Thermalization vs Localization in a Solvable Circuit Model

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arXiv:1501.01971

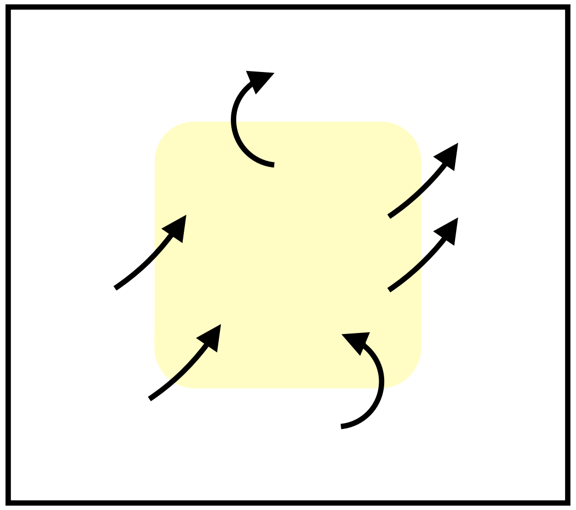
Work with Chris Laumann

Long-time behavior of unitary dynamics

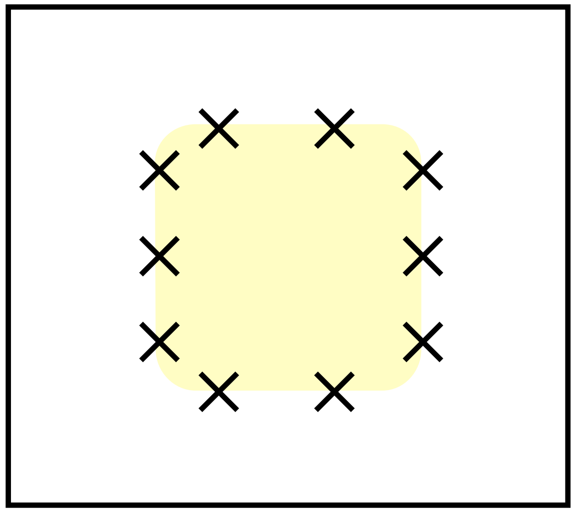
Ergodic

Localized

?



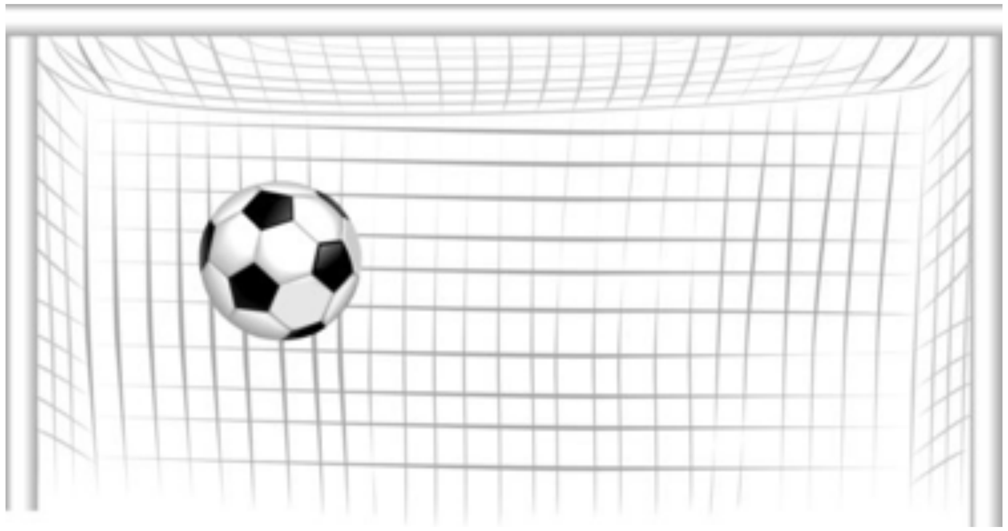
Statistical mechanics ensembles emerge



Remembers initial conditions forever

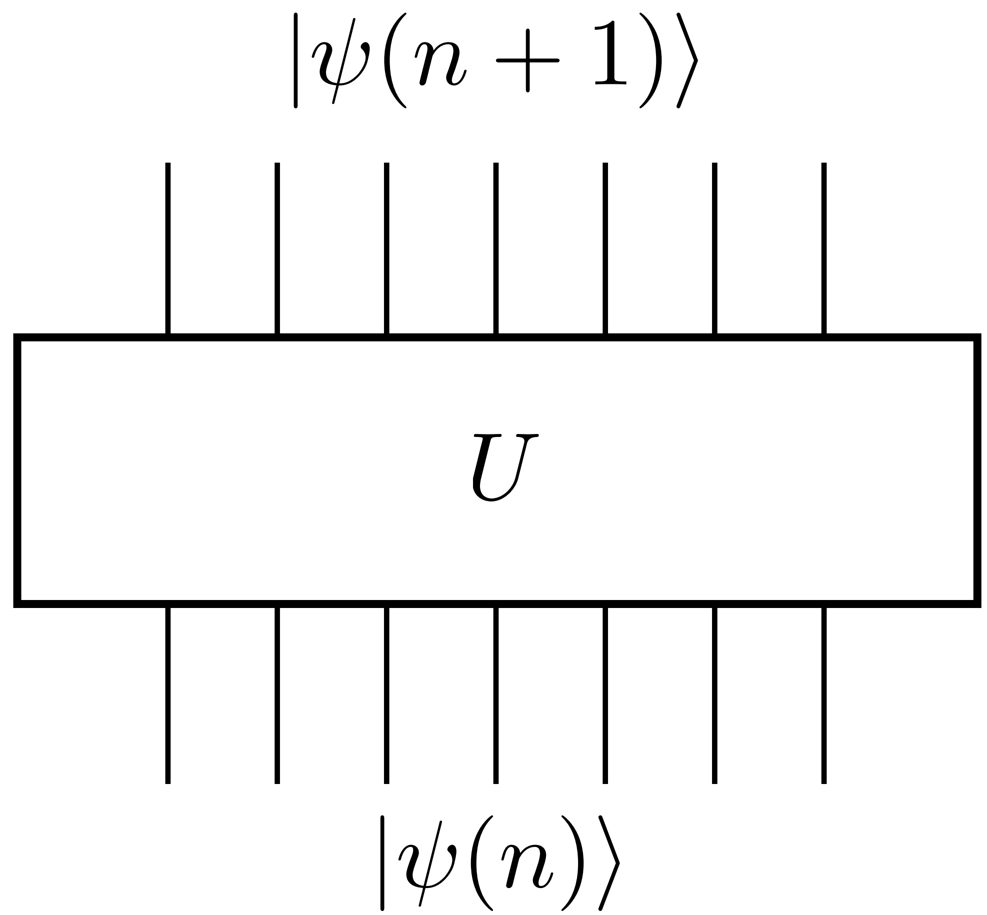
Goal: minimal microscopic model

- Analytically tractable
- Works in d>1
- Delocalized, localized, transition



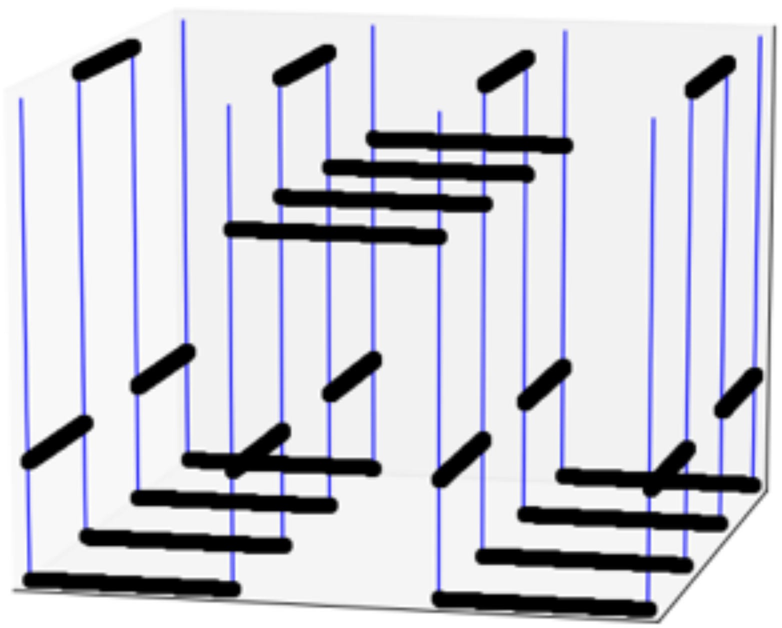
- Focus on entanglement
- No conservation laws (no symmetry)

