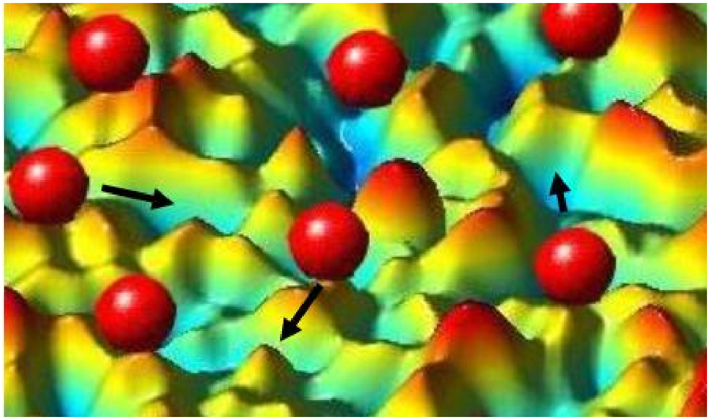
Slow relaxations and aging in electron glasses

Ariel Amir , work with
Yuval Oreg and Yoseph Imry



מכון ויצמן למדע
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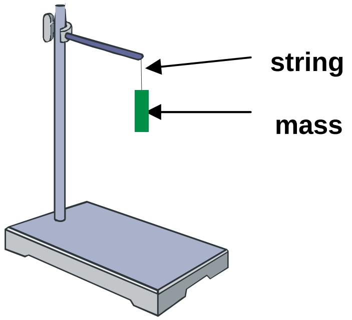
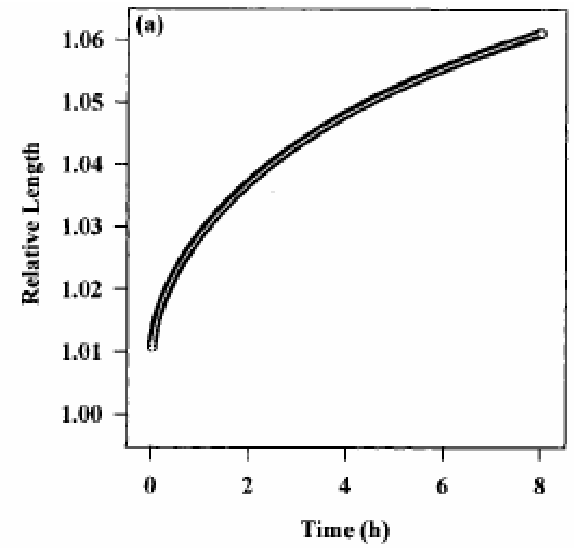
Slow relaxations in nature

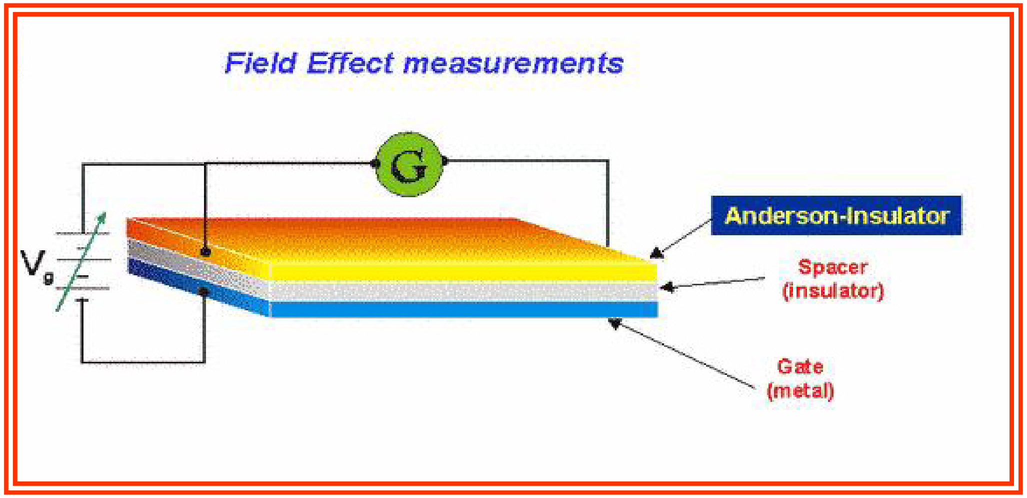
W. Weber, *Ann. Phys.* (1835)

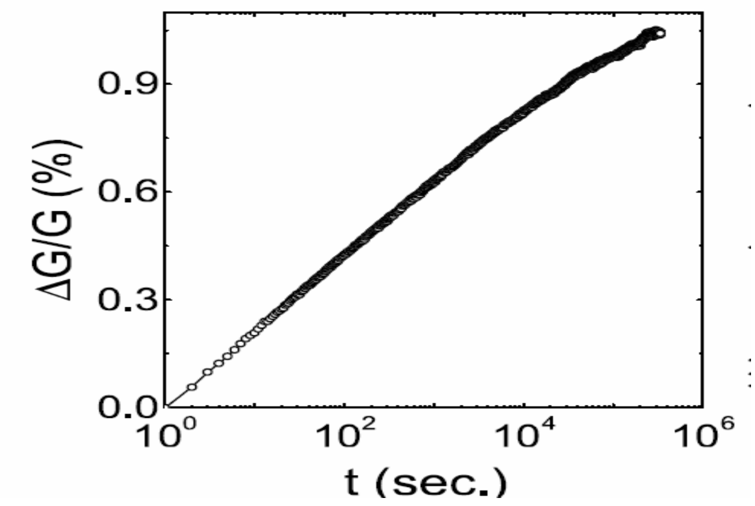
D. S. Thompson, *J. Exp. Bot.* (2001)





Electron glass- Experimental system





What are the ingredients leading to slow relaxations?

Ovadyahu et al.

Logarithmic relaxations for 5 days!

Electron glass aging– experimental protocol

A. Vaknin and Z. Ovadyahu and M. Pollak, PRL 2000

Step I

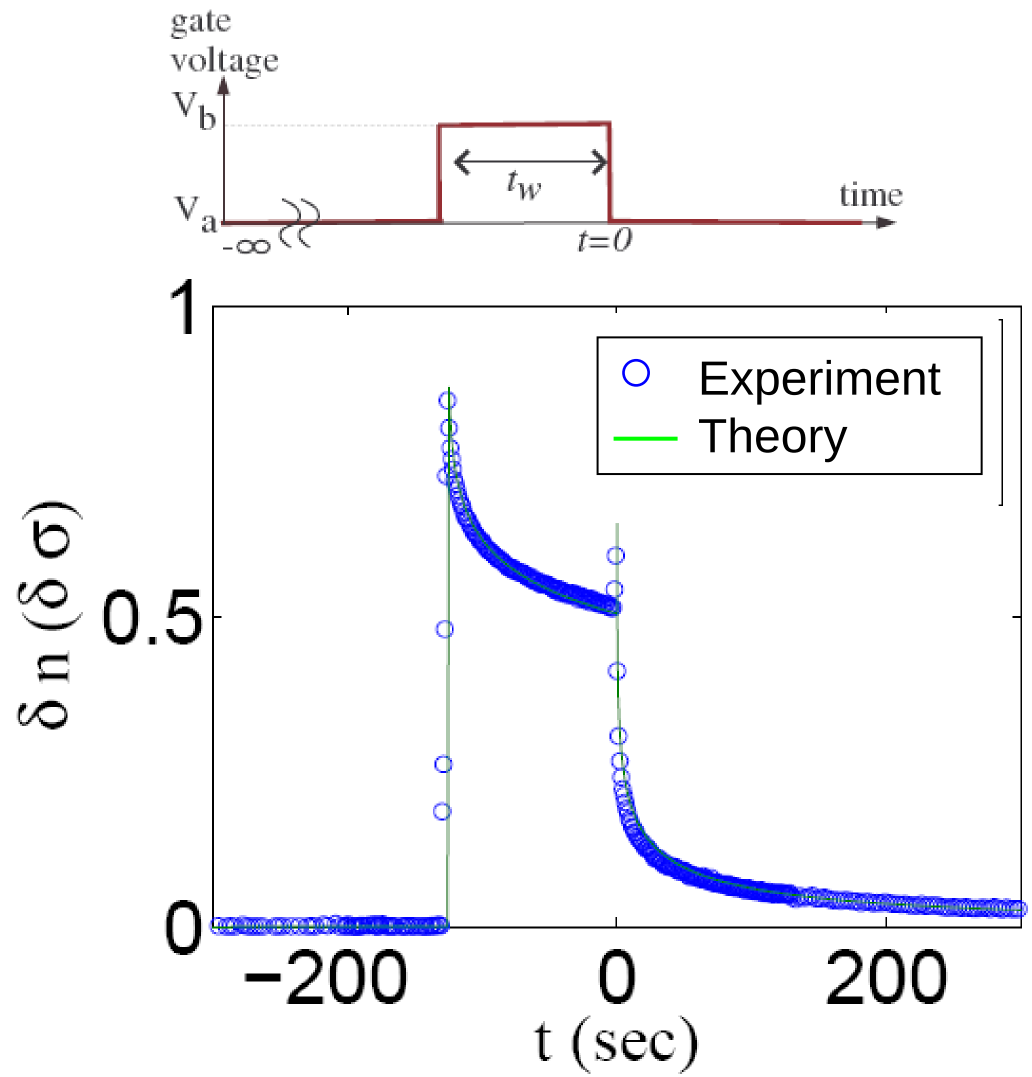
System equilibrates for long time

Step II

V_g is changed, for a time of t_w.

