



GRC Lisa Randall

# BEYOND THE STANDARD COSMOLOGICAL MODEL

# Post-Modern Cosmology

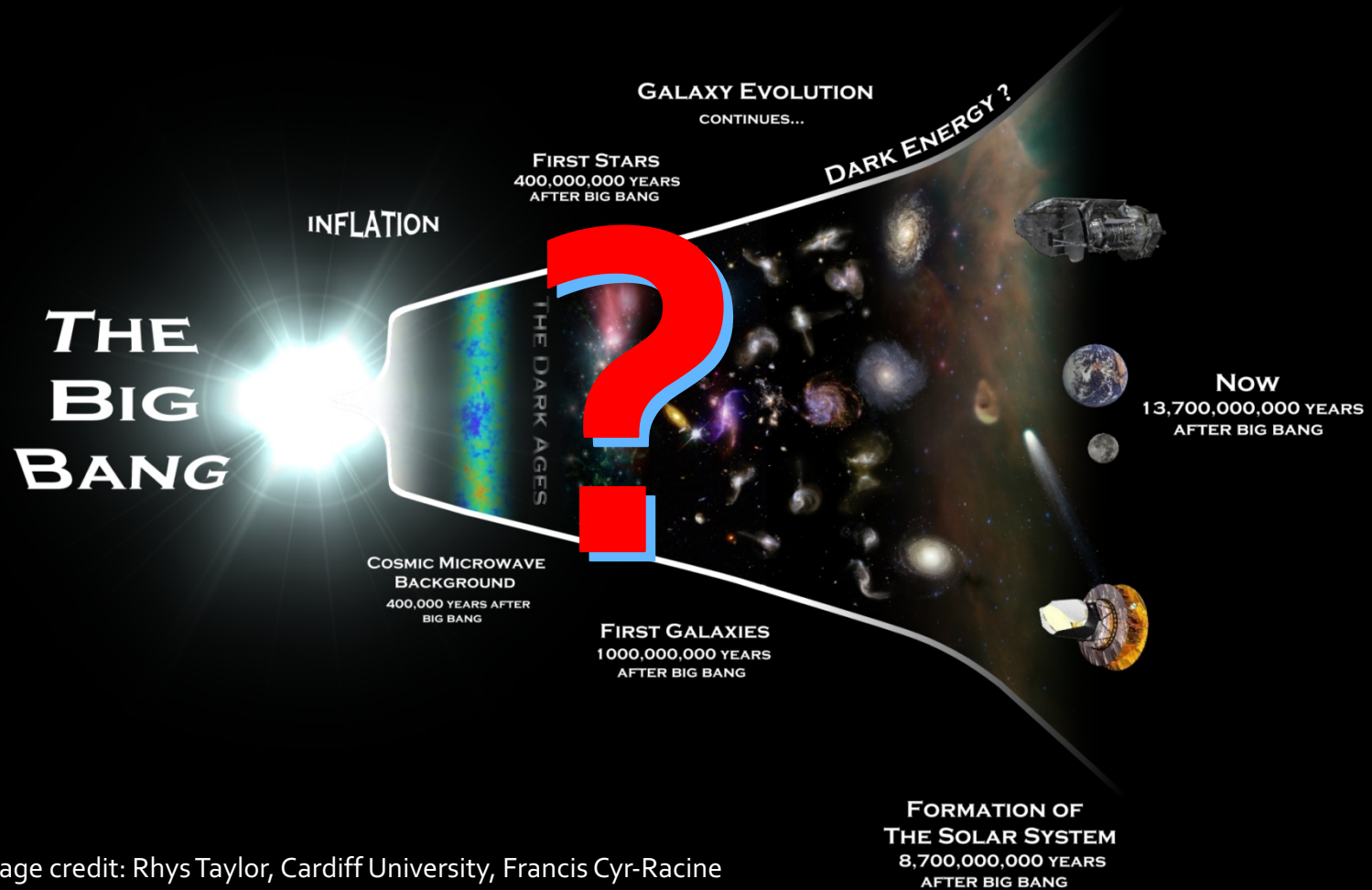


Image credit: Rhys Taylor, Cardiff University, Francis Cyr-Racine

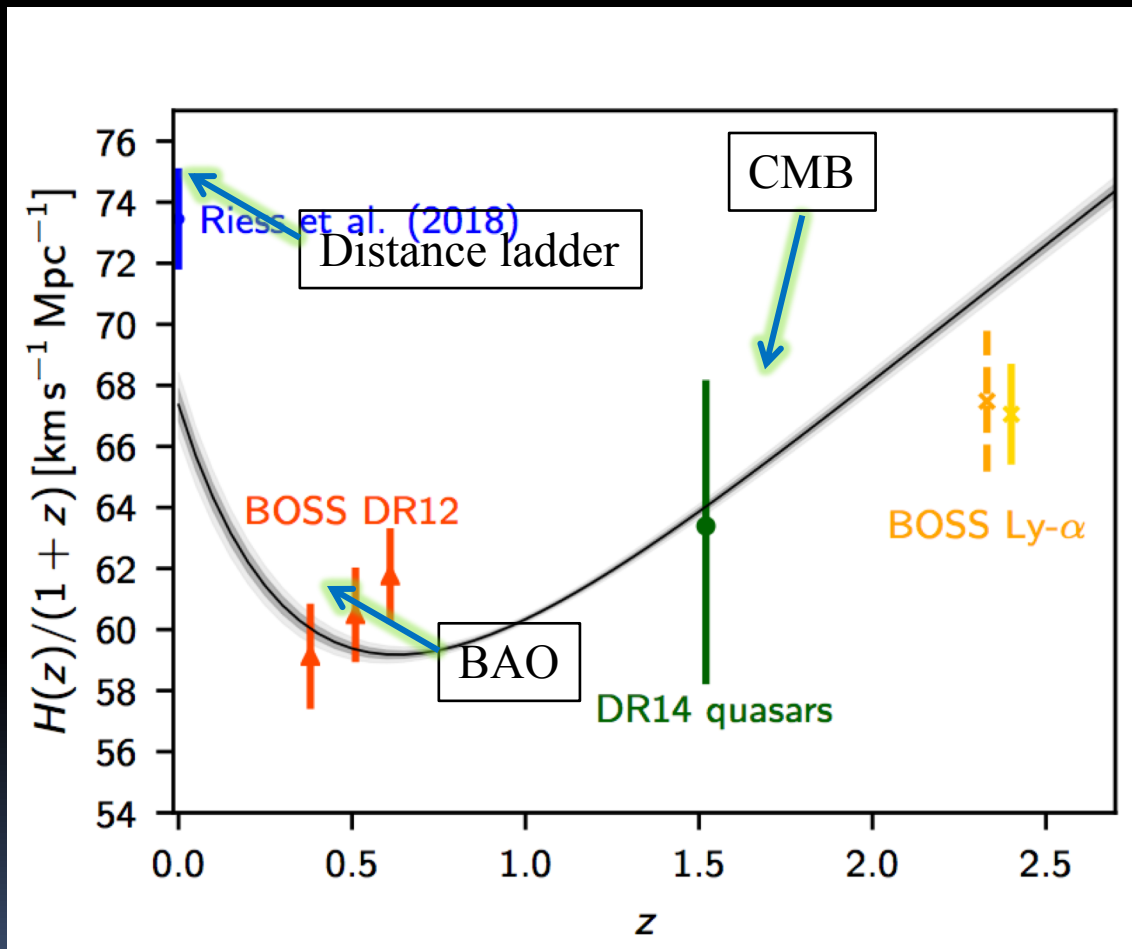
# Why look beyond SCM?

- Why do physics?
- Can measure parameters but interesting in so far as we learn new things
  - We are not curing cancer
  - Our goal is new knowledge for its own sake
- Cosmology already amazingly refined
- Model works brilliantly
  - But some potential holes
  - Important to know whether and how can be accommodated
  - I briefly mention one

# Precision Cosmological Measurements

- Local measurements:  $74.03 \text{ km/s/Mpc} \pm 1.42$  vs CMB +BAO alone:  $67.66 \pm 0.42$
- $4.4\sigma$ , 9 % difference
- **Challenging to resolve in expected theories**
- Why pursue?
  - Has become stronger with time
  - Why measure unless a possibility for unexpected?
- What I show here
  - We find field-theoretically consistent potentials with correct behavior
    - That we can track explicitly
  - With full data sets,  $H_0$  up to 72.3 (at 2 sigma)
  - Future measurements will definitely have the last word

