Unusual ground-states formed in very disordered superconducting films

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Publications: *Phys Rev Lett.* **98**, 027001(2007) (M.F., Ioffe, Kravtsov, Yuzbashyan)
arXiv:0909.2263 (L.Ioffe and M.Mezard)
Annals of Physics **325,** 1368 (2010) (M.F., L.Ioffe, V.Kravtsov, E.Cuevas)
arXiv:1006.5767 (M.F., L.Ioffe and M.Mezard)

Plan of the talk

1. Motivation from experiments: insulators and pseudo-gap superconductors

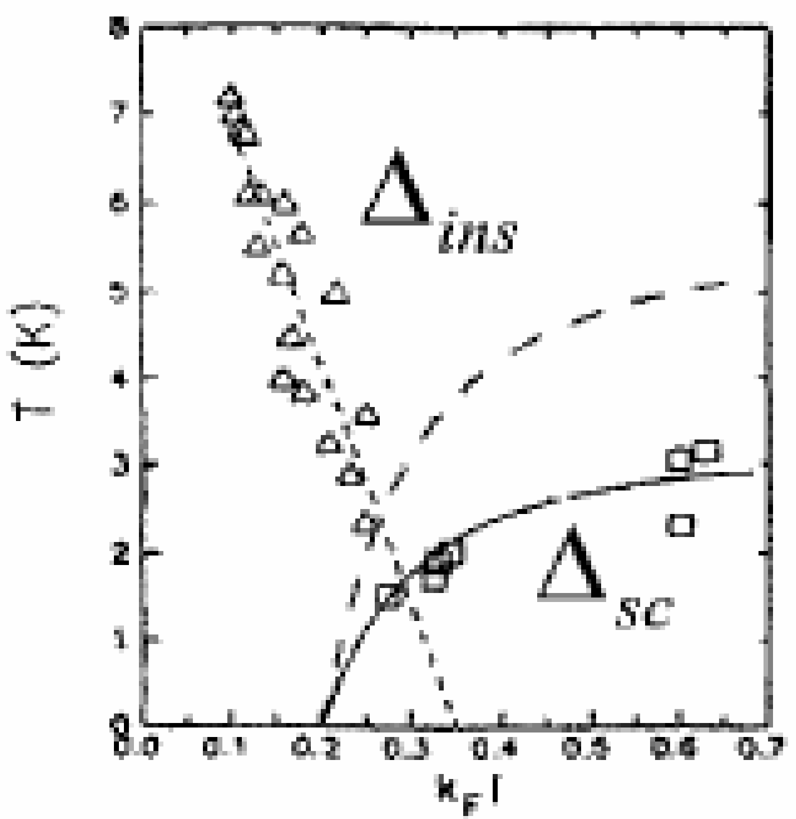
2. Cooper pairing for near-critical eigenstates

3. Interacting pseudospins: model for a new quantum phase transition

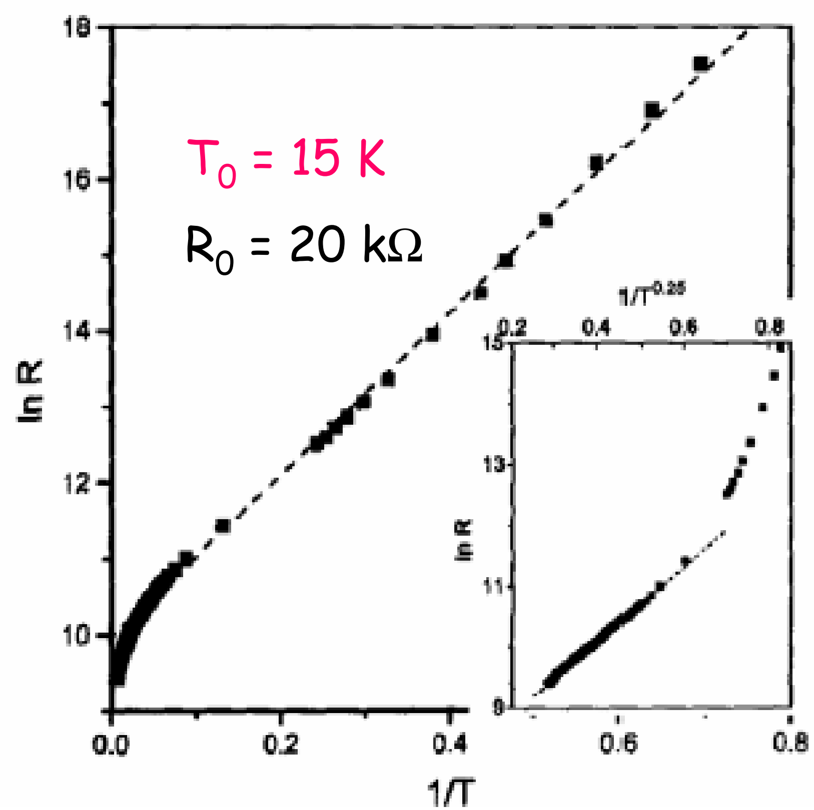
4. Conclusions

Brief introduction into experimental results on direct S-I transtions

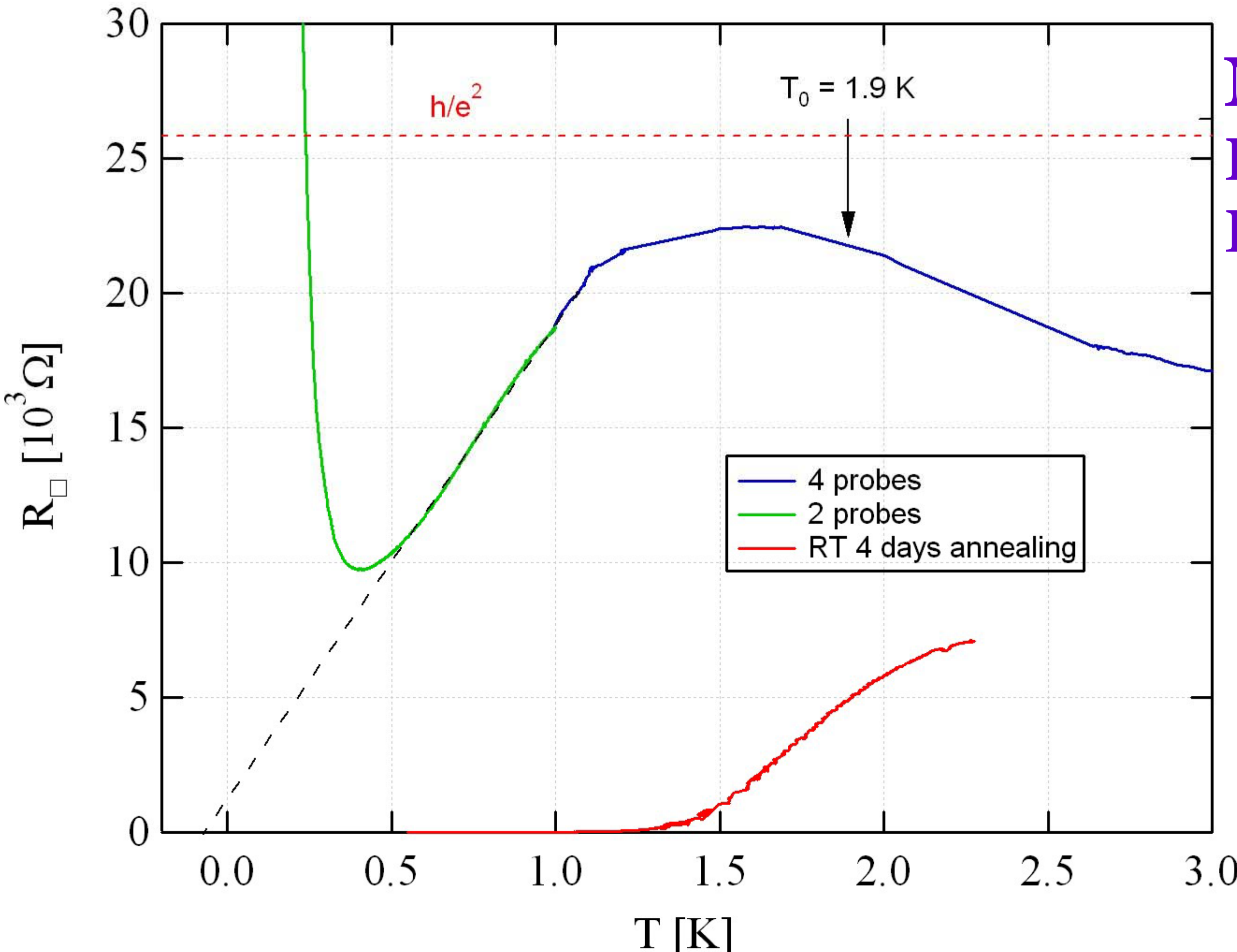
D.Shahar & Z.Ovadyahu
amorphous InO 1992



