From Circuit QED to Mechanical Cooling: TLS Fluctuators in Solid-state Resonators

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What we have heard in this conference

- decoherence of qubit due to TLS and 1/f noise
- material property and TLS distribution
- protect qubits from noise, error correction
- ……

What I will be discussing:

quantum manipulation and quantum processes related with TLS's

- circuit QED of TLS's - quantum gates and more
- effect of TLS on "laser cooling" of nanomechanical modes

stimulated by long (de)coherence time of TLS in phase qubits, and strong coupling of TLS to junctions and mechanical strain in solids

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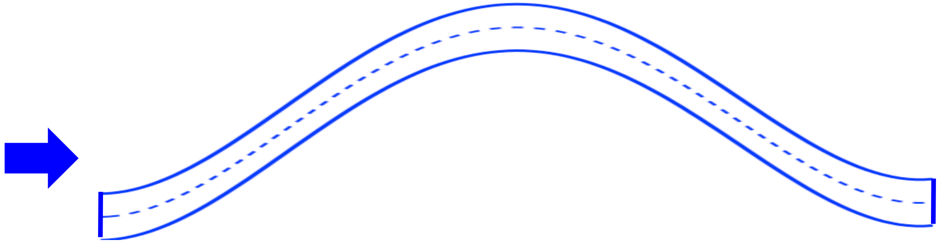
TLS and Nanomechanical Resonators

Nanomechanical systems in quantum limit:

- reaching resolved sideband regime: resonator freq. >> cavity damping
- ultra-high Q nanomechanical modes: Q>10,000,000
- frequency in a wide range: between kHz to GHz
- improving measurement to approach quantum limited detection



flexural mode

Why go to the quantum limit?

- macroscopic quantum effects and fundamental questions in quantum physics
- (quantum) metrology and new concepts in small force detection
- quantum information and technology e.g. Tian and H. Wang, arXiv 1007.1687 (optical frequency conversion for quantum states – quantum network with NEMS)

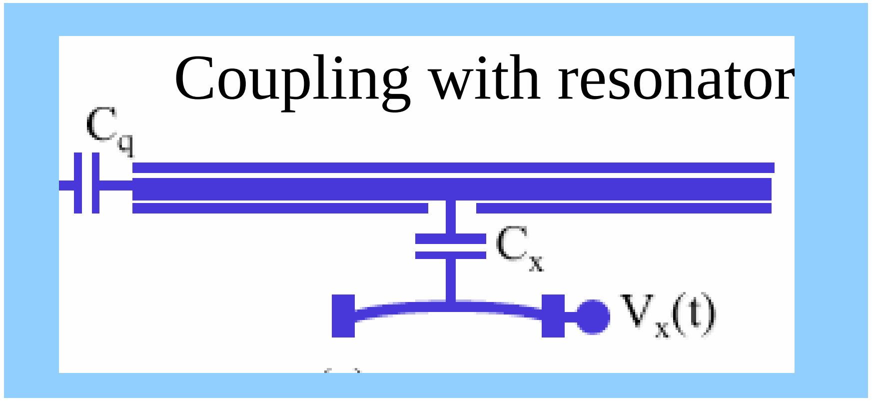
Ground State Cooling

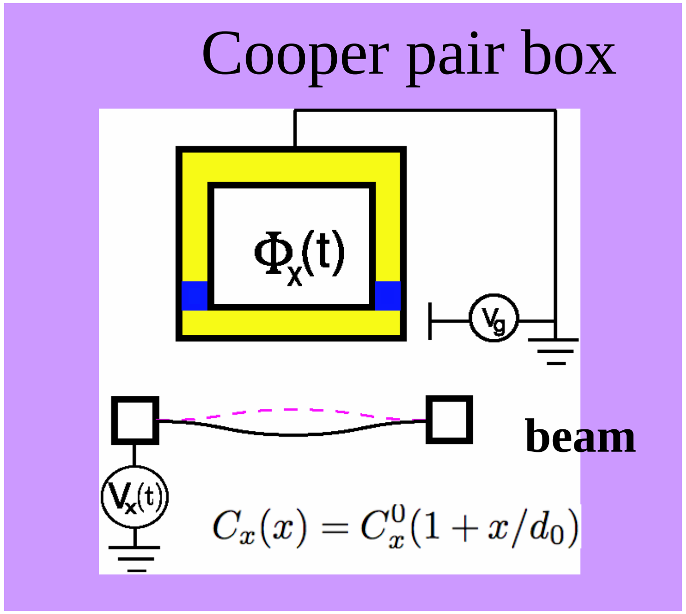
Quantum engineering tasks can be achieved via coupling with

- solid-state electronic devices – superconductors, quantum dots …
- atomic systems – atoms, ions, and condensates
- external control sources to implement quantum protocols

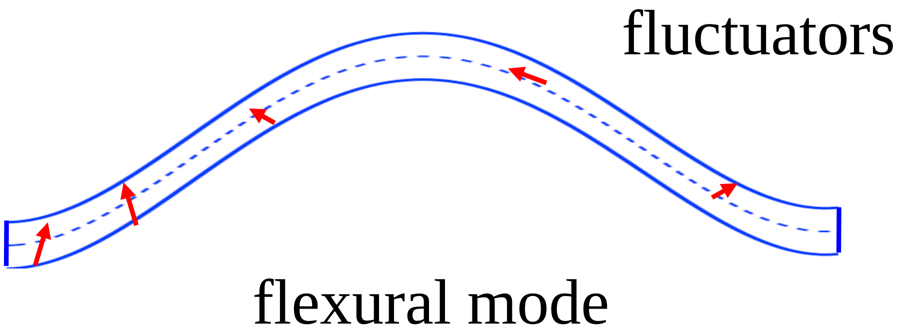
Cooling via coupling to resonators and qubits

- recently, resolved-side band regime reached – a few MHz modes





TLS and Nanomechanical Resonators



Coupling

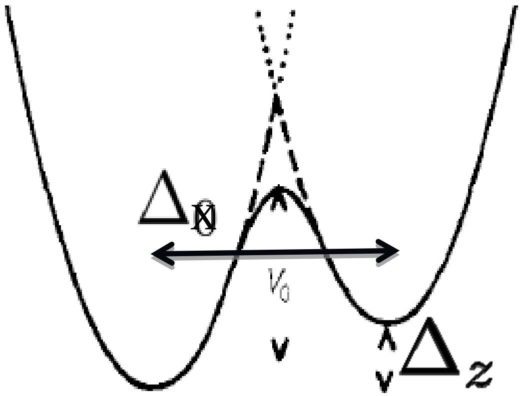
- TLS fluctuators due to defects in the amorphous materials
- TLS energy depends on deformation potential and strain tensor
- mechanical vibration generates strains on beam

Remus and Blencowe, PRB 2009, and many previous works

Effects – <u>this talk</u>

- coupling modulates energy spectrum
- affects cooling process
- can be used to study microscopic picture of TLS

