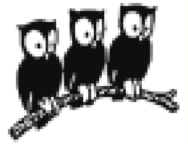
Criticality and avalanches in spin and electron glasses

Markus Müller

The Abdus Salam International Center of Theoretical Physics

In collaboration with
Pierre Le Doussal (LPT-ENS Paris)
Kay Wiese (LPT-ENS Paris)

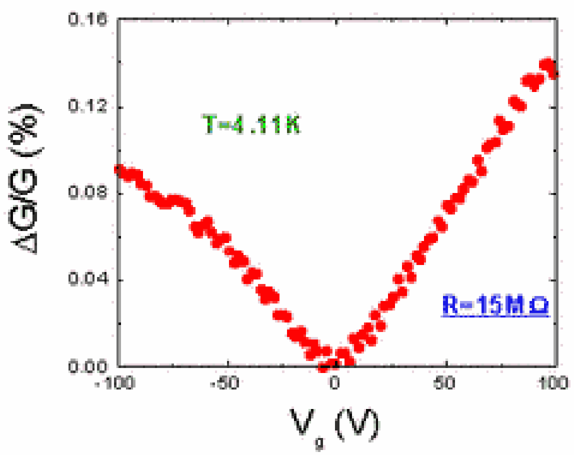


Out of equilibrium quantum systems, KITP 23-27 August, 2010

Motivation I

Memory dip in electron glasses after change of gate voltage

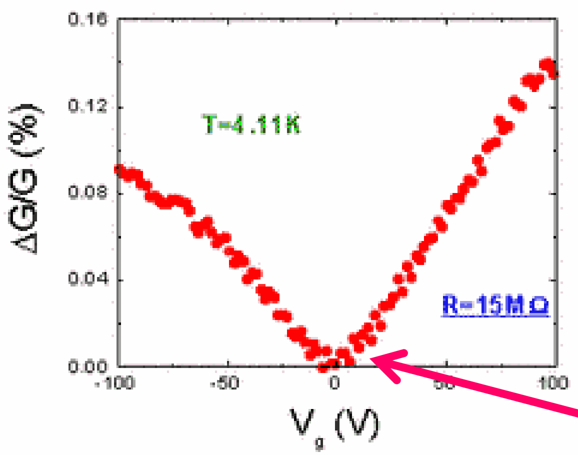
M. Ben-Chorin et al., PRL ***84****, 3402 (2000)*



Motivation I

Memory dip in electron glasses after change of gate voltage

*M. Ben-Chorin et al., PRL **84**, 3402 (2000)*



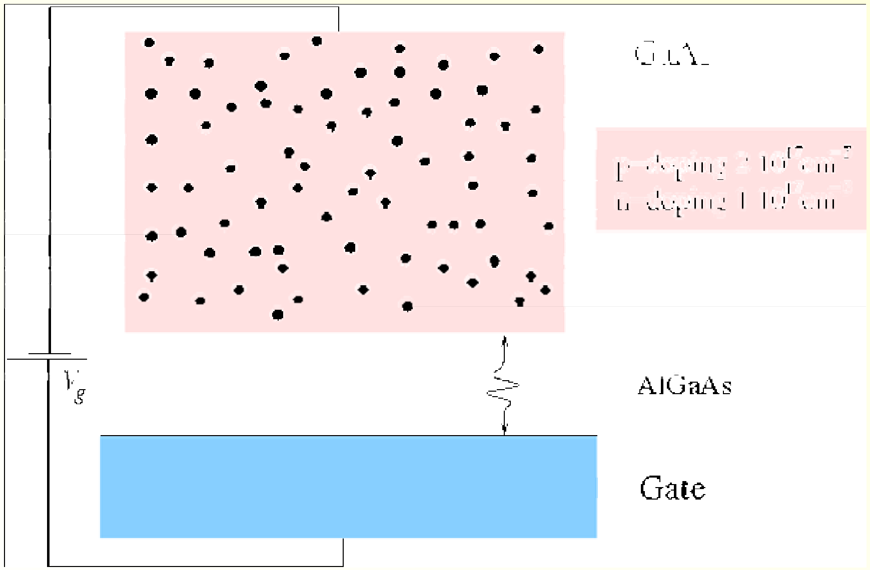
What happens actually, as new electrons come into the sample?

Or - what happens after P. Armitage's or D. Popovic's excitations?

(cf. yesterday's talks)

II. Charging a glassy capacitor

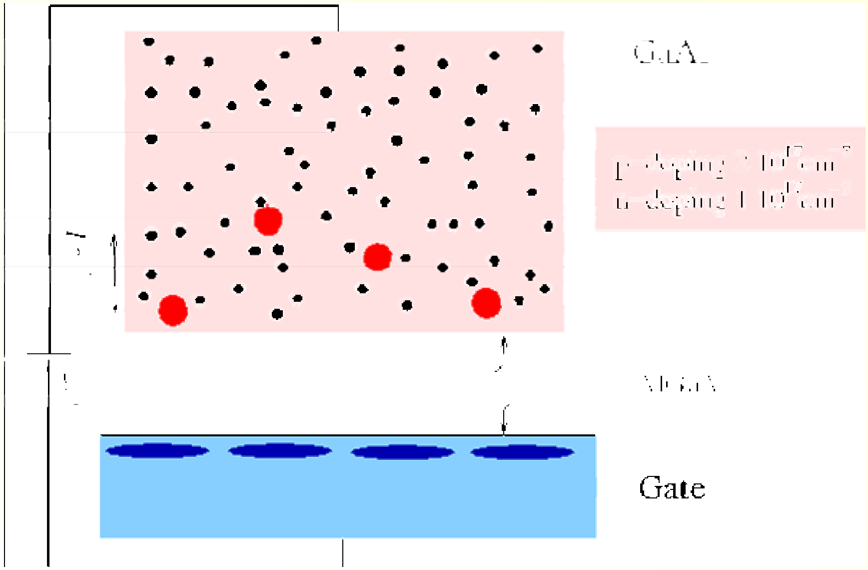
Introducing charge in a strongly insulating Coulomb glass



D. Monroe et al.,
PRL ***59****, 1148 (1987)*

II. Charging a glassy capacitor

Introducing charge in a strongly insulating Coulomb glass

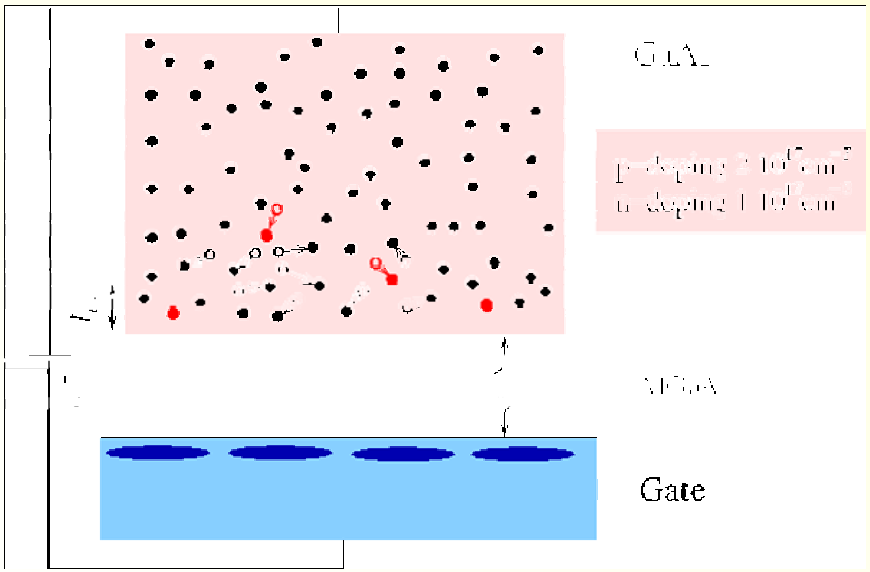


D. Monroe et al.,
*PRL **59**, 1148 (1987)*

Injection from leads (or better from a tunnel tip) and …

II. Charging a glassy capacitor

Introducing charge in a strongly insulating Coulomb glass



D. Monroe et al.,
*PRL **59**, 1148 (1987)*

… avalanche-like relaxation, or **“crackling”.**

A.k.a. “non-linear screening” (Baranovskii, Shklovskii, Efros 1984)

Outline

- Crackling, avalanches, “shocks” in disordered, non-linear systems; Self-organized criticality

- Avalanches in the magnetizing process (“Barkhausen noise”)

- The criticality of spin glasses at equilibrium – why to expect scale free avalanches

- Magnetization avalanches in the Sherrington-Kirkpatrick spin glass – an analytical study.

- Applications/perspectives: Finite dimensions, electron glasses, avalanches in quantum systems

Crackling

*Review: Sethna, Dahmen, Myers, Nature **410**, 242 (2001).*

Crackling = Response to a slow driving which occurs in a discrete set of avalanches, spanning a wide range of sizes.

Occurs often but not necessarily out of equilibrium.

Examples:

- Earthquakes
- Crumpling paper
- Charging an electron glass (presumably)
- Disordered magnet in a changing external field magnetizes in a series of jumps

Crackling

Review: Sethna, Dahmen, Myers, Nature ***410****, 242 (2001).*

Crackling = Response to a slow driving which occurs in a discrete set of avalanches, spanning a wide range of sizes.

Occurs often but not necessarily out of equilibrium.

Examples:

- Earthquakes
- Crumpling paper
- Charging an electron glass (presumably)
- Disordered magnet in a changing external field magnetizes in a series of jumps

But: Not everything crackles!

Intermediate between snapping (e.g., twigs, chalk, weakly disordered ferromagnets, nucleation in clean systems)

and popping (e.g., popcorn, strongly disordered ferromagnets)

Crackling

Review: Sethna, Dahmen, Myers, Nature ***410****, 242 (2001).*

Crackling = Response to a slow driving which occurs in a discrete set of avalanches, spanning a wide range of sizes.

Occurs often but not necessarily out of equilibrium.

Examples:

- Earthquakes
- Crumpling paper
- Charging an electron glass (presumably)
- Disordered magnet in a changing external field magnetizes in a series of jumps

But: Not everything crackles!