On insulating side
(far enough):
Kowal-Ovadyahu 1994



Disorder-controlled SIT: nonzero gap at the transition



Near-critial InO_x

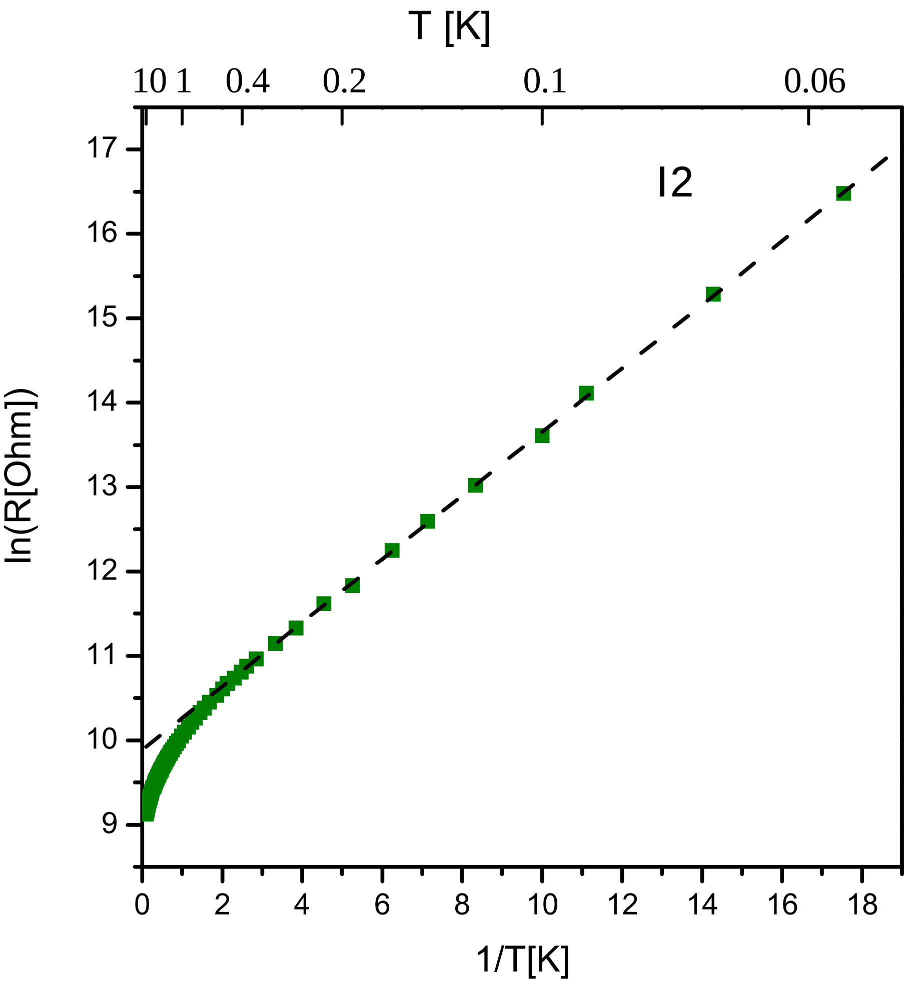
B.Sacepe M.Ovadia D.Shahar (2009)

T_o = 1.9 K

Activation gap

Red curve: same sample after 4 days annealing at 300K but **not presented** in the following slides

