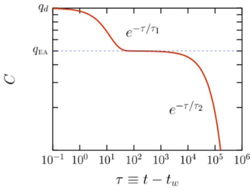
Quantum mechanical view on dynamical glass transitions



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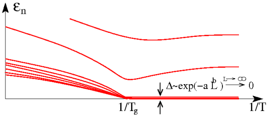

EPSRC
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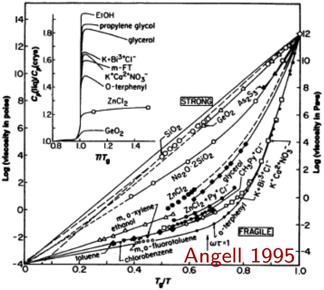
Outline

- (very!) brief overview of classical glassy systems
- quantum mechanical perspective: why is it interesting?
  - **classical dynamical transition**
    $\Rightarrow$ **static (zero-temperature) quantum transition**
  - quantum measures that do not require an order parameter:
    - **fidelity susceptibility** $\Leftrightarrow$ heat capacity
    - **von Neumann entanglement entropy** $\Leftrightarrow$ detects glass transition and growing correlation length
- analogies between topological spin Hamiltonians and kinetically-constrained models
- conclusions

Classical glass transition

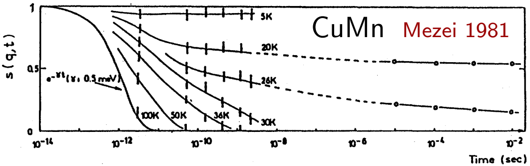
experimentally:

$T < T_g \Leftrightarrow$ viscosity larger than 10^{13} Poise



$$C_c(t, +\tau, t) \equiv \langle \mathcal{O}(t+\tau)\mathcal{O}(t)\rangle_{\rm th} - \langle \mathcal{O}(t)\rangle_{\rm th}^2$$

$$q_{\rm EA}(\mathcal{O}) \equiv \lim_{\tau\to\infty} \lim_{t\to\infty} \mathcal{C}_c(t, +\tau, t)$$

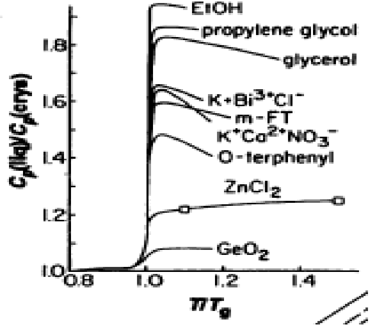


theoretically:

actual singularity in the time scales at $T = T_g$

Open issues

- how do glass transitions compare to thermodynamic ones? no (local) order parameter
- do concepts like scaling and universality apply?
- there is evidence in support of a divergent dynamical length scale at T_g – what about singularities in the free energy?



Open issues

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  no (local) order parameter
- do concepts like scaling and universality apply?
- there is evidence in support of a divergent dynamical length scale at T_g – what about singularities in the free energy?

quantum mechanical perspective

- 'unifies' space and time into a static quantum mechanical system at zero temperature
- provides new angles to investigate dynamical phenomena (e.g., fidelity and entanglement measures not based on an order parameter)

Markov processes with detailed balance (I)

configs: $\{\mathcal{C}\}$, energy $E_{\mathcal{C}}$

$$P_{\mathcal{C}}^{(\mathrm{eq})} = \frac{e^{-\beta E_{\mathcal{C}}}}{Z} \qquad Z = \sum_{\mathcal{C}} e^{-\beta E_{\mathcal{C}}} \qquad \left(\beta = \frac{1}{k_B T}\right)$$

$$\frac{d}{dt} P_{\mathcal{C}}(t) = \sum_{\mathcal{C}' \neq \mathcal{C}} \left[W_{\mathcal{C},\mathcal{C}'} P_{\mathcal{C}'}(t) - W_{\mathcal{C}',\mathcal{C}} P_{\mathcal{C}}(t) \right]$$

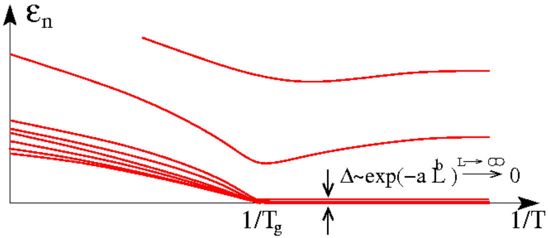
$$W_{\mathcal{C},\mathcal{C}'} e^{-\beta E_{\mathcal{C}'}} = W_{\mathcal{C}',\mathcal{C}} e^{-\beta E_{\mathcal{C}}}$$

