The Unusual Behavior of

Excess Low-Frequency Flux Noise

at

Temperatures Below 1K

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Outline

- I. Introduction to 1/f flux noise
- II. Experimental arrangements
- III. Some basic facts about excess flux noise
- IV. Some details:
 - how we know its flux noise,how we know the electron temperature,distribution of amplitudes,device parameters and other devices
- V. Conclusions

Nakamura et al. results on Flux Qubit with best coherence



$\Gamma_{\varphi Ramsey}$, $\Gamma_{\varphi echo}$ vs n_{ϕ} : flux bias dependence

Nakamura et al



F. Yoshihara et al. PRL 97, 167001 (2006

Measured flux noise at 1Hz for several SQUIDs

"Low Frequency Noise in dc Superconducting Quantum Interference Devices Below 1K", F. C. Wellstood, C. Urbina, and J. Clarke, Appl. Phys. Lett., 50, 772 (1987).



- noise increases as T decreases below 1 K and then levels off
- noise roughly independent of SQUID size
- different devices with same shape produce similar noise

Nakamura et al.



charge/Josephson-energy fluctuations?

critical current fluctuations

The accepted explanation for 1/f noise (Dutta and Horn)

(1) Assume many two-level (magnetic dipole) fluctuators with equal well depths



(3) The magnetic moment of each fluctuator gives telegraph noise that produces a (1-sided) power spectrum (Machlup, J.A.P. 1954)):

$$S_{mi}(f) = \frac{4\sigma_m^2 \tau_i}{1 + \omega^2 \tau_i^2}$$

Lorentzian spectrum

Note that: the mean square standard deviation in the magnetic moment from the i-th fluctuator is:

$$\sigma_{m}^{2} = \int_{0}^{\infty} S_{mi}(f) df = \int_{0}^{\infty} \frac{4\sigma_{m}^{2}\tau_{i}}{1+\omega^{2}\tau_{i}^{2}} df = \frac{4\sigma_{m}^{2}}{2\pi} \int_{0}^{\infty} \frac{2\pi\tau_{i} df}{1+\omega^{2}\tau_{i}^{2}}$$
$$= \frac{2\sigma_{m}^{2}}{\pi} \int_{0}^{\infty} \frac{dy}{1+y^{2}} = \frac{2\sigma_{m}^{2}}{\pi} (a \tan(\infty) - a \tan(0)) = \sigma_{m}^{2} \qquad \text{independent}$$

The accepted explanation for 1/f noise (Dutta and Horn)



need equal number of Lorentzians in each decade of frequency or to get 1/f $D(\tau)d\tau \sim \frac{d\tau}{\tau}$ (4) Assume that there is a uniform distribution of barriers U, *i.e.* D(U) dU = C" dU notice that $U = kT \ln(2\tau/\tau_o)$ $D(\tau) = D(U) \frac{dU}{d\tau} = \frac{C''kT}{\tau}$

(5) The total noise spectrum is

$$S_{m}^{tot}(f) = \int_{0}^{\infty} S_{mi}(f)D(U)dU = 4\sigma_{m}^{2}C''\int_{0}^{\infty} \frac{\tau_{i}}{1+\omega^{2}\tau_{i}^{2}}dU$$

Now substitute: $\tau = \frac{\tau_{o}}{2}e^{U/kT}$
and find: $S_{m}^{tot}(f) = 4\sigma_{m}^{2}C''\int_{0}^{\infty} \frac{\frac{\tau_{o}}{2}e^{U/kT}}{1+\omega^{2}\frac{\tau_{o}^{2}}{4}e^{2U/kT}}dU$
change variables to $y = \frac{\omega\tau_{o}}{2}e^{U/kT}$
 $S_{m}^{tot}(f) = \frac{4\sigma_{m}^{2}C''kT}{\omega}\int_{0}^{\infty} \frac{\frac{\omega\tau_{o}}{2}e^{U/kT}}{1+\omega^{2}\frac{\tau_{o}^{2}}{4}e^{2U/kT}}dU$
 $S_{m}^{tot}(f) = \frac{4\sigma_{m}^{2}C''kT}{\omega}\int_{0}^{\infty} \frac{\frac{\omega\tau_{o}}{2}e^{U/kT}}{1+\omega^{2}\frac{\tau_{o}^{2}}{4}e^{2U/kT}}dU$
 Tf noise goes away as T decreases
 $S_{m}^{tot}(f) = \frac{4\sigma_{m}^{2}C''kT}{\omega}\int_{0}^{\infty} \frac{2\tau_{o}^{2}}{1+\omega^{2}\frac{\tau_{o}^{2}}{4}e^{2U/kT}}dU$

(4) Assume that there is a uniform distribution of barriers U, *i.e.* D(U) dU = C" dU notice that $U = kT \ln(2\tau/\tau_o)$ $D(\tau) = D(U) \frac{dU}{d\tau} = \frac{C''kT}{\tau}$

(5) The total noise spectrum is

expected noise to scales as T/f, which is not seen!

