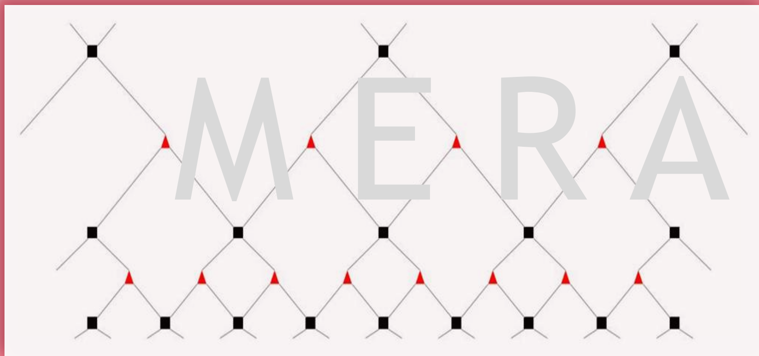
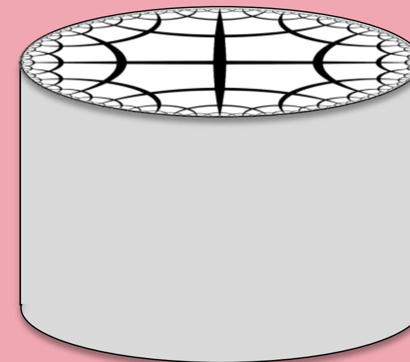
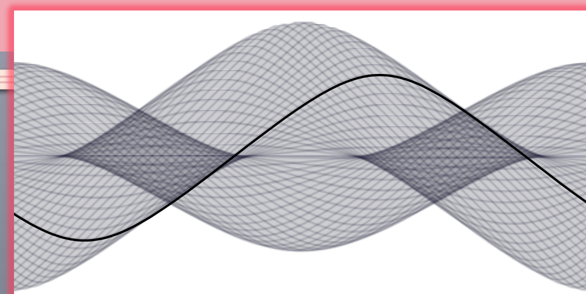
Tensor Networks ⟶ Holography ?

via ⟶ Integral Geometry







Bartłomiej Czech

with Lampros Lamprou, Sam McCandlish, James Sully

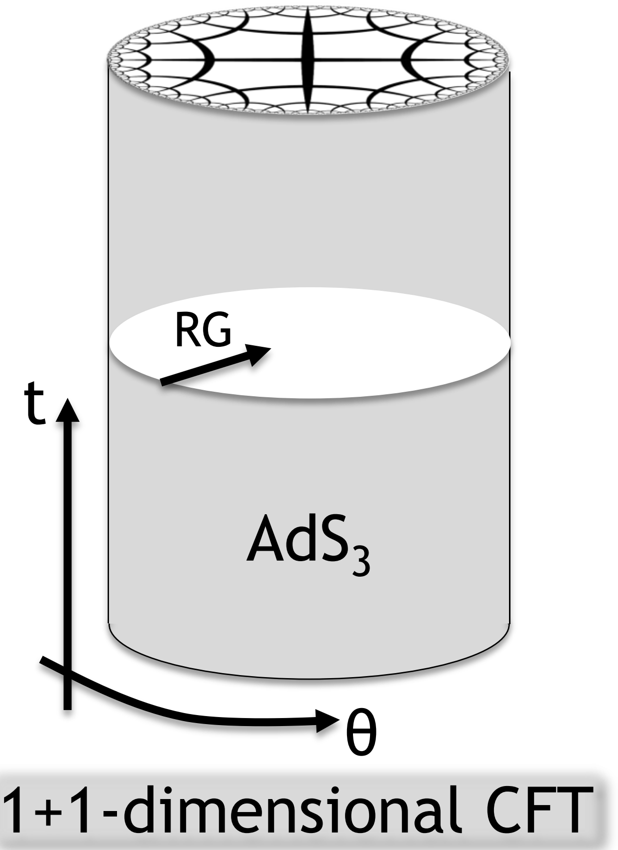
Stanford University

Closing the Entanglement Gap, KITP, 3 June 2015

Why should tensor networks relate to holography?

What is holographic duality?

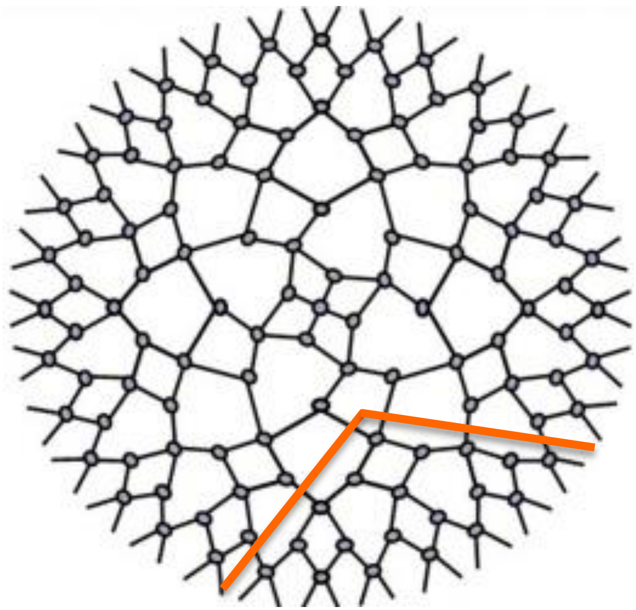
- an auxiliary representation of a conformal field theory as a gravity theory
- which makes scale transformations (RG) manifest
- by introducing an additional dimension