$P^{(\mathrm{eq})}$ is the *null* right eigenvector of W: $\sum_{\mathcal{C}'} W_{\mathcal{C},\mathcal{C}'} P_{\mathcal{C}'}^{(\mathrm{eq})} = 0$

$\rightarrow$ no decay

Markov processes with detailed balance (II)

- in a *finite* system all other eigenvalues $-\varepsilon_n$ are negative, $\varepsilon_0 = 0 < \varepsilon_1 \leq \varepsilon_2 \leq \ldots$ $\longrightarrow$ exponential decay $\sim e^{-\varepsilon_n t}$
- glass transition $\Rightarrow \lim_{L\to\infty} \varepsilon_n = 0, \quad \exists\, n$
  (*not sufficient*: 2D Ising model)



discrete systems without conserved quantities

understanding a glass transition $\Longleftrightarrow$ understanding the nature of the collapsing relaxation rates

QM perspective (I)

- symmetrise W by similarity transformation — Felderhof 1970

$$H_{\mathcal{C},\mathcal{C}'} \equiv -\exp\left(\beta E_{\mathcal{C}}/2\right) W_{\mathcal{C},\mathcal{C}'} \exp\left(-\beta E_{\mathcal{C}'}/2\right)$$

- promote $\mathcal{C}$ to orthonormal basis $|\mathcal{C}\rangle$, $\langle\mathcal{C}|\mathcal{C}'\rangle = \delta_{\mathcal{C}\mathcal{C}'}$
- quantum mechanical interpretation $\langle\mathcal{C}|\hat{H}|\mathcal{C}'\rangle \equiv H_{\mathcal{C},\mathcal{C}'}$:

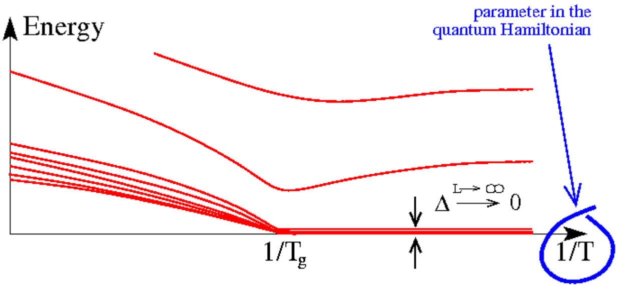
Rokhsar + Kivelson 1998; Henley 2004
Ardonne *et al.* 2004; Castelnovo *et al.* 2005-2006

$$\hat{H} = \sum_{\mathcal{C}\neq\mathcal{C}'} w_{\mathcal{C}\mathcal{C}'} \left[e^{\beta(E_{\mathcal{C}}-E_{\mathcal{C}'})/2} |\mathcal{C}\rangle\langle\mathcal{C}| - |\mathcal{C}\rangle\langle\mathcal{C}'| \right]$$

$$\hat{H}\,|\psi_n\rangle = \varepsilon_n |\psi_n\rangle \qquad \varepsilon_0 = 0 < \varepsilon_1 \leq \varepsilon_2 \leq \ldots$$

$$|\psi_0\rangle = \frac{1}{\sqrt{Z}} \sum_{\mathcal{C}} e^{-\beta E_{\mathcal{C}}/2} |\mathcal{C}\rangle \qquad Z = \sum_{\mathcal{C}} e^{-\beta E_{\mathcal{C}}}$$

QM perspective (II)



- *local* energy $E_{\mathcal{C}}$ + *local* dynamics $\Rightarrow$ *local* Hamiltonian
- the dynamical classical problem reduces to a static zero-temperature quantum system
- at some 'critical' coupling T_g, a spectral collapse occurs
- understanding glassiness $\Leftrightarrow$ understanding the collapse

What happens at T_g?

details

local static (zero-frequency) susceptibility of the quantum system at zero temperature

$$\hat{H}'(\beta,\lambda) \; = \; \hat{H}(\beta) + \lambda\hat{\mathcal{O}} \qquad (\neq \text{classical field})$$

$$\chi^{\mathrm{loc}}(\omega=0) \; \equiv \; \int_0^\infty d\tau \; C_c(\tau) = \sum_{n\neq 0} \frac{\left|\langle\psi_n|\hat{\mathcal{O}}|\psi_0\rangle\right|^2}{\varepsilon_n}$$