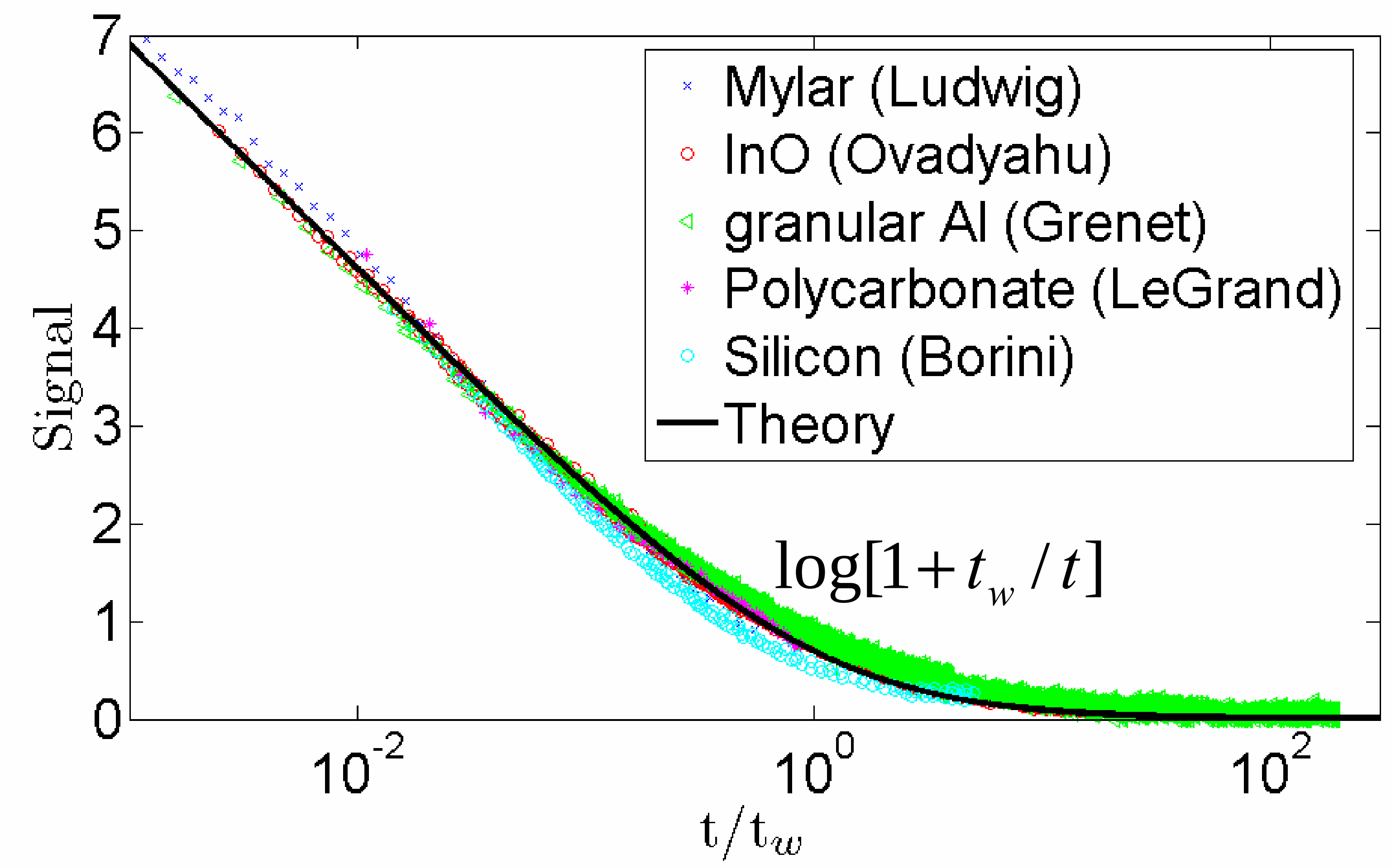
Throughout the experiment

Conductance is measured as a function of time.



Data: *Ovadyahu et al.*

Aging and universality



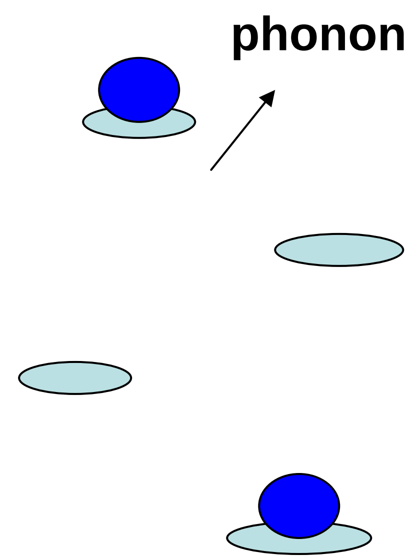
Amir, Oreg and Imry, to be published

The model

- Strong localization due to disorder

  → randomly positioned sites, on-site disorder.

- Coulomb interactions are included
- “Phonons” induce transitions between configurations.
- Interference (quantum) effects neglected.



e.g:

Pollak (1970)

Shklovskii and Efros (1975)

Ovadyahu and Pollak (2003)

Muller and Ioffe (2004)

“Local mean-field” approximation - Dynamics

AA, Oreg and Imry, PRB (2008)

$$n_i \to \langle n_i \rangle, \qquad \frac{dn_i}{dt} = \sum_j -\gamma_{i,j} + \gamma_{j,i}$$

$$\gamma_{i,j} = \exp(-2r_{ij}/\xi) n_i (1-n_j)[N(|\Delta E|) + \theta(\Delta E)]$$

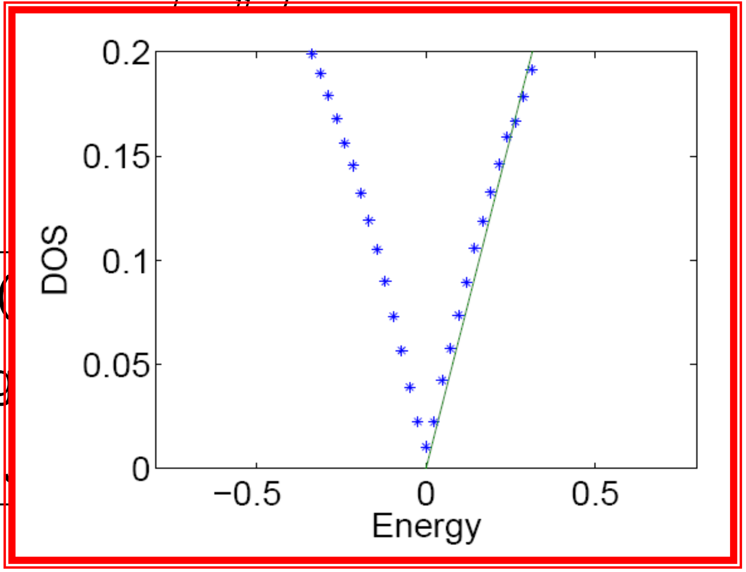
- ΔE includes the interactions
- N is the Bose-Einstein distribution
- ξ - the localization length

$$\left(E_i = \varepsilon_i + \sum_j \frac{n_j}{r_{ij}} \right)$$

At long times (Statics):

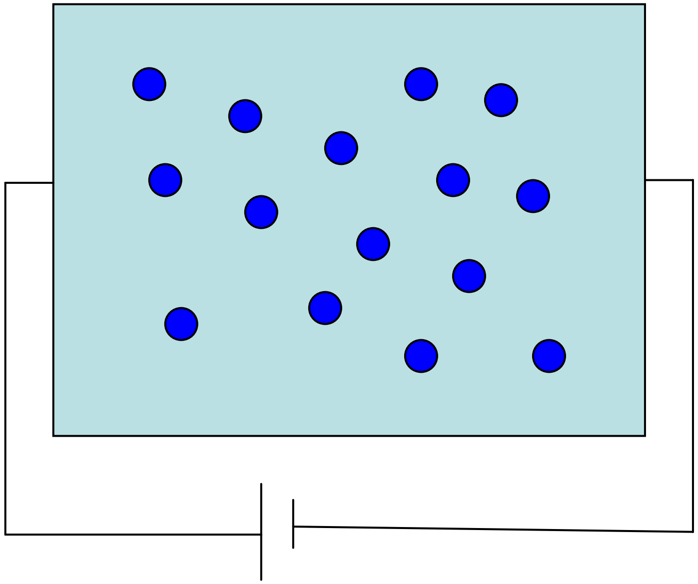
- The system reaches a locally stable point (
- Many metastable states, each manifesting

(“Pseudo-ground-states”, *Baranovski et al.,*



“Local mean-field” approximation – Steady State

Miller-Abrahams resistance network (no interactions)



$$R_{ij} = \frac{T}{e^2 \gamma_{ij}^0}$$

Equilibrium rates,

obeying detailed balance

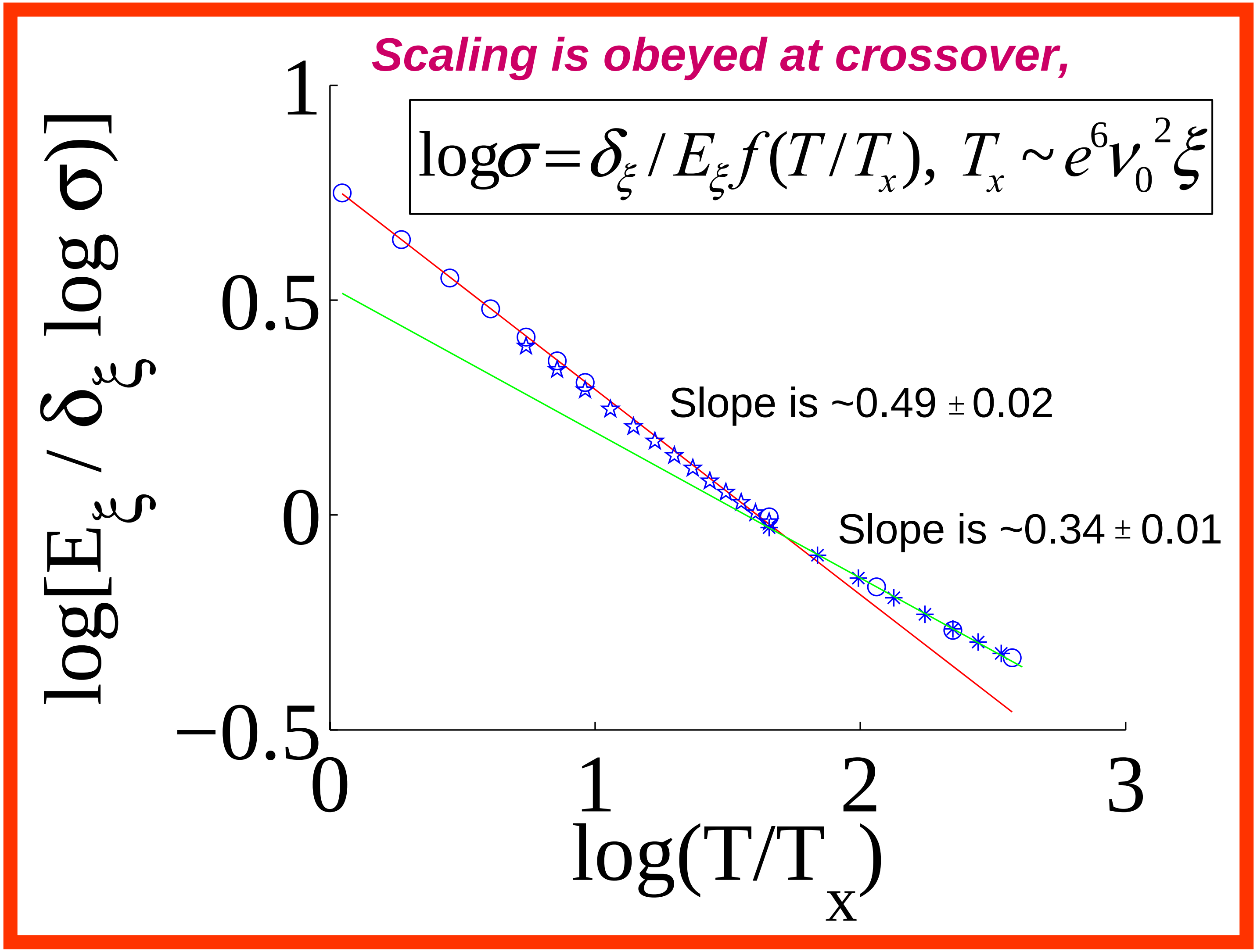
A. Miller and E. Abrahams, (Phys. Rev. 1960)

Generalization

1) Find n_i and E_i such that the systems is in steady state.

2) Construct resistance network.