TLS and Nanomechanical Resonators



- Model for TLS in solids

$$H_{TLS} = \frac{\Delta_z}{2}\sigma_z + \frac{\Delta_0}{2}\sigma_x$$

- distribution

$$\propto d\Delta_z \frac{d\Delta_0}{\Delta_0}$$

- coupling depends on strain $\Delta_z(y''(x))$

$$\Delta_z \to \Delta_z + 2\sum \nu_{kl}\epsilon_{kl}$$

$$\epsilon_{kl} \propto d^2y(x)/dx^2$$

deformation potential ν_{kl}

strain tensor ϵ_{kl}

TLS and Nanomechanical Resonators

$$H_t = \frac{\Delta_z}{2}\sigma_z + \frac{\Delta_x}{2}\sigma_x + \lambda(a+a^\dagger)\sigma_z + \hbar\omega_m(a^\dagger a + 1/2) + (-\Delta_0)b^\dagger b + g_0(a+a^\dagger)(b+b^\dagger)$$

TLS

coupling

NEMS

linearized cooling terms

can be a qubit too

- coupling from strain in solids
- changes energy of NEMS
- affects cooling

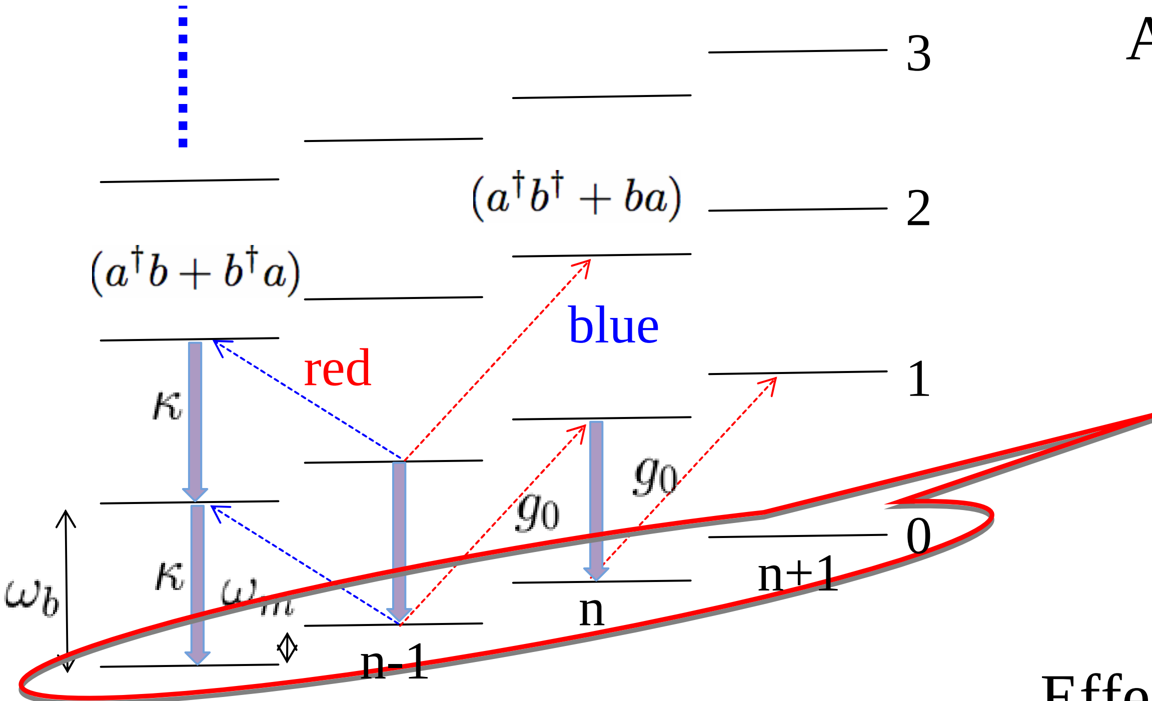
- optomechanical coupling is linearized
- parametric linear coupling

  L. Tian, PRB 79, 193407 (2009)
- cooling by adiabatic elimination
- detuning is crucial for cooling

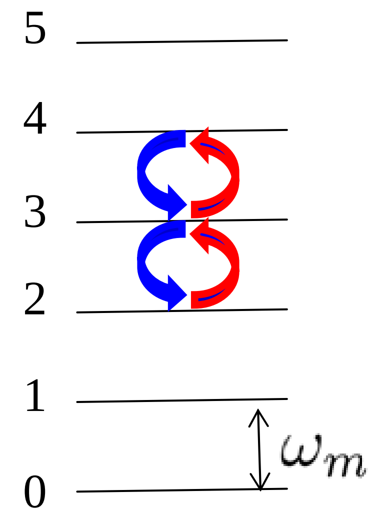
$$\omega_d = \omega_b - \omega_a$$
$$-\Delta_0 = \omega_a$$

Ground State Cooling

$$n_s = \frac{\Gamma_+}{\Gamma_- - \Gamma_+}$$



Adiabatic elimination



linearized coupling $g_0(a + a^\dagger)(b + b^\dagger)$

cooling rate

heating rate

$$\Gamma_\pm = \frac{g^2 \kappa_0}{(\kappa_0/2)^2 + (\hbar\omega_m \pm \Delta_0)^2}$$

Effective master equation:

$$\frac{d\rho}{dt} = \Gamma_- \mathcal{L}(a)\rho + \Gamma_+ \mathcal{L}(a^\dagger)\rho$$

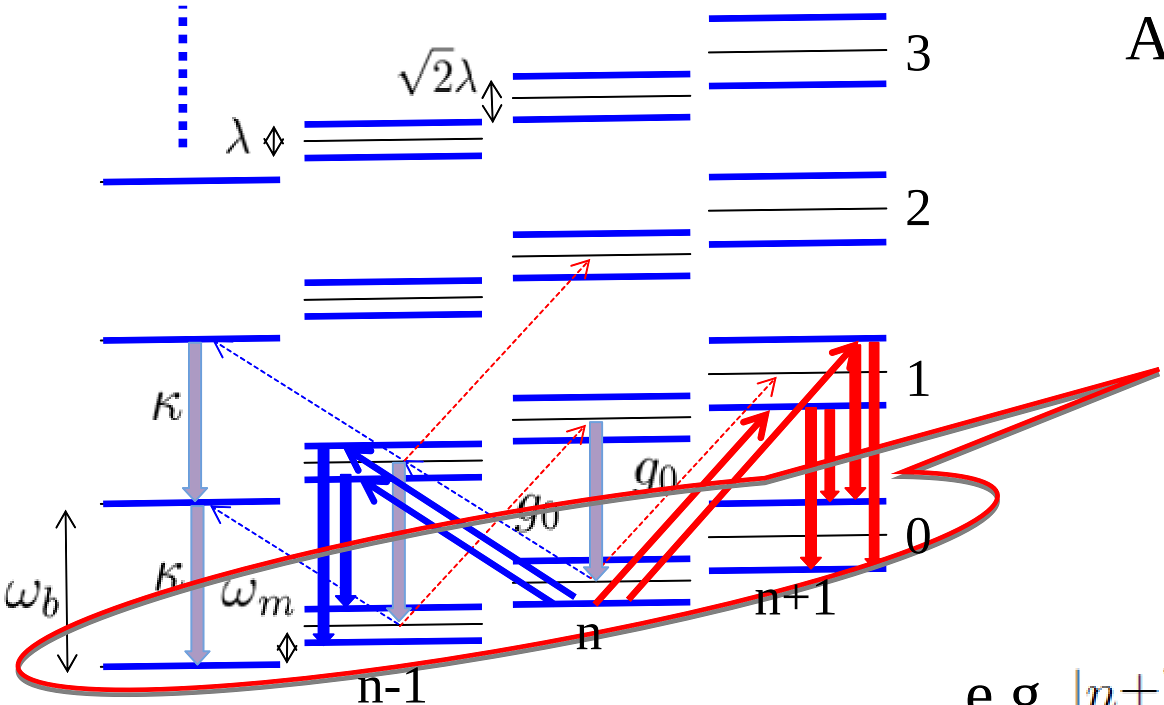
$$\mathcal{L}(O)\rho = 2O\rho O^\dagger - \rho O^\dagger O - O^\dagger O \rho$$