Henley 2004

At $T = T_g$, $q_{\mathrm{EA}} = \lim_{\tau\to\infty} C_c(\tau)$ becomes finite

$\Rightarrow$ $\chi^{\mathrm{loc}}(\omega=0)$ diverges

Fidelity susceptibility

quantum measure <u>not</u> based on an order parameter!

$$|\psi_0(\beta)\rangle = \sum_{\mathcal{C}} \frac{e^{-\beta E_{\mathcal{C}}/2}}{\sqrt{Z}} |\mathcal{C}\rangle$$

new tools to study the dynamical transitions as QPTs:

fidelity Zanardi *et al.* 2007

$$\mathcal{F}(\beta, \delta\beta) \equiv \langle \psi_0(\beta - \delta\beta/2) \,|\, \psi_0(\beta + \delta\beta/2) \rangle$$

fidelity susceptibility Zanardi *et al.* 2007, You *et al.* 2007

$$\chi_{\mathcal{F}}(\beta) \equiv \lim_{\delta\beta \to 0} \left[-2 \frac{\ln \mathcal{F}(\beta, \delta\beta)}{\delta\beta^2} \right] = \frac{1}{4\beta^2} C_V(\beta)$$

Castelnovo, Chamon, Sherrington 2010

local Hamiltonian + closing of a gap $\longrightarrow$ singularity in $\chi_{\mathcal{F}}(\beta)$
specific heat singularity expected at a (local) glass transition!

Fidelity susceptibility

quantum measure <u>not</u> based on an order parameter!

$$|\psi_0(\beta)\rangle = \sum_{\mathcal{C}} \frac{e^{-\beta E_{\mathcal{C}}/2}}{\sqrt{Z}} |\mathcal{C}\rangle$$

new tools to study the dynamical transitions as QPTs:

fidelity

$$\mathcal{F}(\beta, \delta\beta) \equiv \langle \psi_0(\beta - \delta\beta/2) \,|\, \psi_0(\beta$$

fidelity susceptibility

Za

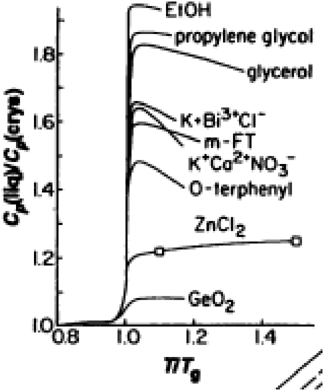
$$\chi_{\mathcal{F}}(\beta) \equiv \lim_{\delta\beta \to 0} \left[-2 \frac{\ln \mathcal{F}(\beta, \delta\beta)}{\delta\beta^2} \right]$$

$$= \frac{1}{4\beta^2} C_V(\beta)$$

Cast

local Hamiltonian + closing of a gap —

specific heat singularity expected at a

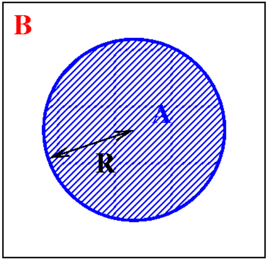


von Neumann entanglement entropy

given the ground state density matrix $\hat{\rho} = |\psi_0\rangle\langle\psi_0|$, and a bipartition (A, B)

$$\hat{\rho}_A = \mathrm{Tr}_B\, \hat{\rho}$$

$$S_{AB} = -\mathrm{Tr}\left[\hat{\rho}_A \log \hat{\rho}_A\right] = \alpha L^{d-1} + \ldots$$



$$\hat{\rho}(\beta) = \frac{1}{Z(\beta)} \sum_{\mathcal{C},\mathcal{C}'} e^{-\beta(E_{\mathcal{C}}+E_{\mathcal{C}'})/2} |\mathcal{C}\rangle\langle\mathcal{C}'|$$

$$\Rightarrow \quad S_{AB}(T) = \beta F_A + \beta F_B - \beta F_{A\cup B} + \beta\langle E^{\partial}\rangle_{\mathrm{th}}$$

$$\beta F_{A,B} = -\ln Z_D^{A,B} \quad \text{and} \quad \beta F_{A\cup B} = -\ln Z$$