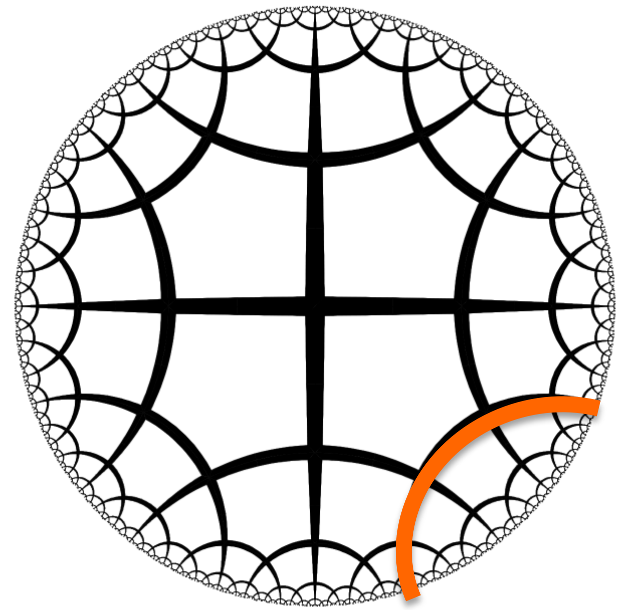
Sounds just like tensor networks!

Two descriptions of a state

MERA network



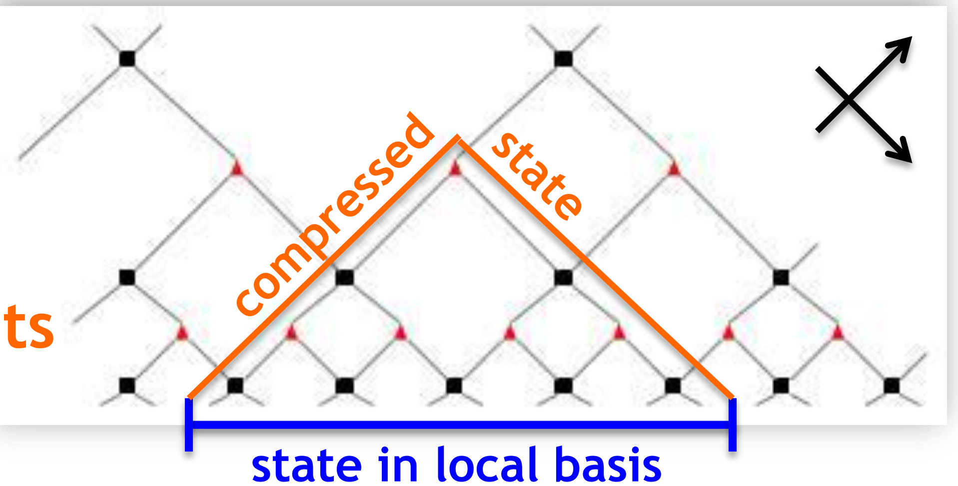
spatial slice
of holographic geometry



- an additional dimension
- which encodes RG transformations
- entropies represented by **minimal cuts** ↔ **geodesics**
  (Ryu-Takayanagi, 2006)

Swingle, 2009

Goal: Quantify this relation



Ingredients:

- the causal structure of MERA
- the minimal cut prescription
- entanglement entropy ⟷ #cuts

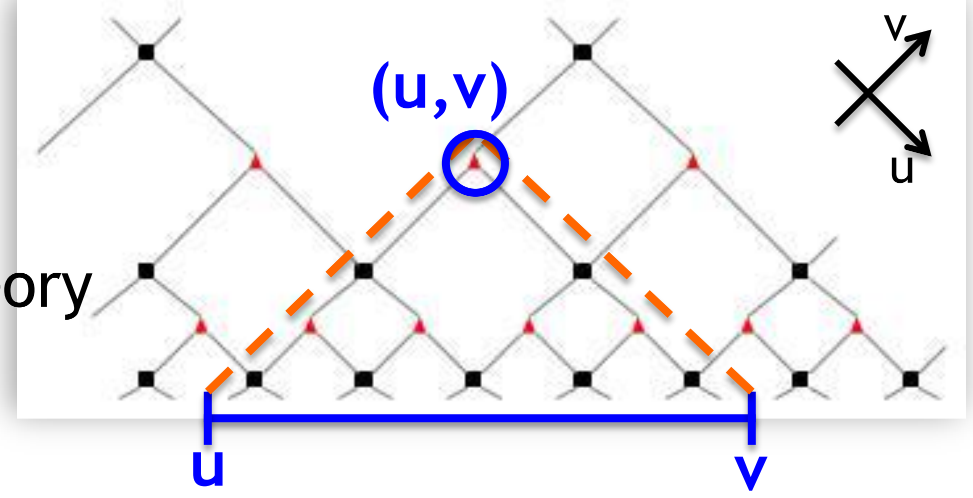
Strategy:

- construct a **metric** that captures this structure of MERA
- relate that **metric** to the holographic geometry

Kinematic Metric

Choosing a metric: causal structure

- identify null coordinates
- point **(u,v)** sits on top of a unique minimal cut,
- which identifies a field theory interval **(u,v)**
- the isometry at **(u,v)** completes the compression of the state on interval **(u,v)**

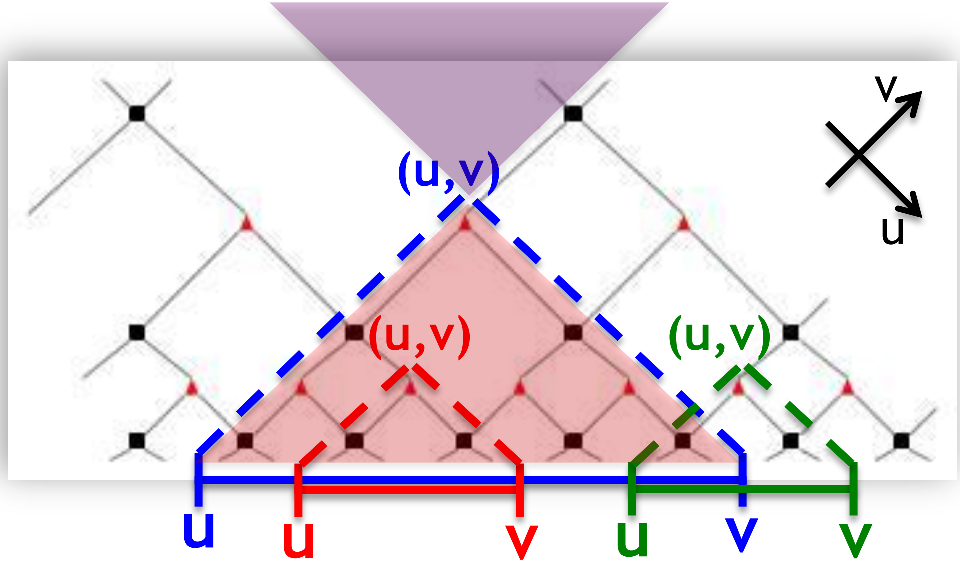


Kinematic Metric:

$$ds^2 = (\ldots)\, du dv$$

Why is the metric Lorentzian?

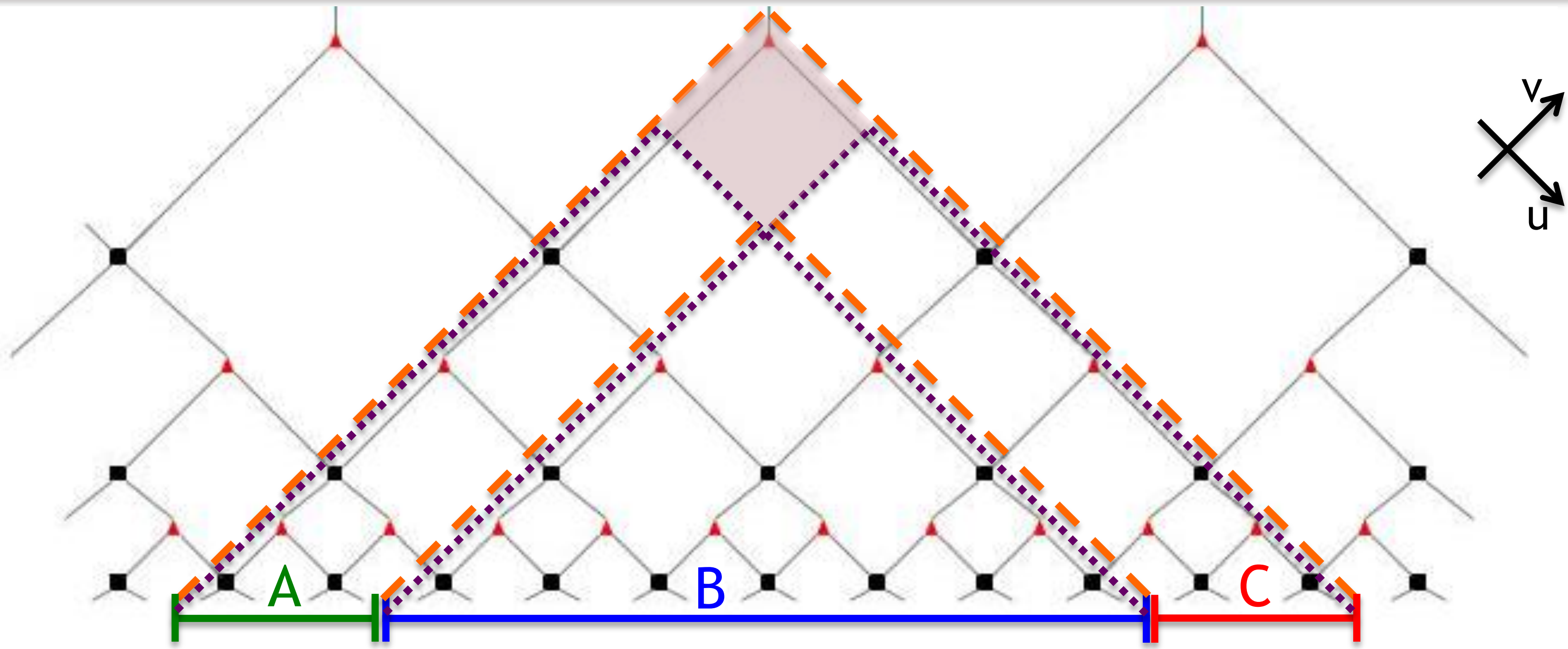
- timelike separated (u,v): (u,v) contains (u,v)
- spacelike separated (u,v): neither contains the other
- null separated: common endpoint left (u = u) or right (v = v)



