Indications for finite-temperature phase transitions connected with the apparent metal-insulator transition in 2D disordered systems

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1. Introduction / motivation for reanalysis
   of Lai et al., PRB 75 (2007) 033314
2. Empirical situation
3. Comparison with other experiments
4. Scaling analysis
5. Peculiarity at MIT
6. Conclusions

Out of Equilibrium Quantum Systems, KITP, Santa Barbara, 27.10.2010

1. Introduction / motivation

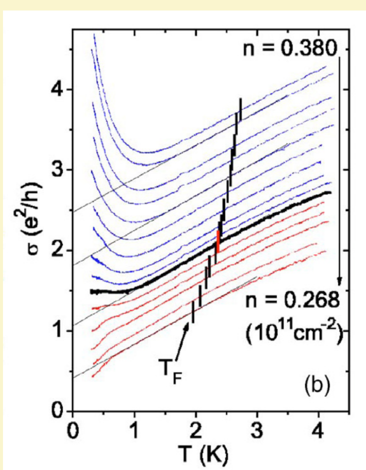
Fundamental: Is conduction in two-dimensional system always nonmetallic?

First: theoretical YES by Abrahams et al., 1979, then: experimental NO by Kravchenko et al., 1994. Recent reviews: Kravchenko, Sarachik, 2004, 2010, Spivak et al., 2010. Here: thinking on

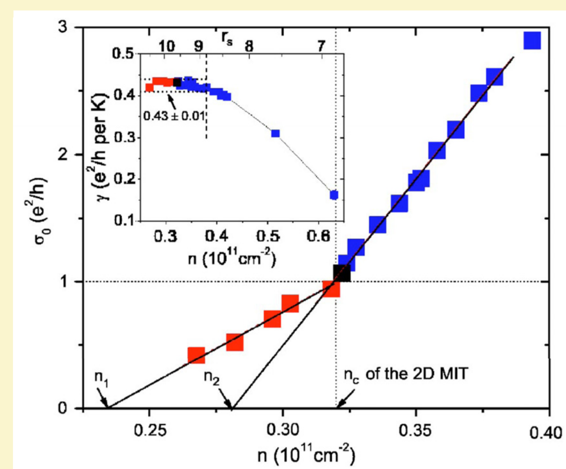
PHYSICAL REVIEW B **75**, 033314 (2007)

Linear temperature dependence of the conductivity in Si two-dimensional electrons near the apparent metal-to-insulator transition

K. Lai,[1,*] W. Pan,[2] D. C. Tsui,[1] S. Lyon,[1] M. Mühlberger,[3] and F. Schäffler[3]
[1]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*

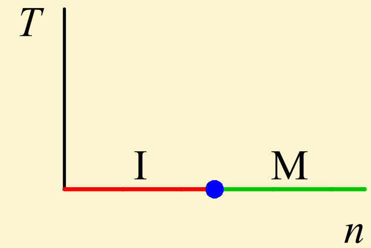


extrapolation
to $T = 0$



Results by Lai et al. for $Si_{0.75}Ge_{0.25}/Si/Si_{0.75}Ge_{0.25}$ qu. well:

- Extrapolation $\sigma_0(n)$ exhibits sharp bend (knee), which coincides with critical concentration defined by $d\sigma/dT = 0$ at lowest T. Thus existence of different phases likely.



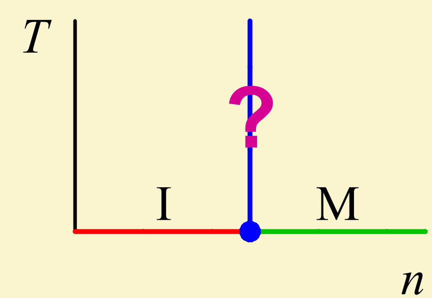
- Slope of $\sigma(T, n = \text{const})$ around T_F is almost constant.

Two question are suggested:

- If $\sigma_0(n)$ has a knee, and the slope is roughly constant, should not $\sigma(T = \text{const}, n)$ exhibit a knee for finite T?

- If yes, is its existence restricted to the regions of linear $\sigma(T, n = \text{const})$, or is it more general phenomenon?

That means: Might there be a phase transition at finite T?



Why would a positive answer be important?

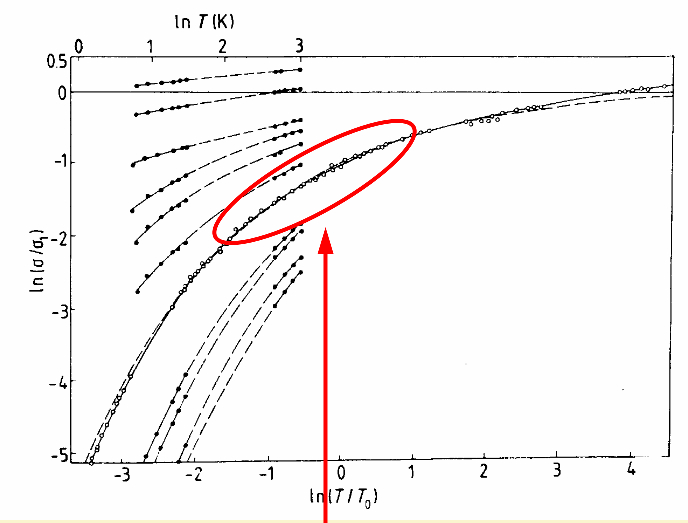
Personal experience from three-dimensional (3D) systems, in particular amorphous transition-metal semiconductor alloys:

Qualitatively, the $\sigma(T,n)$ curve sets for **2D** and homogeneous **3D** systems (for example Si:P) have a series of **common features**:

- On the "insulating" side, stretched exponential vanishing of $\sigma(T,n)$ with decreasing T, which indicates variable-range hopping
- As $T \to 0$, sign change of $\mathrm{d}\sigma/\mathrm{d}T$ at and "close to" the metal-insulator transition (MIT) for 2D and 3D systems, respectively.
- Upturn of $\sigma(T,n)$ with decreasing T for apparently metallic conduction.
- For arbitrary fixed T, continuous variation of $\mathrm{d}\sigma/\mathrm{d}T$ with n.