Fradkin, Moore 2006, Castelnovo, Chamon 2007

von Neumann entanglement entropy

Castelnovo, Chamon, Sherrington 2010

$$S_{AB}(T) \;=\; \beta\left[\Delta F_A(T) + \Delta F_B(T)\right] + S^F_{AB}(T)$$

$$\Delta F_A(T) = -\frac{1}{\beta}\ln\left(\frac{Z^D_A}{Z^F_A}\right)$$

$$S^F_{AB}(T) = \ln\left\langle \exp\left[\beta\left(E^\delta - \left\langle E^\delta\right\rangle_{\mathrm{th}}\right)\right]\right\rangle_{\mathrm{th}}$$

von Neumann entanglement entropy (I)

consider $S^F_{AB}(T) = \ln \left\langle \exp \left[\beta \left(E^\delta - \left\langle E^\delta \right\rangle_{\mathrm{th}} \right) \right] \right\rangle_{\mathrm{th}}$

- cumulant-generating function for the fluctuations of the boundary energy
- e.g., second moment $\propto$ boundary heat capacity

$\Rightarrow$ detects thermodynamic singularities
(like the fidelity susceptibility)

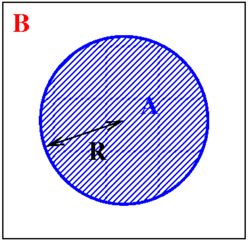
What if there are truly no thermodynamic singularities?

von Neumann entanglement entropy (II)

consider $\Delta F_A(T) \sim -T \ln[Z_A^D / Z_A^F]$

- above T_g: the system adapts to the fixed B.C. $\rightarrow \Delta F_A \sim E_\delta^*$
- below T_g: $\mathcal{N} > 1$ distinct free energy minima
  - one minimum preferred by the fixed B.C. ($E_\delta \simeq E_\delta^*$)
  - all others equally disfavoured ($E_\delta \simeq E_\delta^* + \Delta E$)

Biroli, Bouchaud 2004



$$\Delta F_A \simeq -T \ln \left[\frac{\sum_{i=1}^{\mathcal{N}} e^{-\beta E_i}}{\mathcal{N}} \right]$$

$$\simeq -T \ln \left[e^{-\beta E_\partial^*} \frac{(\mathcal{N}-1) e^{-\beta \Delta E} + 1}{\mathcal{N}} \right]$$

$$\simeq E_\partial^* - T \ln \left[e^{-\beta \Delta E} + e^{-S^*} \right], \quad S^* \equiv \ln \mathcal{N}$$

von Neumann entanglement entropy (II)

consider $\Delta F_A(T) \sim -T \ln[Z_A^D / Z_A^F]$

- $\underline{\text{above } T_g}$: the system adapts to the fixed B.C. $\rightarrow \Delta F_A \sim E_\delta^*$
- $\underline{\text{below } T_g}$: $\mathcal{N} > 1$ distinct free energy minima
  - one minimum preferred by the fixed B.C. $(E_\delta \simeq E_\delta^*)$
  - all others equally disfavoured $(E_\delta \simeq E_\delta^* + \Delta E)$

$$\Delta F_A \sim E_\delta^* - T \ln\left(e^{-\Delta E/T} + e^{-S^*}\right)$$

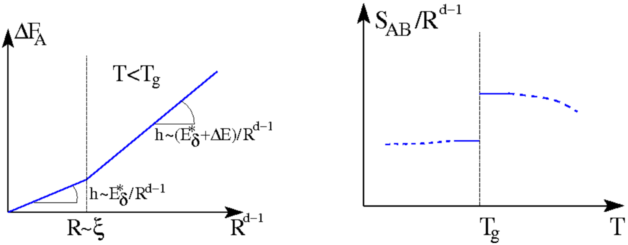
$$\Delta E \sim R^{d-1} \qquad \text{vs} \qquad S^* = \ln \mathcal{N} \sim R^d$$

correlation length ξ identified by $\Delta E(R = \xi) \sim S^*(R = \xi)$

$$\begin{cases} R \ll \xi \\ \Delta F_A \sim E_\delta^* + S^* T \sim E_\delta^* \end{cases} \qquad \begin{cases} R \gg \xi \\ \Delta F_A \sim E_\delta^* + \Delta E \end{cases}$$

von Neumann entanglement entropy (II)

$$\begin{cases} R \ll \xi \\ \Delta F_A \sim E^*_\delta + S^* T \sim E^*_\delta \end{cases} \qquad \begin{cases} R \gg \xi \\ \Delta F_A \sim E^*_\delta + \Delta E \end{cases}$$



classical argument: requires well-defined metastable states $\longrightarrow$
dependent on separation of time scales