Kenyon et al. found T^2/f for uniform distribution of well barrier heights and well asymmetry...

T²/f or T/f works well for critical current noise, charge $^{\land}$ noise and might explain flux noise above 1 K but not below 1 K

playing around with D(U) can yield different slopes, but the temperature dependence doesn't look like data

$$S_{m}^{tot}(f) = \frac{4\sigma_{m}^{2}C''kT}{\omega} \int_{0}^{\infty} \frac{\frac{\omega\tau_{o}}{2}e^{U/kT}}{1+\omega^{2}\frac{\tau_{o}^{2}}{4}e^{2U/kT}} \frac{dU}{kT} = \frac{4\sigma_{m}^{2}C''kT}{\omega} \int_{0}^{\infty} \frac{dy}{1+y^{2}} = \frac{\sigma_{m}^{2}C''kT}{f}$$

 $U_1 \quad U_2$

Flux noise power from fluctuators uniformly spread in 3D

consider flux coupled into i-th small loop when current I_s flows in SQUID



current fluctuating in i-th small loop

$$\Phi_{\rm sa} = \mathbf{I}_{\rm s} M_i = \overline{B}_s(r,r') \cdot \overline{A}_a$$

mutual inductance between SQUID and i-th small loop $\longrightarrow M_i = \frac{B_s(r, r') \cdot A_a}{I}$

Flux coupled into SQUID by i-th small loop is:

 $\Phi_i = M_i I_a = \frac{B_s(r, r') \cdot \overline{A}_a}{I_s} I_a$ Let $I_a = |I_a|\sigma$, where $\sigma = +/-1$. Flux noise power spectral density from $S_{\Phi i}(f) = \left| \frac{\overline{B}_s(\overline{r}, \overline{r'}) \cdot \overline{A}_{aj}}{I} \right|^2 I_a^2 S_{\sigma i}(f)$

Total flux noise power spectral density from all small loops in half-space below the SQUID

$$S_{\Phi} = \sum_{i=1}^{N} S_{\Phi i} = \frac{1}{2} \int \left[\frac{\overline{B}_{s}(\overline{r},\overline{r}') \cdot \overline{A}_{aj}(\overline{r}')}{I_{s}} \right]^{2} I_{a}^{2} S_{\sigma i} n dV'$$
$$= \frac{n}{2} \left(\frac{I_{a} A_{a}}{I_{s}} \right)^{2} \left(\int \left| \overline{B}_{s}(\overline{r},\overline{r}') \right|^{2} \left\langle \cos^{2}(\theta(\overline{r}')) \right\rangle dV' \right) S_{\sigma i} = \frac{1}{6} n \mu_{o} \mu_{B}^{2} L \left\langle S_{\sigma} \right\rangle$$

Flux noise measurement technique





Flux noise measurement technique









Surround cell with: Vaccum can aluminum dewar 2 mu-metal cylinders cu-mesh shielded room in second basement lab

Cell filled with liquid helium Measurements made from 4.2 K to 0.02 K,

on shunted SQUIDs in the continuously running state (Power ~ few pW),

in low-field,

in '84-87



Clarke's dilution refrigerator lab - Second Basement of Birge Hall ~1985

Nb/NbOx/PbIn SQUID

Type D



Typical Fabrication Procedure:

- Clean 2" wafer (Si, Oxidized Si, or sapphire), pattern photoresist, deposit Cr or Ti and then AuCu, liftoff in acetone
- 2) clean, sputter Nb pattern photoresist plasma etch in SF6O2
- 3) clean, pattern photoresist deposit Cr or Ti and then SiO, liftoff in acetone
- 4) clean, pattern photoresist deposit Cr or Ti and then SiO liftoff in acetone
- 5) clean, pattern photoresist
- 6) Dice wafer into 5 mm chips
- 7) Use Ar-Ion mill to clean Nb surface, transfer chip in air to oxidation chamber, oxidize in Ar+O2 plasma evaporate Pb+ 5%In liftoff in acetone







10

1

0.1

0.1

10000 -**--** 0.51 K 1 mm AuCu shunt 100 µm 1000 S_{Φ} ($\mu \Phi_{o}/Hz^{1/2}$)² 100 10 1

100

f (Hz)

1000

10000







RMS flux noise power spectral density at 1Hz vs T in device D2

Type D

1 mm

"Big" SQUIDs

Type D

.

