Noise in SQubits

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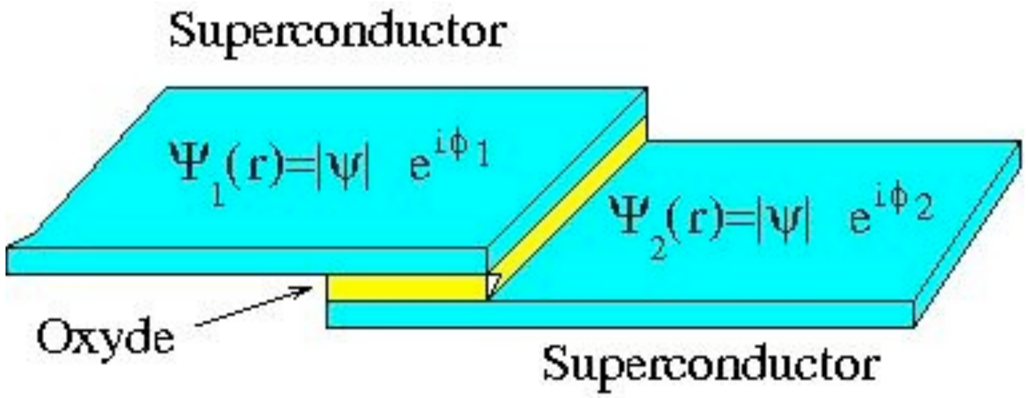
Trap noise in Josephson

- SQubits and noise
- Surface roughness
- Noninteracting traps
- Interacting traps
- quasiparticles

Superconducting qubits 101

Superconducting qubits 101

The Josephson effect



Superconducting qubits 101

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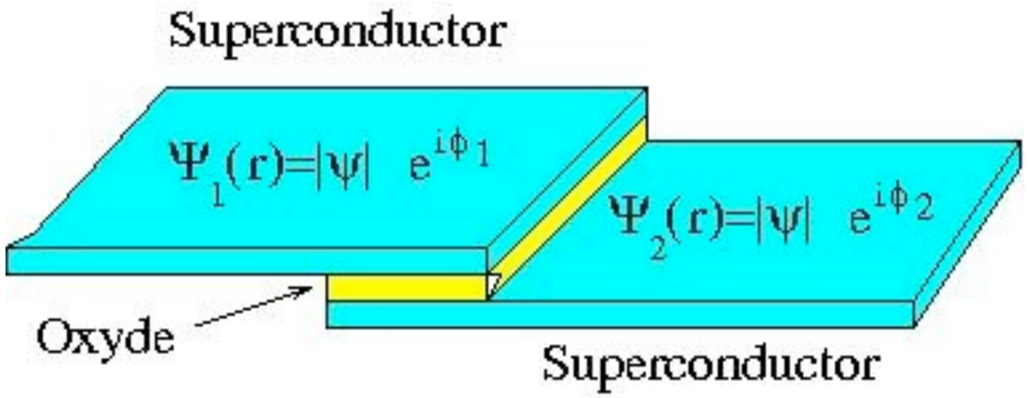

Josephson relations

$$I = I_c \sin\phi \quad \phi = \phi_1 - \phi_2$$

$$2eV = \hbar\dot{\phi}$$

Superconducting qubits 101

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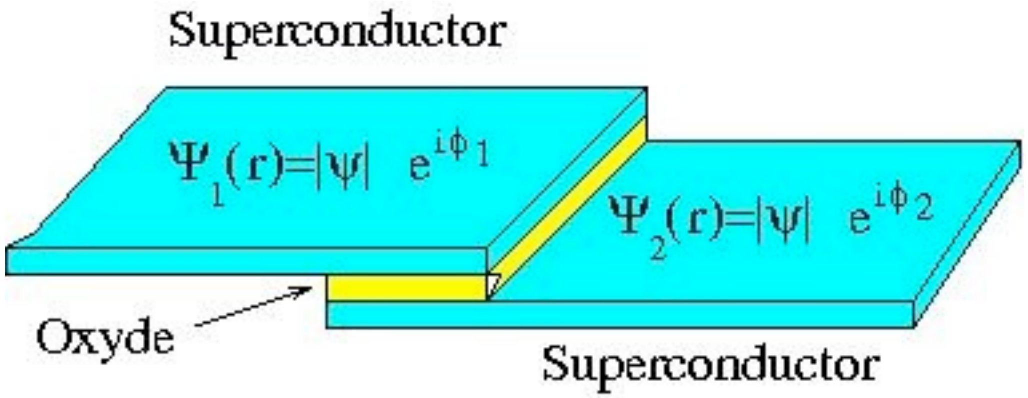
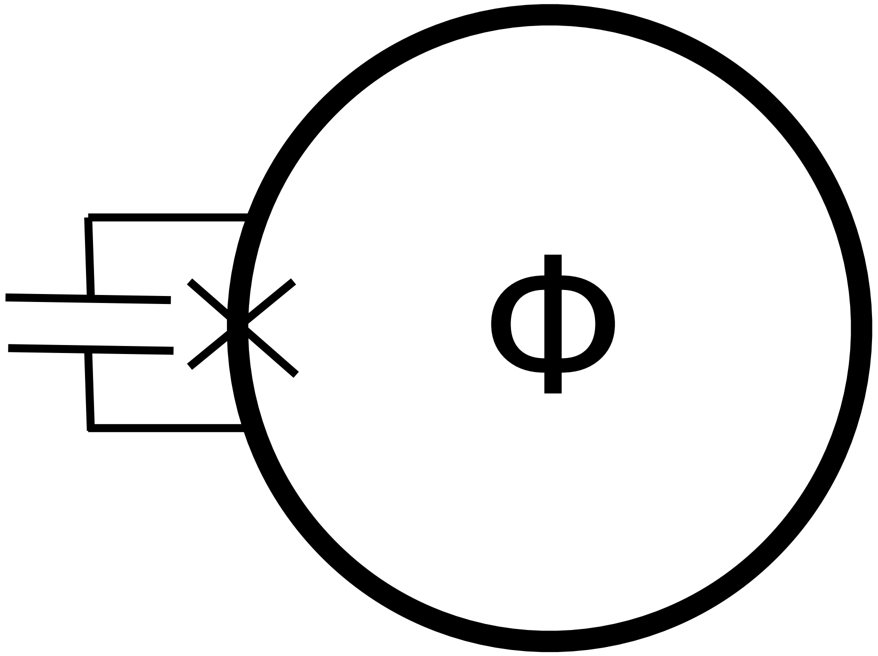


Flux qubit



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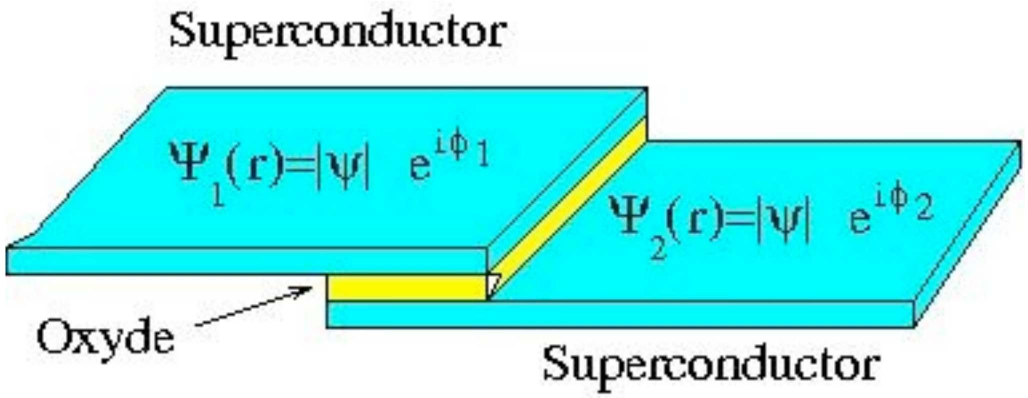

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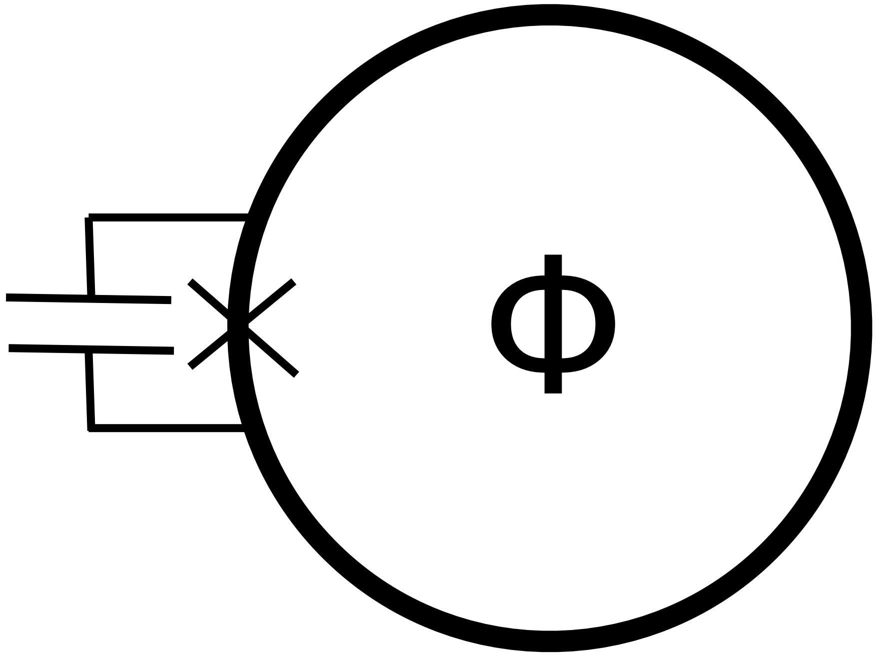

Also:

Phase and charge qubits

Superconducting qubits 101

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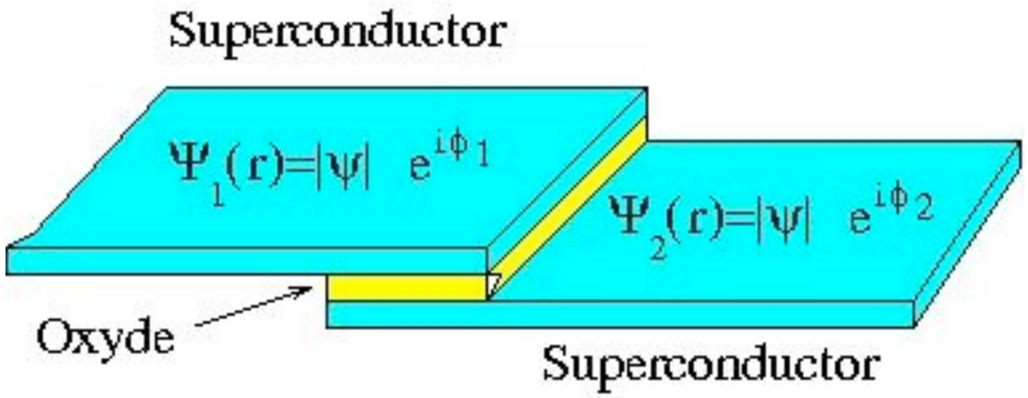

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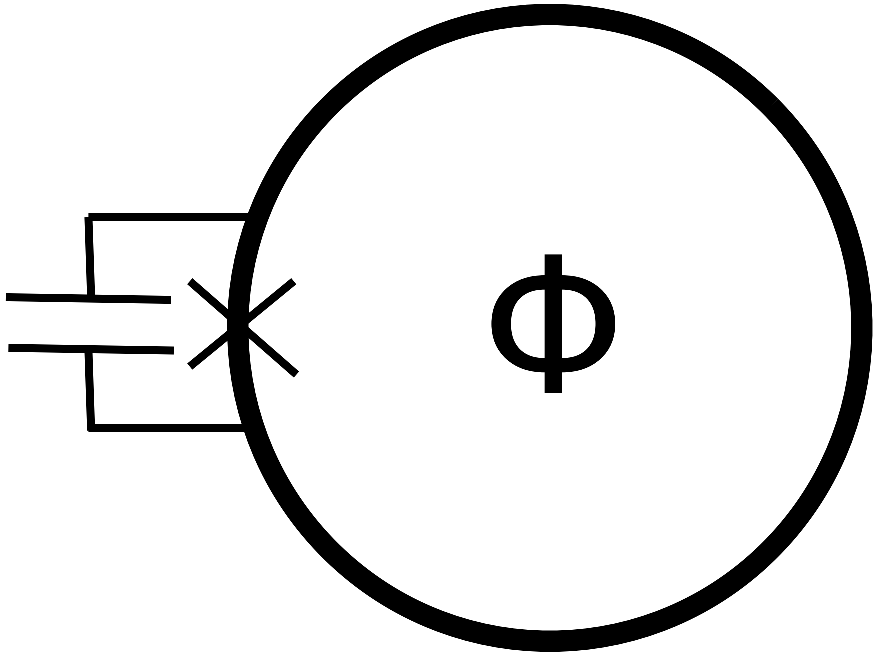
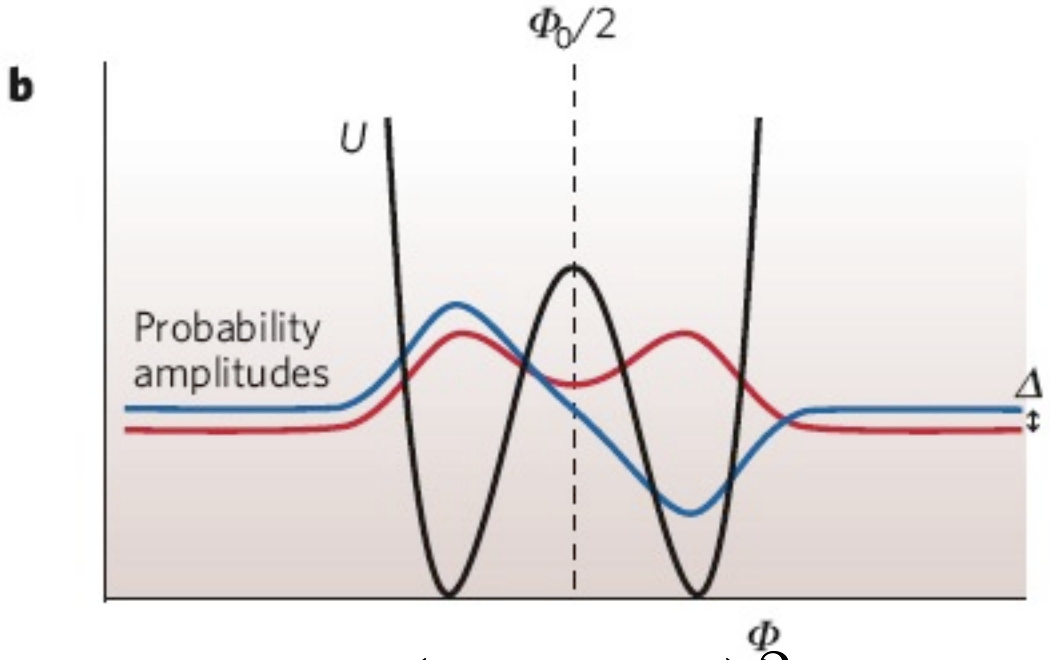


Quantize circuit equations



$$E_J = I_c \frac{\Phi_0}{2\pi}$$

$$V(\Phi) = \frac{(\Phi - \Phi_x)^2}{2L} - E_J \cos\left(2\pi\frac{\Phi}{\Phi_0}\right)$$

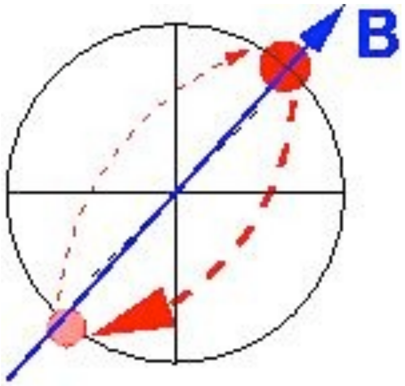
Also:
Phase and charge qubits

Fingerprints of noise

Fingerprints of noise

T_1 processes: Relaxation

Energy exchange with noise



Rate: $\frac{1}{T_1} \propto S\left(\frac{E}{\hbar}\right)$

Fingerprints of noise

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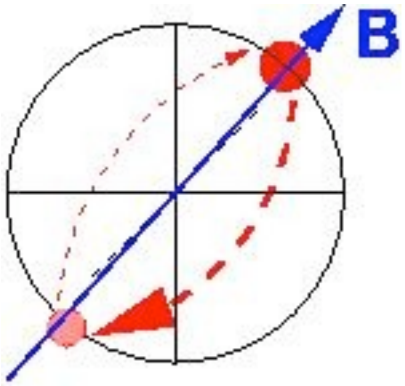

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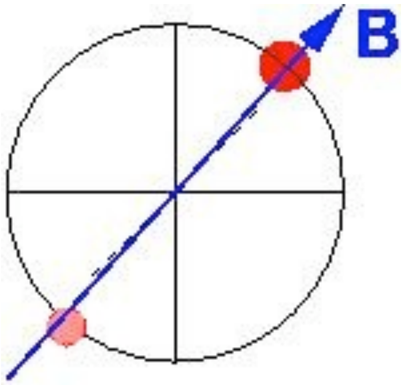
Pure dephasing:

No energy exchange



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Fingerprints of noise

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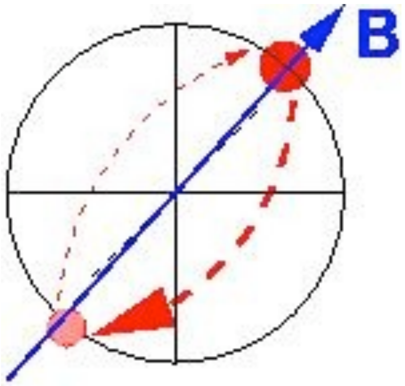

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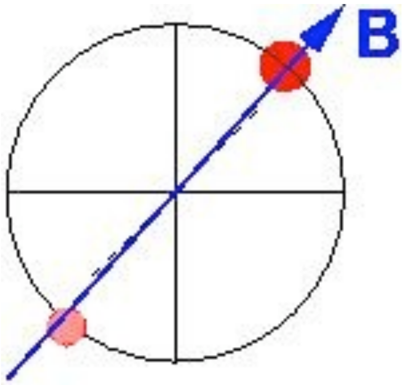

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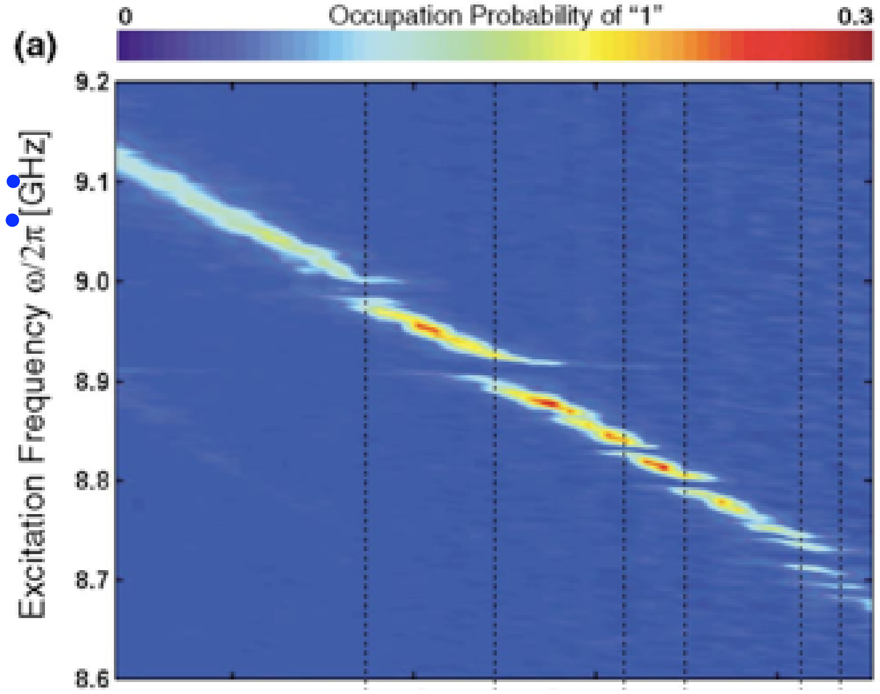
Junction resonators:

Simmonds et al, PRL 2003



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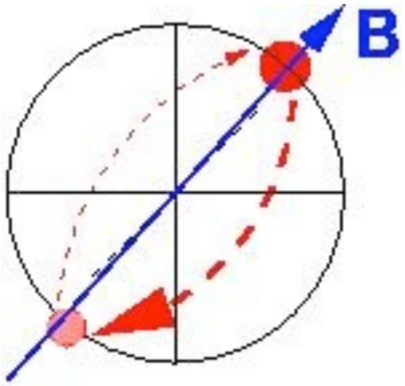

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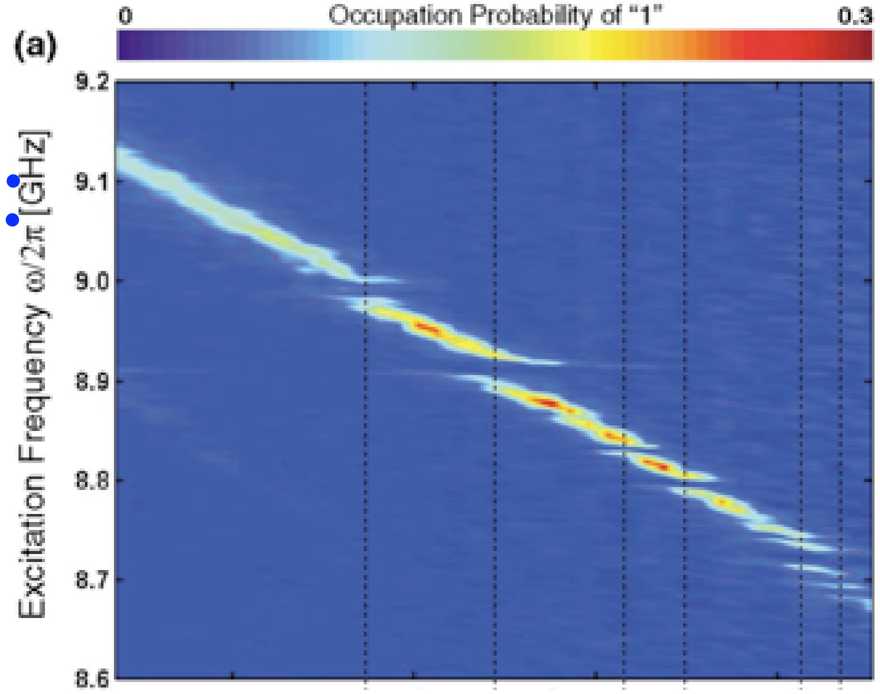

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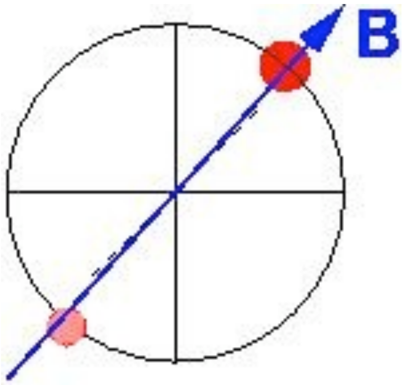

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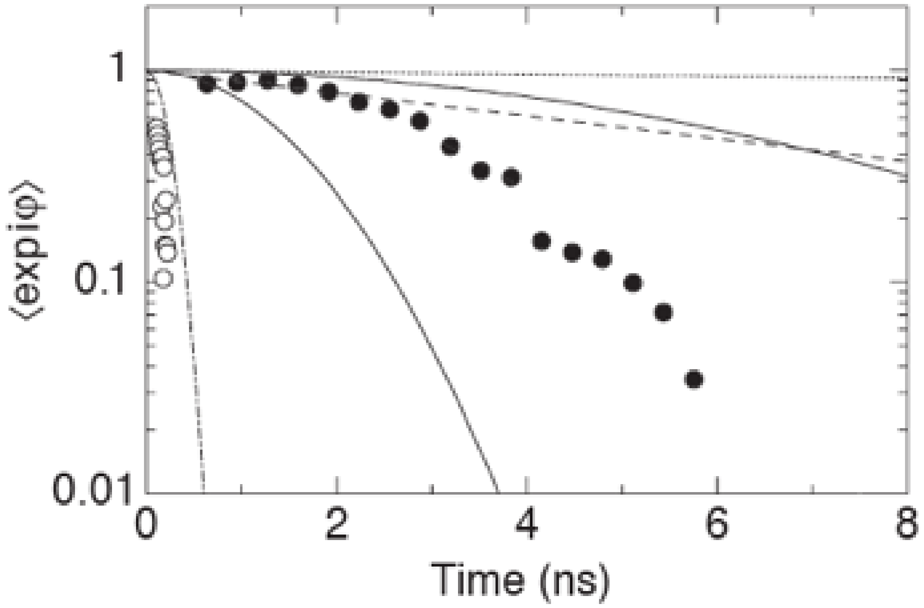
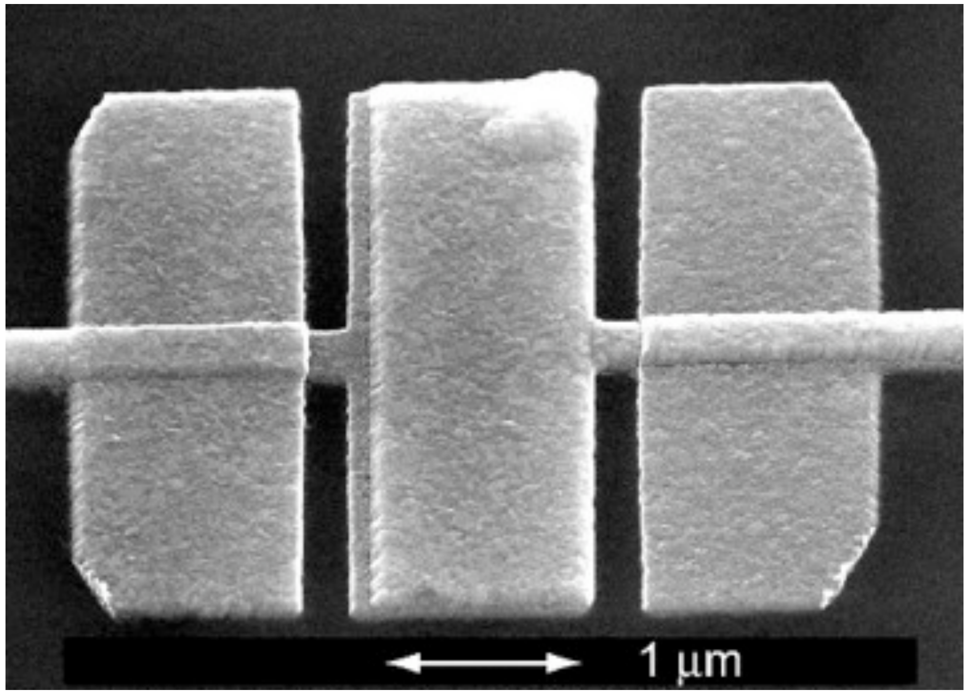
Echo: Nakamura et al., PRL 2001