Common believe in discontinuous MIT for 2D systems, defined by $\mathrm{d}\sigma/\mathrm{d}T = 0$ for $T \to 0$, and continuous MIT in 3D materials, defined by $\sigma(T \to 0, n) = 0$.

First doubts in common believe concerning 3D case arise from scaling of T dependence of σ in hopping region for several materials: Provided $T < T^*$, if $\sigma(T,n) < \sigma_0$ then $\sigma(T,n) = \sigma_0\, \varphi(T/T_0(n))$, where $T_0(n) \to 0$ as $n \to n_c - 0$.



Master curve construction for a-$Si_{1-x}Cr_x$ (AM et al., 1983)

Constancy of prefactor of ES hopping as $n \to n_c$ for Ge:Ga (A.G Zabrodskii, K.N. Zinov'eva, 1984)

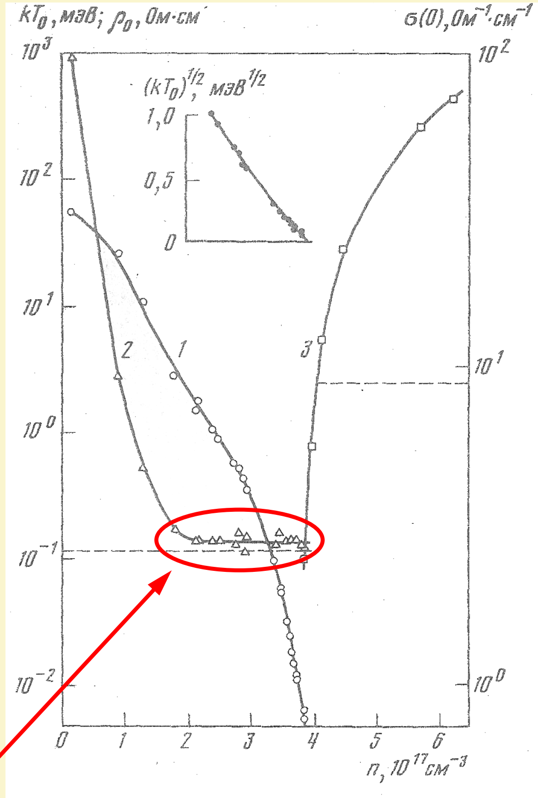


Рис. 3. Критическое поведение проводимости: 1 – T_0, 2 – ρ_0, 3 – $\sigma(0)$. Пунктир: σ_M^{-1} (слева) и σ_M (справа)

Consequences of $\sigma(T,n) = \sigma_0\, \varphi(T/T_0(n))$:

(i) existence of a minimum metallic conductivity,

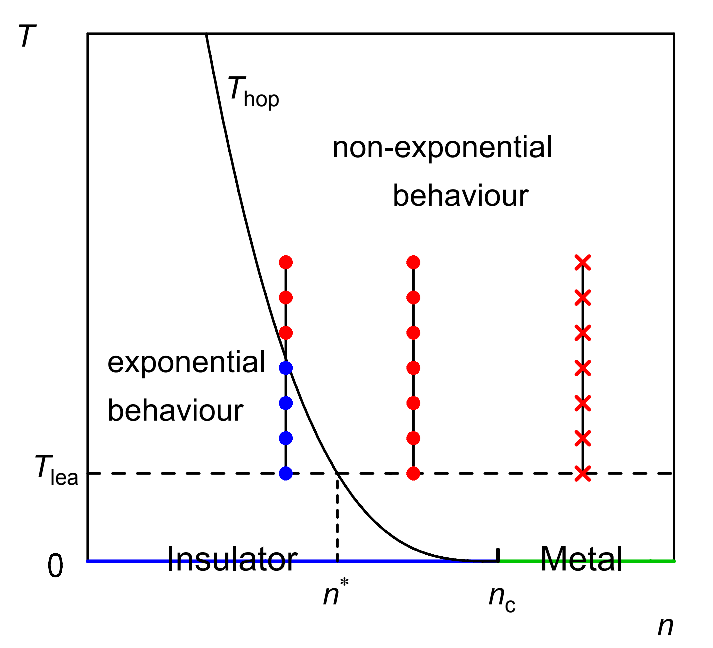
(ii) $\sigma_0\, \varphi(\infty)$ has a special meaning for arbitrary $T < T^*$.

However, there is a series of papers claiming continuity of the MIT at $T = 0$.

Be cautious:

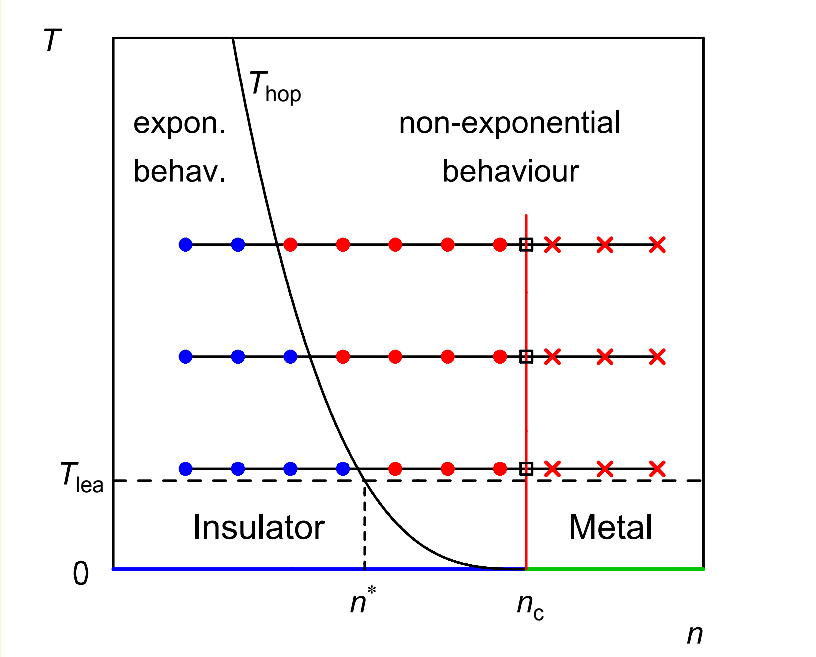
MIT cannot be studied at low T, since, as it is approached, mean activation energy vanishes.

