Many-body localization (overview)

Dima Abanin
Perimeter Institute
University of Geneva

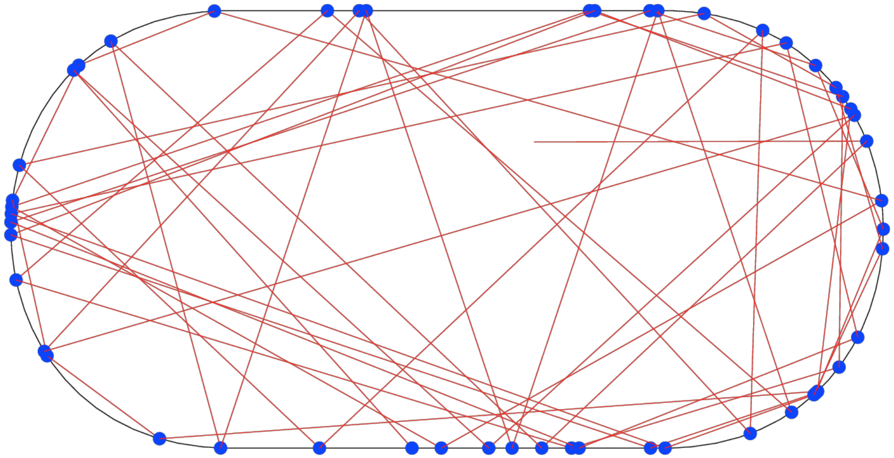
Closing The Entanglement Gap
Santa Barbara, June 4, 2015

Ergodicity and its breaking in classical systems

Ergodicity:

System explores full phase space

Chaotic systems



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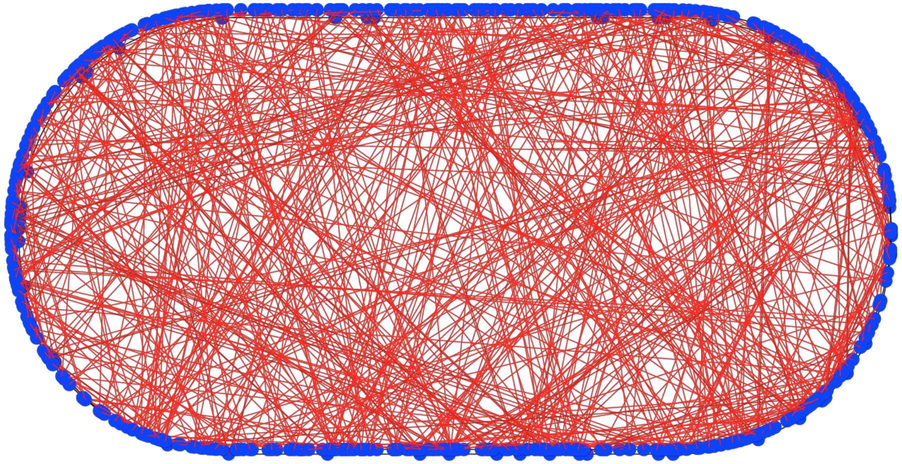

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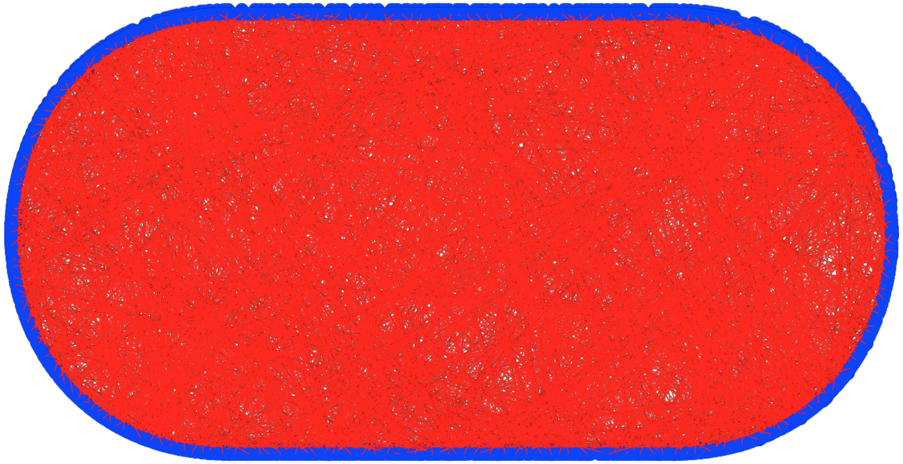

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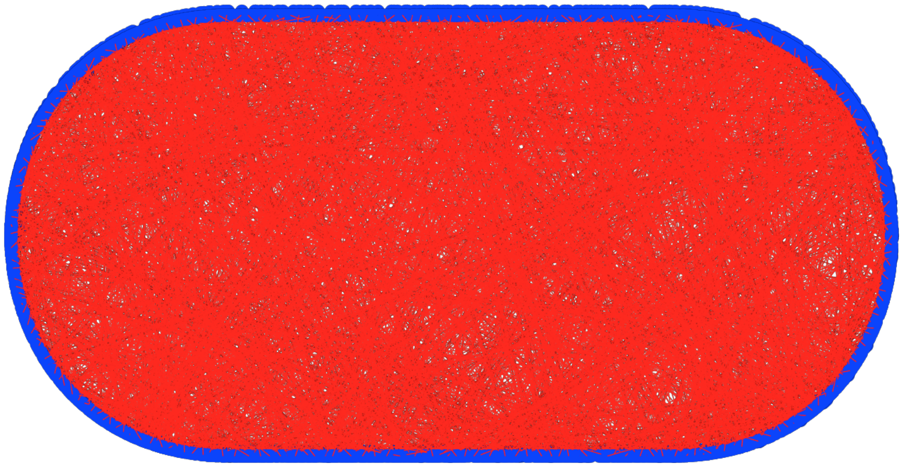

Described by statistical mechanics

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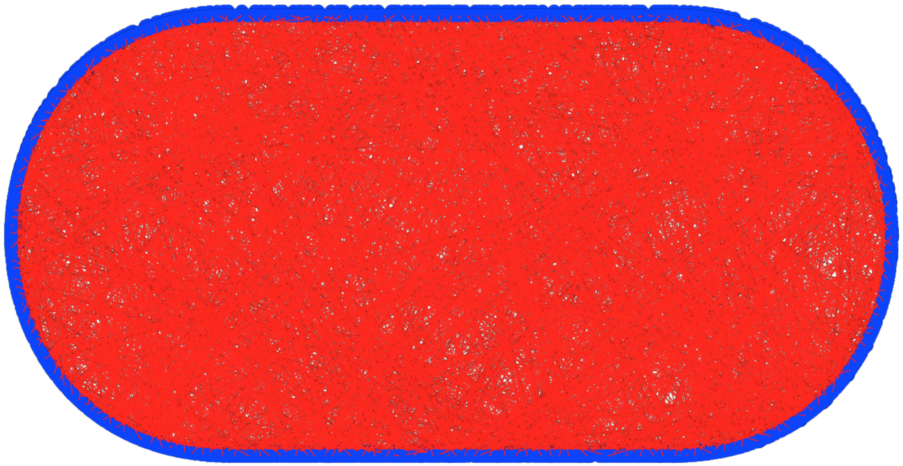


Described by statistical mechanics

Ergodicity breaking

Classically integrable systems

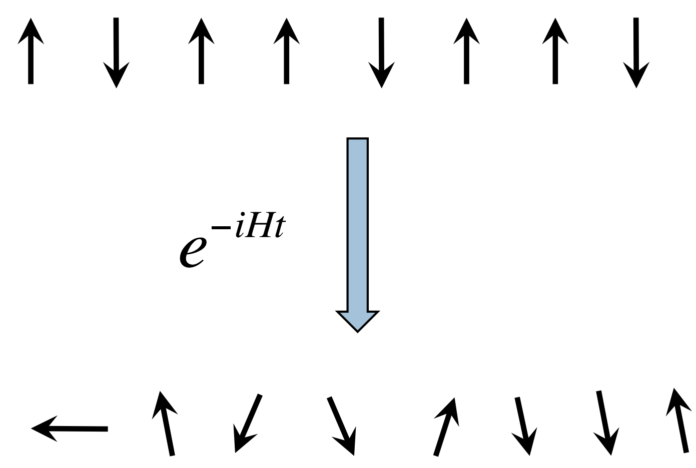
Regular motion

Do not explore full phase space

Ergodicity in quantum many-body systems

Prepare system in some state $\left|\psi\right\rangle$

Unitary evolution



Ergodicity in quantum many-body systems

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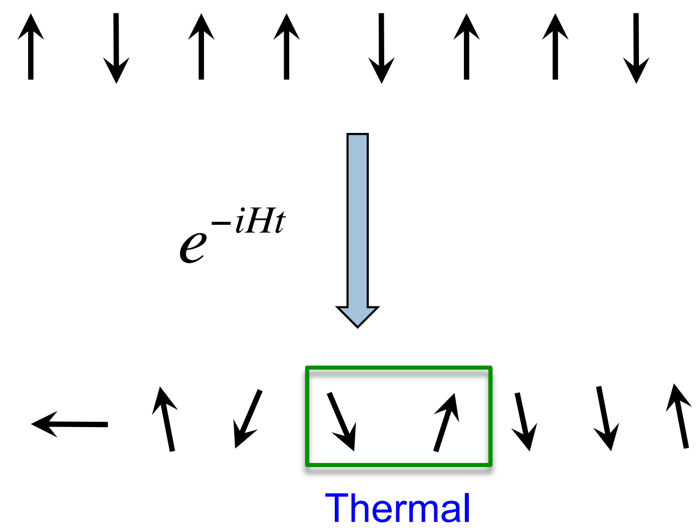
Unitary evolution

At long times, any sub-system thermalizes

(but the system is in a pure quantum state)



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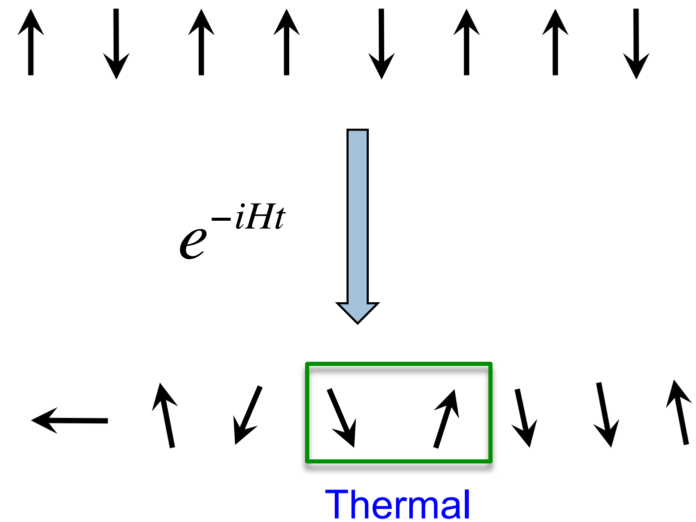

Eigenstate thermalization hypothesis:

*In ergodic systems, **individual many-body eigenstates** are thermal.*

Observables are given by microcanonical ensemble

Deutsch'91, Srednicki'94, Rigol et al'08

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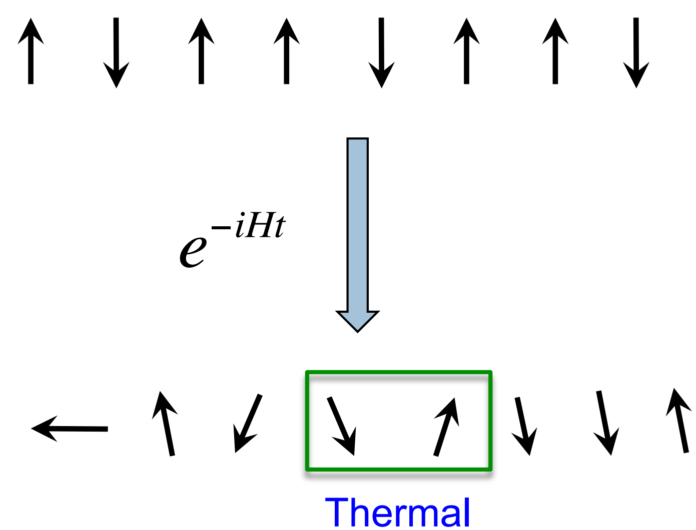

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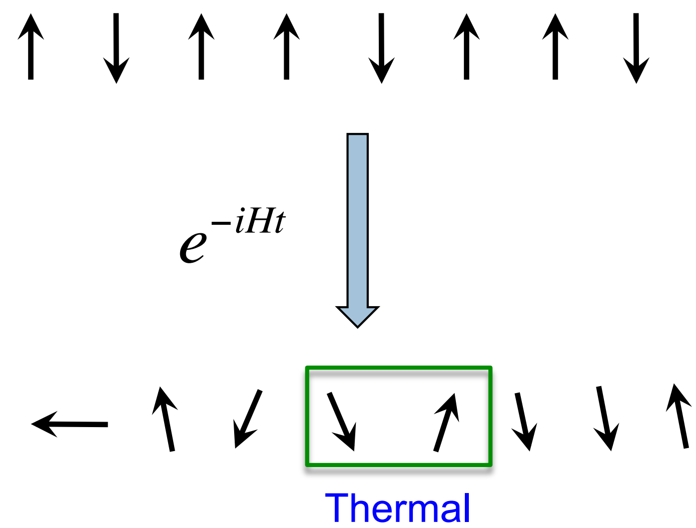

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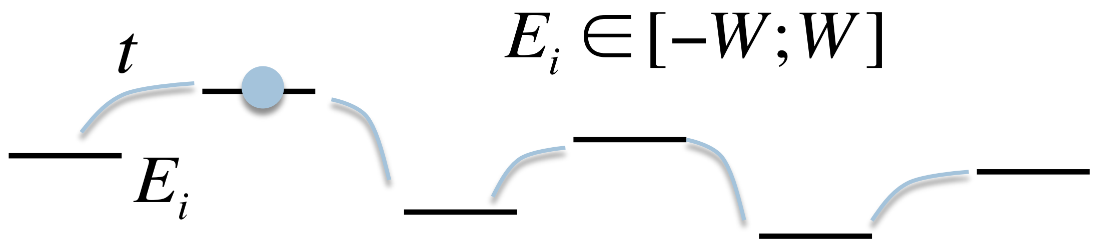
Are all many-body systems ergodic? NO!