detailed balance

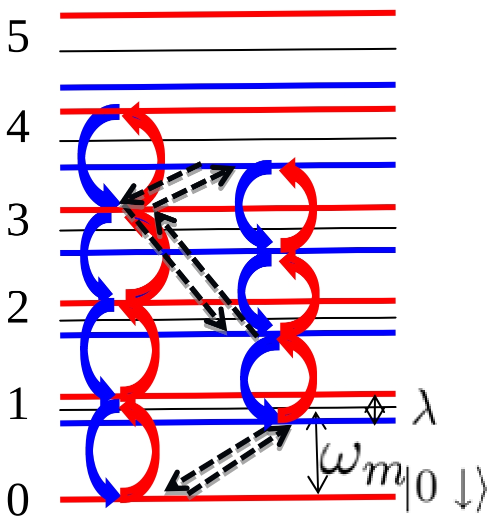
$$P_n / P_{n+1} = \Gamma_+ / \Gamma_-$$

$$\frac{dP_n}{dt} = \Gamma_-(n+1)P_{n+1} - \Gamma_- n P_n + \Gamma_+ n P_{n-1} - \Gamma_+(n+1)P_n$$

TLS and Nanomechanical Resonators



Adiabatic elimination



e.g. $|n\pm\rangle = |n\downarrow\rangle \pm |(n-1)\uparrow\rangle$

- energy levels are shifted due to coupling to TLS
- doublet states of Jaynes-Cummings model – CQED
- resonance not in red-side frequency(s) any more
- cooling can be affected

Adiabatic Elimination with TLS

1. Eigenbasis - polarization $E_{n\alpha} = n\hbar\omega - \frac{\delta E}{2} + \alpha\sqrt{(\frac{\delta E}{2})^2 + \lambda^2 n}$

resonance $|n\pm\rangle = |n\downarrow\rangle \pm |(n-1)\uparrow\rangle$ $\alpha = \pm 1$

dispersive $|n+\rangle = |n\downarrow\rangle + (\lambda\sqrt{n}/\delta E)|(n-1)\uparrow\rangle$

2. Operators in eigenbasis

$$a = \sum_{n,\alpha,\beta} A^{(n)}_{\beta\alpha} |(n-1)\beta\rangle\langle n\alpha|$$

3. Adiabatic elimination in eigen-basis – rate equation

$$\frac{dP_{n\alpha}}{dt} = \sum_{\beta} \Gamma^{n+1}_{1,\beta\alpha}|A^{(n+1)}_{\alpha\beta}|^2 P_{(n+1)\beta} - \Gamma^{n}_{1,\alpha\beta}|A^{(n)}_{\beta\alpha}|^2 P_{n\alpha} + \Gamma^{n}_{2,\alpha\beta}|A^{(n)}_{\beta\alpha}|^2 P_{(n-1)\beta} - \Gamma^{n+1}_{2,\beta\alpha}|A^{(n+1)}_{\alpha\beta}|^2 P_{n\alpha}$$

cooling heating

previous ➡ $\frac{dP_n}{dt} = \Gamma_-(n+1)P_{n+1} - \Gamma_- n P_n + \Gamma_+ n P_{n-1} - \Gamma_+(n+1)P_n$

$$\Gamma^{n}_{z,\alpha\beta} = \frac{g^2\kappa_0}{(\kappa_0/2)^2 + (E_{n\alpha} - E_{(n-1)\beta} \pm \Delta_0)^2}$$

Dispersive Regime

Eigenstates: Stark shifts, polarization=spin

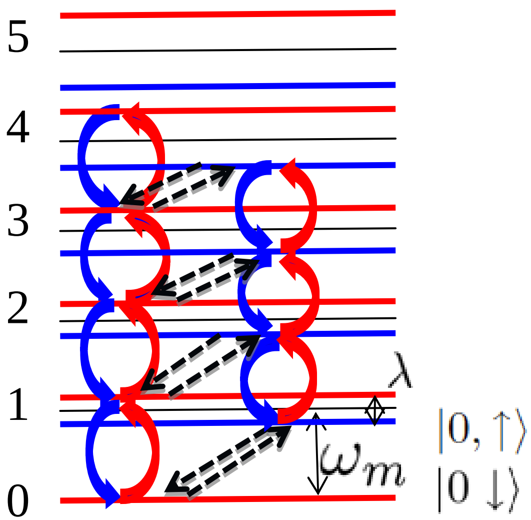
$$|n+\rangle = |n\downarrow\rangle + (\lambda\sqrt{n}/\delta E)|(n-1)\uparrow\rangle$$

$$E_{n+} = \hbar\omega_m + \frac{\lambda^2 n}{\delta E}$$

$$|n-\rangle = (\lambda\sqrt{n}/\delta E)|n\downarrow\rangle - |(n-1)\uparrow\rangle$$

$$E_{n-} = E_{tls} - \frac{\lambda^2 n}{\delta E}$$

$$\Gamma^n_{z,\alpha\alpha} = \Gamma_{z,\alpha}$$



Cooling rates:

same spin transition $A^{(n)}_{1,1} = \sqrt{n}(1 - \frac{1}{2}(\frac{\lambda}{\delta E})^2)$

$$\Gamma_{\pm,\alpha} = \frac{g^2\kappa_0}{(\kappa_0/2)^2 + (\hbar\omega_m + \alpha\frac{\lambda^2}{\delta E} \pm \Delta_0)^2}$$

opposite spin transition $A^{(n)}_{1,-1} = (\frac{\lambda}{\delta E})$

two separate cooling process for two spins under Stark shift

$$\hbar\omega_m \rightarrow \hbar\omega_m - (\lambda^2/\Delta)\langle\sigma_z\rangle$$

extra relaxation (cooling) of spin states with small rate $A^{(n)}_{1,-1} = (\frac{\lambda}{\delta E})$

Resonator-TLS on Resonance

Eigenstates: $|n\pm\rangle = |n\downarrow\rangle \pm |(n-1)\uparrow\rangle$

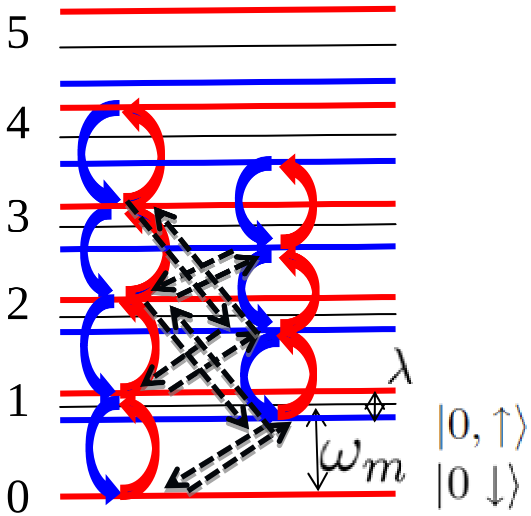
$$E_{n\alpha} = \hbar\omega_m + \alpha\lambda\sqrt{n}$$