$\Rightarrow$ Not surprising that, in various cases, study of $w = d \ln \sigma / d \ln T$ causes doubts in n_c value.



Dream

Situation will be enormously simplified if the MIT at $T = 0$ is the endpoint of a line of transitions at finite T:



⇒ Question: Might nature be so “nice”, at least in the 2D case?

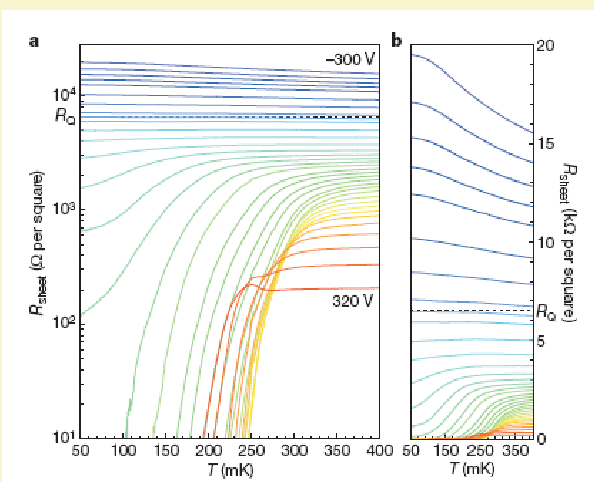
Additional motivation

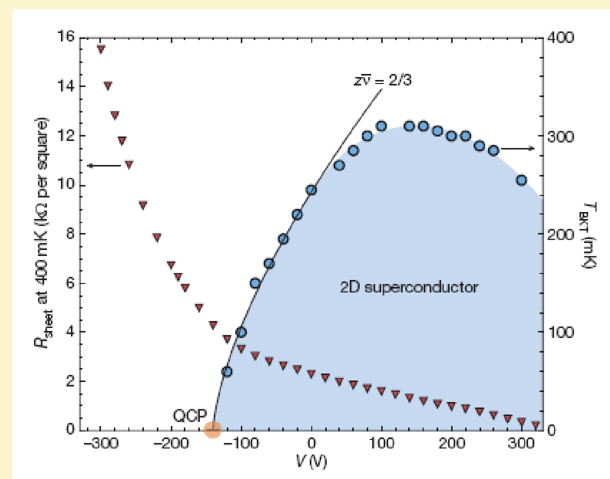
nature Vol 456| 4 December 2008|doi:10.1038/nature07576

LETTERS

Electric field control of the $LaAlO_3/SrTiO_3$ interface ground state

A. D. Caviglia[1], S. Gariglio[1], N. Reyren[1], D. Jaccard[1], T. Schneider[2], M. Gabay[3], S. Thiel[4], G. Hammerl[4], J. Mannhart[4] & J.-M. Triscone[1]





2. Empirical situation for 2D sample by Lai et al.

Compare two empirical 4-parameter fits:

Piecewise linear,

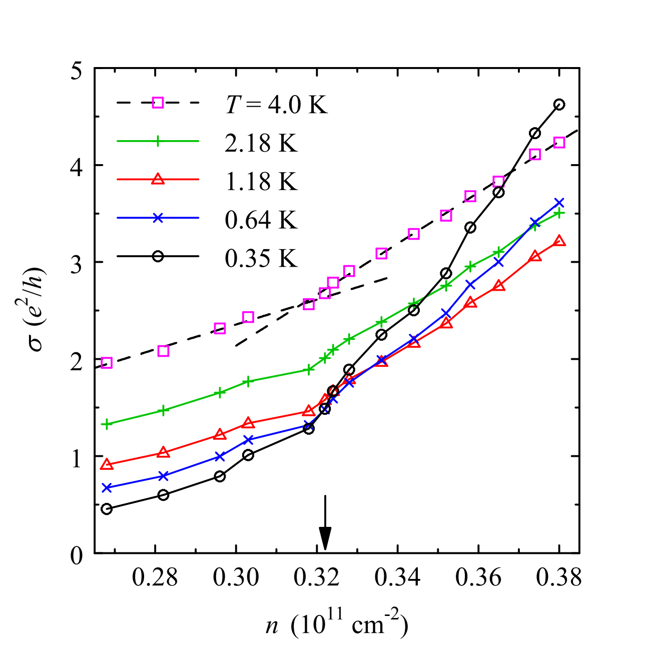
$$f_{\text{plf}} = a + b\,n + c\,(n - n_{\text{k}})\ \theta(n - n_{\text{k}})\ ,$$

and polynomial of third order,

$$f_{\text{pto}} = p + q\,\text{n} + r\,n^2 + s\,n^3$$

Results for complete data set:

T(K)	$n_{\text{k}}(10^{-11}\text{cm}^{-2})$	$\chi_{\text{plf}}^2(e^4/h^2)$	$\chi_{\text{pto}}^2(e^4/h^2)$
4.0	0.318	0.010	0.024
2.18	0.318	0.006	0.019
1.18	0.318	0.010	0.019
0.64	0.318	0.028	0.031
0.35	0.315	0.097	0.063



⇒ **Clear advantage for piecewise linear fit at high *T***

In detail

Problem of comparison: σ range broadens when $T \rightarrow 0$ and $\sigma(n)$ is nonlinear at insulator side, vanishes exponentially. $\Rightarrow$ Fixed σ range is more appropriate.