# What is Hubble Tension and Why Worry?

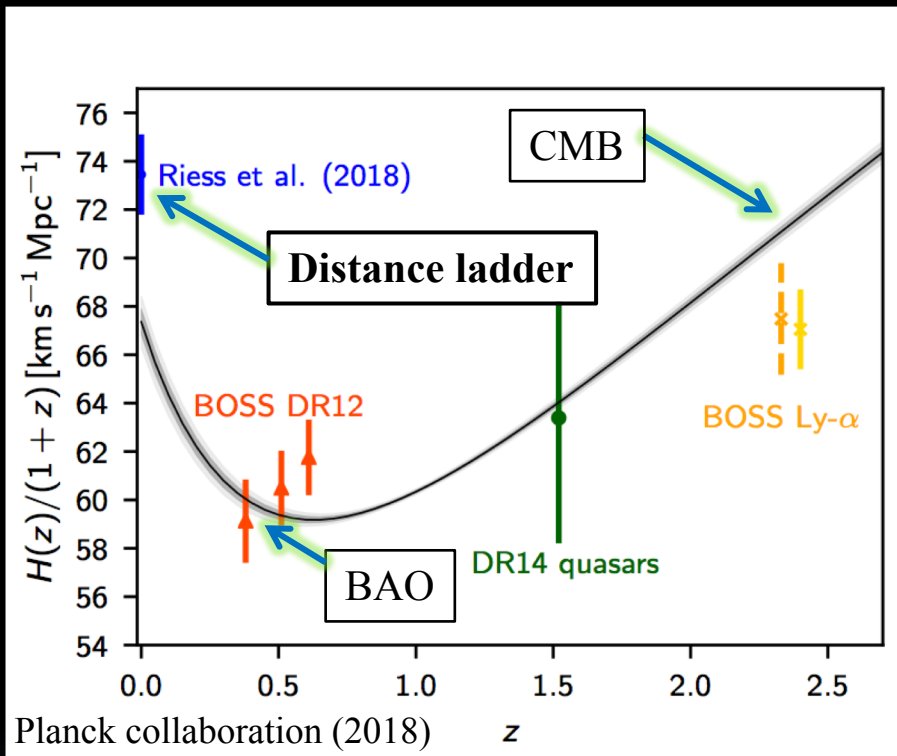


With CMB and SN alone  
can fit with late physics  
Difficult however to fit  
(low  $z$ ) BAO as well

Is this possible?  
Is there room for new  
physics?  
Role for model builders?

Planck collaboration (2018)

# How to Proceed?



Riess et al. (2018) provides a *direct* measurement of the current Hubble rate.

Other measurements require knowledge of the baryon-photon sound horizon,  $r_s$ .

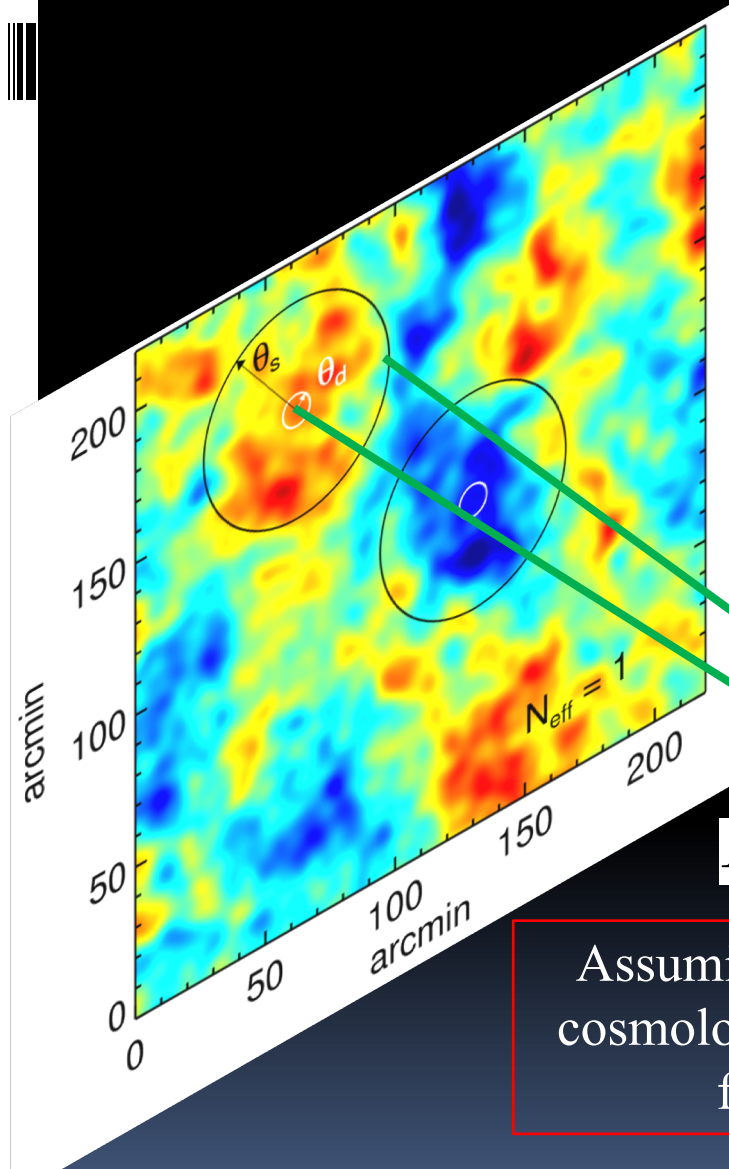
Time of baryon decoupling

$$r_s = \int_0^{t_d} c_s dt / a = \int_0^{a_d} c_s \frac{da}{a^2 H(a)}$$

$$r_s = \int_{z_*}^{\infty} \frac{c_s dz'}{H(z')},$$

$$D_M(z) = \int_0^z \frac{dz'}{H(z')}.$$

# Cosmic Microwave Background



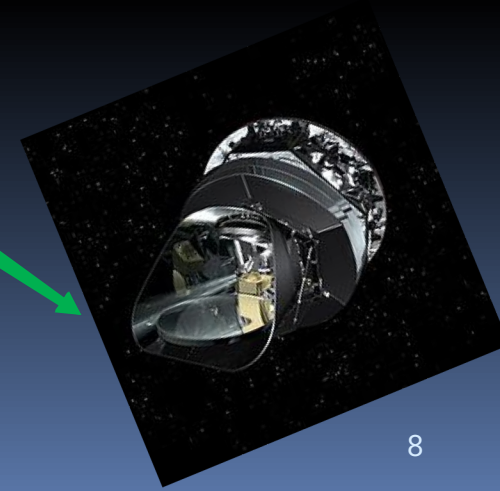
The CMB primarily measures angles on the sky.

$$\theta_s = r_s / D_A(z_d)$$

$$D_A(z) = \int_0^z dz' / H(z')$$

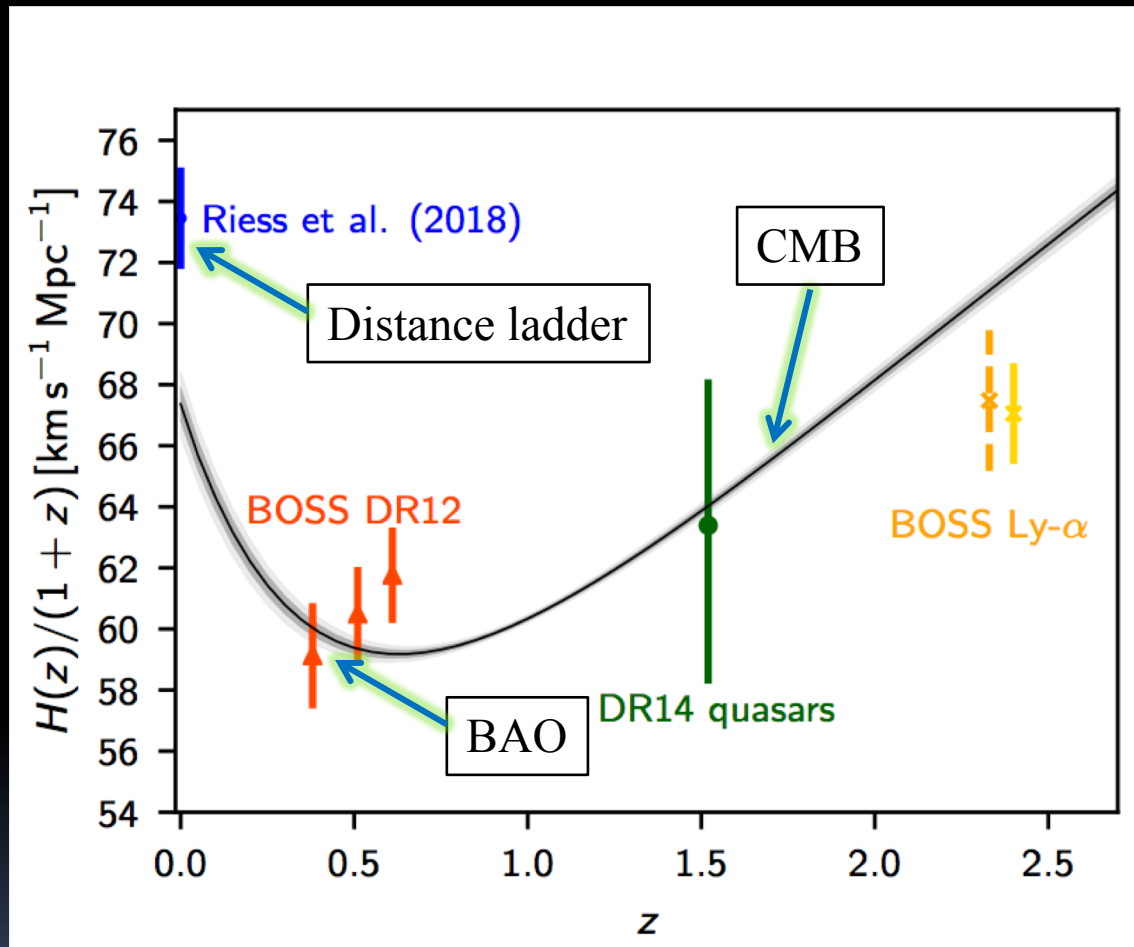
$$D_A(z_d)$$

Assuming a late-time cosmology, can infer  $r_s$  from  $\theta_s$ .