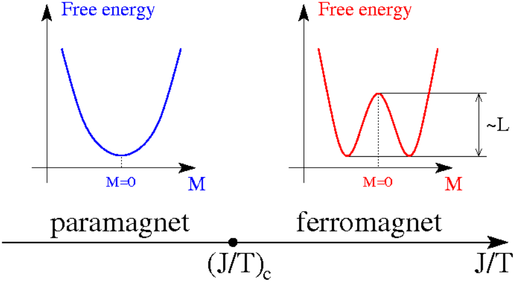
quantum entanglement: static + generic

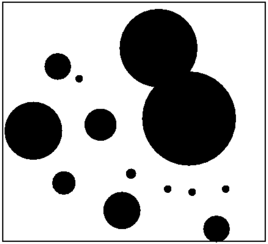
Castelnovo, Chamon, Sherrington 2010

Example I: the 2D Ising ferromagnet

$$\mathcal{C} \equiv \{\sigma_i = \pm 1\}$$

$$E_{\mathcal{C}} \equiv J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$





under single spin flip dynamics (Glauber)

$$W_{\mathcal{C}'\mathcal{C}} = \frac{e^{-\beta(E_{\mathcal{C}'} - E_{\mathcal{C}})/2}}{2\cosh[\beta(E_{\mathcal{C}'} - E_{\mathcal{C}})/2]}$$

energy barriers $\sim L$ from positive to negative magnetisation

Example I: the 2D Ising ferromagnet

$$\hat{H} = \sum_{(\mathcal{C},\mathcal{C}')} \frac{1}{2\cosh[\beta(E_{\mathcal{C}'} - E_{\mathcal{C}})/2]} \left\{ e^{-\beta(E_{\mathcal{C}'}-E_{\mathcal{C}})/2} |\mathcal{C}\rangle\langle\mathcal{C}| - |\mathcal{C}\rangle\langle\mathcal{C}'| \right\}$$

single spin flip: $E_{\mathcal{C}'} - E_{\mathcal{C}} = 2\sum_j J_{ij}\,\sigma_i\sigma_j$

$$\hat{H} = \sum_{i,\mathcal{C}} \frac{1}{2\cosh[\beta\sum_j J_{ij}\sigma_j]} \left\{ e^{-\beta\sum_j J_{ij}\sigma_j\sigma_i} |\mathcal{C}\rangle\langle\mathcal{C}| - |\mathcal{C}\rangle\langle\mathcal{C}|\hat{\sigma}_i^x \right\}$$

$$\Longrightarrow \qquad \hat{H} = \sum_{i} \frac{1}{2\cosh[\beta\sum_j J_{ij}\hat{\sigma}_j^z]} \left\{ e^{-\beta\sum_j J_{ij}\hat{\sigma}_j^z\hat{\sigma}_i^z} - \hat{\sigma}_i^x \right\}$$

Castelnovo, Chamon, Pujol, Mudry 2005

Conclusions

dynamical glass transition $\Leftrightarrow$ *static quantum phase transition*

- massive collapse of eigenstates, divergent local susceptibility
- singularities in the fidelity susceptibility relate directly to the classical heat capacity
- entanglement entropy: static measure to detect glass transitions and growing correlation lengths
- cross-fertilisation between different areas of physics:
  - known glassy systems $\rightarrow$ new exotic quantum Hamiltonians
  - ‘unconventional’ quantum systems devoid of local order (e.g., topological order) $\rightarrow$ insight in glassiness

Conclusions

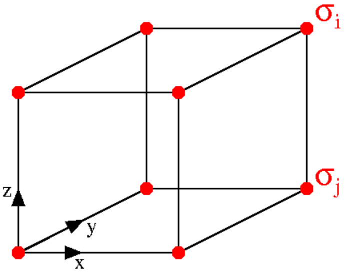
potential advantages and open questions

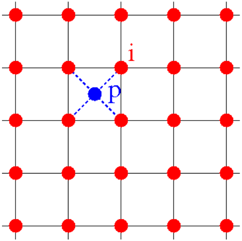
- can we fully characterise dynamical transitions using static (zero-temperature) techniques?
- can we use the scaling exponents of the fidelity susceptibility to classify dynamical glass transitions?
- ability to construct (off-diagonal) order parameters, which correpond to ‘dynamical’ quantities in the original classical system