Results for restricted set, $0.5\,\sigma(T, n_c) < \sigma(T, n) < 2\,\sigma(T, n_c)$:

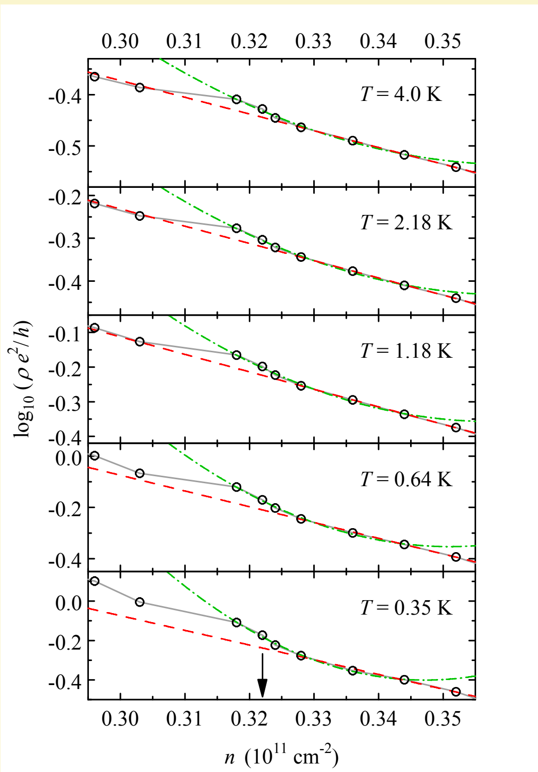
T (K)	N	n_k (10^{-11}cm^{-2})	χ_{plf}^2 (e^4/h^2)	χ_{pto}^2 (e^4/h^2)
4.0	15	0.318	0.010	0.024
2.18	15	0.318	0.006	0.019
1.18	14	0.318	0.009	0.018
0.87	13	0.318	0.009	0.020
0.64	11	0.318	0.013	0.026
0.47	9	0.318	0.023	0.032
0.35	9	0.318	0.024	0.036

$\Rightarrow$ **Piecewise linear function is always clearly better than polynomial.**

$\Rightarrow$ **All knee positions very close to critical concentration from $d\sigma/dT = 0$.**

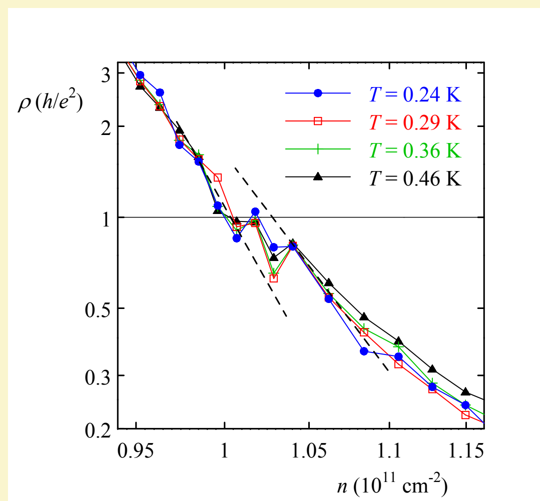
3. Comparison with other experiments

Consider usual log-log plots: Data by Lai et al. exhibit “rounded step” for all *T*.

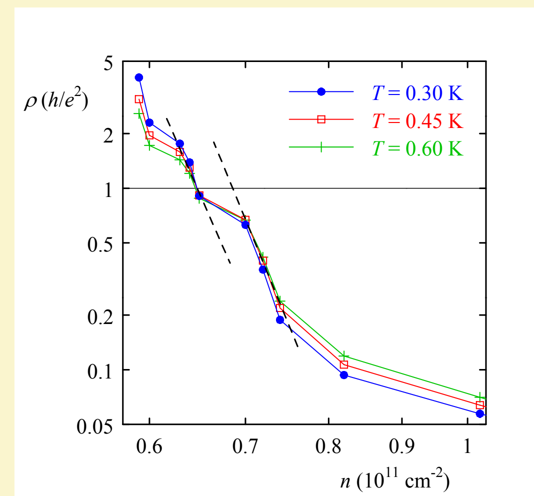


Similar structures were found 2002 in data for

high mobility MOSFET by Kravchenko et al. (1995)



AlAs quantum well by Papadakis and Shayegan (1998)



Significance

Peculiarities might arise from random deviations, but it is very unlikely that all following coincidences occur only by chance:

(a) Observed at **three independent experiments** at different materials.

(b) Peculiarities have **qualitatively same form** in all three cases.

(c) Always in **same resistivity region**, slightly below h/e^{2}.

(d) Always **close to common intersection point** of $\rho(T = \text{const}, n)$ curves.

(e) Features seem to be present for **all considered *T***.

Not found in further experiments, but may be easily overlooked due to inhomogeneities or too small density of the *n* values, compare chapter 4.

4. Scaling analysis

Suppose, $\sigma(T,n)$ scales as found at Si MOSFETs by Kravchenko et al.:

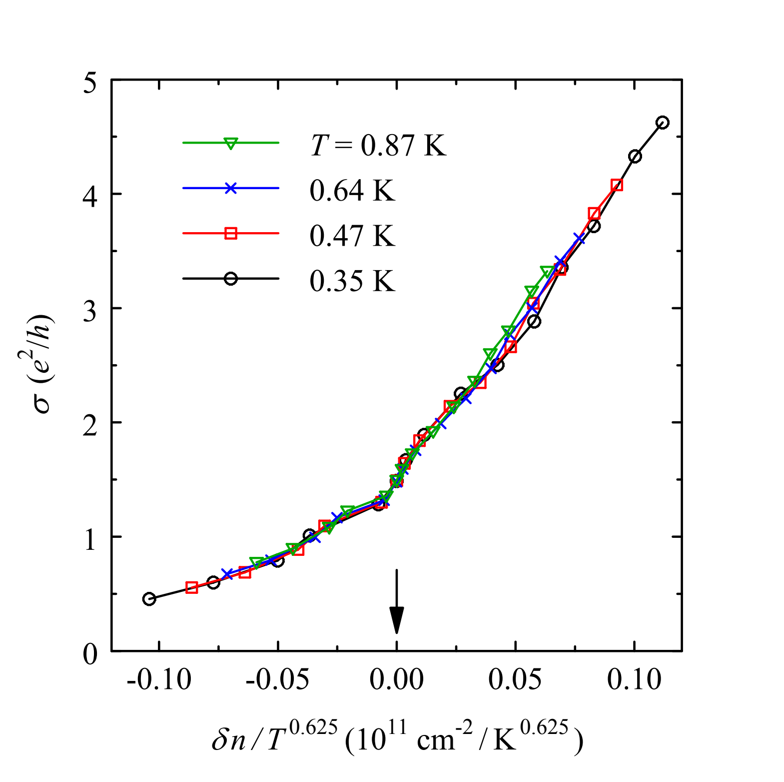
$$\sigma(T,n) = \sigma(t) \quad \text{with} \quad t = T/T_0(n)$$

Assume, scaling holds up to n_c, where T_0 vanishes, and

$$T_0(n) = A\,|\delta n|^\beta \quad \text{with} \quad \delta n = n - n_c\,.$$

Thus, σ should depend only on $T/|\delta n|^\beta$ or rather on $\delta n/T^{1/\beta}$.