VRH (Mott) to E-S Crossover



Amir, Oreg and Imry , PRB (2009)

“Local mean-field” approximation - Dynamics

$$n_i \rightarrow \langle n_i \rangle$$

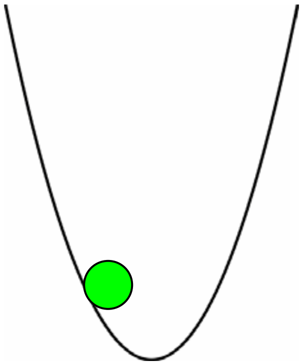
$$\frac{dn_i}{dt} = \sum_j -\gamma_{i,j} + \gamma_{j,i}$$

$$\gamma_{i,j} = \exp(-2r_{ij}/\xi)n_i(1-n_j)[N(|\Delta E|) + \theta(\Delta E)]$$

We saw: approach works well for statics & steady-states

Moving on to dynamics…

Solution near locally stable point



Close enough to the equilibrium (locally) stable point,

one can linearize the equations, leading to the equation:

$$\frac{d\delta\vec{n}}{dt} = A \cdot \delta\vec{n}$$

$$A_{i,j} = \frac{\gamma^0_{i,j}}{n^0_j(1-n^0_j)} - \frac{e^2}{T}\sum_{l\neq i,j}\gamma^0_{i,k}\left(\frac{1}{r_{i,j}} - \frac{1}{r_{i,k}}\right), \qquad (i \neq j)$$

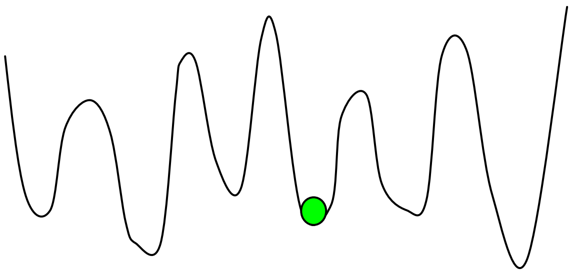
Sum of columns vanishes (particle conservation number)

$A = \gamma \cdot \beta$, β^{-1} is equal-time correlation matrix

$\gamma^0_{i,j} \sim e^{-\frac{2r_{ij}}{\xi}}$ (Anderson Localization)

For low temperatures, near a local minimum,

second term is negligible →

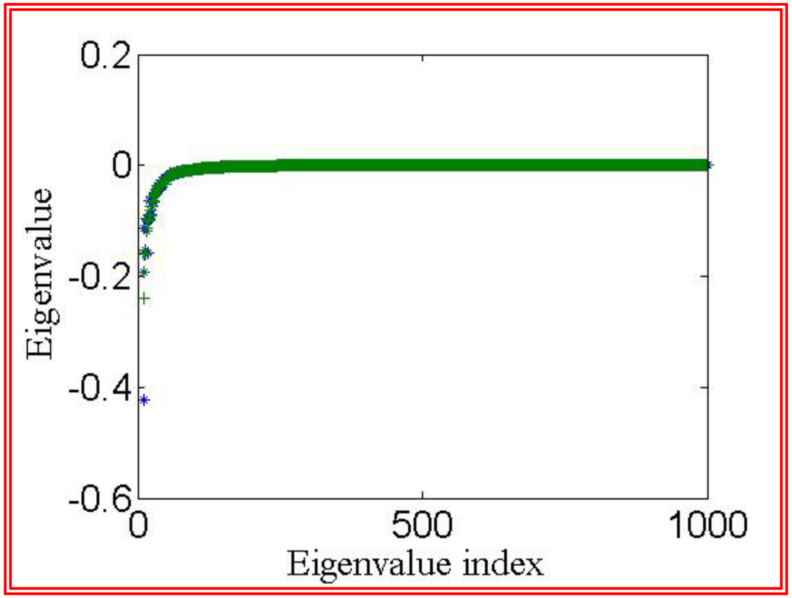


All eigenvalues are real and negative

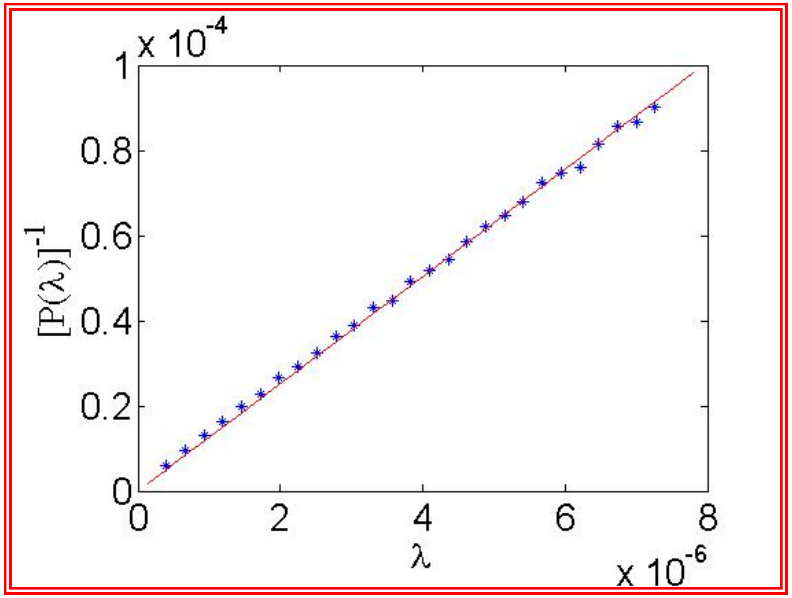
Eigenvalue Distribution

Solving numerically shows a distribution proportional to $1/\lambda$:

Eigenvalues



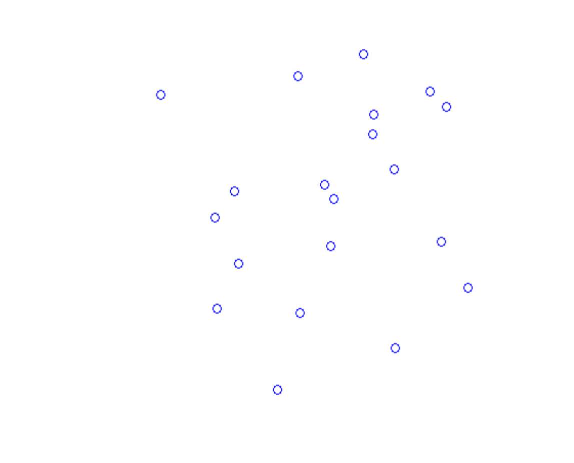
Eigenvalue distribution



$$\sum_{\lambda} e^{-\lambda t} \longrightarrow \int P(\lambda) e^{-\lambda t} d\lambda \sim -\gamma_E - \log(\lambda_{\min} t)$$

Digression: What are Random Distance Matrices?

1) Choose N points randomly and uniformly in a d-dimensional cube.



I. M. Lifshitz, *Adv. Phys* (1964).

Mezard, Parisi and Zee, *Nucl. Phys.* (1999)

Bogomolny, Bohigas, and Schmit, *J. Phys. A: Math. Gen.* (2003).

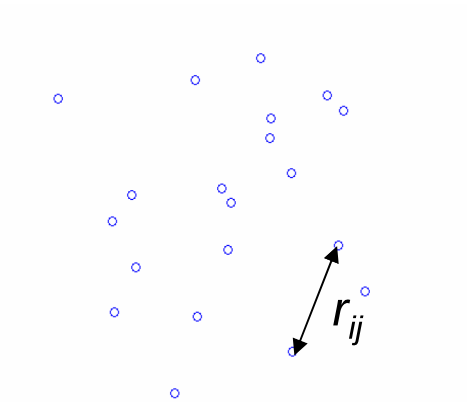
Digression: What are Random Distance Matrices?

1) Choose N points randomly and uniformly in a d-dimensional cube.
2) Define the off-diagonals of our matrix as:

$$A_{i,j} = f(r_{ij}) \ , \ \ f(r) = e^{-r/\xi}$$

(Euclidean distance)

$$\varepsilon = \xi / \langle r \rangle$$



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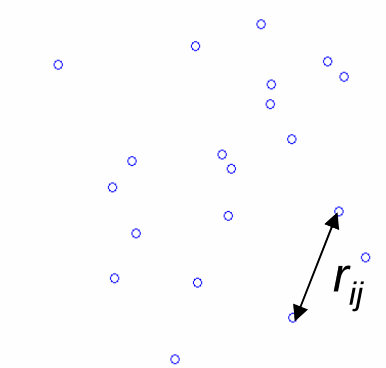

3) Define diagonal as:

$$A_{i,i} = -\sum_{j \neq i} A_{i,j}$$

sum of every column vanishes

(will come from a conservation law)

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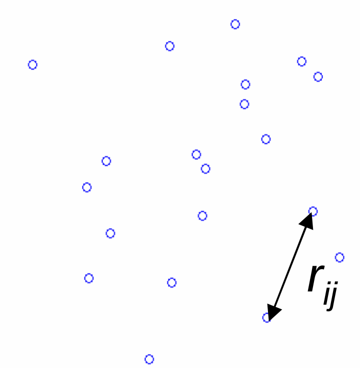

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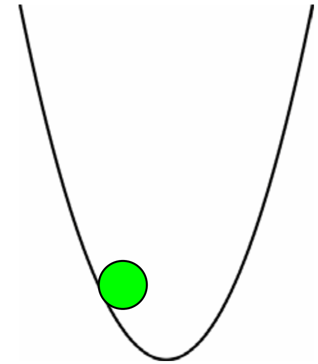
Q: What is the eigenvalue distribution?

What are the eigenmodes?

Solution near locally stable point



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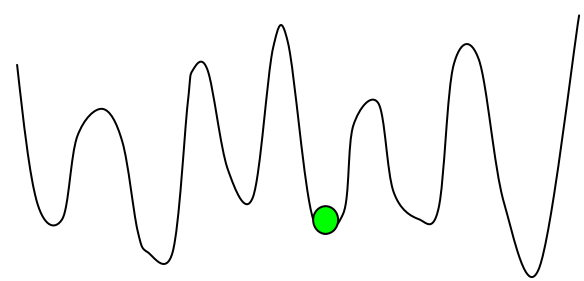

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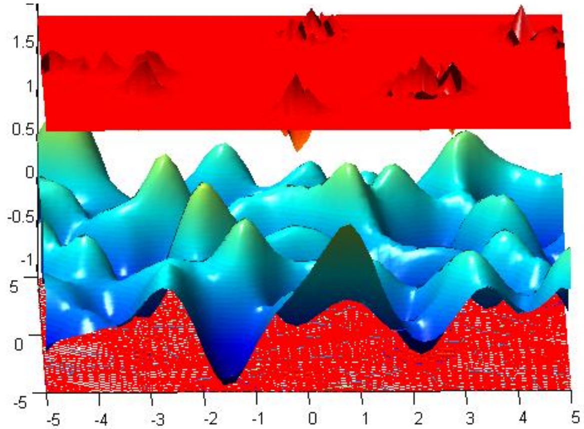
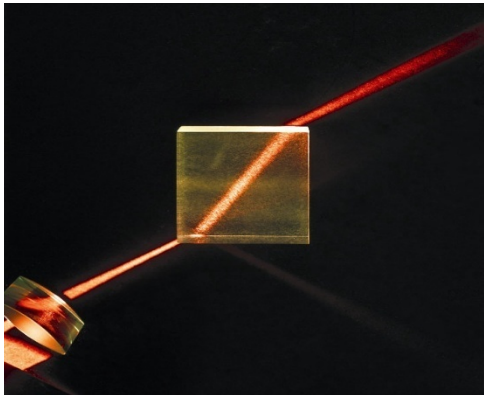
Distance matrices – Motivation