Example II: the gonihedric model

Jonsson, Savvidy 1999-2000, Savvidy 2000

$$E = -J_{xy} \sum_i \sigma_i\, \sigma_{i+\hat{x}}\, \sigma_{i+\hat{x}+\hat{y}}\, \sigma_{i+\hat{y}} - J_{yz} \sum_i \sigma_i\, \sigma_{i+\hat{y}}\, \sigma_{i+\hat{y}+\hat{z}}\, \sigma_{i+\hat{z}} - J_{zx} \sum_i \sigma_i\, \sigma_{i+\hat{z}}\, \sigma_{i+\hat{z}+\hat{x}}\, \sigma_{i+\hat{x}}$$





$J_{xy} = J'$, $J_{yz} = J_{zx} = 0$:
square plaquette kinetically constrained model

$$E = -J' \sum_p \prod_{i \in p} \sigma_i$$

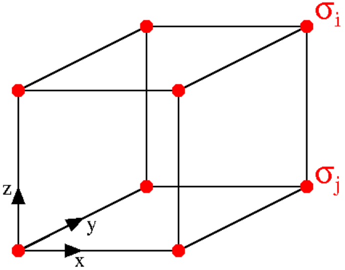
no transition; time scales diverge for $T \to 0$

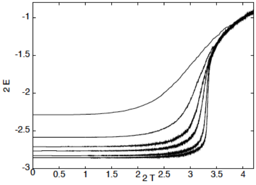
Jack *et al.* 2005

Example II: the gonihedric model

Jonsson, Savvidy 1999-2000, Savvidy 2000

$$E = -J_{xy} \sum_i \sigma_i \, \sigma_{i+\hat{x}} \, \sigma_{i+\hat{x}+\hat{y}} \, \sigma_{i+\hat{y}}$$
$$-J_{yz} \sum_i \sigma_i \, \sigma_{i+\hat{y}} \, \sigma_{i+\hat{y}+\hat{z}} \, \sigma_{i+\hat{z}}$$
$$-J_{zx} \sum_i \sigma_i \, \sigma_{i+\hat{z}} \, \sigma_{i+\hat{z}+\hat{x}} \, \sigma_{i+\hat{x}}$$





Johnston *et al.* 2007

$J_{xy} = J_{yz} = J_{zx} = J$: gonihedric model

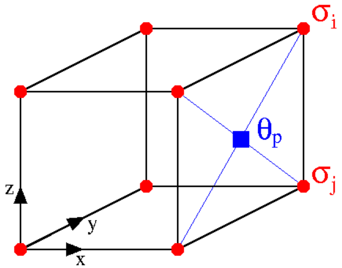
- first order phase transition at finite T_c
- glass transition at $T_g \sim T_c$

Example II: the gonihedric model

convenient description using plaquette dual spins

$$\theta_p \equiv \prod_{i \in p} \sigma_i$$

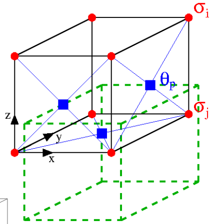
$$E = -J \sum_p \prod_{i \in p} \sigma_i = -J \sum_p \theta_p$$

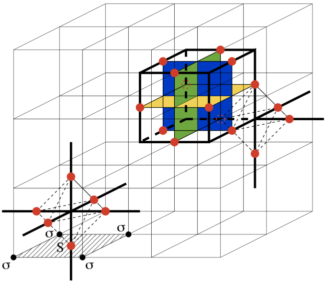


- kinetically constrained model: trivial thermodynamics, cooperative dynamics
- not all θ spin configurations are allowed

Example II: the gonihedric model

the θ spins live on the bonds of the body-centered dual lattice





$$\sum_{\text{cubes } c} \prod_{p \in c} \theta_p^x$$

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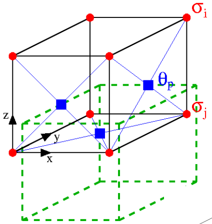
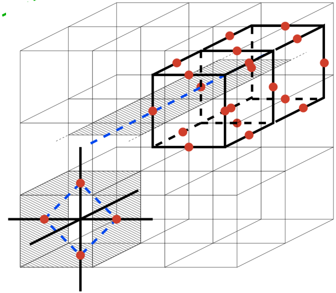


$$-\lambda \left[\sum_{\text{stars } s} \prod_{p \in s} \theta_p^z + \sum_{\text{strip } t} \prod_{p \in t} \theta_p^z \right]$$