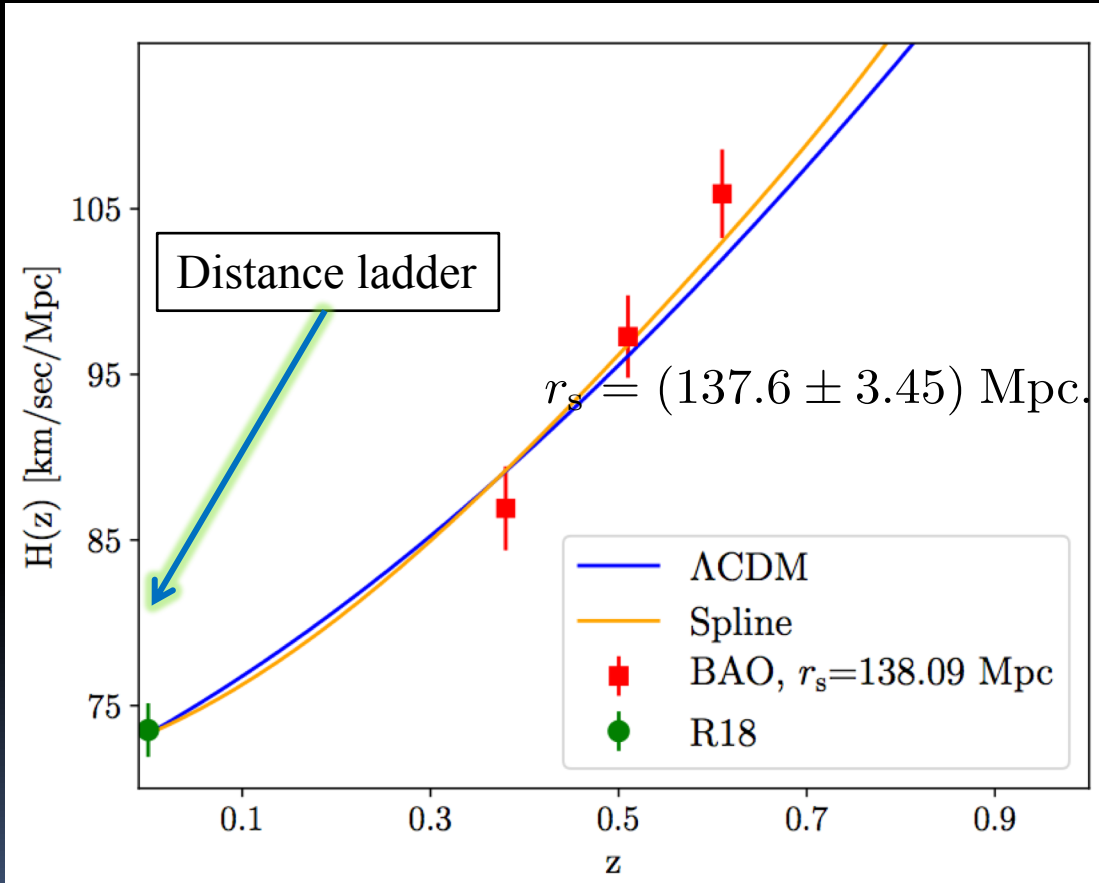
# BAO Calibrated with CMB



BOSS data points on this plot use CMB-measured value of the sound horizon as calibration  
H is function of time  
Feeds into all the measurements

Planck collaboration (2018)

# Instead Calibrate BAO with local distance ladder



BAO compatible with local  $H_0$  measurement with a smaller baryon-photon sound horizon.

For comparison, Planck's CMB value is:

$$r_s = 147.05 \pm 0.30 \text{ Mpc}$$

Aylor et al, (2018)

# Baryon-Photon Sound Horizon

- **Hubble rate** at early times.

$$c_s = \frac{1}{\sqrt{3\left(1 + \frac{3\rho_b}{4\rho_\gamma}\right)}}$$

$$r_s = \int_0^{a_d} da \frac{c_s(a)}{a^2 H(a)}$$

Or sound speed  
-very unlikely

New question:  
Different Hubble rate  
before recombination  
and still match other  
data?

$$H^2(a) = \frac{8\pi G}{3} \sum_i \rho_i(a)$$

# Possible energy injection shape:

Alireza Hojjati<sup>1</sup>, Eric V. Linder<sup>1,2</sup>, Johan Samsing<sup>3</sup>

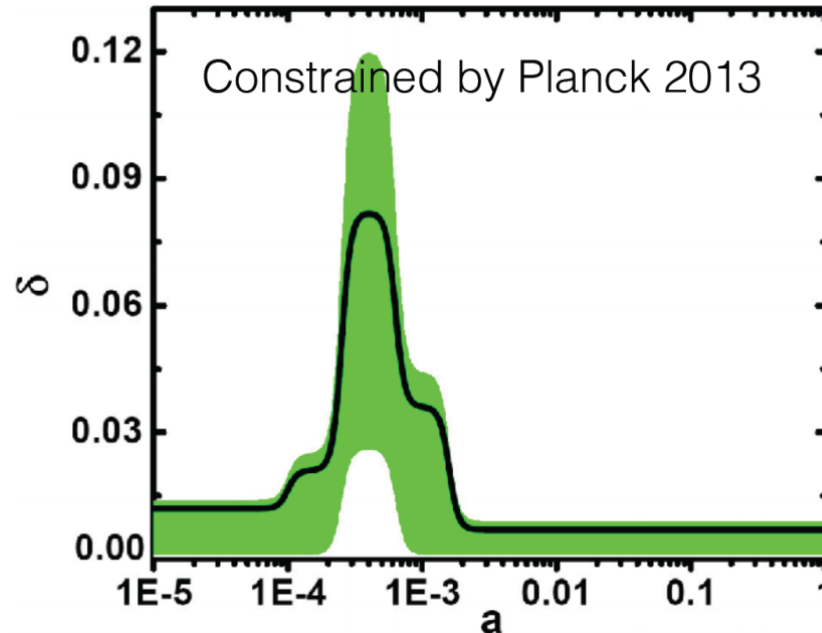
# Constraint on injection from cmb

Designer  $H(a)$

arXiv:1304.3724

New Constraints on the Early Expansion History

Alireza Hojjati<sup>1</sup>, Eric V. Linder<sup>1,2</sup>, Johan Samsing<sup>3</sup>



$$H^2(a) = \frac{8\pi G}{3} [\rho_m(a) + \rho_r(a) + \rho_\Lambda] [1 + \delta(a)]$$

Knox  
slide

# Goal: Potential

- Explicitly accomplish goal
- Allows you to check if it works
  - Background
  - Fluctuations
- Challenge:
  - Speed of Transition
    - Energy that is present too early or too late problematic
  - Need well Localized to Matter-Radiation ,  
Decoupling on Tail

# What We Want for Potential



Agrawal, Cyr-Racine, Pinner, LR

- Note energy densities separately conserved

$$\frac{\rho_\phi}{\rho_b} \propto \exp\left(-\int 3[w_\phi(a) - w_b(a)]d \ln a\right)$$

- Need energy not to dominate early or late

$(w_\phi - w_b)$  must transition from negative to positive.