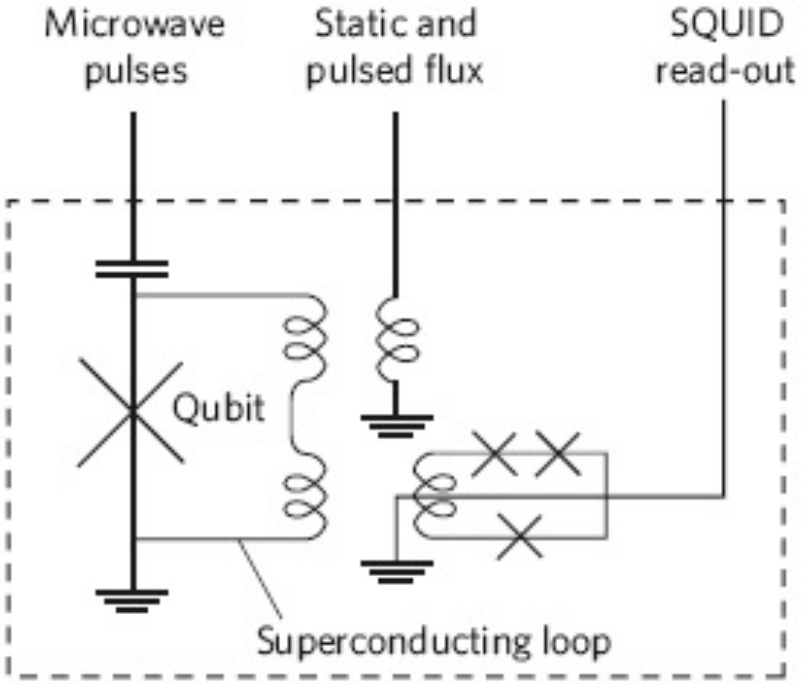


Sources of decoherence

Josephson junction



Circuitry: Under control



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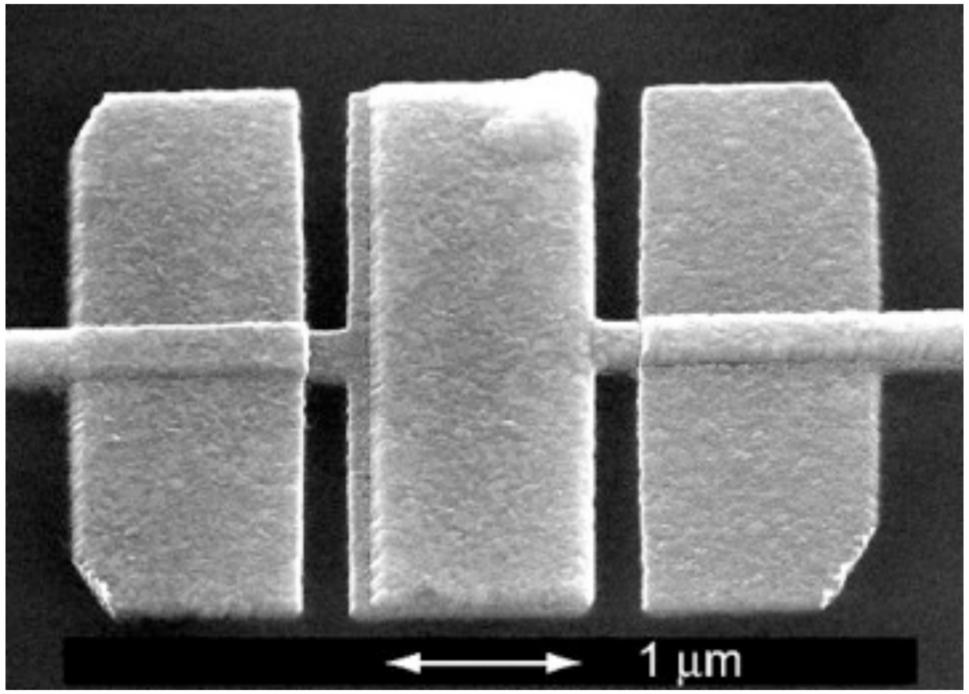

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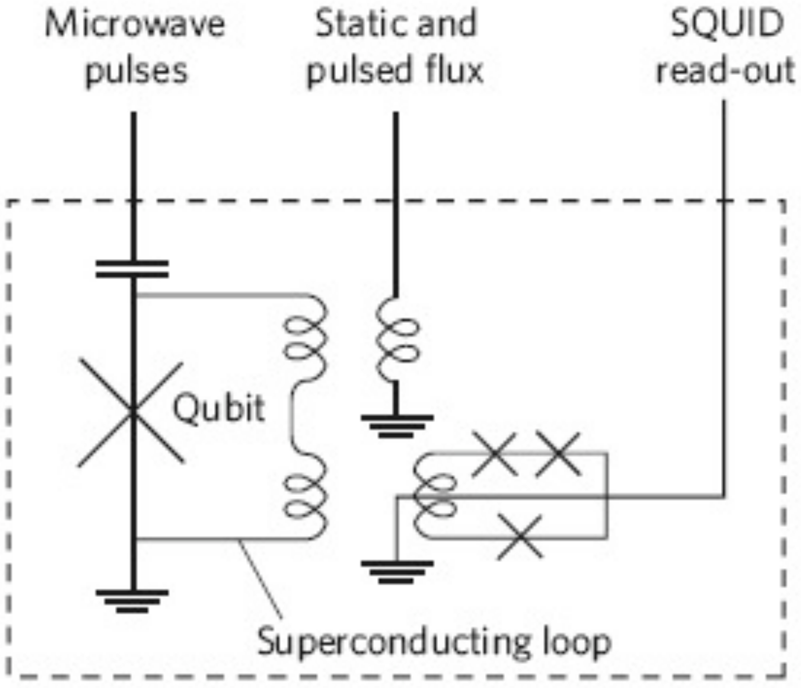
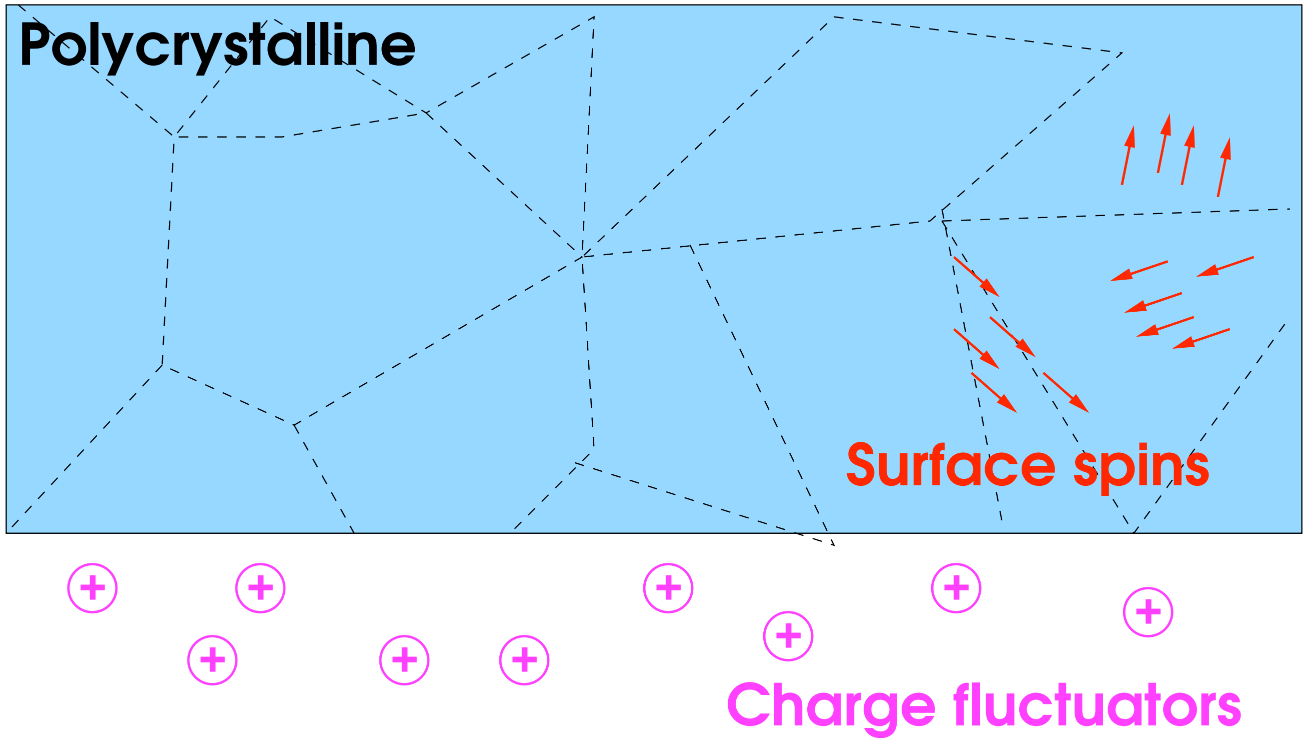


Real superconductor



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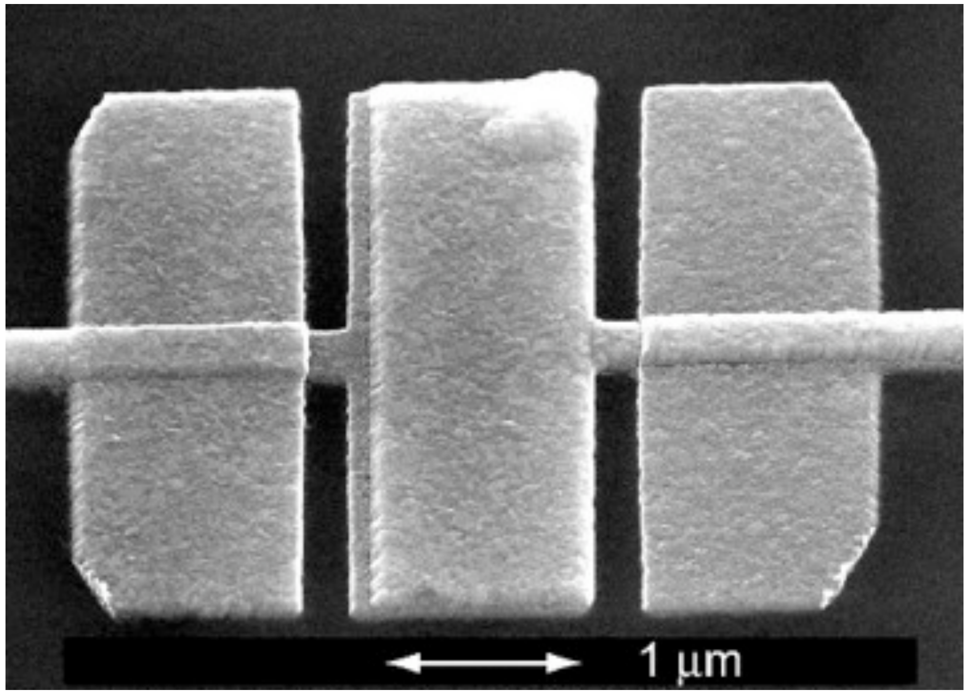

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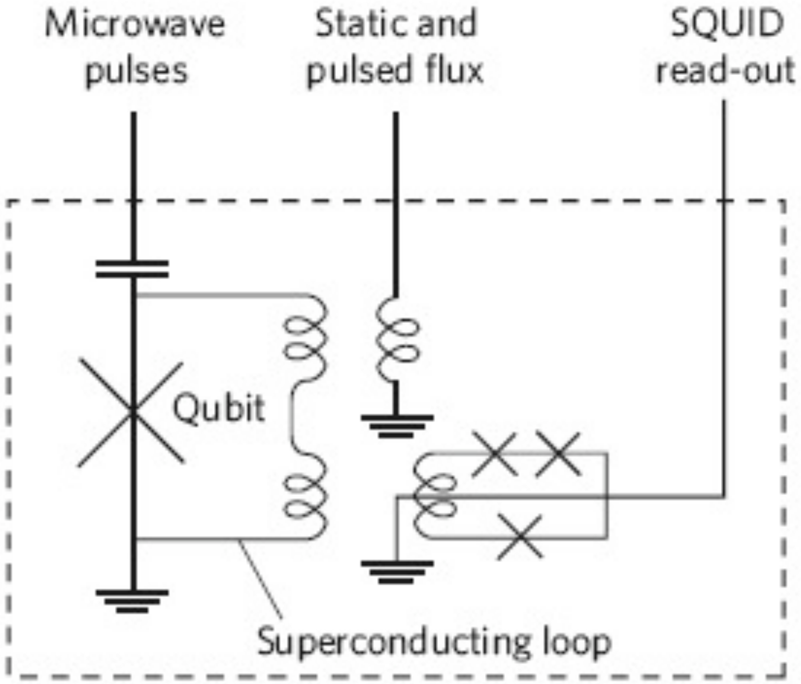

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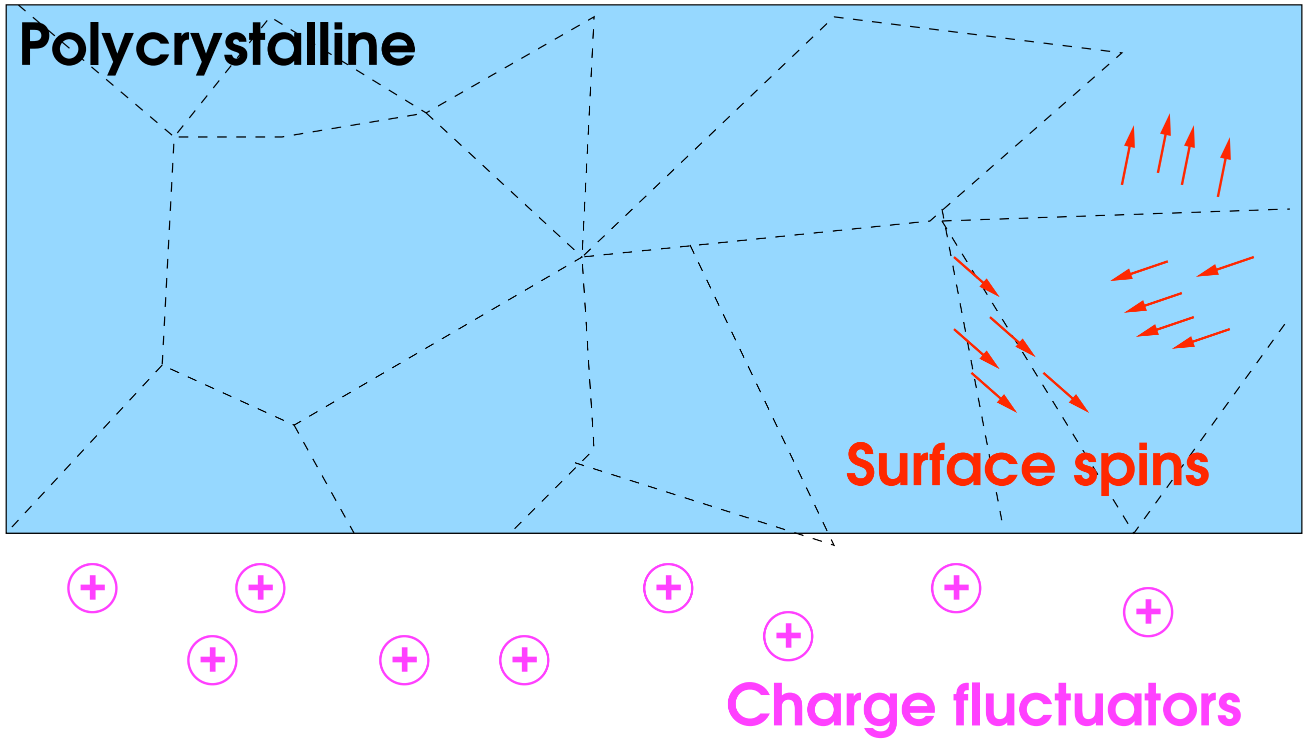
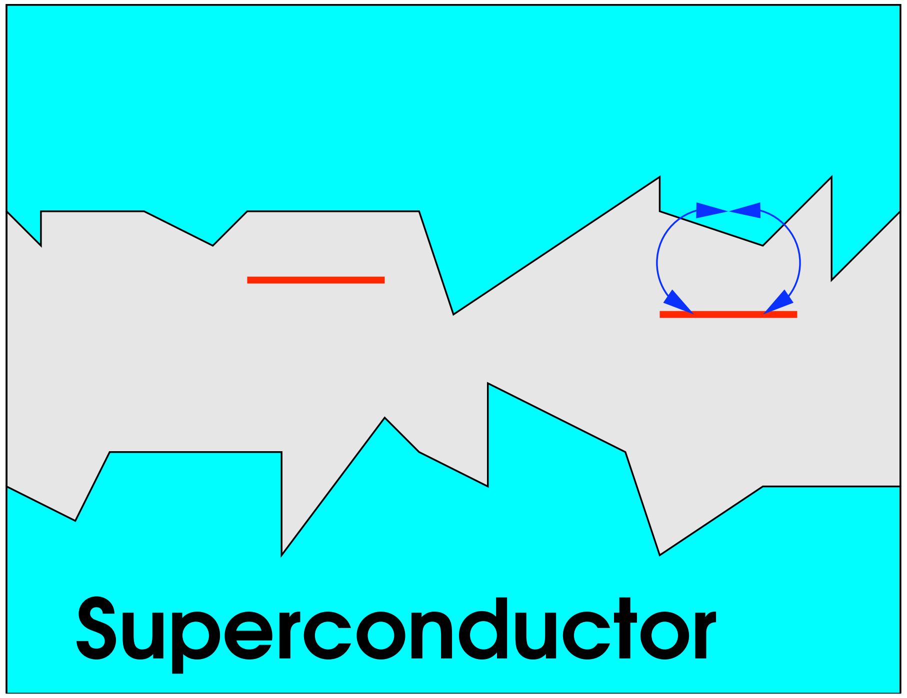
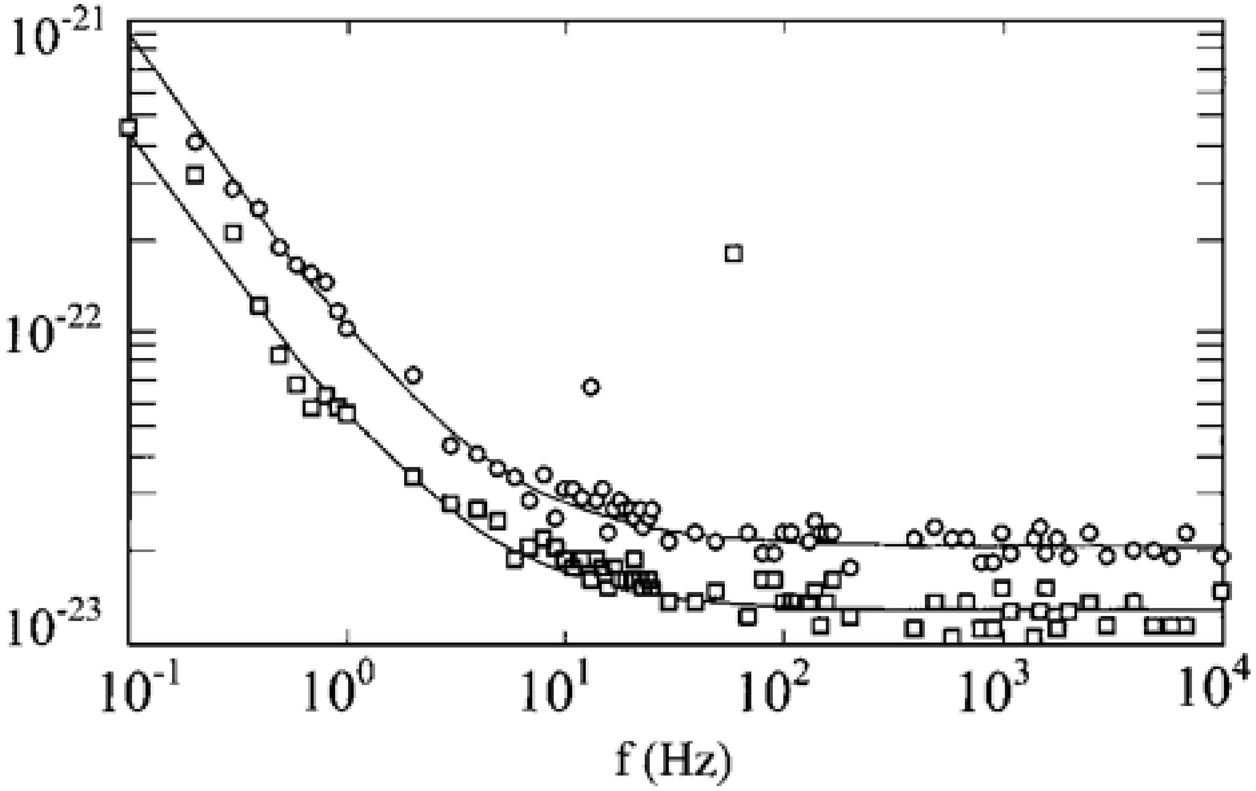
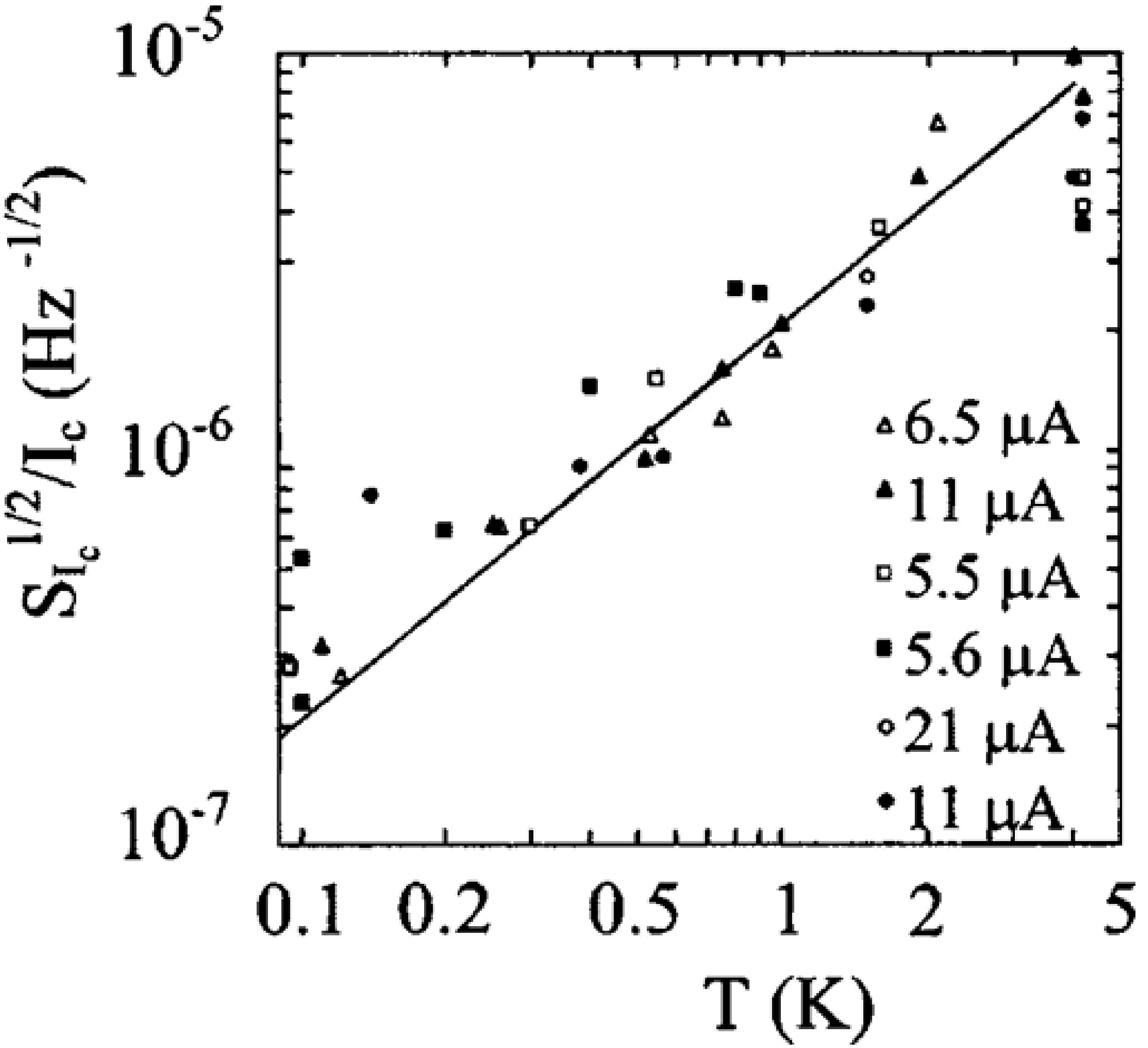