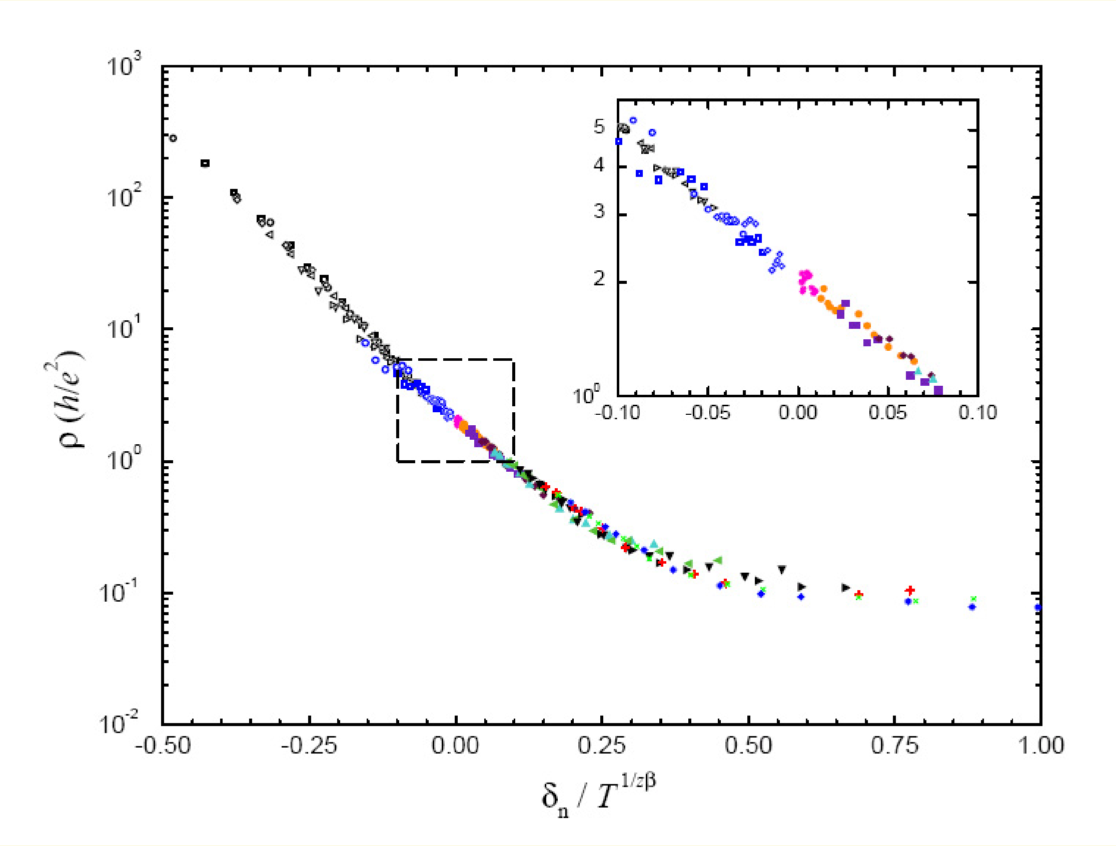
Exponent β should be universal, $\beta = 1.6 \pm 0.1$ for Si MOSFET.