- Implies energy injection when  $w_\phi = w_b$
- Most straightforward:  $w_\phi = -1$  initially
  - Field frozen
  - Find scalar potential with  $w_\phi > w_b$  once field starts moving

# Model: Rolling Solutions (With constant $w$ )

- Search for:

$$w_\phi > w_b,$$

$$\rho_\phi(a) = \rho_0 \left( \frac{a_0}{a} \right)^{3(1+w_\phi)}$$

- Gives potential and its derivative

$1 + w_\phi = a^2 H^2 (\partial_a \phi)^2 / \rho_\phi$ , we can extract formulae for  $V(\phi)$  and  $\partial_a \phi$

$$V(\phi) = \frac{1 - w_\phi}{2} \rho_\phi,$$

$$\partial_a \phi = \frac{\sqrt{(1 + w_\phi) \rho_\phi}}{aH}.$$

Yields power law potential:

Using  $3H^2 M_{\text{Pl}}^2 \approx \rho_b = \rho_{b0} \left( \frac{a_0}{a} \right)^{3(1+w_b)}$ , we can solve for  $\phi(a)$ ,

$$\phi(a) = c \left( \frac{a_0}{a} \right)^{\frac{3}{2}(w_\phi - w_b)}, \quad c = \frac{M_{\text{Pl}}}{(w_\phi - w_b)} \sqrt{\frac{4(1 + w_\phi) \rho_0}{3\rho_{b0}}}$$

# Power law potential

$$w_\phi > w_b,$$

$$\rho_\phi(a) = \rho_0 \left( \frac{a_0}{a} \right)^{3(1+w_\phi)}$$

$$V(\phi) = \frac{1}{2}(1 - w_\phi)\rho_0 \left( \frac{\phi}{c} \right)^{2n}, \quad n = \frac{1 + w_\phi}{w_\phi - w_b}.$$

This is asymptotically rolling

This potential can have oscillating solutions too

And those can also be of interest



# Emden-Fowler Classification: Rocking and Rolling

$$\frac{\partial^2 \phi}{\partial (\log a)^2} + \frac{3}{2}(1 - w_b) \frac{\partial \phi}{\partial (\log a)} + \frac{\partial_\phi V}{H^2(a)} = 0$$

$$s \propto a^{\frac{3}{2}(1-w_b)} \text{ and } y = s\phi$$

$$y''(s) + s^\sigma y^\gamma(s) = 0.$$

$$\sigma = \frac{4}{1-w_b} - 2(n+1) \text{ and } \gamma = 2n - 1.$$

Asymptotic solutions	Emden-Fowler conditions	Translation to scalar field models	Background	
			Radiation	Matter
Osc. only	$\sigma + 2 \geq 0$	$n < \frac{2}{1-w_b}$	$n < 3$	$n < 2$
Osc. + non-osc.	$\sigma + 2 < 0 \leq \sigma + \frac{\gamma+3}{2}$	$\frac{3+w_b}{1-w_b} \geq n > \frac{2}{1-w_b}$	$5 \geq n > 3$	$3 \geq n > 2$
Non-osc. only	$\sigma + \frac{\gamma+3}{2} < 0$	$n > \frac{3+w_b}{1-w_b}$	$n > 5$	$n > 3$

# Rocking vs Rolling

- When both exist, rocking solution more stable

$$w_{\text{osc}} \approx \frac{n-1}{n+1} > 1/(n+1)$$

- Furthermore energy dissipates more quickly
- Challenge for rapidly oscillating is to track increasingly rapid oscillations
- In practice cut off by dark energy domination and early stage most important
- But averaging (as done before) inadequate

# Stability of fluctuations

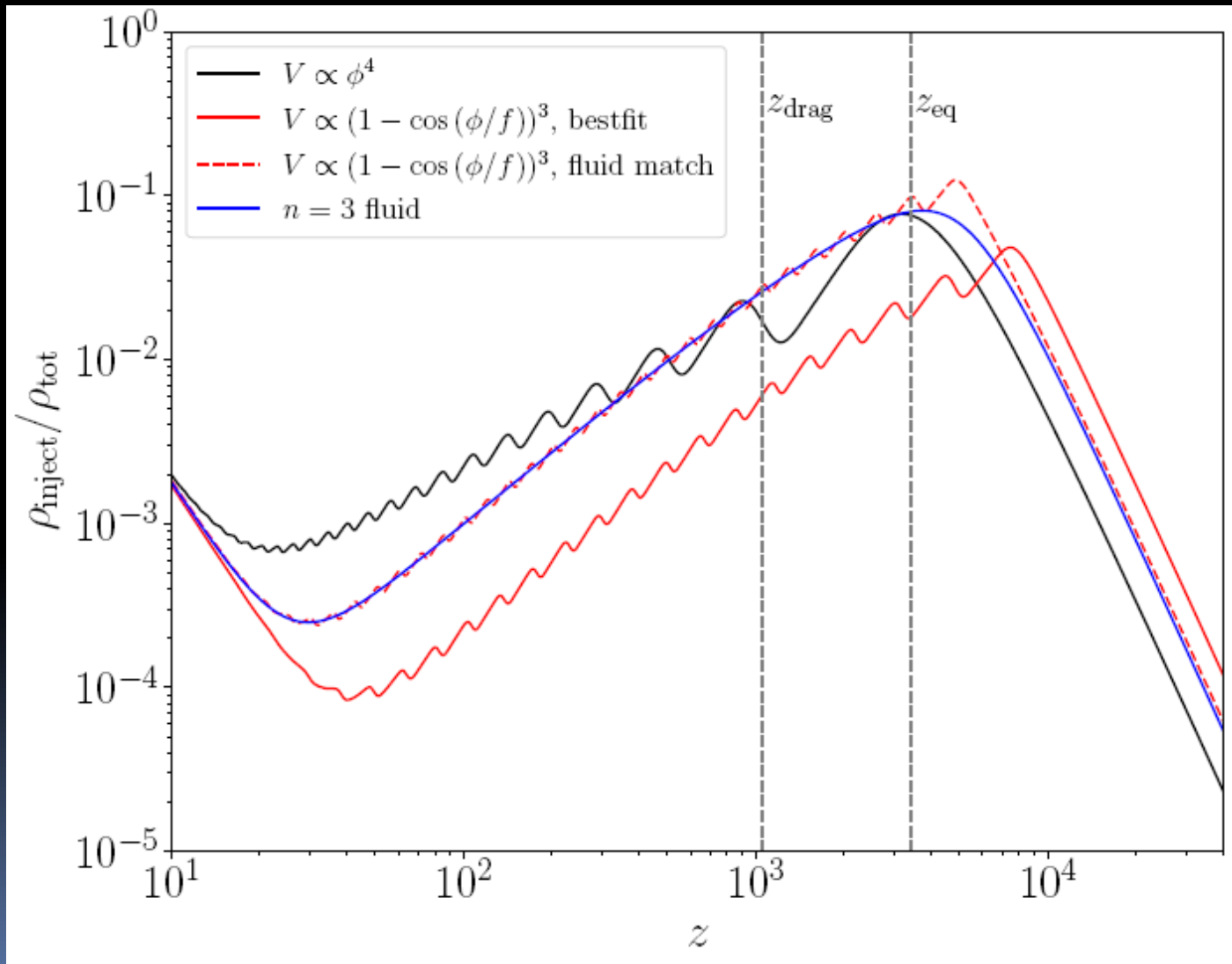
$$\frac{\partial^2 \delta\phi_k}{\partial(\log a)^2} + \frac{3}{2}(1 - w_b) \frac{\partial \delta\phi_k}{\partial(\log a)} + \left[ \frac{k^2}{a^2 H(a)^2} + \frac{9}{4}(1 - w_\phi)(2 + w_\phi + w_b) \right] \delta\phi_k = 0, \quad (2.14)$$