TiN film 5 nm thickness



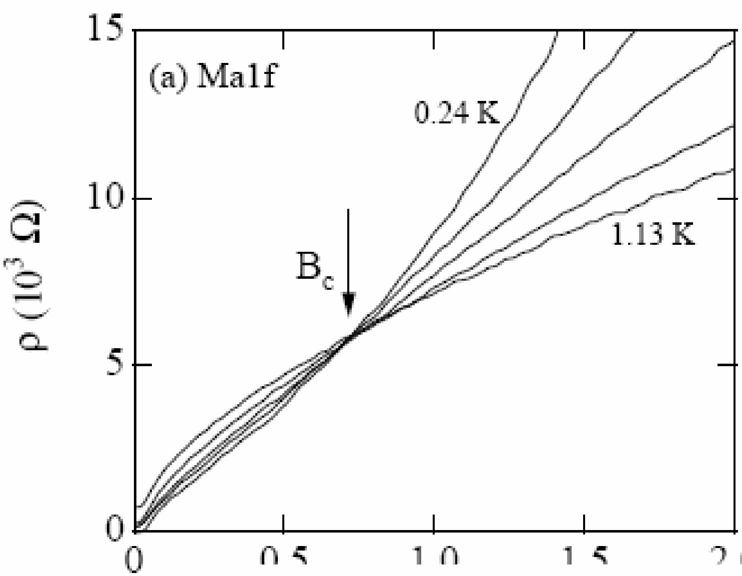
Baturina et al 2007

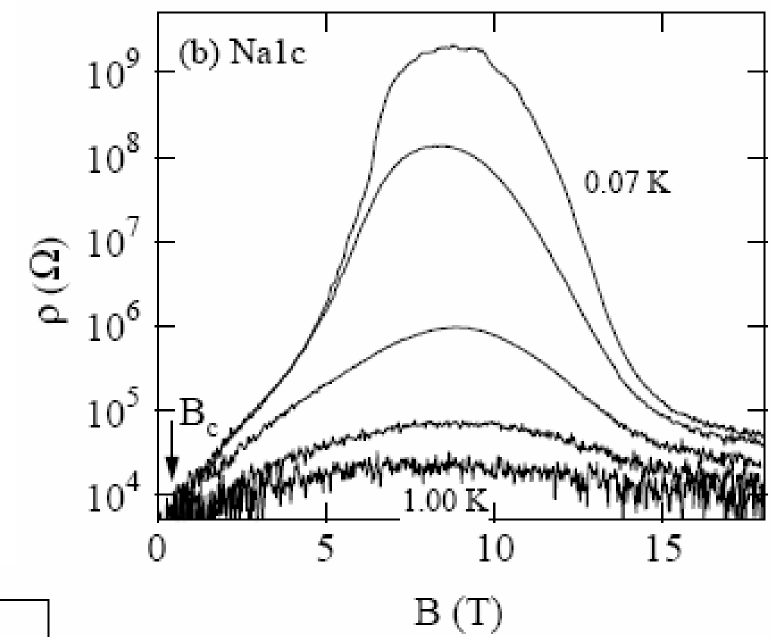
T_0 = 0.38 K

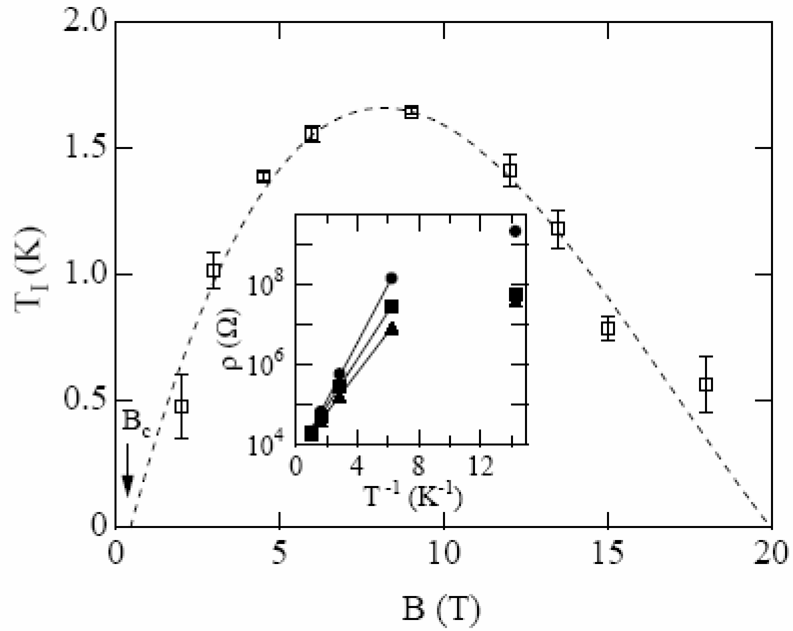
R_0 = 20 kΩ

Magnetic-field induced SIT and giant magnetoresistance

InOx D. Shahar et al (2004)







Transport by pairs

Gap vanishes at B_c

Conclusion 1:

Resistivity follows activation law

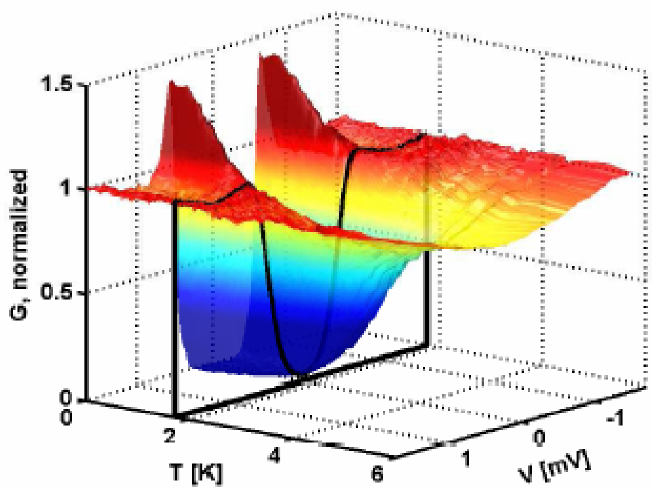
$$R \sim \exp(T_o/T)$$

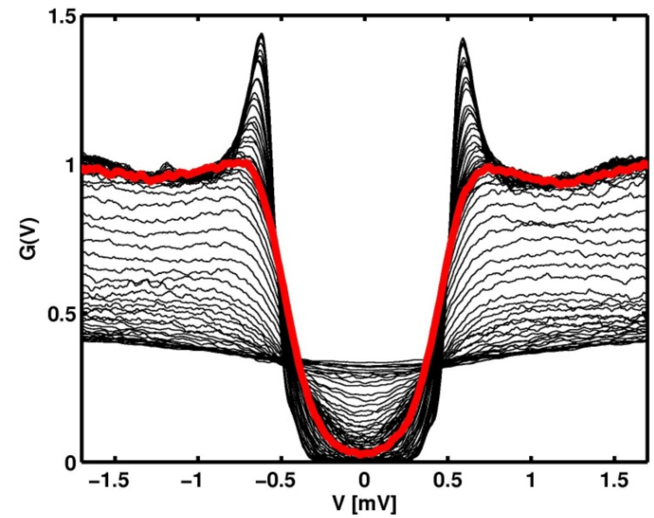
both near SIT and far in the insulating state

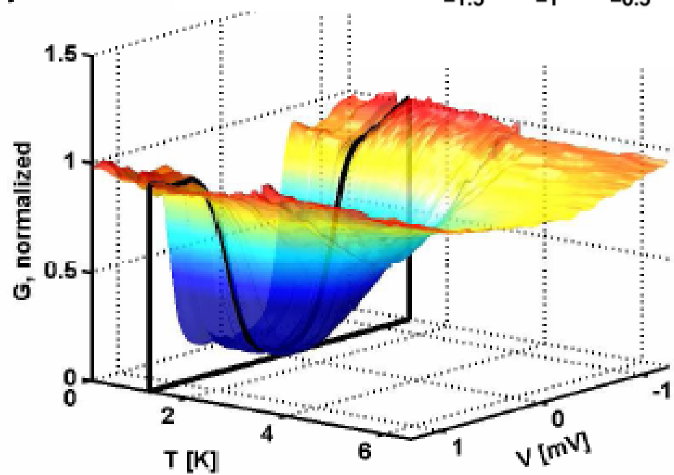
SC side: local tunneling conductance

Spectral signature of localized Cooper pairs in disordered superconductors.

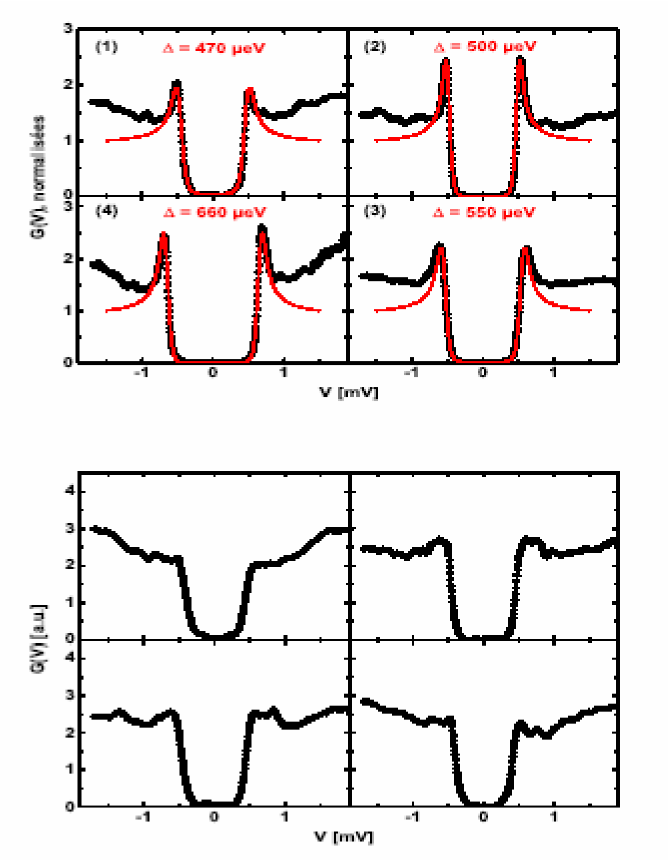
Benjamin Sacépé,[1,*] Thomas Dubouchet,[1] Claude Chapelier,[1] Marc Sanquer,[1] Maoz Ovadia,[2] Dan Shahar,[2] Mikhail Feigel'man,[3] and Lev Ioffe[4,3]