⇒ **Scaling check without fit passed**

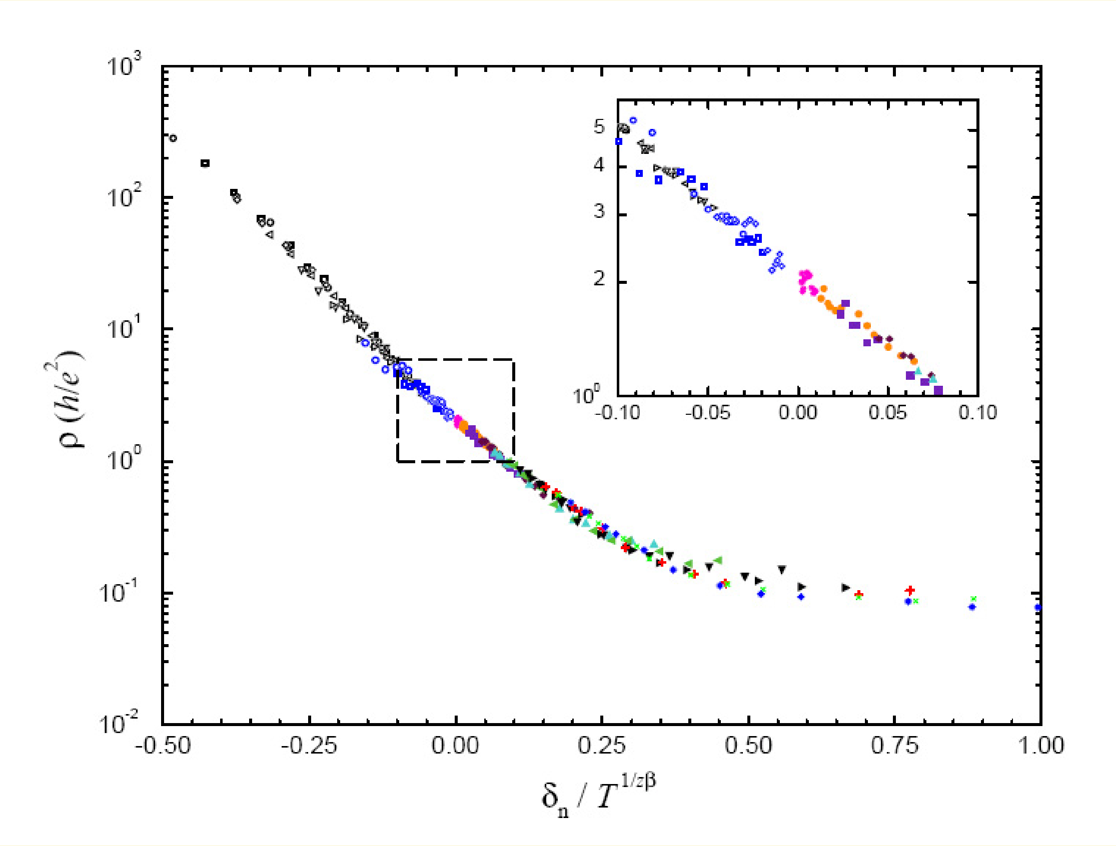
Scaling from previous work

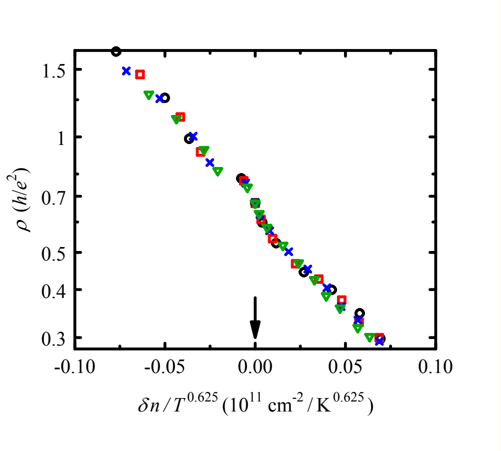
$\rho(T,n)$ instead of $\sigma(T,n)$ for Si-MOSFET by Kravchenko et al. (1995)



Scaling from previous work

$\rho(T,n)$ instead of $\sigma(T,n)$ for Si-MOSFET by Kravchenko et al. (1995), in comparison to data by Lai et al. (2007)





Remark

Data collapse **without parameter adjustment** is quite convincing, but contains hidden message:

Compare to three-dimensional case: Motivation for scaling analysis from own experiments on amorphous $Si_{1-x}Cr_x$ films. There, scaling only in hopping region. Reason for non-scaling at metallic side: *x*-dependence of $\sigma\ (T = 0, x)$

Here, for $d = 2$, hint to scaling at both sides of MIT as in Kravchenko's MOSFET study. This feature might imply new phase.

Why?

In case of conventional metallic conduction, as $T \to 0$:

$$\sigma(T,n) \to \text{finite } \sigma(T=0,n), \text{ which increases monotonously with } n$$

Vanishing T corresponds to diverging $\delta n / T^{0.625}$,

$$\sigma(\delta n / T^{0.625}, n) \to \sigma(\delta n / T^{0.625} = \infty, n) = \sigma(T=0,n)$$

Thus, curves drawn in scaling plot for varying T and fixed δn would split, in contradiction to observed scaling.

$\Rightarrow$ Scaling for $n > n_c$ cannot be understood in terms of conventional metallic conduction. **“Superconductivity” (only) at T = 0** may be an alternative, compare Kravchenko et al. (1995).

$\Rightarrow$ Data collapse at metallic side = **big puzzle**

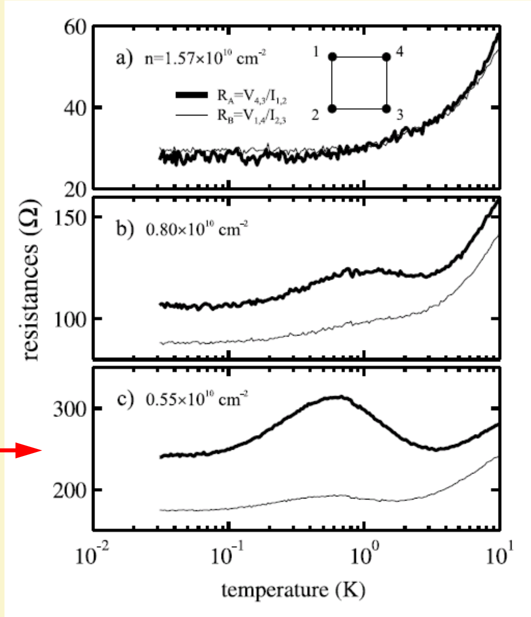
However, saturation at n dependent σ values as $T \to 0$ in many other studies.
Might, nevertheless, the minority experiments exhibit the generic feature?

Principal problem:

- Unknown validity region of scaling

Possible experimental uncertainties:

- Precision / density of data points,
- thermal decoupling as $T \to 0$,
- high-temperature mechanism causing apparent T dependence of n_c,
- inhomogeneities / stress → $\Rightarrow$ desirable check: van der Pauw at samples of different size



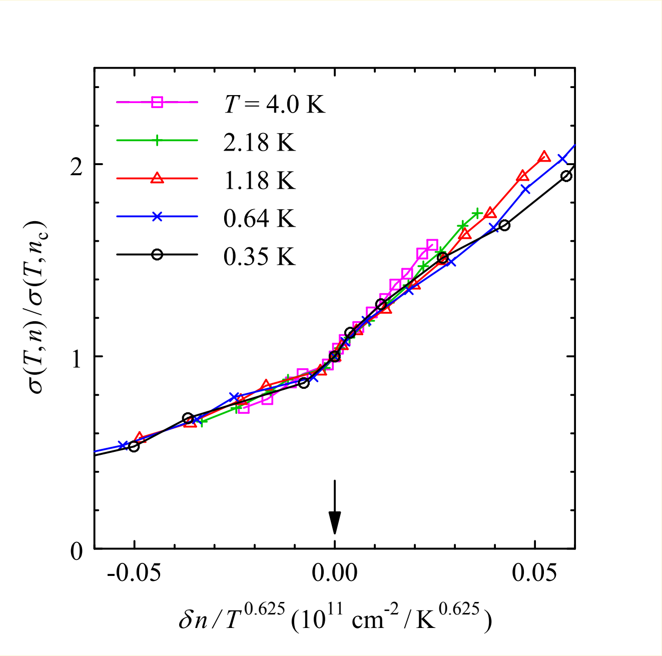
Lilly et al. (2003)