has solutions which scale as

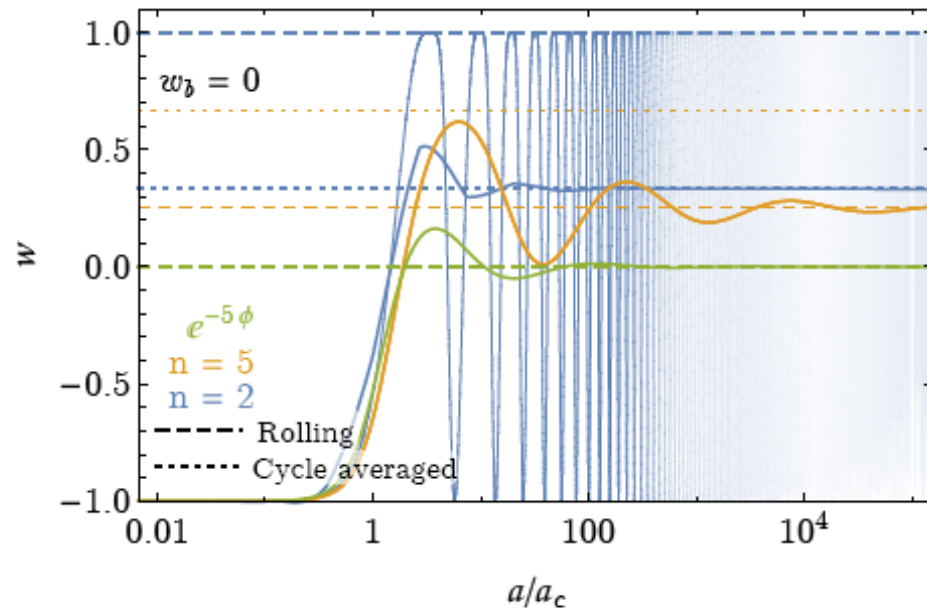
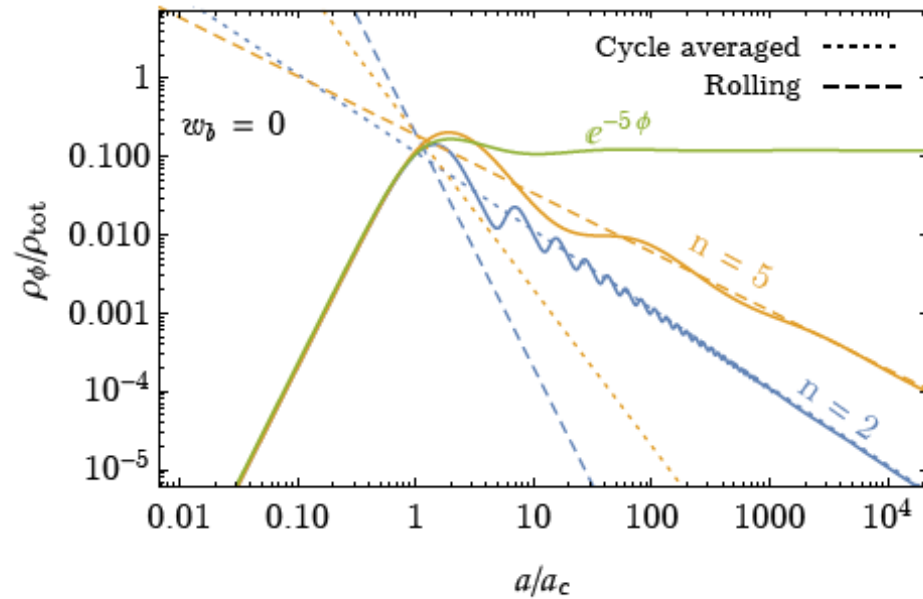
$$\delta\phi_0 \sim a^{-\frac{3}{4}[(1-w_b) \pm \sqrt{4w_\phi^2 + (1+w_b)(4w_\phi + w_b - 7)}]}. \quad (2.15)$$

For  $w_\phi < \sqrt{2}\sqrt{1+w_b} - (1+w_b)/2$ , these solutions are oscillatory, and their envelope redshifts as  $a^{-3(1-w_b)/4}$ , while the rolling solution redshifts as  $a^{-3(w_\phi-w_b)/2}$ . Thus the fluctuations grow relative to the rolling solution for  $1 > w_\phi > (1+w_b)/2$ , corresponding to  $2 < n(1-w_b) < 3+w_b$  and coinciding

# Fluid and Model Disagree Even when we try...



Solutions:  
Rocking  
Or  
Rolling?



Asymptotes to  
constant  $w$   
Or averages to  
constant  $w$   
Will get cutoff  
when  
 $w_b$  becomes -1

## 2.2 Exponential potentials

Exponential potentials are a special limiting case of the rolling solutions which occur when  $w_\phi = w_b$ , corresponding to the limit  $n \rightarrow \infty$ . In this case, the field  $\phi$  depends logarithmically on  $a$ ,

$$\phi(a) = \phi_0 + \sqrt{\frac{3(1+w_b)\rho_0}{\rho_{b0} + \rho_0}} \log \frac{a}{a_0}, \quad (w_\phi = w_b). \quad (2.11)$$

In this case we have kept the back-reaction of the field since it is possible to obtain a simple analytical solution even with the back-reaction included. This trajectory corresponds to an exponential potential,

$$V(\phi) = V_0 \exp\left(-\lambda \frac{\phi}{M_{\text{Pl}}}\right), \quad \lambda = \sqrt{3(1+w_b) \left(1 + \frac{\rho_{b0}}{\rho_0}\right)}, \quad (w_\phi = w_b). \quad (2.12)$$

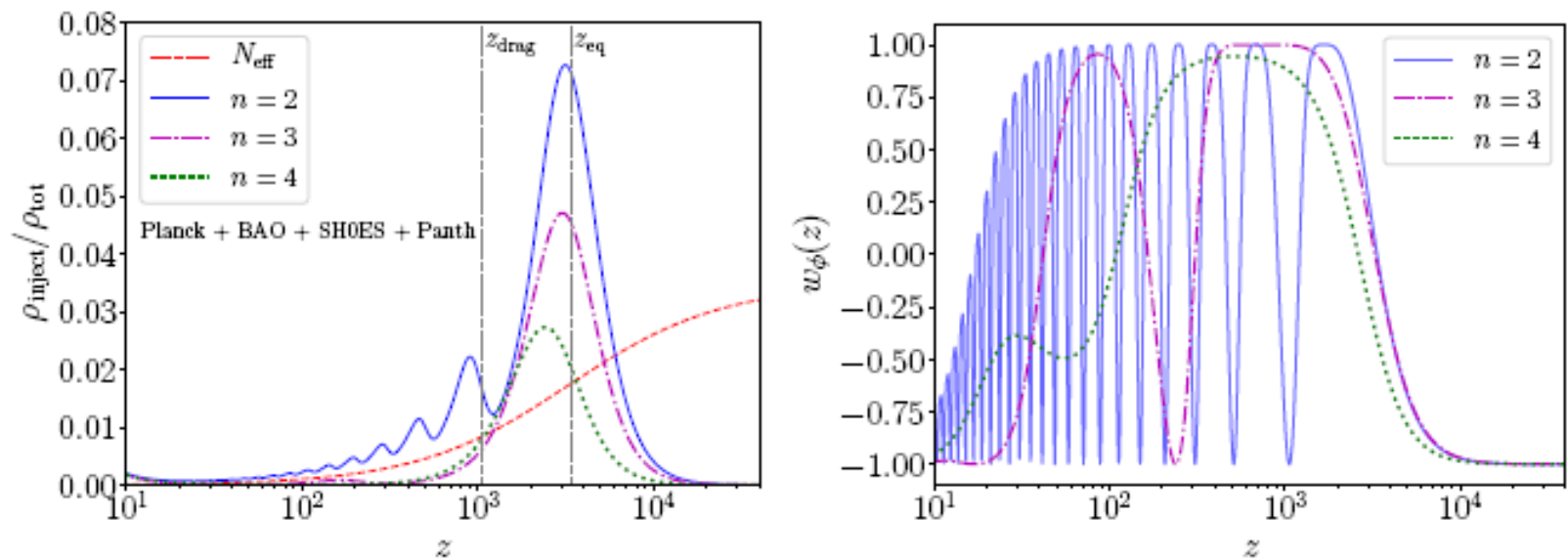
On this solution,

$$\frac{\partial_\phi^2 V(\phi)}{H(a)^2} = \frac{9}{2}(1-w_b^2), \quad (w_\phi = w_b) \quad (2.13)$$

is an  $\mathcal{O}(1)$  constant, as for the non-oscillatory solutions discussed in the previous subsection. These exponential potentials have previously been studied in the context of quintessence models (see [30] for a review). However, since  $w_\phi = w_b$ , the energy injection for this potential does not redshift relative to the background (see figure 1), which prevents these solutions from being ideal candidates to resolve the Hubble tension. Therefore we will focus on the case of monomial potentials at finite  $n$  for the remainder of the paper.

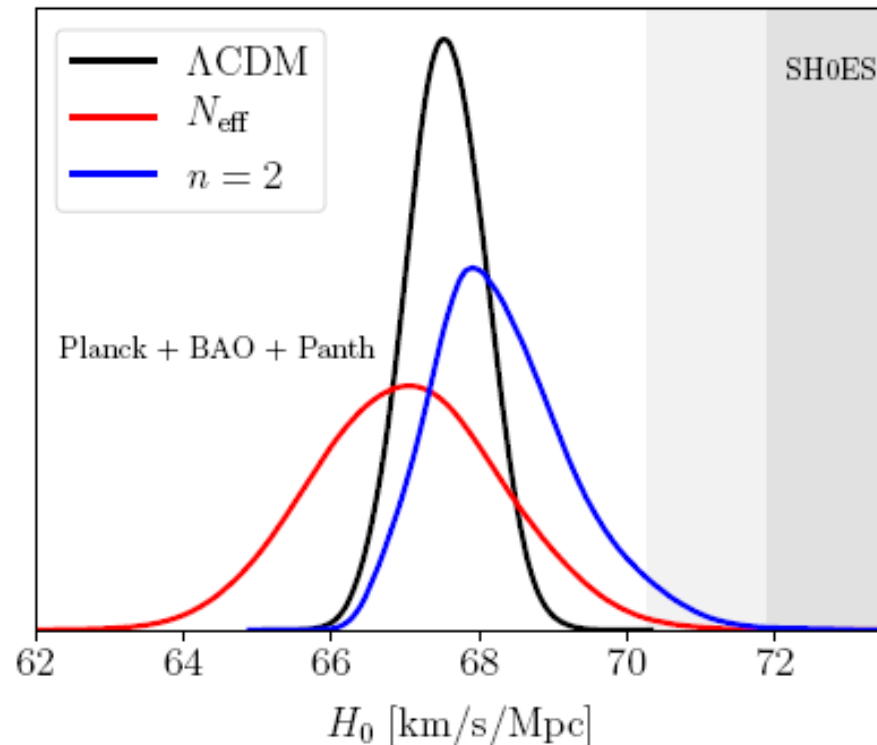
# True Energy Injection profile

## Numerical, Includes backreaction



**Figure 3.** *Left panel:* Energy injection profile for the best-fit models for each value of  $n$  as a function of redshift. Results are shown here for the data combination “Planck + BAO + SH0ES + Pantheon”. For reference, we also show the amount of energy injected as compared to standard  $\Lambda$ CDM for the best fit  $N_{\text{eff}}$  model using the same data combination (corresponding to  $\Delta N_{\text{eff}} = 0.27$ ). To guide the eye, we have indicated by vertical dashed lines the matter-radiation equality and baryon drag epochs in the standard  $\Lambda$ CDM model. *Right panel:* The scalar field equation of state as a function of redshift for each value of  $n$ .