Anderson localization

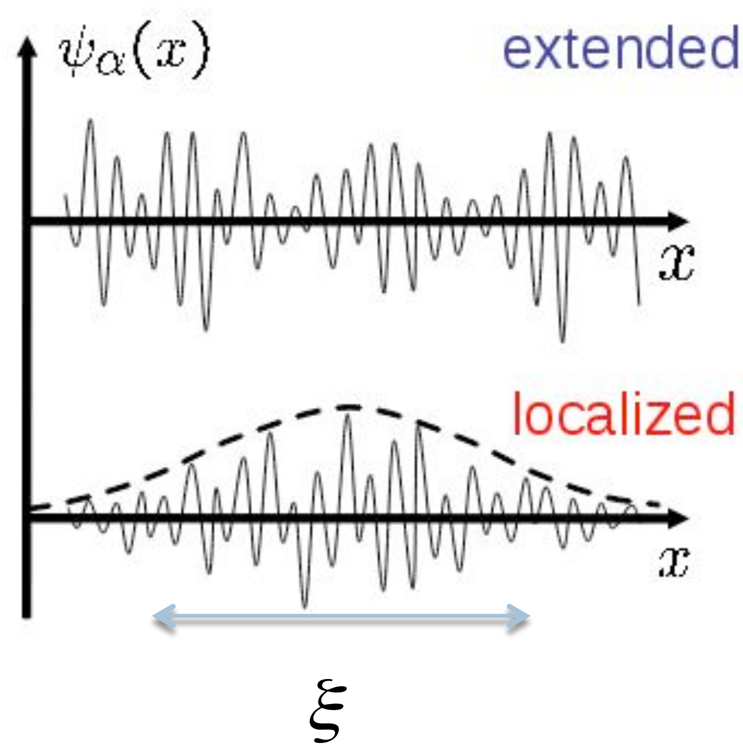
One quantum particle in 1D disordered crystal



Quantum memory effects → Wave functions become localized

$$\psi(x) \sim \exp(-|x - x_0| / \xi)$$

Absence of diffusion → Anderson insulator



Anderson '58

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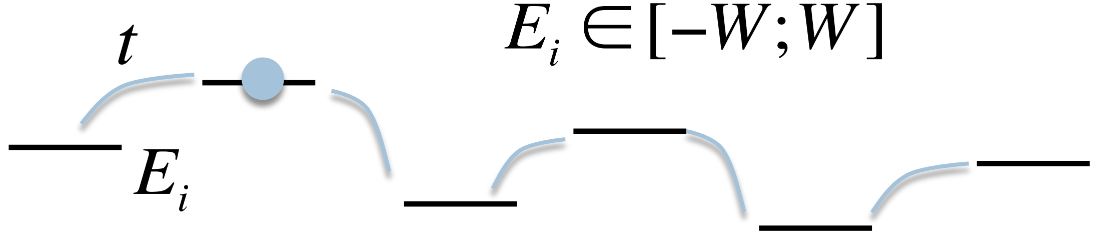

$$E_i \in [-W;W]$$

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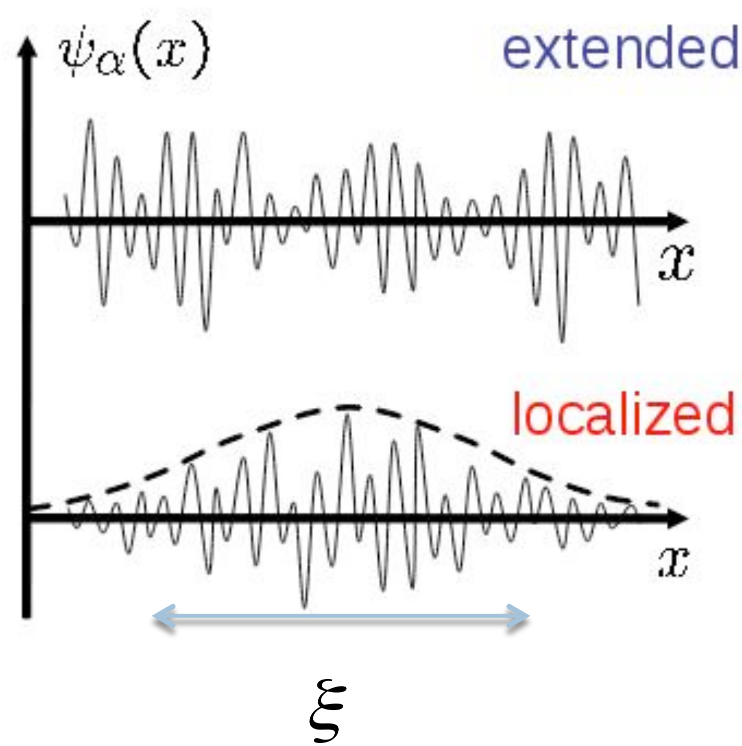


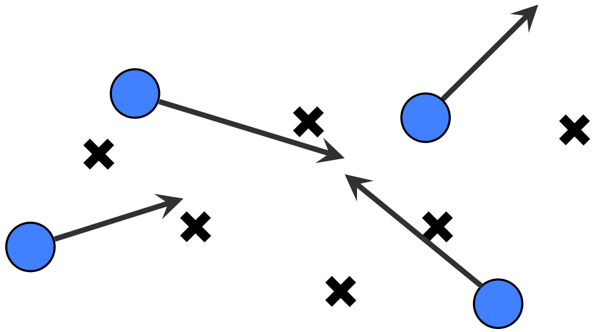
Anderson '58

A toy example of ergodicity breaking

Many-body localization

Ergodicity breaking?

Can localization survive in many-body systems?



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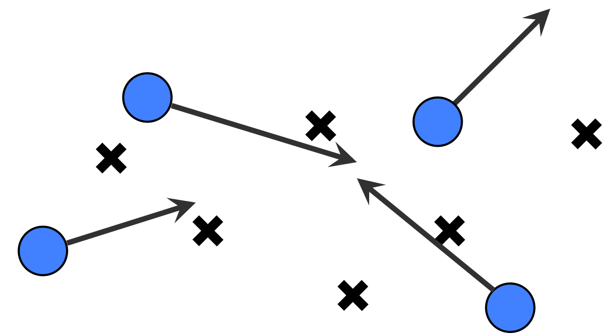

YES! Localization is possible (and inevitable) at strong enoug

<u>Perturbation theory</u>: Polyakov, Gornyi, Mirlin'05; Basko, Aleiner, Altshuler'05

<u>Numerics</u>: Oganesyan, Huse'07; Znidaric et al, 08, Pal, Huse'10; Bardarson et al'12, Serbyn, Papic, DA'13, De Luca, Scardicchio'13, Kjall et al'14, Luitz et al'14, Tang, Iyer, Rigol'15

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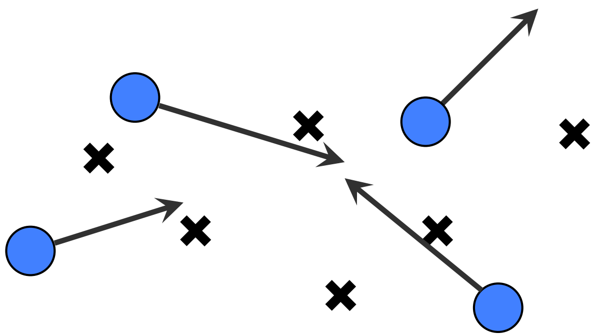

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↓

Many-body localized phase: a non-ergodic phase of matter not described by statistical mechanics

Many-body localization

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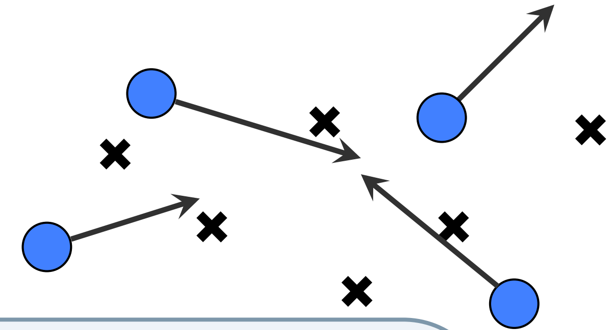

YE

This talk: Insights from entanglement

Describe many-body localized eigenstates

Universal dynamical properties

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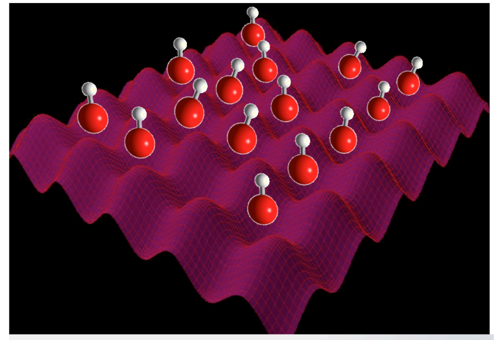
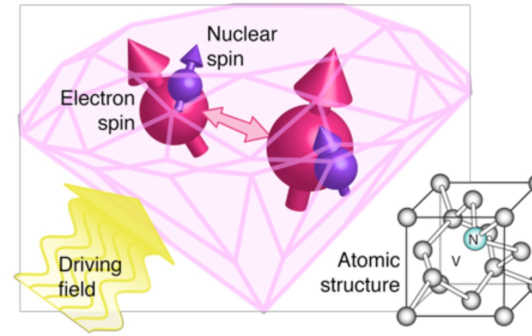
Many-body localized phase: a non-ergodic phase of matter not described by statistical mechanics

New experimental systems

Isolated & quantum-coherent. Tunable interactions and disorder

-Cold atoms, optical lattices





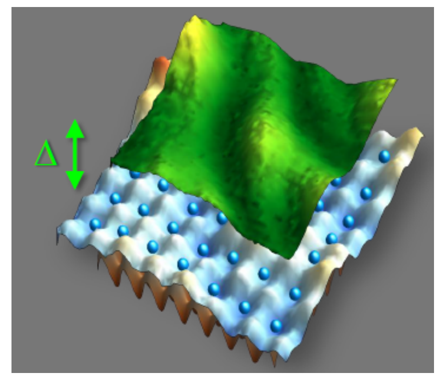
-Polar molecules

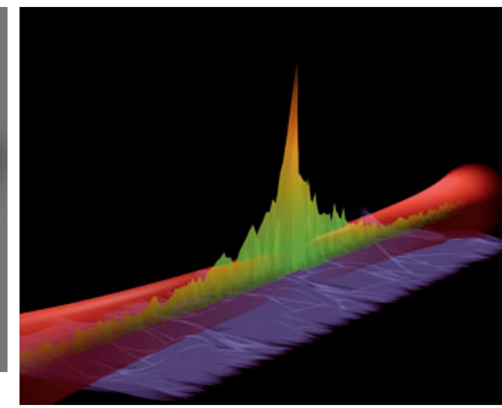
-Spin systems (NV-centers in diamond)

REPORTS

Three-Dimensional Anderson Localization of Ultracold Matter

S. S. Kondov, W. R. McGehee, J. J. Zirbel, B. DeMarco*





EXPERIMENTS: Paris, Florence, Urbana, Munich

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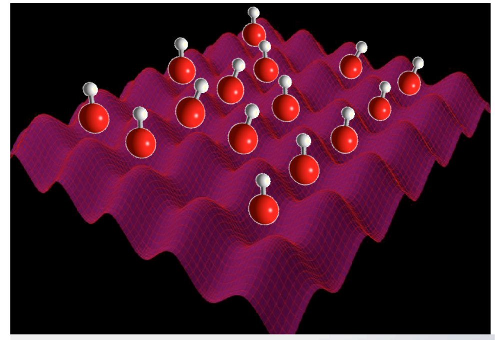

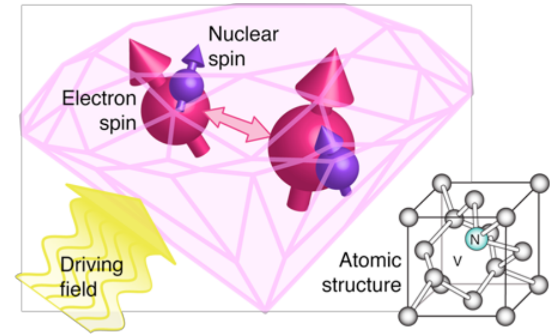

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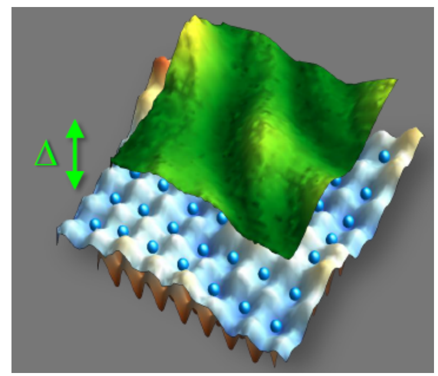

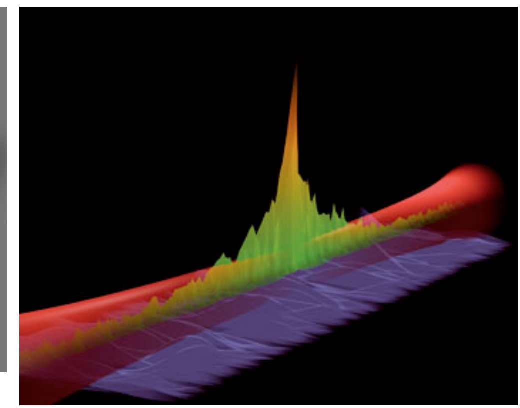

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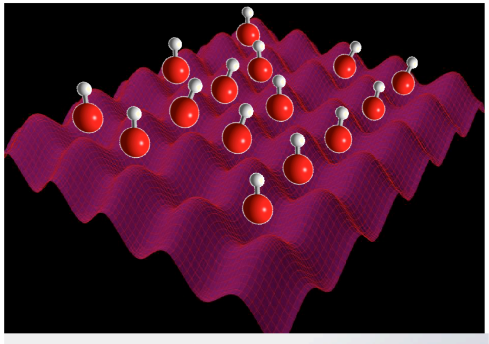

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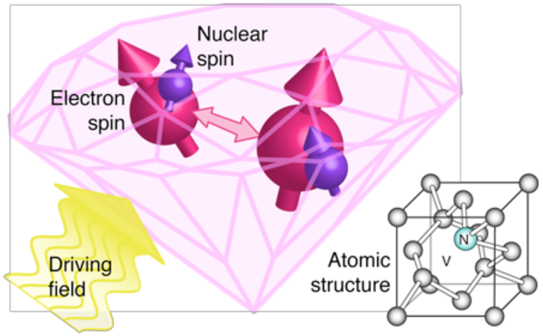


-Spin systems (NV-centers in diamond)

Disorder-Induced Localization in a Strongly Correlated Atomic Hubbard Gas

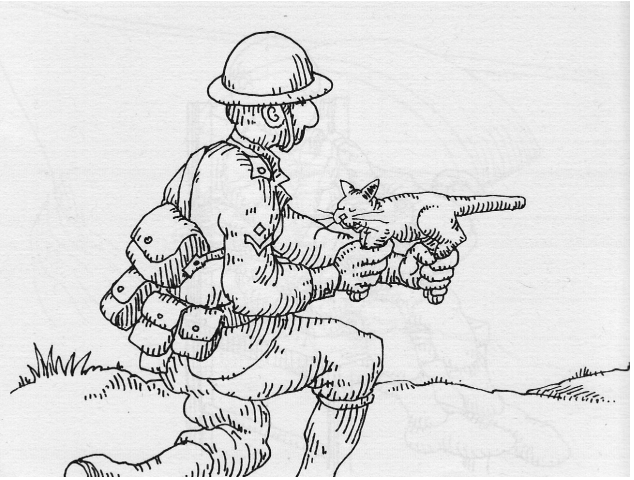
S. S. Kondov,[1,*] W. R. McGehee,[1] W. Xu,[1] and B. DeMarco[1]
Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA

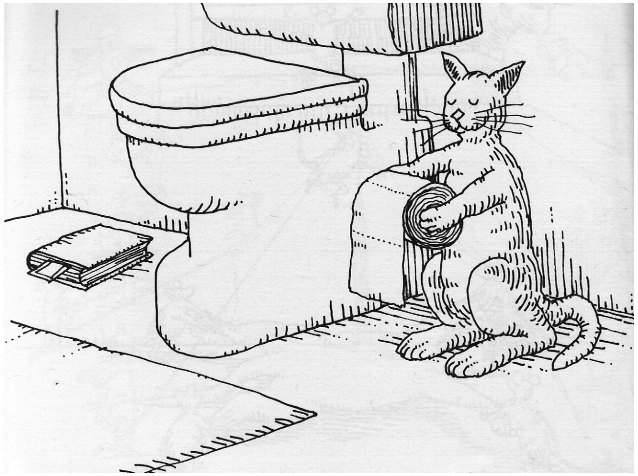
Observation of many-body localization of interacting fermions in a quasi-random optical lattice

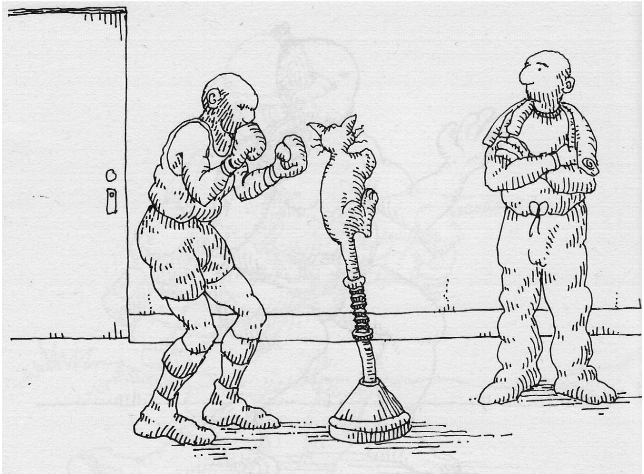
Michael Schreiber[1,2], Sean S. Hodgman[1,2], Pranjal Bordia[1,2], Henrik P. Lüschen[1,2], Mark H. Fischer[3], Ronen Vosk[3], Ehud Altman[3], Ulrich Schneider[1,2] and Immanuel Bloch[1,2]
[1]Fakultät für Physik, Ludwig-Maximilians-Universität München, Schellingstr. 4, 80799 Munich, Germany
[2]Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, 85748 Garching, Germany
[3]Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

Studying many-body localization experimentally now possible!