Cooling rates:

same spin transition $A^{(n)}_{\alpha\alpha} = \dfrac{\sqrt{n}+\sqrt{n-1}}{2}$

$$\Gamma_{\pm} = \frac{g^2\kappa_0}{(\kappa_0/2)^2 + (\hbar\omega_m \pm \Delta_0)^2}$$

opposite spin transition $A^{(n)}_{\alpha-\alpha} = \dfrac{\sqrt{n}-\sqrt{n-1}}{2}$



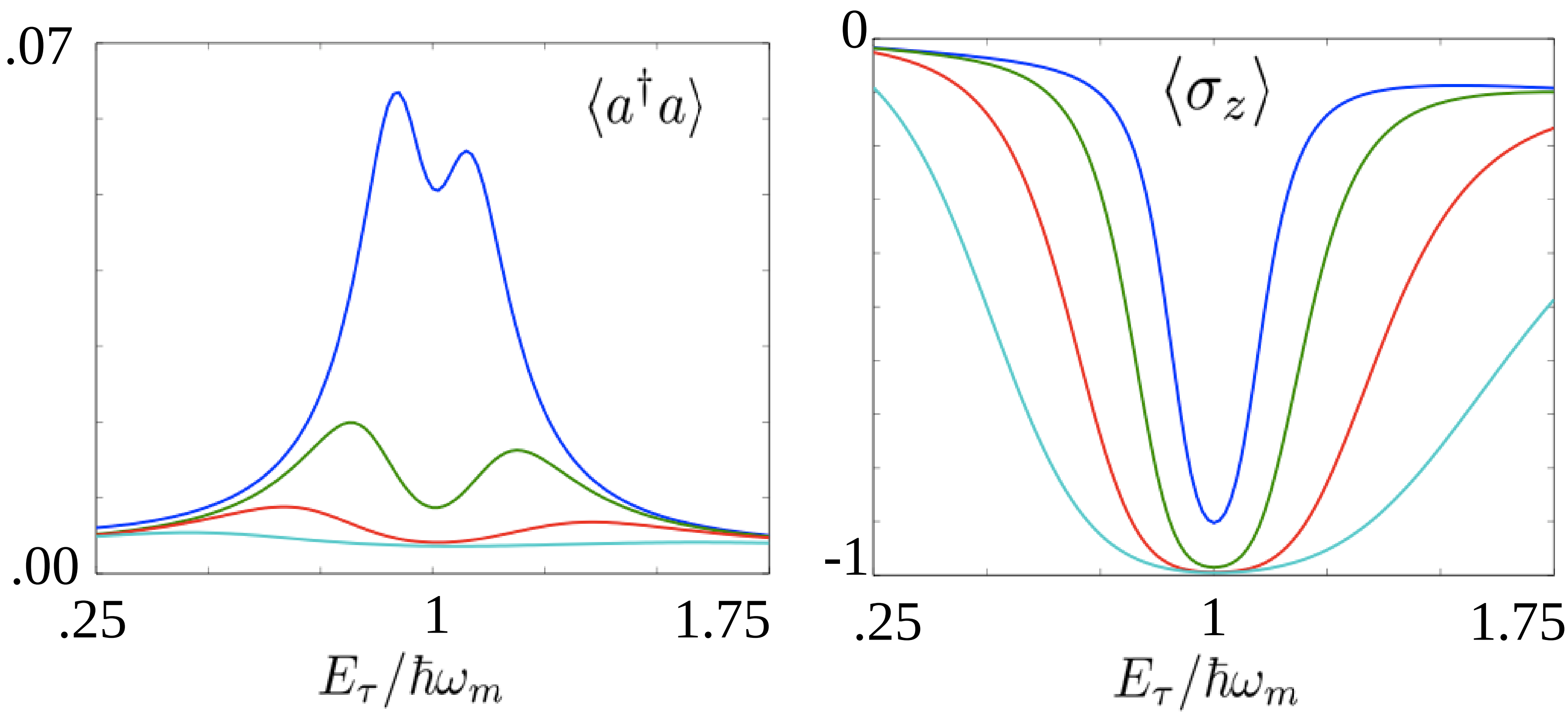
two separate cooling processes for two spins for large n –> (1/16n)

mixed up at small n –> (1/2)

extra relaxation (cooling) of spin states with large rate

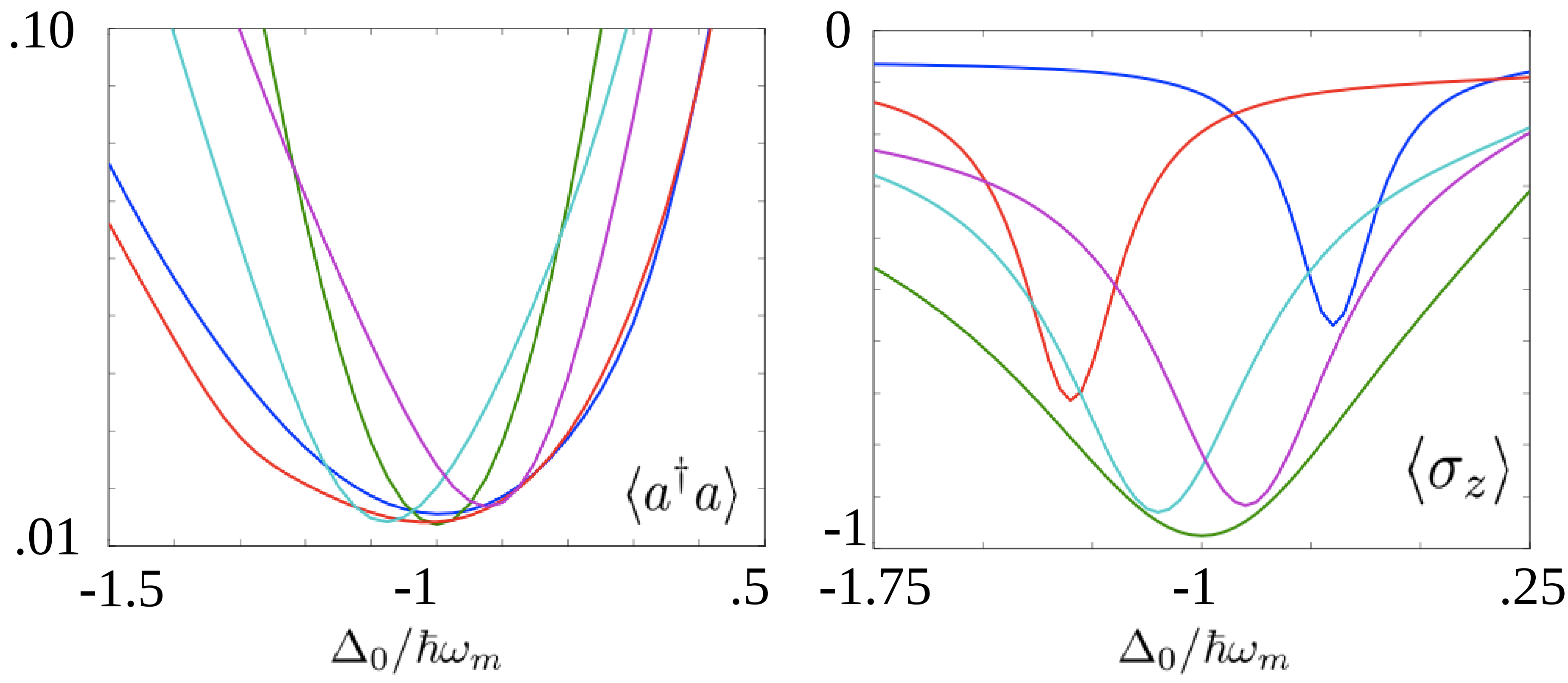
Numerical Results

- intrinsic decay rate of TLS 5e-7, 5e-6, 5e-5, 5e-4 (color CRGB)
- cavity driving at red side band
- <n> depends on TLS decay strongly what at resonance
- cooling of TLS strong at resonance