# Compare to Neutrinos: Results without Riess



**Figure 7.** Normalized  $H_0$  posteriors obtained using the data combination “Planck + BAO + Pantheon”, that is, without including the local Hubble constant measurement from ref. [3].



# Results

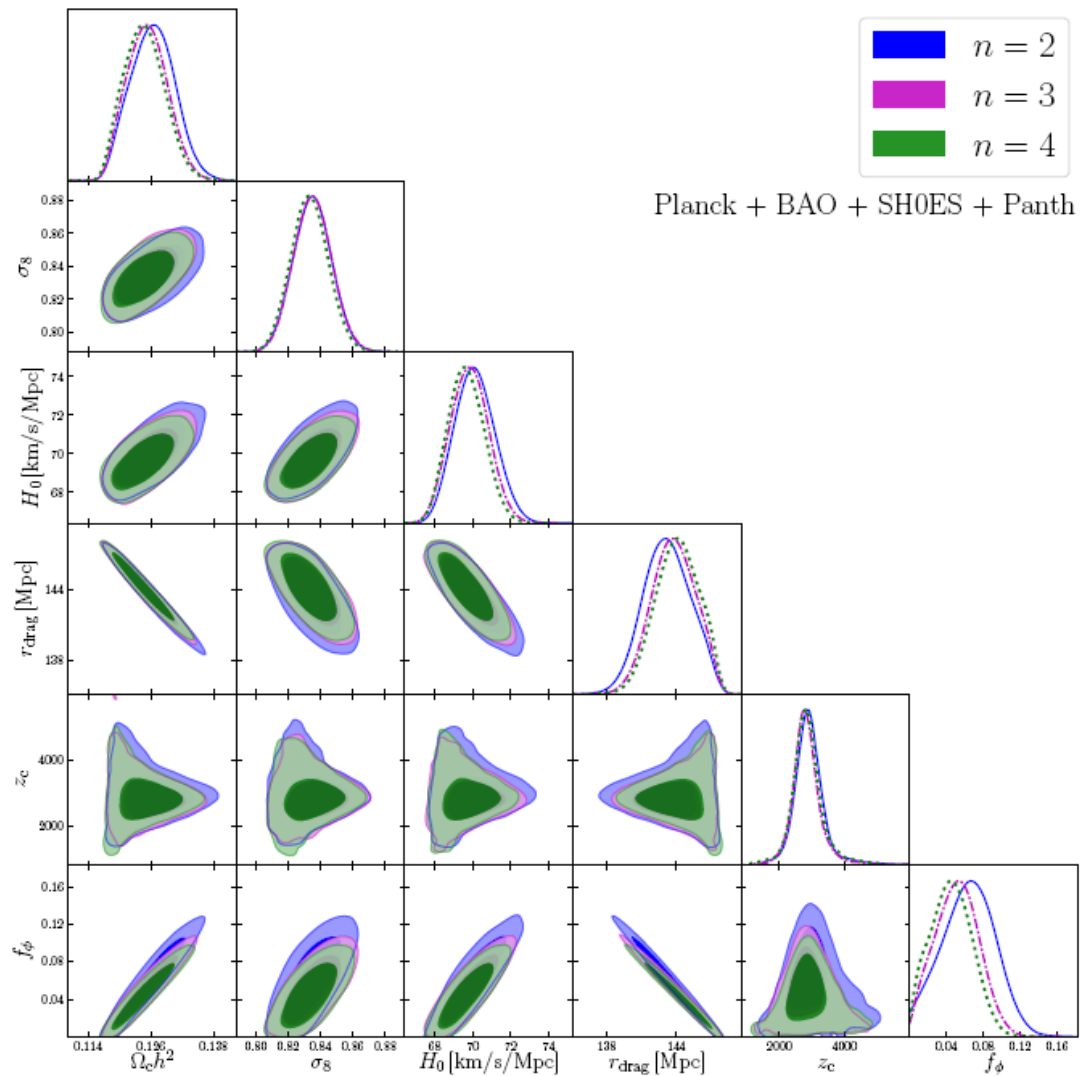


Figure 2. Marginalized posterior distributions for  $V \propto \phi^{2n}$  models for three different values of  $n$ . Results are shown here for the data combination “Planck + BAO + SH0ES + Pantheon”.

$r_s$ ,  $H_0$ ,  $f_{\text{ede}}$ ,  $\sigma_8$

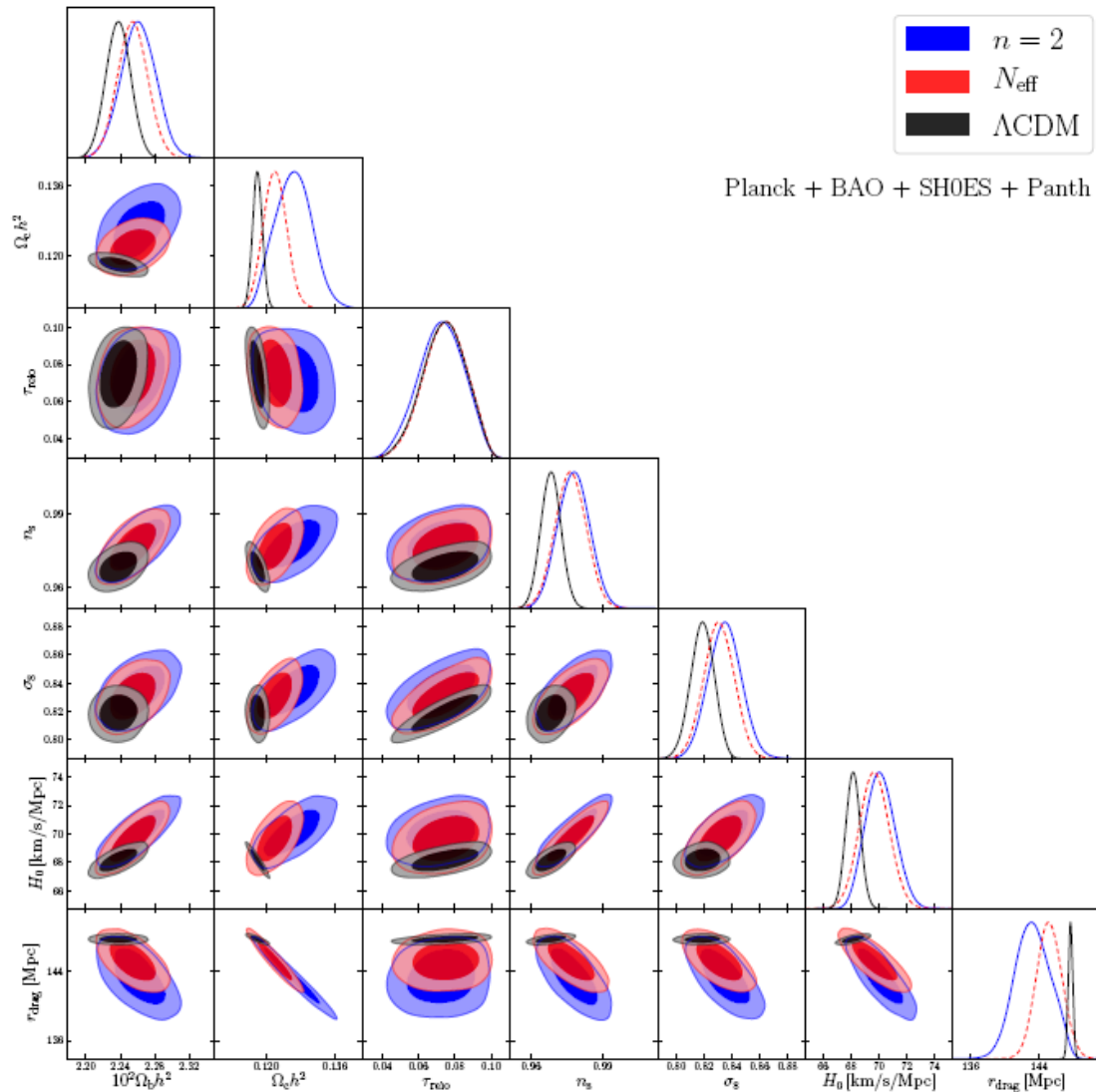


Figure 6. Comparison between the  $n = 2$  model, the  $N_{\text{eff}}$  extension of the standard cosmological model, and plain  $\Lambda\text{CDM}$ . All posterior distributions shown here were obtained using the data combination “Planck + BAO + SH0ES + Pantheon”.