- Past: all intervals contained in (u,v)
- Future: all intervals containing (u,v)

Kinematic Metric: $ds^2 = (...)\, du dv$

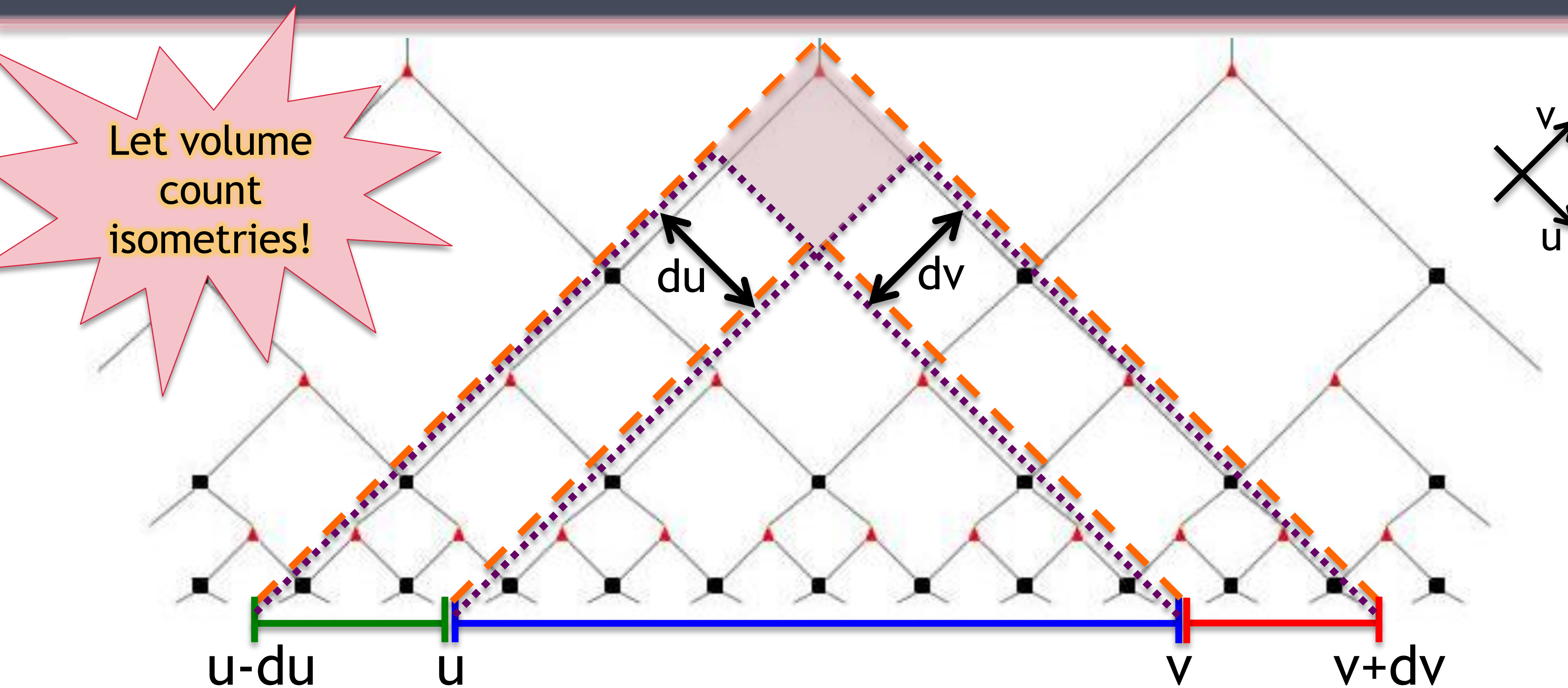
Conditional mutual information



$$I(A,C|B) = S(AB) + S(BC) - S(ABC) - S(B)$$

- strong subadditivity of entropy: $I(A,C|B) \geq 0$ → $\#(\Delta) \geq 0$
- because of cancellations, this quantity localizes in the network
- it counts the number of isometries in a causal diamond