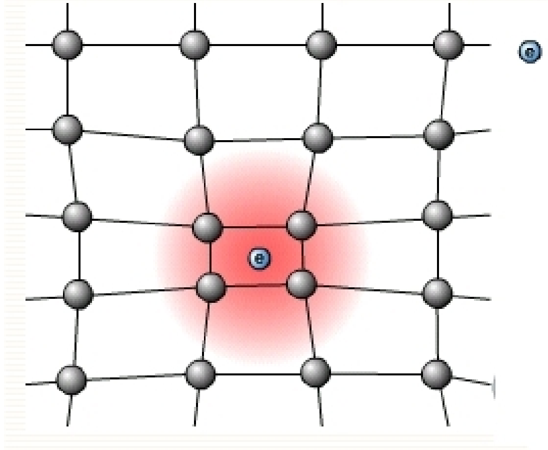
Relaxation in electron glasses
Amir, Oreg and Imry, PRB 2008



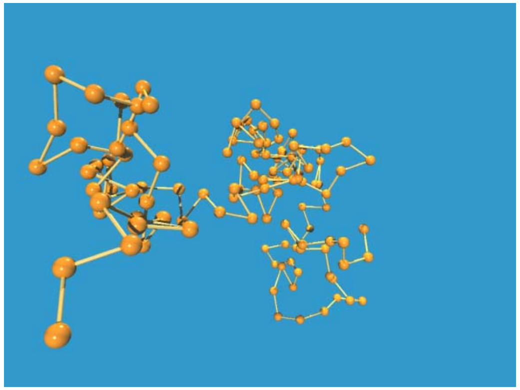
Photon propagation in a gas
Akkermans, Gero and Kaiser, PRL 2008



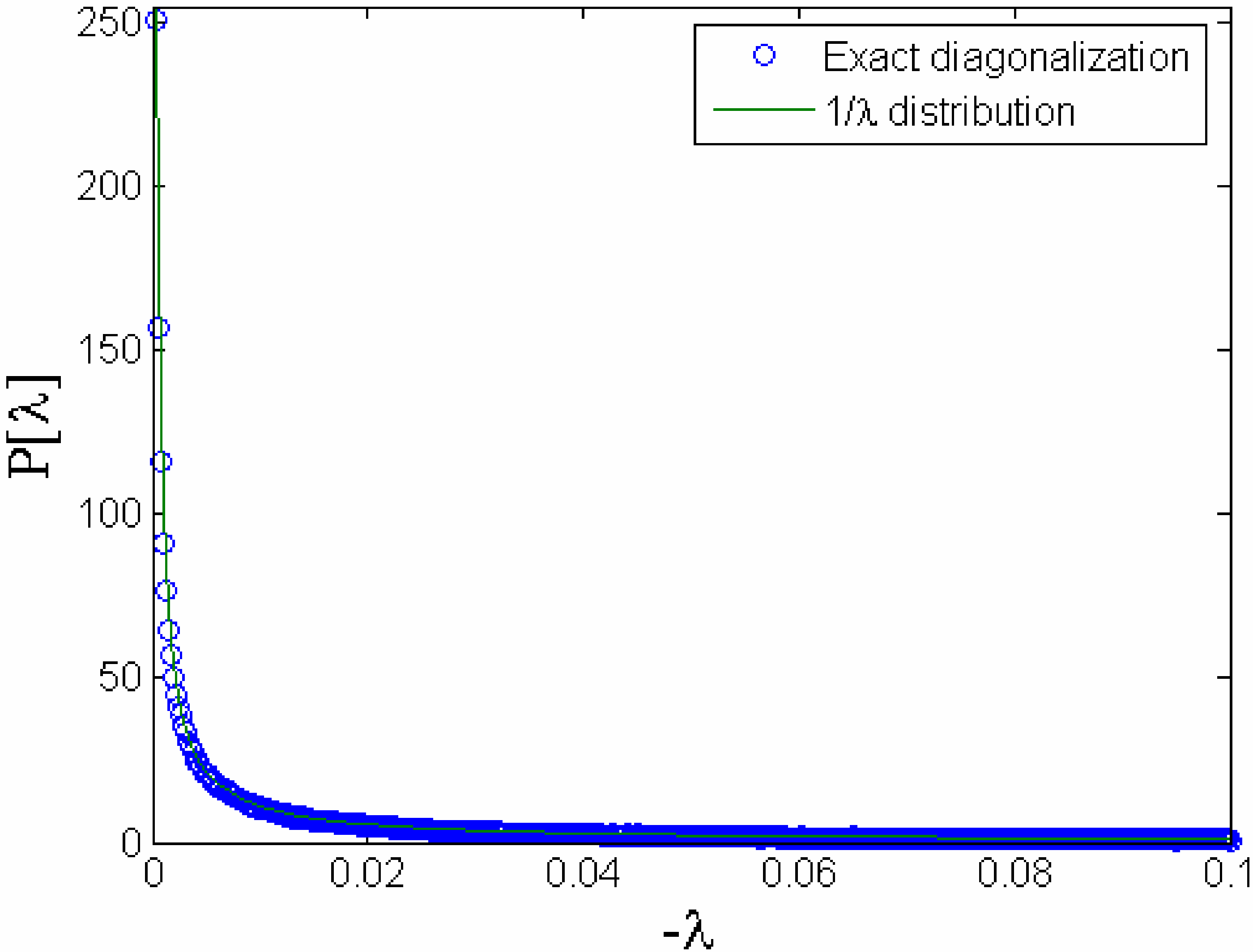
Localization of phonons
Ziman, PRL 1982
Vitelli et al., PRE 2010



Anomalous diffusion
Scher and Montroll, PRB 1975
Metzler, Barkai and Klafter, PRL 1999

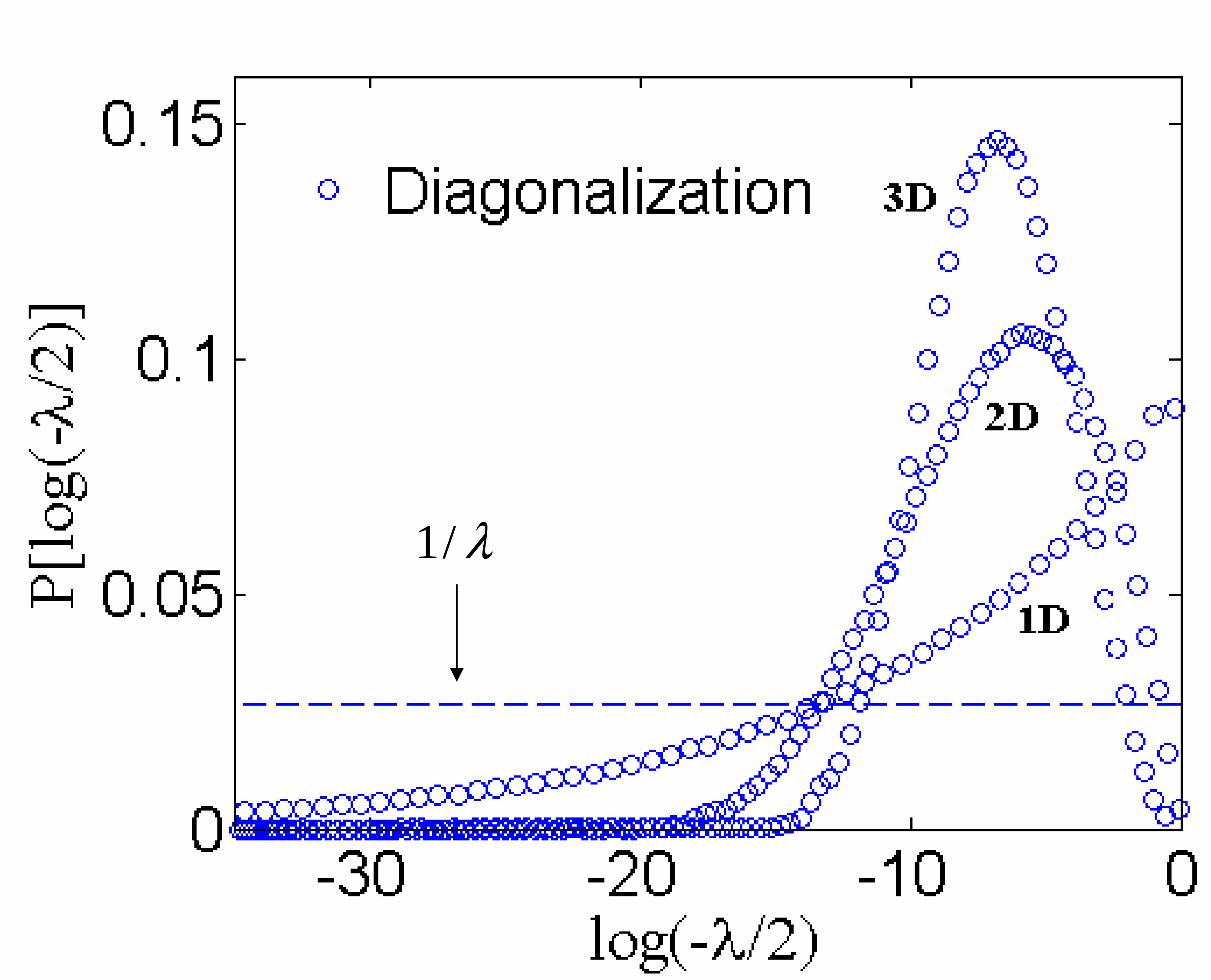


Results – 2D



250
200
150
100
50
0
0
0.02
0.04
0.06
0.08
0.1
P[λ]
-λ
Exact diagonalization
1/λ distribution