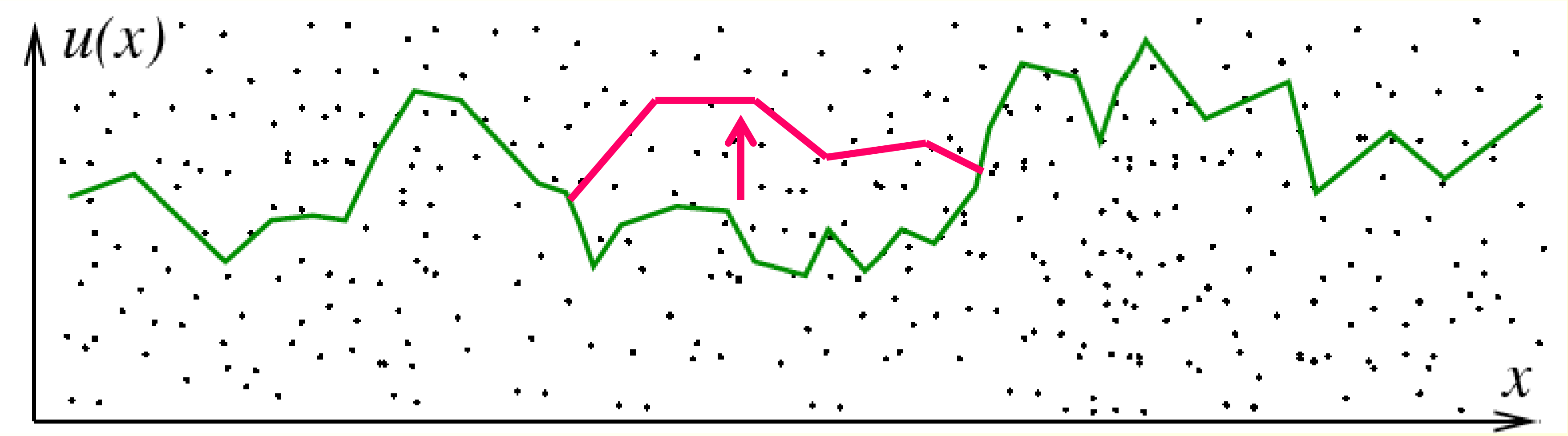
Intermediate between snapping (e.g., twigs, chalk, weakly disordered ferromagnets, nucleation in clean systems)
and popping (e.g., popcorn, strongly disordered ferromagnets)

Crackling on all scales – generally signature of a critical state in driven, non-linear systems. → Can be an interesting *diagnostic tool.*

Examples of crackling I

- Depinning of contact lines, interfaces and other elastic objects

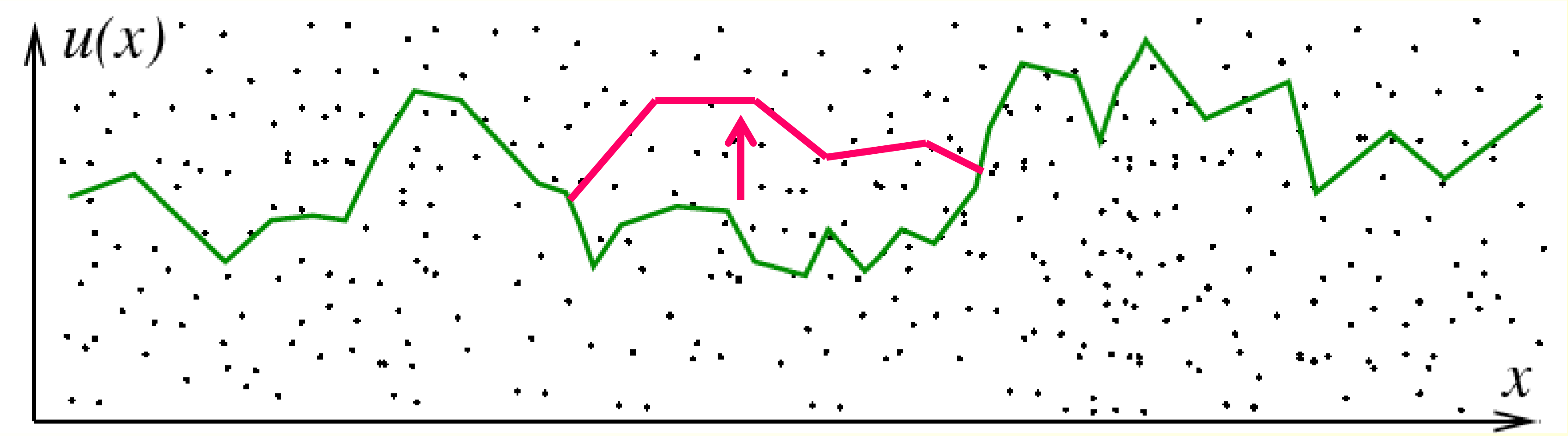
Liquid fronts, domain walls, charge density waves, vortex lattices:



Examples of crackling I

- Depinning of contact lines, interfaces and other elastic objects

Liquid fronts, domain walls, charge density waves, vortex lattices:



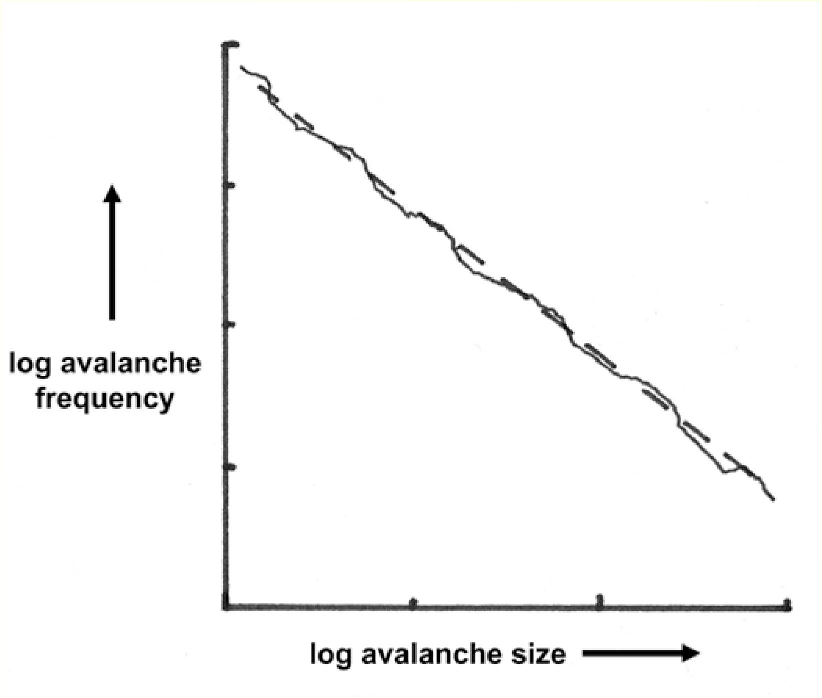
Theoretical approach: functional RG [D. Fisher, Balents, LeDoussal+Wiese, etc]

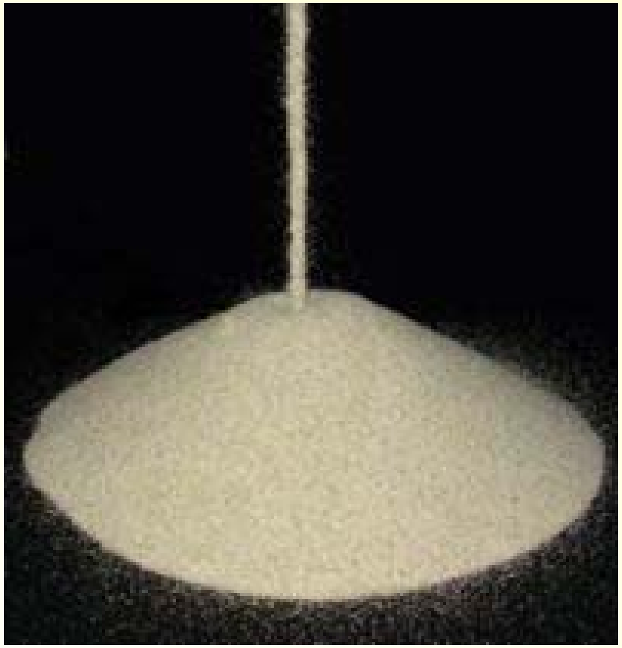
Statistics of avalanches: non-trivial scale-free power laws

Examples of crackling II

• Power laws due to self-organized criticality:
Dynamics is attracted to a critical state, without fine-tuning of parameters

Example: sandpile model by Bak, Tang, and Wiesenfeld





Magnetic systems

- Crackling noise in the hysteresis loop: “Barkhausen noise”
- When does crackling occur in random magnets, and why?

Magnetic systems

- Crackling noise in the hysteresis loop: “Barkhausen noise”

- When does crackling occur in random magnets, and why?

This talk:
Equilibrium avalanches in hysteresis reflect criticality of
glassy magnetic phases!

Experimental proposal:
Barkhausen noise as a diagnostic of glasses!

Avalanches in ferromagnetic films

Direct Observation of Barkhausen Avalanche in (ferro) Co Thin Films

Kim, Choe, and Shin (PRL 2003)

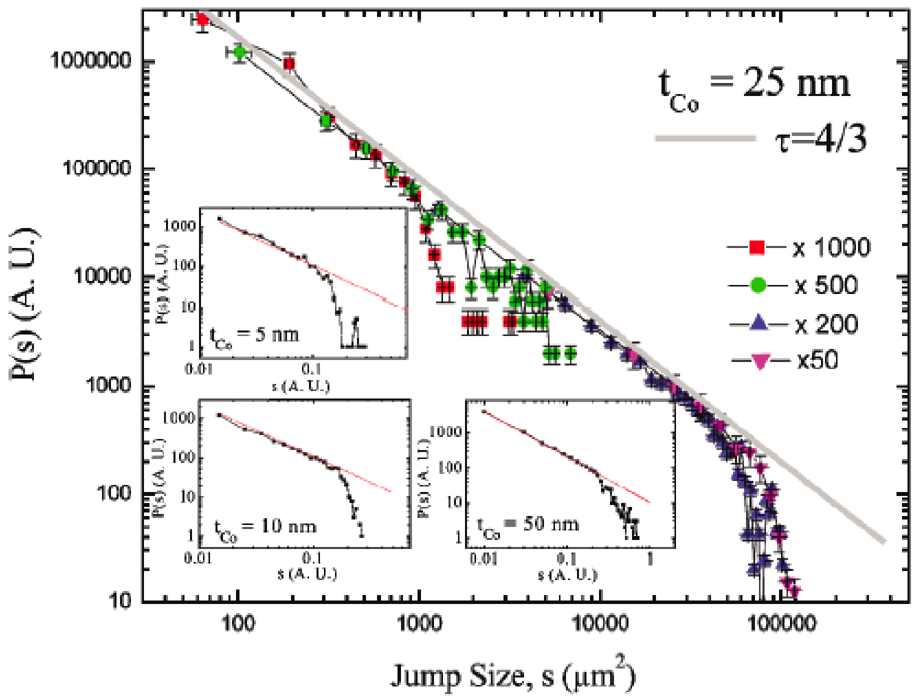


FIG. 3 (color). Distributions of the Barkhausen jump size in 25 and 50-nm Co samples. Distributions in 5, 10, and 50-nm Co samples are shown in the insets. Fitting curve with $\tau = 1.33$ is denoted at each graph.