Entanglement: a tool to characterize/classify ground states







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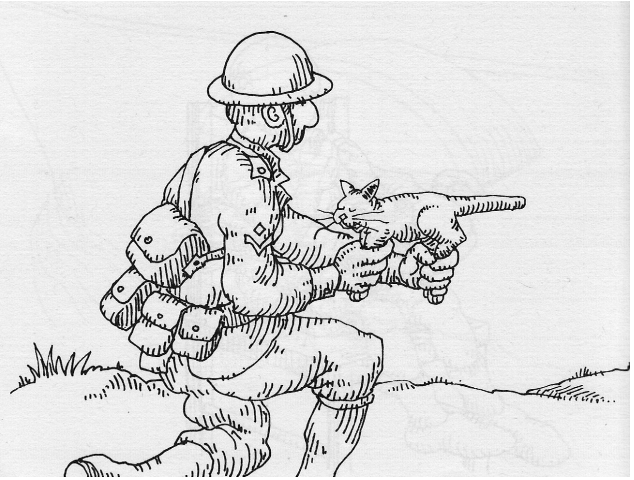
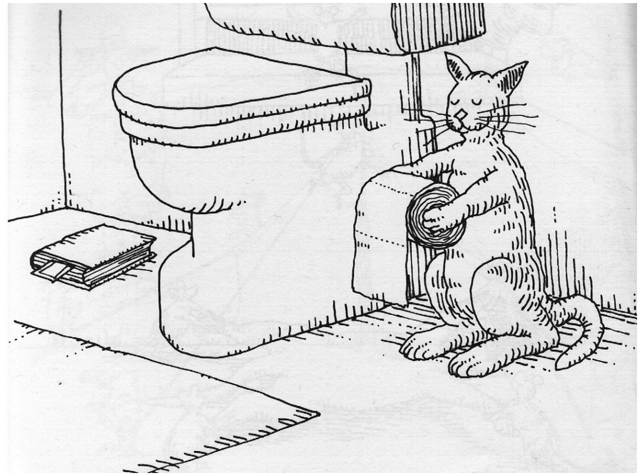
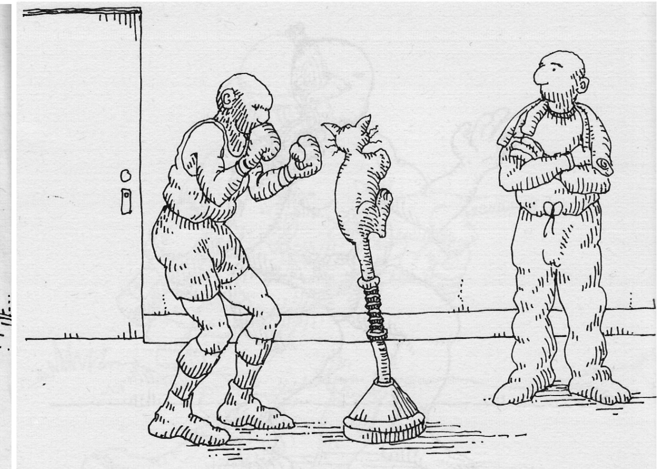
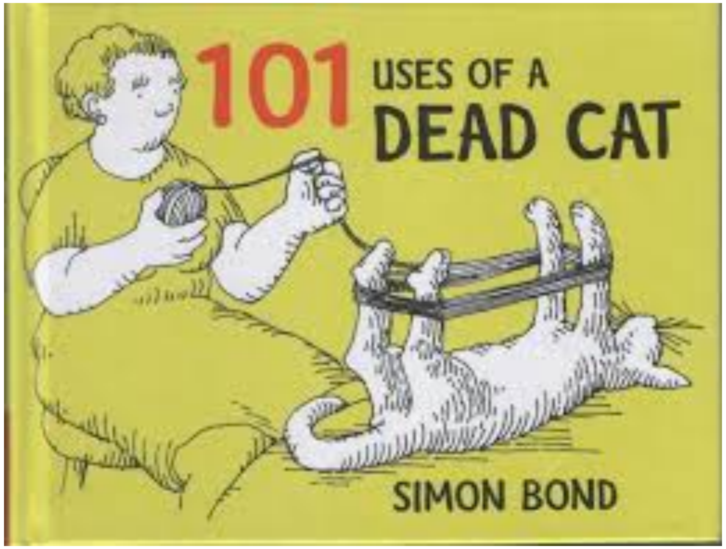






Towards a complete classification..



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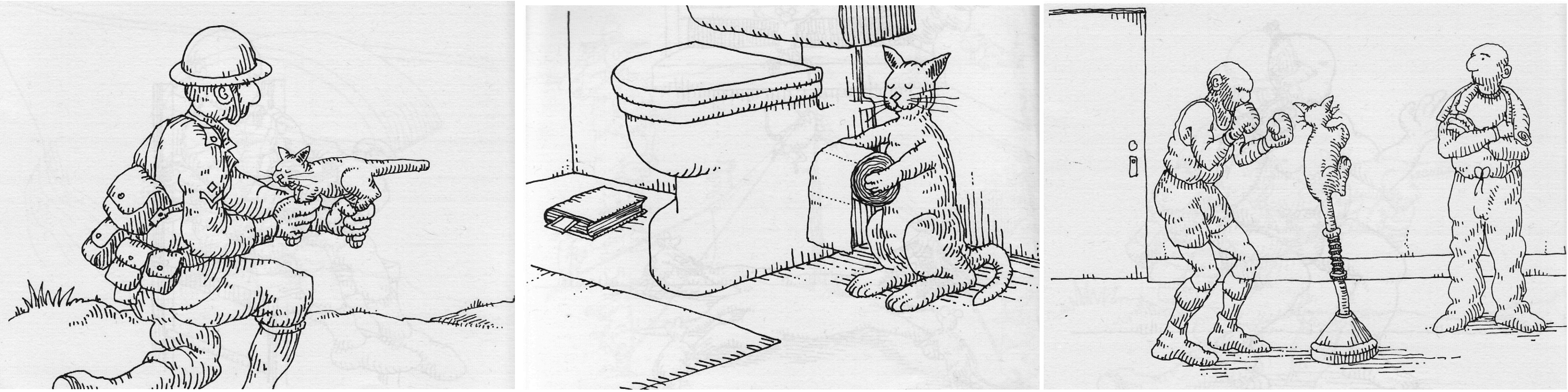

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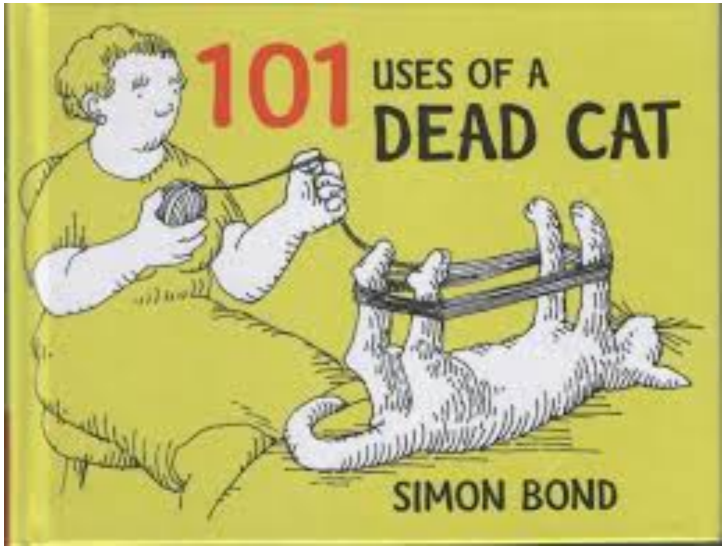


Understand different dynamical regimes?

Need to understand highly excited states

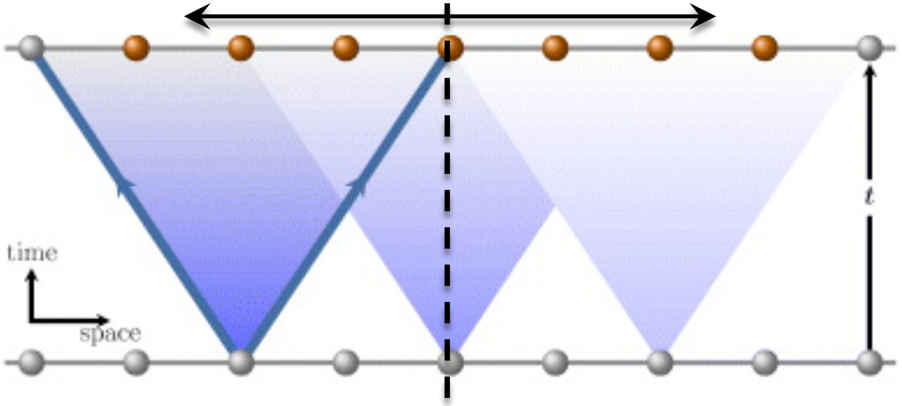
Entanglement plays a central role: eigenstates, dynamics

Entanglement propagation in <u>ergodic</u> systems

Light-cone-like spreading of correlations

Initial product states

$$x(t) = vt$$



Lieb,Robinson'72, Hastings'04, Calabrese,Cardy'05

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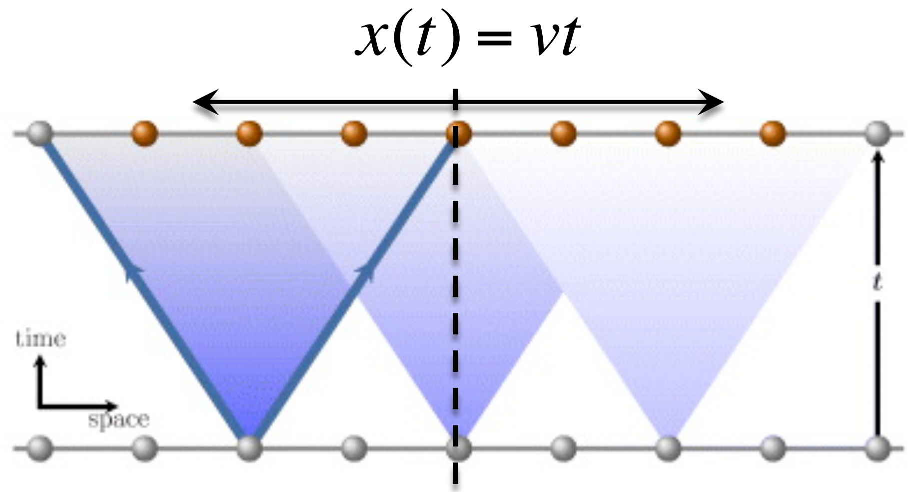
Linear growth of entanglement entropy

$$S_{ent} \sim N(x(t)) \propto t$$



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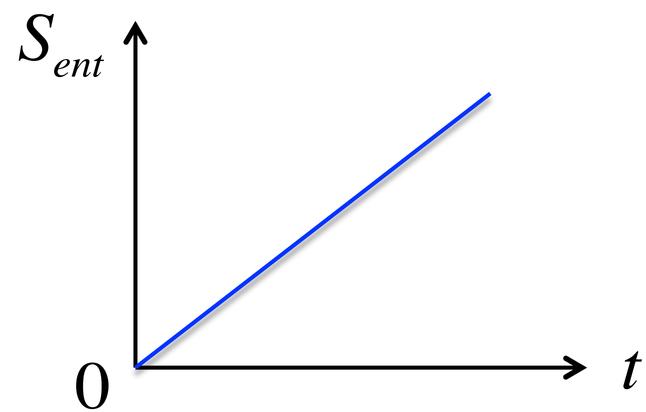

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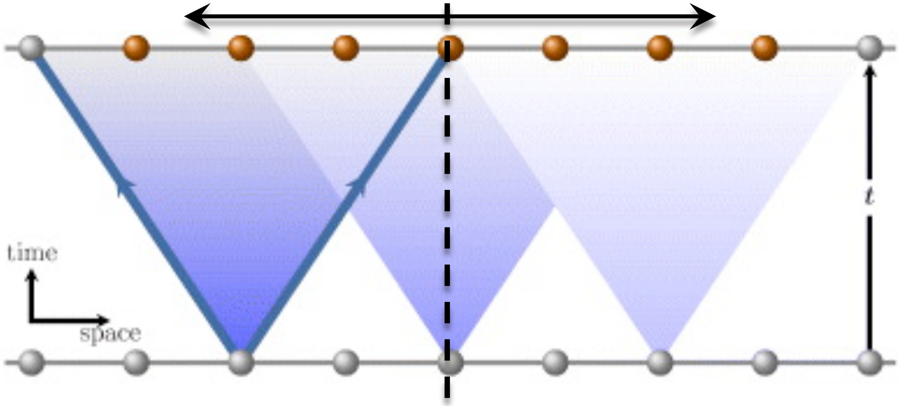

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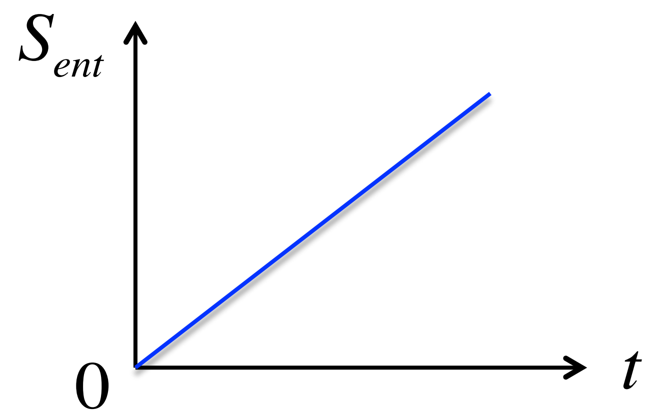