Fig. 1.16 Schematic of a Type M dc SQUID.

RMS flux noise power spectral density at 1Hz vs T

- noise magnitude **increases** as T **decreases** below 1 K

- different devices with same shape produce similar noise

"Big" SQUIDs

Type D

100 μm

Fig. 1.16 Schematic of a Type M dc SQUID.

Type P

Type K

Nb AuCu Shunt

built on sapphire

Type P

100 µm

- noise magnitude **increases** as T **decreases** below 1 K
- different devices with same shape produce similar noise
- noise depends weakly on SQUID "size" (linewidth ? overall length?)
- noise present with sapphire or Si substrates

"Small" SQUIDs

- noise magnitude **increases** as T **decreases** below 1 K
- different devices with same shape produce similar noise
- noise depends weakly on size of SQUID center hole
- noise independent of materials & substrate (Si, sapphire, oxidized Si)

Flux noise power and magnetic loss tangent

Fluctuation-dissipation theorem:

when there is noise there is an associated dissipation For magnetic dissipation in a loop, we can define a complex permeability $\mu = \mu_1 - i\mu_2$ and a complex inductance $L_c = (\mu_1 - i\mu_2)\mu_o F(a, b, ...)$

The impedance of the loop is:

$$Z(\omega) = i\omega L_c = i\omega \left(\mu_1 - i\mu_2\right)\mu_o F(a, b, \dots) = i\omega \mu_1 \mu_o F(a, b, \dots) \left(1 - i\frac{\mu_2}{\mu_1}\right)$$

1

$$=i\omega L\left(1-i\frac{\mu_2}{\mu_1}\right)=i\omega L+\omega L\tan(\delta_m)=i\omega L+R_m(\omega)$$

The flux noise produced in the loop is then:

$$\mathbf{S}_{\Phi}(\mathbf{f}) = \mathbf{S}_{\mathrm{I}}(\mathbf{f})L^{2} = \frac{\mathbf{S}_{\mathrm{v}}(\mathbf{f})L^{2}}{\omega^{2}L^{2} + \mathbf{R}_{\mathrm{m}}^{2}} = \frac{4\mathbf{k}_{\mathrm{B}}TL^{2} \cdot \mathrm{Im}(\mathbf{Z}(\omega))}{\omega^{2}L^{2} + \omega^{2}L^{2}\tan^{2}(\delta_{\mathrm{m}})} = \frac{4\mathbf{k}_{\mathrm{B}}TL^{2} \cdot \omega L\tan(\delta_{\mathrm{m}})}{\omega^{2}L^{2} + \omega^{2}L^{2}\tan^{2}(\delta_{\mathrm{m}})}$$

Flux noise power and magnetic loss tangent

For $\tan(\delta_m) \ll 1$, we find $S_{\Phi}(f) \cong \frac{4k_B TL}{2\pi f} \tan(\delta_m)$ $\tan(\delta_m) \cong \frac{\pi f S_{\Phi}(f)}{2k_B TL}$

If $tan(\delta_m)$ is independent of frequency, then we get $1/f^1$ noise, If flux noise independent of T, then $tan(\delta)$ scales as 1/T.

Fig. 1.10 Schematic of a Type F dc SQUID.

Flux noise power spectrum in two SQUIDs

Temperature dependence of the slope α of the 1/f α flux noise

- devices with similar shapes had similar slope α
- small devices tend to have lower alphas with stronger T dependence
- only biggest devices (D1, D2, M1, M2) had α close to1

An "explanation" for 1/f^{2/3} noise

(1) Assume two-level magnetic dipole fluctuators with energy difference $U_i < < kT$

(2) Assume that transitions from the upper to lower state takes time:

$$T_1 = \tau_{LR} = \frac{\tau_o}{F_i^3} = \tau_{RL} = 2\tau_i$$

(3) Assume F_i is an (unknown) physical parameter that has a uniform distribution;
 D(F) dF = C" dF and note that:

$$F = \left(\frac{\tau_o}{2}\right)^{-1/3} \tau^{-1/3} \longrightarrow D(\tau) = D(F) \left|\frac{dF}{d\tau}\right| = \frac{C''}{3} \left(\frac{\tau_o}{2}\right)^{-1/3} \frac{1}{\tau^{4/3}}$$