Distribution of magnetization jumps

$$P(s) = \frac{A}{s^{\tau}}$$

$$\tau = \frac{4}{3}$$

Cizeau et al.:
Theoretical model with **dipolar long range interactions**
(crucial to get criticality)

Model ferromagnets

Dahmen, Sethna
Vives, Planes

Random field Ising model (short range):

$$H = -J\sum_{<ij>} s_i s_j - \sum_i h_i s_i - h_{ext}\sum_i s_i$$

- Generically non-critical
- Scale free avalanches require fine tuning of disorder $\Delta = \langle h_i^2 \rangle$ *and* field $h_{ext,crit}$

Reason: not enough frustration, no glassy phase!

→ Look at spin glasses

Mean field spin glass

VOLUME 83, NUMBER 5 PHYSICAL REVIEW LETTERS 2 AUGUST 1999

Self-Organized Criticality in the Hysteresis of the Sherrington-Kirkpatrick Model

Ferenc Pázmándi,[1,2,3] Gergely Zaránd,[1,3] and Gergely T. Zimányi[1]

Canonical spin glass model: Sherrington-Kirkpatrick (SK) model - fully connected

$$H = \frac{1}{2}\sum_{i,j} J_{ij} s_i s_j - h_{ext}\sum_i s_i, \qquad J_{ij} : \text{ random Gaussian } \overline{J_{ij}^2} = J^2/N$$

- Mean field version of the Edwards-Anderson model in finite dimensions

Mean field spin glass

VOLUME 83, NUMBER 5 PHYSICAL REVIEW LETTERS 2 AUGUST 1999

Self-Organized Criticality in the Hysteresis of the Sherrington-Kirkpatrick Model

Ferenc Pázmándi,[1,2,3] Gergely Zaránd,[1,3] and Gergely T. Zimányi[1]

Canonical spin glass model: Sherrington-Kirkpatrick (SK) model - fully connected

$$H = \frac{1}{2}\sum_{i,j} J_{ij} s_i s_j - h_{ext}\sum_i s_i, \qquad J_{ij} : \text{ random Gaussian } \overline{J_{ij}^2} = J^2/N$$

- Mean field version of the Edwards-Anderson model in finite dimensions
- Known facts:

- Thermodynamic transition at T_c to glass phase:
- $M_{tot} = 0$, despite of broken Ising symmetry: $\langle s_i \rangle \neq 0$,
- Order parameter $\quad Q_{EA} = \frac{1}{N}\sum_i \langle s_i \rangle^2$

Mean field spin glass

VOLUME 83, NUMBER 5 PHYSICAL REVIEW LETTERS 2 AUGUST 1999

Self-Organized Criticality in the Hysteresis of the Sherrington-Kirkpatrick Model

Ferenc Pázmándi,[1,2,3] Gergely Zaránd,[1,3] and Gergely T. Zimányi[1]

Canonical spin glass model: Sherrington-Kirkpatrick (SK) model - fully connected

$$H = \frac{1}{2}\sum_{i,j} J_{ij} s_i s_j - h_{ext}\sum_i s_i, \qquad J_{ij} : \text{ random Gaussian } \overline{J_{ij}^2} = J^2/N$$

- Mean field version of the Edwards-Anderson model in finite dimensions
- Known facts:

- Thermodynamic transition at T_c to glass phase:
- $M_{tot} = 0$, despite of broken Ising symmetry: $\langle s_i \rangle \neq 0$,
- Order parameter $Q_{EA} = \frac{1}{N}\sum_i \langle s_i \rangle^2$
- Many metastable states

Mean field spin glass

VOLUME 83, NUMBER 5 PHYSICAL REVIEW LETTERS 2 AUGUST 1999

Self-Organized Criticality in the Hysteresis of the Sherrington-Kirkpatrick Model

Ferenc Pázmándi,[1,2,3] Gergely Zaránd,[1,3] and Gergely T. Zimányi[1]

Canonical spin glass model: Sherrington-Kirkpatrick (SK) model - fully connected

$$H = \frac{1}{2}\sum_{i,j} J_{ij} s_i s_j - h_{ext}\sum_i s_i, \qquad J_{ij} : \text{ random Gaussian } \overline{J_{ij}^2} = J^2/N$$

- Mean field version of the Edwards-Anderson model in finite dimensions
- Known facts:

- Thermodynamic transition at T_c to glass phase:
- $M_{tot} = 0$, despite of broken Ising symmetry: $\langle s_i \rangle \neq 0$,
- Order parameter $Q_{EA} = \frac{1}{N}\sum_i \langle s_i \rangle^2$
- Many metastable states

Glass phase is always [self-organized] critical! *(SK: Kondor-DeDominicis)*
Power law correlations also in the droplet model! *(Fisher-Huse)*

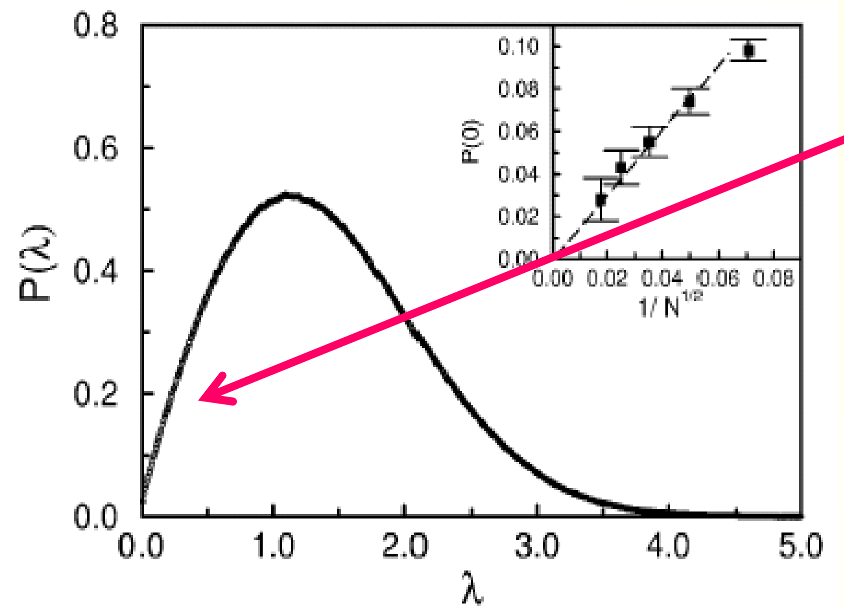
SK criticality – local fields

$$H = \frac{1}{2}\sum_{i,j} J_{ij} s_i s_j - h_{ext} \sum_i s_i$$

Thouless, Anderson and Palmer (1977)
Palmer and Pond (1979)
Parisi (1979)
Bray, Moore (1980)
Sommers and Dupont (1984)
Dobrosavljevic, Pastor (1999)
Pazmandi, Zarand, Zimanyi (1999)
MM, Pankov (2007)

Local field on spin *i*:

$$\lambda_i \equiv -\frac{\partial H}{\partial s_i} = -\sum_{j \neq i} J_{ij} s_j + h_{ext}$$



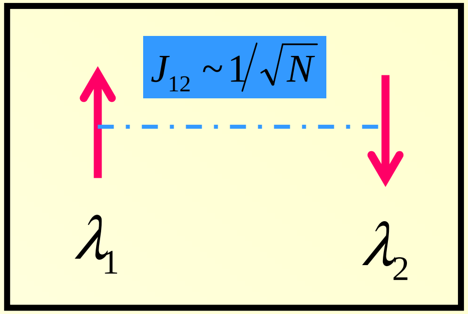
Linear "Coulomb" gap in the distribution of local fields (analogous to Efros-Shklovskii Coulomb gap, 1975)

A first indication of criticality!

The linear pseudogap in SK

Thouless (1977)

Stability of ground state with respect to flipping of a pair:



The distribution of local fields must vanish at λ =0 at T = 0!

- Suppose pseudogap $P(\lambda) \propto \lambda^{\gamma}$

$\rightarrow$ Smallest local fields $\lambda_{\min} \propto N^{-1/1+\gamma}$

- 2-spin flip cost $E_{\cos t} \propto |\lambda_1| + |\lambda_2| - N^{-1/2} \sim N^{-1/1+\gamma} - N^{-1/2} \overset{!}{>} 0$

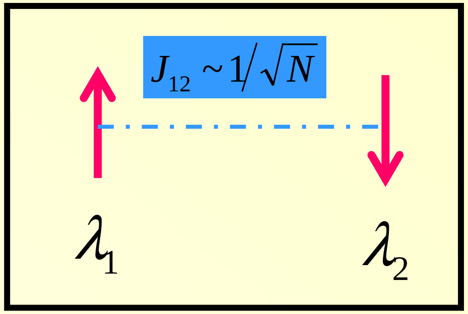
$\gamma \geq 1 \rightarrow$ At least linear pseudogap!

The linear pseudogap in SK

Thouless (1977)

Stability of ground state with respect to flipping of a pair:

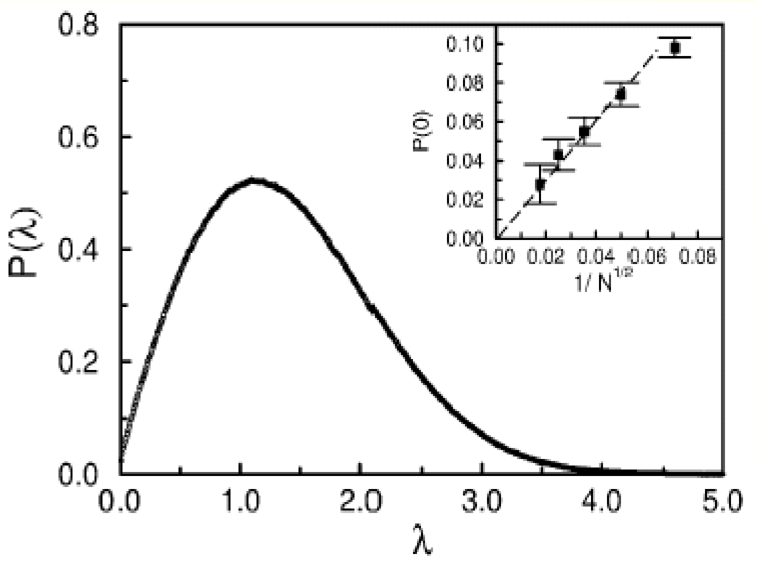
The distribution of local fields must vanish at λ =0 at T = 0!



