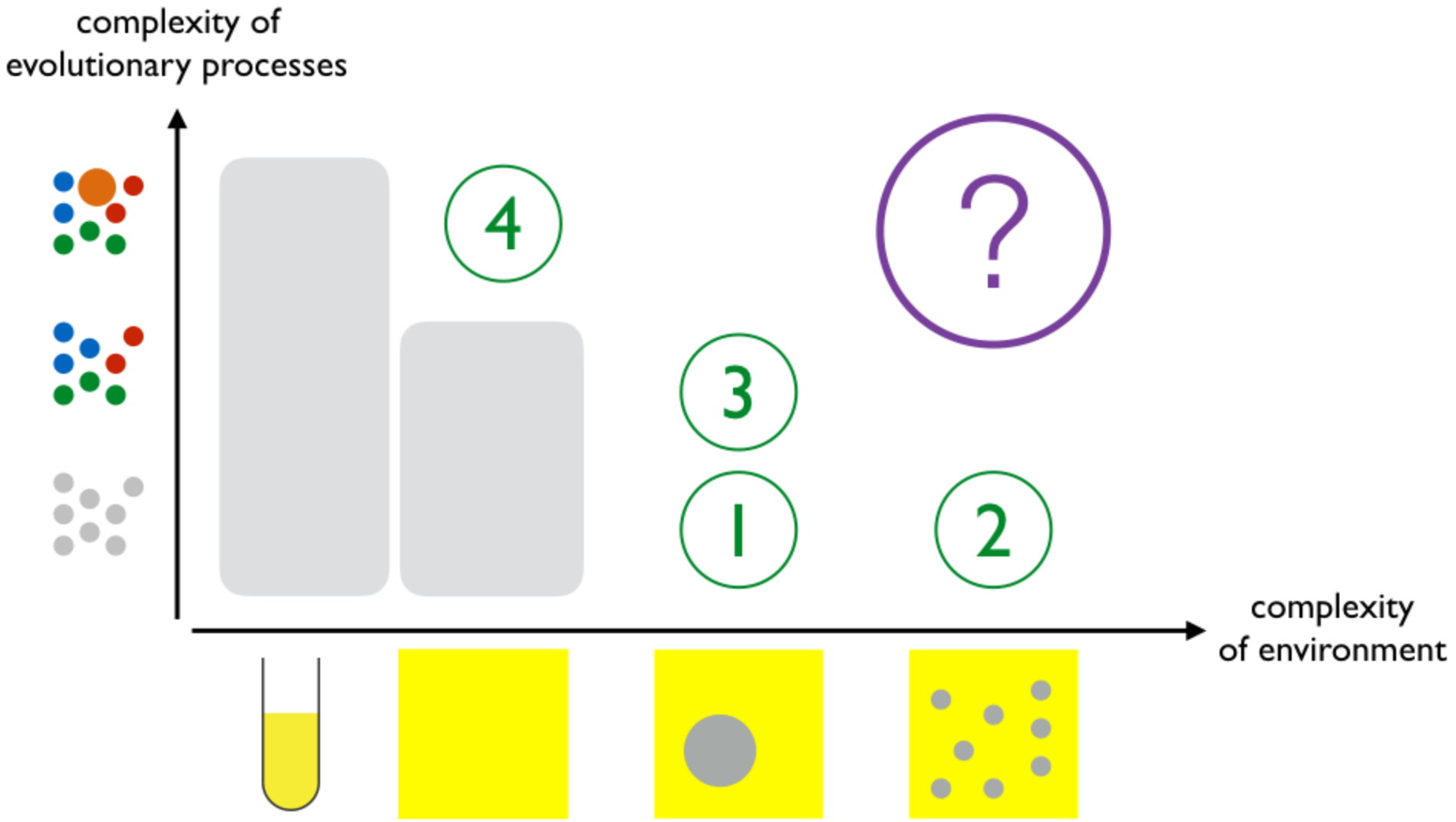


- evolution experiment with bacteriophage T7
- rapid, continuous speed-up

- What causes this, genetically and/or physically?
- What are implications for range expansions more generally?

work in progress with Diana Fusco and Oskar Hallatschek

Front dynamics and evolution associated with spatial spread in heterogeneous environments



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[qb3.berkeley.edu]

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Oskar Hallatschek

Hallatschek Lab members

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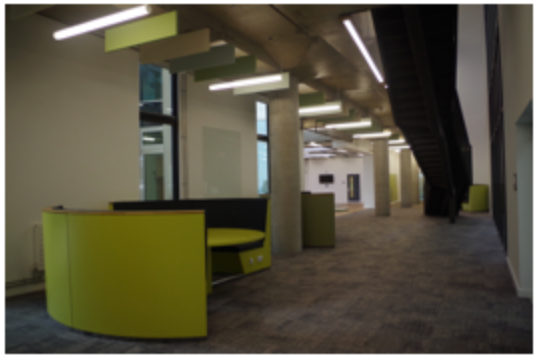
Living Systems Institute, University of Exeter



England, Devon