Periodic circuit on qubits

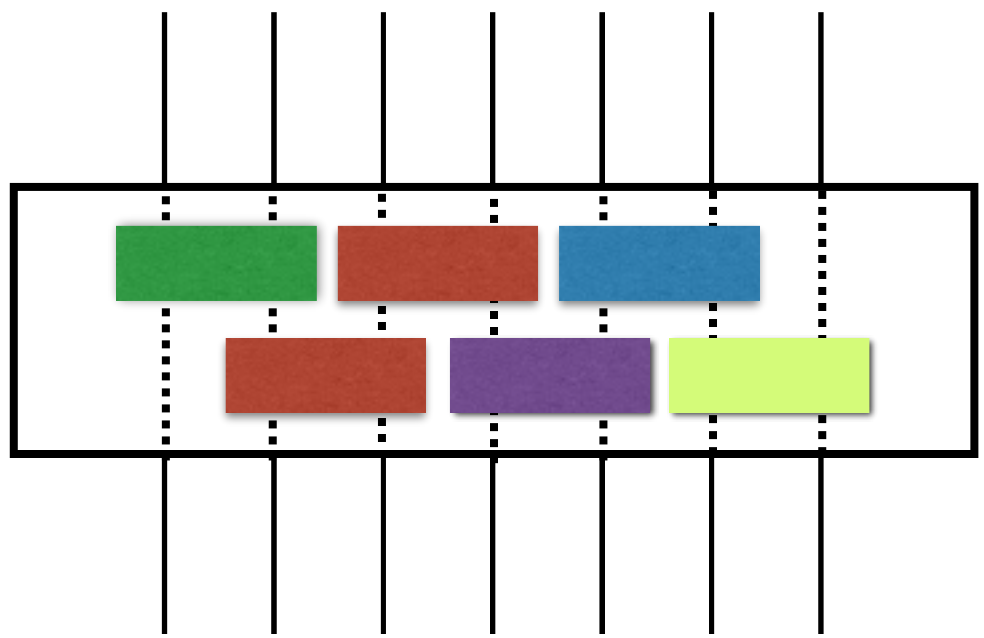


$$|\psi(n)\rangle = U^n |\psi(0)\rangle$$

Spatial locality



$$|\psi(n+1)\rangle$$



$$|\psi(n)\rangle$$

$$|\psi(n)\rangle = U^n|\psi(0)\rangle$$

Heisenberg picture

$$O(n) = (U^\dagger)^n O U^n$$

- Pauli basis generators: $\{X_1, \ldots X_N, Z_1, \ldots Z_N\}$
- O can be decomposed in Pauli basis
- Time evolution of generators ⇒ O(n)

Clifford circuits

$$X_1(n) = \sum_{\sigma \in \{1, X, Y, Z\}} \mathcal{A}_{i_1 \ldots i_N}(n) \sigma_{i_1} \otimes \ldots \otimes \sigma_{i_N}$$

$\big\downarrow$ Clifford

$$X_1(n) = \pm \sigma_{i_1} \otimes \sigma_{i_2} \ldots \otimes \sigma_{i_N}$$

The Clifford group

$$X_1(n) = \pm\sigma_{i_1} \otimes \sigma_{i_2} \ldots \otimes \sigma_{i_N}$$

- A discrete group
- Efficiently classically simulable for a class of initial states

$\mathbb{1}:$ 00 $\quad X:$ 01 $\quad Y:$ 11 $\quad Z:$ 10

$$X_1(n) = X_1 \otimes Y_2 \ldots \otimes Z_N$$

01 11 10

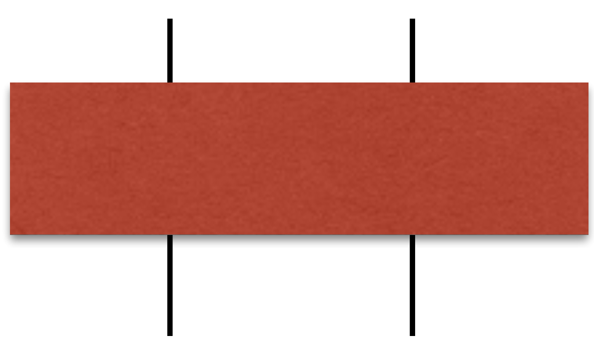
- Contrast to free fermions $c_i(n) = \sum_{j=1}^{N} \mathcal{A}_j(n) c_j$

Gottesman-Knill (1996)
Aaronson & Gotteman (2004)

Entangling example

- Two qubit Clifford gate

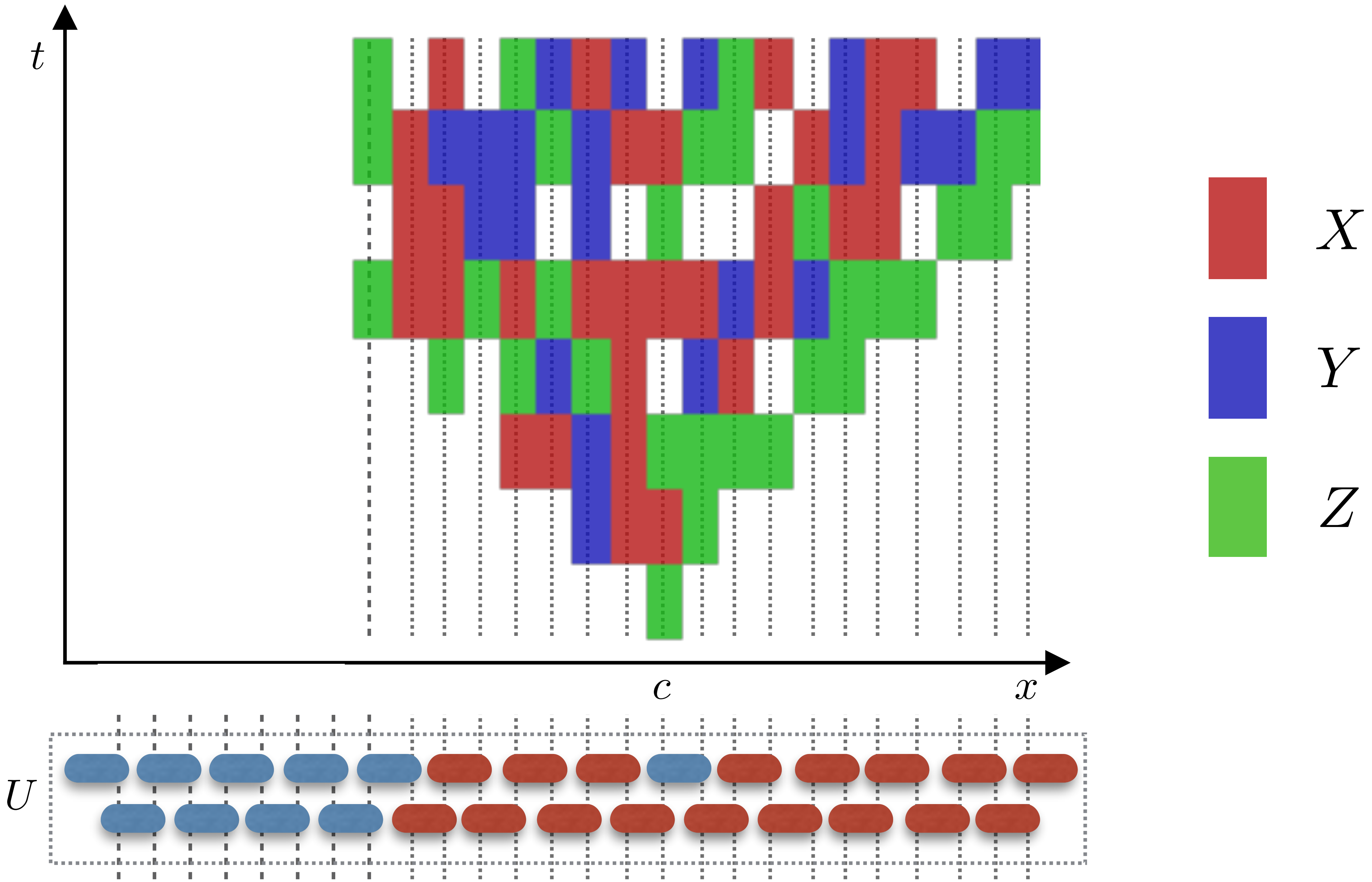
$$\frac{|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle}{\sqrt{2}}$$