- Suppose pseudogap $P(\lambda) \propto \lambda^{\gamma}$

→ Smallest local fields $\lambda_{min} \propto N^{-1/1+\gamma}$

- 2-spin flip cost $E_{cost} \propto |\lambda_1| + |\lambda_2| - N^{-1/2} \sim N^{-1/1+\gamma} - N^{-1/2} \overset{!}{>} 0$



$\gamma \geq 1$ → At least linear pseudogap!

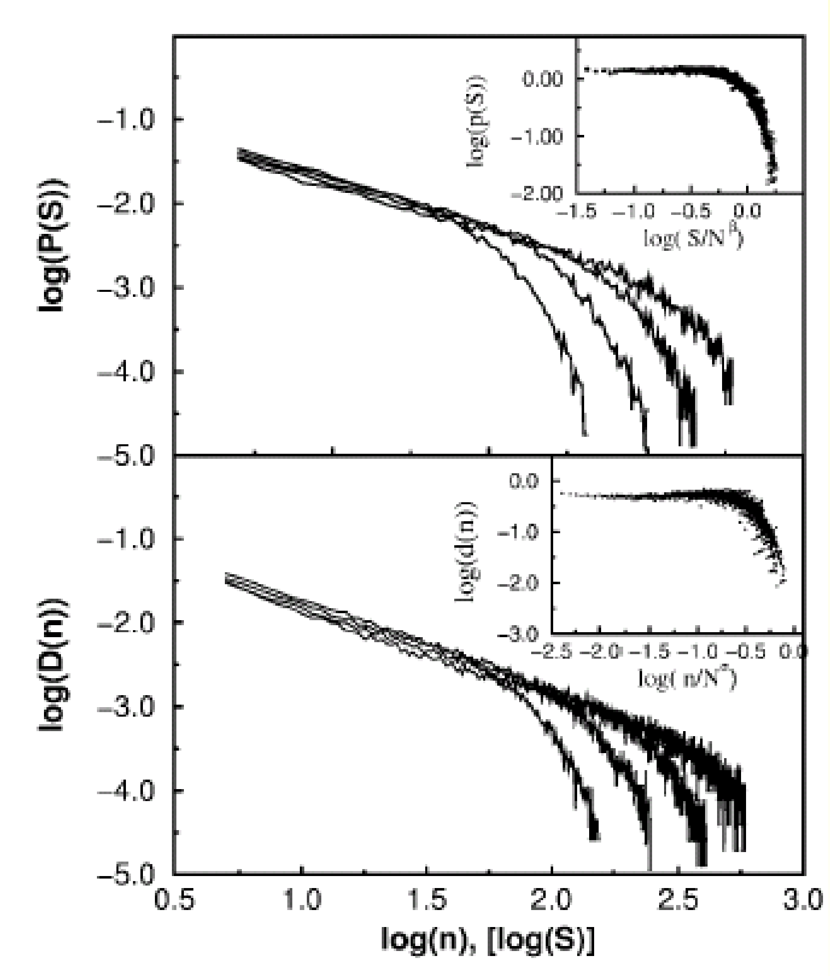
- But: γ = 1! → marginally stable!
  Largest possible density of soft spins!

Distribution is critical: flipping one spin by an increase of $\Delta h_{ext} = \lambda_{min}$ can trigger large avalanche!

"Living on the edge"

Pazmandi, Zarand, Zimanyi (1999)

Numerical analysis of hysteresis in the SK model



Size distribution of avalanches:

- Avalanches are large:
  Only cutoff : system size N

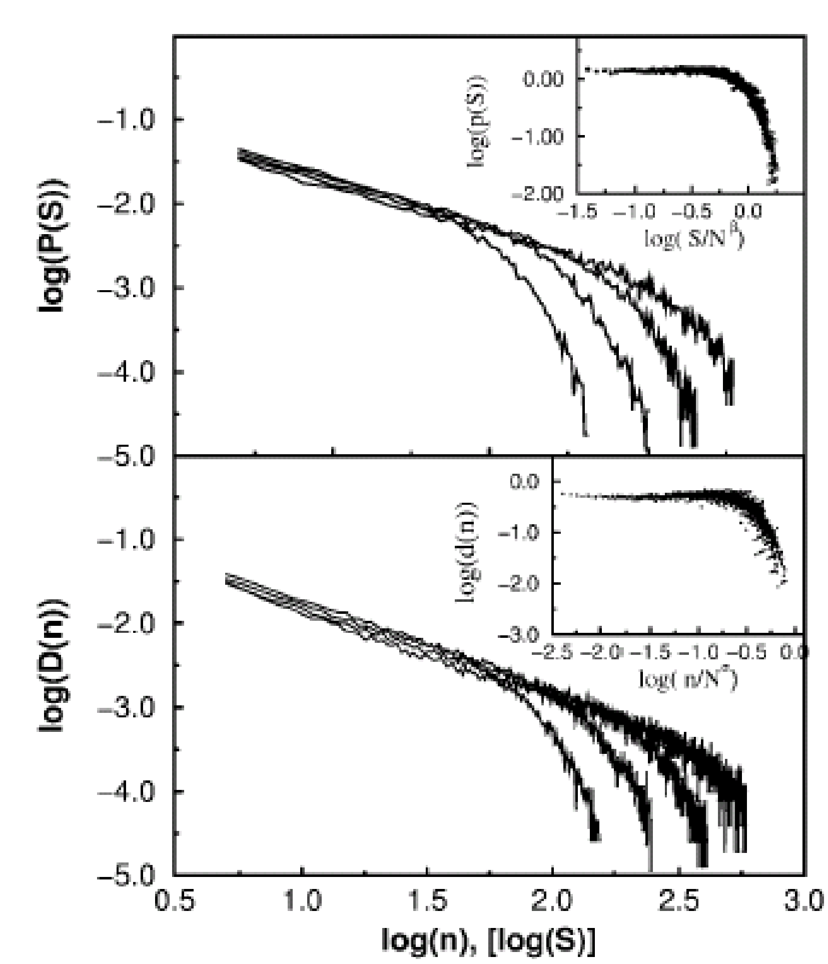
  $S = \Delta M \sim N^{1/2}$
  and
  $N_{flip} \sim N$ [!]

- Power laws:
  Indication of self-organized criticality

“Living on the edge”

Pazmandi, Zarand, Zimanyi (1999)

Numerical analysis of hysteresis in the SK model



Size distribution of avalanches:

- Avalanches are large:
  Only cutoff : system size N

$S = \Delta M \sim N^{1/2}$

and

$N_{flip} \sim N$ [!]

- Power laws:
  Indication of self-organized criticality
- Nearly random up and down flips!
- Typical spins flip ~ $N^{1/2}$ times back and forth during a hysteresis loop!

Theory??

Criticality of the SK model

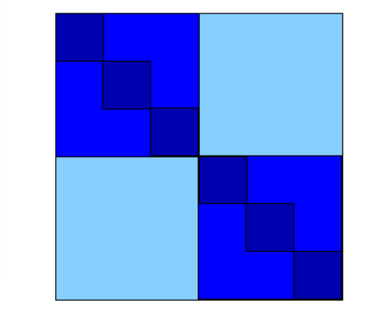
SK-model $$H = \sum_{i<j} J_{ij} s_i s_j$$

Replica trick: $$F = \underset{Q}{\text{ext}} \left[F\{Q_{ab}\} \right] \qquad Q_{ab} = \frac{1}{N} \sum_i s_i^a s_i^b$$

Parisi ansatz for the saddle point:
Hierarchical replica symmetry breaking

Parisi (1979)

$Q_{ab} =$



Criticality of the SK model

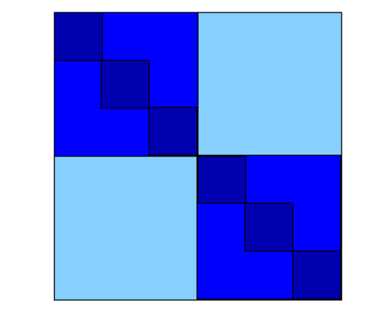
SK-model $$H = \sum_{i<j} J_{ij} s_i s_j$$

Replica trick: $$F = \operatorname{ext}_Q \left[F\{Q_{ab}\} \right] \qquad Q_{ab} = \frac{1}{N} \sum_i s_i^a s_i^b$$

Parisi ansatz for the saddle point:
Hierarchical replica symmetry breaking

Parisi (1979)

$$Q_{ab} =$$



Criticality of the glass: Zero modes of stability matrix $$\frac{\partial^2 F}{\partial Q_{ab} \partial Q_{cd}}$$

- Critical spin-spin correlations in the whole glass phase!
  Numerically also found in finite dimensions
  (also in the droplet model)!

- Criticality is directly related to the linear pseudogap in *P(h)!*
(Sommers-Dupont, Pankov)

Avalanches?