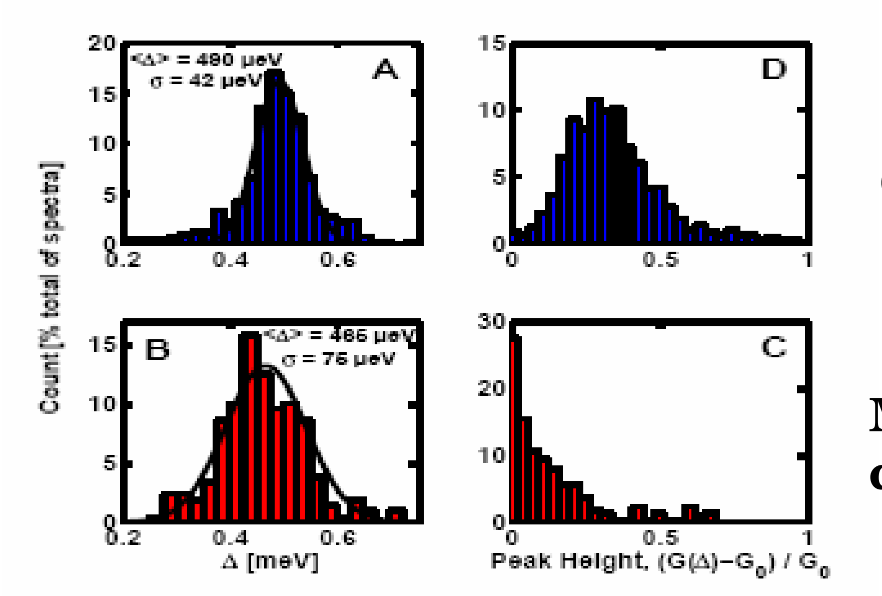




Local tunneling conductance-2



Gap widths Peak heights



Less disorder

More disorder

Conclusion 2

Superconductive state near SIT is very unusual:

1. the spectral gap appears much before (with T decrease) than superconductive coherence does
2. Coherence peaks in the DoS appear together with resistance vanishing

Class of relevant materials

- Amorphously disordered
  (no structural grains)
- Low carrier density
  (around 10^{21} cm^{-3} at low temp.)

Examples:

InO_x TiN thin films Be (ultra thin films)

NbN_x

Special example: nanostructured Bi films
(J.Valles et al)

What about theory ?

Superconductivity v/s Localization

- Coulomb-induced suppression of Tc in uniform films ***“Fermionic mechanism”***

A.Finkelstein *(1987)* et al

- Granular systems with Coulomb interaction

K.Efetov *(1980)* M.P.A.Fisher et al *(1990)* ***“Bosonic mechanism”***

- Competition of Cooper pairing and localization (no Coulomb)

Imry-Strongin, **Ma-Lee**, Kotliar-Kapitulnik, Bulaevskii-Sadovskii(mid-80’s)

Ghosal, Randeria, Trivedi 1998-2001

FERMIONIC MODEL OF SUPERCONDUCTOR-INSULATOR TRANSITION

- Disorder increases Coulomb interaction and thus decreases the pairing interaction (sum of Coulomb and phonon attraction). In perturbation theory:

$$\lambda(\varepsilon) = \lambda_0 - \frac{1}{24\pi g} Log\left(\frac{1}{\varepsilon\tau}\right)$$

$$\frac{\delta T_c}{T_c} = -\frac{\delta\lambda}{\lambda^2}$$

If $\lambda(\omega_D)<0$ – attraction is gone and superconductivity disappears *(Finkelstein 1987)*

→ The state formed is likely to be a bad metal (no reasons for a large insulator gap)
No reason for 'superconducting' gap above T_c sensitive to magnetic fields.

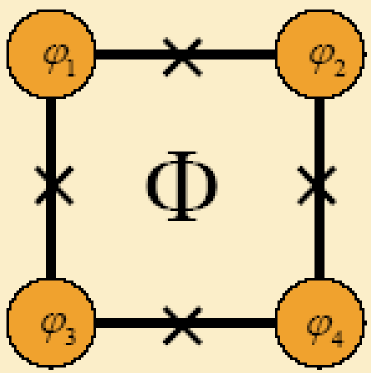
We consider amorphous systems with direct S-I transition
Gap is NOT suppressed at the transition

Bosonic mechanism:

JOSEPHSON ARRAYS

Control parameter

Elementary building block



$$x = E_C/E_J$$

$$E_c = e^2/2C$$

Ideal Hamiltonian:

$$H = \frac{1}{2}\sum_{i,j} C_{ij}^{-1} q_i q_j + E_J \cos(\varphi_i - \varphi_j - 2\pi \frac{\Phi_{ij}}{\Phi_0}) \quad q_i = 2e\, i \frac{d}{d\varphi_i}$$

C_{ij} - capacitance matrix E_J - Josephson energy

Bosonic v/s Fermionic scenario ?

None of them is able to describe data on InO_x and TiN :
Both **scenaria are ruled out by STM data in SC state**

3-d scenario: competition between Cooper pairing and localization

(without any role of Coulomb interaction)

End of Introduction

Theoretical model

Simplest BCS attraction model,
but for critical (or weakly
localized) electron eigenstates

$$H = H_o - g \int d^3r\, \Psi_\uparrow^\dagger \Psi_\downarrow^\dagger \Psi_\downarrow \Psi_\uparrow$$

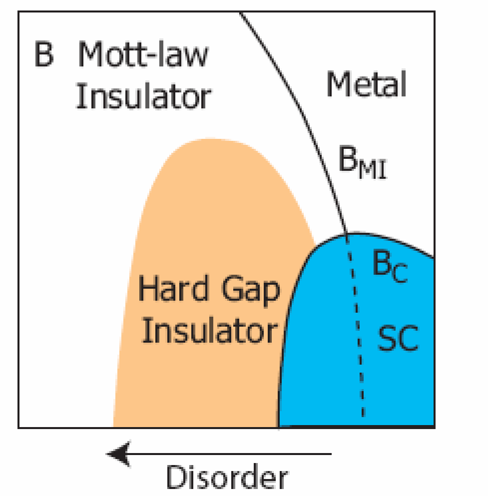
$\boldsymbol{\Psi} = \Sigma\, c_j\, \Psi_j\, (r)$ Basis of localized eigenfunctions

M. Ma and P. Lee (1985) : S-I transition at $\delta_L \approx T_c$

We will see that in fact SC state survives far into the region $\delta_L >> Tc$

Superconductivity at the Localization Threshold: $\delta_L \to 0$

Consider Fermi energy very close to the mobility edge:
single-electron states are extended
but **fractal**
and populate small fraction of the whole volume



How BCS theory should be modified to account for eigenstate's fractality ?

Method: combination of analitic theory and numerical data for Anderson mobility edge model

Mean-Field Eq. for T_c

$$\Delta(r) = \int K_T(r,r')\Delta(r')d^d r' \tag{9}$$

where kernel $\hat{K}_T$ is equal to

$$K_T(r,r') = \frac{\lambda}{2\nu_0}\sum_{ij}\frac{\tanh\frac{\xi_i}{2T} + \tanh\frac{\xi_j}{2T}}{\xi_i + \xi_j}\psi_i(r)\psi_j(r)\psi_i(r')\psi_j(r') \tag{10}$$

Standard averaging over space $\Delta(r) \to \overline{\Delta}$ leads to "Anderson theorem" result: totally incorrect in the present situation.