Numerical Results

- Cooling vs. driving frequency at TLS decay 1e-4
- TLS energies E_τ=2.6, 2.2, 2, 1.8 1.4 (color RCGPB)
- Optimal cooling position shifted, but not monotonic
- TLS optimal cooling when cavity detuning = ω_b - E_τ



This Part

Study coupling between TLS and mechanical mode
Derive cooling equation by adiabatic elimination
Cooling strongly affected by TLS relaxation at resonance
TLS is cooled via coupling
(L. Tian, in preparation)

What's next?
Time evolution of the cooling process
cooling process changes as n --> small
Dynamics with flicker noise

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Coupling to Josephson Junction

1. **cause of strong decoherence in superconducting qubits**
   induces charge/flux/current noise with $1/f^{\alpha}$ spectrum
   with large number of TLS's
2. **ubiquitous in solid-state systems:** defects in amorphous materials – oxide, glass, …
3. **experiments show strong/coherent coupling with qubits:**
   phase qubit measurements show spectroscopic splitting due to TLS fluctuators inside amorphous junctions
   (Simmonds et al. 2004, Martinis et al. 2005, Y. Yu et al, 2008, S.-Y. Han group, 2009, …)
4. **long coherence time demonstrated:** Neeley et al 2008, …
5. **quantum manipulation of TLS:**
   demonstrate microscopic mechanism, improve qubit property,
   lack of direct controlling

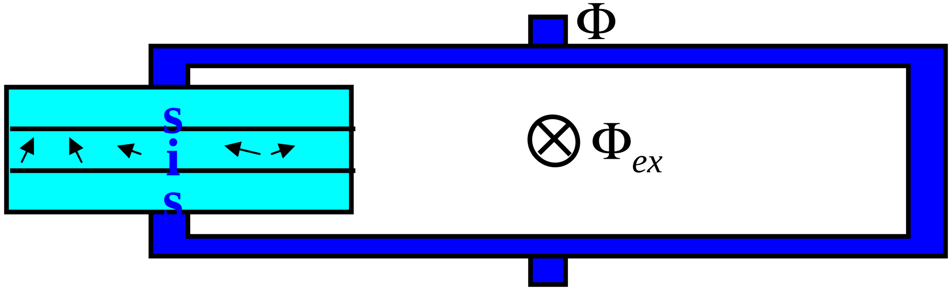
Idea for Quantum Logic Operation

Long coherence time demonstrated in recent experiments

- (de)coherece time longer than that of qubit
- can we test logic operations with TLS's inside amorphous layer?

A circuit QED idea to achieve universal quantum logic

TLS's inside a
driven
Junction resonator



$$\omega_c = \sqrt{\frac{4e^2 E_J^{eff}}{\hbar^2 C_0}}$$

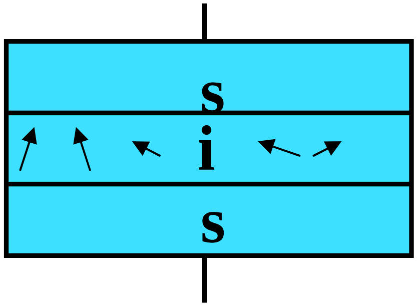
Challenges in the idea – **<u>not trivial</u>**

- TLS's are well spaced in energy – usually off-resonance
- lack of control handle on individual TLS

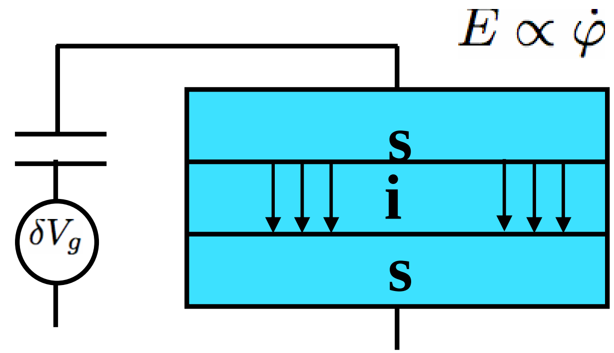
L. Tian & K. Jacobs, PRB 79, 114503 (2009)

Coupling to Josephson Junction

Critical Current Coupling



Dielectric coupling



$$H_{TLS} = \frac{\Delta_z}{2}\sigma_z + \frac{\Delta_0}{2}\sigma_x$$

$$-E_{J1}(1 + \vec{j}_d \cdot \vec{\sigma})\cos\varphi - \frac{\hbar}{2e} I_b \varphi$$

$$g_1 = E_{J1} j_z \sqrt{\frac{2e^2}{C_J \hbar\omega_c}} \sin\varphi_0$$

φ_0: shift in phase by current bias

$$-\frac{2e^2 d_0}{C_J h_0}\frac{\hat{p}_\varphi}{\hbar}\sigma_z$$

$$g_1 = \frac{d_0}{h_0}\sqrt{\frac{e^2\hbar\omega_c}{2C_J}}$$

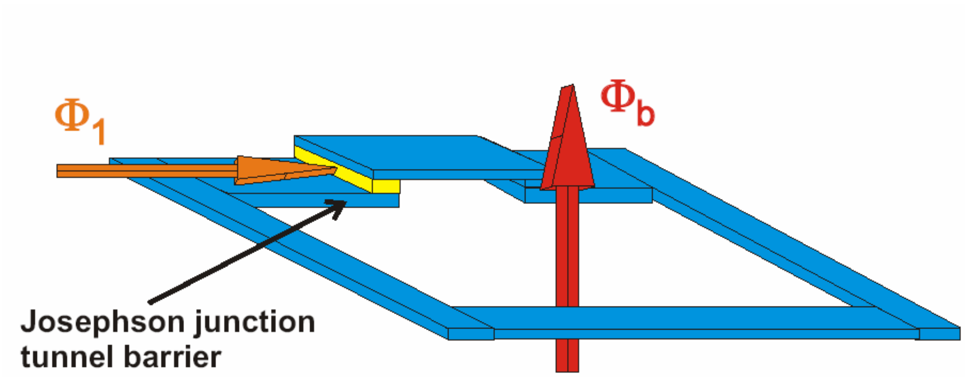
d_0: dipole, h_0: barrier thickness

Circuit QED in JJ Resonator

Josephson junction resonator mode - Microwave cavity mode

- Q-factor ~$10^{3\text{-}4}$
- frequency a few GHz - tunable by RF SQUID circuit
- have been tested experimentally