Ballistic! Unlike diffusive charge/energy transport

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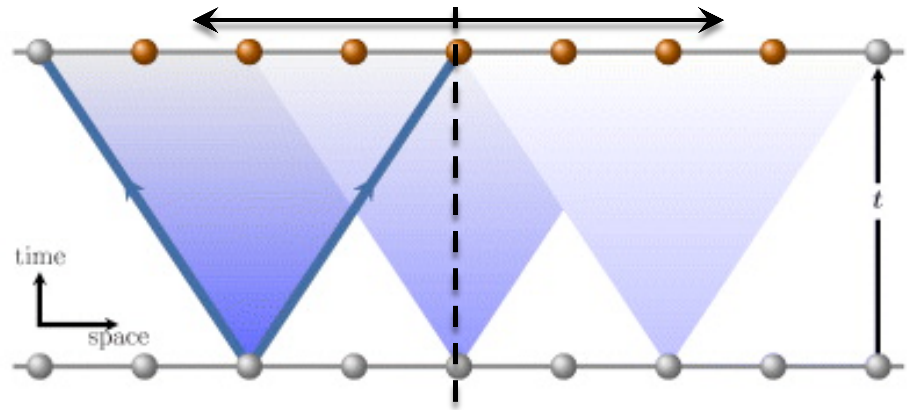

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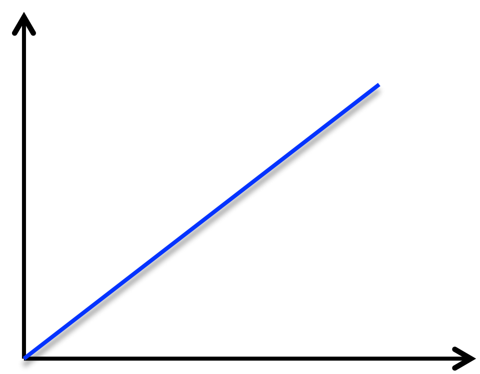


Recent experiments:

Light-cone-like spreading of correlations in a quantum many-body system

Marc Cheneau[1], Peter Barmettler[2], Dario Poletti[2], Manuel Endres[1], Peter Schauß[1], Takeshi Fukuhara[1], Christian Gross[1], Immanuel Bloch[1,3], Corinna Kollath[2,4] & Stefan Kuhr[1,5]

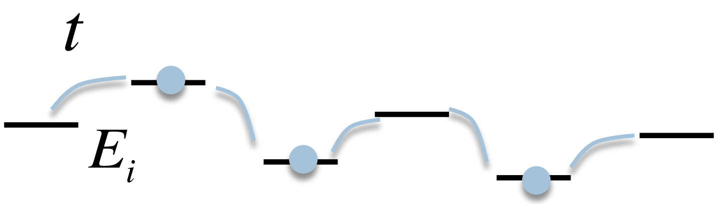
Observation of entanglement propagation in a quantum many-body system

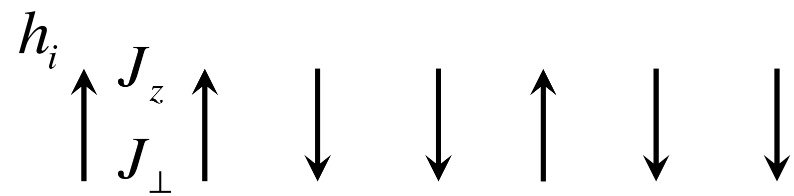
P. Jurcevic,[1,2,*] B. P. Lanyon,[1,2,*] P. Hauke,[1,3] C. Hempel,[1,2] P. Zoller,[1,3] R. Blatt,[1,2] and C. F. Roos[1,2,†]

A simple model of many-body localization

Jordan-Wigner

Spinless interacting 1D fermions $\approx$ Random-field XXZ spin-1/2 chain





$$H=\sum_i E_i c_i^+ c_i + t\sum_i c_i^+ c_{i+1} + h.c. + V\sum_i n_i n_{i+1}$$

$$H=\sum_i h_i S_i^z + J_\perp \sum_i (S_i^+ S_{i+1}^- + h.c) + J_z \sum_i S_i^z S_{i+1}^z$$

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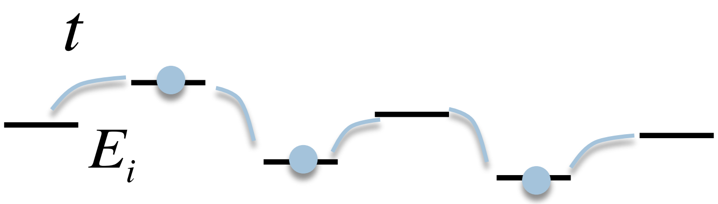

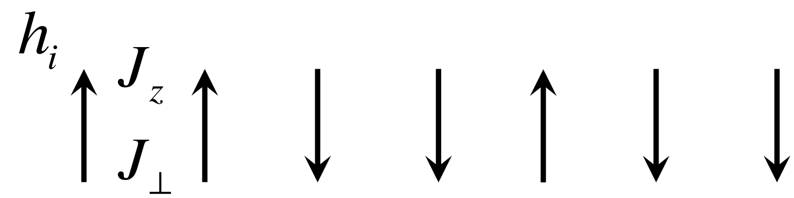

$$H = \boxed{\sum_i E_i c_i^+ c_i} + \boxed{t \sum_i c_i^+ c_{i+1} + h.c.} + \boxed{V \sum_i n_i n_{i+1}}$$

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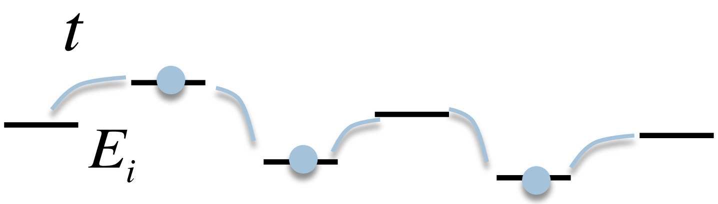
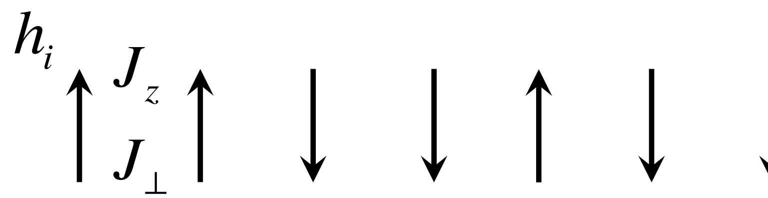




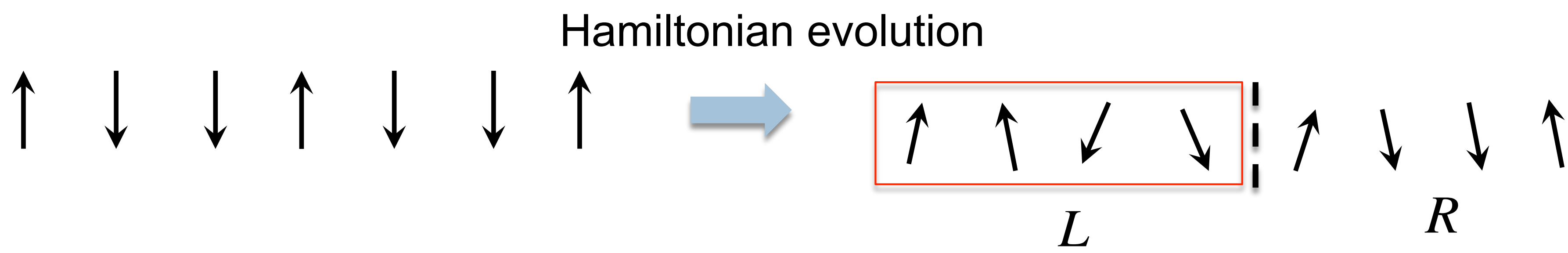
$$H = \boxed{\sum_i E_i c_i^+ c_i} + \boxed{t\sum_i c_i^+ c_{i+1} + h.c.} + \boxed{V\sum_i n_i n_{i+1}}$$

$$H = \boxed{\sum_i h_i S_i^z} + \boxed{J_\perp \sum_i (S_i^+ S_{i+1}^- + h.c)} + \boxed{J_z \sum_i S_i^z S_{i+1}^z}$$

Many-body localization at strong disorder (numerics)
At weaker disorder, ergodic phase

Oganesyan, Huse'07, Prosen et al'08, Pal, Huse'10, Monthus, Garel'10, Bardarson,Pollman,Moore'12, Serbyn, Papic, DA'13, Luca, Scardicchio'13

Recent numerics: entanglement propagation in localized systems



Znidaric et al'08, Bardarson, Pollmann, Moore'12

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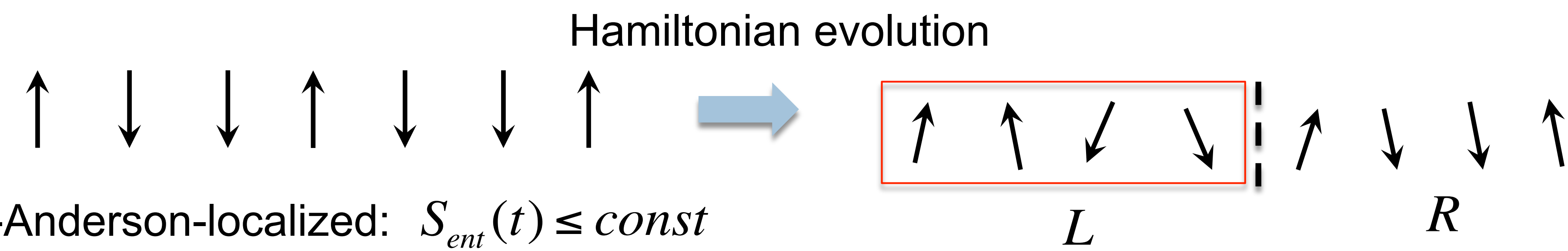
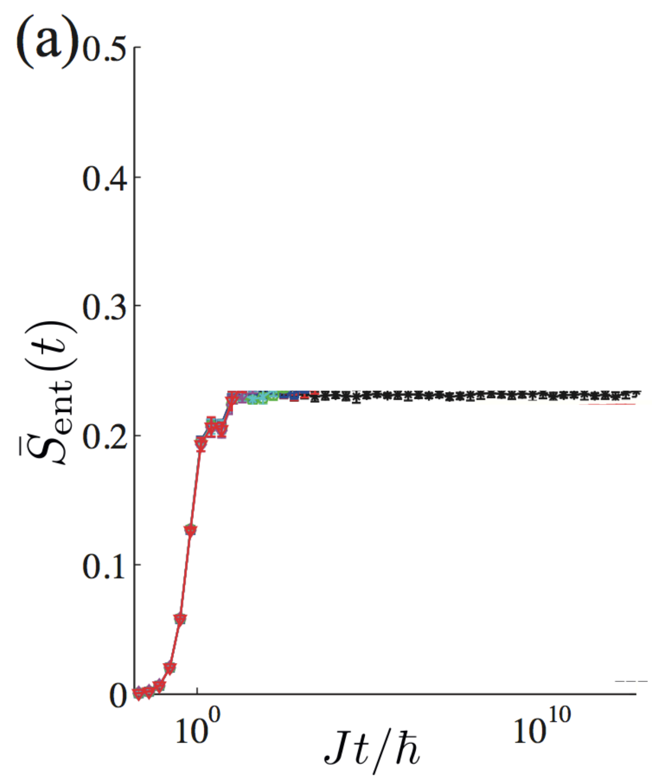


-Anderson-localized: $S_{ent}(t) \leq const$



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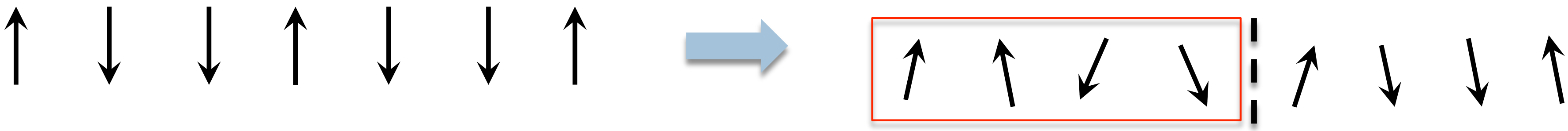
Hamiltonian evolution



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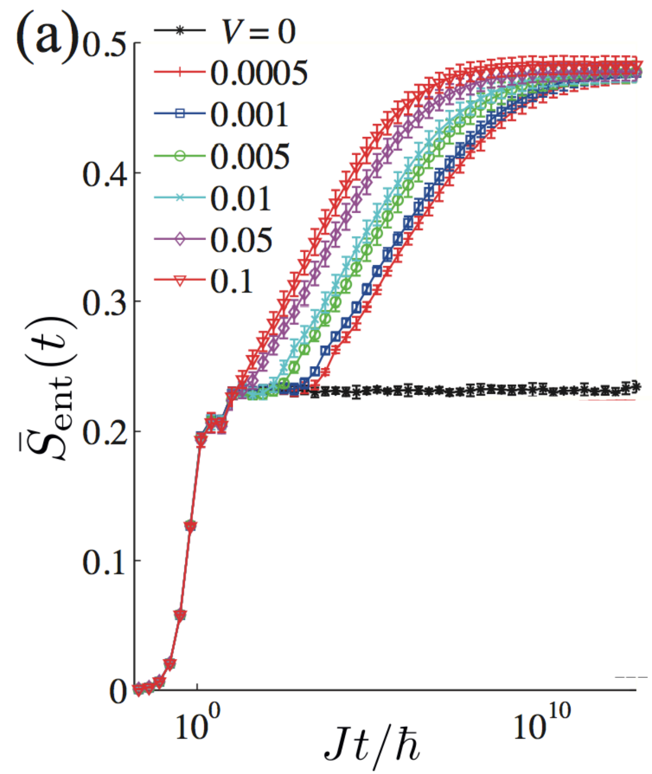
-Many-body localized: slow growth of entanglement

$$S_{ent}(t) \propto \log t$$



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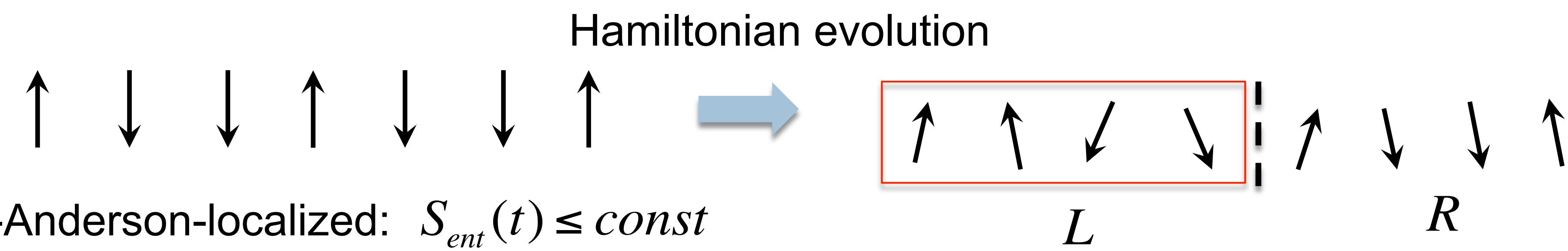