Then the total noise spectrum is

$$S_m^{tot}(f) = \int_0^\infty S_{mi}(f) D(F) dF = C'' 4\sigma_m^2 \int_0^\infty \frac{dF}{1 + \omega^2 (\tau_o/F^3)^2} \frac{\tau_o}{F^3}$$

$$S_{m}^{tot}(f) = C'' 4\sigma_{m}^{2} \int_{0}^{\infty} \frac{dF}{1 + (\omega \tau_{o}/F^{3})^{2}} \frac{\tau_{o}}{F^{3}}$$

now change variables to

$$y = \frac{\omega \tau_o}{F^3}$$

and notice
$$F = (\omega \tau_o)^{1/3} y^{-1/3}$$
 $dF = -\frac{1}{3} (\omega \tau_o)^{1/3} y^{-4/3} dy$

$$S_m^{tot}(f) = \frac{C'' 4\sigma_m^2}{\omega} \int_0^\infty \frac{\frac{1}{3} (\omega \tau_o)^{1/3} y^{-4/3} dy}{1 + y^2} y = \frac{C'' 4\sigma_m^2 \tau_o^{1/3}}{3\omega^{2/3}} \int_0^\infty \left(\frac{1}{y^{1/3}}\right) \frac{dy}{1 + y^2}$$

$$S_m^{tot}(f) = \frac{C'' 4\sigma_m^2 \tau_o^{1/3}}{3(2\pi)^{2/3}} \frac{\pi}{2\sin(\pi/3)} \frac{1}{f^{2/3}}$$

Temperature independent $1/f^{2/3}$ noise...assuming kT>>U. Putting in all the thermal factors one finds the noise falls off rapidly for kT<<U and is flat for kT>>U.... **Key Question:** We assumed $T_1 = \frac{\tau_o}{F^3}$, but what is the factor F?

Answer: Don't know if this is the correct explanation (and its not likely it is!). If F does exist, its not known what it is physically.

How do we know its really a flux noise: I-V and I- Φ : SQUID D2 at 0.95 K

and 0.7 μ A, respectively. (f) Noise vs V for the same flux bias currents as in (e).(9-1-86).

Flux noise: Total noise at the output scales with flux gain

Fig .8.6 (a) Output rms excess current noise at 1 Hz vs. flux gain for SQUID E2 at 110 mK. (b) Output excess noise power at 1 Hz vs. square of flux gain. Solid line is linear plus constant fit.

Critical Current Noise

"Flicker (1/f) noise in the critical current of Josephson junctions at 0.09--4.2 K", F. C. Wellstood, C. Urbina, and John Clarke, Appl. Phys. Lett. **85**, 5296 (2004).

Hot electrons in cold SQUIDs (the electron temperature in the shunts)

Fig. 11.6 Flux noise energy vs. T_0 for SQUIDs \Box D1, \blacksquare D2, \bullet M1, 0 M2. SQUIDs were biased near $\Phi_0/4$, with the operating points given in Table II. Solid line is prediction of Tesche and Clarke.⁽²⁾

Hot electrons in cold SQUIDs (the electron temperature in the shunts)

<u>Table 11.1</u> Operating points for the SQUIDs D1, D2. M1, and M2. V is the voltage at which the SQUID is biased, I is the current flowing through the SQUID. P is the power dissipated in the SQUID, Ω is the shunt volume, A is the shunt area in contact with the substrate.

Device	v	I	P	P/Q	P/A	$T_{min} = (P / \Sigma \alpha)^{1/5}$	
	(<i>W</i>)	(Au)	(pW)	(Wm ⁻³)	(Wm ⁻²)	(K)	
D1	1.1	5	5.4	1.9 X 10 ⁵	5.6 X 10 ⁻³	0.151	
D2	1.1	3.5	3.8	1.3 X 10 ⁵	4. X 10 ⁻³	0.140	
M1	1.2	4.5	5.4	1.2 X 10 ³	3.5 X 10 ⁻⁵	0.055	
M2	4.0	5	20.	1.4 X 10 ²	1.3 X 10 ⁻⁴	0.036	

"<u>Hot-electron effects in metals</u>", F. C. Wellstood, C. Urbina, and J. Clarke, Phys. Rev. B **49**, 5942 (1994).

"Hot-electron limitation to the sensitivity of the dc superconducting quantum interference device", F. C. Wellstood, C. Urbina, and J. Clarke, Appl. Phys. Lett. **54**, 2599 (1989).