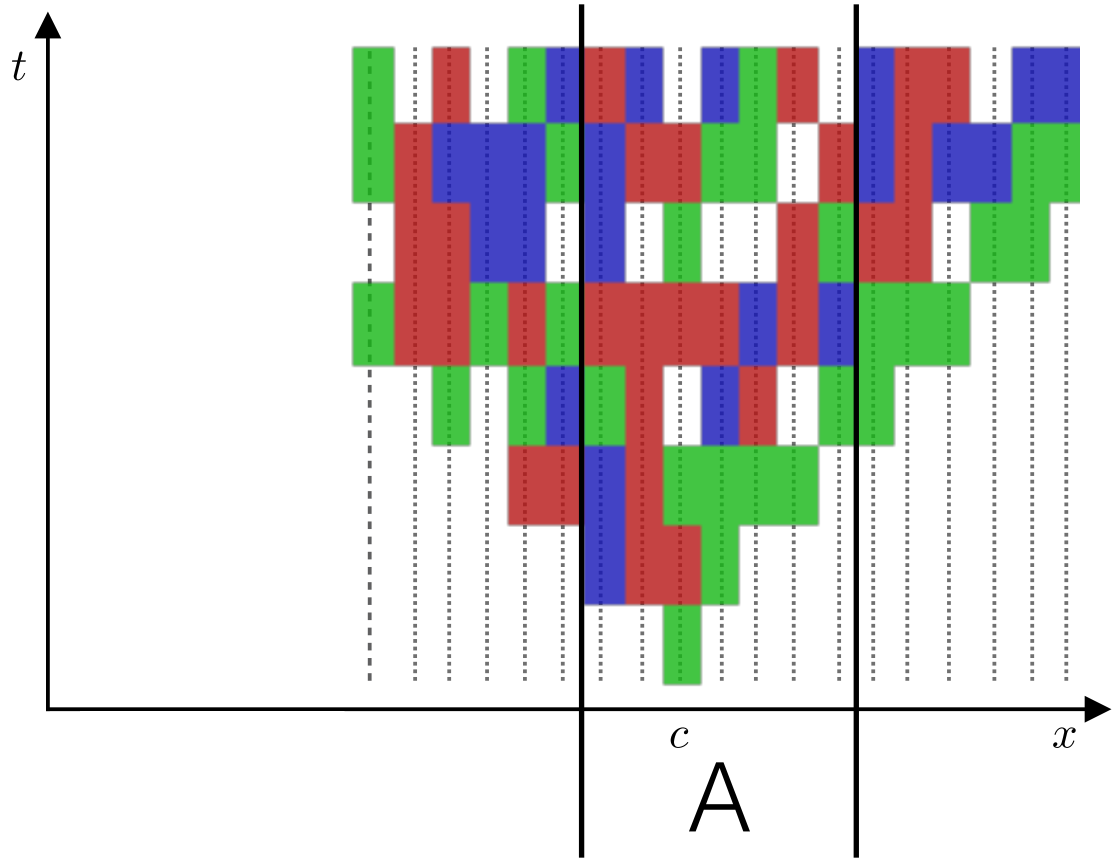
$$|\downarrow\rangle \quad |\downarrow\rangle$$

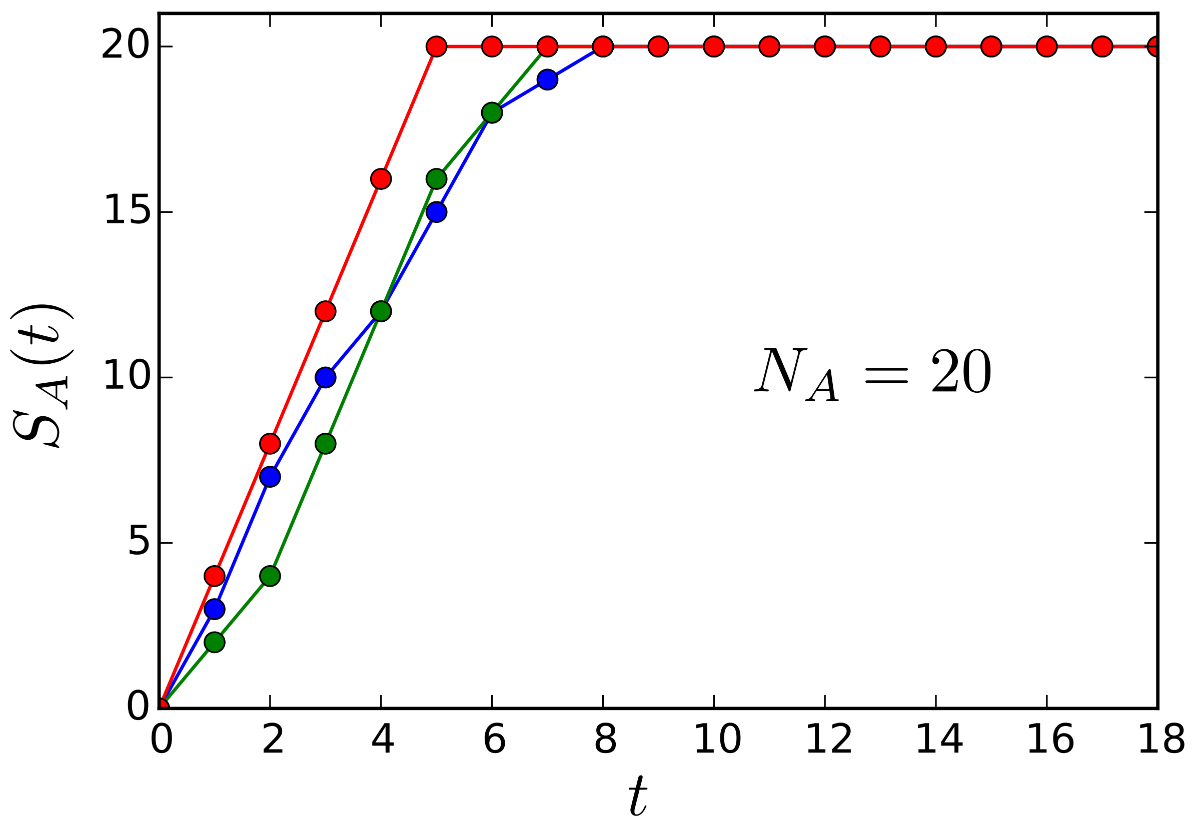
$$1Z \to ZZ$$
$$1X \to X1$$
$$Z1 \to XX$$
$$X1 \to 1Z$$