The reason: critical eigenstates $\psi_j(r)$ are strongly correlated in real 3D space, they fill some small **submanifold** of the whole space only.

In fact one should define T_c as the divergence temperature of the Cooper ladder

$$\mathcal{C} = \left(1 - \hat{K}\right)^{-1}$$

Thus averaging procedure should be applied to $\mathcal{C}$ instead of K

We expand $\mathcal{C}$ in powers of K and average over disorder realizations. Keeping main sequence of resulting diagramms only, we come to the following equation for determination of T_c:

$$\Phi(\xi) = \frac{\lambda}{2} \int \frac{d\xi' \tanh(\xi'/2T)}{\xi'} M(\xi - \xi')\Phi(\xi') \tag{11}$$

$$M(\omega) = \mathcal{V}\overline{M_{ij}} = \int \overline{\psi_i^2(r)\psi_j^2(r)}d^dr \quad \text{for} \quad |\xi_i - \xi_j| = \omega$$

Fractality of wavefunctions

For critical eigenstates

$$L_{\text{loc}} \to \infty$$

IPR: $M_i = \int |\psi_i(\mathbf{r})|^4 d\mathbf{r}$

one finds

$$\langle M_i \rangle \approx 3\ell^{-(d-d_2)} L^{-d_2}.$$

$$M(\omega) = \left(\frac{E_0}{\omega}\right)^\gamma$$

$$E_0 = 1/\nu_0 \ell^3$$

where

$$\gamma = 1 - \frac{D_2}{d}$$

$d_2 \approx 1.3$ in 3D

is a measure of fractality

ℓ is the short-scale cut-off length

Usual "dirty superconductor":

$$M(\omega) = 1 \qquad \gamma = 0$$

3D Anderson model: $\gamma = 0.57$

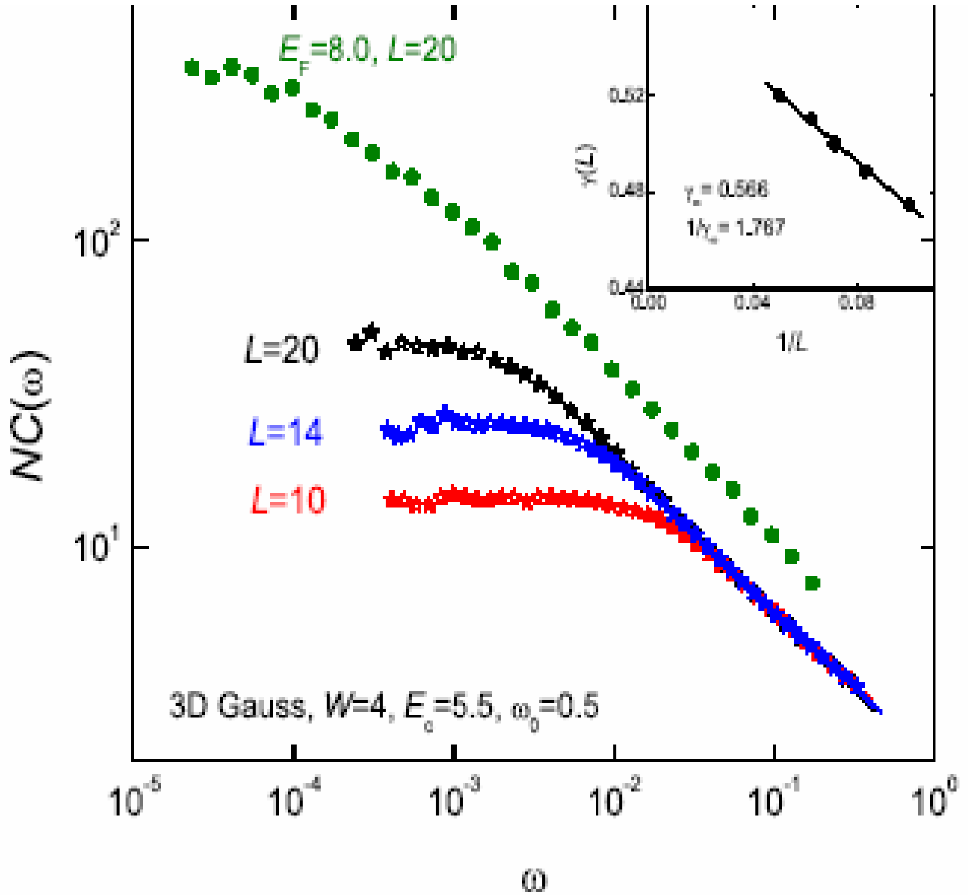


FIG. 2: (Color online) Correlation function $M(\omega)$ for 3DAM with Guassian disorder and lattice sizes $L = 10, 14, 20$ at the mobility edge $E = 5.5$ (red, blue and black points) and at the energy $E = 8$ inside localized band (green points). Inset shows γ values for $L = 10.12.14.16.20$.

Modified mean-field approximation for critical temperature T_c

$$\Delta(\xi) = \frac{\lambda}{2}\int d\zeta \eta(\zeta) M(\xi-\zeta)\Delta(\zeta)$$

$$\eta_i \equiv \eta_{ii} = \xi_i^{-1}\tanh(\xi_i/2T).$$

$$T_c^0(\lambda,\gamma) = E_0 \lambda^{1/\gamma} C(\gamma)$$

For small λ this $\mathbf{T_c}$ is higher than BCS value !

Alternative method to find Tc:

Virial expansion

(A.Larkin & D.Khmelnitsky 1970)

$$F = \sum_{n=1}^{\infty} \mathcal{F}^{(n)} = \sum_i F_i + \sum_{i>j} (F_{ij} - F_i - F_j)$$

$$+ \sum_{i>j>k} (F_{ijk} - F_{ij} - F_{jk} - F_{ik} + F_i + F_j + F_k) + \ldots$$

$$V_\Delta = -\sum_j (\Delta S_j^+ + \Delta^* S_j^-)$$

$$\chi(T) = -\frac{\partial^2 F}{\partial \Delta \partial \Delta^*} = \sum_{M=1} \chi_M(T)$$

$$\chi_1 = \sum_i \chi_i^{(1)} \qquad (148)$$

$$\chi_2 = \sum_{n>m} (\chi_{nm}^{(2)} - \chi_n^{(1)} - \chi_m^{(1)})$$

$$\chi_3 = \sum_{n>m>l} (\chi_{nml}^{(3)} - \chi_{nl}^{(2)} - \chi_{ml}^{(2)} - \chi_{nm}^{(2)} + \chi_n^{(1)} + \chi_m^{(1)} + \chi_l^{(1)})$$

$$\lim_{n\to\infty} \frac{\chi^{(n+1)}(T_c)}{\chi^{(n)}(T_c)} = 1 \quad \Longrightarrow \quad \frac{\chi^{(3)}(T_c)}{\chi^{(2)}(T_c)} = 1$$