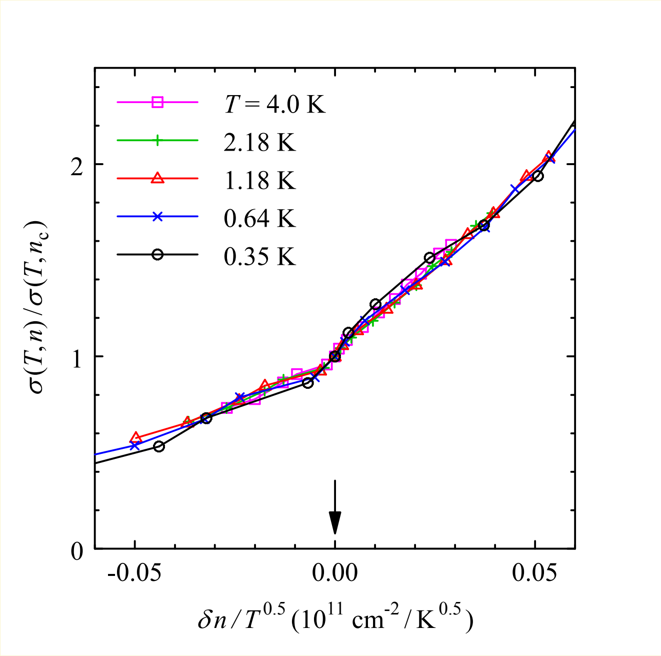
Generalisation to $T > 1$ K

Let T increase: Scaling $\sigma(T,n) = \sigma(t)$ breaks down when separatrix $\sigma(T,n_c)$ becomes T dependent. Experience from 3D systems suggests hypothesis

$$\sigma(T,n) = \sigma_{scal}(T/T_0(n)) \cdot \xi(T/T_1) \quad \text{with} \quad T_1 = \text{const} .$$

$\Rightarrow$ $s(T,n) = \sigma(T,n) / \sigma(T,n_c)$ may be scaleable. Tests for $\beta = 1.6$ and 2:

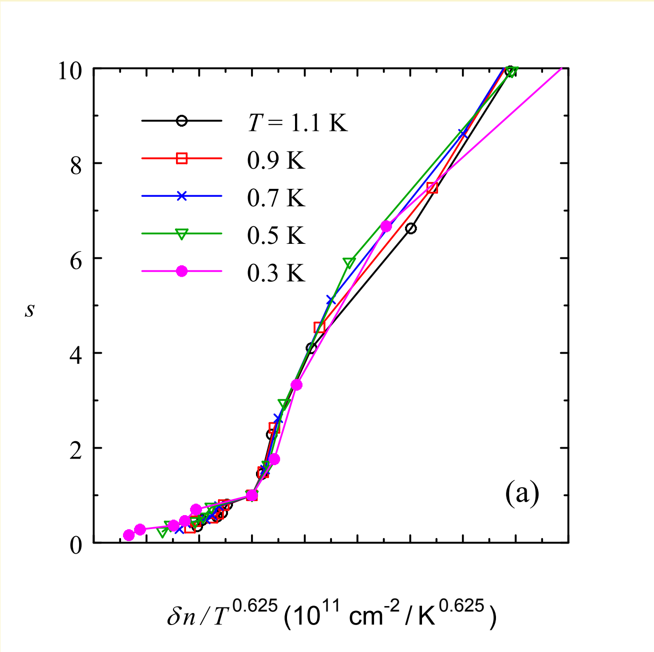


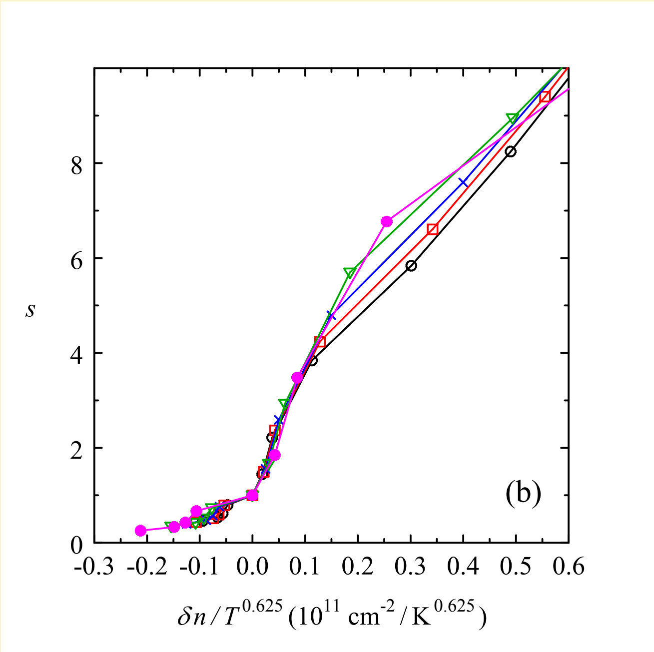


Comparison to AlAs

Turn once more to data by Papadakis and Shayegan (1998). Focus on $s(T,n) = \sigma(T,n) / \sigma(T,n_c)$.