Gaussian distribution of amplitudes

bin #

Fig. 8.10 Noise amplitude distribution from SQUID M1.: number of times the output acheived a certain amplitude vs. the amplitude. Solid line is gaussian fit to data.

Parameters of measured devices

Device	Date	T	210	4	r	α	β	L
		(mK)	(Au)					(nH)
A1	7- 2-85	150	3.66	0.484	0.632	0.45	0.7	0.396
A2 (*)	7-21-85	130	10.8	0.328	0.61	0.6	1.35	0.26
A3 (**)	5-25-85	160	5.78	0.635	0.779	0.22	0.495	0.18
A4	8-10-85	120	0.71	0.89	0.99	0.0	0.16	0.47
A'5	9-16-85	170	6.23	0.392	0.754	0.39	1.2	0.399
A6 (***) 2-19-87	1390	2.04	0.6	0.83	0.18	0.58	0.59
A7 (***) 2-16-87	4200	17.2	0.176	0.953	0.25	4.0	0.48
A8 (***) 2-12-87	4200	3.84	0.53	0.9 05	0.11	0.79	0.43
B1	10-27-87	160	3.00	0.304	0.709	0.55	1.7	1.17
°C1	9-21-85	120	6.15	0.32	0.60	0.68	0.52	0.18
C2	11-15-85	150	5.32	0.618	0.826	0.18	0.55	0.214
C3	1-31-86	1400	21.2	0.292	1.0	0.0	2.1	0.205
C4	2- 6-86	1400	7.1	0.55	0.954	0.06	0.75	0.219
C5	2- 8-86	1400	3.07	0.379	0.766	0.62	0.24	0.16
D1	5-30-86	110	6.34	0.313	0.597	0.64	1.35	0.44
D2	8-29-86	50	4.55	0.414	0.772	0.35	1.1	0.50
E1	2-19-86	95	5.51	0.563	0.96	0.05	0.725	0.27
E2	5-21-86	510	15.1	0.291	0.96	0.13	2.1	0.29
F1	2-27-86	140	5.73	0.32	0.945	0.15	1.93	0.70
G1	3-12-86	110	4.15	0.39	0.76	0.61	0.24	0.12
I1	6-27-87	105	4.13	0.668	0.905	0.09	0.45	0.225
J1	8- 5-86	50	21.6	0.283	0.919	0.25	2.1	0.20
K1	8-14-86	35	11.42	0.23	0.939	0.23	2.9	0.53
M1	10-31-87	25	5.64	0.39	0.99	0.02	1.4	0.514
M2	5- 6-88	20	6.2	0.35	0.71	0.55	1.4	0.47
N2	12-11-87	20	17.7	0.196	0.886	0.5	3.4	0.398
01	5-19-87	30	1.36	0.97	0.964	0.05	0.03	0.046
P1	6- 5-87	23	1.77	0.352	0.665	0.55	1.25	1.46
NBS1	7-21-87	20		(see Chapt	ter 8)			0.08
FIN1	4-10-88	43		(see Chapt	ter 8)			0.04

(*) 20 turn input coil, left open for measurement of β.

(**) Magnetometer configuration, (geometry described in Chapter 3)

(***) These were variants of the type A, discussed in Chapter 7.

Gradiometric micro-SQUID susceptometer for scanning measurements of mesoscopic samples

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Investigation of Low-frequency Excess Flux Noise in dc SQUIDs at mK Temperatures

D. Drung, J. Beyer, J.-H. Storm, M. Peters, and T. Schurig *Physikalisch-Technische Bundesanstalt (PTB), Berlin, Germany*

 $4.2 \text{ K} \rightarrow \langle 320 \text{ mK} \rangle$: white noise falls, but 1/f noise rises

For this example: mK noise higher than 4.2 K noise above ≈40 Hz

Conclusions

Low-frequency excess flux noise has some unusual properties below 1 K:

- smooth, featureless spectra (except maybe near 0.7K)
- does not depend on materials... but comes from them
- does not behave like excess charge or critical current noise
- increases in magnitude as T decreases below 1 K.... Why?
- may level out to 5-15 $\mu \Phi_o/Hz^{1/2}$ below about 0.5 K
- depends weakly on the SQUID area (can't be B noise)
- implies significant tan(δ) if 1/f noise extends to GHz

The slope" α of the 1/f^{α} noise depends on the loop geometry

- α can be significantly less than 1
- α depends on temperature
- α depends on geometry of the SQUID... How is this possible?
- α tended to be smaller for smaller SQUIDs.... Why?
- recent data on small AI flux qubits suggests α =1, but is it?