T_c from 3 different calculations

Modified MFA equation leads to:

$$T_c = (6.5 \pm 0.8)\lambda^{1.77}$$

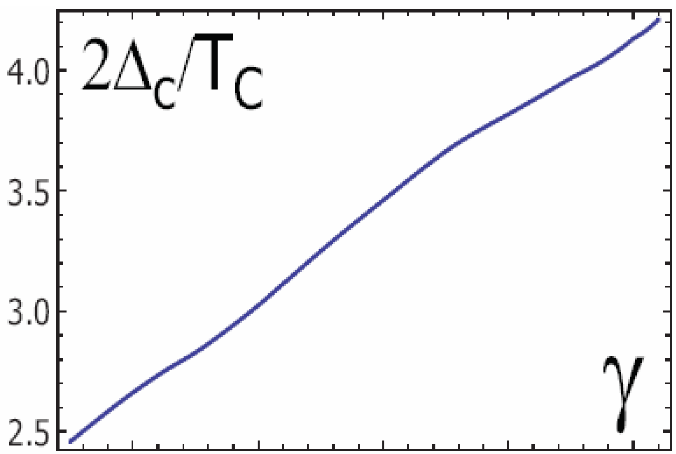
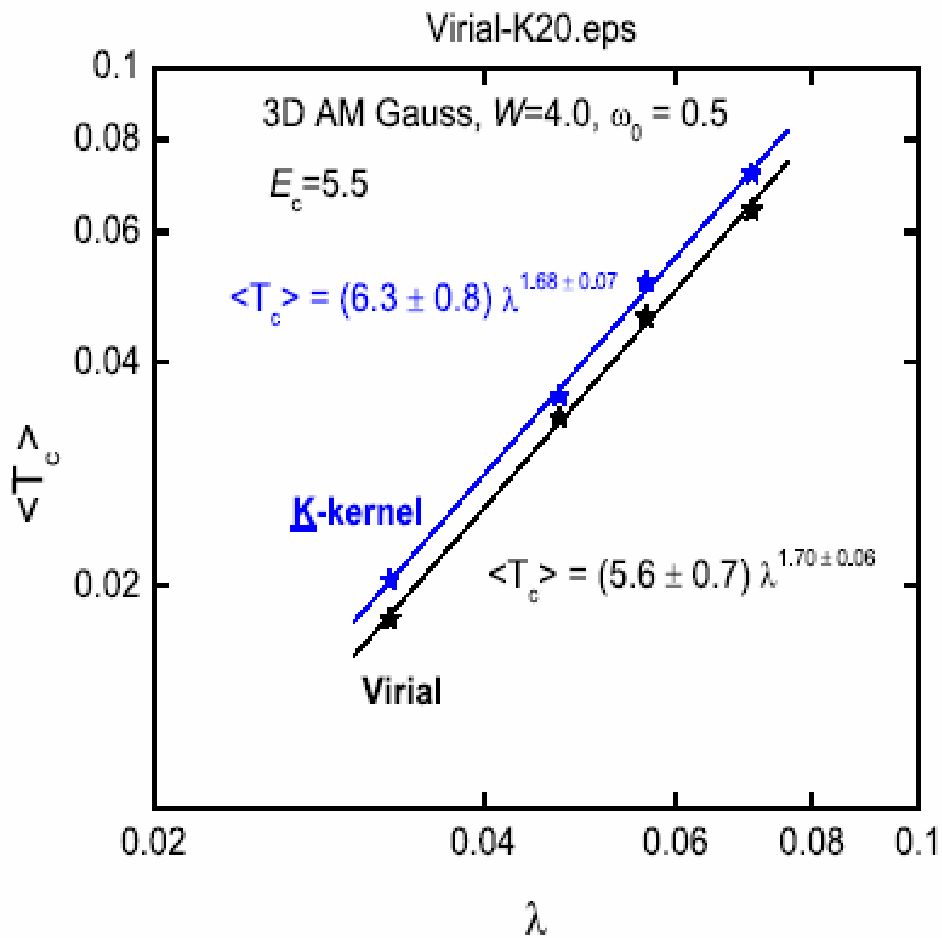


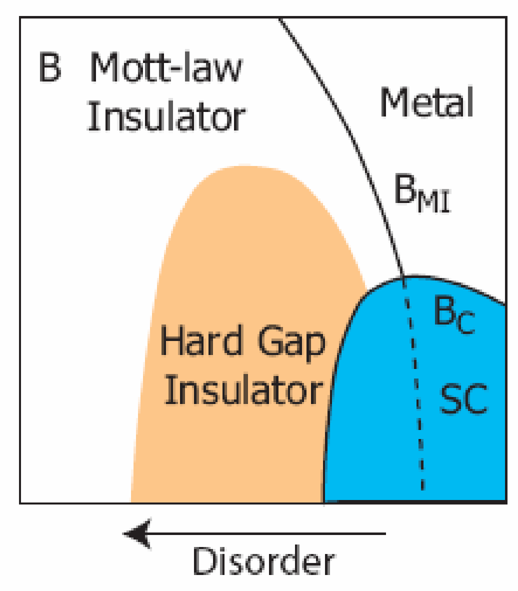
FIG. 16: Ratio $2\Delta(0)/T_c$ as function of γ.



BCS theory: $T_c = \omega_D \exp(-1/\lambda)$

Insulator and Superconductor with Pseudogap

Now we move
Fermi- level into the
range of localized eigenstates

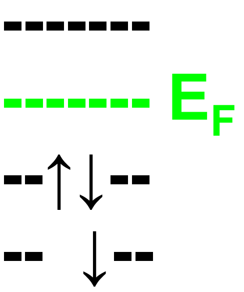


Local pairing appears
in addition to
collective pairing

Local pairing energy

1. Parity gap in ultrasmall grains

K. Matveev and A. Larkin 1997



$$\Delta \ll \delta:$$

No many-body correlations

$$\Delta_P = \frac{1}{2}\lambda\delta \qquad \lambda_R = \lambda/(1-\lambda\log(\epsilon_0/\delta)). \qquad \Delta_P = \frac{\delta}{2\ln\frac{\delta}{\Delta}}$$

Correlations between pairs of electrons localized in the same "orbital"

2. Parity gap for Anderson-localized eigenstates

The increase of thermodynamic potential Ω due to addition of *odd* electron to the ground-state is

$$\delta\Omega_{\rm oe} = \xi_{m+1} = \xi_{m+1} - \tilde{\xi}_{m+1} + \tilde{\xi}_{m+1} = \frac{g}{2} M_{m+1} + O(\mathcal{V}^{-1})$$

$\tilde{\xi}_j = \xi_j - \frac{g}{2} M_j$.

Energy of two single-particle excitations after depairing:

$$2\Delta_P = \xi_{m+1} - \xi_m + gM_m = \frac{g}{2}(M_m + M_{m+1}) + O(\mathcal{V}^{-1})$$