Representation of evolution



Ergodic

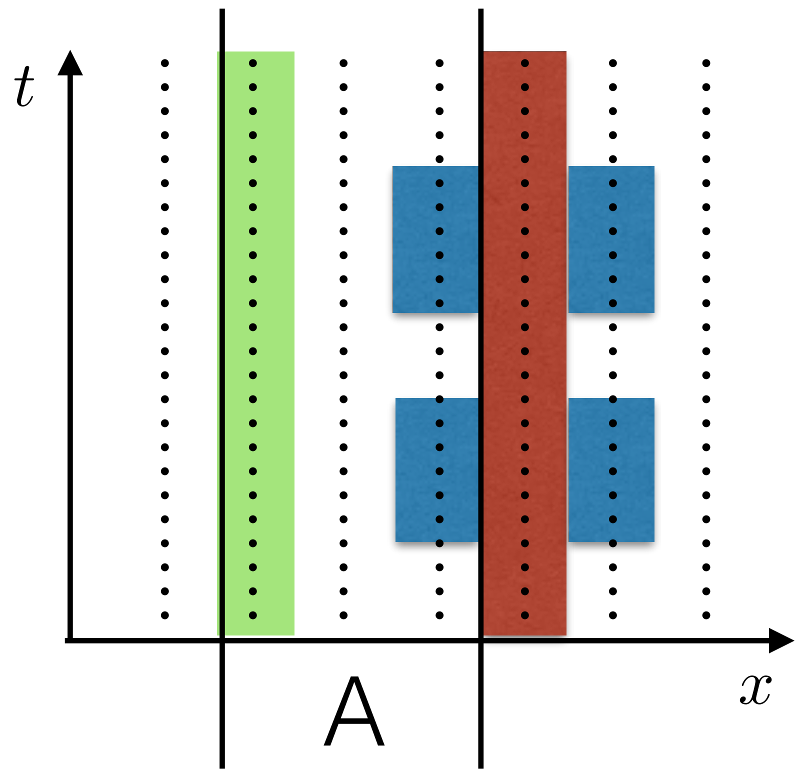


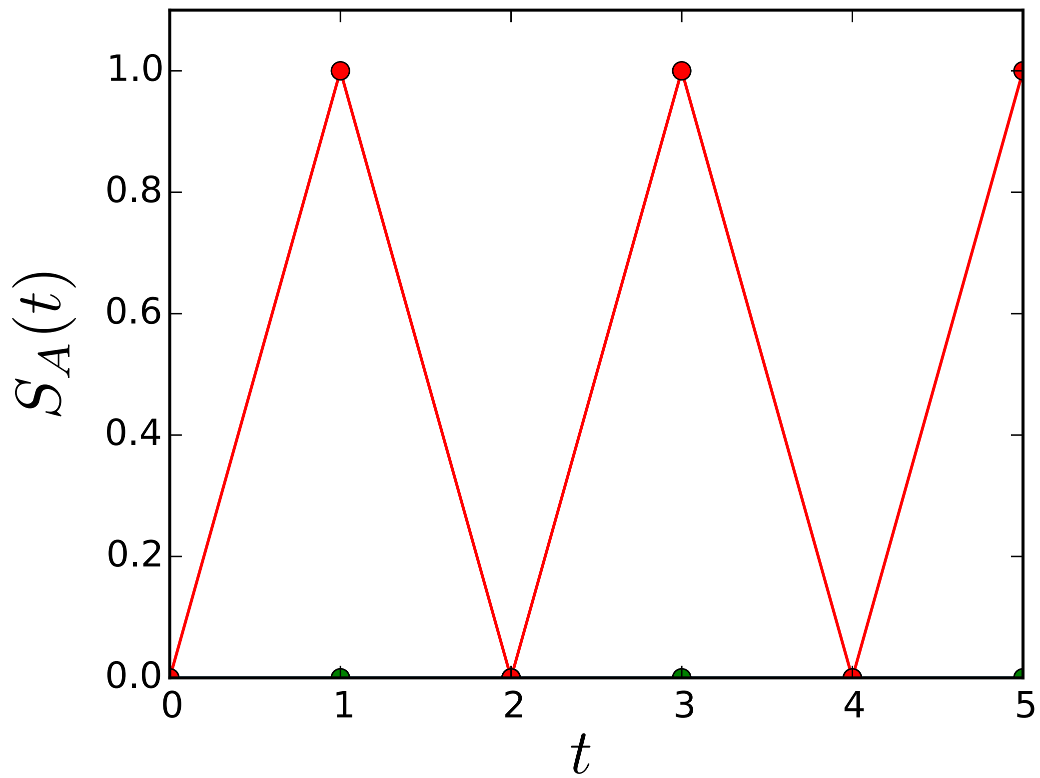


$$\rho_A = 1 \text{ for } t > vN_A$$

Simulable system that thermalizes!