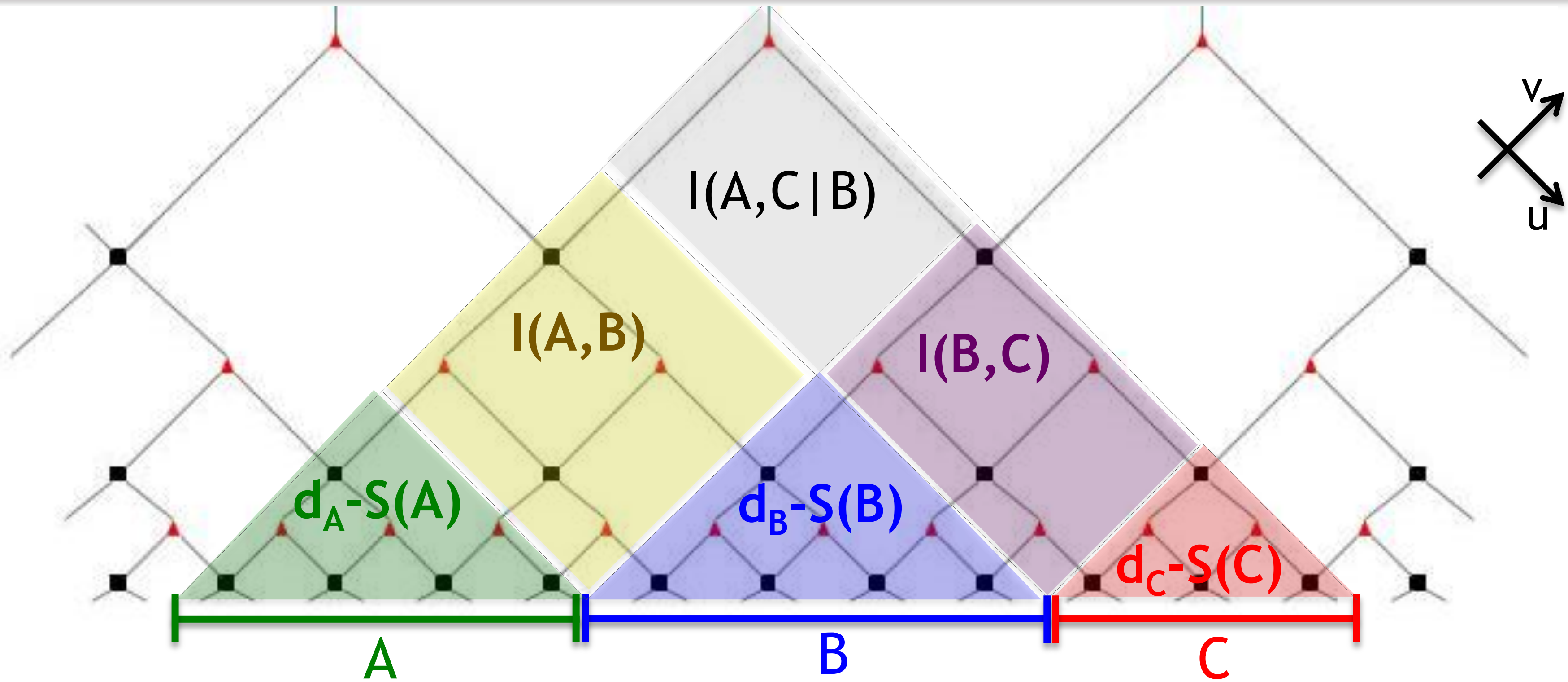
Choosing a metric: volume element



I(A,C|B) = S(AB) + S(BC) - S(ABC) - S(B) → $\dfrac{\partial^2 S(u,v)}{\partial u\,\partial v}$