$$\langle M_i \rangle = 3\ell^{-(d-d_2)} L_{\rm loc}^{-d_2},$$

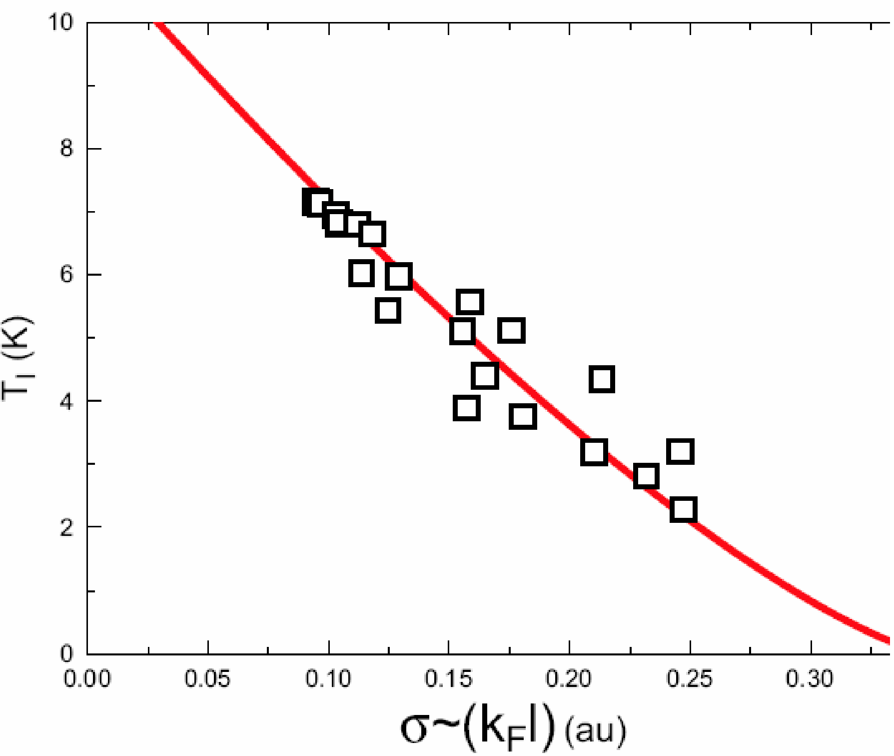
$$\Delta_P = \frac{3}{2} g \ell^{-3} (L_{loc}/\ell)^{-d_2} = \frac{3\lambda}{2} E_0 \left(\frac{E_c - E_F}{E_0}\right)^{\nu d_2}$$

Δ_P plays the role of the activation gap

Insulating state:
parity gap scaling
near mobility edge

Activation energy T_I from Shahar-Ovadyahu exp. and fit to the theory

$$T_I = A(1-\sigma/\sigma_c)^{\nu d_2}, \quad A \approx 0.5\lambda E_0$$



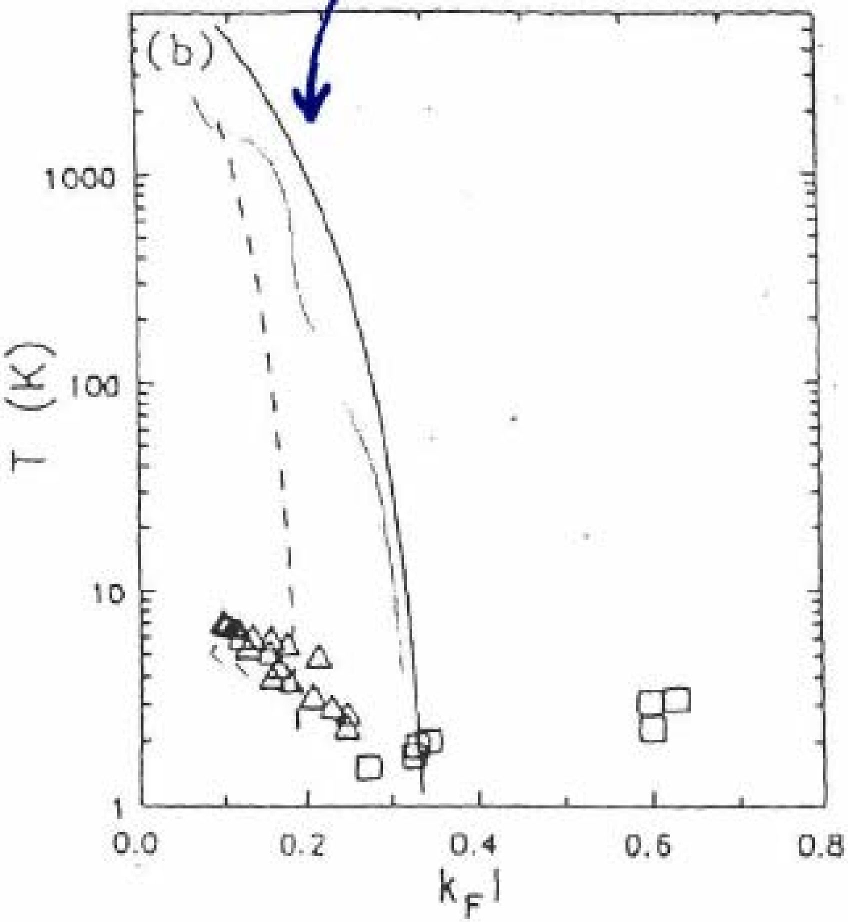
The fit was obtained with single fitting parameter

Example of consistent choice:

λ **= 0.05** E_0 **= 400 K**

Shahar- Ovadyahu 1992

$$\delta_L \sim \frac{1}{\nu} L_{loc}^{-3} \sim (n-n_c)^3$$



Superconductive state with a pseudogap

Critical temperature in the pseudogap regime

MFA:

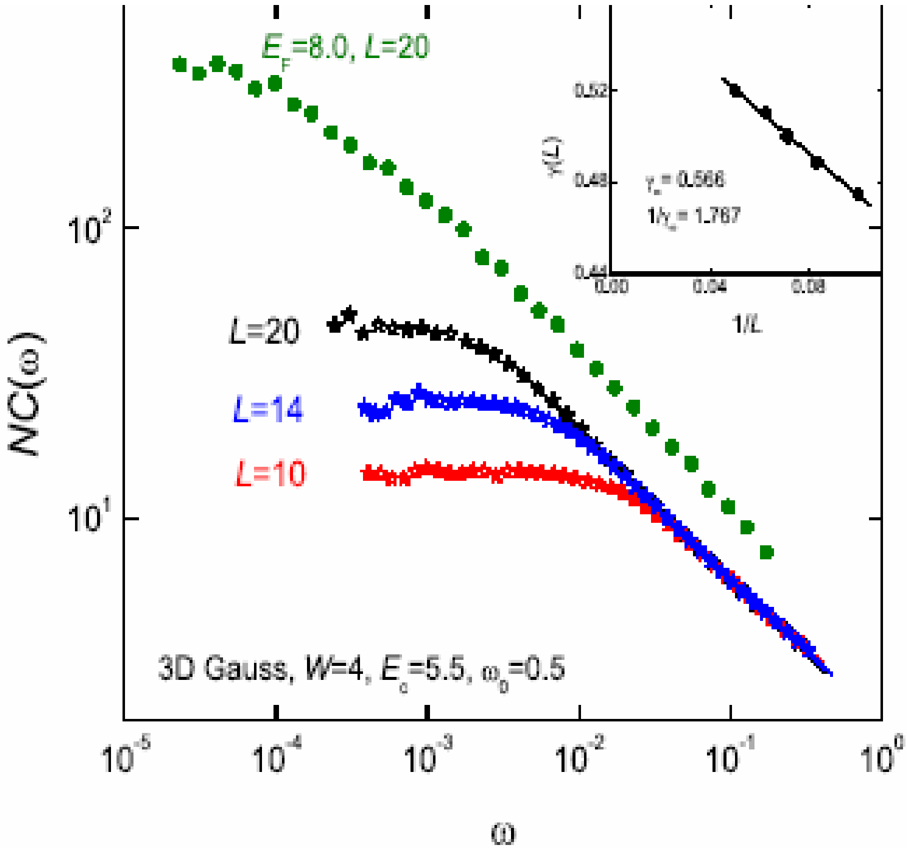
$$\Delta(\xi) = \frac{\lambda}{2}\int d\zeta\eta(\zeta)M(\xi-\zeta)\Delta(\zeta)$$

$$\eta_i \equiv \eta_{ii} = \xi_i^{-1}\tanh(\xi_i/2T).$$

Here we use M(ω) specific for localized states

MFA is OK as long as $Z \sim \nu_0 T_c L_{loc}^d$ **is large**

Correlation function M(ω)



No saturation at **$\omega < \delta_L$** :

$M(\omega) \sim \ln^2 (\delta_L / \omega)$

(Cuevas & Kravtsov PRB,2007)

Superconductivity with **$T_c < \delta_L$** is possible

This region was not found previously

Here "local gap" exceeds **SC gap :**

$$\Delta_P = \frac{1}{2D^\gamma(\gamma)}\delta_L \left(\frac{\Delta(0)}{\delta_L}\right)^\gamma$$

FIG. 2: (Color online) Correlation function $M(\omega)$ for 3DAM with Guassian disorder and lattice sizes $L = 10, 14, 20$ at the mobility edge $E = 5.5$ (red, blue and black points) and at the energy $E = 8$ inside localized band (green points). Inset shows γ values for $L = 10.12.14.16.20$.