Localized

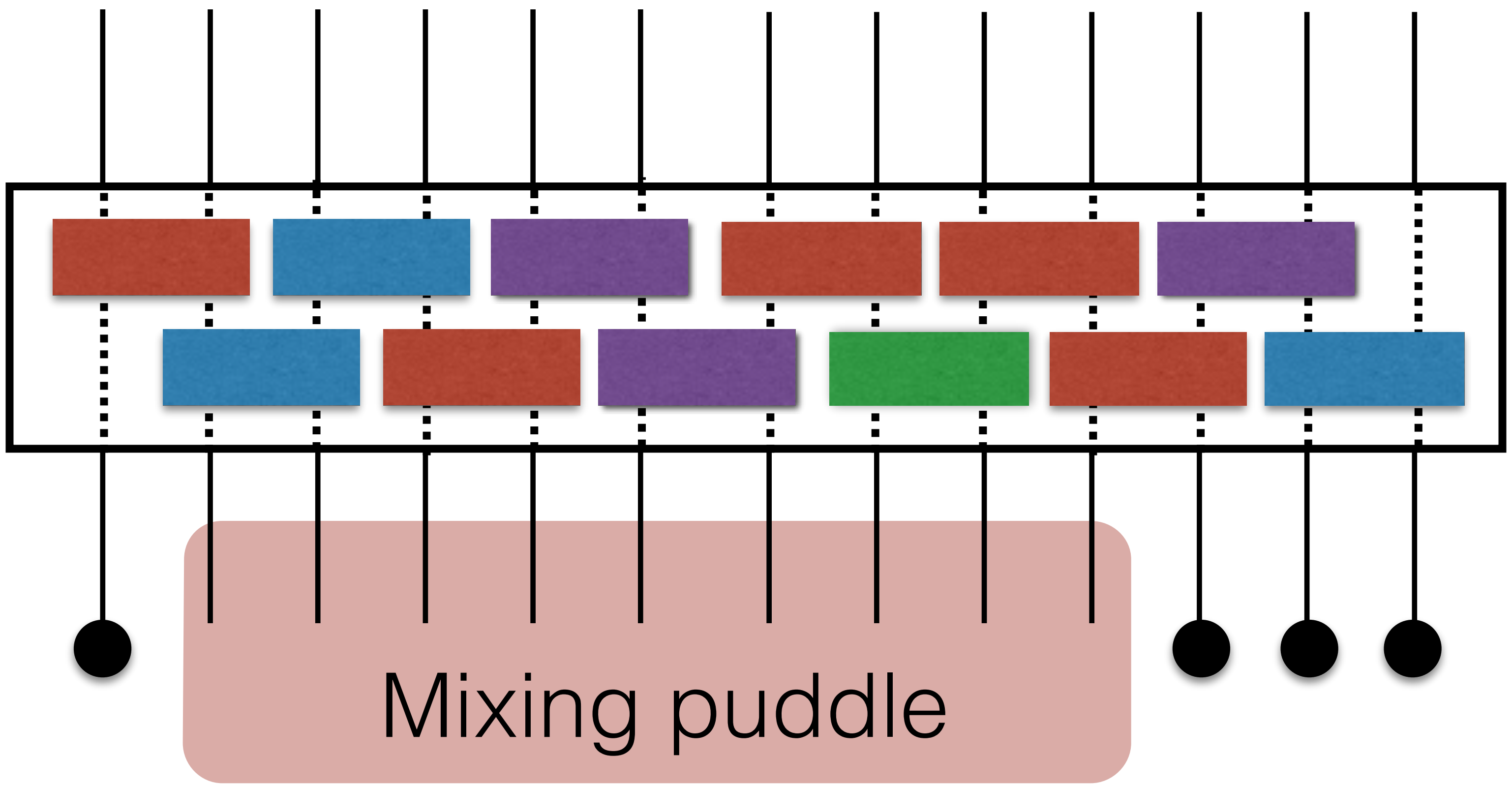




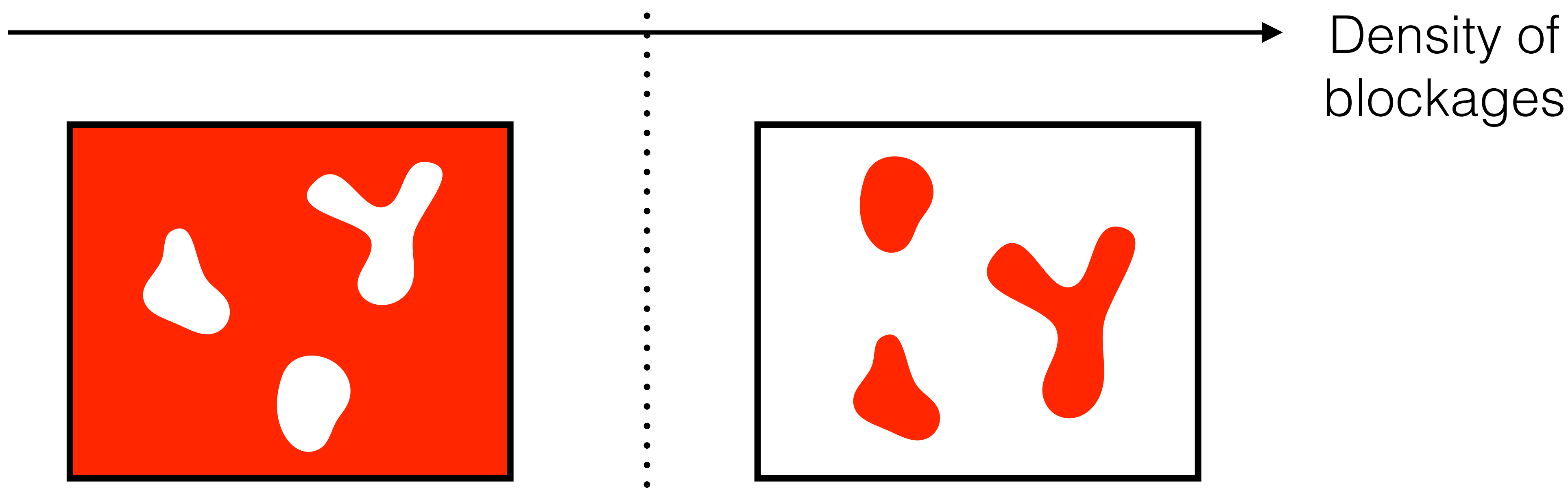
- Strictly local integrals of motion: Z_i
- Block spread of entanglement (blockages)

A.C., J. Carrasquilla, I. H. Kim, D. A. Abanin, G. Vidal (2014)

Locally distinguishable



Percolation of entanglement



Delocalized

Volume law

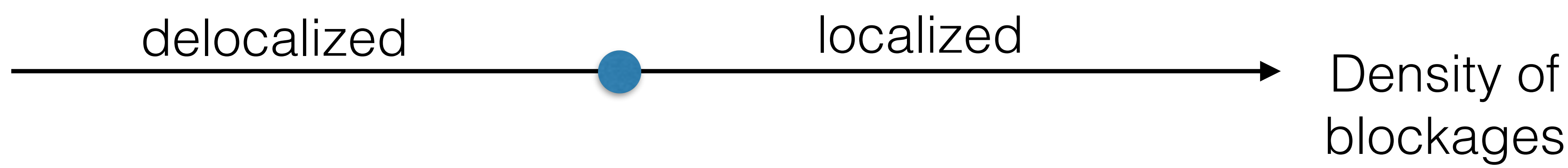
Partial set of
local integrals of motion

Localized

Area law

Complete set of
local integrals of motion

Nature of transition



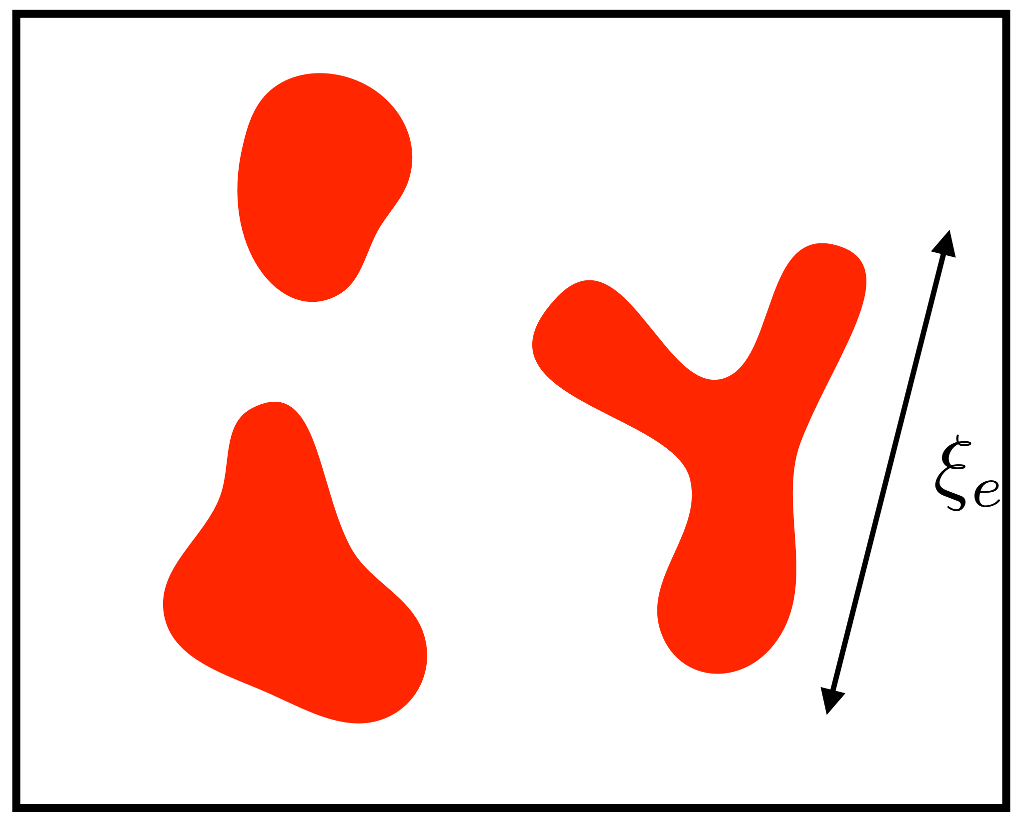
- Continuous
- Classical percolation universality class
  - E.g. $\nu=4/3$ in $d=2$, $\nu\cong0.88$ in $d=3$ etc.
- Entanglement entropy obeys “fractal law”

$$S_A \sim N_A^{d_f/d}$$

Grover (2014)