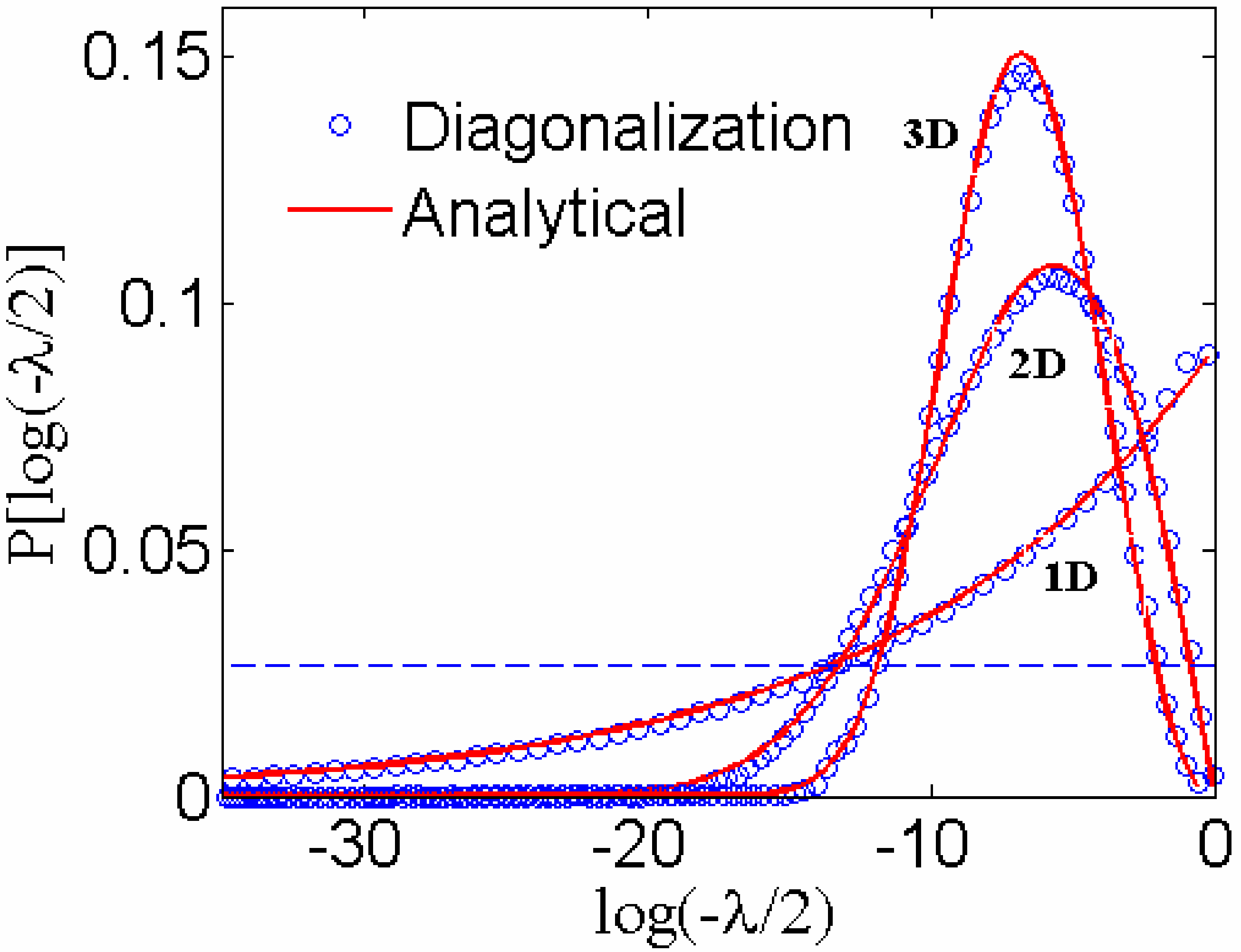
Results



Diagonalization
3D
2D
1D
$1/\lambda$
0.15
0.1
0.05
0
-30
-20
-10
0
P[log(-λ/2)]
log(-λ/2)

Results

(no fitting parameters)



Results

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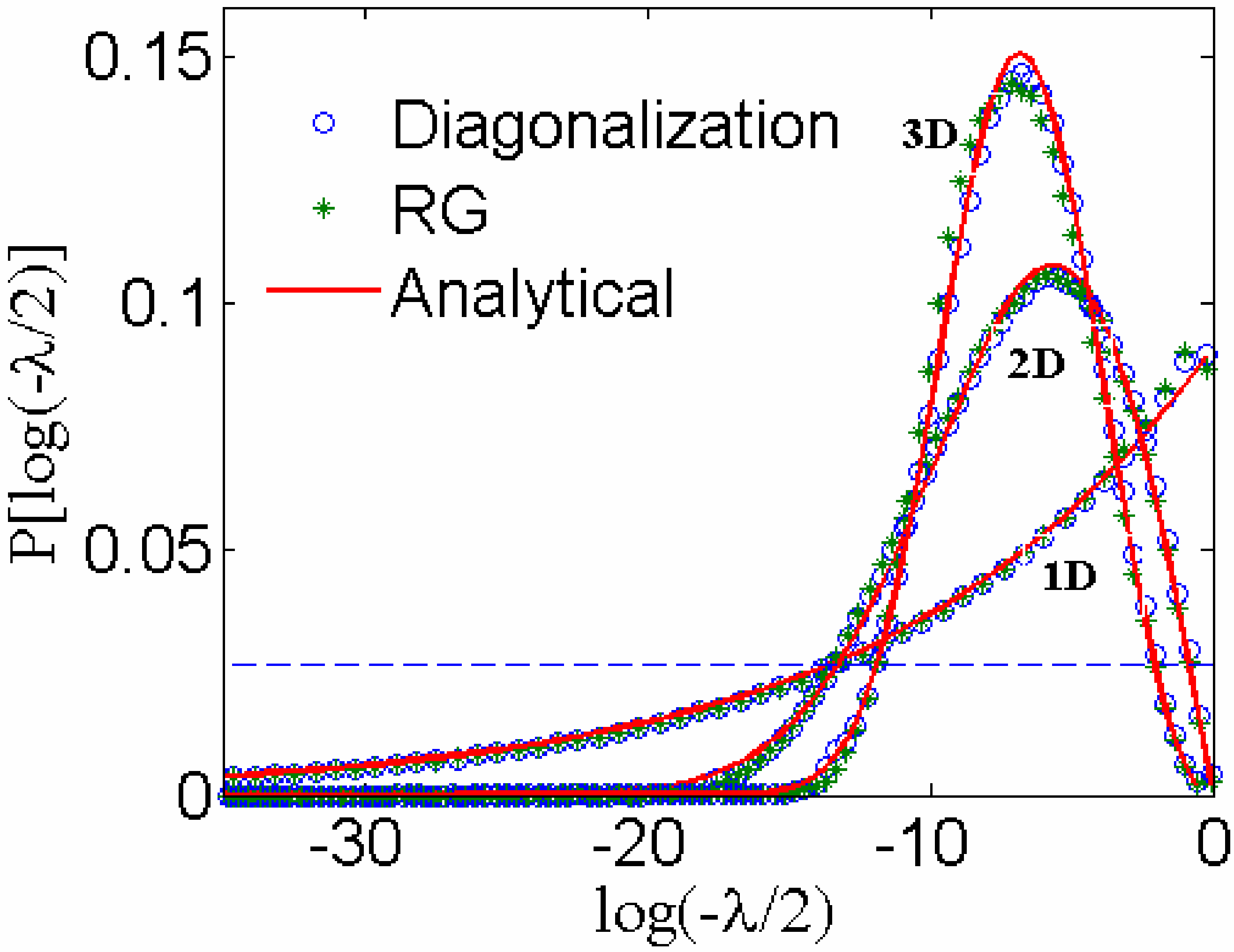


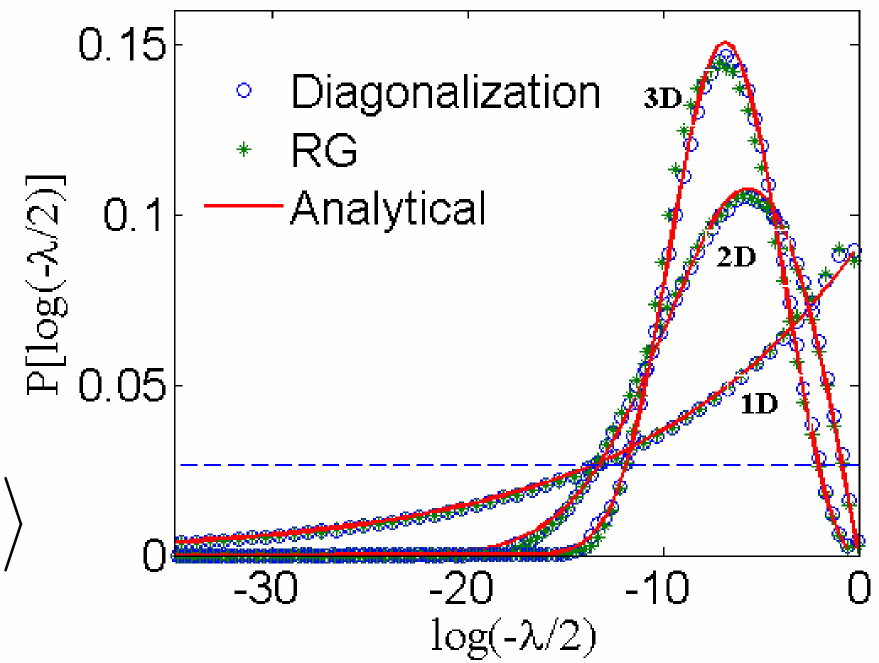
Exponential Distance Matrices- results

$$P(\lambda)=\frac{dC_d\varepsilon^d\log^{d-1}(\lambda/2)e^{-\frac{C_d}{2}\varepsilon^d\log^d(\lambda/2)}}{2\lambda}$$

(arbitrary dimension *d*)

$\varepsilon=\xi/\langle r\rangle,$ *C_d=volume of a d-dimensional sphere*

- Logarithmic corrections to $1/\lambda$
- In dimensions > 1: cutoff at $e^{-C/\varepsilon^{d/(d-1)}}$

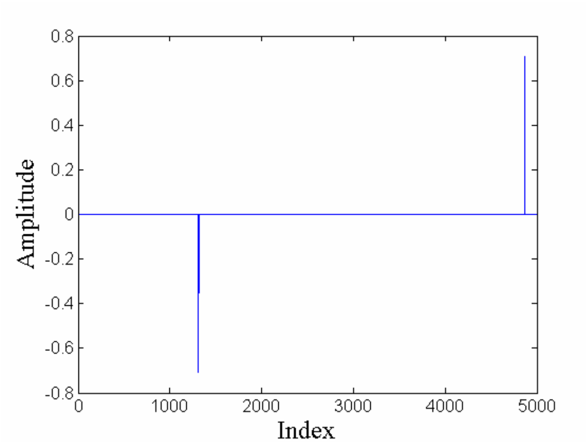


Calculation of moments:

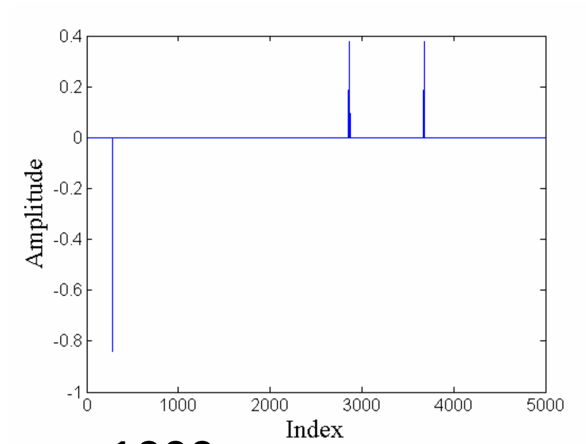
$$I_k=\int\lambda^kP(\lambda)d\lambda=\frac{1}{N}\left\langle A_{i_1,i_2}A_{i_2,i_3}\dots A_{i_k,i_1}\right\rangle$$