Real junction layer



Critical current noise in SQUIDs

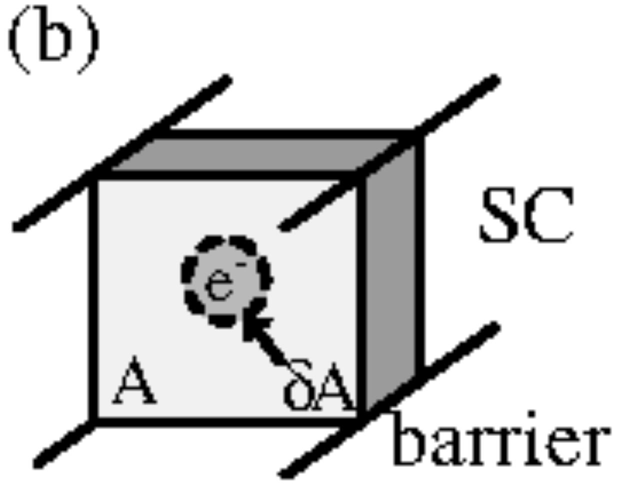




Wellstood, Urbina, and Clarke, APL 2004 (data taken in 1980s)

Impact on qubits

Trapping mechanism



van Harlingen et al., PRB 2004

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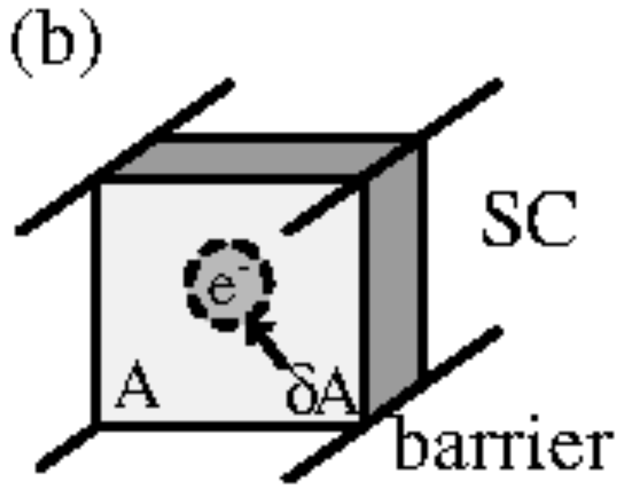
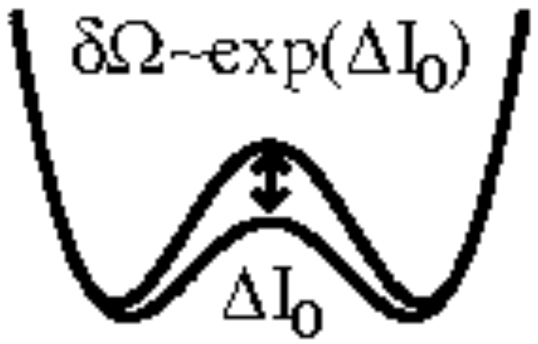


Fluctuation of barrier



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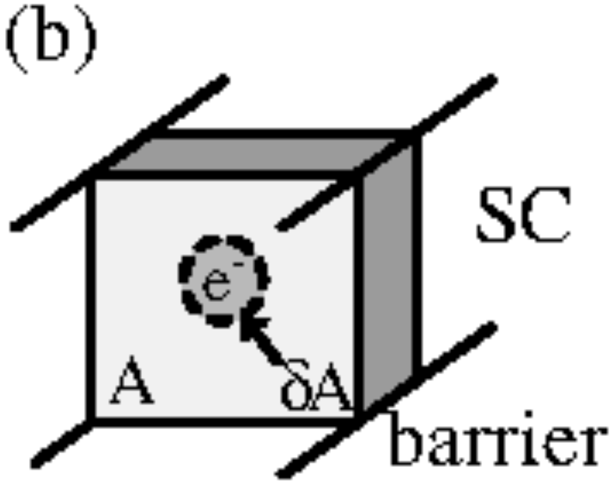

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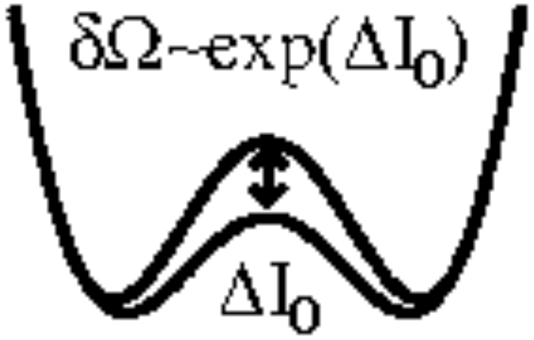

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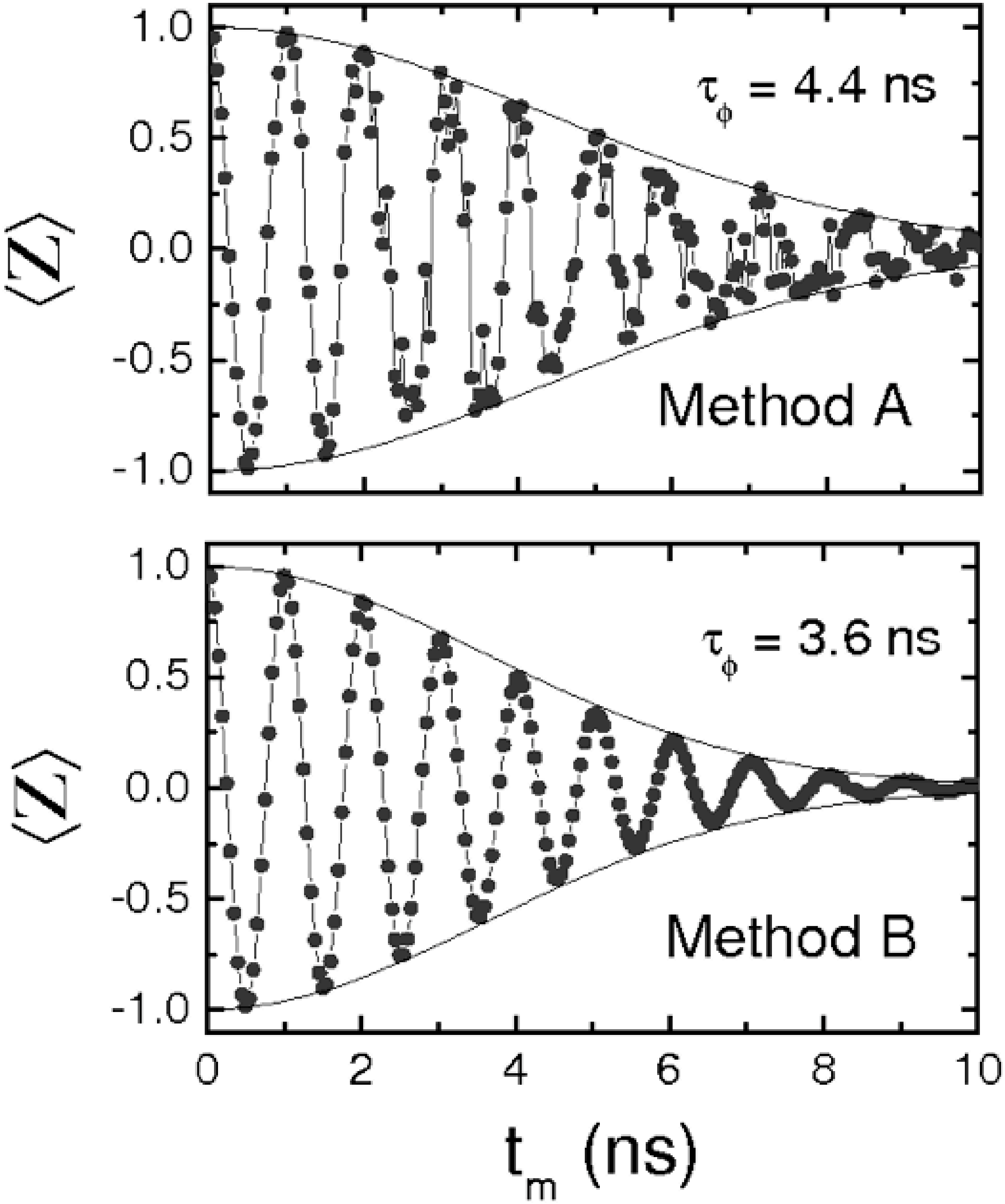
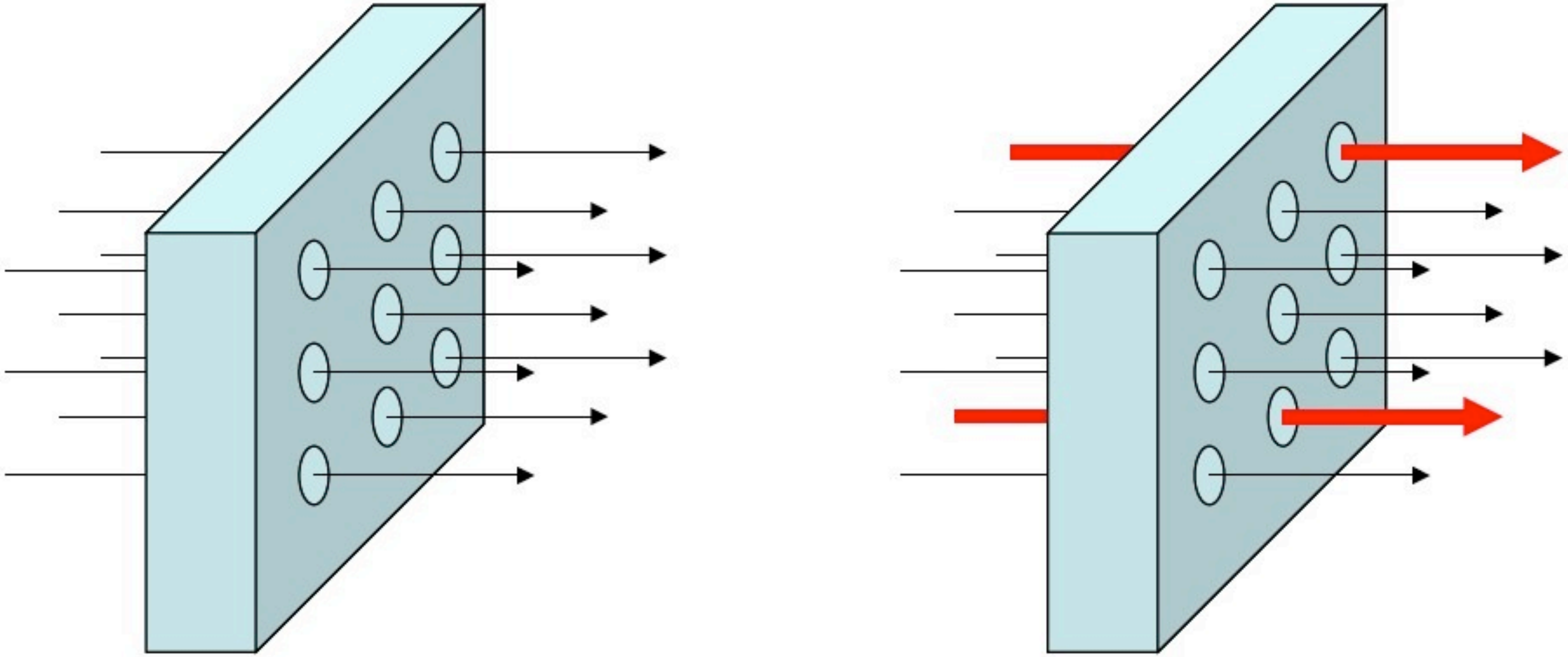


Trap noise in Josephson

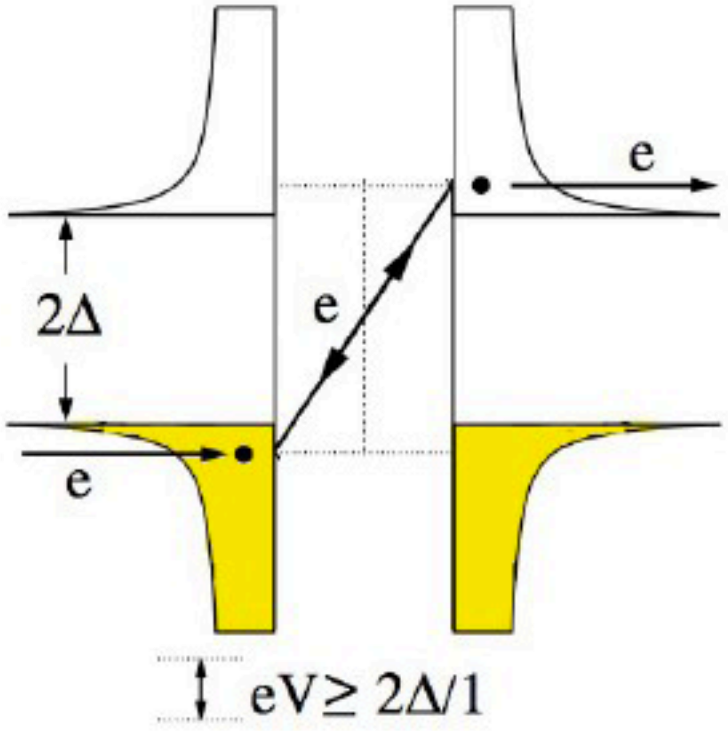
- SQubits and noise
- Surface roughness
- Noninteracting traps
- Interacting traps
- quasiparticles

Junction roughness

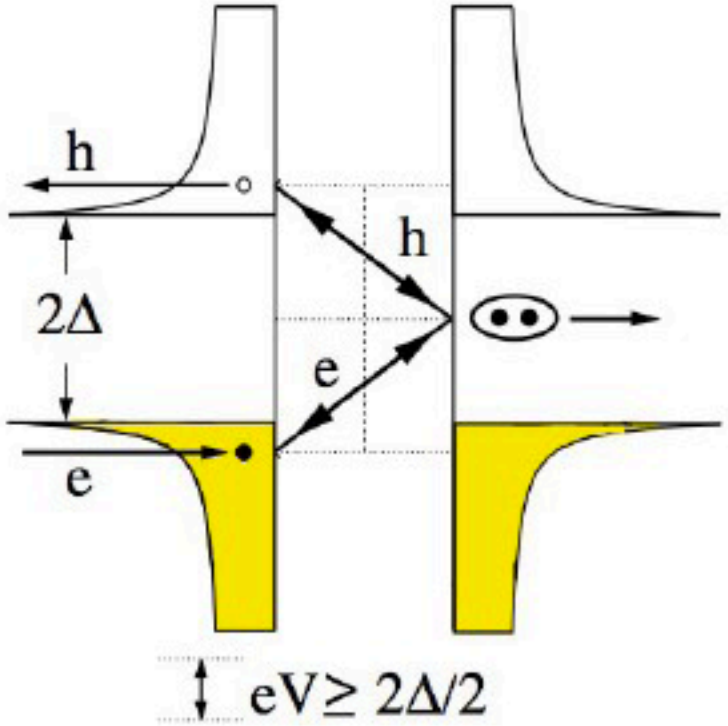


- Transmission coefficients for tunnel jct $T \ll 1$
- phase qubit: $\simeq 10^6$ channels, $T \approx 0.003$
- Rough junction $T \propto e^{-d\sqrt{2m(U-E)}} \lesssim 1$ disordered *d*

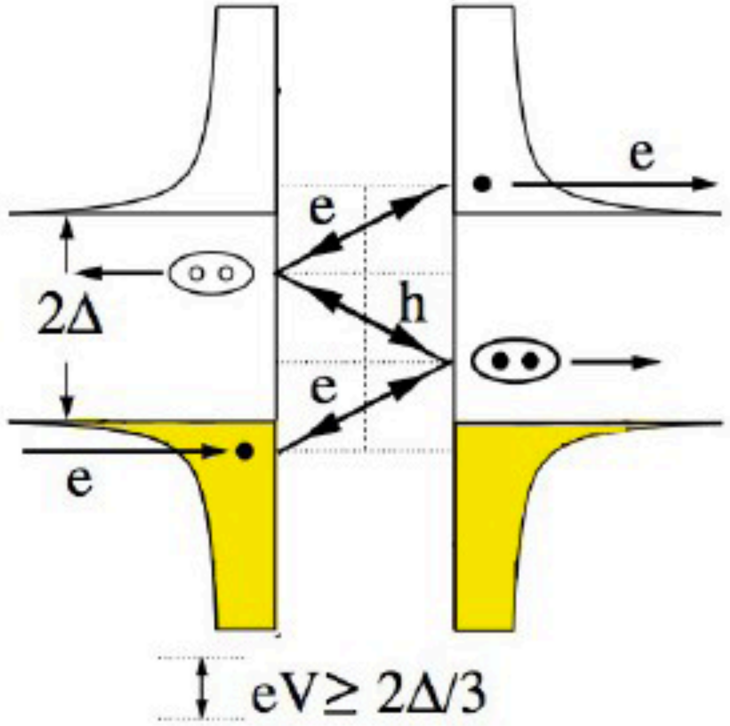
Multiple Andreev reflection



transferred charge 1e

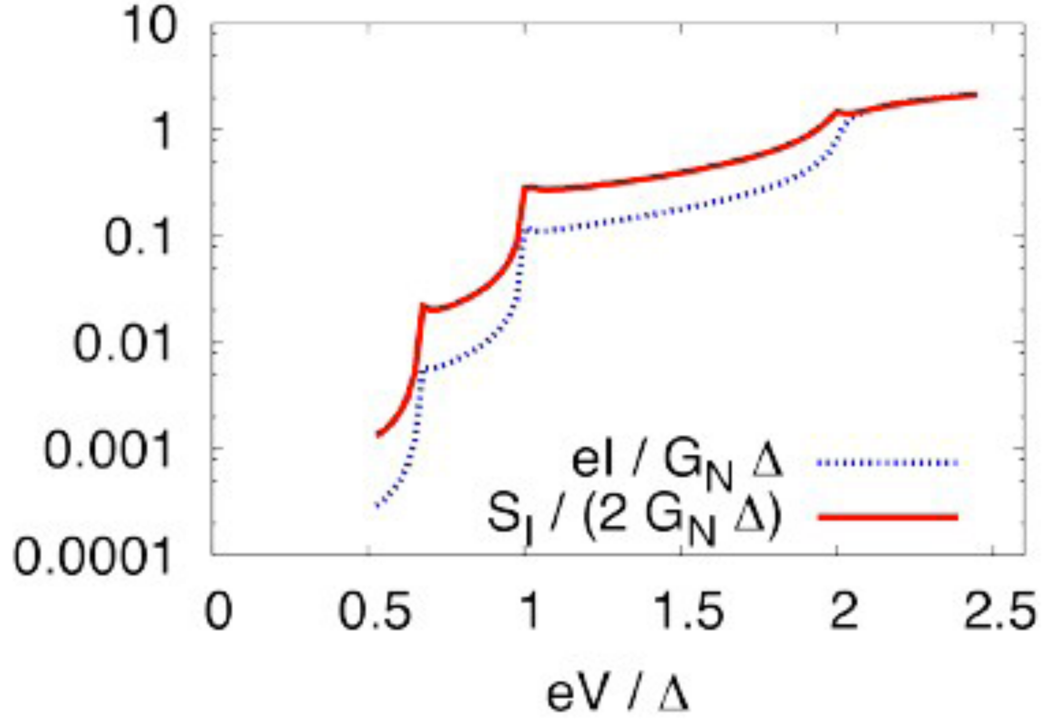


transferred charge 2e



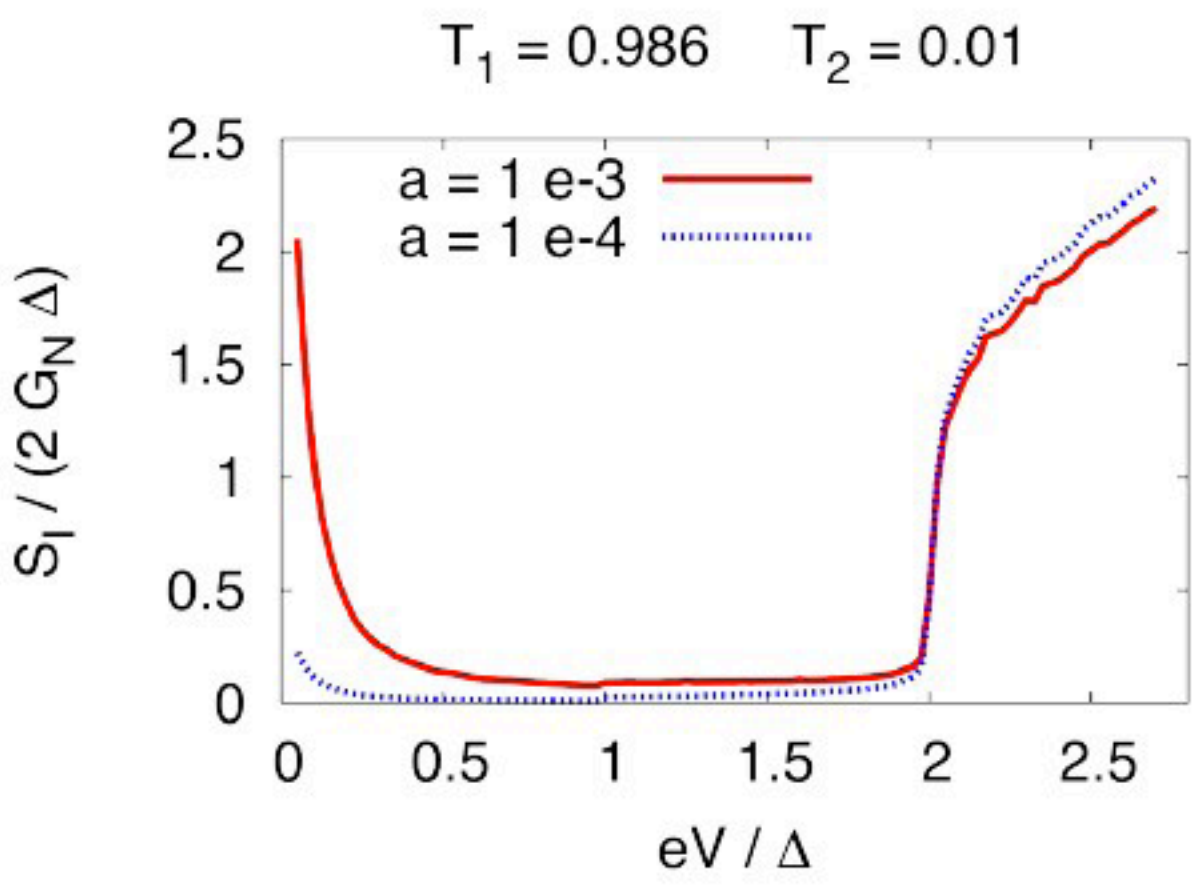
transferred charge 3e

- Higher order process T^{2n}
- subgap voltage $V_n = \dfrac{2\Delta}{en}$
- size of charge packet $q^* = en$
- Poissonian shot noise $S_I = 2q^* I$

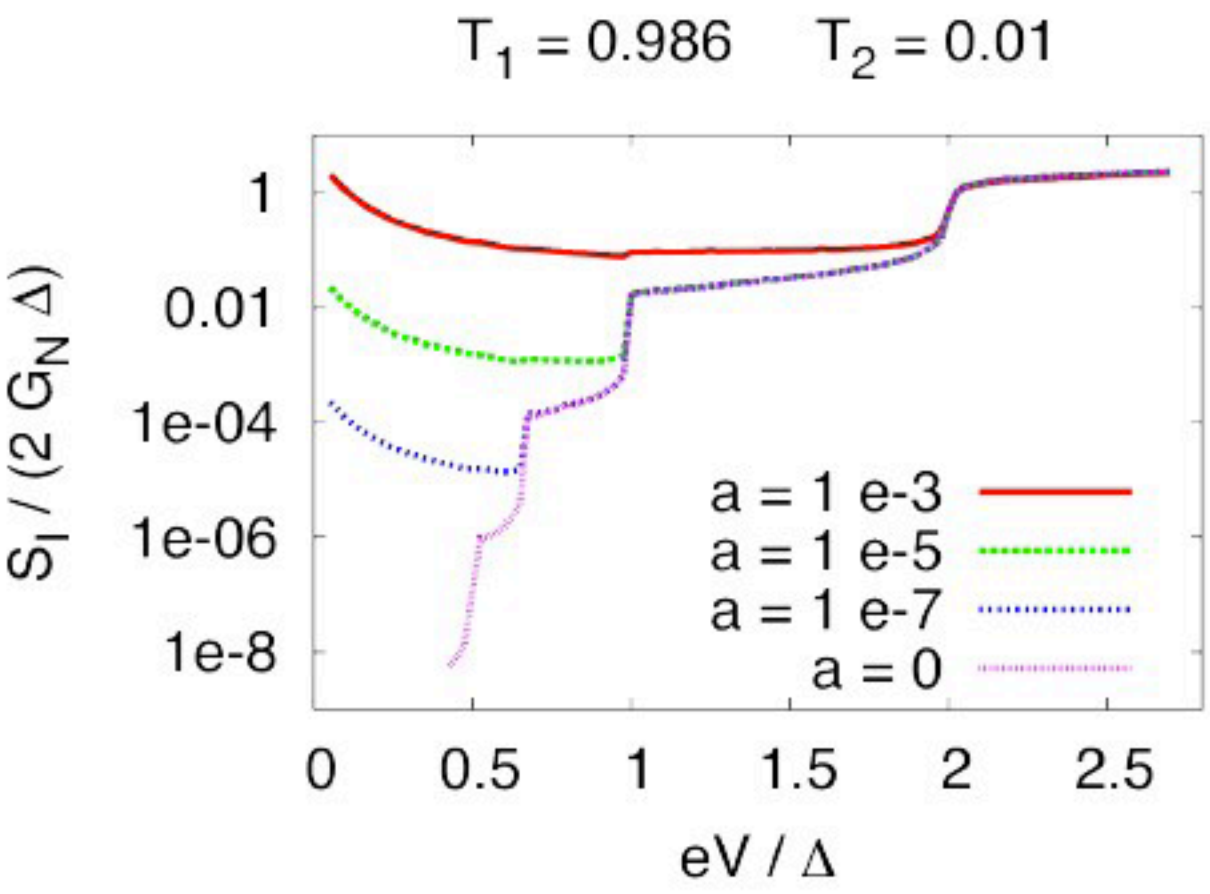


T = 0.1

Shot noise at high T



$T_1 = 0.986$ $T_2 = 0.01$



$T_1 = 0.986$ $T_2 = 0.01$

- low V → large n → large q* → strong noise
- dominates noise even with low pinhole fraction a
- todo: $V=0$, $\omega>0$