Experimental Realization - Circuit - Adjustable frequency

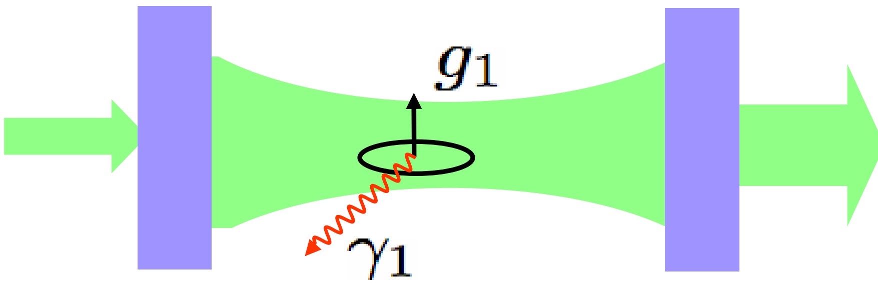


$$\omega_c = \sqrt{\frac{4e^2 E_J^{eff}}{\hbar^2 C_0}}$$

1. frequency tunable by adjusting Φ_b
2. coupling adjustable by e.g.controlling Φ_1

L. Tian & RW Simmonds, PRL (2007)

Circuit QED in JJ Resonator



- atoms, ions in cavity
- quantum dot photonic devices
- superconducting quantum circuit

$$\hbar\omega_c \hat{a}^\dagger\hat{a} + \hbar\omega_1\sigma_{1z} + g_1(\hat{a}\sigma_{1+} + \hat{a}^\dagger\sigma_{1-}) + \epsilon(\hat{a}+\hat{a}^\dagger) + H_\kappa + H_\gamma$$

cavity - Josesphon junction resonator

atom – qubit (TLS) in eigenbasis

coupling

microwave driving

cavity damping

TLS noise

Δ_c - detuning of microwave mode

Δ_a - detuning of qubit (TLS)

g_1 - coupling, $g_1 = g_c$, g_d

cavity QED in solid-state devices

- qubit
- TLS

$$g_1(\hat{a}^\dagger\sigma_{1-} + \sigma_{1+}\hat{a})$$

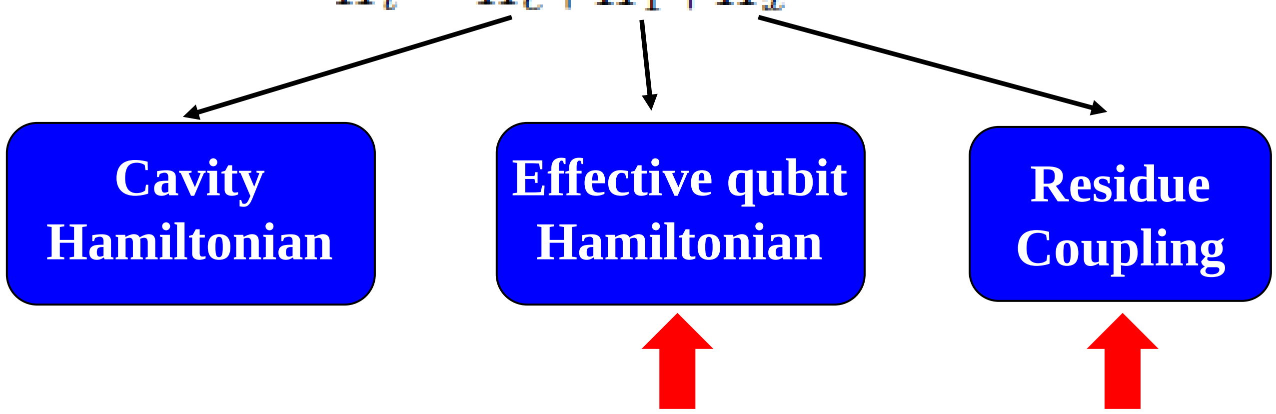
Effective Hamiltonian of TLS's

- dispersive regime: resonator off-resonance with TLS
- driving on resonator
- applying unitary transformation: $\widetilde{H}_t = U H_t U^\dagger$

$$U = e^{-\epsilon(a-a^\dagger)/\Delta_c} \prod_n e^{-g_n(a^\dagger\sigma_{n-}-\sigma_{n+}a)/\Delta_{nc}}$$

- effective Hamiltonian:

$$\widetilde{H}_t = H_c + \widetilde{H}_1 + \widetilde{H}_x$$



Effective Hamiltonian of TLS's

$$\widetilde{H}_1 = \sum \frac{\bar{\Delta}_n}{2}\sigma_{nz} + \Omega_n \sigma_+ + \Omega_n^\star \sigma_- + \sum_{\langle n,m\rangle} \lambda_{nm}\sigma_{n+}\sigma_{m-} + \lambda_{nm}^\star \sigma_{m+}\sigma_{n-} + \widetilde{H}_k$$

- resonator and TLS are decoupled in $\widetilde{H}_1$
- TLS parameters are controllable via resonator – single qubit
- extra noise induced in $\widetilde{H}_k$ ⇨ $(g_n/\Delta_{nc})^2$
- TLS's are <u>off resonance – how to perform gates?</u>

Residue Coupling

$$\widetilde{H}_x = \sum_n \frac{g_n^2}{\Delta_{nc}}\sigma_{nz}\left[a^\dagger a + \epsilon\left(\frac{\Delta_c - 2\Delta_{nc}}{2\Delta_{nc}\Delta_c}\right)(a + a^\dagger)\right]$$

- Residue coupling is "small" (numerical simulation)

Single Qubit Gate

- TLS parameters depend on driving and detunings

$$\tilde{\Delta}_n = \Delta_n + (g_n^2/\Delta_{nc})(1 - 2\epsilon/\Delta_c) \qquad \Omega_{nx} = 2\epsilon g_n/\Delta_{nc}$$

- different TLS's effective decoupled by off-resonance
- arbitrary single qubit gates performed

	$\Delta_c(2\pi\times\text{MHz})$	$\epsilon(2\pi\times\text{MHz})$	$\Omega_{1x}(2\pi\times\text{MHz})$	Time (ns)
X	120	–60	60	8.3
H	160	–32	21.3	16.6