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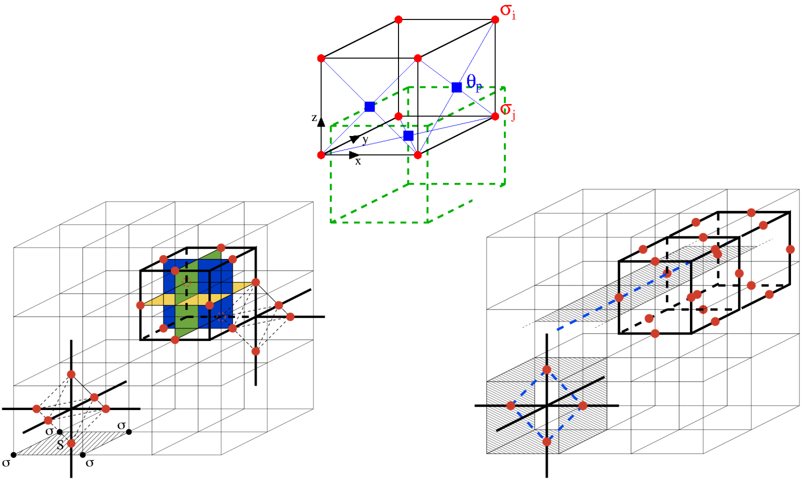

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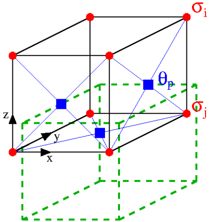
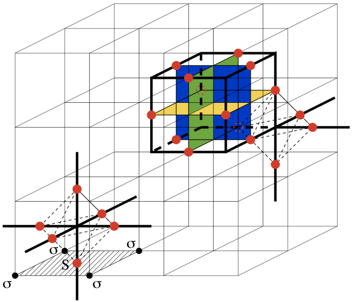


$$\hat{H} = \sum_c \frac{1}{2\cosh(\beta J M_c)} \left\{ \exp\left(-\beta J M_c\right) - \prod_{p \in c} \theta_p^x \right\}$$
$$-\lambda \left[\sum_s \prod_{p \in s} \theta_p^z + \sum_t \prod_{p \in t} \theta_p^z \right]$$

Example II: the gonihedric model

the high temperature ($\beta \to 0$) limit gives an odd toric code



$$\hat{H} = -\sum_c \prod_{p \in c} \theta_p^x - \lambda \sum_s \prod_{p \in s} \theta_p^z - \lambda \sum_t \prod_{p \in t} \theta_p^z$$

- (sub)*extensive* topological degeneracy
- the last term selects a unique topological sector

Variational approach to the lowest energy states

$$|\psi_0\rangle = \sum_{\mathcal{C}} \frac{e^{-\beta E_{\mathcal{C}}/2}}{\sqrt{Z}} |\mathcal{C}\rangle = \sum_{\mathcal{C}} \frac{\exp\left\{\frac{\beta}{2}\sum_{ij} J_{ij}\sigma_j^z\sigma_i^z\right\}}{\sqrt{Z}} |\mathcal{C}\rangle$$

variational approach to the collapsing excited states:

- find $|\psi_1\rangle$ s.t. $\langle\psi_0|\psi_1\rangle = 0$
- compute $\langle\psi_1|H|\psi_1\rangle$
- $\Rightarrow \exists$ at least one excited state with energy $\Delta_1 < \langle\psi_1|H|\psi_1\rangle$

for the 2D Ising ferromagnet:

$$|\psi_1\rangle \propto \sum_{\mathcal{C}} \tanh\left[\sum_i \sigma_i^z\right] \exp\left\{\frac{\beta}{2}\sum_{ij} J_{ij}\sigma_j^z\sigma_i^z\right\} |\mathcal{C}\rangle$$

$$\text{and} \quad \Delta_1 < \frac{\sum e^{-|M(\mathcal{C})|}\, e^{-\beta E_{\mathcal{C}}}}{\sum e^{-\beta E_{\mathcal{C}}}} \sim \exp(-\alpha L) \quad \text{for} \quad T < T_c$$