Amir, Oreg and Imry, **PRL (2010)**

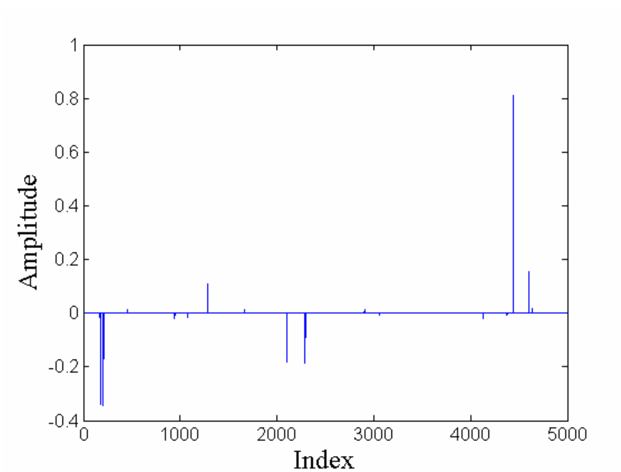
Structure of eigenmodes



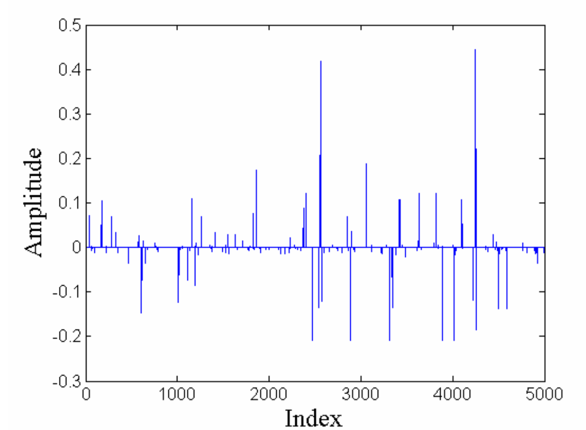
3'rd $\lambda \sim -1.86$



1000 $\lambda \sim -0.05$



3000 $\lambda \sim -9.6 \cdot 10^{-4}$



4000 $\lambda \sim -8.5 \cdot 10^{-5}$

Examples of eigenmodes of a 5000X5000 matrix

Renormalization group approach

Eigenvalue distribution + RG approach

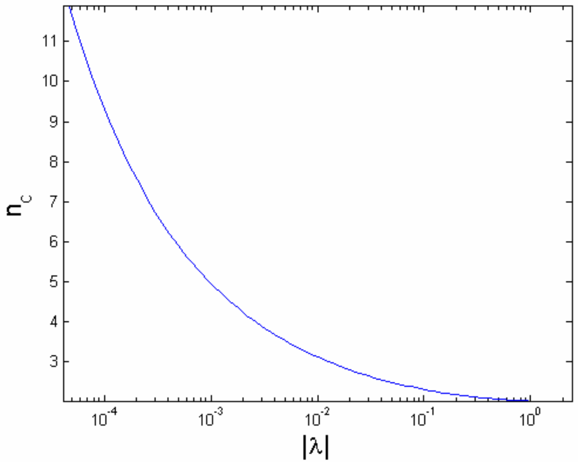
$$\longrightarrow \quad n_c \sim e^{\frac{C_d}{2}\varepsilon^d |\log^d(-\lambda/2)|}$$

Number of points in a cluster of a given eigenvalue

Renormalization group approach

Eigenvalue distribution + RG approach

→ $$n_c \sim e^{\frac{C_d}{2}\varepsilon^d |\log^d(-\lambda/2)|}$$



Number of points in a cluster of a given eigenvalue

- Eigenmodes are localized clusters (“phonon localization”)
- Size of clusters diverges at low frequencies

Amir, Oreg and Imry**,** *Localization, anomalous diffusion and slow relaxations: a random distance matrix approach***, PRL (2010)**

Electron glass aging– experimental protocol

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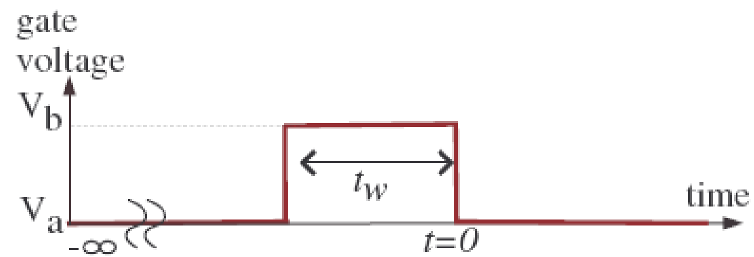
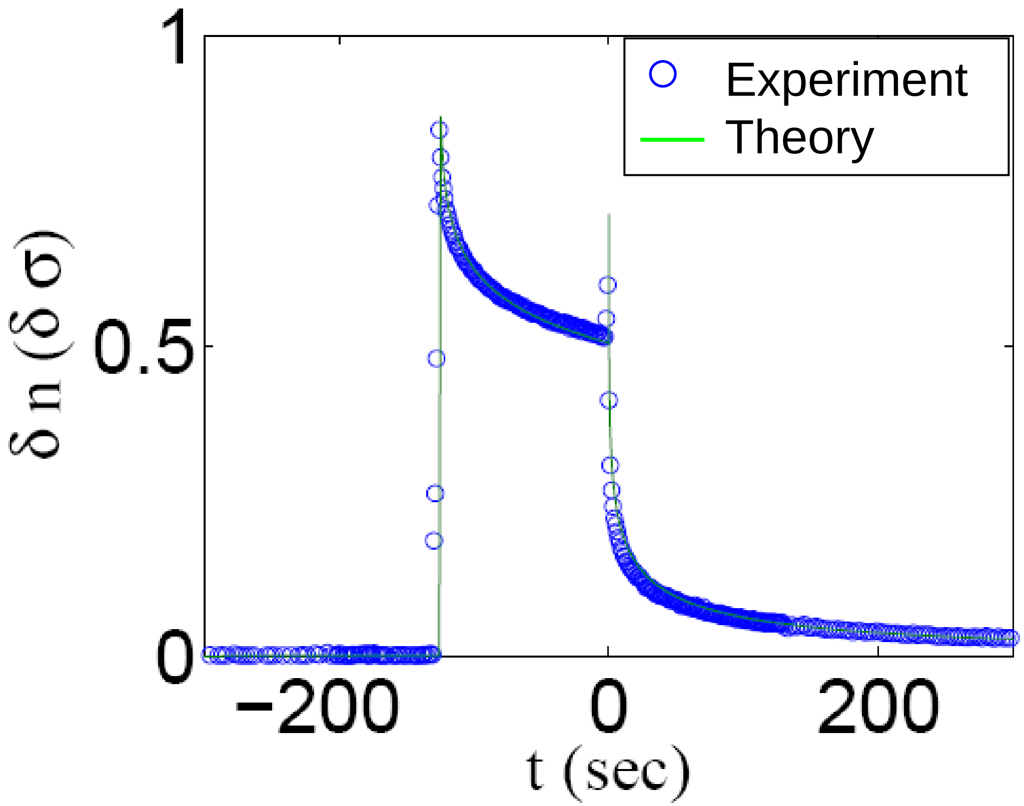
Throughout the experiment

Conductance is measured as a function of time.



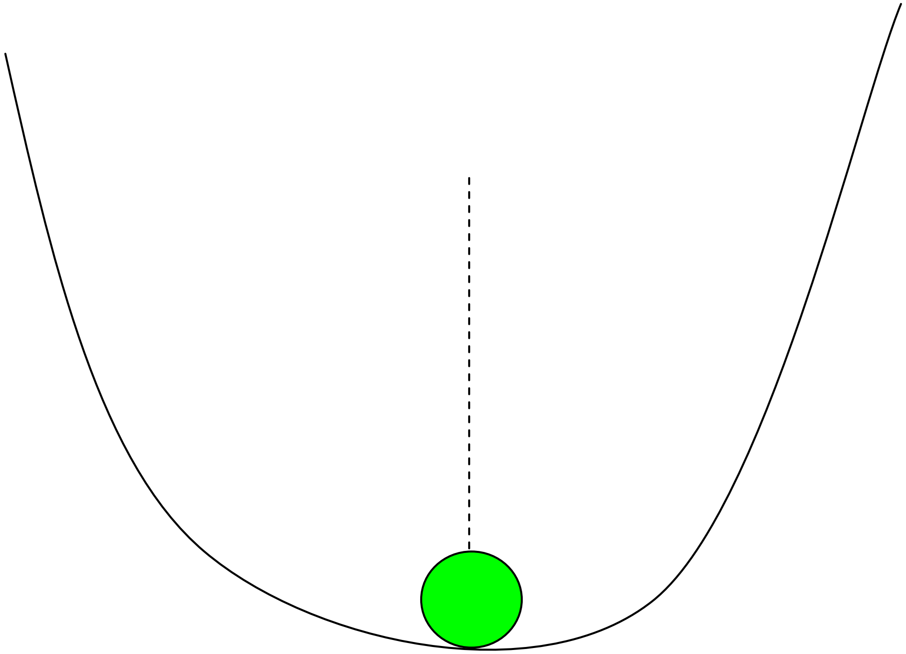


Data: *Ovadyahu et al.*

Aging – physical picture

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the repsonse?



Initially, system is at some local minimum

Aging – physical picture

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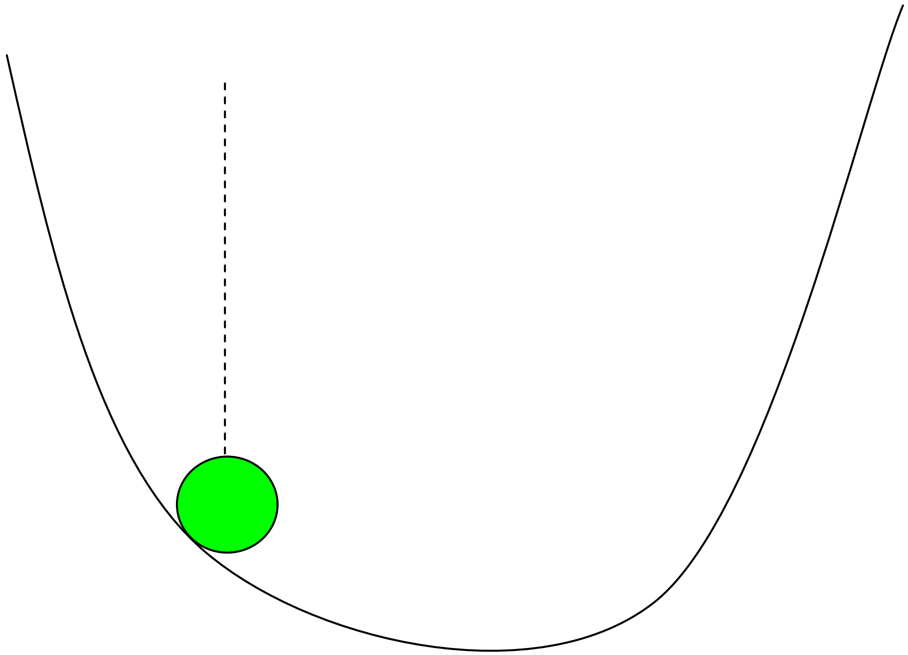


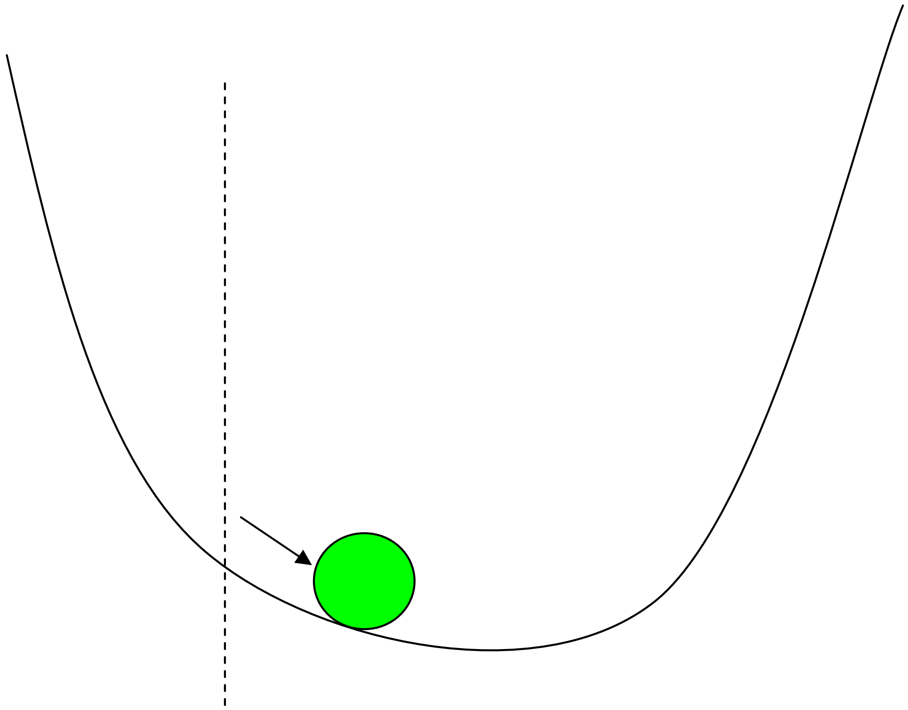
At time t=0 the potential changes,

and the system begins to roll towards the new minimum

Aging – physical picture

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the repsonse?