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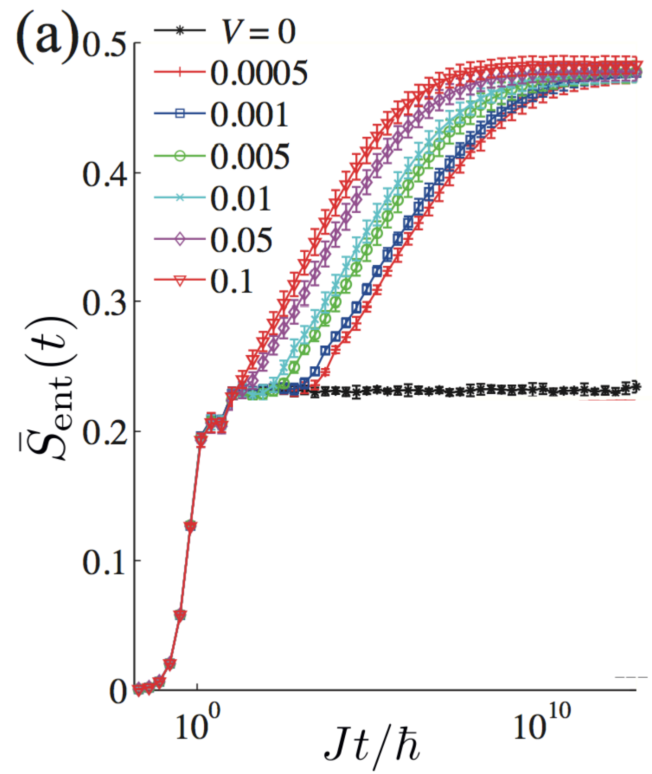
-"Glassy" spread of entanglement

-Very long time scales



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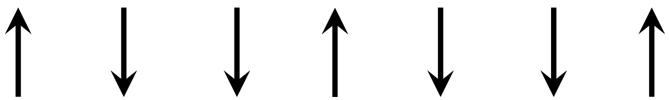

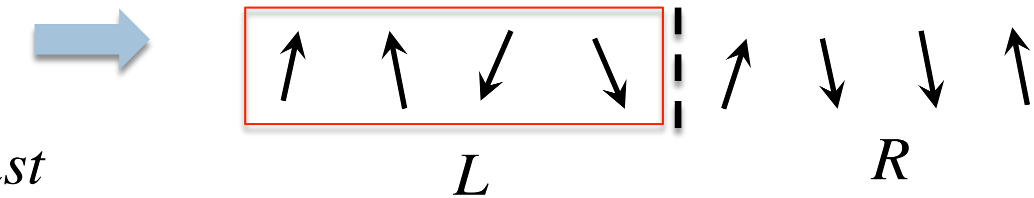

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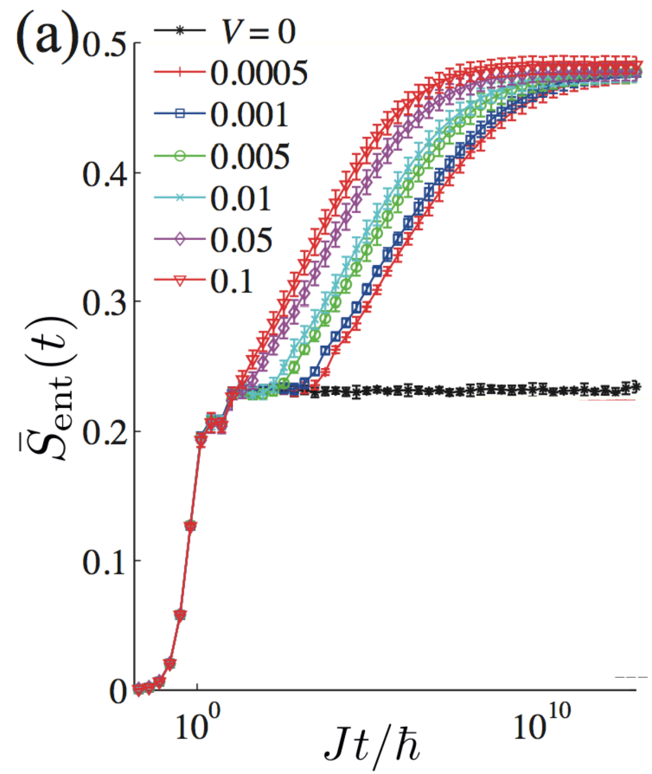

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-Entanglement extensive in system size

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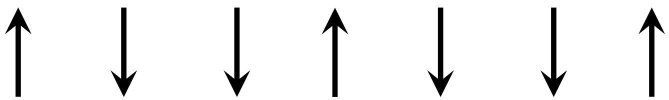

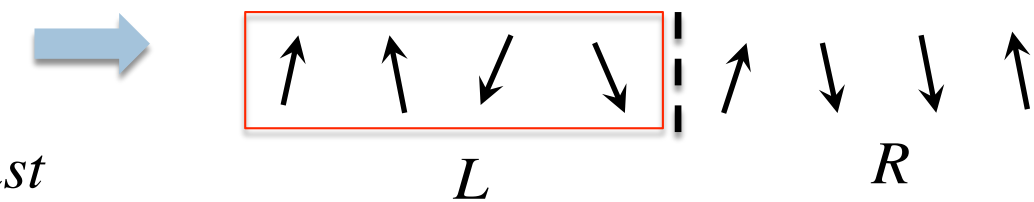

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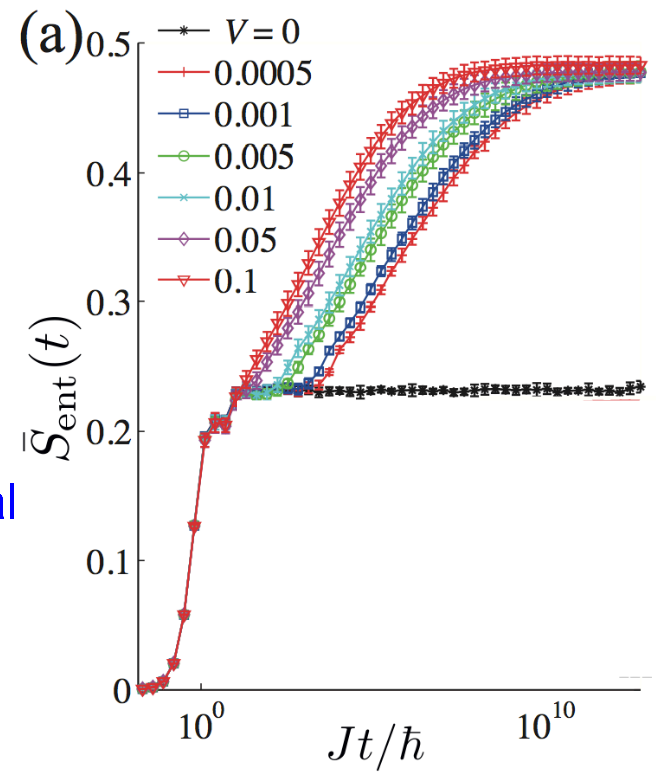
-Entanglement <u>extensive in system size,</u> non-thermal

Very slow equilibration? Slow particle transport?



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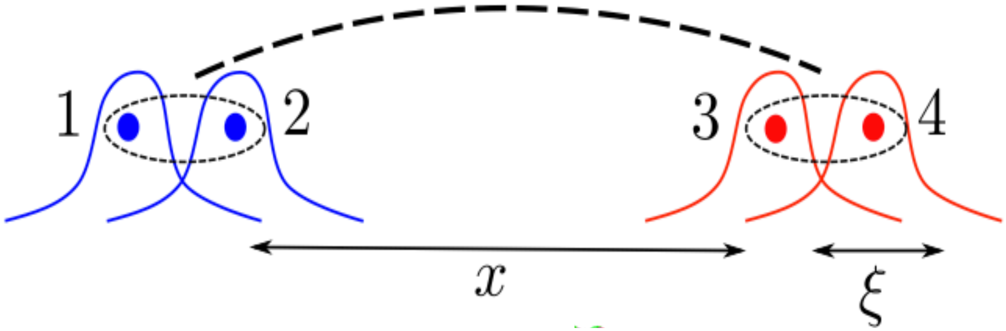
The mechanism of entanglement growth: Toy model

$$|\psi_0\rangle = \frac{1}{2}(c_1^+ + c_2^+)(c_3^+ + c_4^+)|0\rangle$$



Serbyn, Papic, Abanin PRL '13

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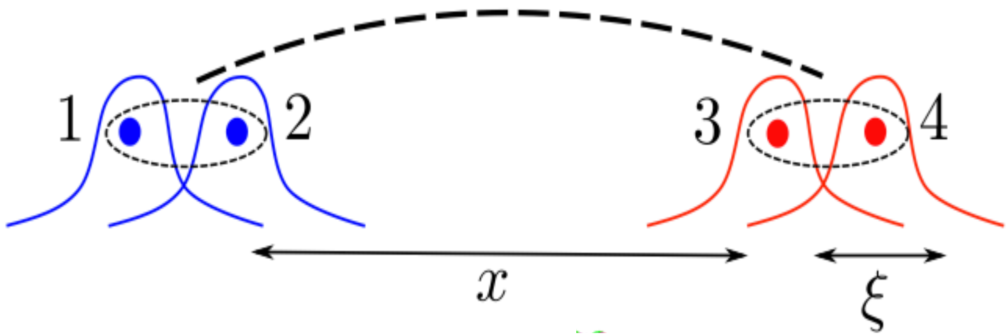

Assume weak interactions

Eigenstate $|\alpha\beta\rangle = c_\alpha^+ c_\beta^+ |0\rangle + O(e^{-x/\xi})$

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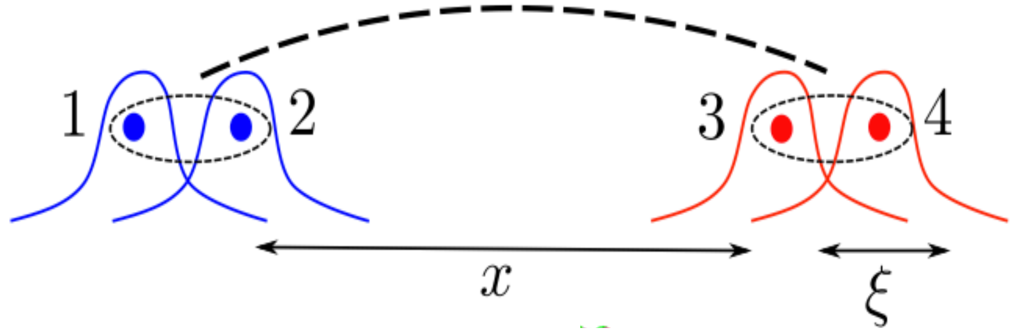

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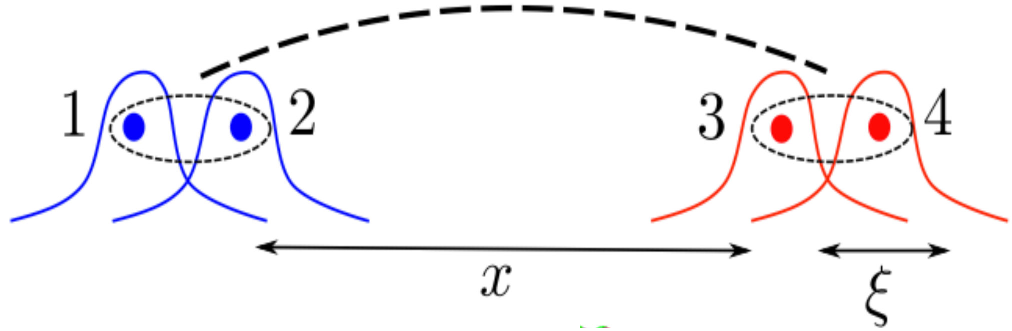

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<u>Reduced density matrix</u>

$$\rho(t) = \frac{1}{2}\begin{bmatrix} 1 & \cos\omega t \\ \cos\omega t & 1 \end{bmatrix}$$

$$\omega \sim \frac{V}{\hbar} e^{x/\xi} \qquad \boxed{t_{deph} \sim \frac{2\pi}{\omega} \sim \frac{\hbar}{V} e^{x/\xi}}$$

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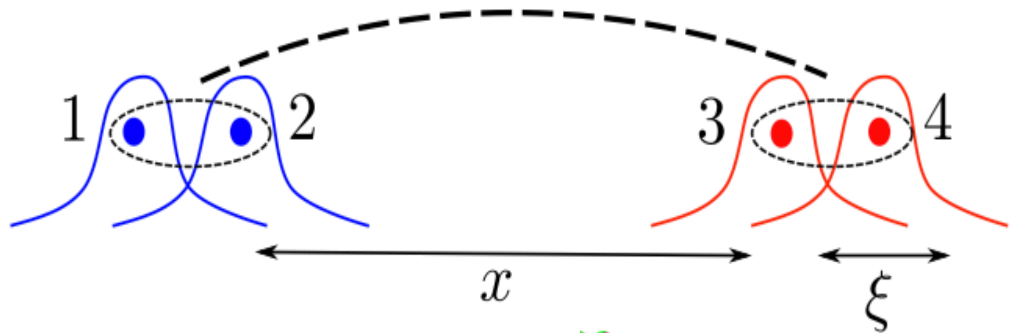

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Interaction-induced dephasing → entanglement generation

Particles can create entanglement without moving

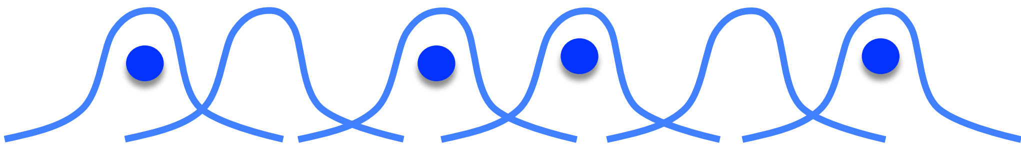
Serbyn, Papic, Abanin PRL '13

Case of many particles

Intuition: Eigenstates at **small *V*** are "close" to non-interacting eigenstates

Non-interacting: occupation numbers

$$n_\alpha = \langle c_\alpha^+ c_\alpha \rangle = 0,1$$



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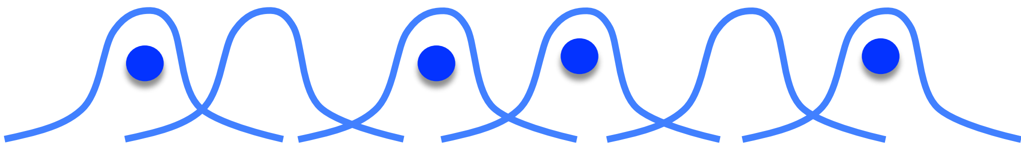
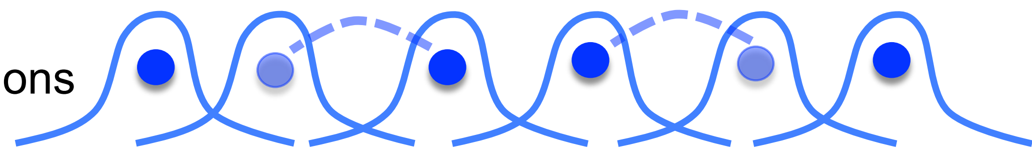


Interacting: obtain by small local deformations

$$\tilde{n}_\alpha = \langle c_\alpha^+ c_\alpha \rangle \approx 0,1$$



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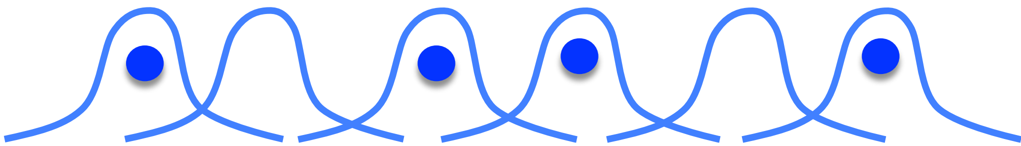