Perturbing away

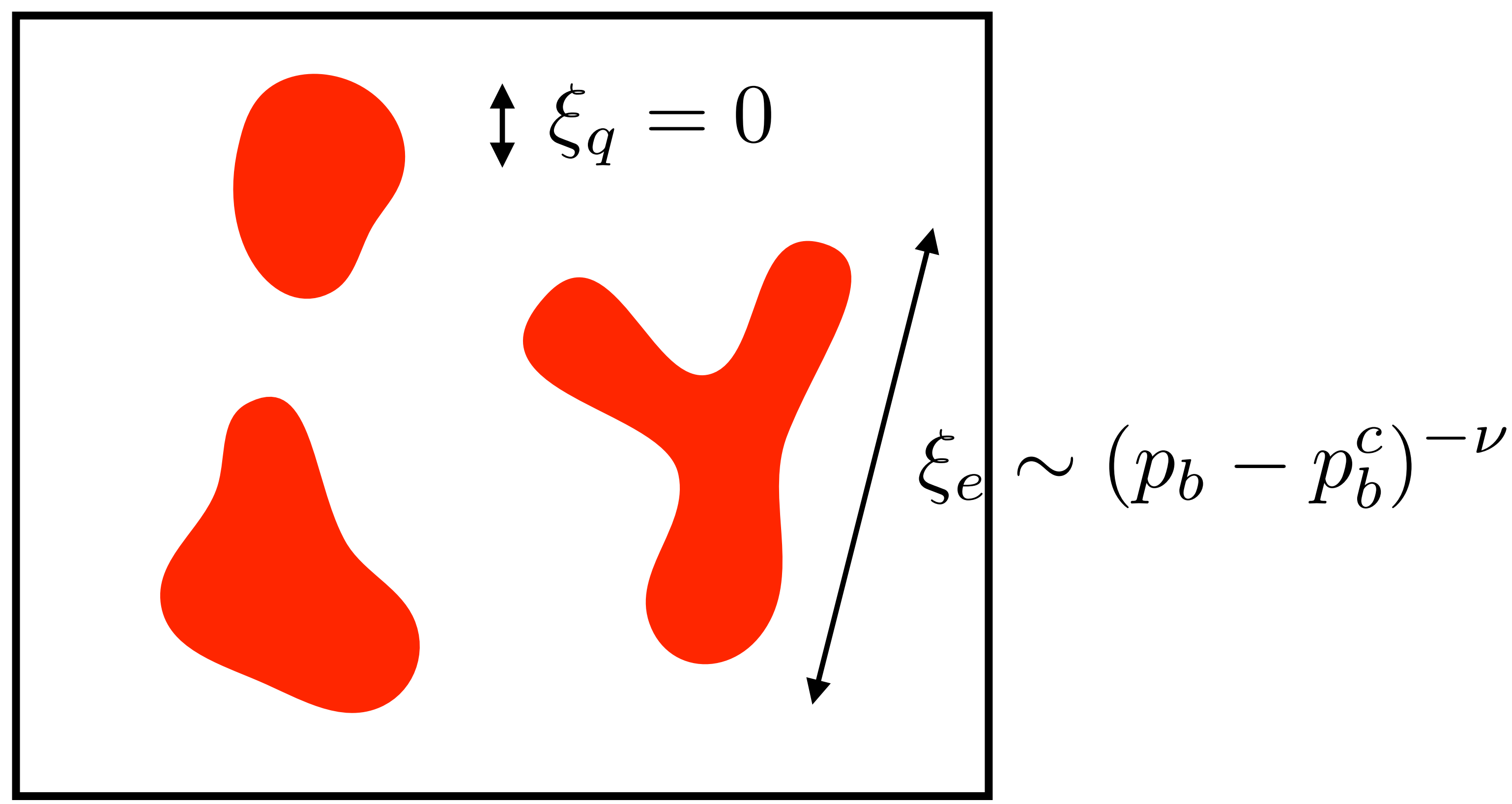
- ☑ Ergodic regions



$$\xi_e \sim (p_b - p_b^c)^{-\nu}$$

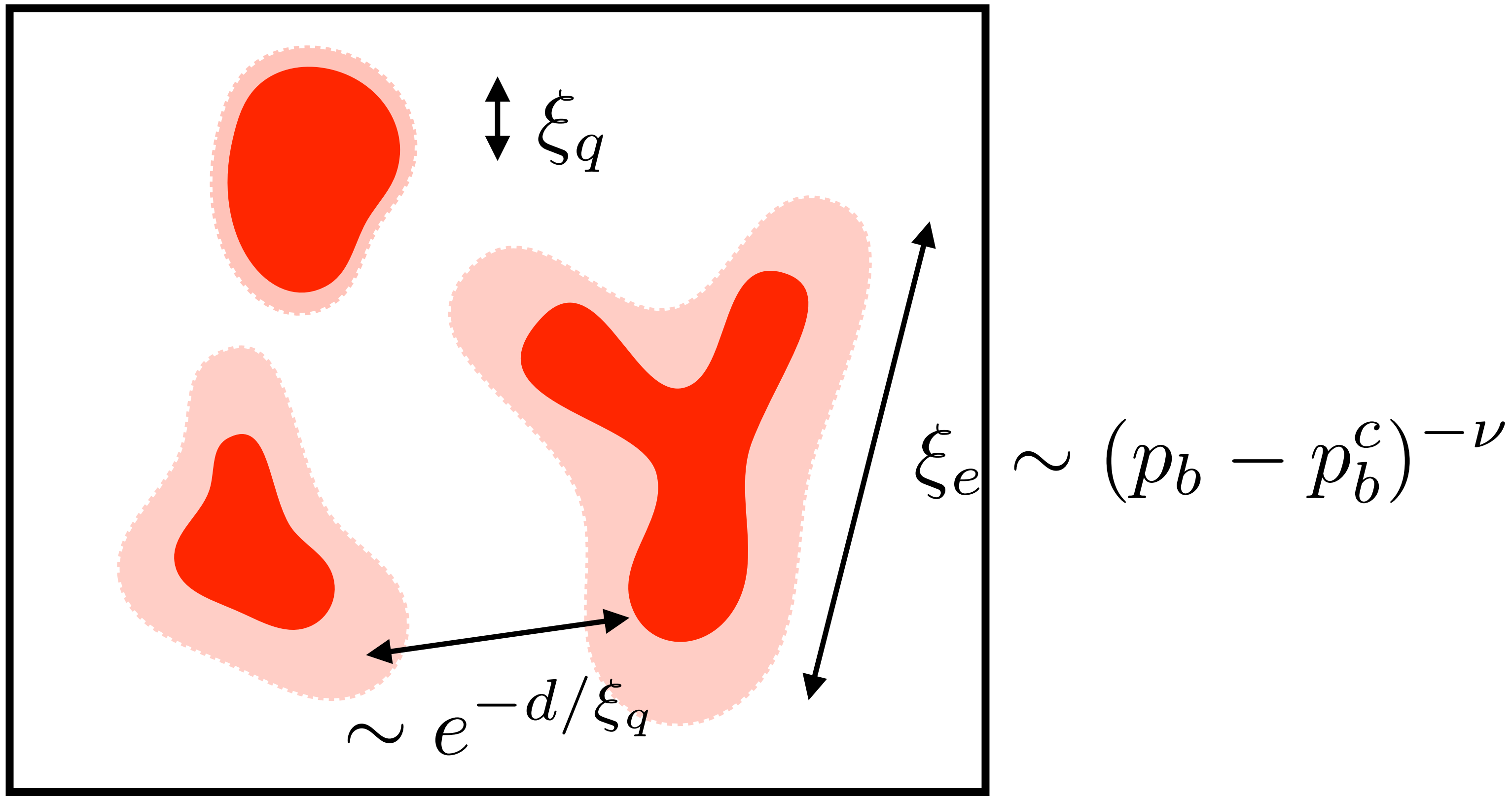
Perturbing away

- ☑ Ergodic regions
- ☒ Generic localized regions



Perturbing away

Microscopically, add small rotations about X axis



Perturbing away