Hidden problem: slight uncertainty of critical concentration from measurements for orthogonal directions (a) and (b). Thus, presume $n_c = 0.70 \cdot 10^{11}$ cm^{-2}





5. Peculiarity at MIT

Possible explanation of indentation in $\sigma(T = \text{const}, n)$ around MIT:

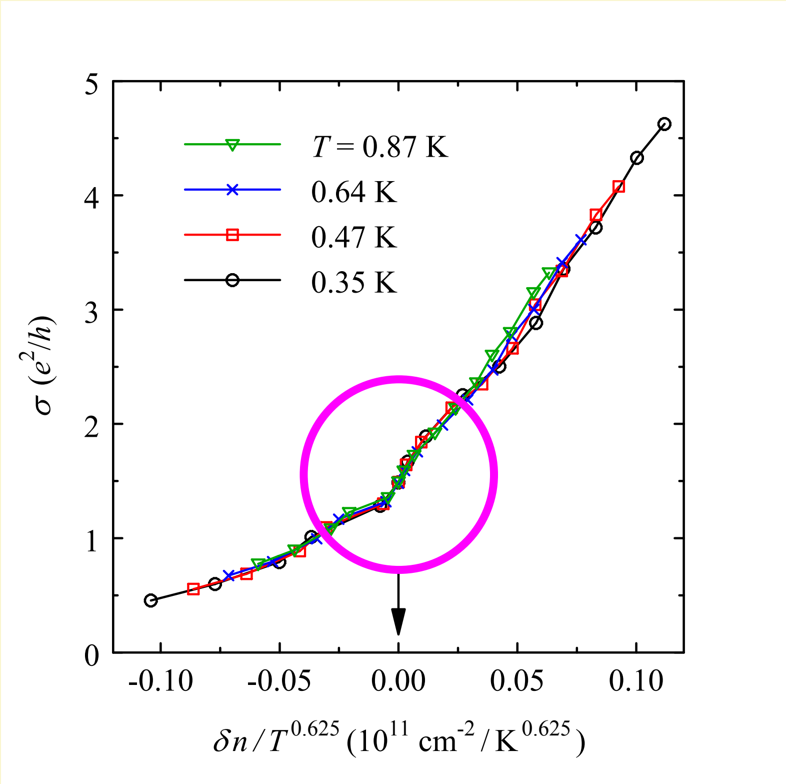
Approach MIT from "insulating" side, $n \to n_c - 0$, where $T_0 \to 0$ so that $t = T/T_0 \to \infty$ for all T. Suppose,

$$\sigma(t) = \sigma_c \cdot (1 - B\, t^{-\nu})$$

with $0 < \nu < ½$ (hopping). Thus,

$$\sigma = \sigma_c \cdot (1 - C\, T^{-\nu}\, |\delta n|^{\beta\nu})$$

with $\beta\nu < 1$. Due to this root-like peculiarity, $d\sigma / dn$ should diverge, as well as $d\log_{10}\sigma / dn$.



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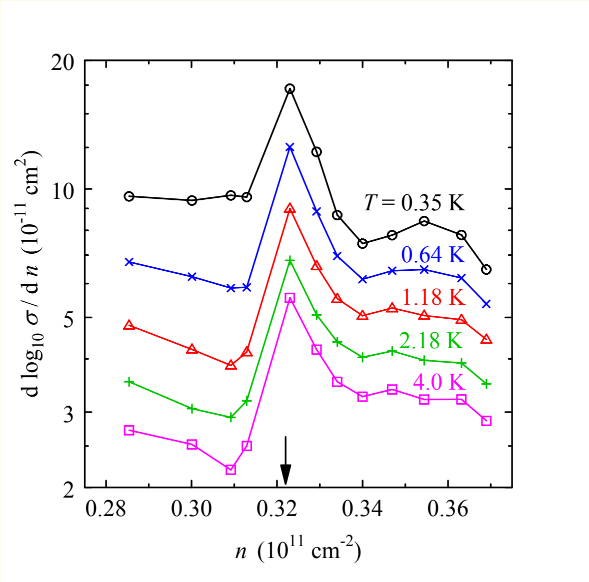


⇒ **Sharp peaks for all *T* present** (logarithmic scale of ordinate)

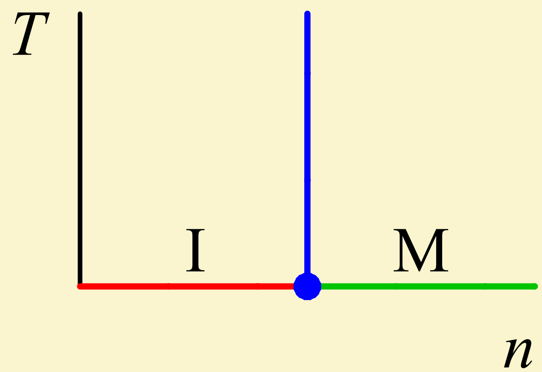
For previous analysis of $d\sigma/dn\,(n)$ compare Mack et al. (1998).

6. Conclusions

Six indications for line of phase transitions at finite T connected with the apparent MIT at $T = 0$:

- ► Advantage of piecewise linear fit compared to power law approximation
- ► Knee position close to n_c and independent of T
- ► Similarity to two previous experiments concerning offset in log-log plots
- ► Scaling analysis leads to data collapse without fit
- ► Quotient $s(T,n) = \sigma(T,n)/\sigma(T,n_c)$ scales even up to far larger T than $\sigma(T,n)$
- ► Sharp peaks in $\mathrm{d}\log_{10}\sigma/\mathrm{d}n$ as function of n for all considered T values

⇒ **Summary:**



⇒ **Challenges to colleagues:**

? Check by independent experiment, **precision matters, not lowest *T*:**

Detection of peculiarity in $\sigma(T = \text{const}, n)$ should at larger T be simpler.

Consider argument of scaling function $\sigma(x)$, $x = \delta n / T^{1/\beta}$, experimental uncertainties Δn and ΔT. Thus: $|\Delta x| = |\Delta n| / T^{1/\beta} + (|\Delta T| / T)\,|x| / \beta$

? Nature of apparently metallic phase

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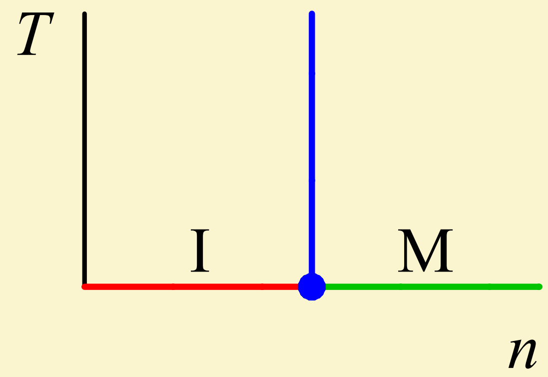

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? Nature of apparently metallic phase

For details: AM, Phys. Rev B **77** (2008) 205317; Physica E **42** (2010) 1243

Thank you for your attention!