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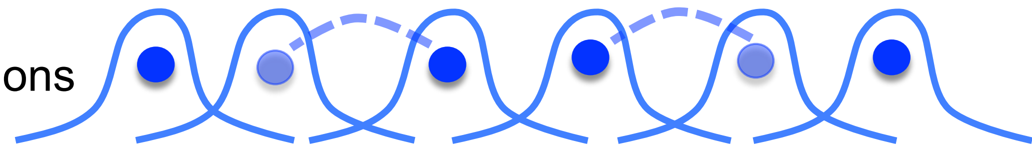

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Energy: perturbation theory in *V*

$$E = \sum E_\alpha n_\alpha + V \sum C_{\alpha\beta} n_\alpha n_\beta e^{-\frac{|R_\alpha - R_\beta|}{\xi}} + V^2 \sum C_{\alpha\beta\gamma} n_\alpha n_\beta n_\gamma e^{-\frac{|R_\alpha - R_\beta| + |R_\gamma - R_\beta|}{\xi}} \ldots$$

1-body energy

2-body interactions

3-body interactions

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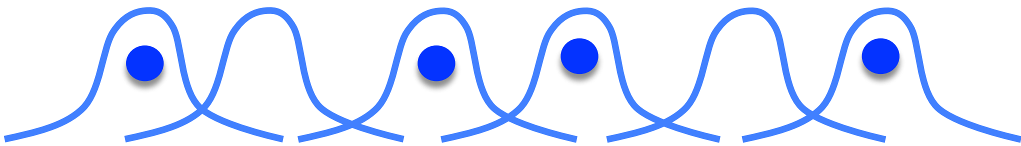

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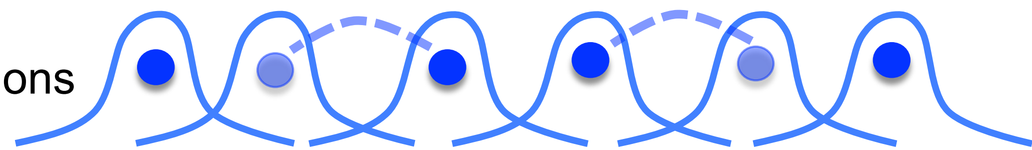

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1-body energy

2-body interactions

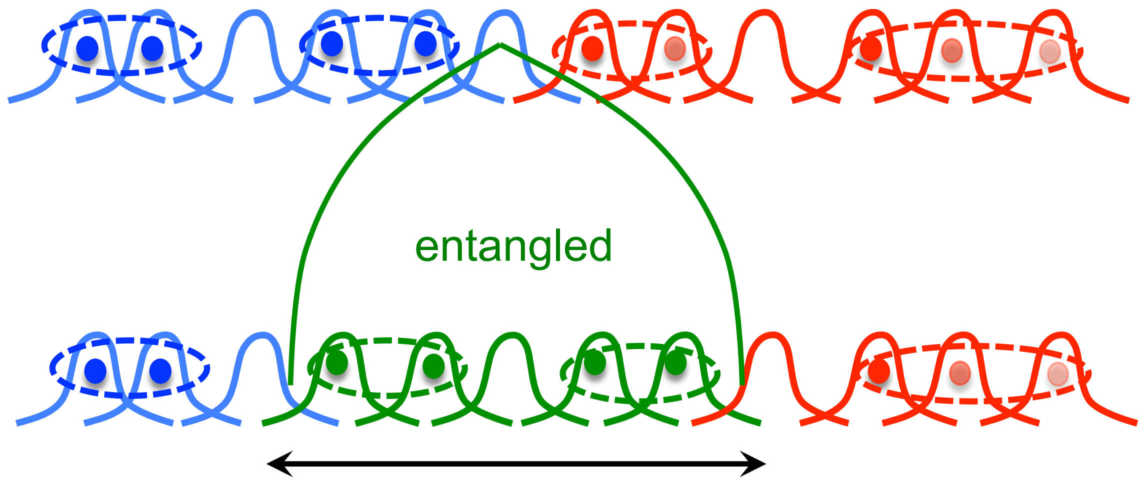
3-body interactions

Interactions of far-away particles are exponentially small

The laws of entanglement growth

Initial product state is a superposition of many eigenstates

$$t(x) \sim \frac{\hbar}{V} e^{x/\xi}$$



$$x(t) \sim \log \frac{Vt}{\hbar}$$

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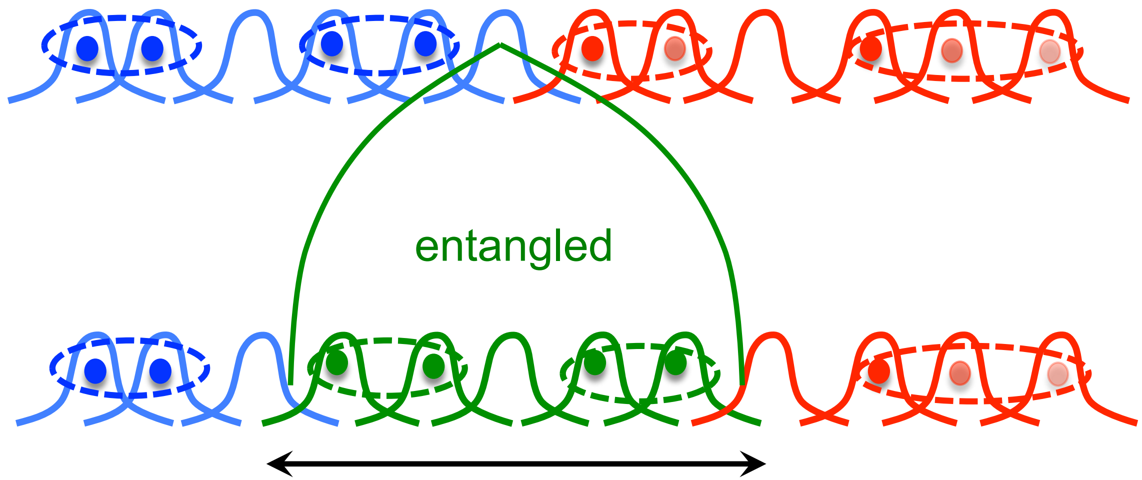

$$\boxed{S_{ent}(t) = C \log \frac{Vt}{\hbar}}$$

C Initial-state dependent

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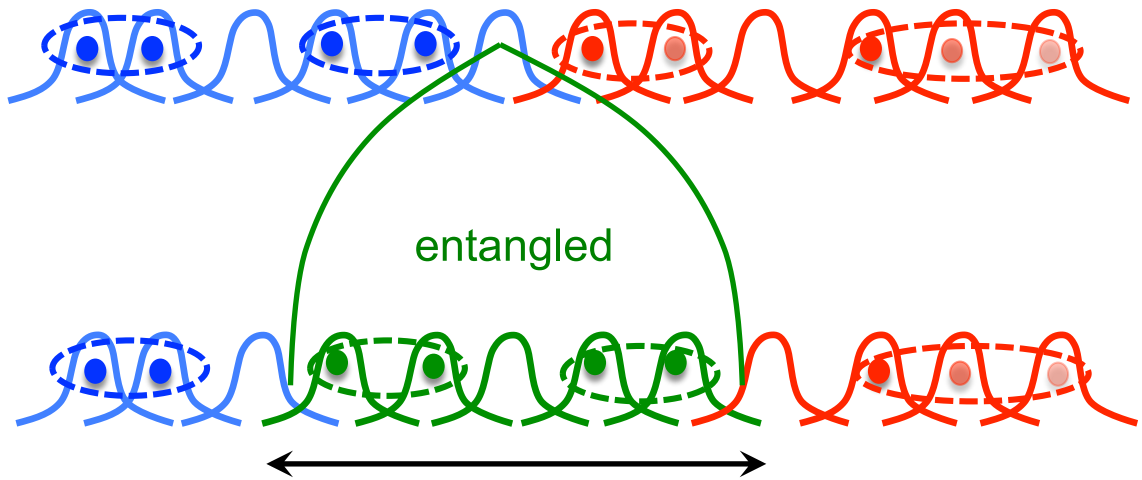

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<u>Predict disorder, interactions, initial state dependence</u>

Confirmed by numerics

We considered weak interactions (starting from single-body-localized phase)

Can we describe localized phase at strong interactions?
Is dynamics universal?

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Can we describe localized phase at strong interactions?
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YES. Key: In the MBL phase there are infinitely many local integrals of motion

Constructing local integrals of motion

$$H_0 = \sum_i h_i s_{\boldsymbol{i}}^z + J_z s_{\boldsymbol{i}}^z s_{\boldsymbol{i}+1}^z$$

$$\uparrow \quad \uparrow \quad \downarrow \quad \downarrow \quad \uparrow \quad \downarrow \quad \uparrow$$

$$s_i^z = \pm 1$$

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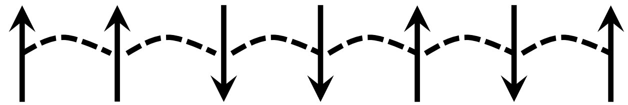

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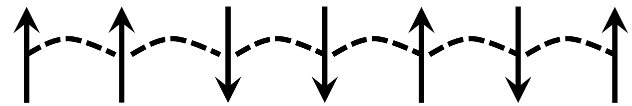

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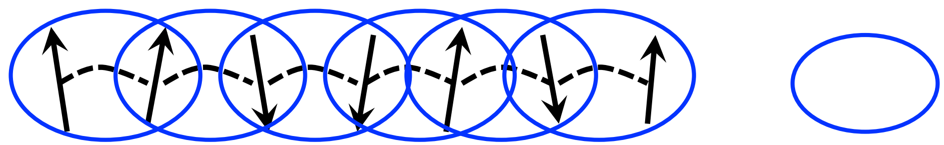

Local unitary

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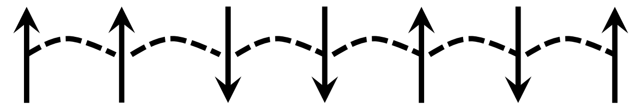

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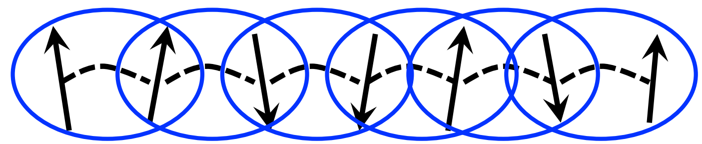
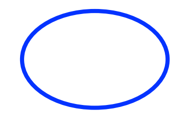


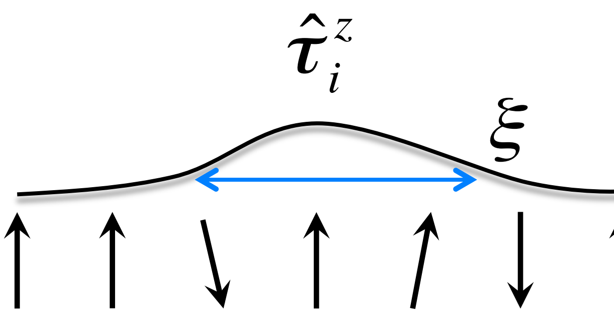


Local unitary

Hamiltonian diagonalized by a sequence of local unitary transformations

$$U^+ H U = H_{diag}$$

Local integral of motion $\hat{\tau}_i^z = U \hat{s}_i^z U^+$



$$[\hat{\tau}_z^i, H] = 0 \qquad [\hat{\tau}_z^i, \hat{\tau}_z^j] = 0$$

"Effective spins", form a complete set

Universal Hamiltonian of many-body localized phase

$[\hat{\tau}_z^i, H] = 0$ → Hamiltonian depends only on $\hat{\tau}_z^i$'s

$$H = \sum_i H_i \tau_z^i + \sum_{ij} H_{ij} \tau_z^i \tau_z^j + \sum_{ijk} H_{ijk} \tau_z^i \tau_z^j \tau_z^k + ..$$

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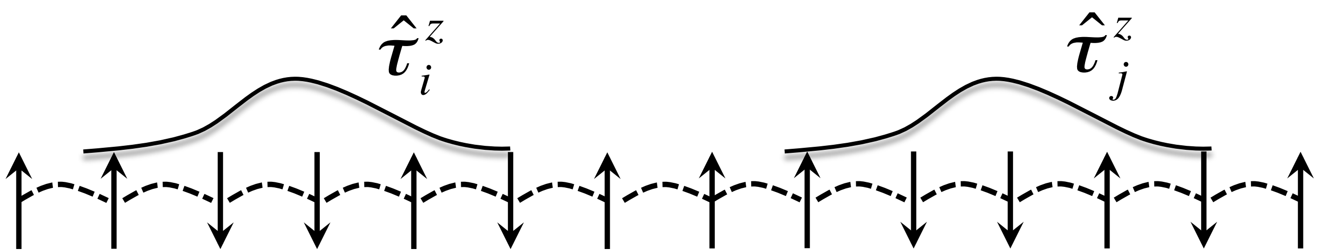
$H_{ij} \propto \exp(-|i-j|a/\xi)$, random



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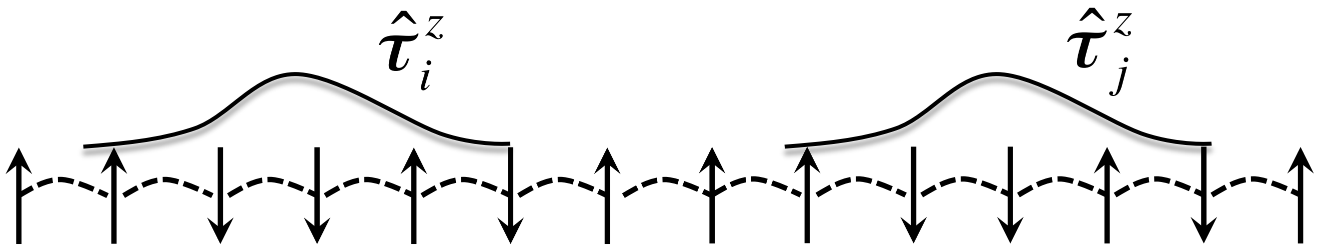

Quantum bits which cannot relax

Exponentially decaying (random) interactions → dephasing

Serbyn, Papic, Abanin PRL'13
Huse, Oganesyan, arXiv'13

Structure of localized eigenstates

Conjecture: MBL eigenstates are obtained from product states by quasi-local unitary transformations

Implication 1: MBL phase is robustly integrable

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Conjecture: MBL eigenstates are obtained from product states by quasi-local unitary transformations

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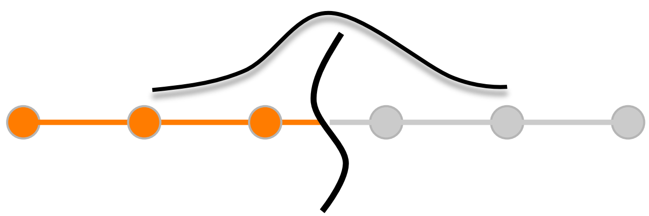
Implication 2: Eigenstates have low entanglement entropy, "area-law"