T_c versus Pseudogap

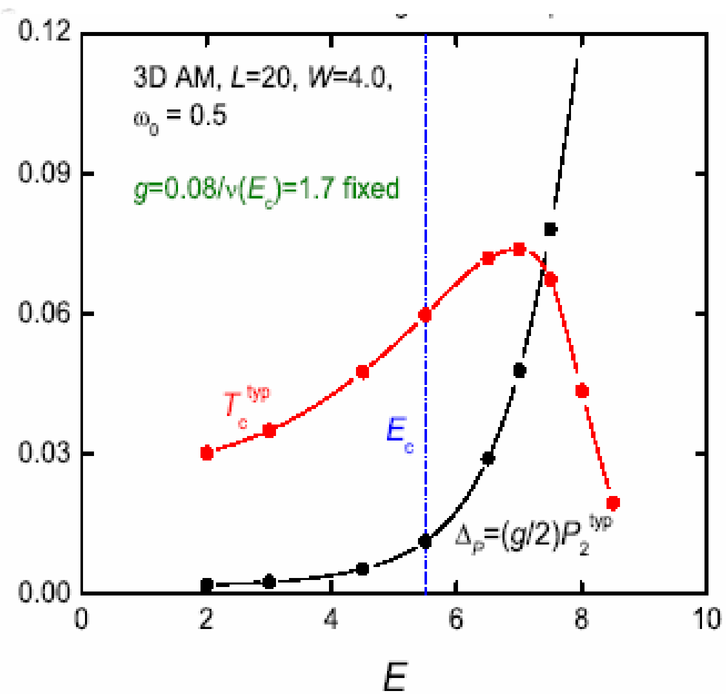


FIG. 25: (Color online) Virial expansion results for T_c (red points) and typical pseudogap Δ_P (black) as functions of E_F. The model with fixed value of the attraction coupling constant $g = 1.7$ was used; pairing susceptibilities were calculated using equations derived in Appendix B.

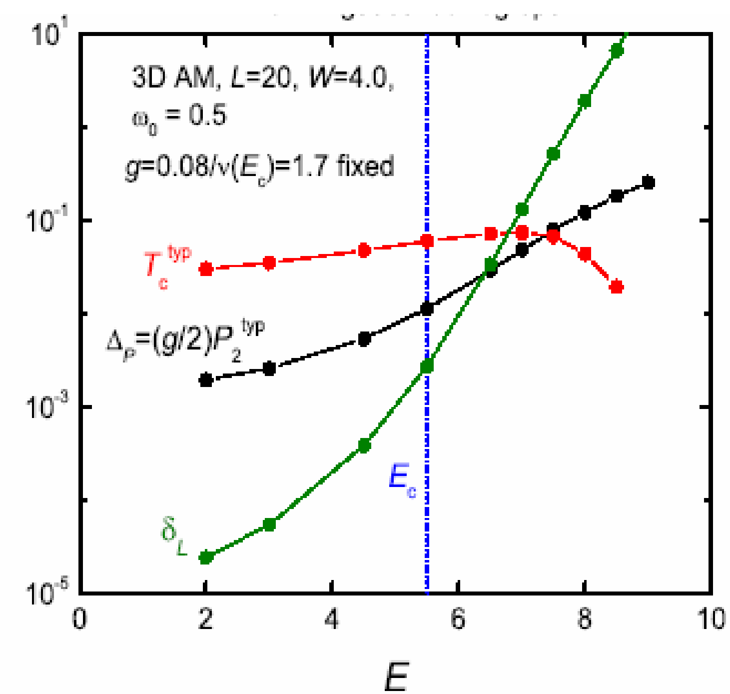


FIG. 26: (Color online) Virial results for T_c (red points), typical pseudogap Δ_P (black) and the corresponding level spacing δ_L (green), as functions of E_F on semi-logarithmic scale.

Transition exists even at $\delta_L >> T_{co}$

Low-energy effective Hamiltonian for pseudogaped SC state

$$\Delta_P >> T_c$$

Population of single-occupancy electron states is suppressed by the pseudogap

„Pseudo spin" representation:

$$S^+_\mu = a^+_{\mu\uparrow} a^+_{\mu\downarrow} \qquad S^-_\mu = a_{\mu\uparrow} a_{\mu\downarrow}$$

$$2S^z_\mu = a^+_{\mu\uparrow} a_{\mu\uparrow} + a^+_{\mu\downarrow} a_{\mu\downarrow}$$

H_{BCS} acts on Even sector:
all states which are
2-filled or empty

$$\hat{H} = \sum_\mu 2\xi_\mu S^z_\mu - g \sum_{\mu,\nu} M_{\mu\nu} S^+_\mu S^-_\nu + \sum_{B_\mu} \left(\xi_\mu + \frac{G_\mu}{2}\right)$$

B: „blocked" states

$$\bar{M}_{\mu\nu} = \frac{1}{\nu V} M(\xi_\mu - \xi_\nu)$$

ν: D.o.S; V: total volume

"Pseudospin" approximation

$$Z \sim \nu_0 T_c L^d_{loc}$$

Effective number of interacting neighbours

S-I-T: Third Scenario

- Bosonic mechanism: preformed Cooper pairs + competition Josephson v/s Coulomb – S I T in arrays
- Fermionic mechanism: suppressed Cooper attraction, no paring – S M T

- Pseudospin mechanism: individually localized pairs
 - S I T in amorphous media

SIT occurs at small Z and lead to paired insulator

$$H = 2\sum_i \xi_i s_i^z - \sum_{ij} M_{ij}(s_i^x s_j^x + s_i^y s_j^y)$$

How to describe this quantum phase transition ?

How to understand hard-gap insulator right after S-I transition ?

Conclusions

Pairing on nearly-critical states produces fractal superconductivity with relatively high T_c but very small superconductive density

Pairing of electrons on localized states leads to hard gap and Arrhenius resistivity for 1e transport

Pseudogap behaviour is generic near S-I transition, with "insulating gap" above T_c

For the theory of S-I transition see talk by Lev Ioffe on Thursday, 08/26

Coulomb enchancement near mobility edge ??

Normally, Coulomb interaction is overscreened, with universal effective coupling constant ~ 1

Condition of universal screening: $2\sigma/(T_c\kappa) \sim (\xi_0/a_{\rm scr})^2/\kappa \gg 1$

$a_{\rm scr}$ is the Thomas-Fermi screening length, $\sigma = (e^2k_F/6\pi^2)(k_F l)$

Example of a-InO_x : bad dirty metal with $k_F l \sim 0.3$

$e^2 k_F \sim 5000K$ the ratio $\sigma/T_c \sim 10$

dielectric constant $\kappa \geq 30$

Effective Coulomb potential is weak:

$$\mu \sim 2\sigma/(T_c\kappa) < 1$$

P(M) distribution