At time t_w the system reached some new configuration

Aging – physical picture

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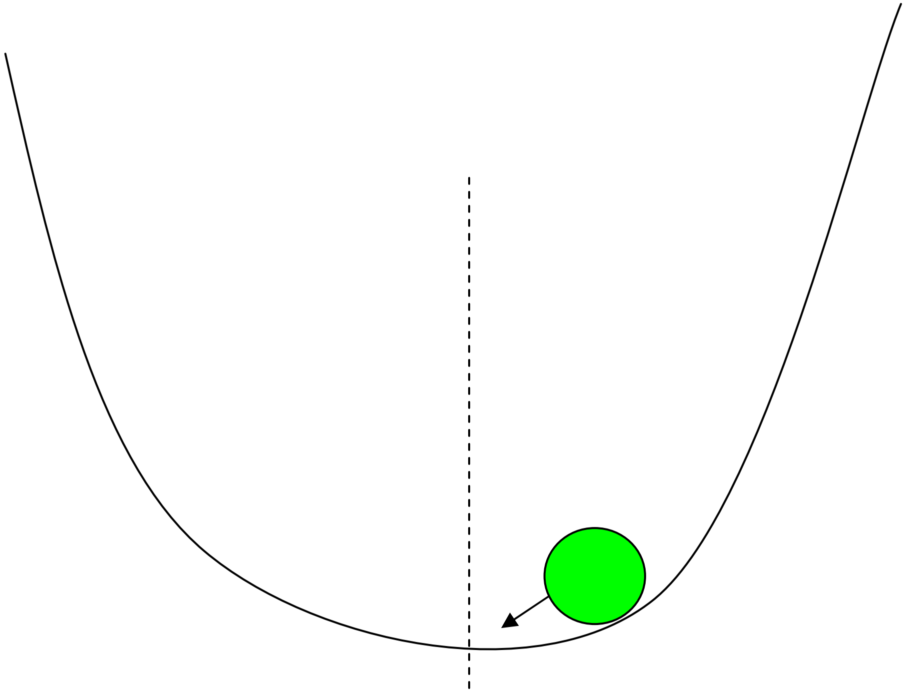


Now the potential is changed back to the initial form-

the particle is not in the minimum!

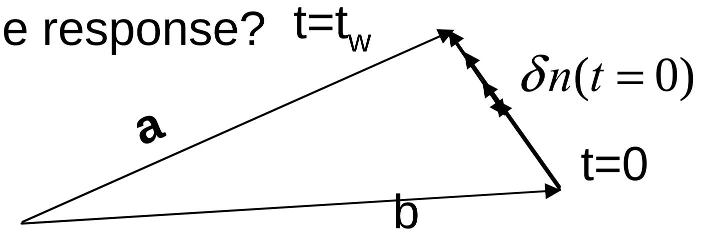
The longer t_w, the further it got away from it.

It will begin to roll down the hill.

Aging – Analysis

Assume a parameter of the system is slightly modified (e.g: V_g)

After time t_w it is changed back. What is the response?



Sketch of calculation

If a and b configurations are close enough in phase space:

$$\delta n(t=t_w) \sim \sum_{\text{eigenmodes}_\alpha} \chi_\alpha e^{-\lambda_\alpha t_w} \left| V_\alpha \right\rangle \Rightarrow \sum_{\text{eigenmodes}_\alpha} e^{-\lambda_\alpha t_w} =$$

modes are independent and contribute uniformly

Logarithmic relaxation during step II

Time *t* after the perturbation is switched off:

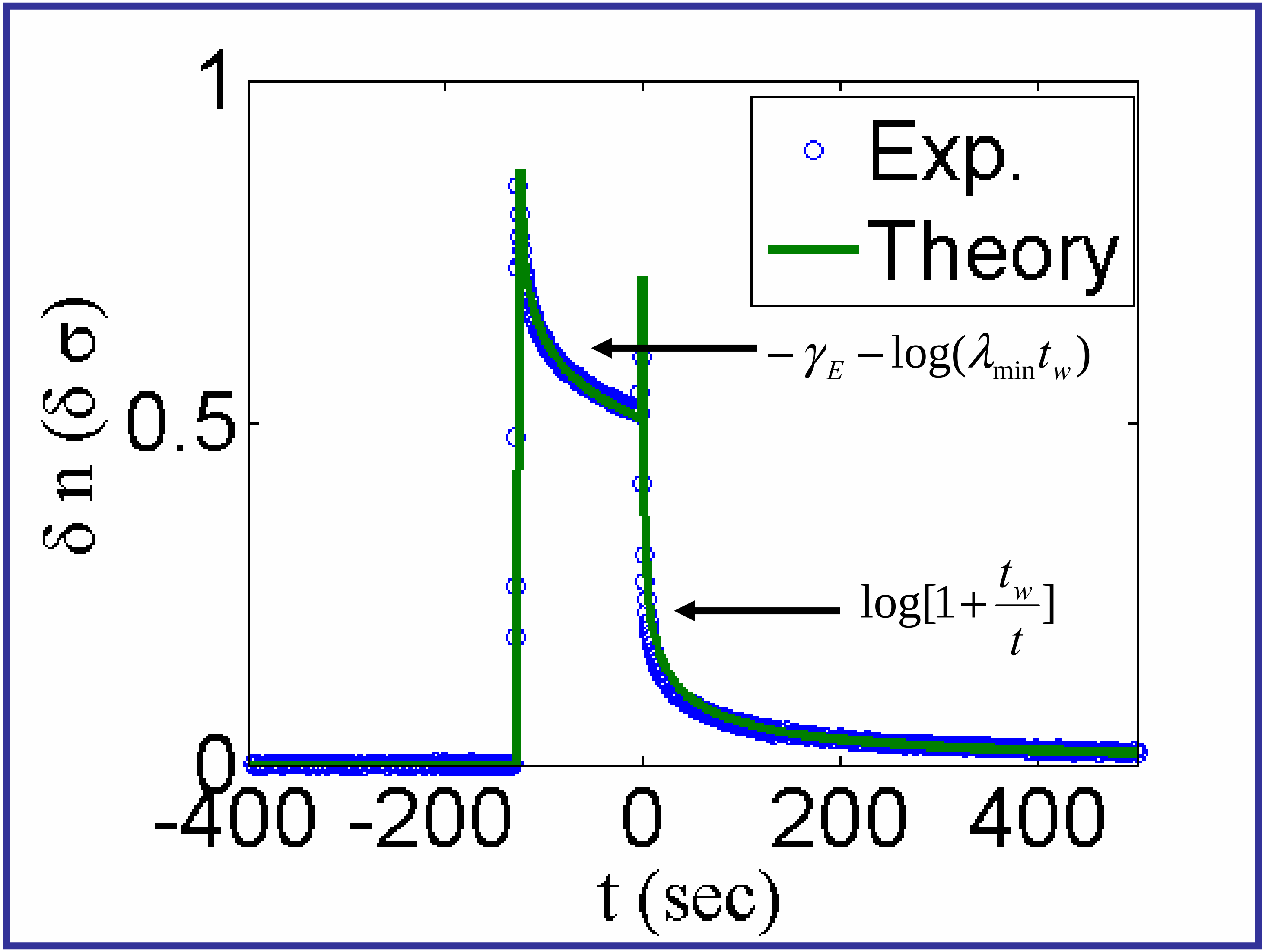
$$\delta n(t) \sim \sum_{eigenmodes\ \alpha} \chi_\alpha (1-e^{-\lambda_\alpha t_w}) e^{-\lambda_\alpha t} \,|V_\alpha> = f(t+t_w) - f(t)$$

Full aging

Only $1/\lambda$ distribution yields full aging!

See also: T. Grenet et al. Eur. Phys. J B 56, 183 (2007)