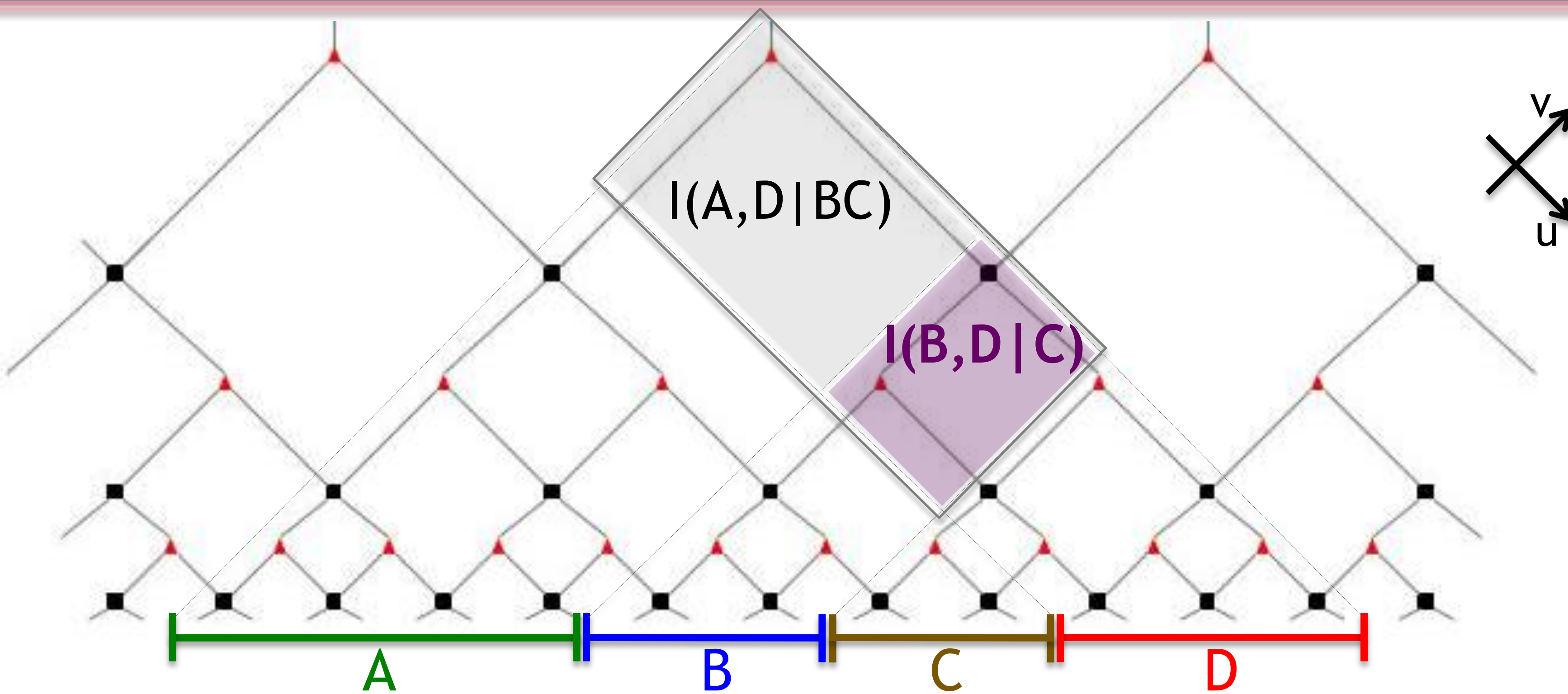
Kinematic Metric: $ds^2 = \dfrac{\partial^2 S_{\text{ent}}(\ldots)}{\partial u\,\partial v}\,du\,dv$

Structure of kinematic space



- Causal diamonds are conditional mutual informations
- Diamonds that extend all the way to the bottom are mutual informations
- Past causal diamonds of kinematic points characterize the isometric embedding of a compressed state in the Hilbert space