G. Heinrich and FKW, PRB 2009

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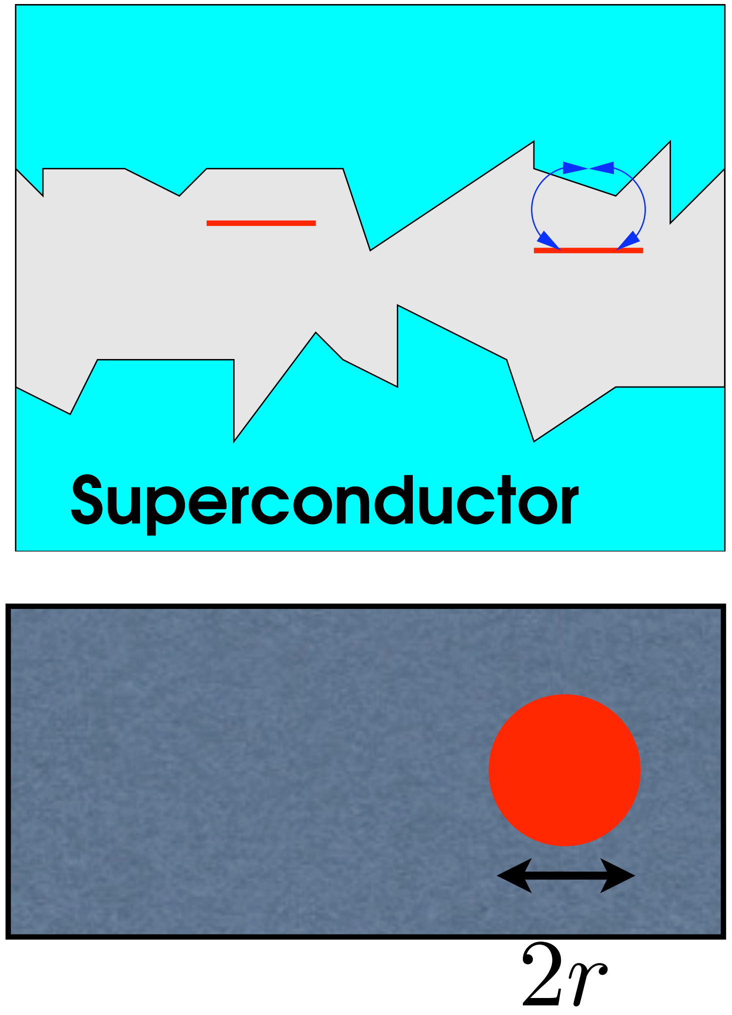
Trap noise physics



- Trap occupation $\hat{n}$
- occupied trap charges barrier: blocks charge
- radius $\delta A = \pi r^2$
- Critical current noise

$$S_I = \langle \delta I_c(t) \delta I(t) \rangle \propto \langle \hat{n}(t) \hat{n}(0) \rangle$$

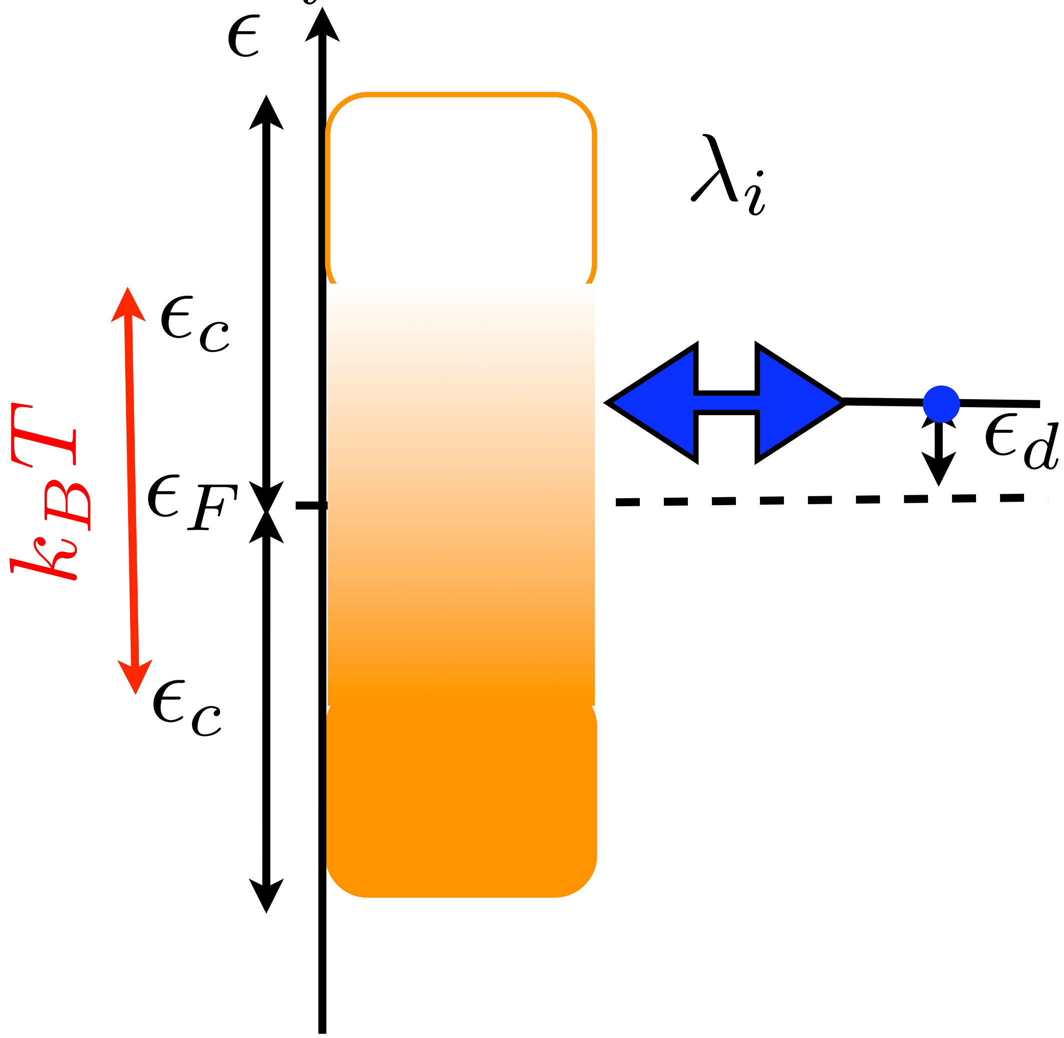
- Also: Charge noise

Multi-trap mechanism: Faoro et al., 2005; Shnirman et al., 2004

Normal conducting charge trap

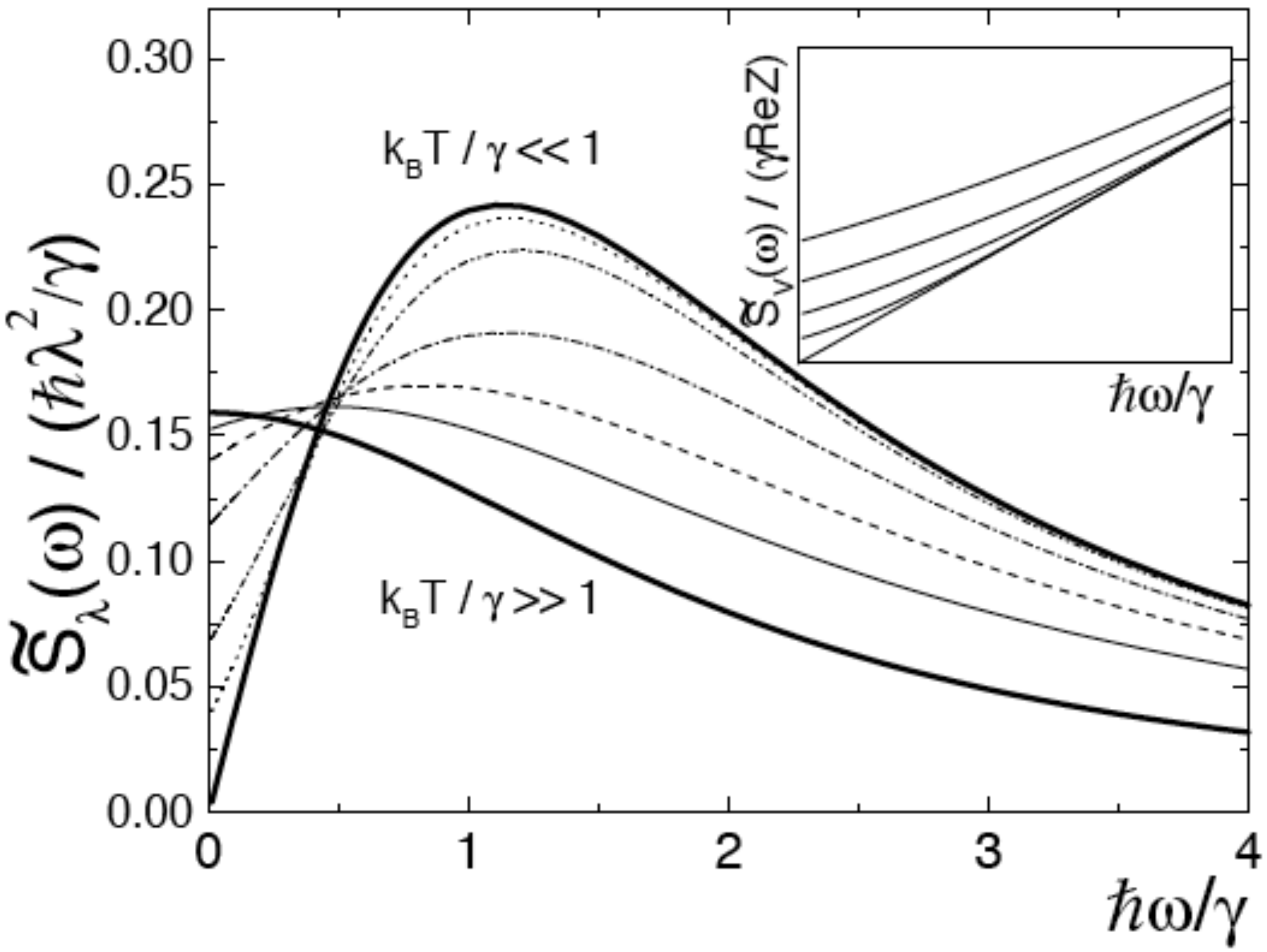
$$\hat{H} = (\epsilon_d - \epsilon_F)\hat{d}^\dagger \hat{d} + \sum_i \epsilon_i \hat{c}_i^\dagger \hat{c}_i + \sum_i \left(\lambda_i \hat{d}^\dagger \hat{c}_i + \text{h.c.}\right)$$

- Fano-Anderson Hamiltonian
- Bath Fermions $\hat{c}_i$
- Local level, $\hat{d}$ infinite repulsion
- Bandwidth $|\epsilon_i - \epsilon_F| < \epsilon_c$



Solution: Bogoliubov transform, compound Fermions

Noise crossover



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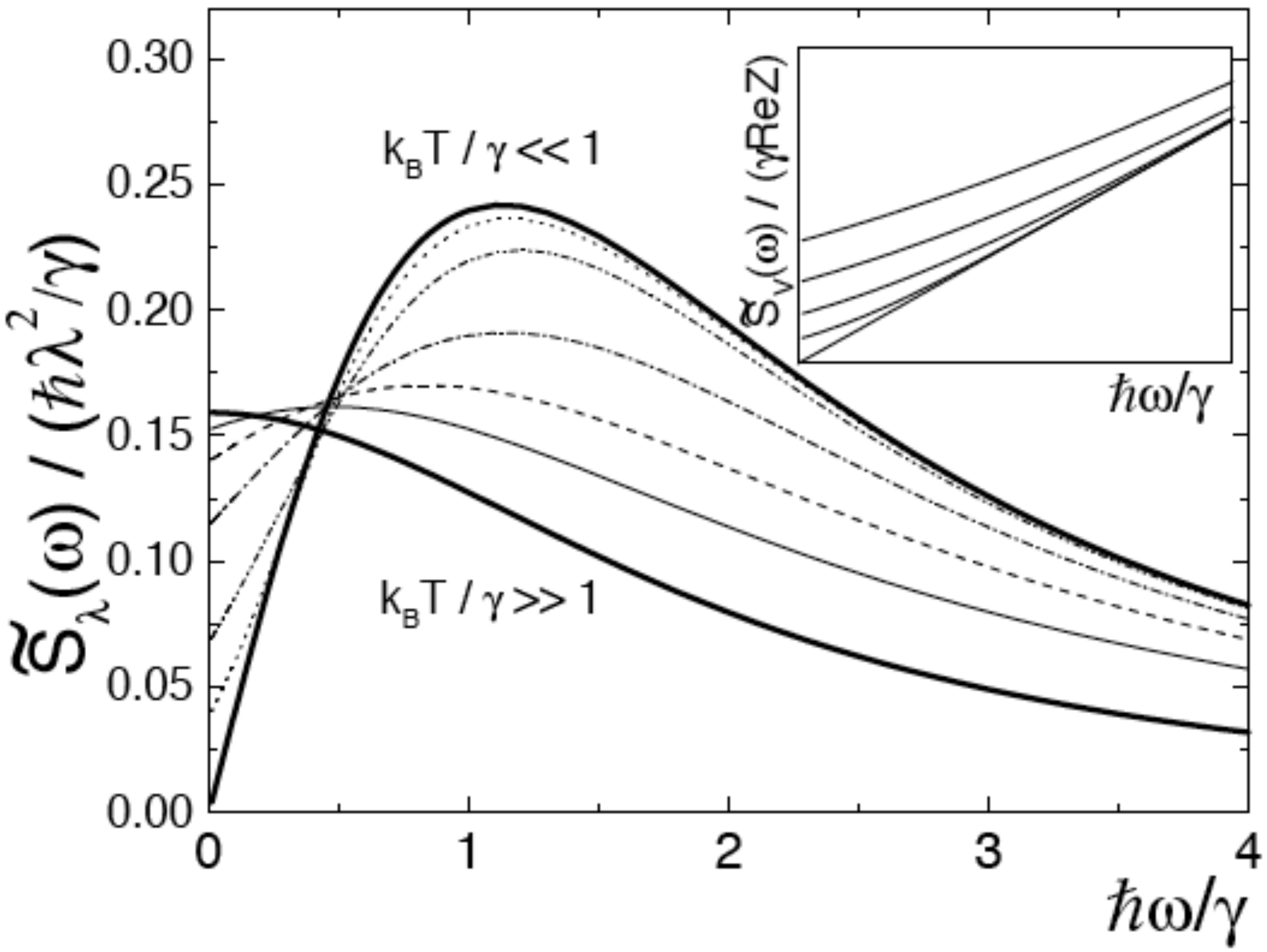

- Weak coupling: Lorentzian centered at f=0
- Strong coupling: f-noise

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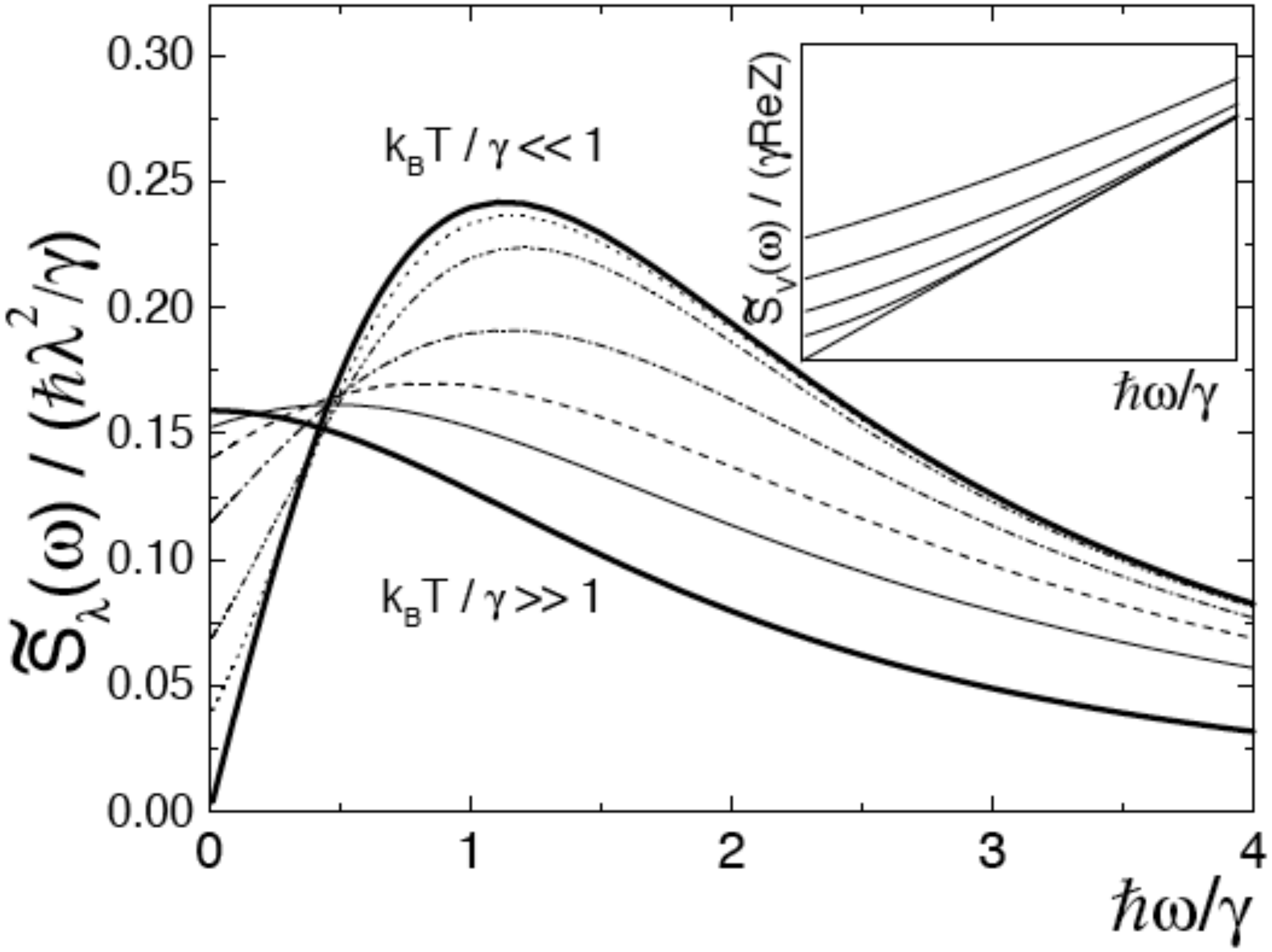

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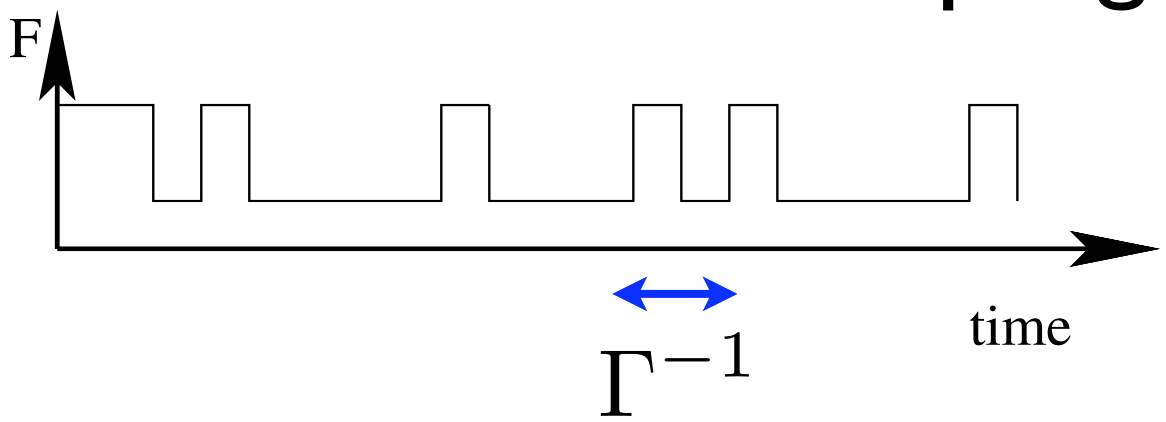

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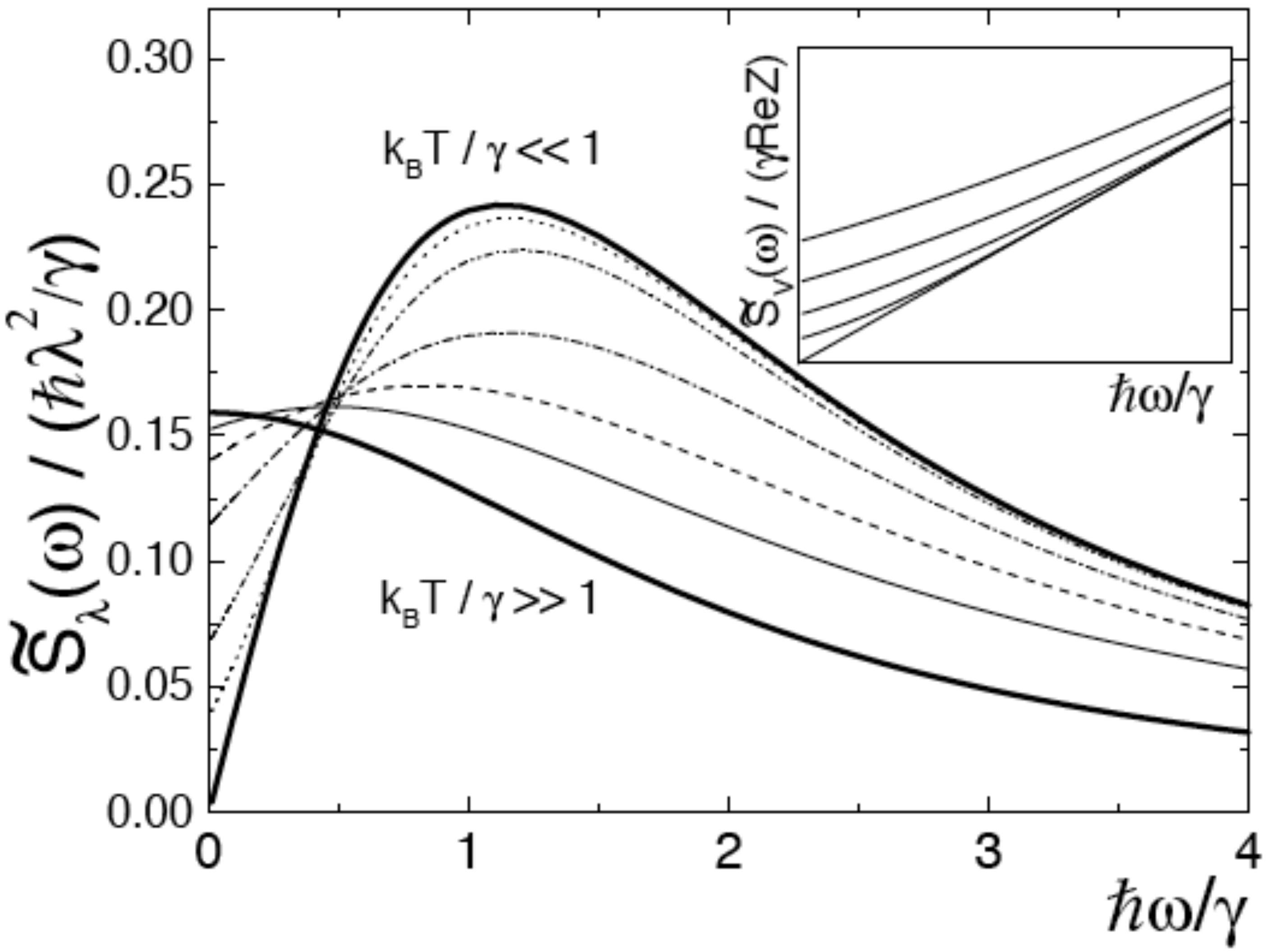