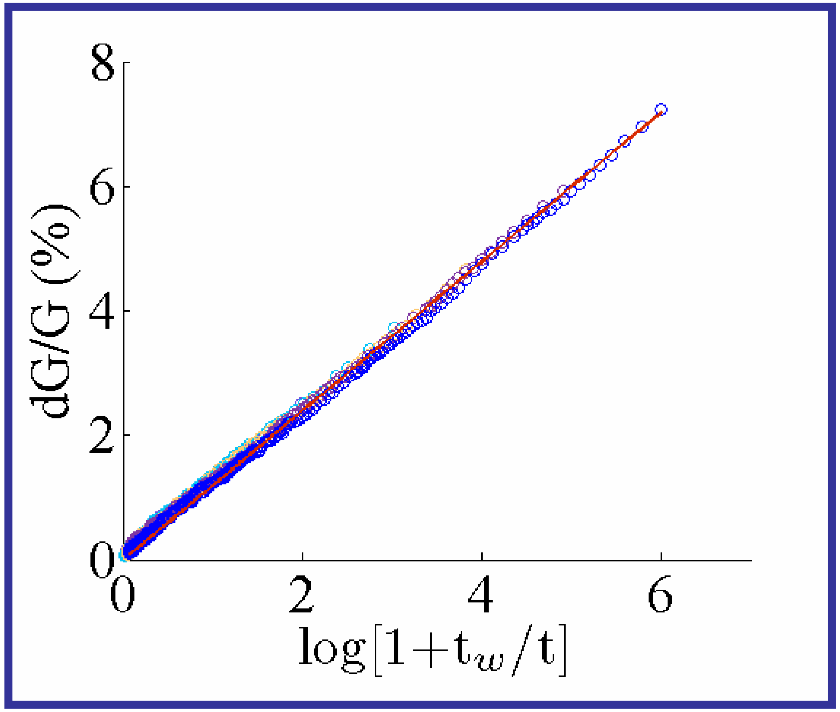
Aging Protocol - Results

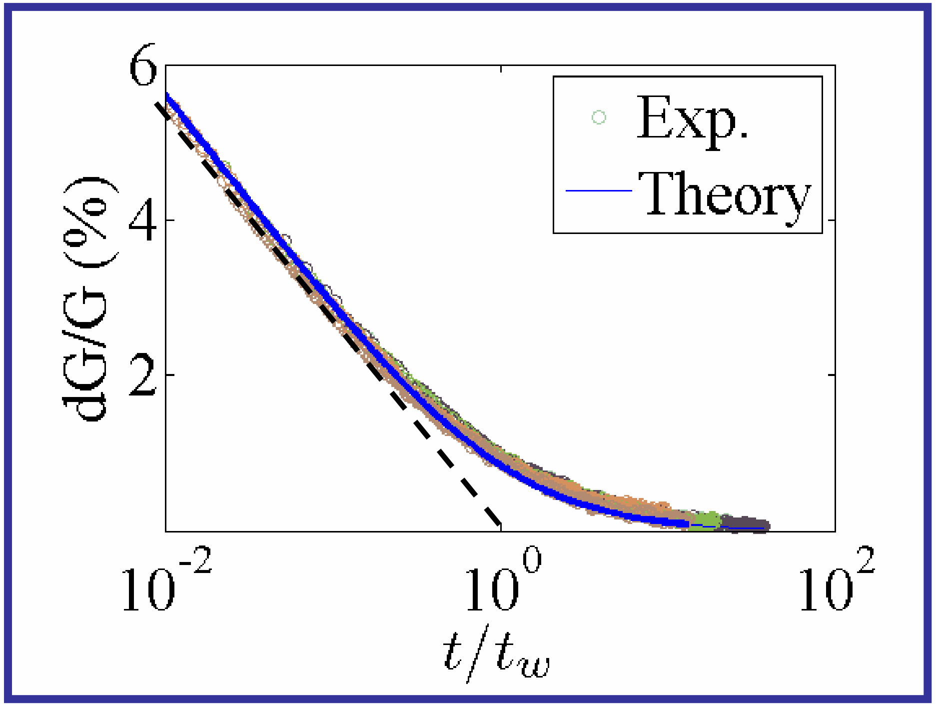


Amir, Oreg and Imry, PRL 2009

Detailed fit to experimental data

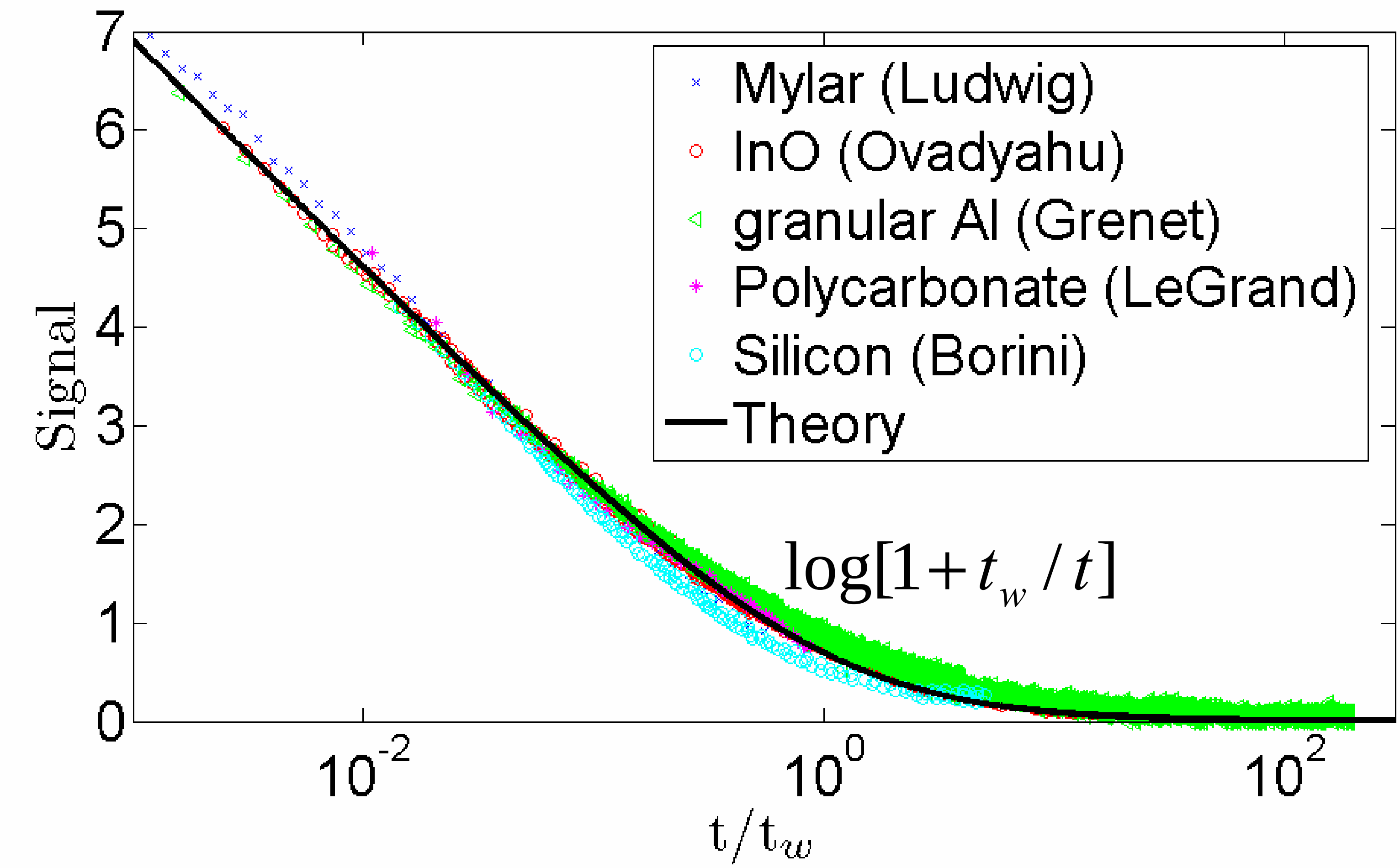
- Full aging
- Deviations from logarithm start at t/t_w





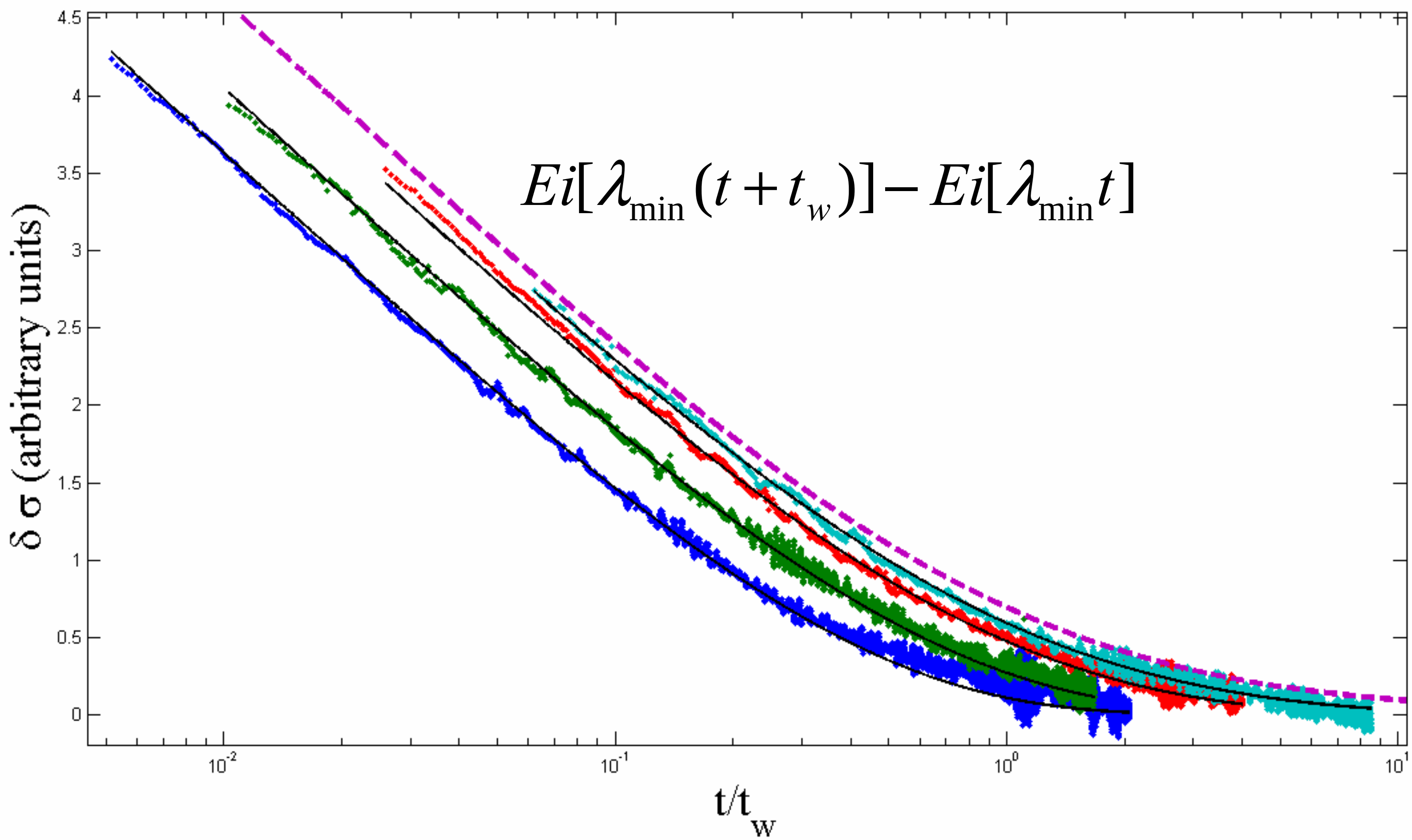
Amir, Oreg and Imry, **PRL** (2009)

Full aging and universality



Amir, Oreg and Imry, to be published

Deviations from full aging



Porous Silicon data (S. Borini)

Connection to 1/f noise?

Amir, Oreg, and Imry
arXiv:0911.5060, Ann. Phys. 2009

Langevin Noise $\longrightarrow \dfrac{d\delta\vec{n}}{dt} = A\cdot\delta\vec{n} + \vec{f}$

Equipartition theorem: each eigenmode should get $\langle E\rangle = kT/2$

The mean-field equations can be derived from a free energy:

$$F = \sum_i \varepsilon_i n_i + \sum_{i\neq j} e^2 \frac{n_i n_j}{r_{ij}} + \sum_i n_i \log n_i + (1-n_i)\log(1-n_i) + \mu N$$

From this we can find the noise correlations matrix:

$$\langle f_i f_j\rangle = -A\cdot W,\ \ W_{ij} = \delta_{ij} n_i^0 (1-n_i^0)$$

The $1/\lambda$ spectrum then leads to a 1/f noise spectrum:

$$\langle \delta n^2\rangle_f = \frac{1}{N}\sum_\lambda \frac{1/\lambda}{1+\left(\omega/\lambda\right)^2} \longrightarrow 1/f$$

B.I. Shklovskiı,

Solid State Commun (1980)

K. Shtengel et al.,

PRB (2003)

Conclusions

• Statics: Coulomb gap, Steady-state: Variable Range Hopping

• Dynamics near locally stable point: many slow *localized* modes, $\sim 1/\lambda$ distribution.

How universal? **We believe: a very relevant RMT class.**

• One obtains full aging, with relaxation approximately of the form :

$$\delta\sigma \sim \log[1+\frac{t_w}{t}]$$

More details:

Phys. Rev. B 77, 1, 2008 (local mean-field model)

Phys. Rev. Lett. 103, 126403 (2009) (aging properties)

Phys. Rev. B 80, 245214 2009 (variable-range hopping)

Ann. Phys. 18, 12, 836 (2009) (*1/f* noise)

Phys. Rev. Lett. 105, 070601 (2010) (exponential matrices – solution)

Electron glass dynamics – Review (soon online)