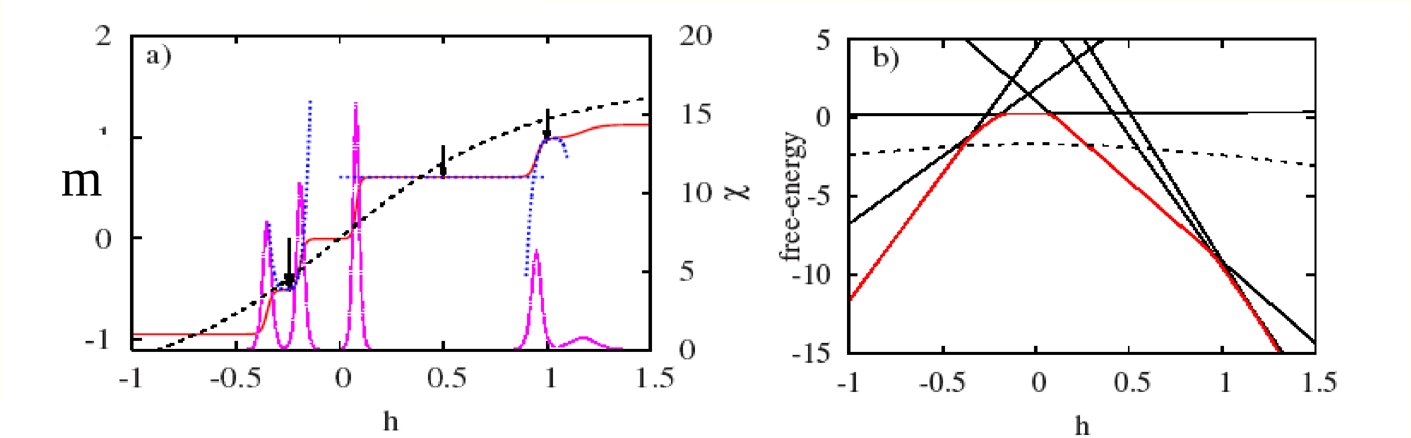
- Understand shocks in spin glasses
- Calculate equilibrium avalanche distribution analytically
- $\rightarrow$ Power law - a consequence of thermodynamic criticality

Stepwise response and shocks in spin glass models

Young, Kirkpatrick 1982, Krzakala, Martin (2003)

Free energy of metastable state α: $F_\alpha(h) = F_\alpha(h=0) - hM_\alpha$

Equilibrium jump/shock when two states cross: $F_\alpha(h_{shock}) = F_\beta(h_{shock})$



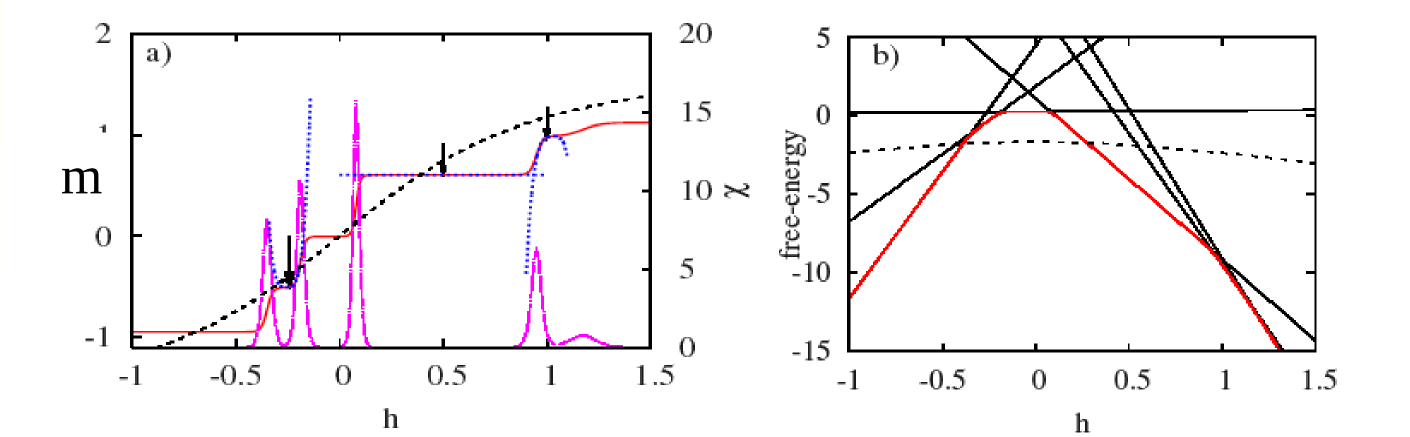
Mesoscopic effect: Susceptibility has spikes and does **not** self-average!

Stepwise response and shocks in spin glass models

Young, Kirkpatrick 1982, Krzakala, Martin (2003)

Free energy of metastable state α: $F_\alpha(h) = F_\alpha(h=0) - hM_\alpha$

Equilibrium jump/shock when two states cross: $F_\alpha(h_{shock}) = F_\beta(h_{shock})$



Mesoscopic effect: Susceptibility has spikes and does **not** self-average!

Yoshino, Rizzo (2008)

First steps of theory in p-spin models

[physics similar as in supercooled liquids]

→ Glassy, but much simpler than SK and non-critical

How to detect avalanches

2^{nd} cumulant of the magnetization ($T = 0$) *Yoshino, Rizzo (2008)*

$$\overline{M(h+\delta h)M(h-\delta h) - M(h)^2} \propto |\delta h|$$

Non-analytic cusp!

Reflects the probability of shocks.

How to detect avalanches

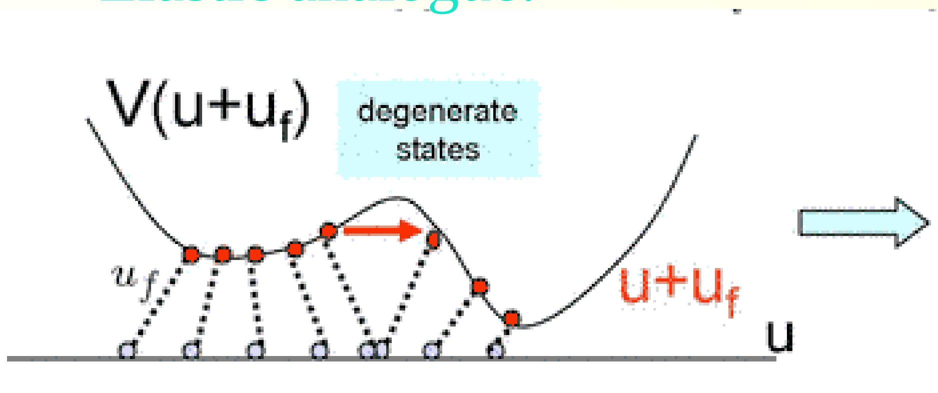
2nd cumulant of the magnetization ($T = 0$) *Yoshino, Rizzo (2008)*

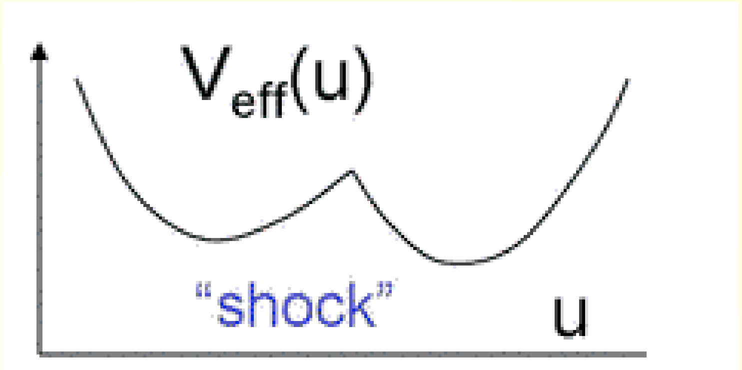
$$\overline{M(h+\delta h)M(h-\delta h) - M(h)^2} \propto |\delta h|$$

Non-analytic cusp!

Reflects the probability of shocks.

Elastic analogue:





For experts: Shocks are direct analogs of the cusp in the FRG beyond the collective pinning scale

Larkin, Fisher
LeDoussal, Wiese
Balents, Bouchaud, Mézard
LeDoussal, MM, Wiese

How to obtain shocks and their distribution for the SK model?

Strategy of calculation

k[th] cumulant of magnetization difference

$$\overline{[M(h) - M(h+\delta h)]^k} = \mathrm{Prob}(\mathrm{shock} \in [h, h+\delta h])\, \overline{\Delta M^k_{\mathrm{shock}}}^{h} + O\left(\delta h^2\right)$$

Shock density

$$\mathrm{Prob}(\mathrm{shock} \in [h, h+\delta h]) = \rho_0 |\delta h|$$

Avalanche size cumulants

$$\overline{\Delta M^k_{\mathrm{shock}}}^{h} = \int_0^\infty d\,\Delta M\, P(\Delta M; h)\, \Delta M^k$$

Strategy of calculation

k[th] cumulant of magnetization difference

$$\overline{[M(h) - M(h+\delta h)]^k} = \mathrm{Prob}(\mathrm{shock} \in [h, h+\delta h])\, \overline{\Delta M^k_{\mathrm{shock}}}^{\,h} + O\left(\delta h^2\right)$$

Shock density

$$\mathrm{Prob}(\mathrm{shock} \in [h, h+\delta h]) = \quad |\delta h|$$

Avalanche size cumulants

$$\overline{\Delta M^k_{\mathrm{shock}}}^{\,h} = \int_0^\infty d\,\Delta M \quad \Delta M^k$$

Calculate $\overline{[M(h) - M(h+\delta h)]^k} \quad \rightarrow \quad ,$

Strategy of calculation

k^th cumulant of magnetization difference

$$\overline{[M(h)-M(h+\delta h)]^k} = \text{Prob}(\text{shock} \in [h, h+\delta h])\, \overline{\Delta M^k_{\text{shock}}}^{\,h} + O\left(\delta h^2\right)$$

Shock density

$$\text{Prob}(\text{shock} \in [h, h+\delta h]) = \quad |\delta h|$$

Avalanche size cumulants

$$\overline{\Delta M^k_{\text{shock}}}^{\,h} = \int_0^\infty d\,\Delta M \qquad \Delta M^k$$

Calculate $\overline{[M(h)-M(h+\delta h)]^k} \quad \rightarrow \quad ,$

Natural scales:

$\delta h \sim \lambda_{\min} \sim N^{-1/2}$ Distance between shocks

$\Delta M \sim \chi N \Delta h \sim N^{1/2}$ Magnetization jumps

Strategy of calculation

- Calculate $\overline{[M(h) - M(h+\delta h)]^k} \quad \rightarrow$,

$$\overline{M(h_1)\dots M(h_k)} = (-1)^k \partial_{h_1} \dots \partial_{h_k} \overline{F(h_1)\dots F(h_k)}$$

- Calculate effective potential of n replicas:

Strategy of calculation

→ Calculate $\overline{[M(h) - M(h+\delta h)]^k}$ $\rightarrow$,

$$\overline{M(h_1)\dots M(h_k)} = (-1)^k \partial_{h_1}\dots\partial_{h_k}\overline{F(h_1)\dots F(h_k)}$$

→ Calculate effective potential of n replicas:

Easy to extract in the replica limit $n \rightarrow 0$

$$\exp\left[W[\{h_a\}]\right] := \overline{\exp\left[-\beta\sum_{a=1}^{n}F(h_a)\right]}^J$$

$$= \exp\left[-\beta\sum_{a=1}^{n}\overline{F(h_a)}^J + \frac{\beta^2}{2}\sum_{a,b=1}^{n}\overline{F(h_a)F(h_b)}^{J,c} - \frac{\beta^3}{3!}\sum_{a,b,c=1}^{n}\overline{F(h_a)F(h_b)F(h_c)}^{J,c} + \dots\right]$$

$$= \sum_{\{S_a^i\}}\int\prod_{a\neq b}\mathrm{d}Q_{ab}\prod_i\exp\left[nN\frac{\beta^2J^2}{2} + \beta^2J^2\sum_{a\neq b}\left(-\frac{N}{2}Q_{ab}^2 + Q_{ab}S_a^iS_b^i\right) + \sum_a\beta h_aS_a^i\right].$$