Bauer, Nayak, JSM'13; Serbyn, Papic, DA PRL'13

$$S_{ent}(L) \leq Const$$



Entanglement limited to boundary, similar to ground states in gapped systems

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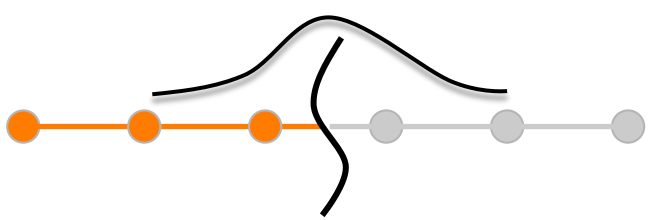

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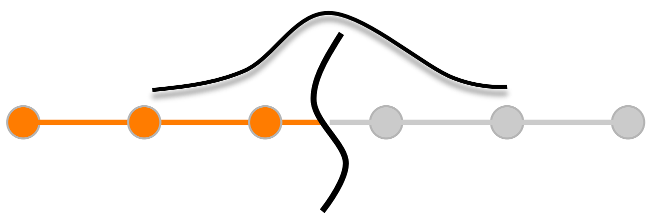

Entanglement limited to boundary,
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Ergodic systems:
“volume-law” of excited states

$$S_{ent}(L) \sim L$$

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Bauer, Nayak, JSM'13; Serbyn, Papic, DA PRL'13

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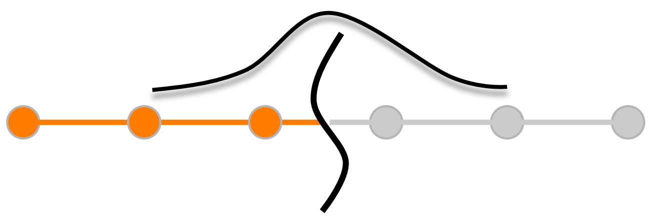


Ergodic systems:
"volume-law" of excited states

$$S_{ent}(L) \sim L$$

Entanglement limited to boundary,
similar to ground states in gapped systems

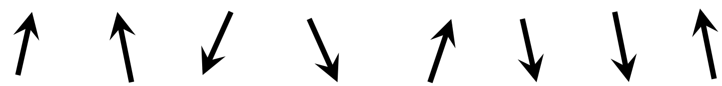
MBL eigenstates can be efficiently simulated clasically
Matrix-product states, tensor networks

Chandran, Carrasquilla, Kim, DA, Vidal, arXiv'14
Pekker, Clark'14

Universal dynamics & experimental signatures

$$H = \sum_i H_i \tau_z^i + \sum_{ij} H_{ij} \tau_z^i \tau_z^j + \sum_{ijk} H_{ijk} \tau_z^i \tau_z^j \tau_z^k + ..$$

Quantum quench (e.g. from a product state)



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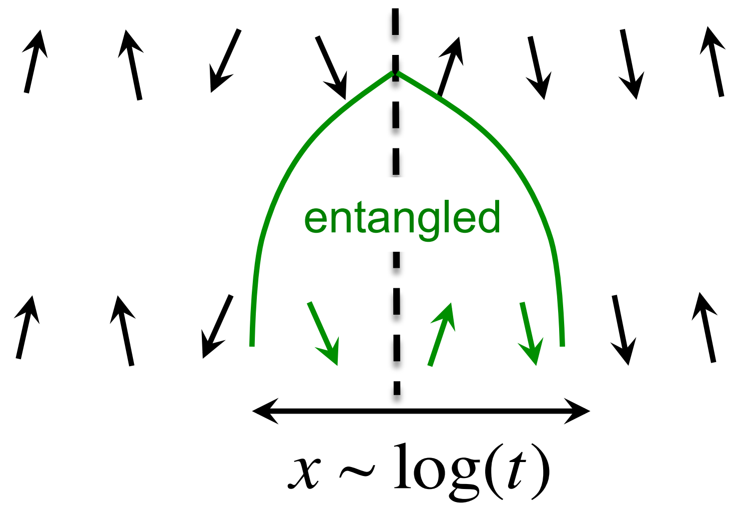
Quantum quench (e.g. from a product state)

-At long times, steady <u>non-thermal</u> state

"Local diagonal ensemble"

$$\left\langle \tau_z^i(t) \right\rangle = Const$$

<u>Memory retained, ergodicity breaking</u>

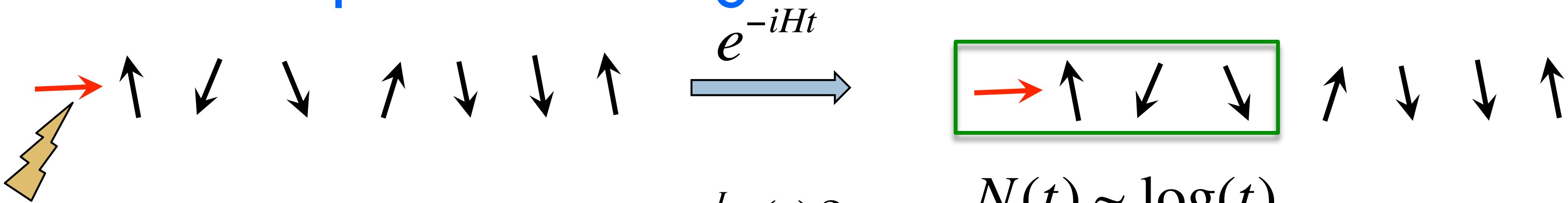


-Logarithmic spreading of correlations

$$S_{ent}(t) \sim \log(t)$$

Universal logarithmic growth of entanglement

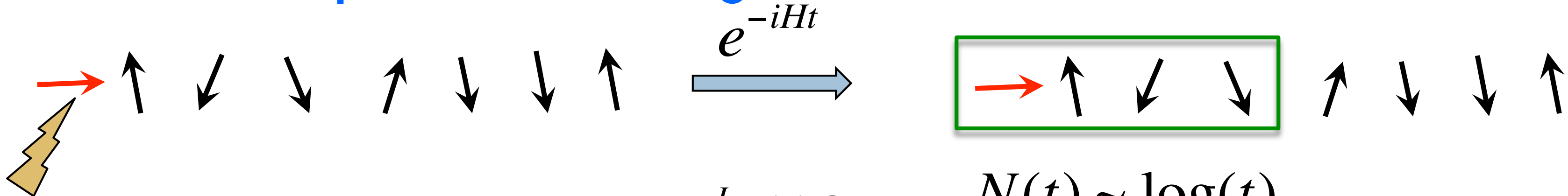
Experimental signature: local observables



Eff. Spin I: reduced density matrix $\rho^{I}_{\alpha\beta}(t)$?

$N(t) \sim \log(t)$

Experimental signature: local observables



$$N(t) \sim \log(t)$$

Eff. Spin I: reduced density matrix $\rho^{I}_{\alpha\beta}(t)$?

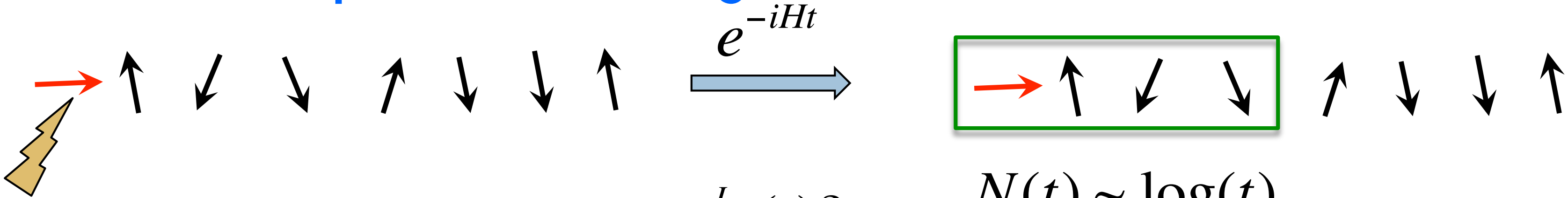
Diagonal $\rho^{I}_{\uparrow\uparrow}(t) = Const$

$$\left\langle \tau^{i}_{z}(t) \right\rangle = Const$$

Off-diagonal $\rho^{I}_{\uparrow\downarrow}(t)$ a sum of $2^{N(t)}$ random terms

$$\left|\rho^{I}_{\uparrow\downarrow}(t)\right| \sim 2^{N(t)/2} \propto \frac{1}{t^{a}}$$

Experimental signature: local observables



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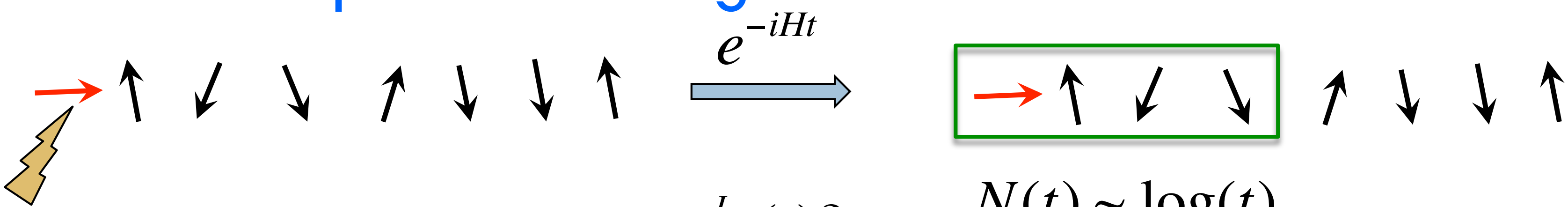
Power-law!

Experimental signature: local observables



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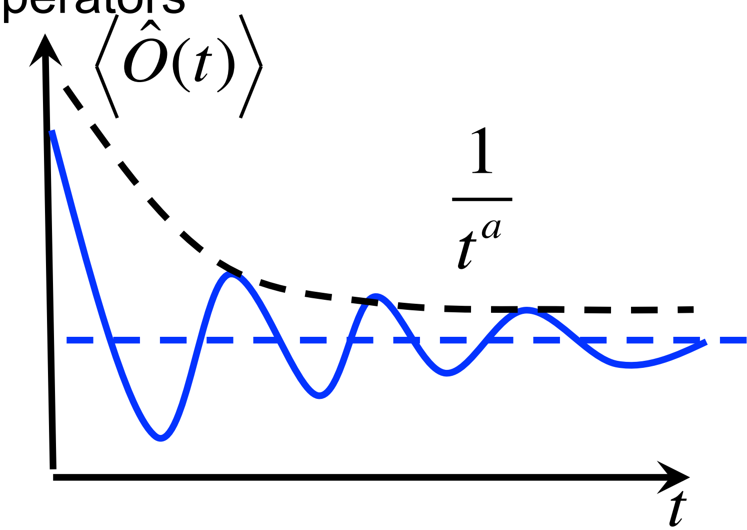
Power-law!

Physical observables are superpositions of $\tau^{i}_{x,y,z}$ operators

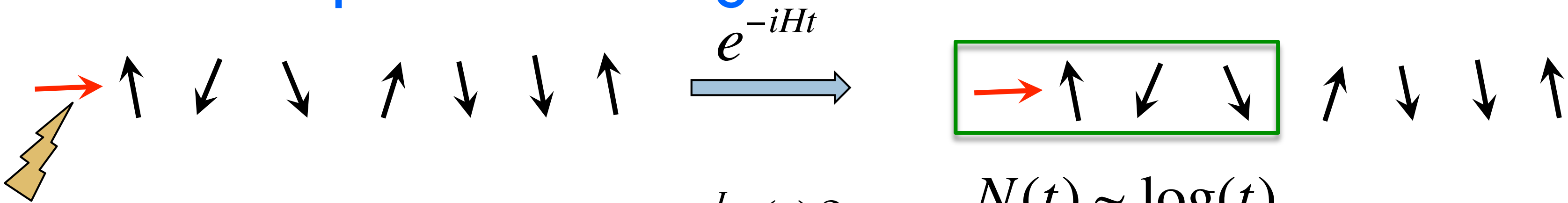
$$\left\langle \hat{O}(\infty) \right\rangle \neq 0 \quad \left|\left\langle \hat{O}(t) \right\rangle - \left\langle \hat{O}(\infty) \right\rangle\right| \propto \frac{1}{t^{a}}$$



Local observables decay as power-law to steady values

Serbyn, Papic, DA PRB'14

Experimental signature: local observables



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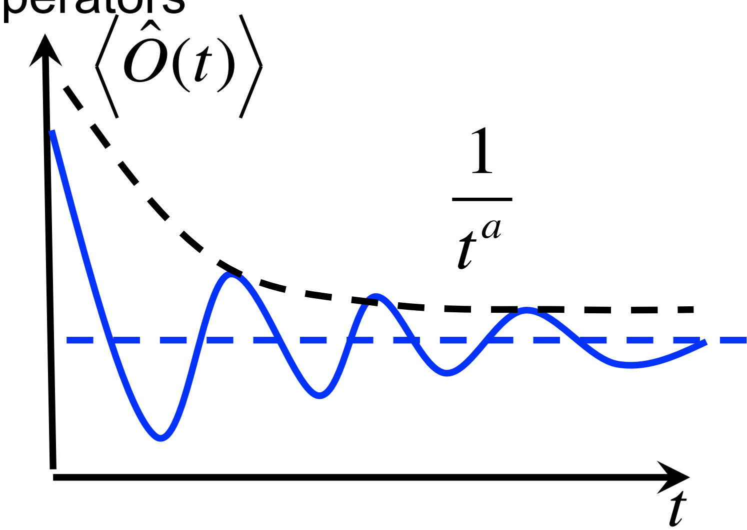

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Serbyn, Papic, DA PRB'14

Alternatives: revivals of local observables Vasseur, Parameswaran, Moore, arXiv'14

Modified spin echo Serbyn et al. PRL'14

Distinct localized phases at high energy

Disordered transverse-field 1D Ising model Z_2 symmetry

$$H = \sum_i J_i \sigma_i^z \sigma_{i+1}^z + h \sum_i \sigma_i^x + J_2 \sum_i \sigma_i^z \sigma_{i+2}^z \qquad J_i = J \pm \delta J_i$$

<u>Ground state:</u>

$J >> h$

Spin glass

Breaks Z_2

$J << h$

Paramagnet

Does not break Z_2

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<u>MBL protects quantum order at finite energy density</u>

Huse et al PRB'13
Kjall, Bardarson, Pollmann, PRL'14

Two distinct MBL phases:

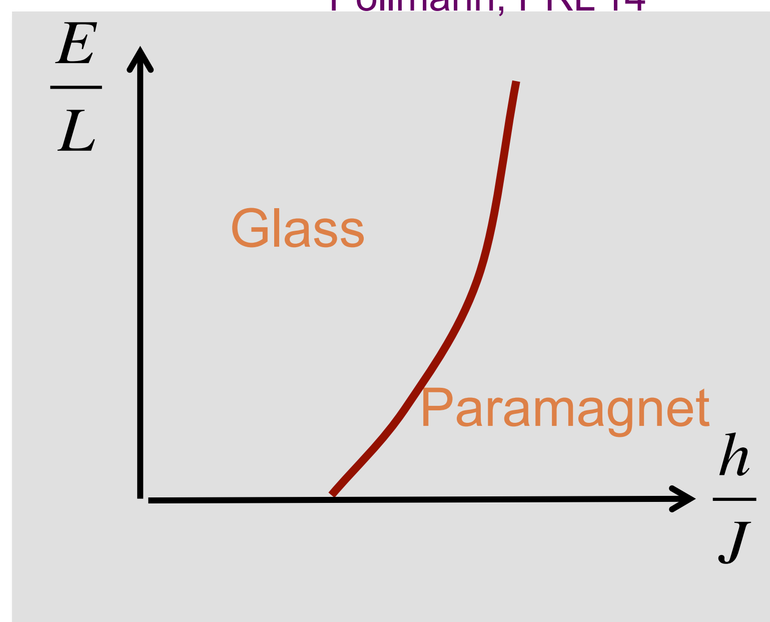
Spin-glass:
Integrals of motion $\sim \sigma_i^z$

Paramagnet:
I.O.M are $\sim \sigma_i^z$

Symmetry broken in individual eigenstates, but not in thermal ensemble

Dynamical critical points characterized using strong-disorder RG

Vosk, Altman PRL'14;
Pekker et al PRX'14

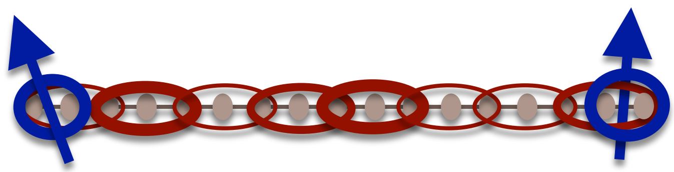


Localization-protected topological order

MBL can protect topological order at finite energy density

Huse et al PRB'13
Bauer, Nayak JSM'13

Topological localized states with
Protected **coherent** edge modes at high energy



Bahri et al, arXiv'13
Chandran et al, PRB'14

Not all symmetry-protected/topological phases can be fully MBL
(e.g., chiral states)

Slagle et al, arXiv'15
Potter, Vishwanath, arXiv'15

Open questions

Phase transition from MBL to ergodic phase? Ehud Altman's talk

Is disorder necessary? Localization in translationally invariant systems?