# Best Fit Values

Parameter	$\Lambda$ CDM	$n = 2$	$N_{\text{eff}}$
$100 \Omega_b h^2$	2.238 (2.236) $^{+0.014}_{-0.015}$	2.261 (2.264) $^{+0.021}_{-0.020}$	2.254 (2.269) $\pm 0.018$
$\Omega_c h^2$	0.1180 (0.1177) $\pm 0.0012$	0.1264 (0.1267) $^{+0.0044}_{-0.0043}$	0.1220 (0.1213) $^{+0.0027}_{-0.0028}$
$100 \theta_s$	1.0420 (1.0422) $\pm 0.0003$	1.0415 (1.0417) $\pm 0.0004$	1.0414 (1.0413) $^{+0.0004}_{-0.0005}$
$\tau_{\text{reio}}$	0.074 (0.077) $^{+0.013}_{-0.012}$	0.072 (0.081) $^{+0.013}_{-0.012}$	0.075 (0.080) $^{+0.013}_{-0.012}$
$\ln(10^{10} A_s)$	3.079 (3.080) $^{+0.024}_{-0.021}$	3.091 (3.105) $^{+0.026}_{-0.023}$	3.089 (3.100) $^{+0.025}_{-0.022}$
$n_s$	0.968 (0.969) $\pm 0.004$	0.978 (0.981) $\pm 0.007$	0.977 (0.977) $^{+0.006}_{-0.007}$
$f_\phi / \Delta N_{\text{eff}}$	-	0.064 (0.073) $^{+0.031}_{-0.028}$	0.26 (0.27) $\pm 0.16$
$z_c$	-	3040 (3160) $^{+330}_{-630}$	-
$\sigma_8$	0.819 (0.819) $^{+0.009}_{-0.008}$	0.835 (0.841) $\pm 0.012$	0.831 (0.832) $\pm 0.011$
$\Omega_m$	0.304 (0.301) $\pm 0.007$	0.304 (0.302) $\pm 0.007$	0.299 (0.293) $^{+0.007}_{-0.008}$
$r_{\text{drag}}$ [Mpc]	147.6 (147.7) $\pm 0.3$	143.2 (142.9) $^{+2.0}_{-2.3}$	145.1 (145.1) $\pm 1.5$
$H_0$ [km/s/Mpc]	68.2 (68.3) $\pm 0.5$	70.1 (70.5) $^{+1.0}_{-1.2}$	69.7 (70.2) $\pm 1.1$

Table 1: Mean values and 68% confidence intervals for key cosmological parameters using the data combination “Planck + BAO + SH0ES + Pantheon”. The numbers in parentheses are the best-fit values for each model.

# Compare fluctuations

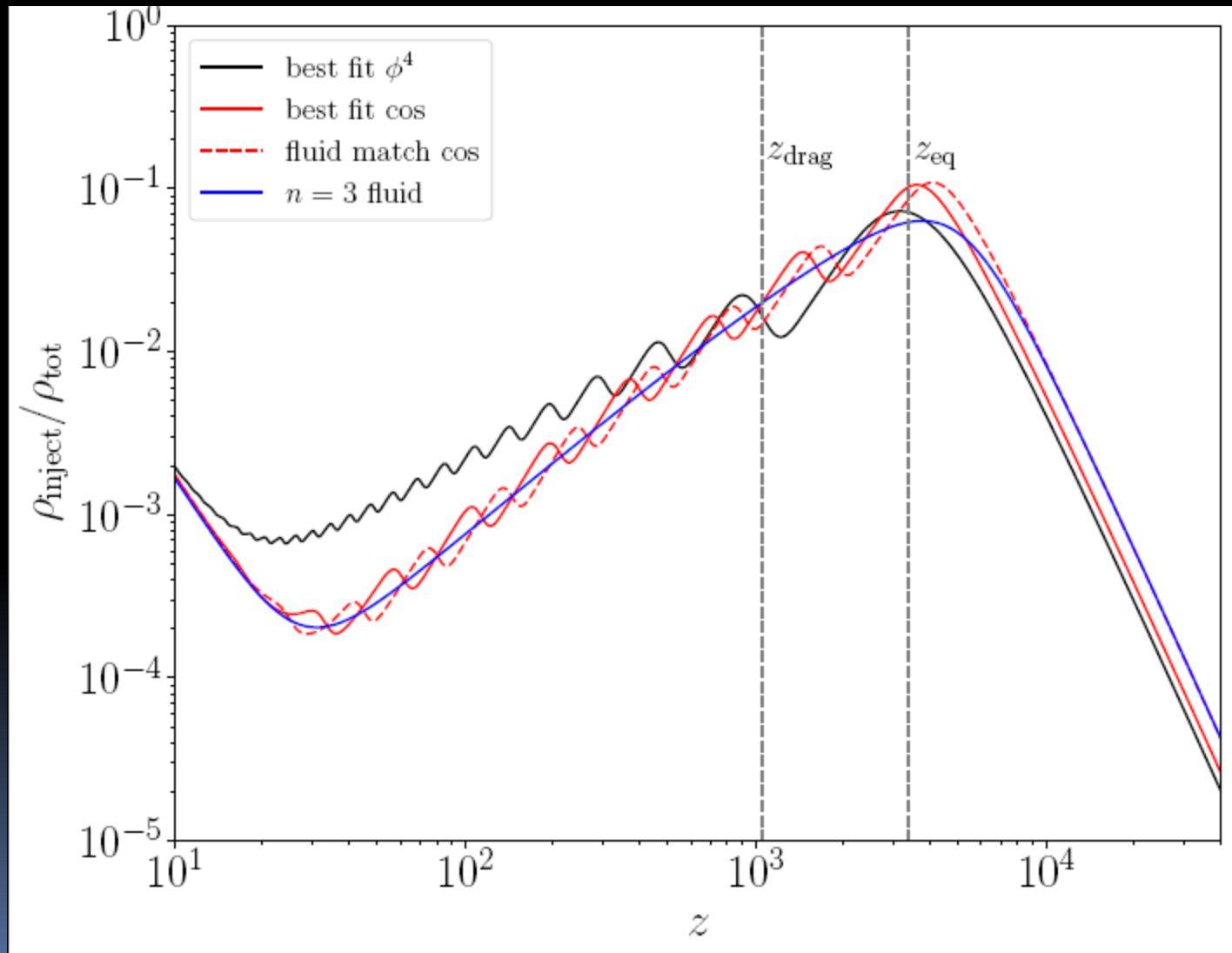
Datasets	$\Lambda$ CDM	$n = 2$	$n = 3$	$n = 4$	$N_{\text{eff}}$
<i>Planck</i> high- $\ell$	2448.6	2449.3	2447.3	2446.2	2449.2
<i>Planck</i> low- $\ell$	10495.6	10494.4	10494.9	10495.6	10495.0
<i>Planck</i> lensing	9.3	9.9	10.2	9.2	10.1
BAO - low $z$	1.9	1.8	1.4	1.8	2.7
BAO - high $z$	1.8	1.9	1.9	1.8	2.0
Pantheon	1027.1	1027.0	1027.1	1027.0	1027.2
SH0ES	10.3	3.5	6.5	7.4	4.2
Total $\chi^2_{\text{min}}$	13994.7	13987.8	13989.2	13989.0	13990.3
$\Delta\chi^2_{\text{min}}$	0	-6.9	-5.5	-5.7	-4.4

Table 1: Best-fit  $\chi^2$  values for each individual dataset used in our cosmological analysis.