… … … …

→ Extract non-analytic part ~|dh| in the limit *T* → *0*

Strategy of calculation

→ Calculate $\overline{[M(h)-M(h+\delta h)]^k}$ $\rightarrow$,

$$\overline{M(h_1)\dots M(h_k)} = (-1)^k \partial_{h_1}\dots\partial_{h_k}\overline{F(h_1)\dots F(h_k)}$$

→ Calculate effective potential of n replicas:

Easy to extract in the replica limit $n \to 0$

$$\exp\left[W[\{h_a\}]\right] := \overline{\exp\left[-\beta\sum_{a=1}^{n}F(h_a)\right]}^J$$

$$= \exp\left[-\beta\sum_{a=1}^{n}\overline{F(h_a)}^J + \frac{\beta^2}{2}\sum_{a,b=1}^{n}\overline{F(h_a)F(h_b)}^{J,c} - \frac{\beta^3}{3!}\sum_{a,b,c=1}^{n}\overline{F(h_a)F(h_b)F(h_c)}^{J,c} + \dots\right]$$

$$= \sum_{\{S_a^i\}}\int\prod_{a\neq b}\mathrm{d}Q_{ab}\prod_i \exp\left[nN\frac{\beta^2J^2}{2} + \beta^2J^2\sum_{a\neq b}\left(-\frac{N}{2}Q_{ab}^2 + Q_{ab}S_a^iS_b^i\right) + \sum_a \beta h_a S_a^i\right].$$

… … … …

→ Extract non-analytic part ~|dh| in the limit $T \to 0$

Final result: (for any mean field glass)

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \Delta m\int_{q(0)}^{q(u_c)} dq\frac{d\hat{u}(q)}{dq}\frac{\exp[-\frac{(\Delta m)^2}{4(q(u_c)-q)}]}{\sqrt{4\pi(q(u_c)-q)}}\, d(\Delta m)d\tilde{h}.$$

Equilibrium saddle point $T^{-1}/(dQ/du)$

Calculation

Result for SK spin glass

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \Delta m \int_{q(0)}^{q(u_c)} dq \frac{d\hat{u}(q)}{dq} \frac{\exp[-\frac{(\Delta m)^2}{4(q(u_c)-q)}]}{\sqrt{4\pi(q(u_c)-q)}}\, d(\Delta m) d\tilde{h}.$$

$$= \Delta m \int_{C\tilde{h}^{2/3}}^{1} dq \frac{\sqrt{c^*}}{2(1-q)^{3/2}} \frac{\exp[-\frac{(\Delta m)^2}{4(1-q)}]}{\sqrt{4\pi(1-q)}}\, d(\Delta m) d\tilde{h}$$

$$= \frac{\sqrt{c^*/\pi}}{\Delta m} \exp[-(\Delta m)^2/4(1 - C\tilde{h}^{2/3})]$$

Zero T solution of the SK model, and its marginal stability!

Avalanche exponent

$$\tau = 1$$

Calculation

Result for SK spin glass

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \Delta m \int_{q(0)}^{q(u_c)} dq \frac{d\hat{u}(q)}{dq} \frac{\exp[-\frac{(\Delta m)^2}{4(q(u_c)-q)}]}{\sqrt{4\pi(q(u_c)-q)}}\, d(\Delta m) d\tilde{h}.$$

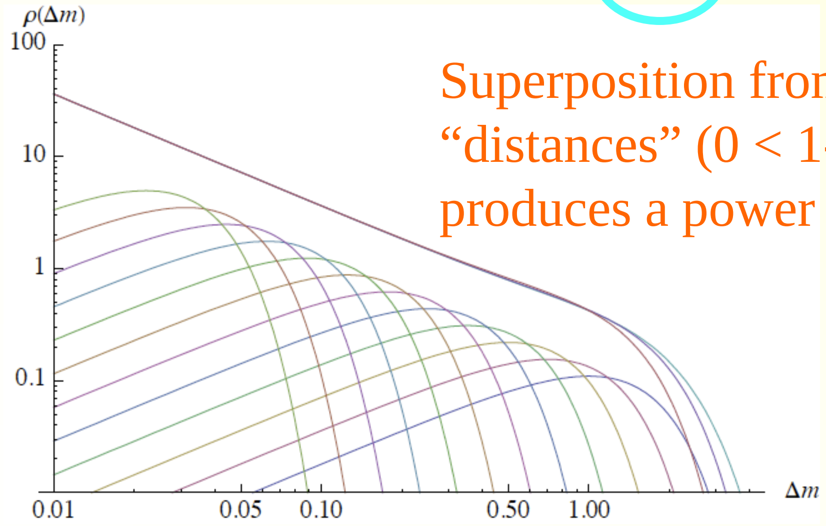
$$= \Delta m \int_{C\tilde{h}^{2/3}}^{1} dq \frac{\sqrt{c^*}}{2(1-q)^{3/2}} \frac{\exp[-\frac{(\Delta m)^2}{4(1-q)}]}{\sqrt{4\pi(1-q)}}\, d(\Delta m) d\tilde{h}$$

$$= \frac{\sqrt{c^*/\pi}}{\Delta m} \exp[-(\Delta m)^2/4(1-C\tilde{h}^{2/3})]$$

Zero T solution of the SK model, and its marginal stability!

Avalanche exponent

$\tau = 1$



Superposition from all "distances" (0 < 1-q <1) produces a power law!

Overlap: $q_{12} = \frac{1}{N} \sum_i s_i^1 s_i^2$

Calculation

Result for SK spin glass

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \Delta m \int_{q(0)}^{q(u_c)} dq \frac{d\hat{u}(q)}{dq} \frac{\exp[-\frac{(\Delta m)^2}{4(q(u_c)-q)}]}{\sqrt{4\pi(q(u_c)-q)}} d(\Delta m) d\tilde{h}.$$

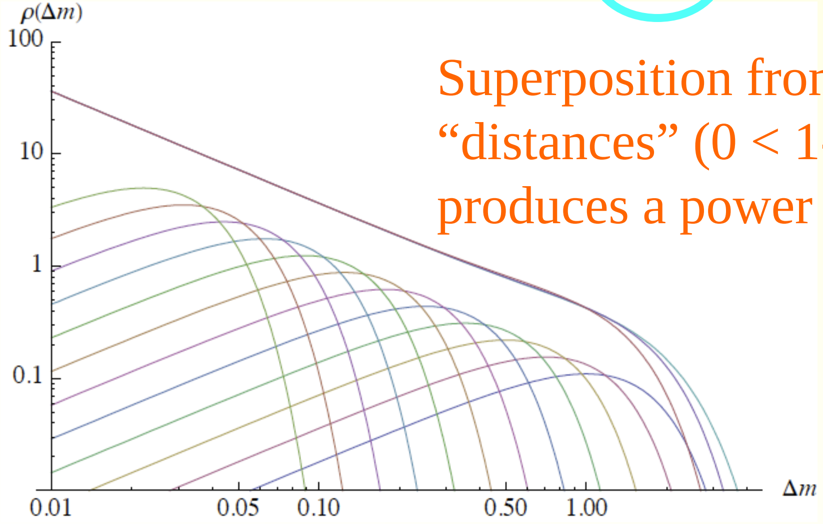
$$= \Delta m \int_{C\tilde{h}^{2/3}}^{1} dq \frac{\sqrt{c^*}}{2(1-q)^{3/2}} \frac{\exp[-\frac{(\Delta m)^2}{4(1-q)}]}{\sqrt{4\pi(1-q)}} d(\Delta m) d\tilde{h}$$

$$= \frac{\sqrt{c^*/\pi}}{\Delta m} \exp[-(\Delta m)^2/4(1-C\tilde{h}^{2/3})]$$

Zero T solution of the SK model, and its marginal stability!

Avalanche exponent

$\tau = 1$



Superposition from all "distances" ($0 < 1-q < 1$) produces a power law!

Overlap: $q_{12} = \frac{1}{N} \sum_i s_i^1 s_i^2$

- Mesoscopic avalanches ~ $N^{1/2}$ fully confirmed
- Critical probability distribution of avalanche sizes

Calculation

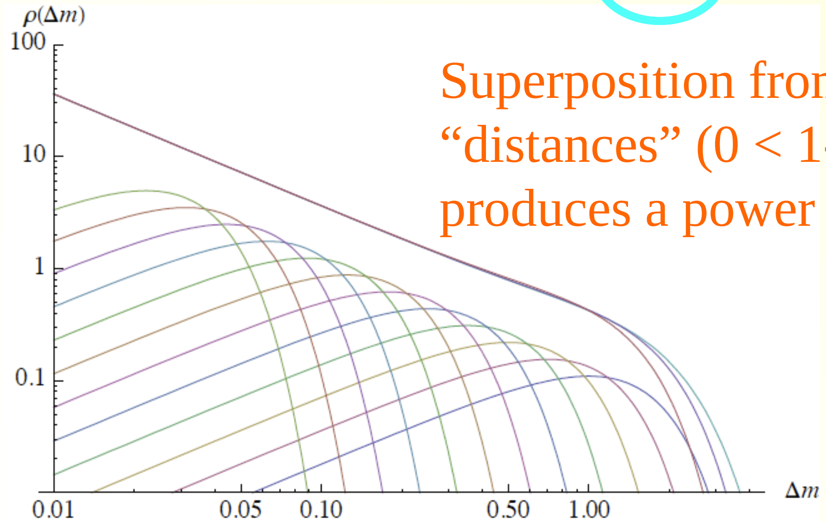
Result for SK spin glass

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \Delta m \int_{q(0)}^{q(u_c)} dq \frac{d\hat{u}(q)}{dq} \frac{\exp[-\frac{(\Delta m)^2}{4(q(u_c)-q)}]}{\sqrt{4\pi(q(u_c)-q)}}\, d(\Delta m) d\tilde{h}.$$

$$= \Delta m \int_{C\tilde{h}^{2/3}}^{1} dq \frac{\sqrt{c^*}}{2(1-q)^{3/2}} \frac{\exp[-\frac{(\Delta m)^2}{4(1-q)}]}{\sqrt{4\pi(1-q)}}\, d(\Delta m) d\tilde{h}$$

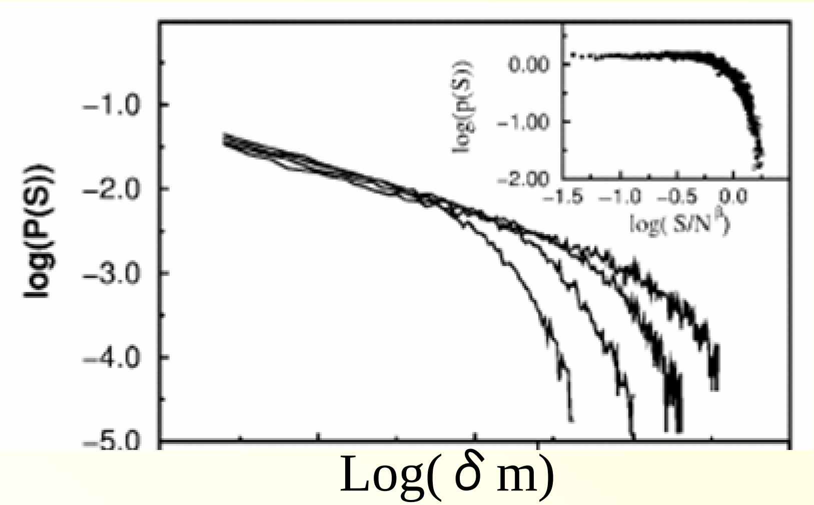
$$= \frac{\sqrt{c^*/\pi}}{\Delta m} \exp[-(\Delta m)^2/4(1-C\tilde{h}^{2/3})]$$

Zero T solution of the SK model, and its marginal stability!