Huveneers, De Roeck'13, Shiulaz, Muller'13,
Yao et al'14, Papic, Stoudenmire, DA'15

Other mechanisms of ergodicity breaking?
Non-ergodic phases which are not fully MBL?

Altshuler et al'06-,
Grover, Fisher'13, Pino, Altshuler, Ioffe'15

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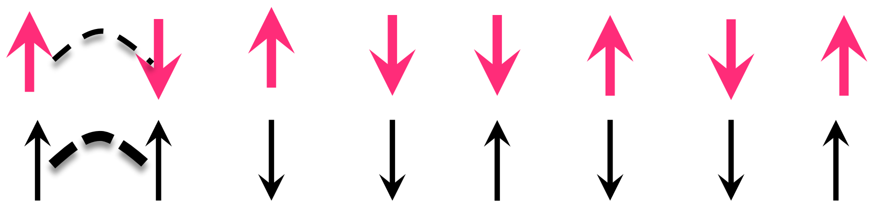
Localization without quenched disorder?

(Huveneers, de Roeck'13, Schiulaz, Muller'13)

Two coupled XXZ spin chains, "fast" σ_i^z and "slow" s_i^z

$\lambda << 1$



$J \sim 1$

$$H_{\text{int}} = W \sum_i \sigma_i^z s_i^z$$

Localization at $\lambda << 1$?

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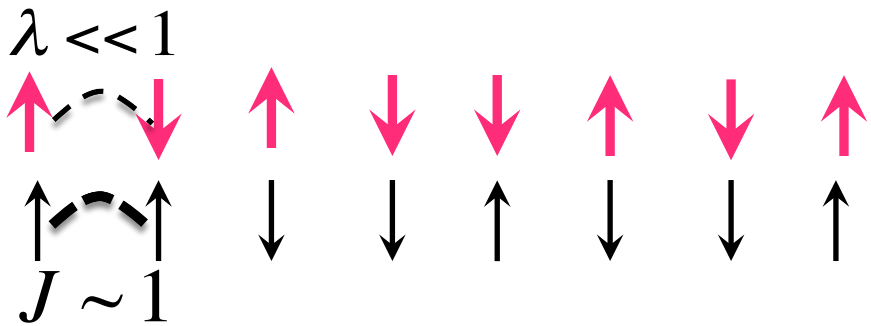

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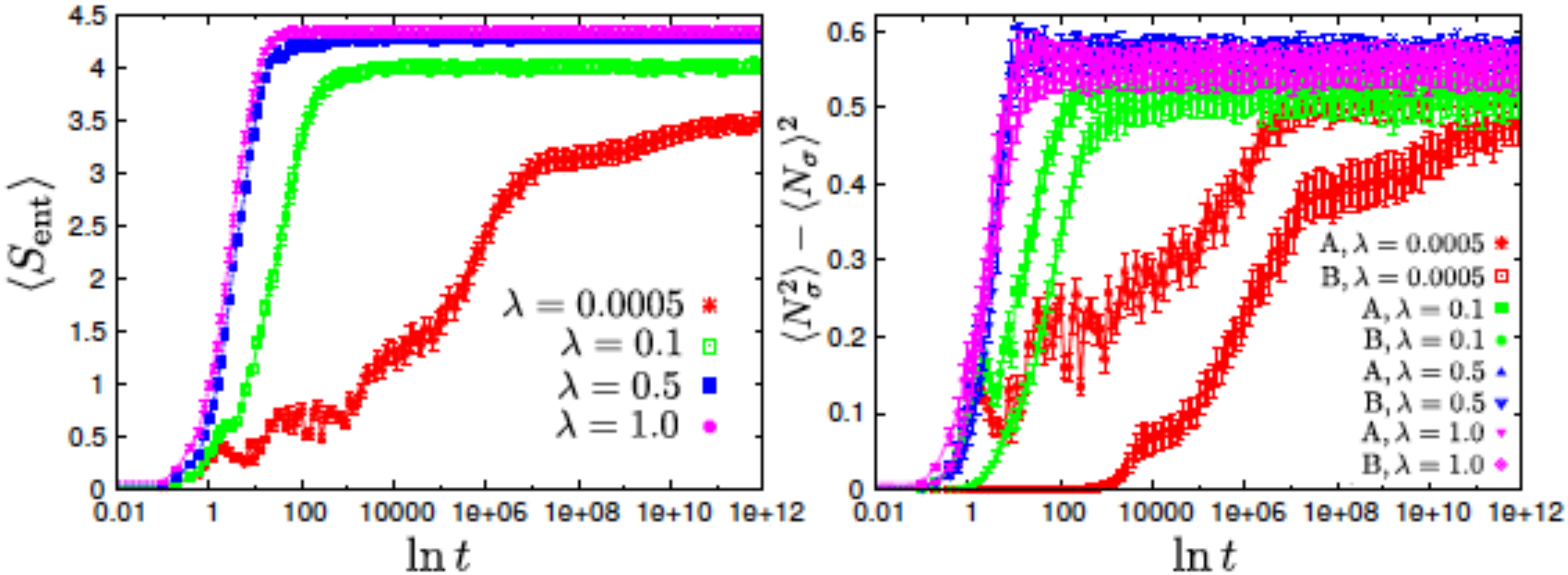
Localization at $\lambda << 1$?

An argument for ergodicity breaking:

Configuration of slow spins creates "disorder" for fast spins

Have to make multiple moves to resonantly couple states

Numerical results: no MBL, but slow dynamics



Papic, Stoudenmire, DA, arXiv'15

Numerical studies: If MBL exists, it only exists at tiny $\lambda \sim 0.01$

Consistent with Yao et al'14

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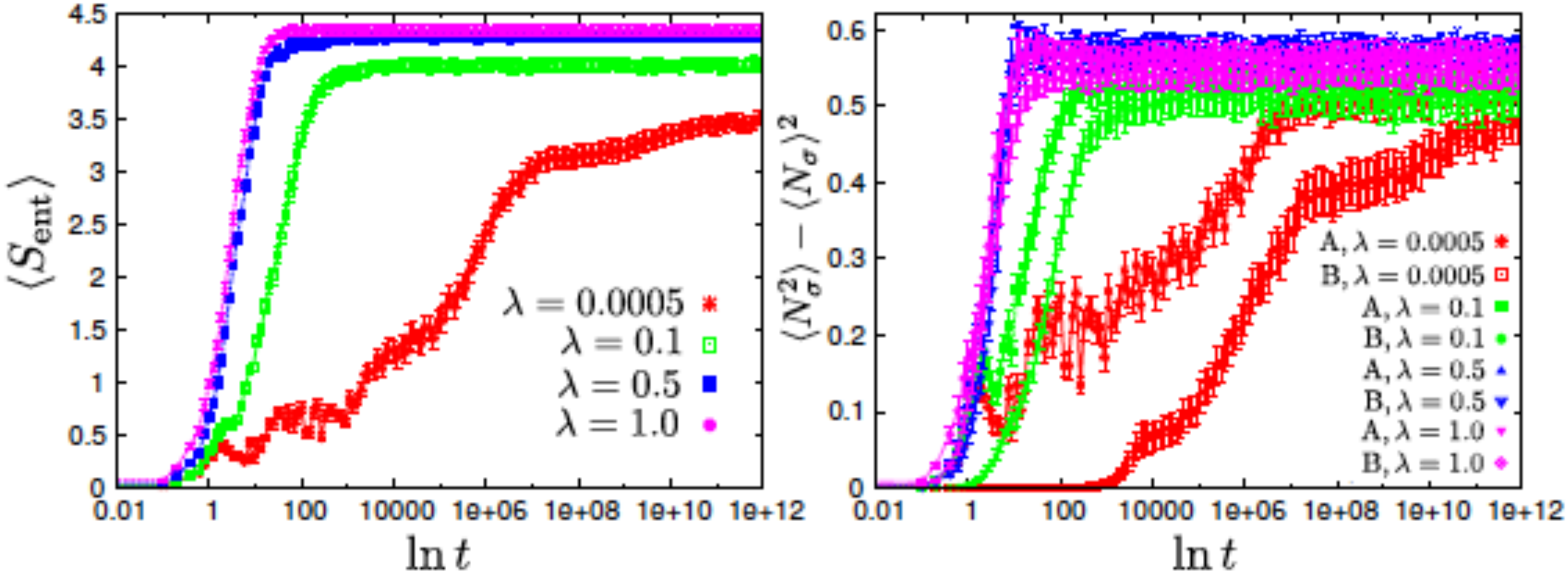

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MBL in translationally inv. systems does not appear robust
BUT: equilibration can be still very slow

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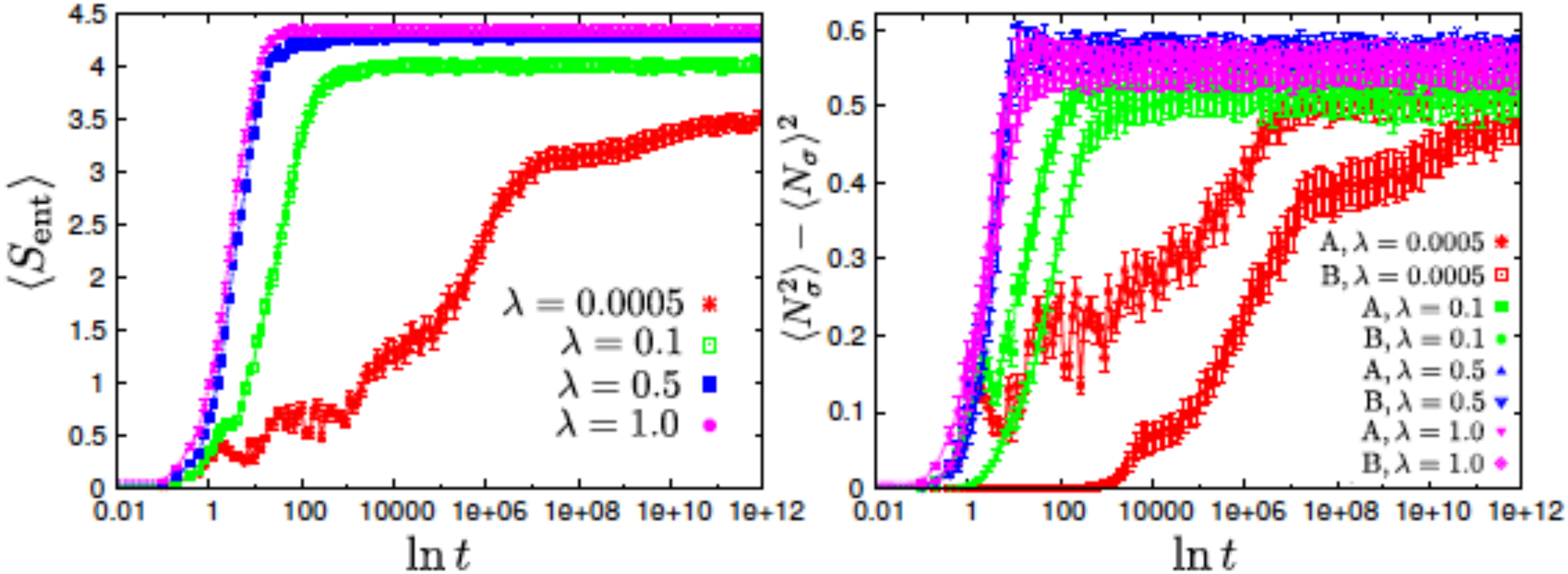

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A rigorous result for a Bose-Hubbard-type model (high *T* → many non-resonant configurations)

$\sigma(T)$ decays faster than any power-law as a function of *T*

Huveneers, De Roeck, CommMathPhys'14

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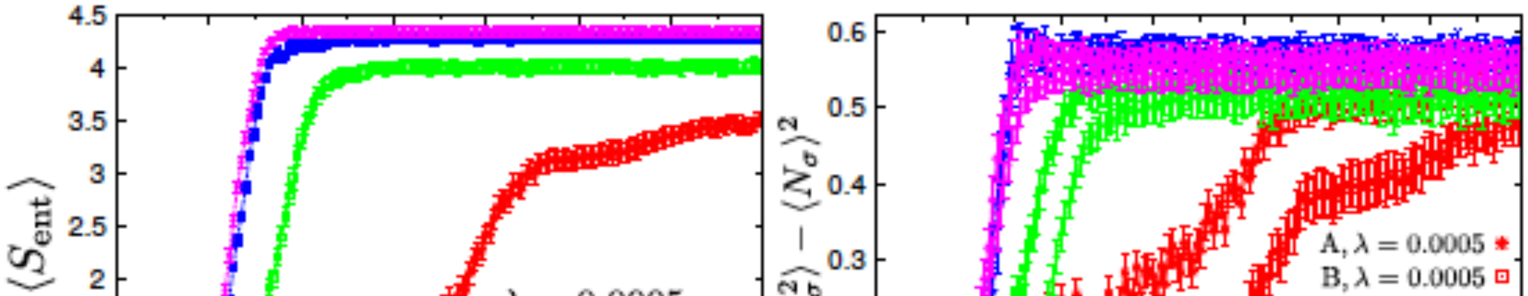

Find systems with exponentially slow equilibration?

Role of geometric frustration?

Connections to glasses?

Nur … th

MBL …

BUT: equilibration can be still very slow

A rigorous result for a Bose-Hubbard-type model (high T → many non-resonant configurations)

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Huveneers, De Roeck, CommMathPhys'14

Summary

-Many-body localization: a mechanism for ergodicity breaking

-Integrability, area-law for eigenstates

-<u>Dynamics is universal</u>:"glassy" entanglement growth, power-law relaxation of physical observables

-Many open questions: Transition; Transl. inv. systems; Other mechanisms of ergodicity breaking?

Summary

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<u>MESSAGE:</u> ENTANGLEMENT GIVES INSIGHTS INTO ERGODICITY AND ITS BREAKING

Acknowledgements

Zlatko Papic

Max Serbyn

Anushya Chandran

Isaac Kim

Discussions: Ehud Altman, David Huse, Guifre Vidal

The End