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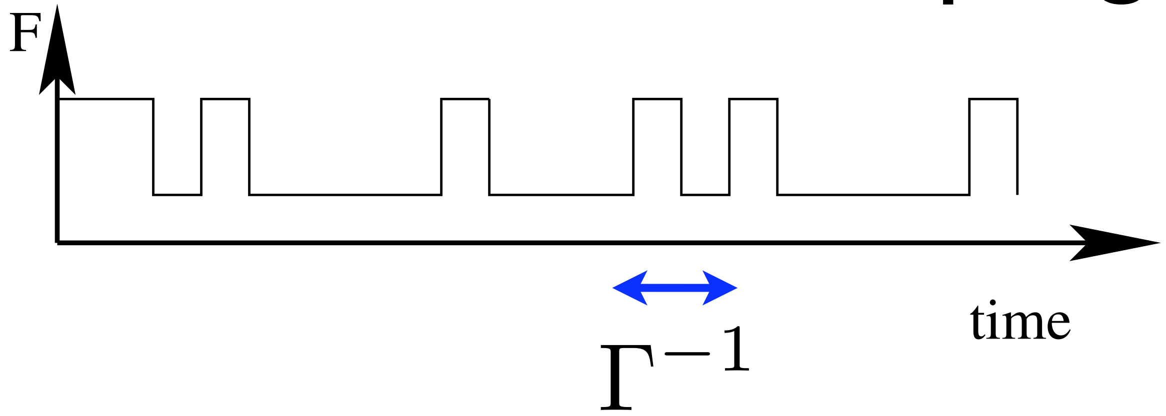
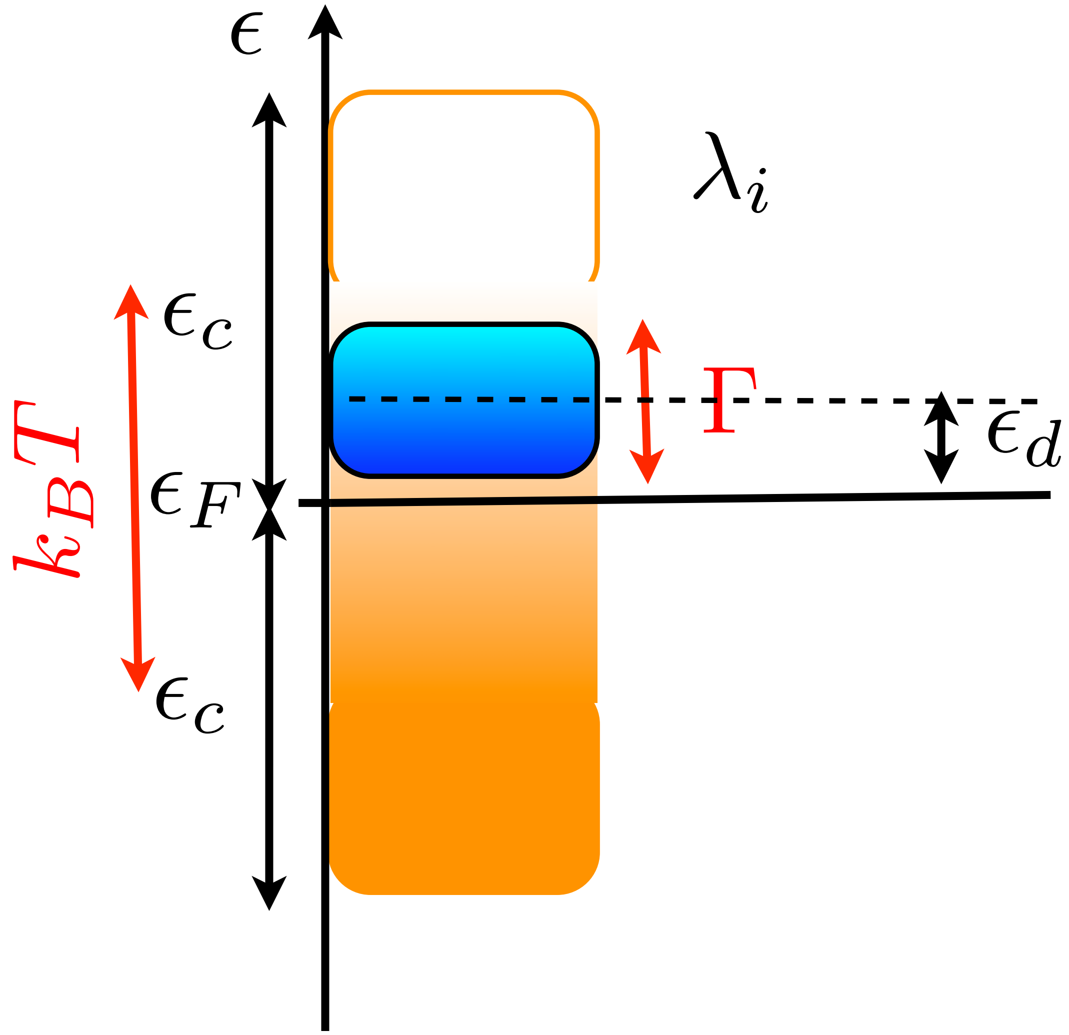


Strong coupling



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Superconductor+Trap

$$\hat{H} = \sum_{\sigma} \epsilon_d \hat{n}_\sigma + \sum_{k,\sigma} \epsilon_k \hat{n}_{k,\sigma}$$

$$-\sum_{k} \Delta \hat{c}^\dagger_{k\uparrow} \hat{c}^\dagger_{-k\downarrow} + \mathrm{h.c.} + \sum_{k,\sigma} \lambda_k \hat{d}^\dagger_\sigma \hat{c}_{k\sigma} + \mathrm{h.c.}$$

- Mean field BCS
- two couplings: BCS +hopping
- Still quadratic Hamiltonian
- Noninteracting impurity

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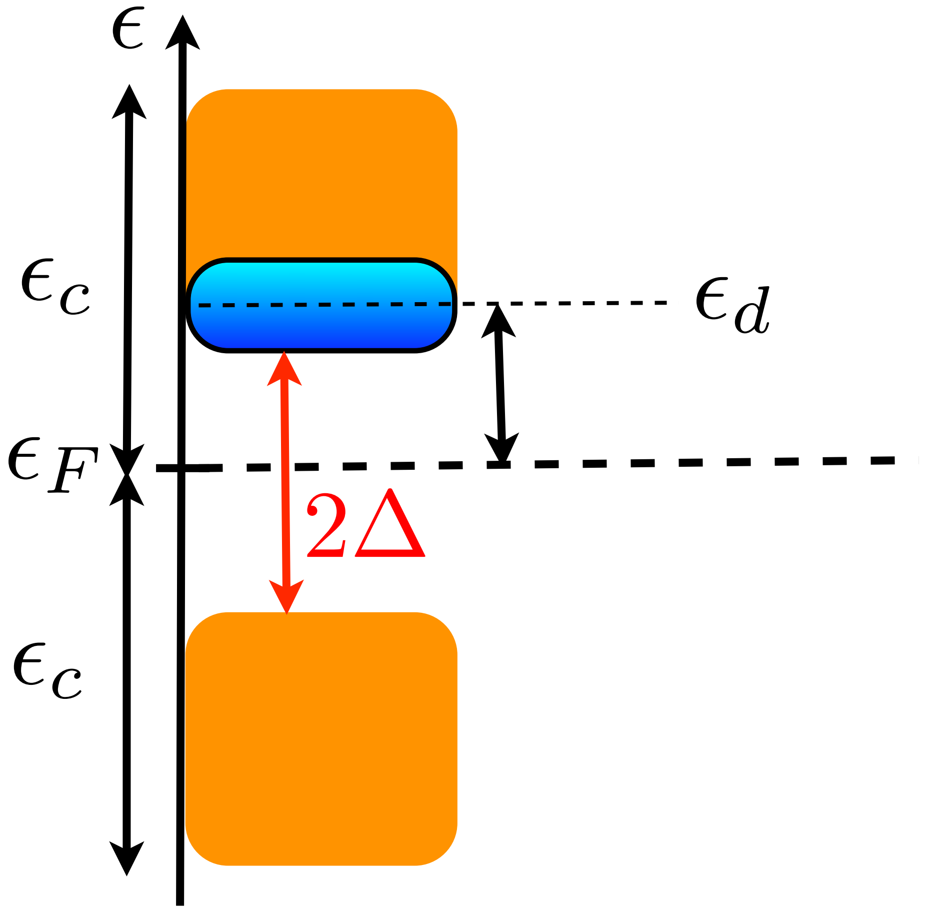

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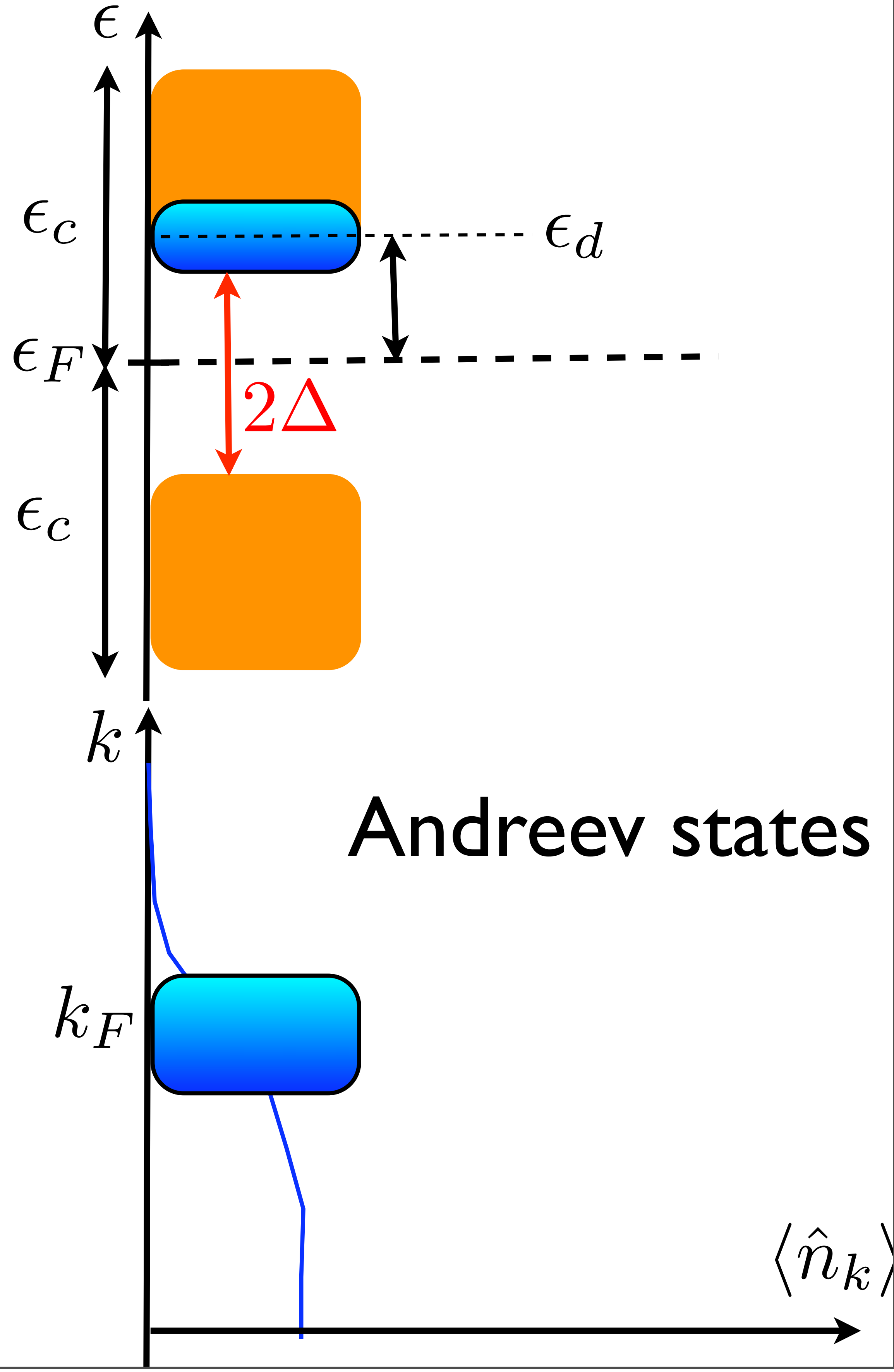
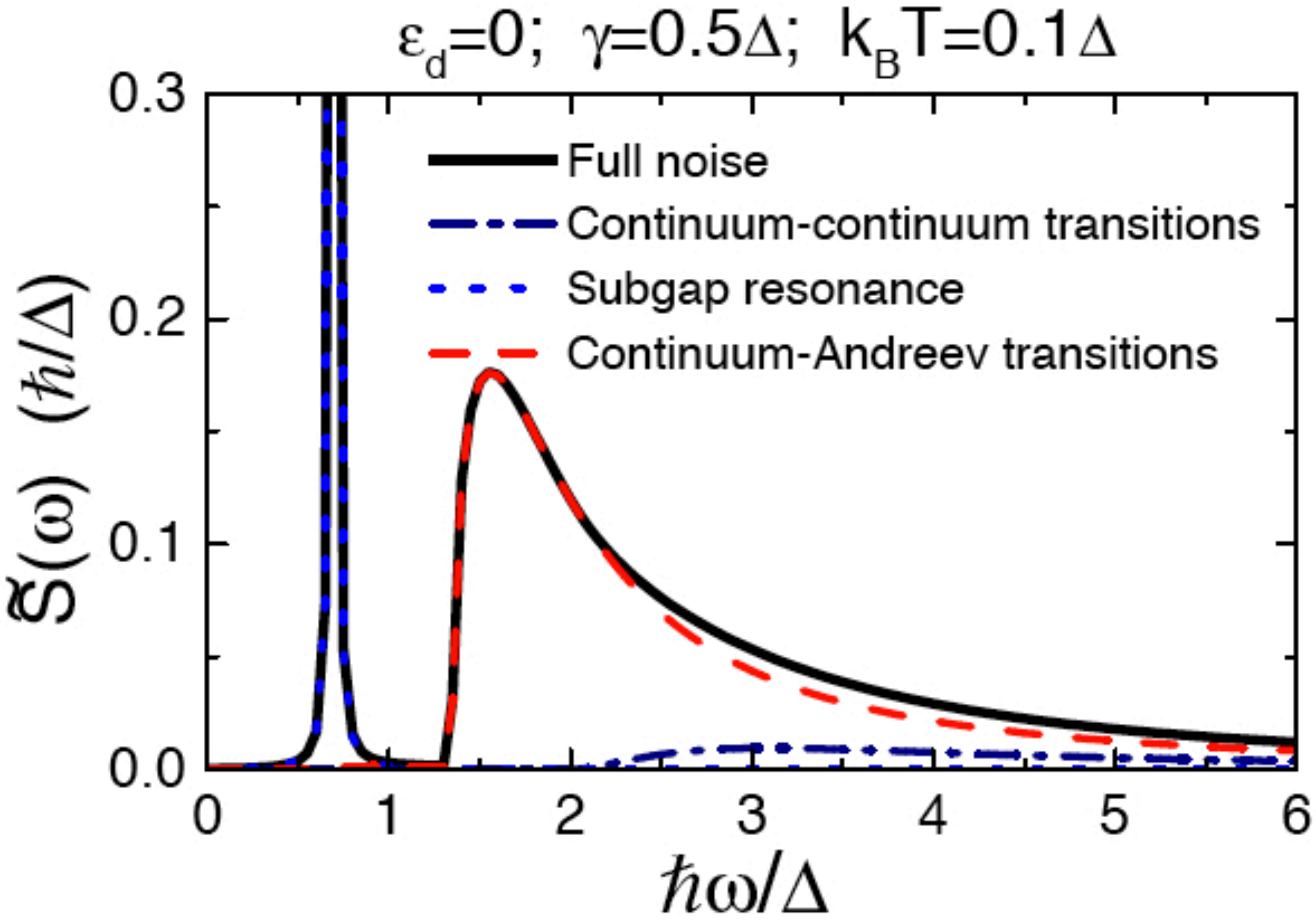
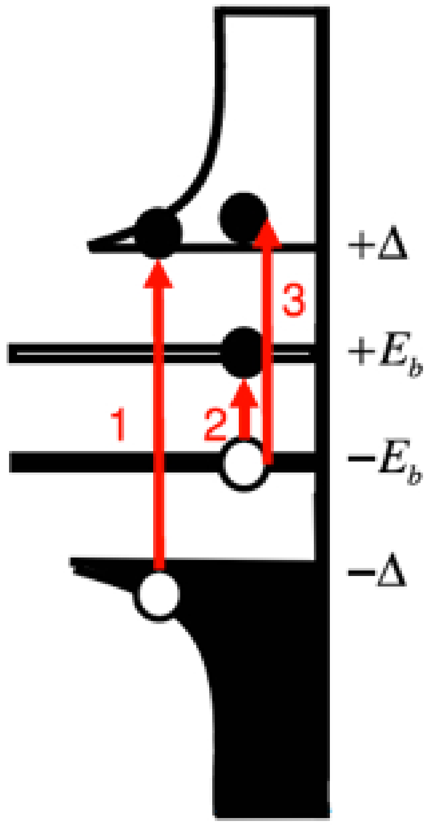


Andreev states

Noise channels

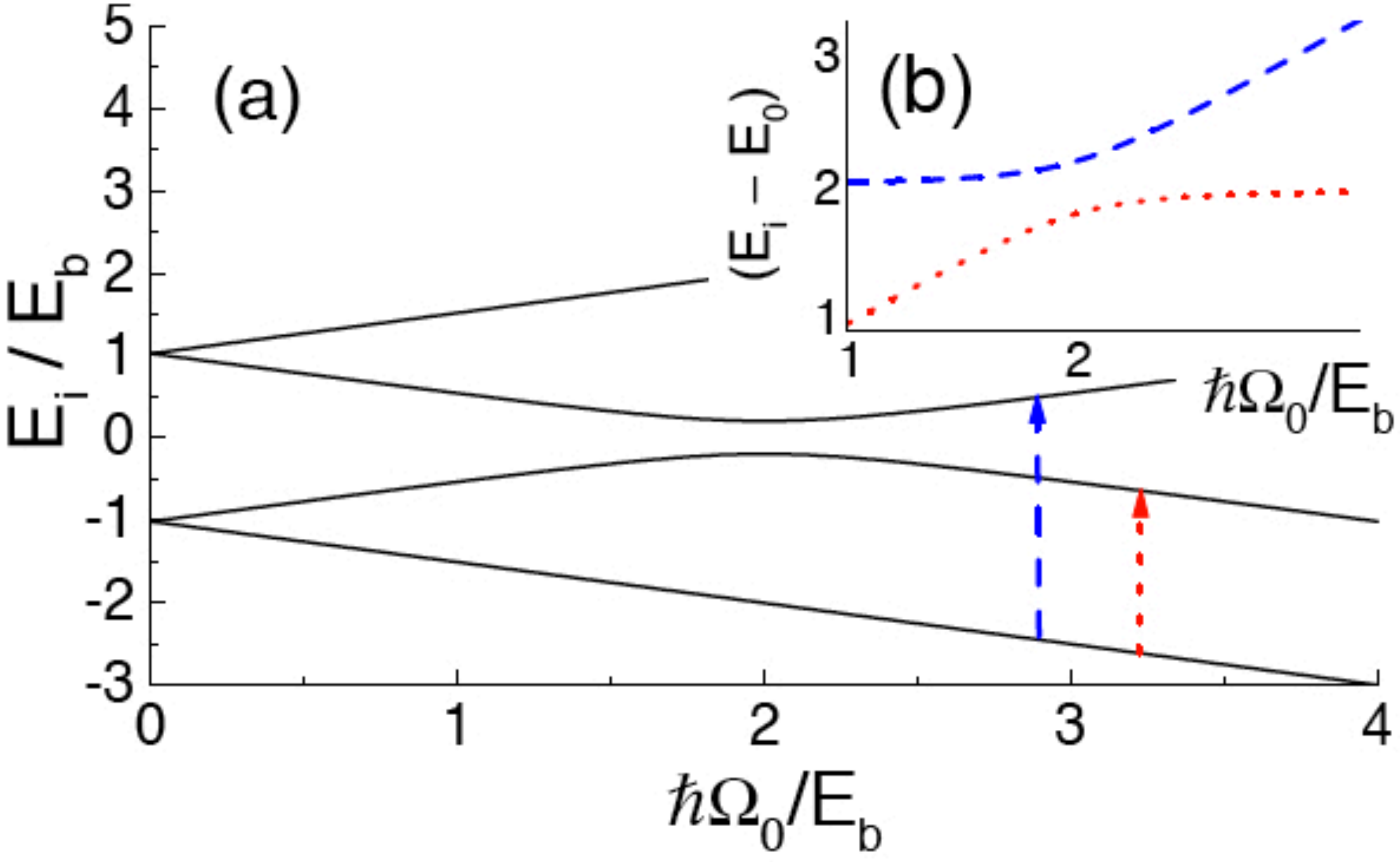




1. Continuum-continuum
2. Subgap resonance
3. Andreev-continuum

- Noise channels between Andreev Bound States and continuum
- analytical results

Junction resonators from traps



- Impact of noise peak on qubit
- Avoided crossing between qubit and pair of Andreev states

de Sousa, Hecht, von Delft, Whaley, FKW, PRB 2009

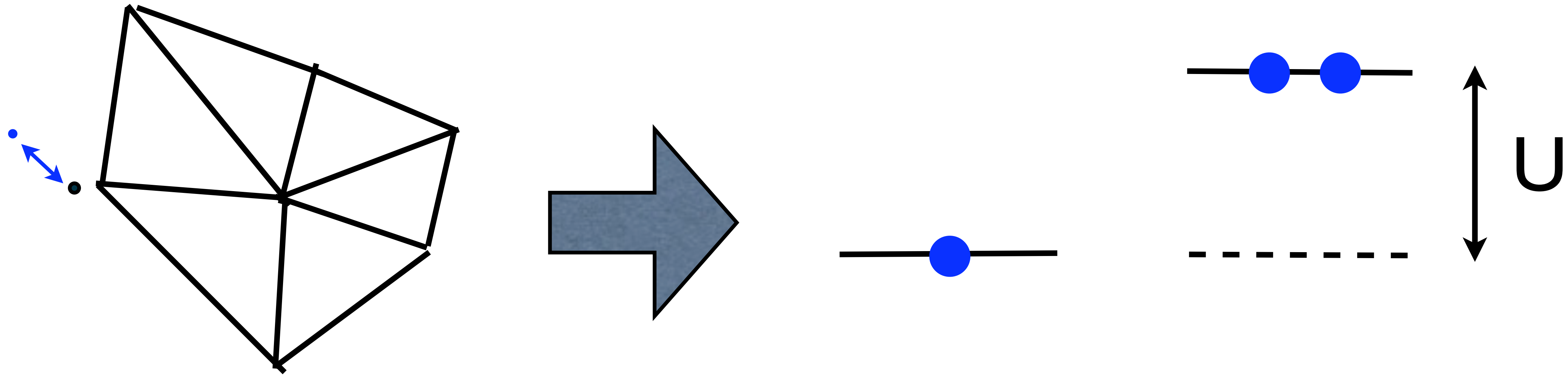
Trap noise in Josephson

- SQubits and noise
- Surface roughness
- Noninteracting traps
- Interacting traps
- quasiparticles

Need to include U

Dangling bond

Large on-site repulsion



- Interacting Hamiltonian:

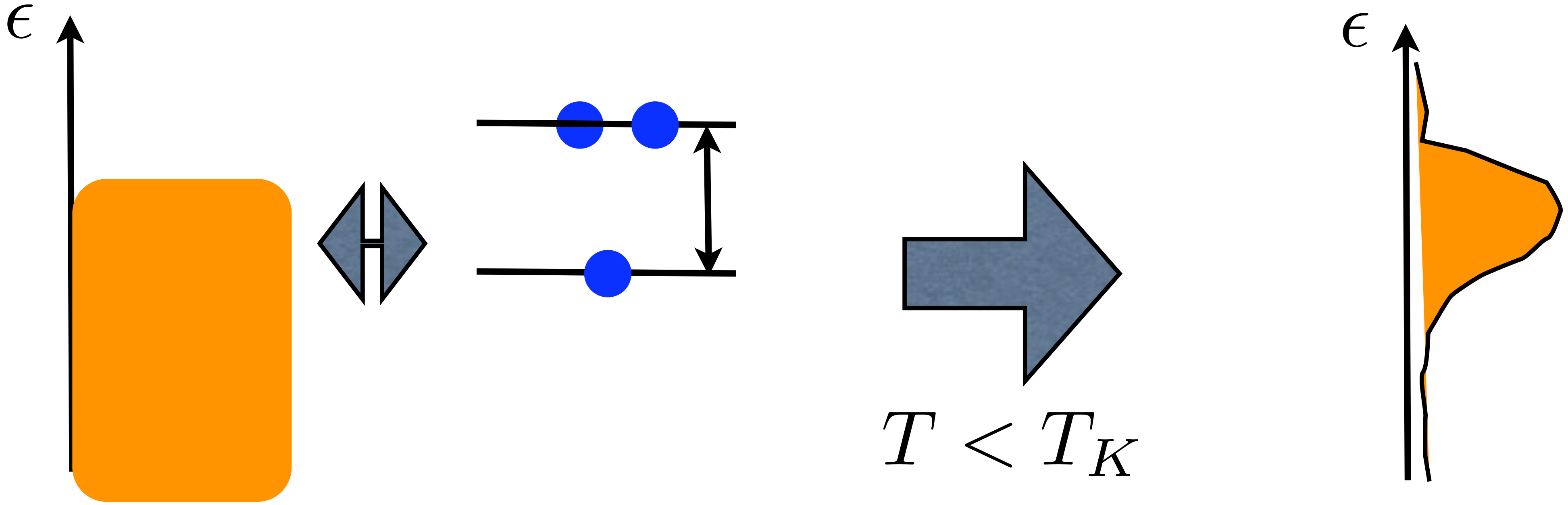
$$\hat{H} = \hat{H}_0 + U\hat{n}_{\downarrow}\hat{n}_{\uparrow}$$