Superposition from all "distances" ($0 < 1-q < 1$) produces a power law!

↔ Avalanches in the hysteresis loop (slowly driven, out-of-equilibrium)



Pazmandi, Zarand, Zimanyi (1999)

A posteriori: a simple derivation !

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \Delta m \int_{q(0)}^{q(u_c)} dq \frac{d\hat{u}(q)}{dq} \frac{\exp[-\frac{(\Delta m)^2}{4(q(u_c)-q)}]}{\sqrt{4\pi(q(u_c)-q)}}\, d(\Delta m) d\tilde{h}.$$

A heuristic derivation/interpretation – a posteriori

A posteriori: a simple derivation !

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \Delta m \int_{q(0)}^{q(u_c)} dq \frac{d\hat{u}(q)}{dq} \frac{\exp[-\frac{(\Delta m)^2}{4(q(u_c)-q)}]}{\sqrt{4\pi(q(u_c)-q)}}\, d(\Delta m) d\tilde{h}.$$

A heuristic derivation/interpretation – a posteriori

Density of states at distance 1-q $\rho(E=0,q) = \frac{1}{T} P(q) = \frac{1}{T}\frac{du}{dq} \equiv \frac{d\hat{u}}{dq}$

Relation between jump in q and M $N_{\text{flip}} = N(1-q)/2$

$$\overline{\Delta m^2} = \overline{\Delta M^2}/N = 4N_{\text{flip}}/N = 2(1-q)$$

Shock location: $\Delta \tilde{h} = \sqrt{N}\Delta h = E/\Delta M$

A posteriori: a simple derivation !

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \Delta m \int_{q(0)}^{q(u_c)} dq \frac{d\hat{u}(q)}{dq} \frac{\exp[-\frac{(\Delta m)^2}{4(q(u_c)-q)}]}{\sqrt{4\pi(q(u_c)-q)}} d(\Delta m) d\tilde{h}.$$

A heuristic derivation/interpretation – a posteriori

Density of states at distance 1-q $\quad \rho(E=0,q) = \frac{1}{T} P(q) = \frac{1}{T}\frac{du}{dq} \equiv \frac{d\hat{u}}{dq}$

Relation between jump in q and M $\quad N_{\text{flip}} = N(1-q)/2$

$$\overline{\Delta m^2} = \overline{\Delta M^2}/N = 4 N_{\text{flip}}/N = 2(1-q)$$

Shock location: $\quad \Delta\tilde{h} = \sqrt{N}\Delta h = E/\Delta M$

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \int_{q_m}^{q_c} dq \int_0^\infty dE \; \cdots \; d(\Delta m) d\tilde{h}$$

A posteriori: a simple derivation !

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \Delta m \int_{q(0)}^{q(u_c)} dq \frac{d\hat{u}(q)}{dq} \frac{\exp[-\frac{(\Delta m)^2}{4(q(u_c)-q)}]}{\sqrt{4\pi(q(u_c)-q)}} d(\Delta m) d\tilde{h}.$$

A heuristic derivation/interpretation – a posteriori

Density of states at distance 1-q $\quad \rho(E=0,q) = \frac{1}{T} P(q) = \frac{1}{T}\frac{du}{dq} \equiv \frac{d\hat{u}}{dq}$

Relation between jump in q and M $\quad N_{\text{flip}} = N(1-q)/2$

$$\overline{\Delta m^2} = \overline{\Delta M^2}/N = 4 N_{\text{flip}}/N = 2(1-q)$$

Shock location: $\quad \Delta\tilde{h} = \sqrt{N}\Delta h = E/\Delta M$

$$\rho(\Delta m)\, d(\Delta m) d\tilde{h} = \int_{q_m}^{q_c} dq \int_0^\infty dE \; [illegible] \; d(\Delta m) d\tilde{h}$$

→ Distribution of jump in m AND number of flipping spins:

$$\rho(\Delta m) d\Delta m d\tilde{h} = \frac{2C}{\sqrt{\pi}} \frac{d\Delta m}{\Delta m} d\tilde{h} \qquad \mathcal{D}(N_{\text{flip}})\, dN_{\text{flip}} d\tilde{h} = \frac{C}{\sqrt{\pi}} \frac{dN_{\text{flip}}}{N_{\text{flip}}} d\tilde{h}$$

Static calculation yields same power laws as out-of-eq. dynamics!

Nature of avalanches

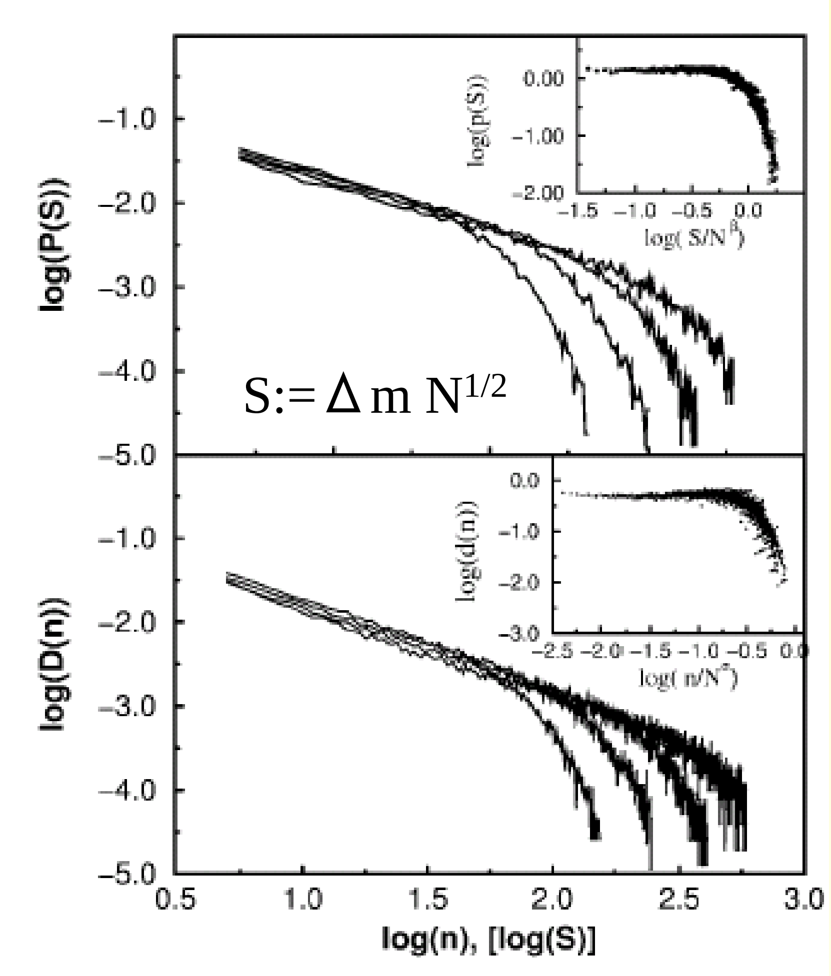
T=0 statics (analytical) ⟷ T=0 dynamics (numerical)

$$\rho(\Delta m) d\Delta m d\tilde{h} = \frac{2C}{\sqrt{\pi}} \frac{d\Delta m}{\Delta m} d\tilde{h}$$

$$\Delta m^{\max} \sim 1 \quad \Delta M^{\max} \sim \sqrt{N}$$

$$\mathcal{D}\left(N_{\text{flip}}\right) dN_{\text{flip}} d\tilde{h} = \frac{C}{\sqrt{\pi}} \frac{dN_{\text{flip}}}{N_{\text{flip}}} d\tilde{h}$$

$$\Delta N_{\text{flip}}^{\max} \sim N$$



Possible reason for similarity:
Statics and dynamics are closely related in marginal glasses, such as SK

Applications and extensions

Finite dimensions

Assuming droplet picture (with critical power law correlations)

Analogous argument as above for droplets in finite dimensions:

$$\rho_{\overline{h}}(\Delta M) \approx \int_1^{L_h} \frac{dL}{L} \int_0^\infty \frac{\nu_0 dE}{L^{d_f+\theta}} \delta\left(\delta h - \frac{E}{\Delta M}\right) P_L(\Delta M)$$

$$= \frac{1}{(\Delta M)^\tau} \frac{\nu_0}{d_m} \int_{\Delta M L_h^{-d_m}}^{\Delta M} dz\, \psi_M(z) z^\tau,$$

→ Power law! With:

Avalanche exponent $\tau = \frac{d_f + \theta}{d_m}$

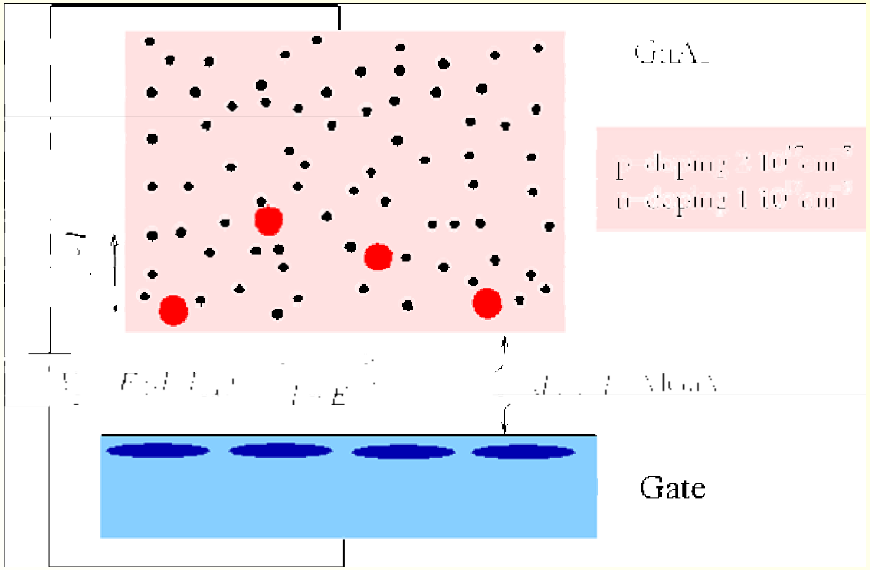
Droplet fractal dimension d_f

Droplet magnetization $\Delta M \sim L^{d_m}$

Droplet energy $\Delta E \sim L^\theta$

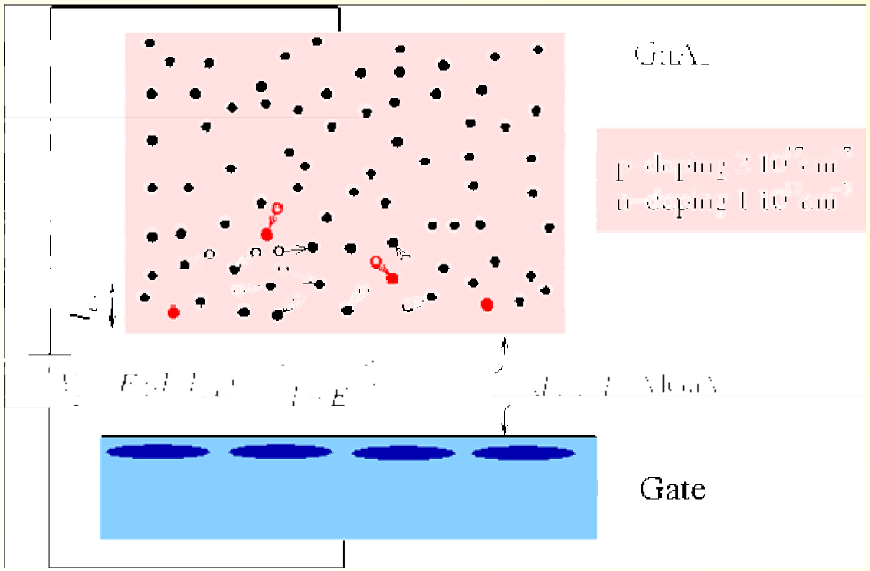
New exponent relation!

Avalanches in the classical Coulomb glass



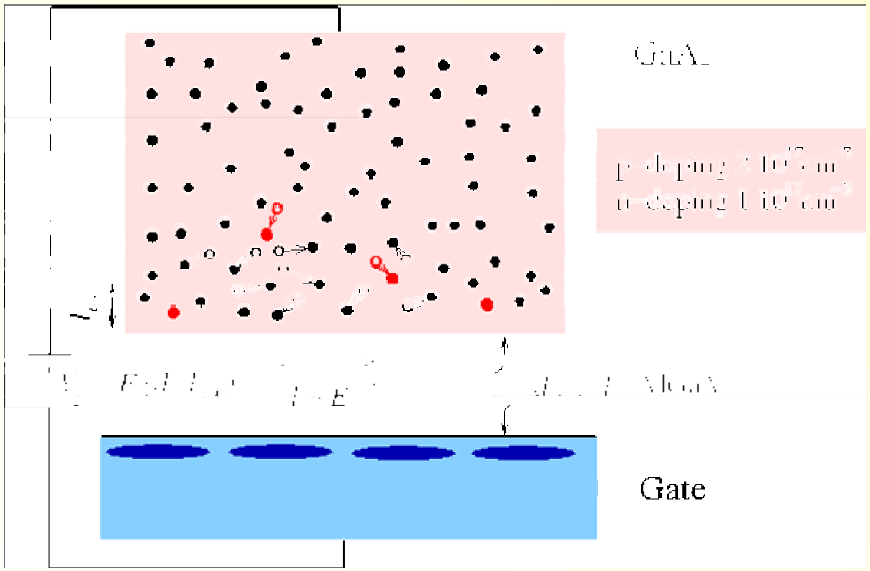
$$H = \frac{1}{2}\sum_{i \neq j} n_i \frac{e^2}{r_{ij}} n_j + \sum_i n_i \varepsilon_i$$

Avalanches in the classical Coulomb glass



$$H = \frac{1}{2}\sum_{i \neq j} n_i \frac{e^2}{r_{ij}} n_j + \sum_i n_i \varepsilon_i$$

Avalanches in the classical Coulomb glass



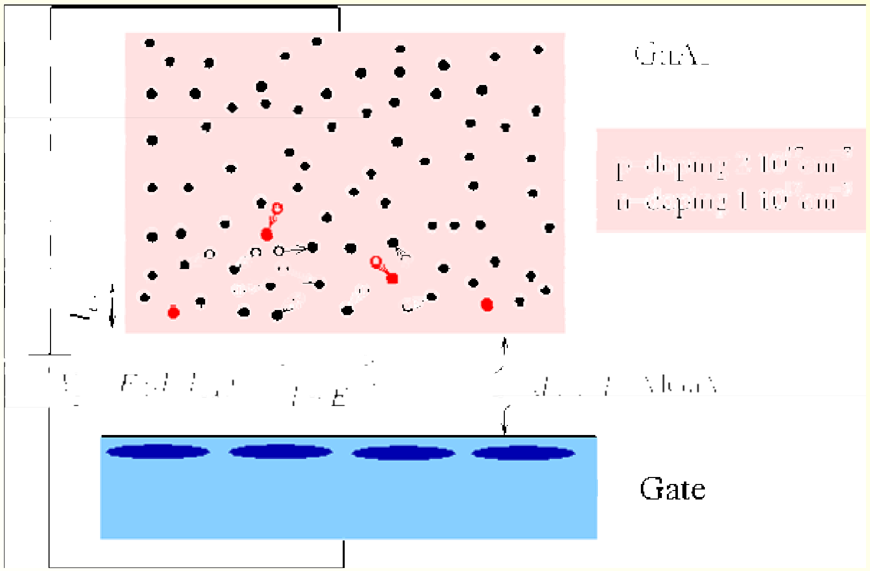
$$H = \frac{1}{2}\sum_{i \neq j} n_i \frac{e^2}{r_{ij}} n_j + \sum_i n_i \varepsilon_i$$

Add one particle on a given site: n_i: 0 → 1

→ Trigger avalanche of “non-linear screening events”

At T=0: no screening → easy to show: at least O(L) induced jumps

Avalanches in the classical Coulomb glass



$$H = \frac{1}{2}\sum_{i\neq j} n_i \frac{e^2}{r_{ij}} n_j + \sum_i n_i \varepsilon_i$$

Add one particle on a given site: n_i: 0 → 1

→ Trigger avalanche of “non-linear screening events”

At T=0: no screening → easy to show: at least O(L) induced jumps

Locator approximation predicts critical glass state

Müller, Ioffe 04
Pankov, Dobrosavljevic 04
Müller, Pankov 07

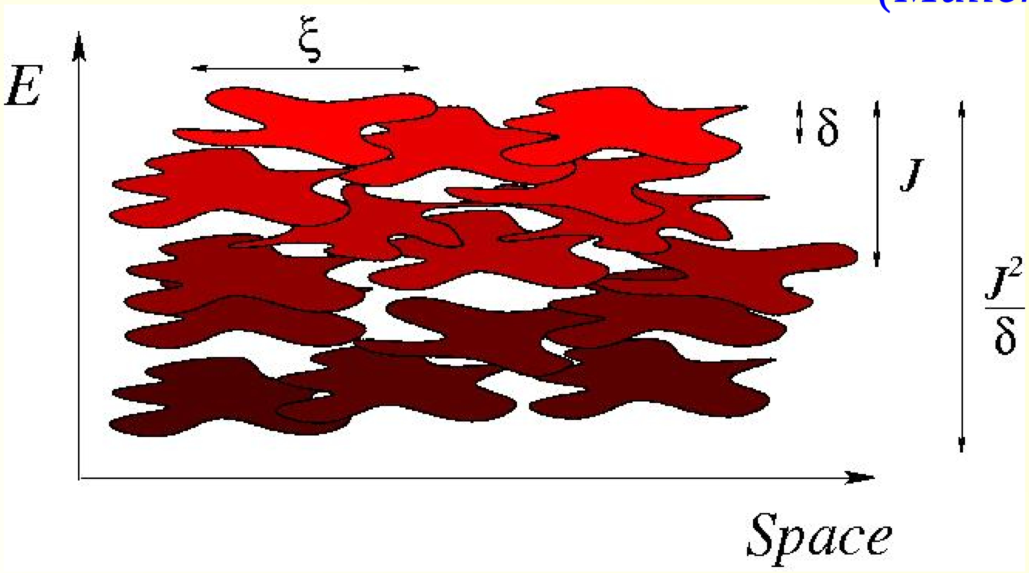
→ Expect power law distribution with cutoff ~ min(1/T, L)

Physical realization of the (quantum) SK model in quantum critical electron glasses?!

(Müller, Ioffe 07)

Quantum electron glasses: close to metal-insulator-criticality

(Müller, Ioffe 07)



$$\delta \equiv \delta_\xi$$

$$J \equiv \frac{e^2}{\kappa\xi}$$

$$J \; ? \;\; \delta$$

Electrons in localization volume behave like a quantum SK model

Adding a charge → avalanches (polarons): affect transport and relaxations.
Static shocks: Rounding of shocks by tunneling!
→ Extract transition rates, avoided level crossing, etc etc…

First step: full solution of quantum SK *(Andreanov, Müller in preparation)*

Conclusion

Spin glass criticality (in the SK model) $\rightarrow$ scale free response to a slow magnetic field change.

Connection between manifestations of criticality:

Soft “Coulomb” gap – avalanches –
algebraic spin-spin correlations

Similar effects expected for electron glasses

Avalanches in Barkhausen noise, fast charge relaxation:
An interesting experimental diagnostic for spin glass criticality?!