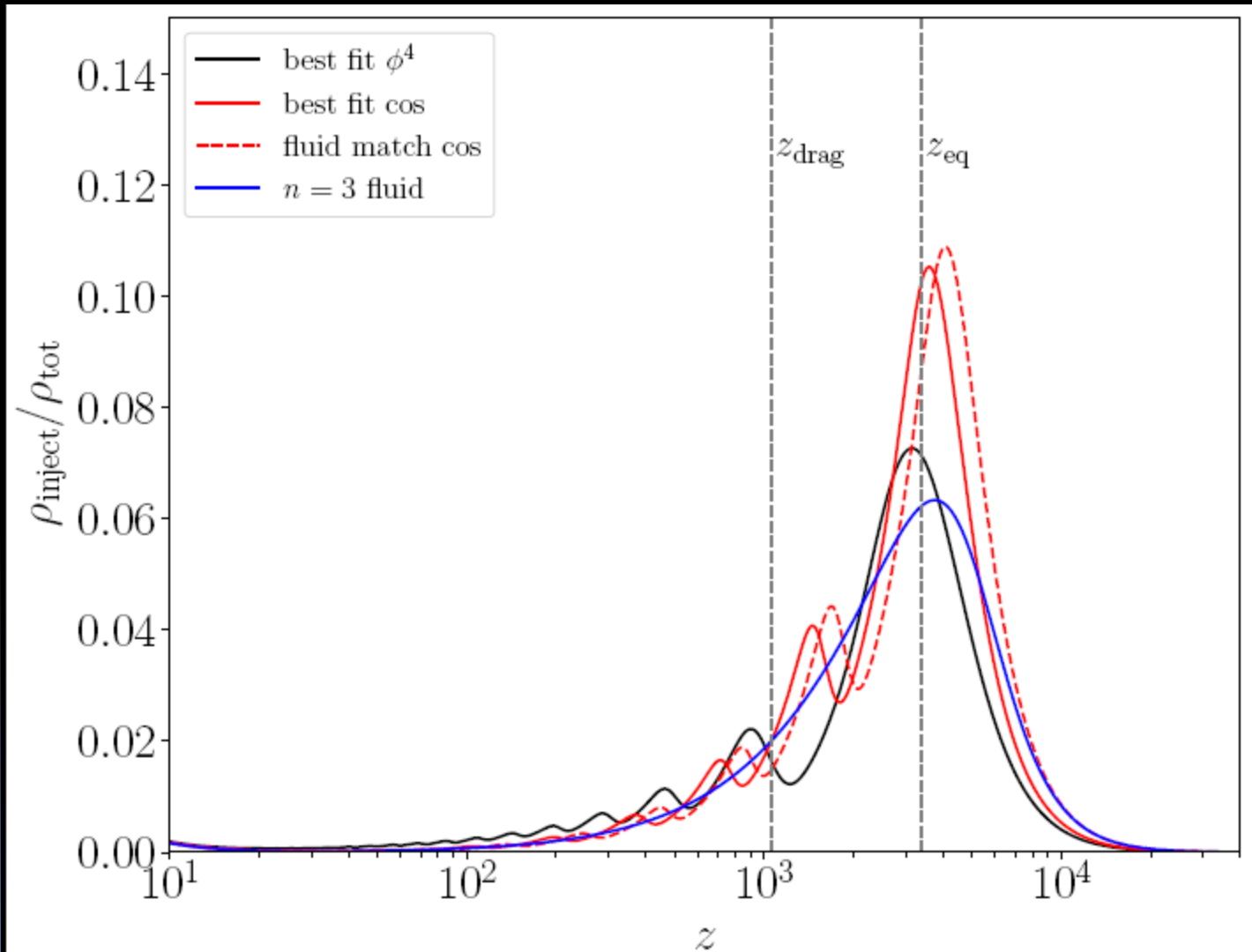
# Result

- $\Phi^4$  model the best of our models
  - With funny initial conditions
- Neutrinos most natural
  - But doesn't agree at high  $l$
- Fluid models agree better
  - Faster drop off
  - No oscillations
- But not obvious which models they match to
  - Certainly nothing obvious
- Already a stretch...

# Better?: $(1 - \cos\Phi/f)^n$ model




# $(1 - \cos\Phi/f)^n$ model





# Lessons

- There are better models
  - But they are hard to find
    - Fluid approximation gives a good model
    - But it's not exactly the model they say
    - Without scanning through actual potential, can't even trust that it works at all
- 



# Other Lessons

- We want
  - Cosmologically reasonable
  - Field theoretically reasonable
- Cosmologically: need energy injection to happen at M/Rad equality scale
- Field theory:
  - Why  $\cos^3$ ? Eg dropping  $\phi^2$ ,  $\phi^4$
  - $f \sim 0.15$ ; where  $\cos$  turns over to power law and 10% detuned from peak region
    - Smaller  $f$ : tachyon develops in fluid
    - Larger  $f$ : power law model
  - Very sensitive to higher order terms
    - See whole potential
    - More generally shape sensitivity

# Other models?

Lin Benevento Hu Raveri

- Get rid of oscillation
  - By fiat!

To make these considerations concrete, consider the following class of potentials:

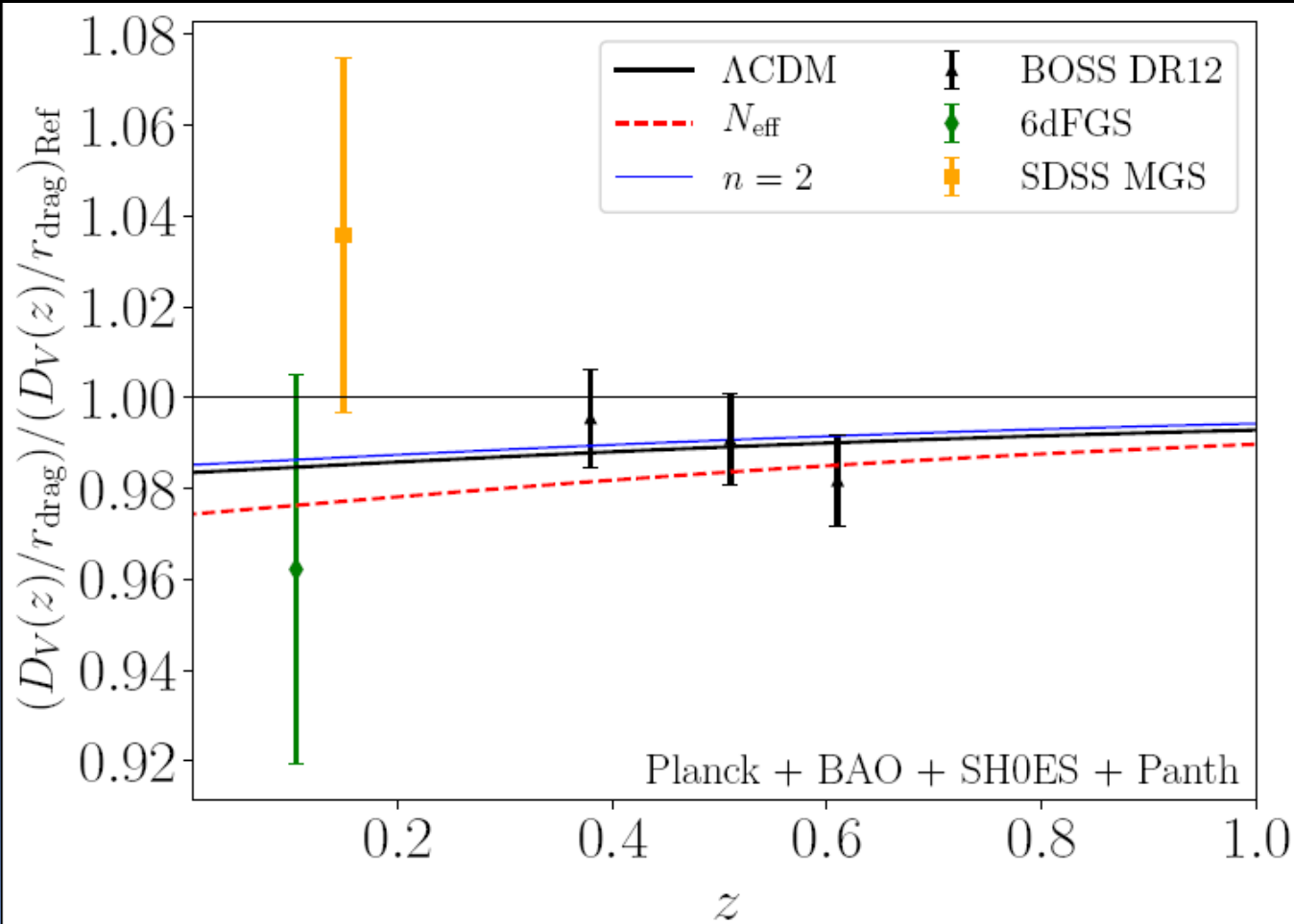
$$V(\phi) = \begin{cases} A \phi^m, & \phi > 0, \\ 0, & \phi \leq 0. \end{cases}$$

Then for  $\phi > 0$

$$2\epsilon_V = \left(\frac{m}{\phi}\right)^2, \quad \eta_V = \frac{m(m-1)}{\phi^2},$$



# For future: Late time measurements



Related  
to  
matterb

# Conclusions and Future Directions

- Clearly could be systematics
- Will be important to see how measurements of  $H$  evolve
- But also late time studies
  - BAO
  - Large scale structure
  - $\sigma_8$  is worse (bigger):  $A_s n_s$  increased to absorb damping tail, (increase  $H$ , more diffusion, less power high  $l$ ),  $\rho_m$  bigger, but  $\Omega_m$  smaller
  - Lyman $_{\alpha}$
- Ultimately we want to know is energy density of universe what we think it is
- So far, the jury is out
- Which is a nice time for theorists