- e-h-Symmetry point $\epsilon_d = -\frac{U}{2}$
- Generalization by interpolation

Cuevas et al., PRB 2001; Vecino et al., PRB 2003

Perturbative solution

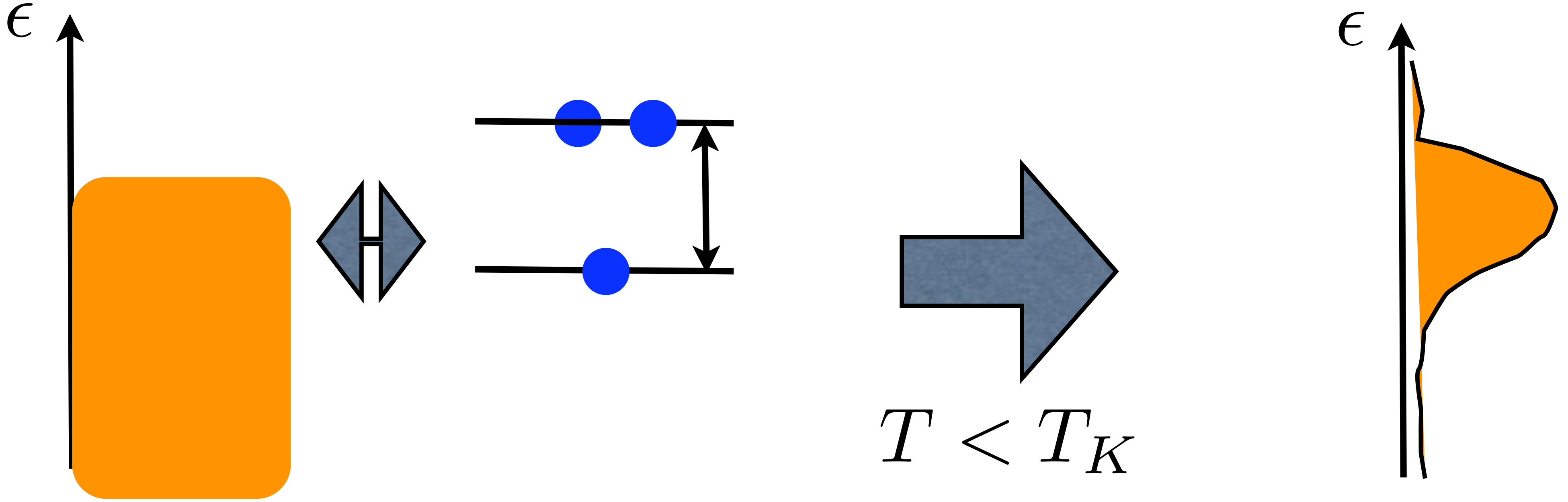
Normal electrodes: Kondo effect



Manybody
Kondo
resonance
at E_F

Perturbative solution

Normal electrodes: Kondo effect



Manybody
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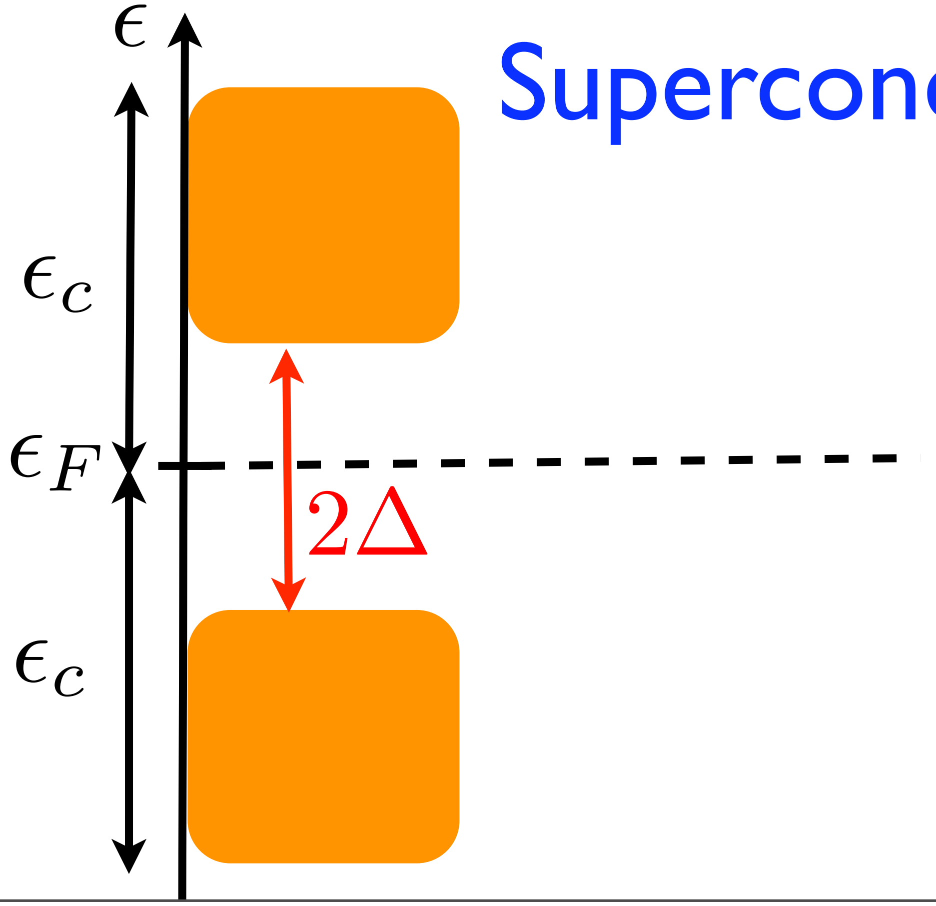
Superconducting electrodes: Low DOS at E_F

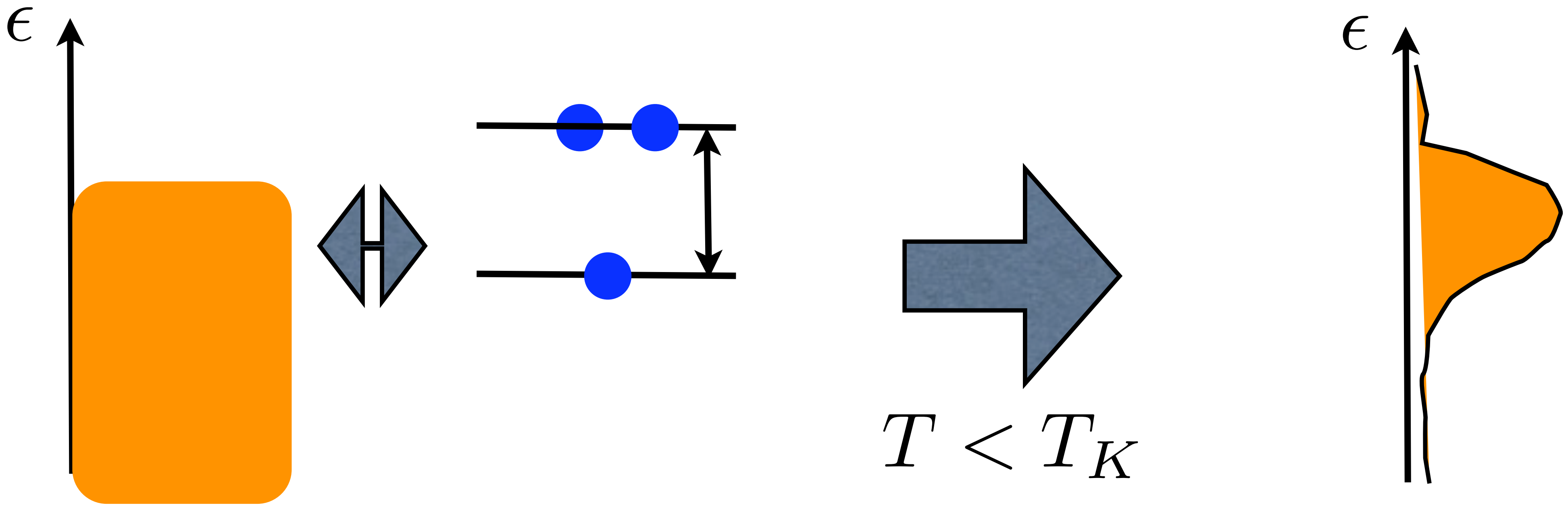


Extremely low Kondo temperature as long as $\Gamma \ll \Delta$

Perturbation theory in $\frac{U}{\Gamma}$ or $\frac{U}{\Delta}$

Perturbative solution

Normal electrodes: Kondo effect



Manybody
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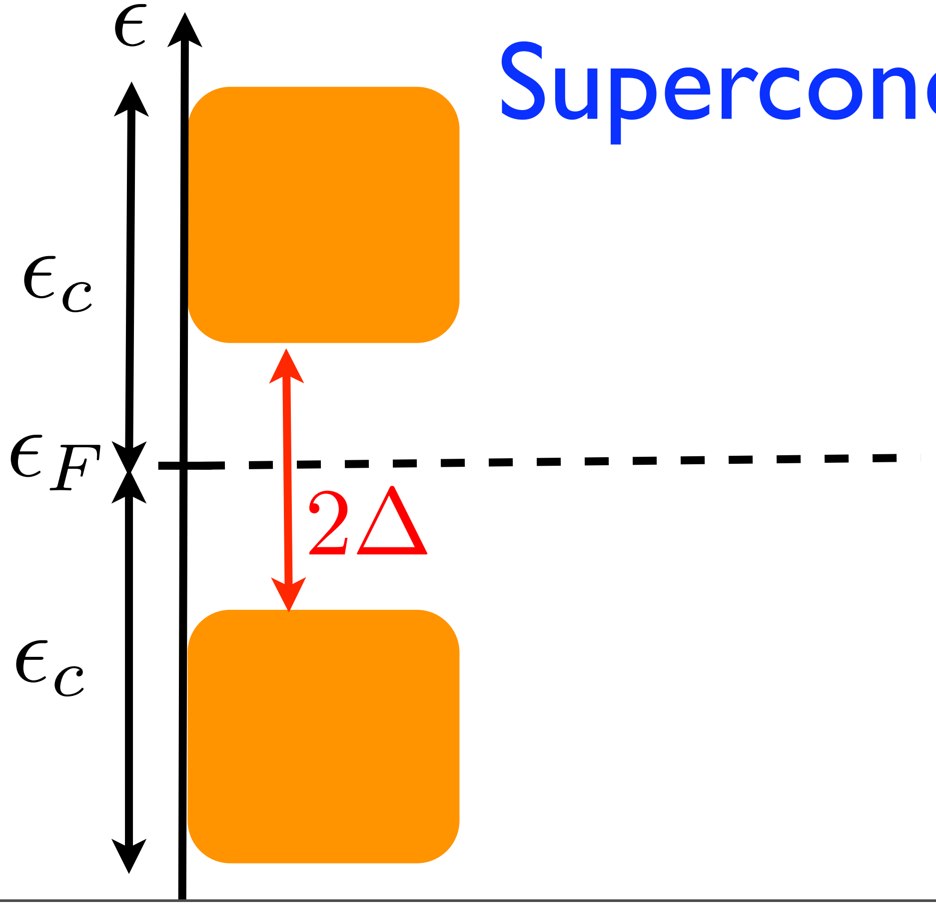


Superconducting electrodes: Low DOS at E_F

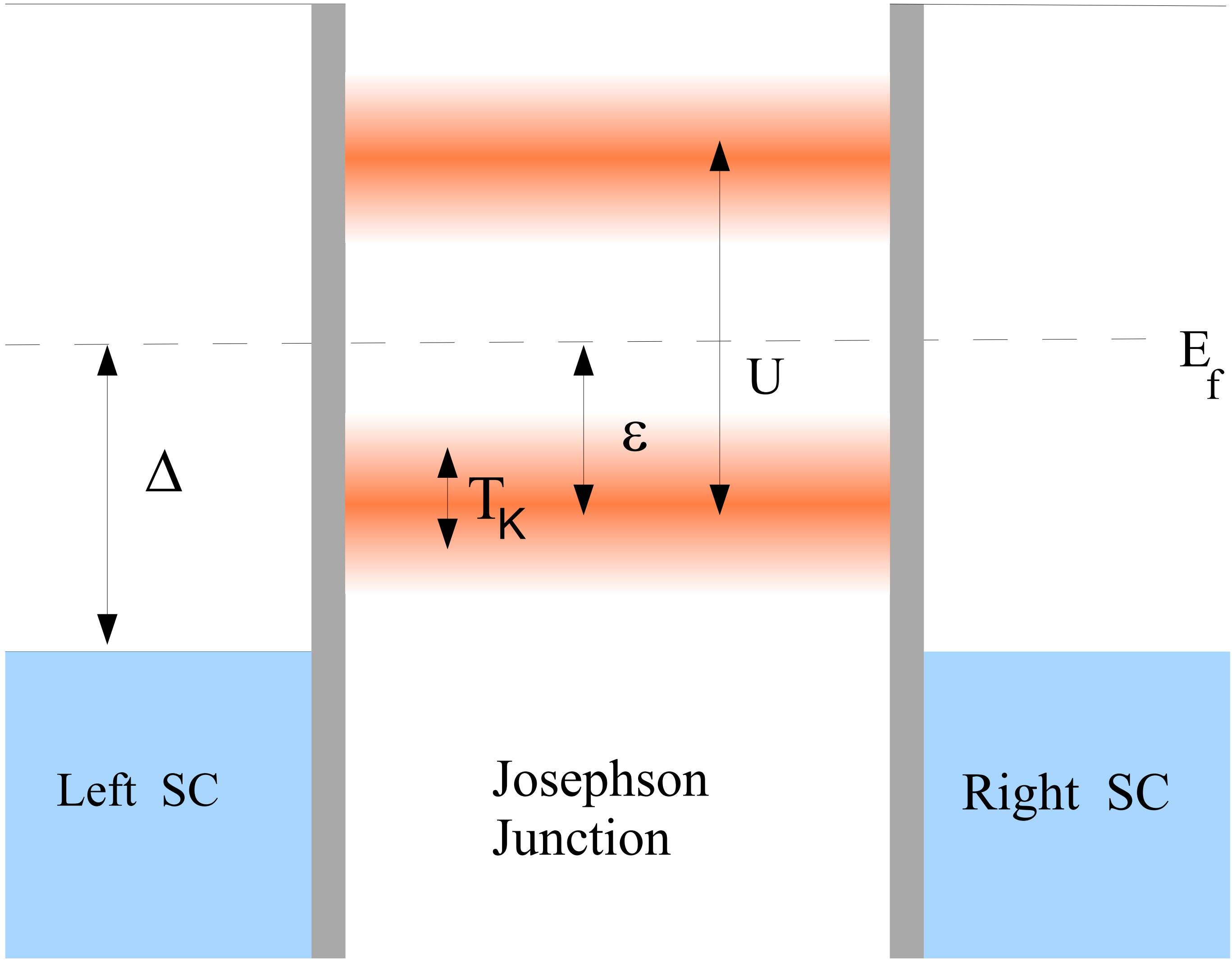
Extremely low Kondo temperature as long as $\Gamma \ll \Delta$

Perturbation theory in $\frac{U}{\Gamma}$ or $\frac{U}{\Delta}$

$T_K \simeq \Delta$

Faoro et al., 2006, 2007, 2008

Hamiltonian and energetics



$$H = \sum_{k,s} \epsilon_k \hat{n}^{(1)}_{k,s} + \sum_{k} \Delta e^{i\phi_1} \hat{c}^{(1)}_{k\uparrow} \hat{c}^{(1)}_{k\downarrow} + \text{h.c.} + (1 \leftrightarrow 2) +$$

$$\sum_{s} \epsilon_d \hat{n}_s - U \hat{n}_\uparrow \hat{n}_\downarrow + t \sum_{k,s} \left(\hat{c}^{(1)}_{k,s} \hat{d}^\dagger_s + \text{h.c.} \right) + (1 \leftrightarrow 2)$$

Problems with Hartree-Fock

Lowest order in U = Hartree Fock

- Enforces artificial symmetries, breaks others
- no Kondo scale
- gets a number of features right for SCs

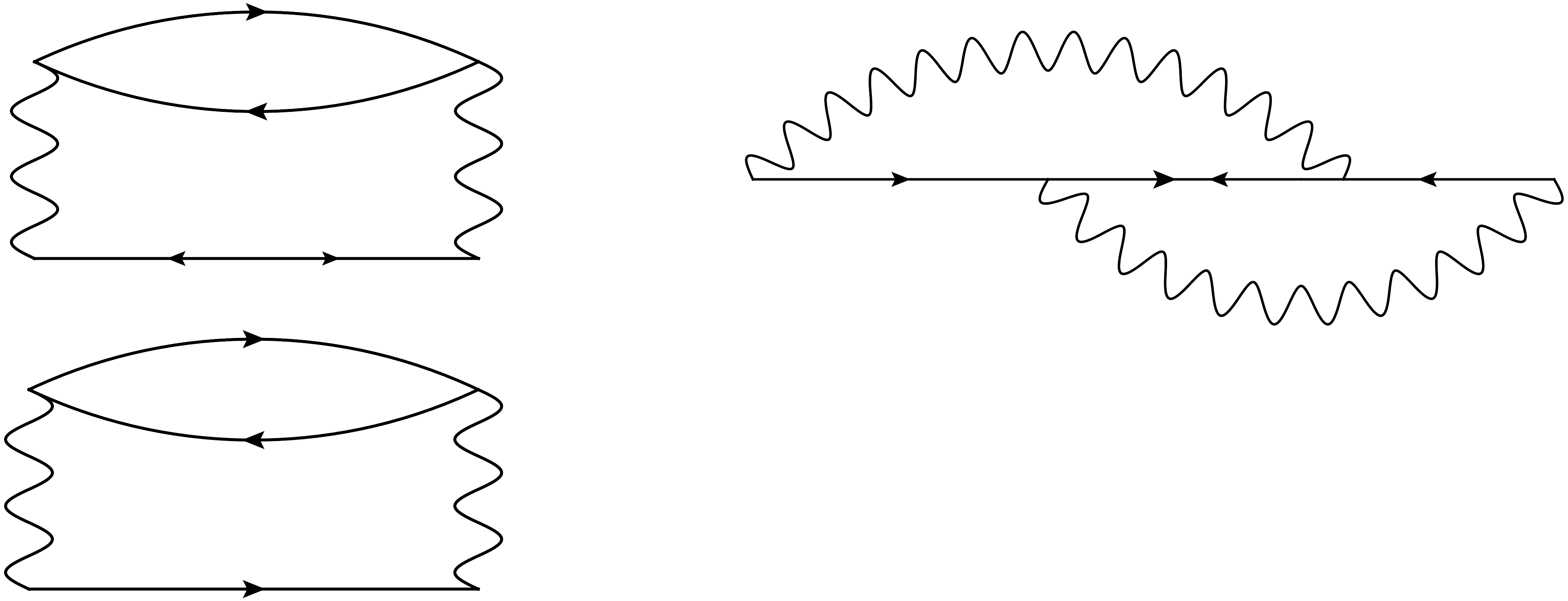
Our approach:
Second order perturbation theory, one order beyond HF

Rozhkov and Arovas, PRL 1999; Yoshioka and Ohashi, J. Phys. Soc. Jpn. 2000

Second-order self energy

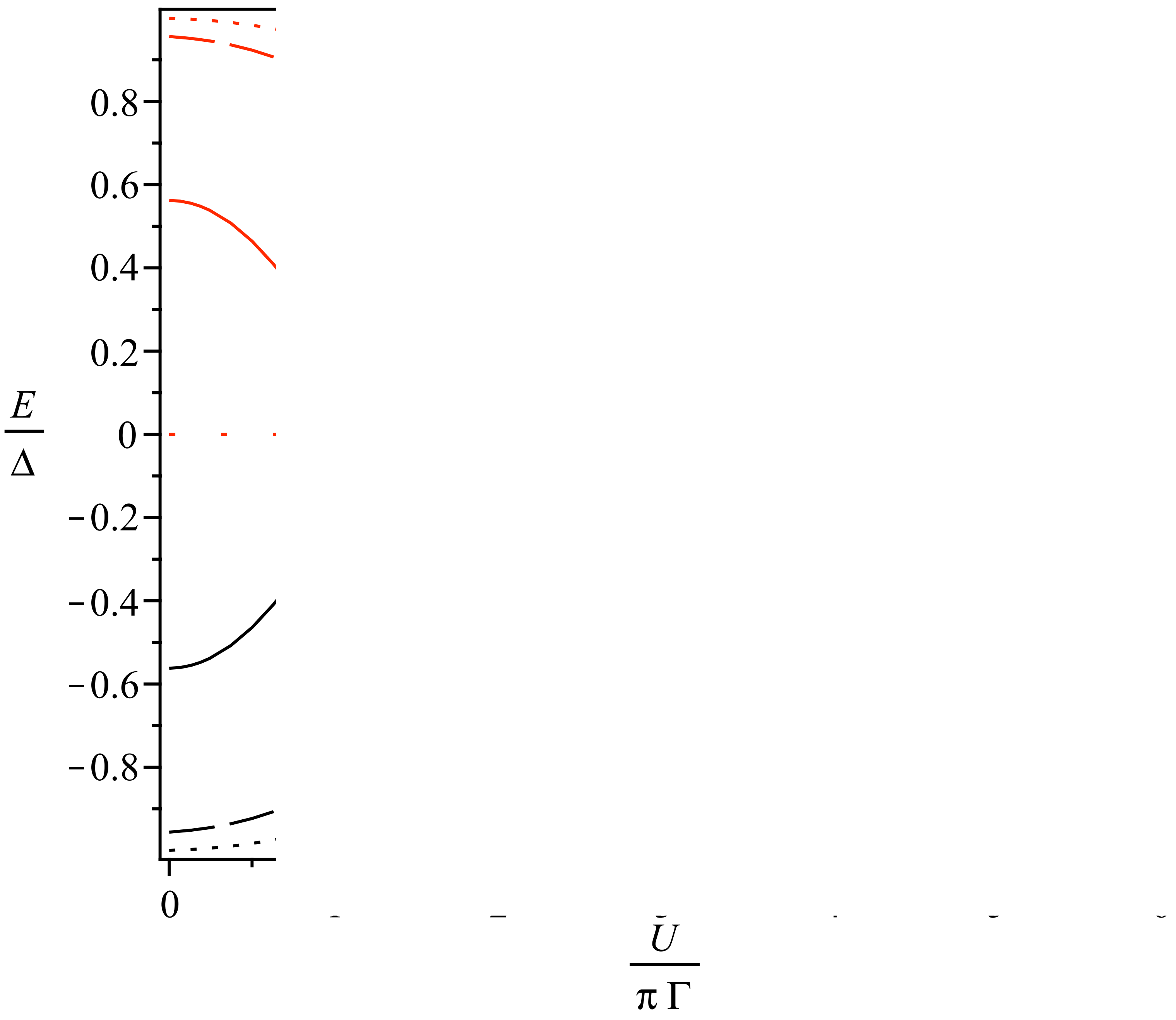
Definitions: $\Sigma(U) = \frac{U}{2} + \alpha(U)\omega$ $\qquad \Sigma_\Delta = \beta(U,\phi)\Delta$

$$\alpha(U) = a\left(\frac{U}{2\pi\Gamma}\right)^2 \qquad \beta(U,\phi) = \frac{\chi_d}{\Delta} + b(\phi)\left(\frac{U}{2\pi\Gamma}\right)^2$$



Compare to FRG: Constant self-energies - artefact?