Other advantages of the quantum language

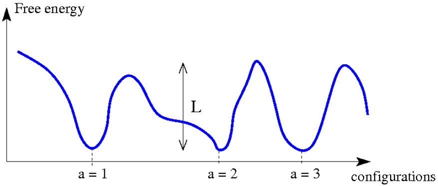
- access to "*off-diagonal*" observables in a static formalism (e.g., using a rotated basis: $\langle \psi_0 | \hat{\sigma}_i^x | \psi_0 \rangle$, ...)
  $\longrightarrow$ new order parameters?
- different angle to look at conventional techiques for glassy systems: large deviation functions ($\Leftrightarrow$ quantum perturbaton theory); four-point dynamical correlation functions; ...
- intriguing connections between very diverse areas of physics
  - glassiness '=' massive collapse of states that are statistically 'similar'
  - topological order '=' finite collapse of states that are exactly indistinguishable by local operators

Compare with the Sherrington-Kirkpatrick model

$$E(\{\sigma\}) \equiv -\sum_{i,j} J_{ij}\,\sigma_i\sigma_j \qquad \overline{J_{ij}} = 0, \quad \overline{J_{ij}^2} = J^2/N$$

using Parisi's picture at low temperatures:

$$Z = \sum_{a=1}^{\mathcal{N}} Z_a \qquad \begin{cases} \tau_{a\to b} \overset{L\to\infty}{\longrightarrow} \infty \\ \mathcal{N} \sim \exp L \end{cases}$$



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$$|\psi_0\rangle = \sum_{a=1}^{\mathcal{N}} \sqrt{\frac{Z_a}{Z}}|\phi_a\rangle \qquad |\phi_a\rangle = \frac{1}{\sqrt{Z_a}} \sum_{\mathcal{C}\in a} e^{-\beta E_{\mathcal{C}}/2}|\mathcal{C}\rangle$$

$$\langle\phi_a|\phi_b\rangle = \delta_{ab}$$

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$\Rightarrow$ extensive ($N \sim exp(L)$) set of collapsing eigenstates
(the known result $q_{\mathrm{EA}}(\mathcal{O}) = \sum_{a=1}^{\mathcal{N}} \sqrt{Z_a/Z}\langle\sigma_i^z\rangle_a^2$ is indeed recovered)

What happens at T_g? (I)

back

the gap closes: is it a quantum phase transition?

- take $\mathcal{O}$ such that $q_{\rm EA}(\mathcal{O}) \neq 0$ for $T < T_g$
- classical $\mathcal{O}$ $\rightarrow$ quantum operator $\hat{\mathcal{O}} \equiv \sum_{\mathcal{C}} |\mathcal{C}\rangle \mathcal{O}_{\mathcal{C}} \langle \mathcal{C}|$
- we can write correlators $C(t+\tau, t)$ and $q_{\rm EA}(\mathcal{O})$ in the quantum mechanical language

$$C(t+\tau,t) = \sum_n e^{-\varepsilon_n \tau} \langle \psi_0 | \hat{S}^{-1} \hat{\mathcal{O}} \hat{S} | \psi_n \rangle \langle \psi_n | \hat{S}^{-1} \hat{\mathcal{O}} | P(t) \rangle$$

$$C_c(\tau) = \lim_{t\to\infty} C(t+\tau,t) = \sum_{n\neq 0} e^{-\varepsilon_n \tau} \left| \langle \psi_n | \hat{\mathcal{O}} | \psi_0 \rangle \right|^2$$

$$q_{\rm EA}(\mathcal{O}) = \lim_{\tau\to\infty} C_c(\tau) = \sum_{n \in \mathcal{D}, n\neq 0} \left| \langle \psi_n | \hat{\mathcal{O}} | \psi_0 \rangle \right|^2$$

What happens at T_g? (II)

back

local static (zero-frequency) susceptibility of the quantum system at zero temperature

$$\hat{H}'(\beta,\lambda) = \hat{H}(\beta) + \lambda\hat{\mathcal{O}} \qquad (\neq \text{classical field})$$

$$\chi^{\rm loc}(\omega=0) \equiv \int_0^\infty d\tau\ C_c(\tau) = \sum_{n\neq 0} \frac{\left|\langle\psi_n|\hat{\mathcal{O}}|\psi_0\rangle\right|^2}{\varepsilon_n}$$

At $T = T_g$, $q_{\rm EA} = \lim_{\tau\to\infty} C_c(\tau)$ becomes finite, and $\chi^{\rm loc}(\omega=0)$ diverges