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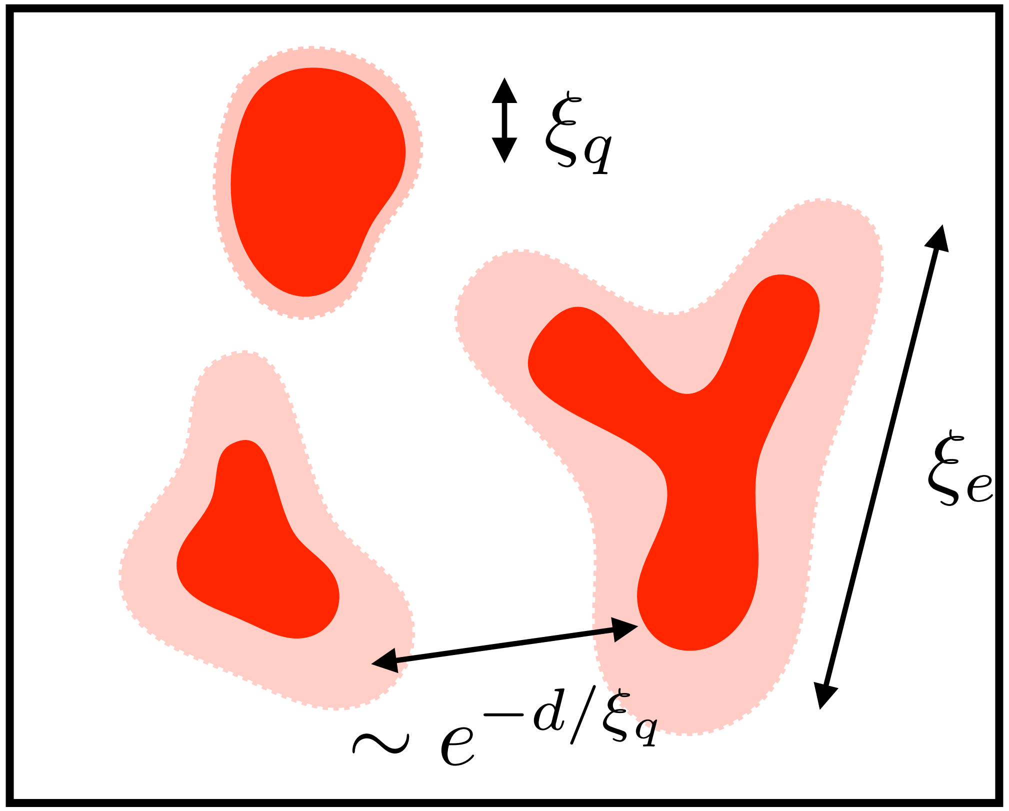
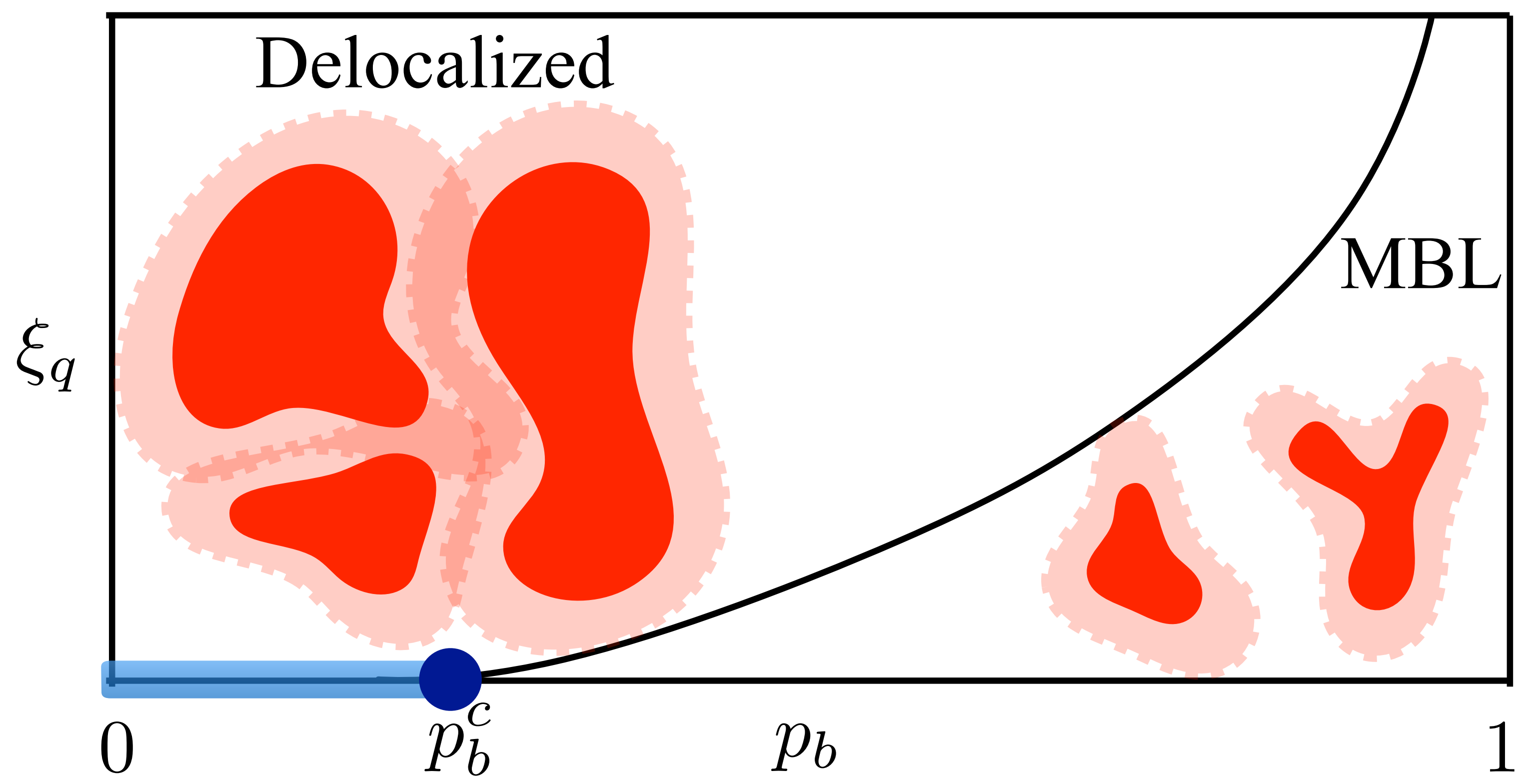


Level spacing on each puddle $\sim e^{-d/\xi_q}$

Estimating phase diagram



- $\xi_q > 0$: delocalized phase is ergodic
- MBL phase: stable and generic (numerics in d=1)
- Conjecture: $\nu \geq \nu_{perc}$