Andreev bound states



Poles of GF:
Fourth order equation

$$E_b \simeq f(U)\Delta\cos\left(\frac{\phi}{2}+\delta\right)$$

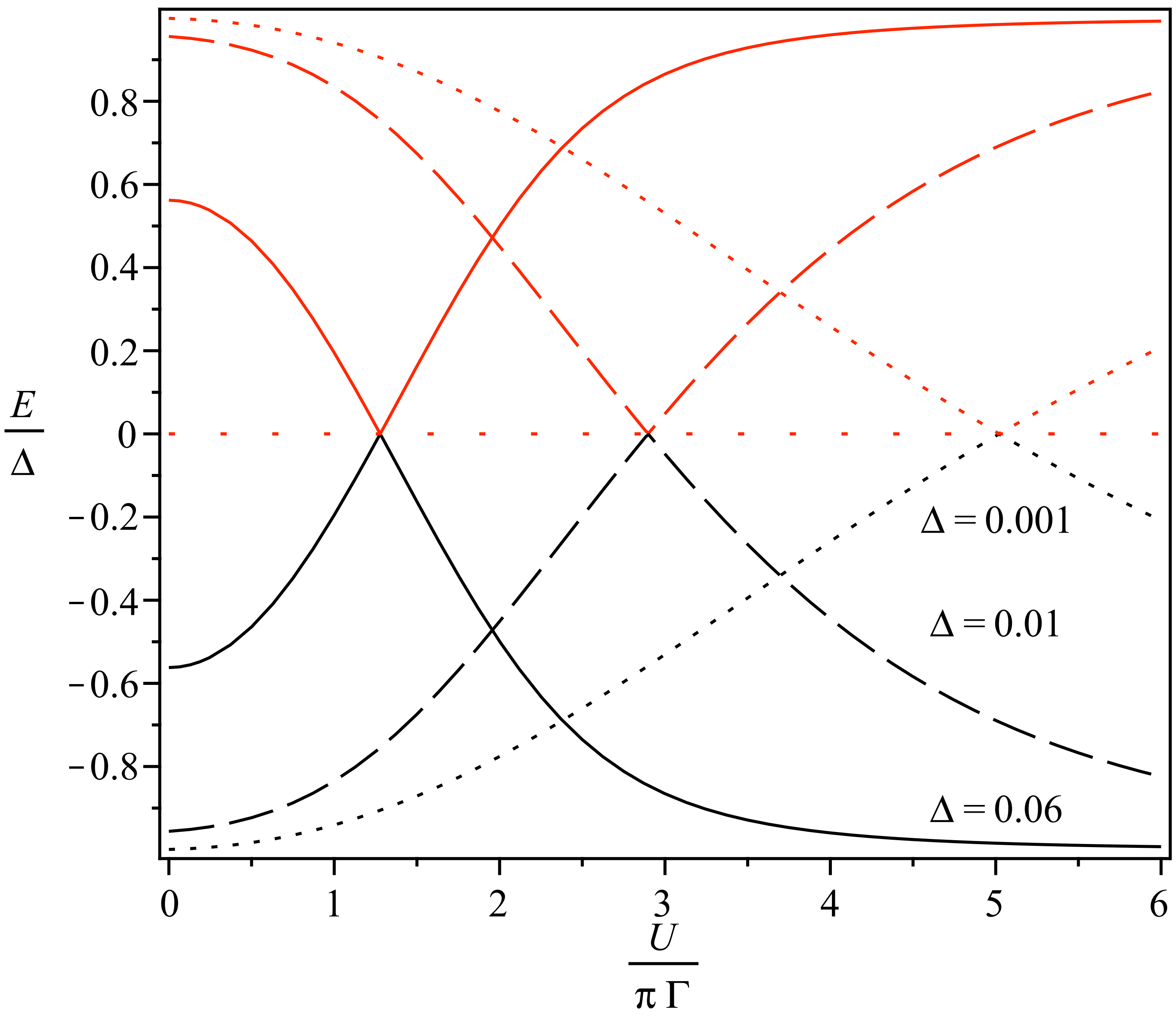
Zero-width states!

Zeroes at singlet-doublet transition

Reproduces NRG results

Bauer, Oguri, Hewson, J. Phys CM 2007

Andreev bound states



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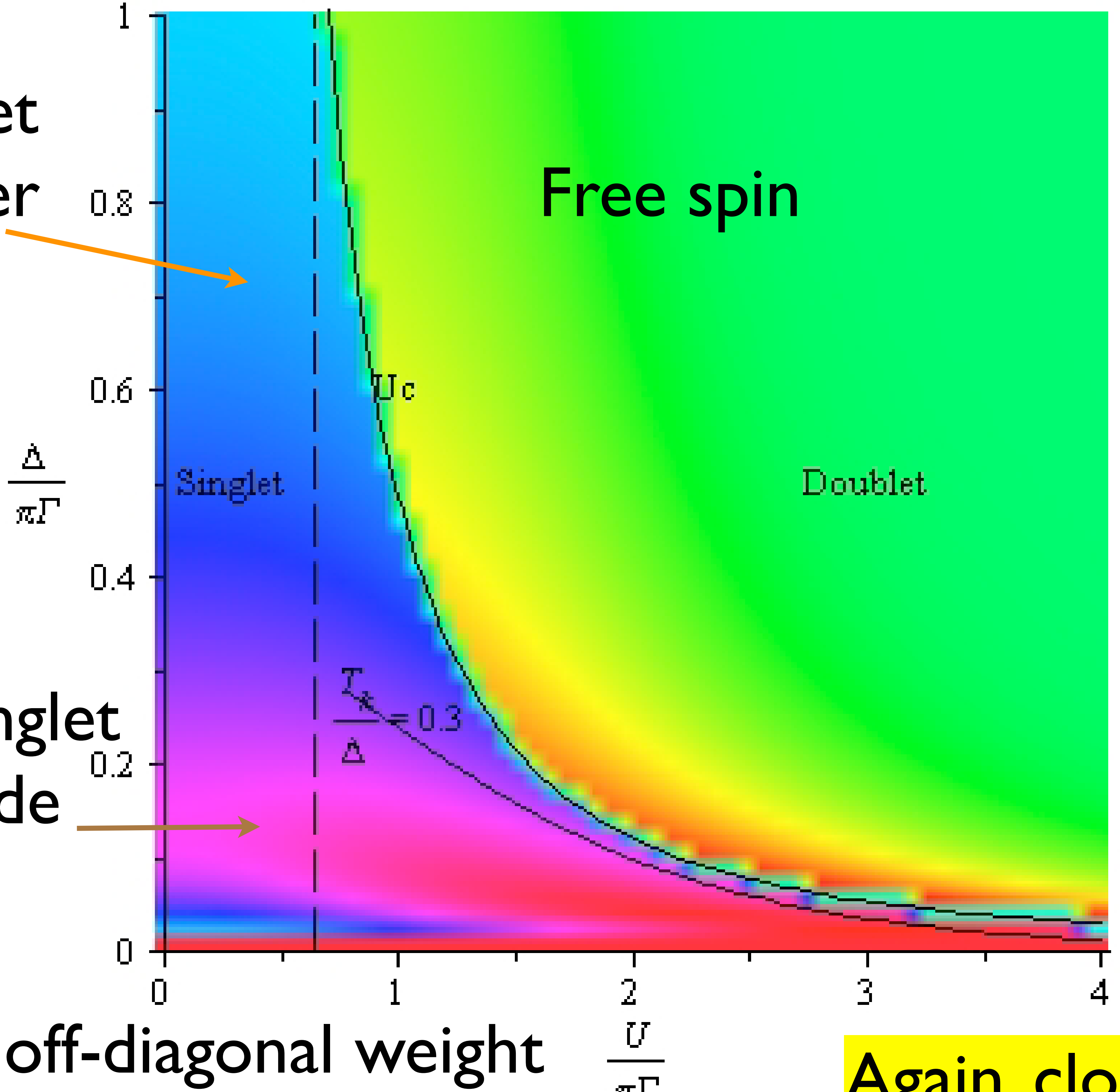
Phase diagram



SC singlet
= Cooper
pair

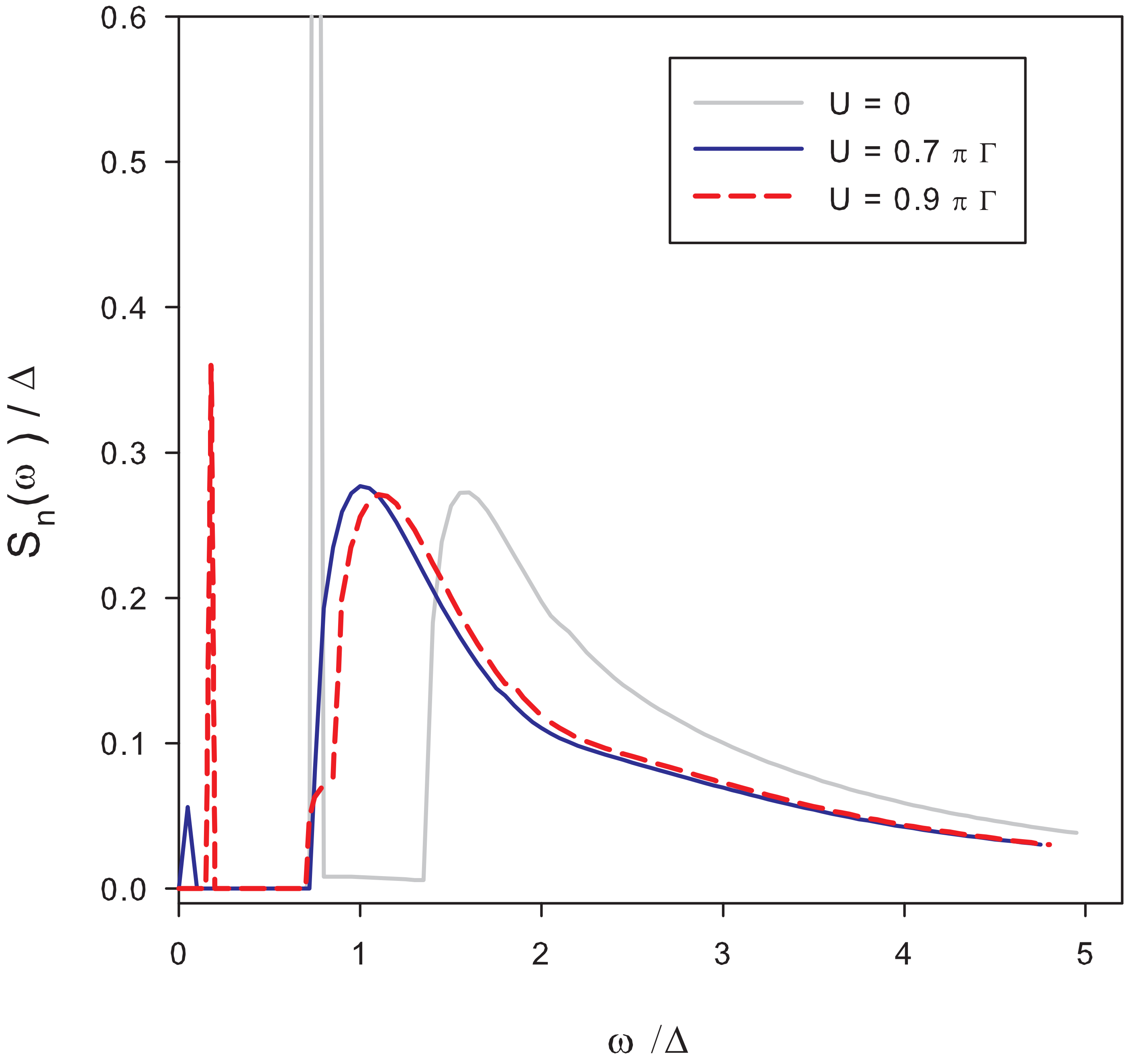
Free spin

Kondo singlet
= electrode
screening

Colour =off-diagonal weight

Again, close to NRG

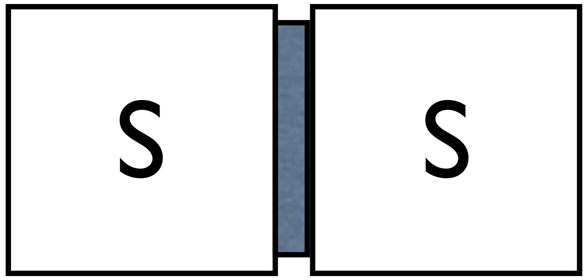
Critical current number noise



- Flows out of noninteracting case
- strong f=0 noise

M. Ansari and FKW, in preparation

Zero- and pi-junctions

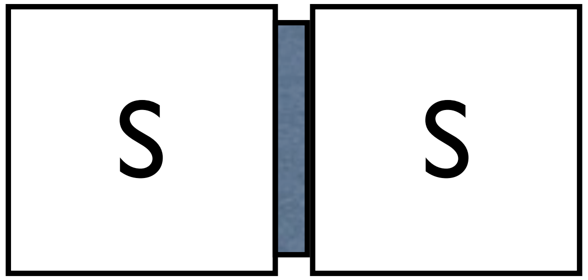


Free Energy $E = E_J(1 - \cos\phi) \quad E_J > 0$

Ground state: $\phi = 0 \bmod 2\pi$

Microscopic expression $I = \dfrac{1}{\Phi_0} \sum_n \dfrac{\partial E_n(\phi)}{\partial \phi}$

Zero- and pi-junctions



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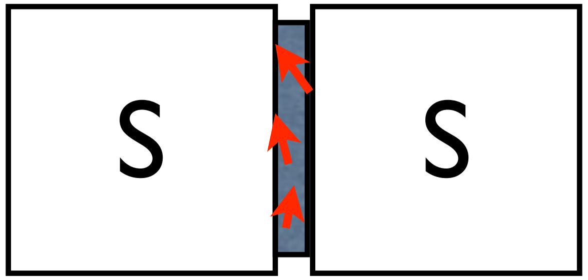
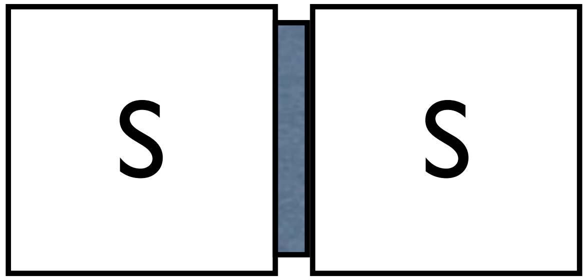


Local moment modifies scattering phase:

More complicated $I(\phi)$ relation

Zero- and pi-junctions



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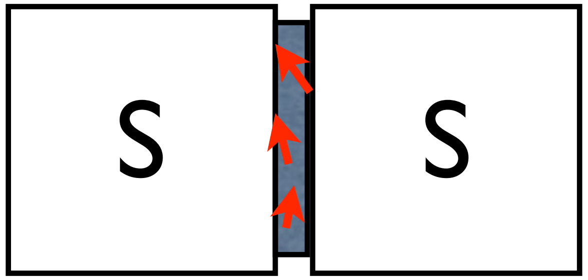
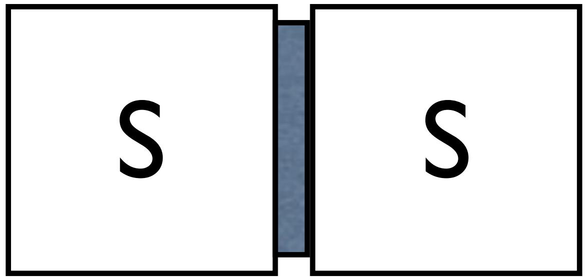


Local moment modifies scattering phase:
More complicated $I(\phi)$ relation

Possible: Ground state at $\phi = \pi \bmod 2\pi$ ‘pi-junction’

Zero- and pi-junctions



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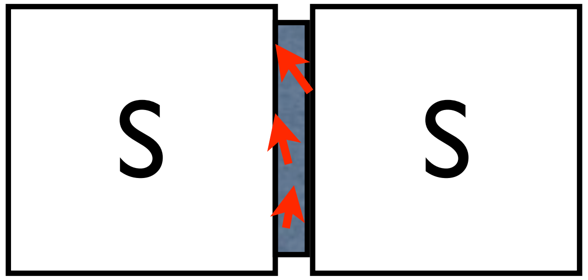
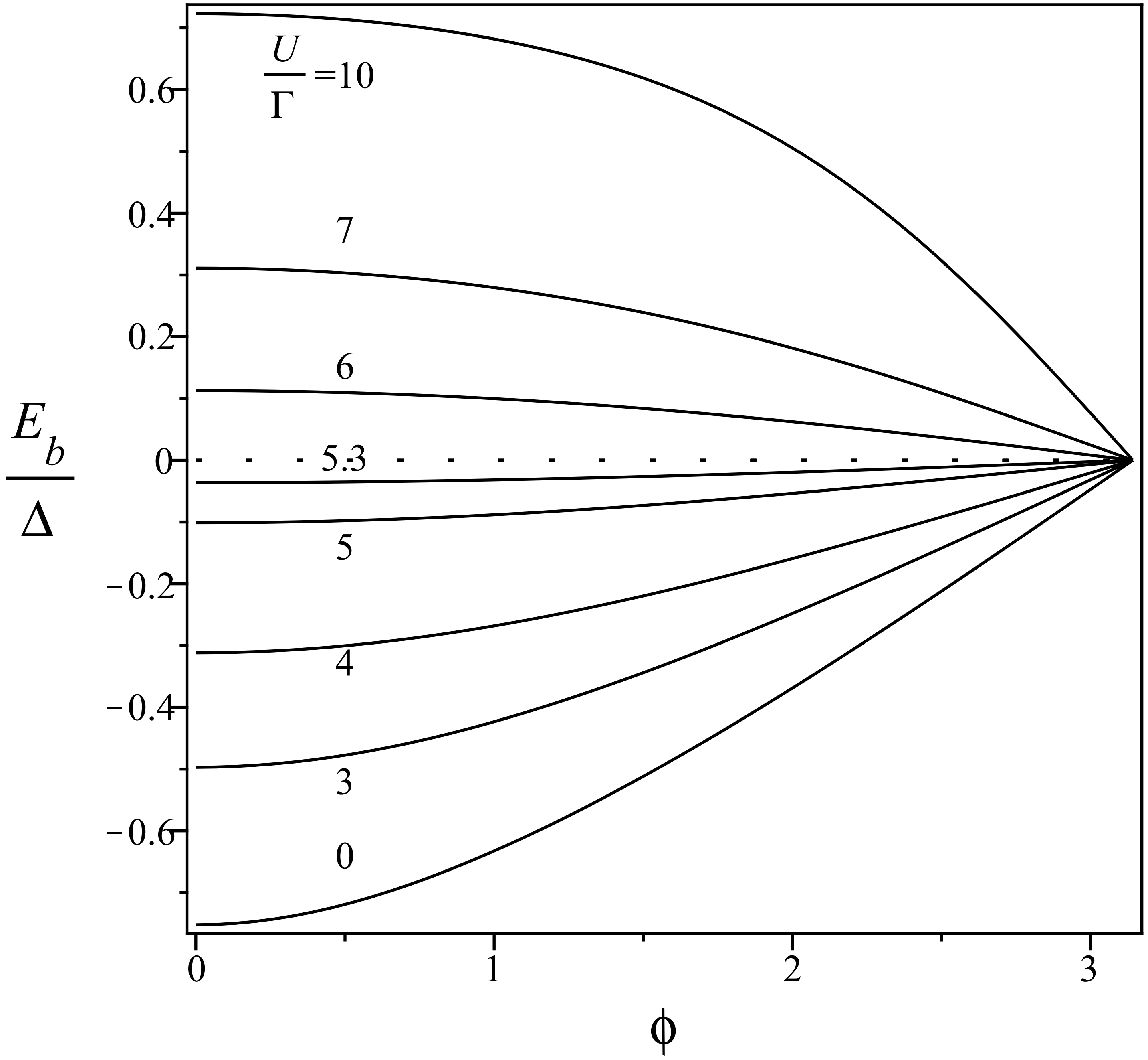


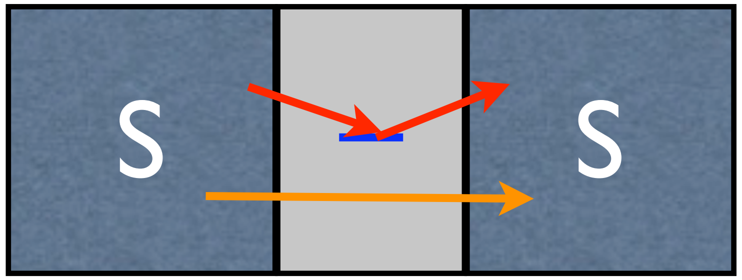
Local moment modifies scattering phase:
More complicated $I(\phi)$ relation

Possible: Ground state at $\phi = \pi \bmod 2\pi$ ‘pi-junction’

Other pi-junction-mechanisms: unconventional superconductor, mesoscopic nonequilibrium

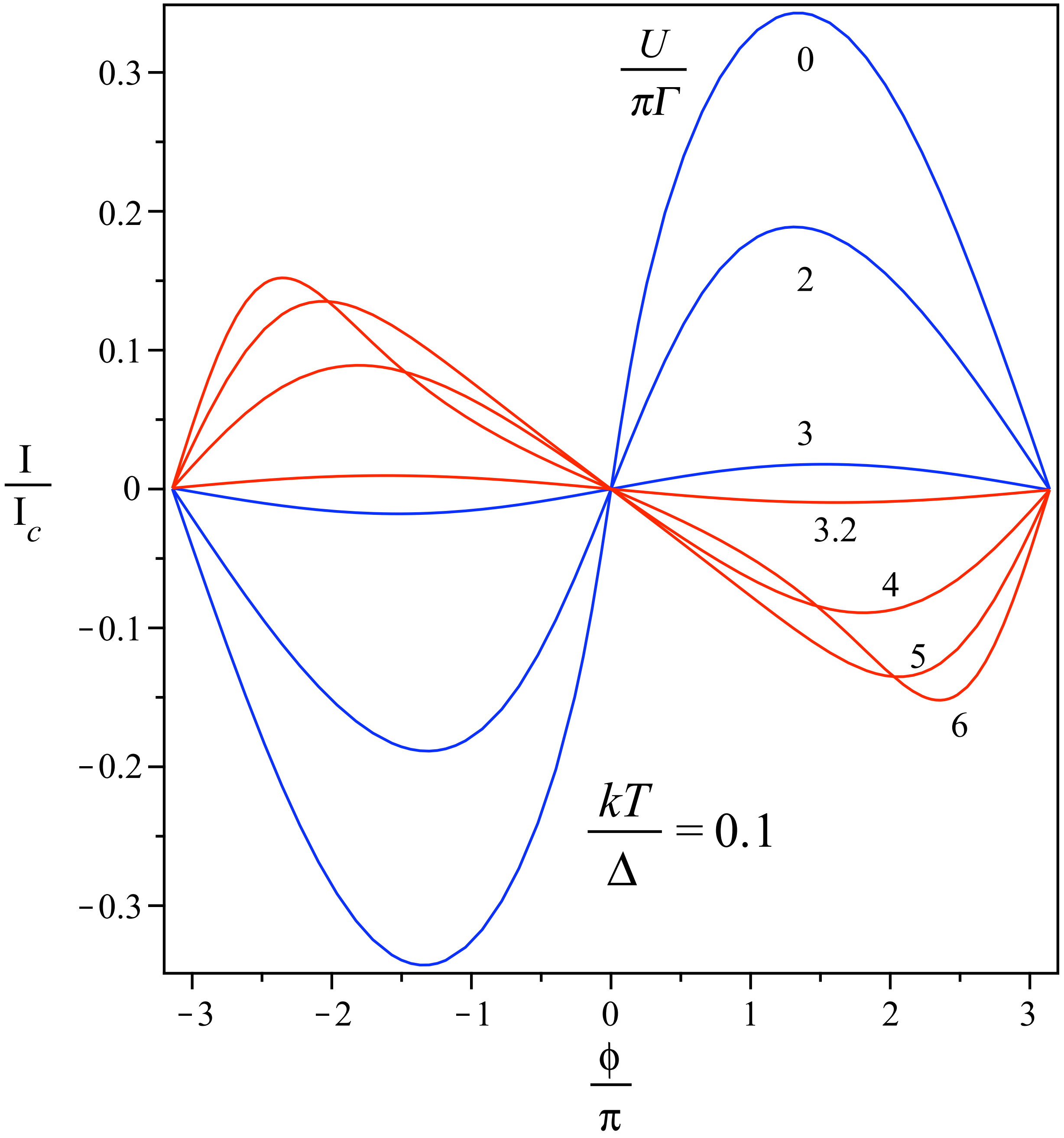
Phase dependence

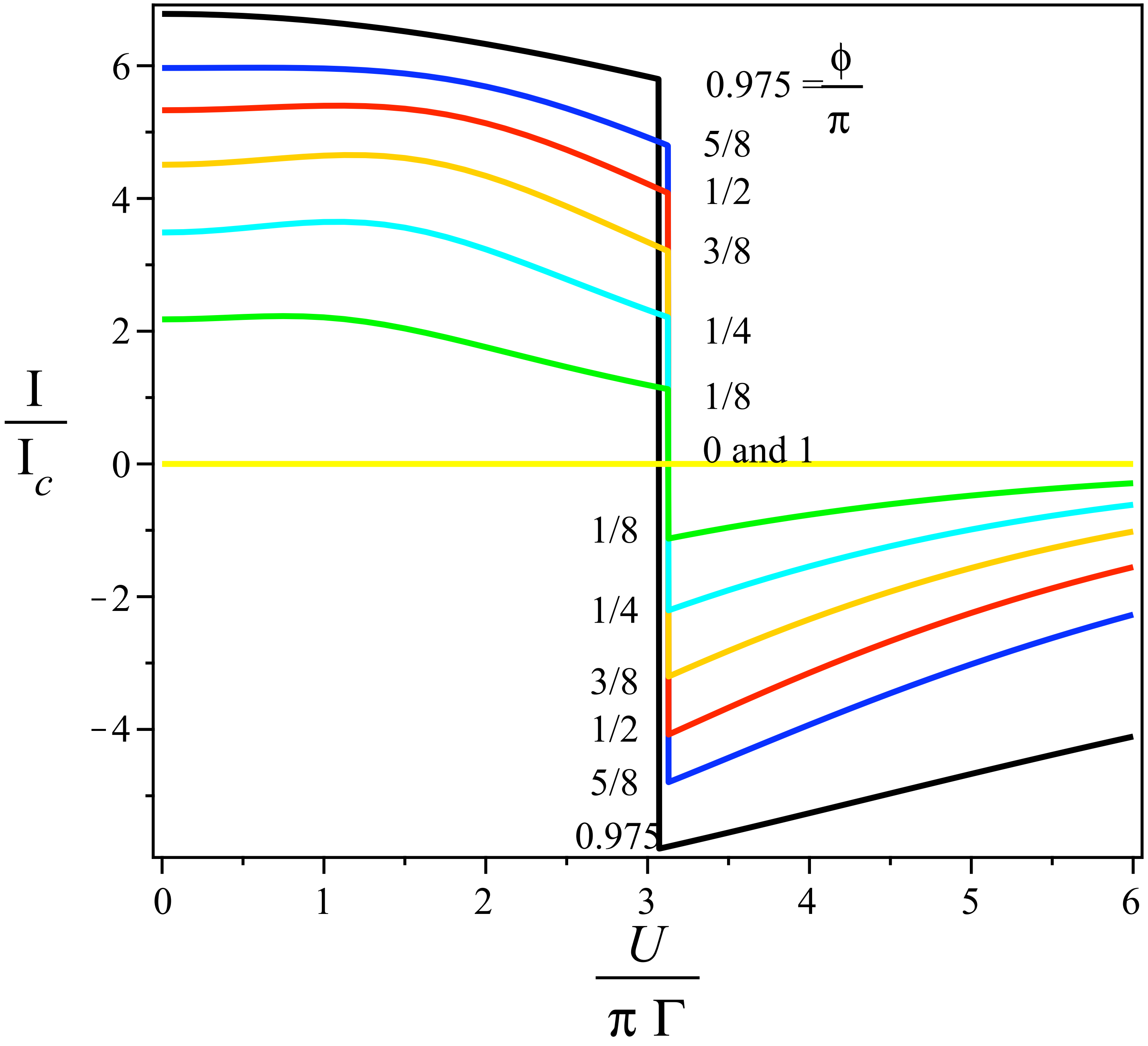




Transition to pi junction with interaction ... but on background of regular junction