coupling/2π ~ 30 - 50 MHz

Controlled Gate

- effective coupling between TLS's

$$\lambda_{mn} = g_m g_n(\Delta_{mc} + \Delta_{nc})/(\Delta_{mc}\Delta_{nc})$$

- but TLS's are usually off-resonance – decoupled

  energy of TLS: $E_n^2 = \widetilde{\Delta}_n^2 + \Omega_{nx}^2$

- adjusting resonator driving to achieve resonance $E_1=E_2$

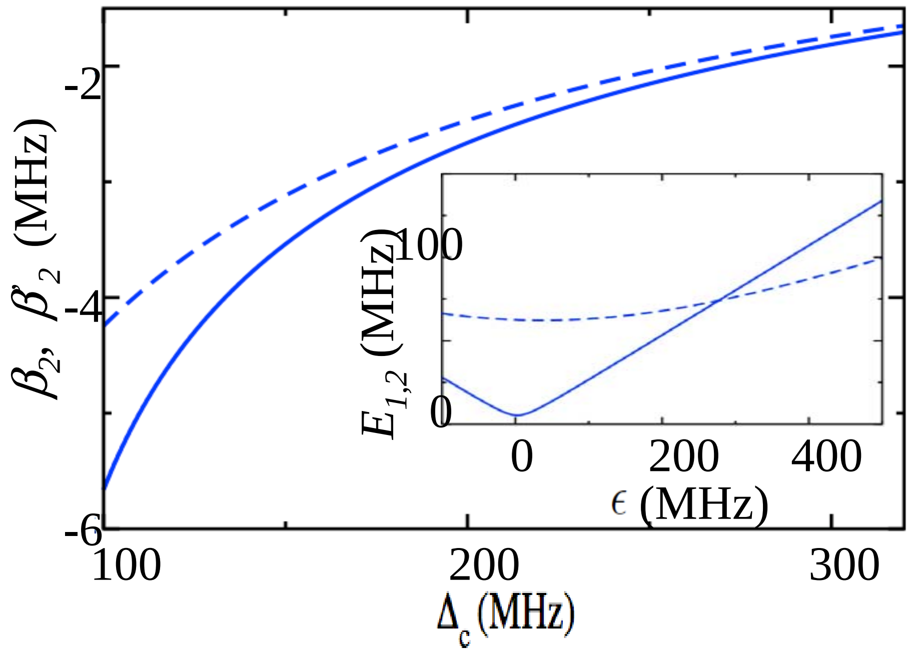
$$H_{12}^{\mathrm{rot}} = \beta_1 \bar{\sigma}_{1z}\bar{\sigma}_{2z} + \beta_2(\bar{\sigma}_{1+}\bar{\sigma}_{2-} + \bar{\sigma}_{1-}\bar{\sigma}_{2+})$$

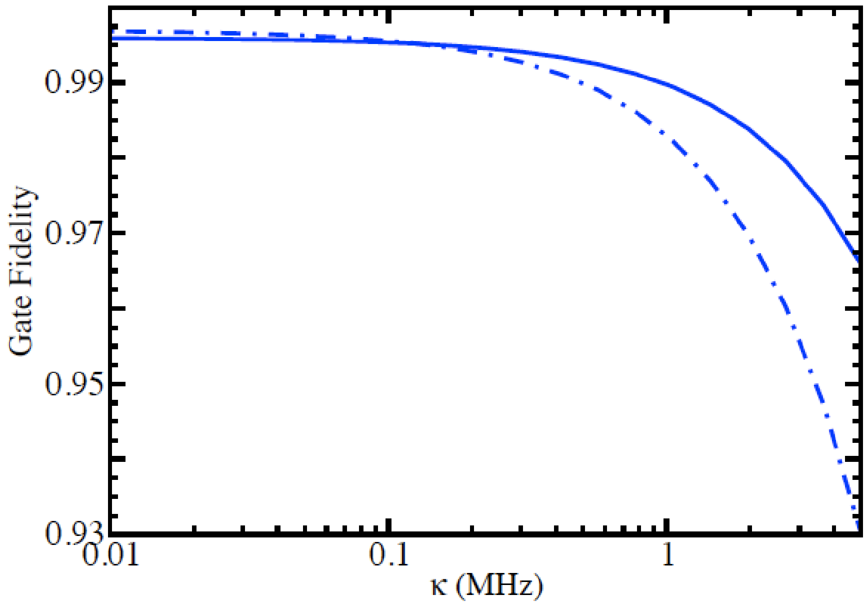
- include another term from residue coupling $\widetilde{H}_x$

  we have effective coupling β_2'

$$\beta_2' = \beta_2 + \frac{f_1 f_2(E_1 + E_2 - 2\Delta_c)}{2(E_1 - \Delta_c)(E_2 - \Delta_c)}$$

Controlled Gate





- effective energy – resonance
- coupling a few MHz
- two-bit gates performed in 150 ns.

- numerical simulation of full Hamiltonian
- at κ=4 MHz, Fid >0.99

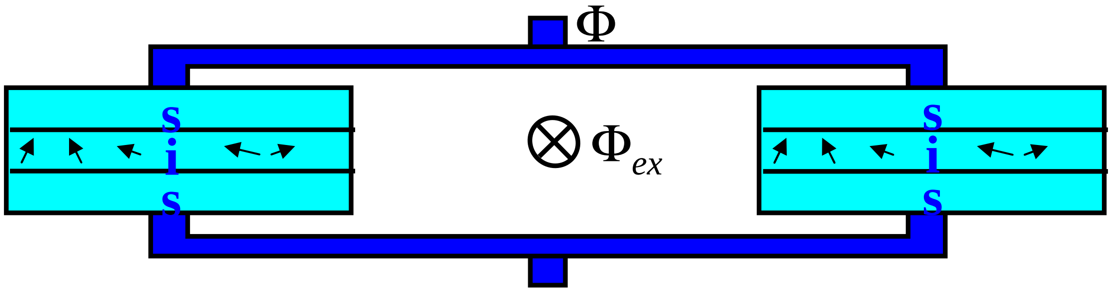
Decoherence

- Resonator decay is the main noise source
- off-resonance protects the TLS's from resonator
- gate time/decoherence time = **0.01**
- system can **test** quantum logic gates for TLS's
- demonstrate microscopic mechanism, improve qubit property

	1-bit	2-bit/dispersive	swap in CZ gate
τ_g	$\pi\Delta_{nc}/g_n\epsilon$	$\pi/2\lvert\beta_2'\rvert$	$\pi/2g_n$
τ_g (ns.)	~ 10	~ 140	~ 10
τ_d^{-1}	$g_n^2\kappa/\Delta_{nc}^2$	$g_n^2\kappa/\Delta_{nc}^2$	$\kappa/2$
τ_g/τ_d	**0.001**	**0.01**	0.02

Scalability

TLS's in different junctions



- Different junctions corresponds to same cavity mode
- TLS's in different junction coupling with same cavity mode
- Controlled logic gates can be performed exactly as before
- Resonator frequency is affected by number of junctions

This Part

Study coupling between TLS and Josephson junction

Cavity QED model for TLS's in junction and effective Hamiltonian

Universal quantum logic gates between TLS's

Fidelity by numerical simulation

L. Tian & K. Jacobs, PRB 79, 114503 (2009)

Summary

We studied coupling between TLS and resonators –

nanomechanical resonator

superconducting JJ resonator

Coherent coupling can induce interesting quantum effects

• TLS affects cooling of NEMS in resolved side-band regime. Cooling of TLS can be resulted. Cooling of NEMS shows dependence on TLS relaxation and coupling constant

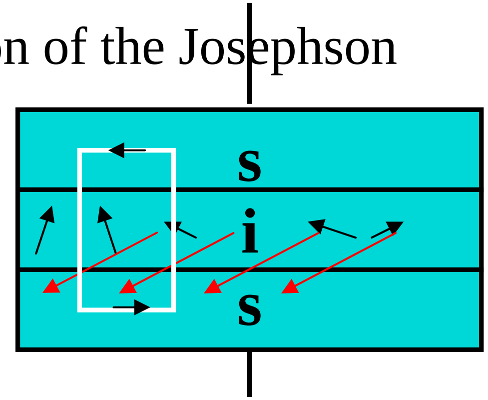
• TLS's are coherent objects as qubit candidates, but it is hard to manipulate or couple them. Using circuit QED with JJR,
university quantum logic gates with high fidelity