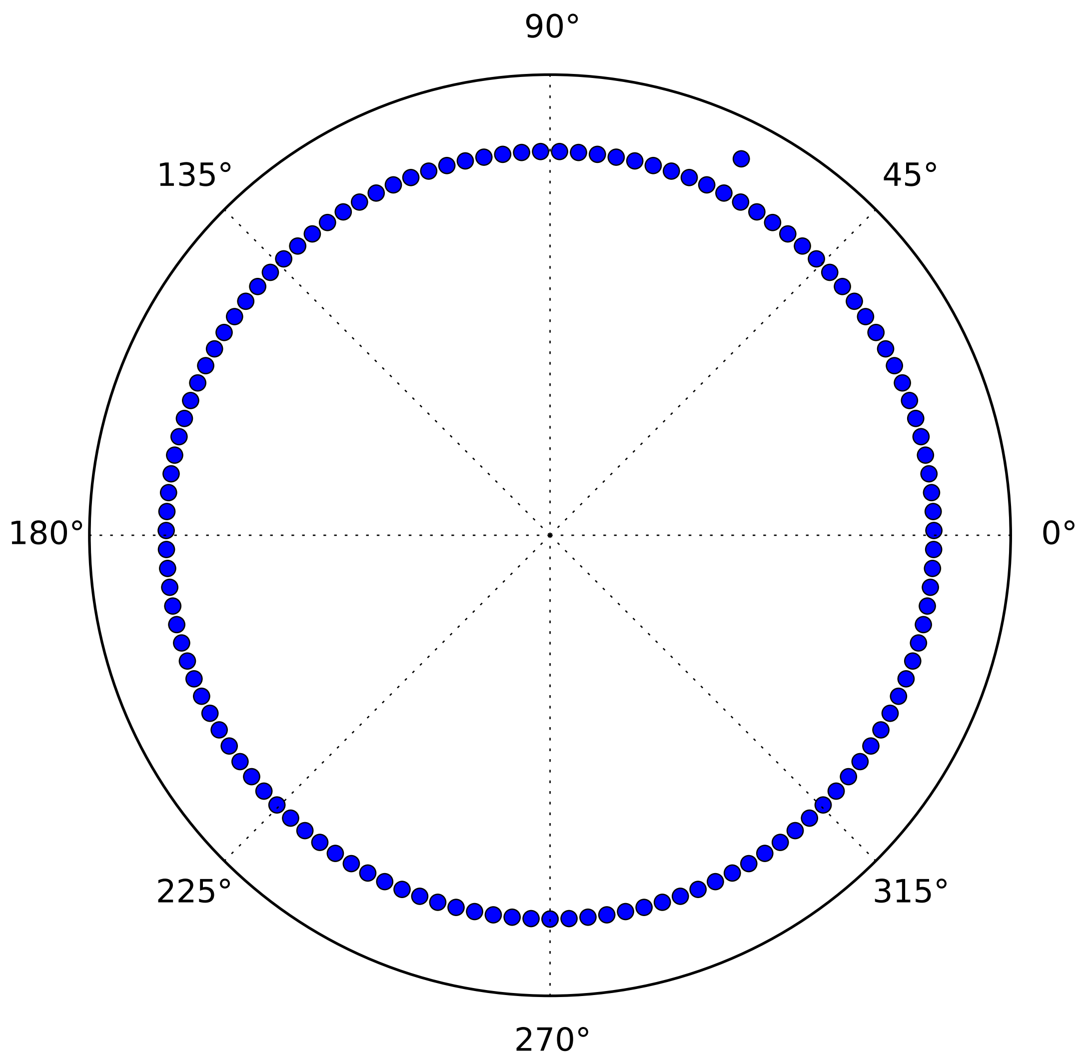
Thermalization

- Classical simulable but quantum ergodic?
- Evidence
  - Analytic proof for $S_A(\infty) = N_A$ in d=1 in translationally invariant circuits + initial stabilizer states
  - Eigenstate structure (ETH)
  - Level repulsion

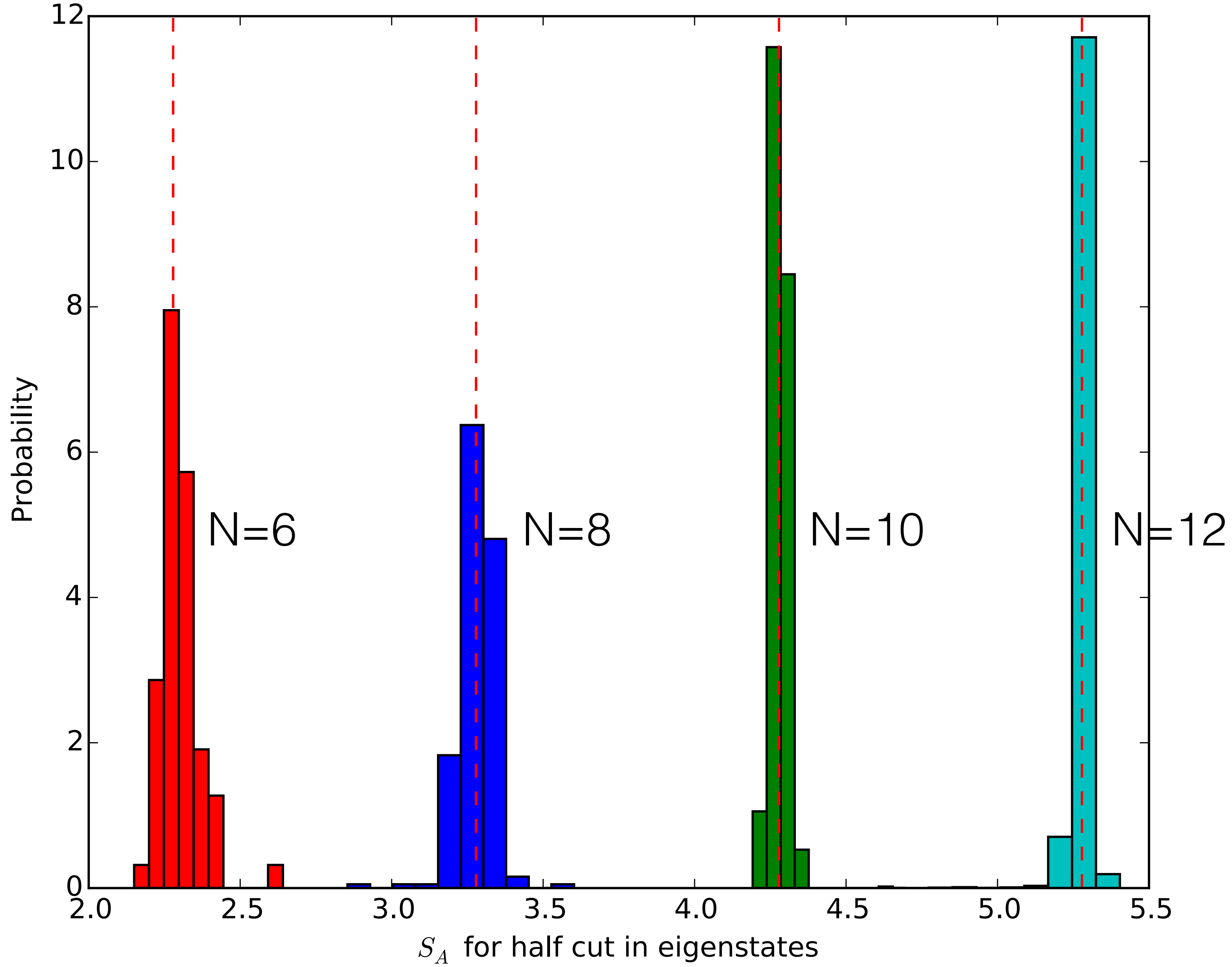
J. Gütschow, S. Uphoff, R. F. Werner, Z. Zimborás (2010)

Spectral statistics

- Clifford group discrete ⇒ $U^M = 1$
- $M \sim \exp(N)$
- Degeneracies don’t scale with N



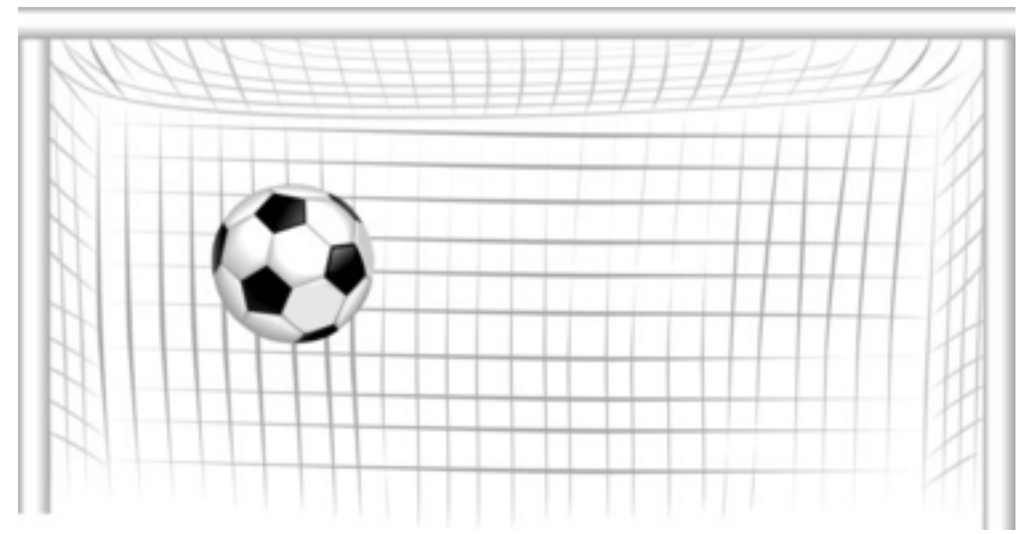
Entanglement entropy of eigenstates



Eigenstates obey Page conjecture: $S_{N/2} = \frac{N}{2} - \frac{1}{2\ln 2}$

Conclusions

- Analytically tractable
- Works in d>1
- Delocalized, localized, transition



- Learn more about thermalization?
- Network model?