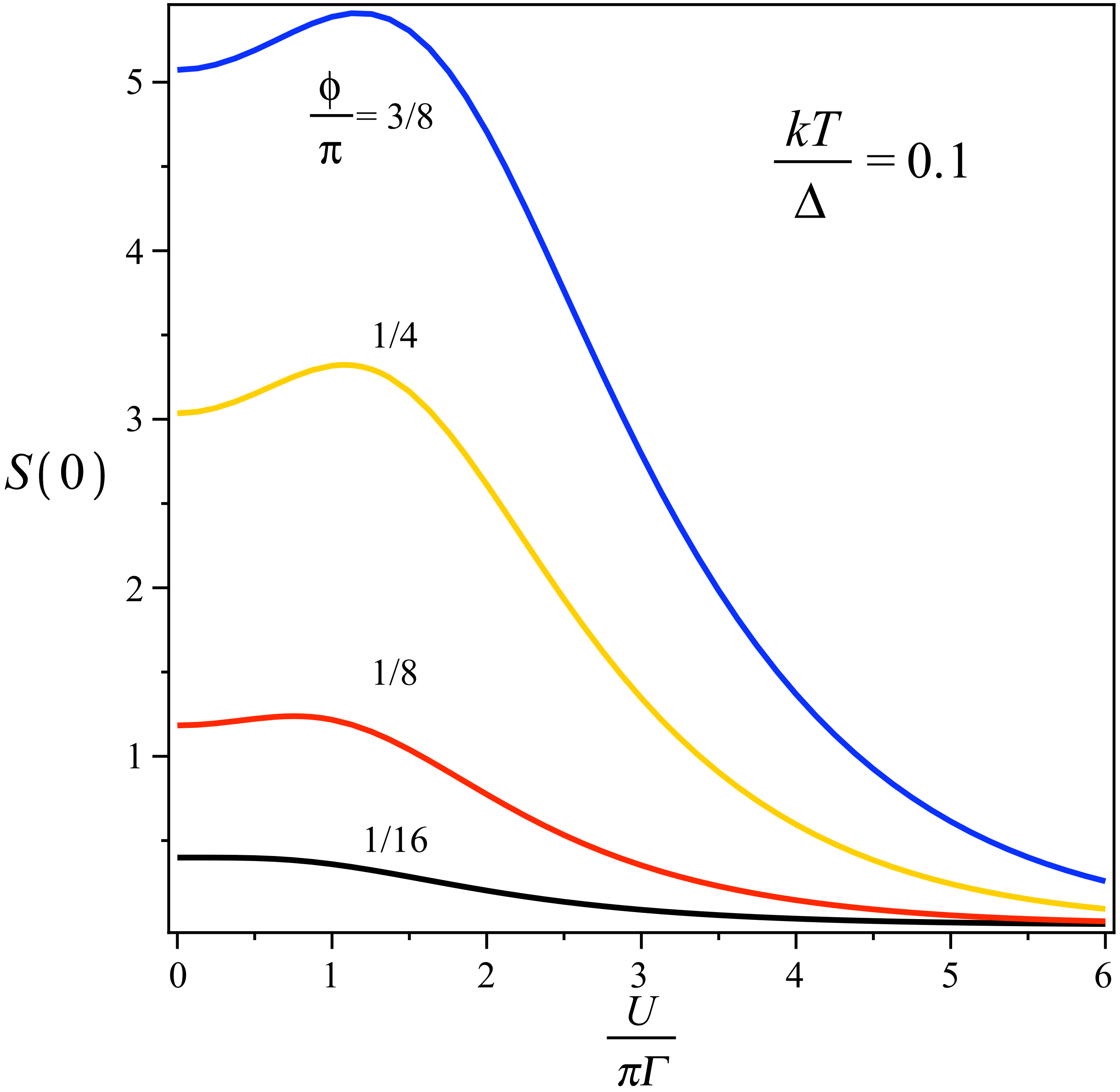
Current





Discontinuous: Potentially noisy transition

Current noise

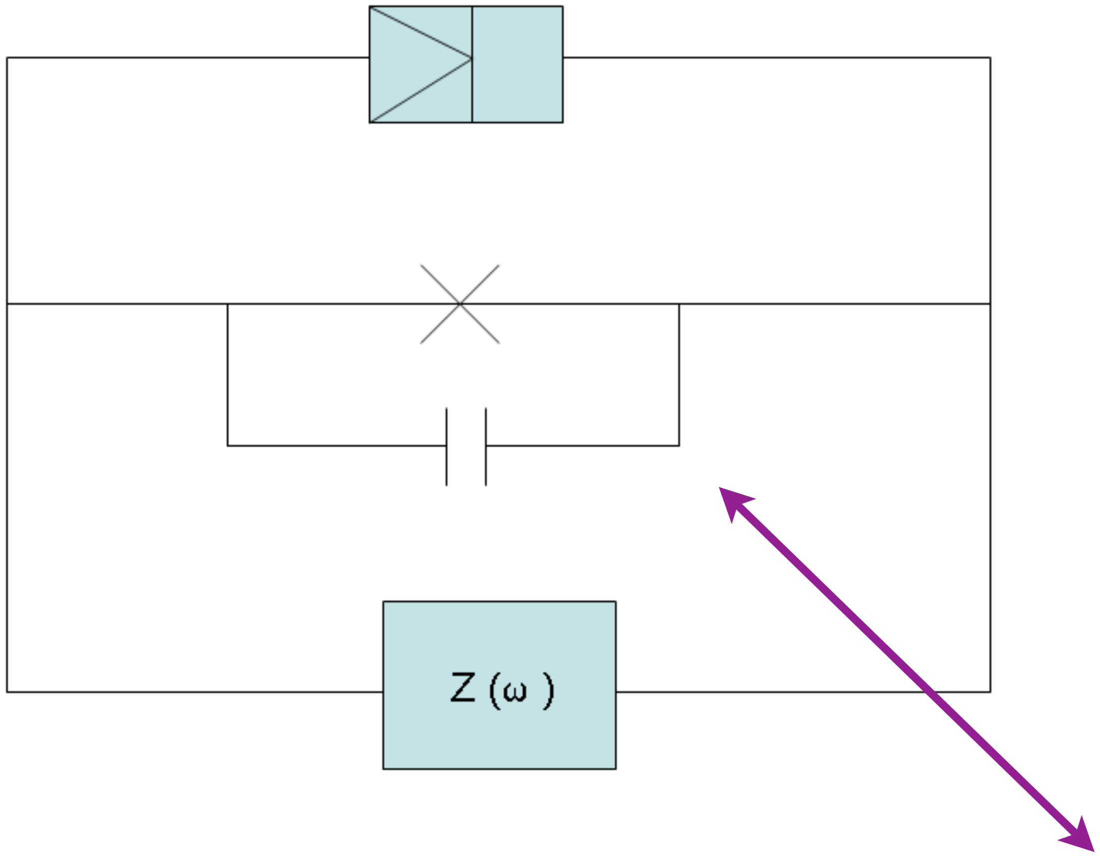


Can go up with interaction before dropping

Trap noise in Josephson

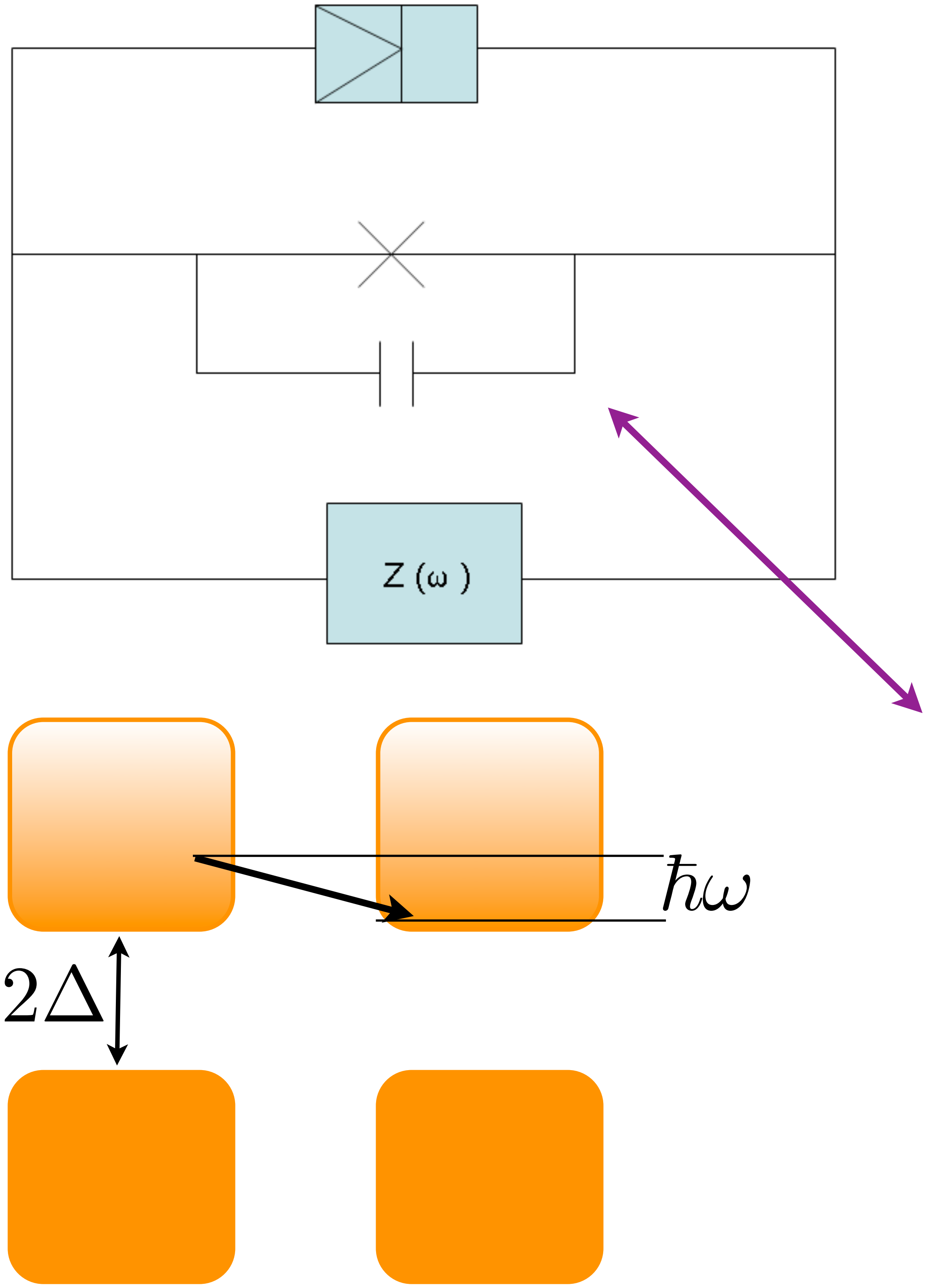
- SQubits and noise
- Surface roughness
- Interacting traps
- quasiparticles

Hot quasiparticle noise



- correlated process: hot quasiparticle - qubit
- contribution to T_1^{-1}
- not sensitive to direction

Hot quasiparticle noise



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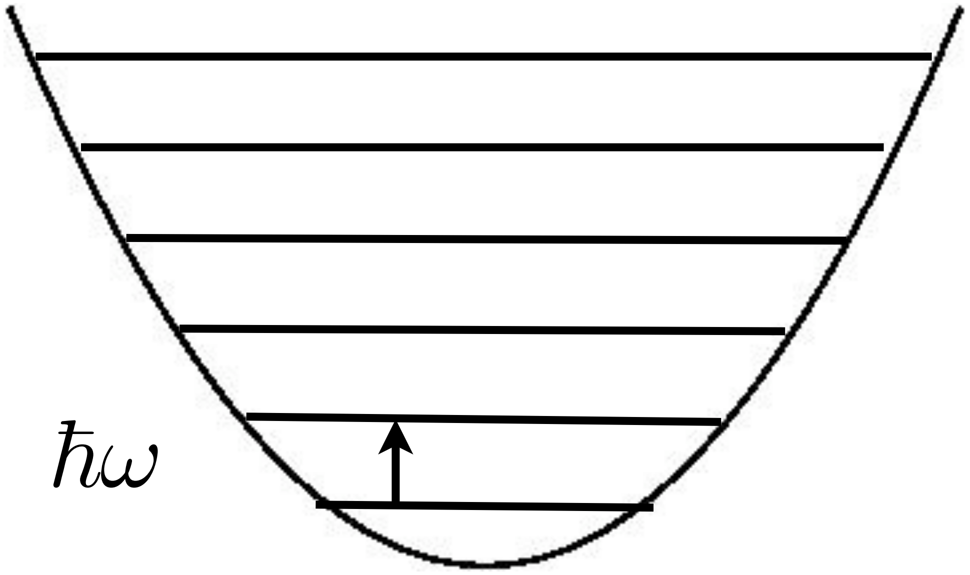

J.M. Martinis *et al.*, PRL 2009

Quasiparticle transition rate

$$\vec{\Gamma}_1 = \frac{4}{R_T e^2} \int dE dE' \, \rho_L(E) \rho_R(E') f_L(E) [1 - f_R(E')] P_{\text{tot}}(E, E')$$

DOS

Quasiparticle transition rate

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DOS

$$P_{\rm tot}(E, E') = \int dt \, e^{-iEt} \left[(u^2 + v^2) - 2uve^{-2S(0)} \cos\phi \right] e^{-[S(t) - S(0)]}.$$

small for phase qubits

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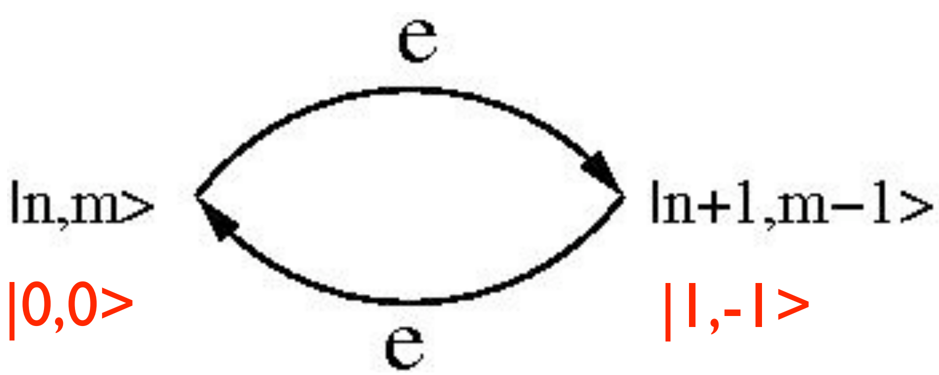

|0,0>

|1,-1>

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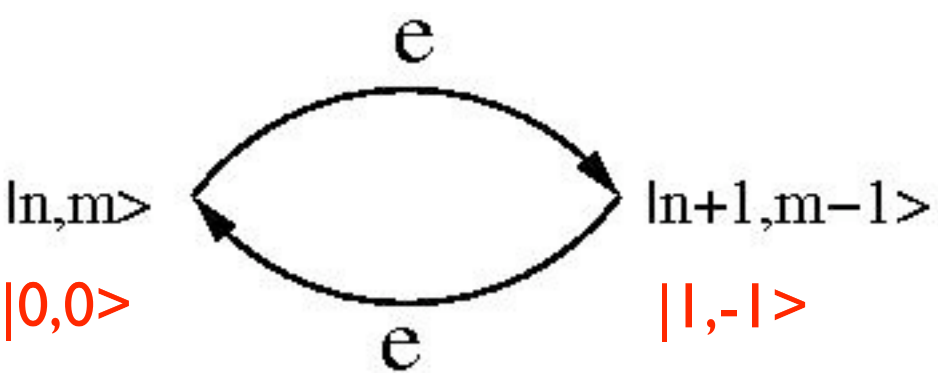

|0,0>

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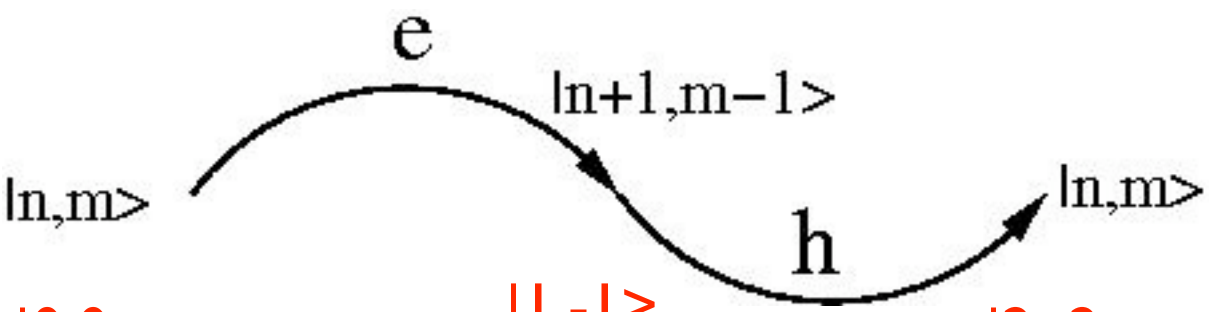

|0,0>

|1,-1>

|2,-2>

Quasiparticle transition rate

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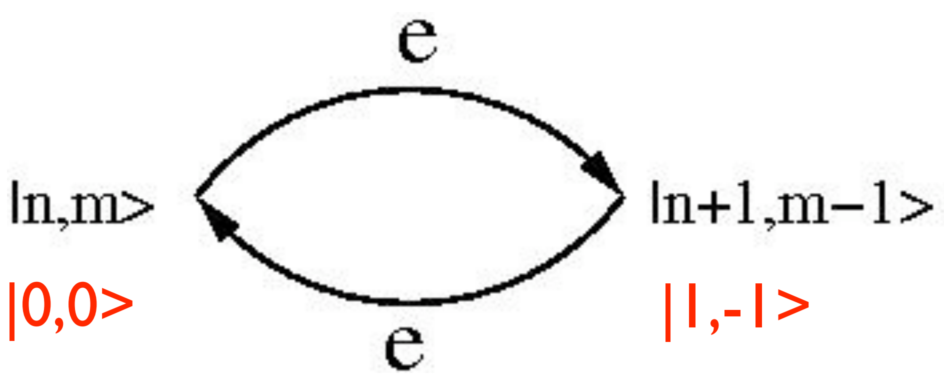
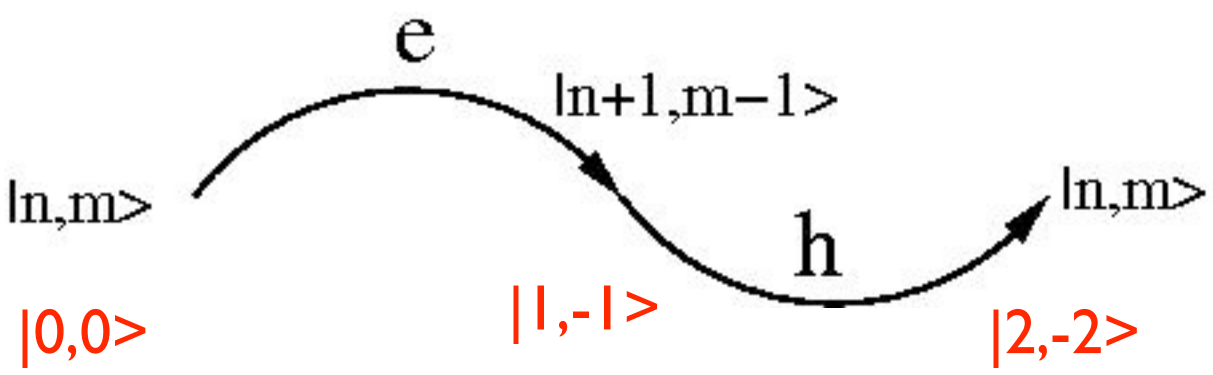
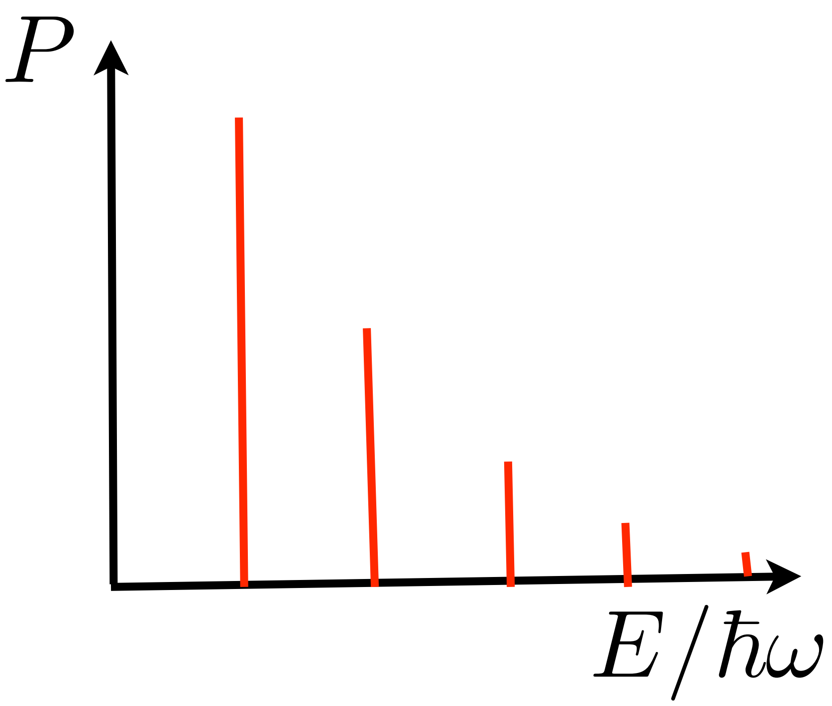




- electron-hole mixed processes: diagonal in qp space, off-diagonal in charge space
- phase sensitivity+dressing by zero point fluctuations

For infinite quality mode

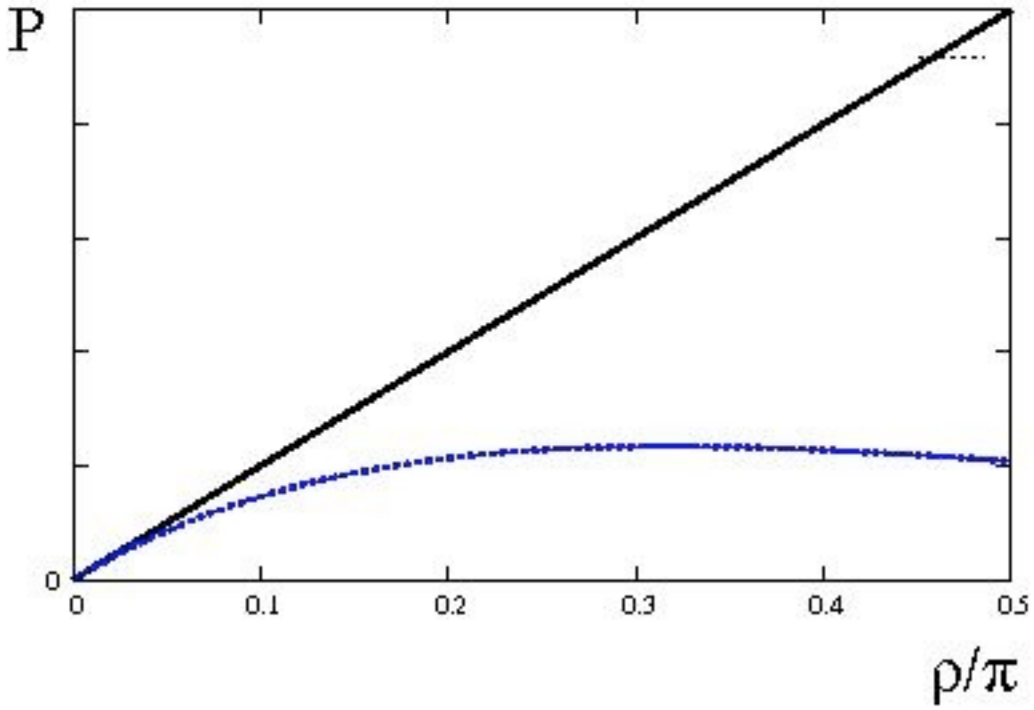
Plasma mode sidebands



$$P(E) = \sum_{k=-\infty}^{\infty} p_k(\rho, \omega_P, T)\delta(\omega - k\omega_p)$$

- rate driven by environment but reduced by dressing

$$\rho = Z/R_K$$



FKW, U. Sinha, A. Sinha, in preparation

Trap noise in Josephson

- low-V shot noise in rough junctions
- junction resonators from traps
- new noises from interactions
- quasiparticle decay