Thank you

Circuit QED in JJ Resonator

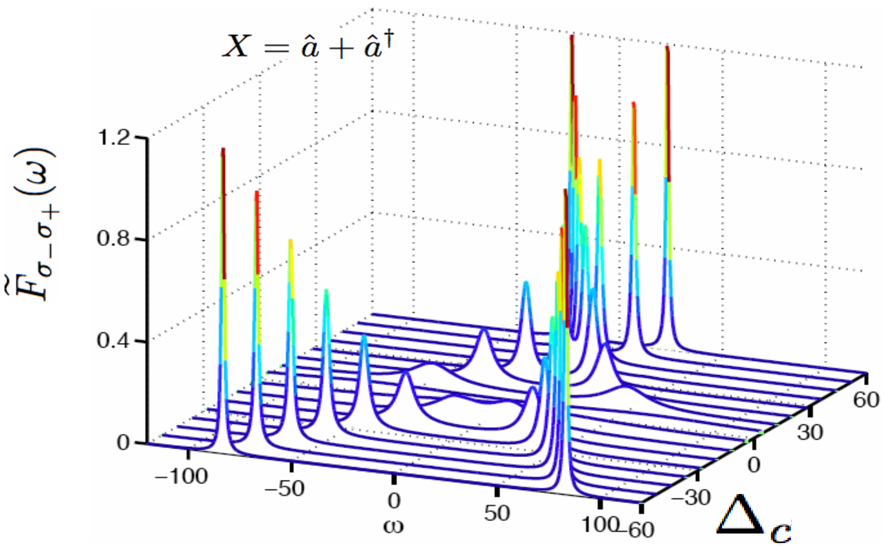
- **CQED can be explored for studying cupling mechanism**

Applying magnetic field to create a spatial modulation of the Josephson energy and the coupling with TLSs



magnetic field B

- Phase variable: $$\varphi(r) = \varphi(0) + \frac{2e}{\hbar} B.A$$



$\Delta_{ac} = 30$ MHz

Tian, Simmonds, PRL (2007)

- **can be used to study** **coupling dependence, coherence of TLS, energy, and spatial distribution** **of TLS**

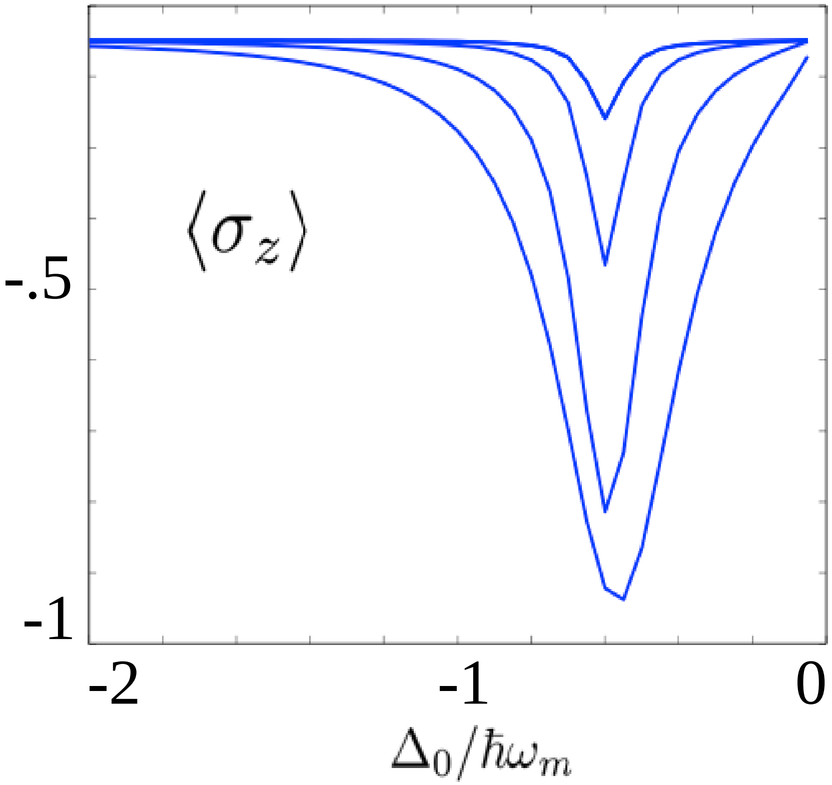
Dispersive Regime

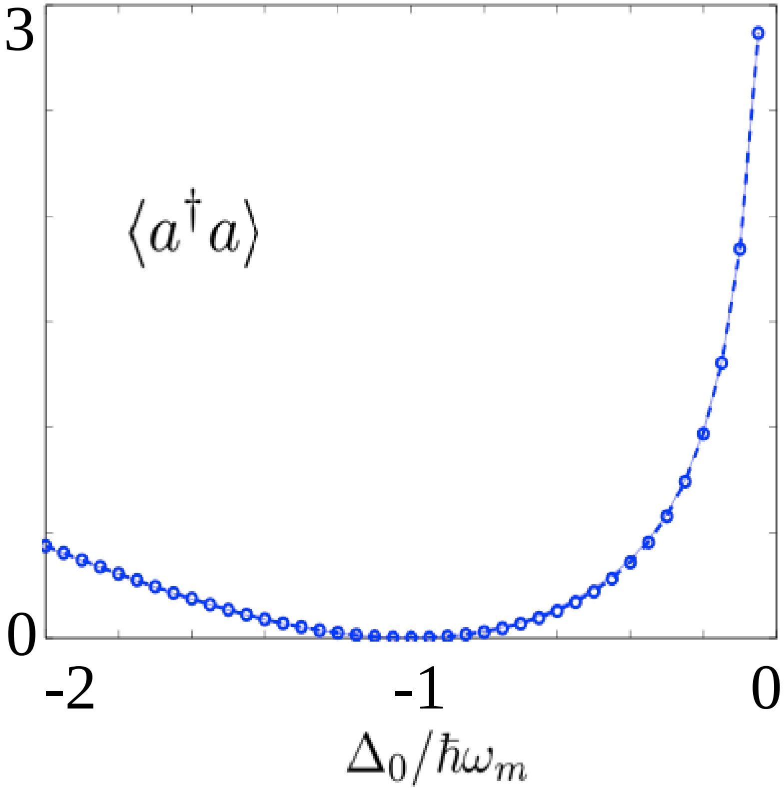
- large detuning between TLS and NEMS $|\Delta| = |\hbar\omega_m - E_\tau| \gg \lambda$
- coupling term approx. as Stark shift
  - NEMS freq shifted by $\hbar\omega_m \rightarrow \hbar\omega_m - (\lambda^2/\Delta)\langle\sigma_z\rangle$
  - TLS energy shifted by $E_\tau \rightarrow E_\tau - (\lambda^2/\Delta)(2\langle a^\dagger a\rangle + 1)$
  - Ref: Blais et al, PRA 2004
- additional coupling between TLS and cavity induces cooling for TLS

$$-g_0(\frac{\lambda}{\Delta})\sigma_x(b + b^\dagger)$$

- apply adiabatic elimination for cavity mode *b*, cooling can be derived
  - cooling of NEMS depends on $\langle\sigma_z\rangle$ effect small
  - cooling of TLS $\langle a^\dagger a\rangle$
  - coupled equations for steady state

Dispersive Regime





- TLS cooled to nearly polarized from thermal bath
- parameters $\hbar\omega_m = 10,\ \lambda = 0.1,\ 0.2,\ 0.5,\ 1,\ \gamma_\tau = 10^{-4}$
- why peak far from red-side band, to be answered