Conditional mutual information as volume

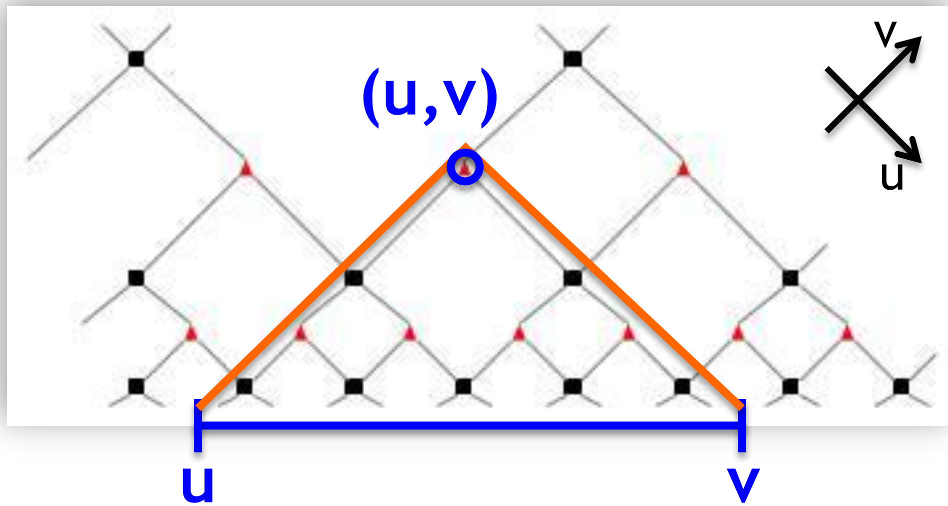


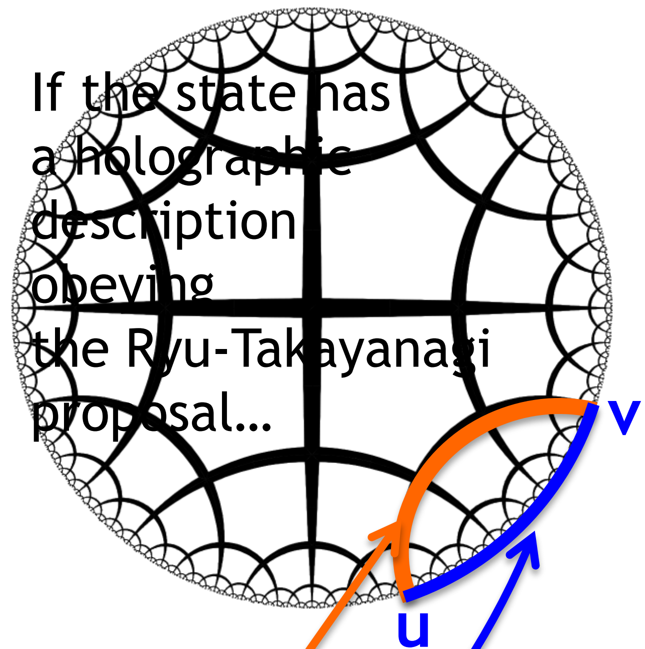
- Because volume is additive, consistency requires:

$$I(A,D|BC) + I(B,D|C) = I(AB,D|C)$$

- This is an identity: the chain rule for conditional mutual information

Kinematic Space ⟶ Holographic Geometry





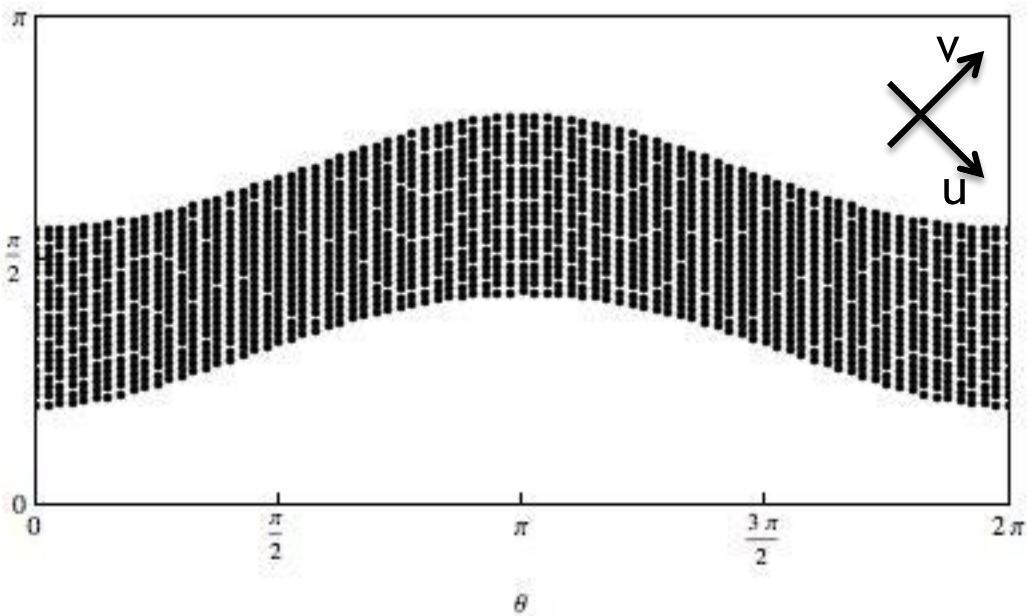
$$ds^2 = \frac{\partial^2 S_{\text{ent}}}{\partial u\, \partial v}\, du dv$$

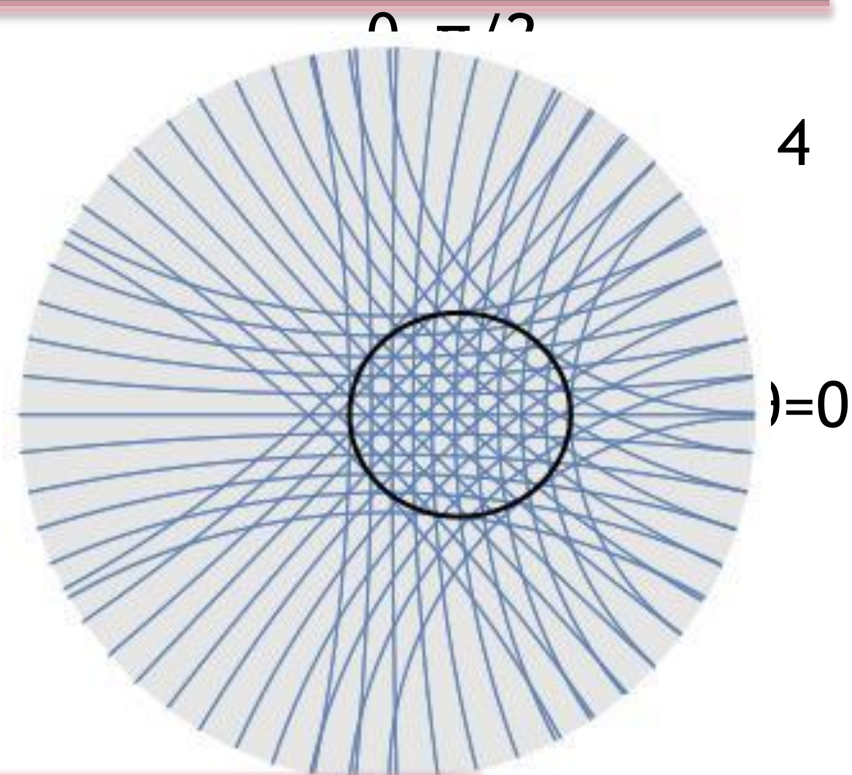
Point **(u,v)** uniquely selects:

- **a minimal cut** ⟷ **an oriented geodesic**
  (dropped along the causal cone)
- a boundary interval **(u,v)**

Kinematic Space is the space of
oriented geodesics ⟷ boundary intervals!

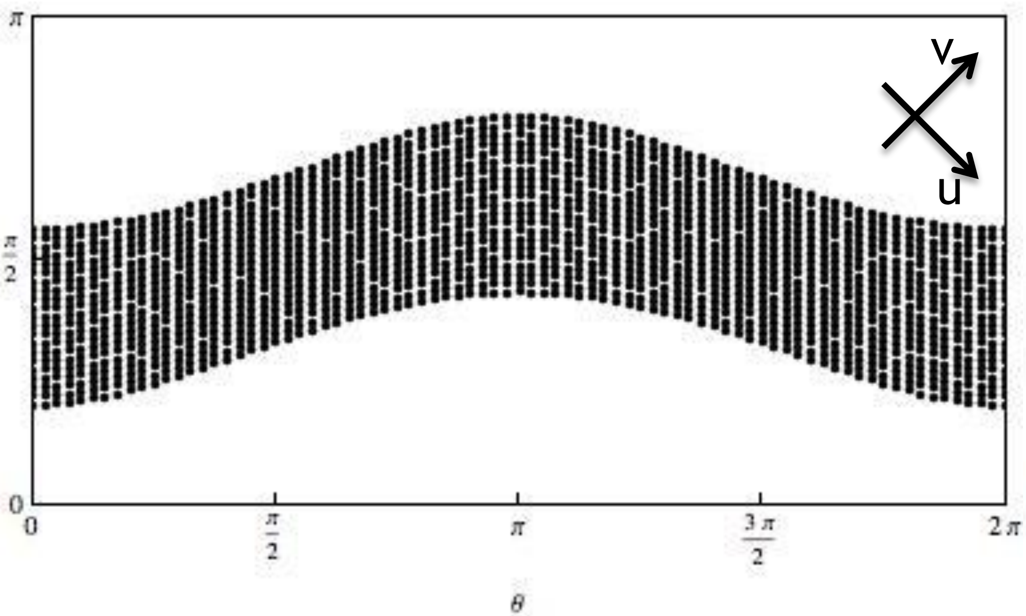
Convex curves are kinematic regions

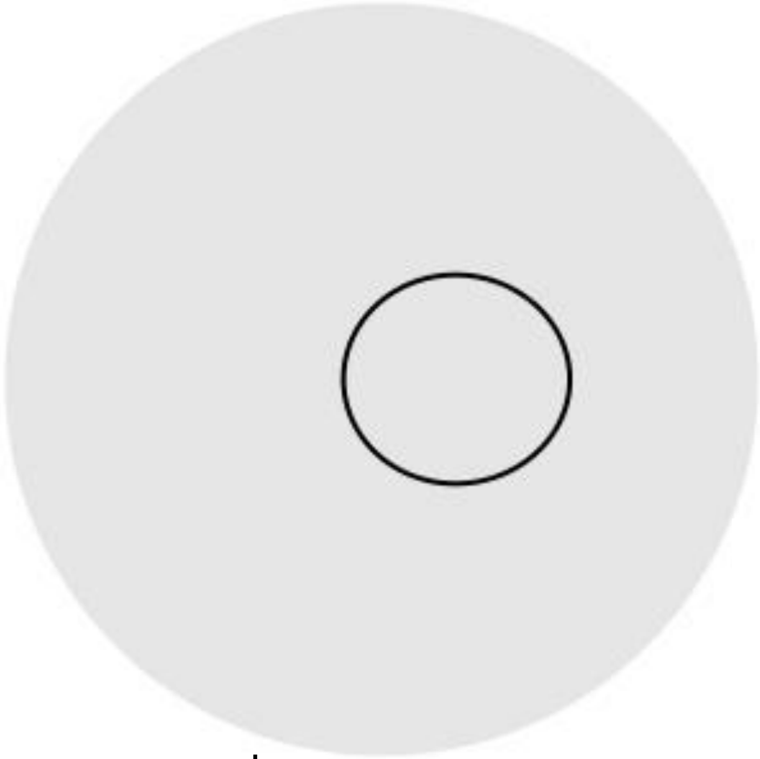




$$\frac{\text{circumference}}{4G} = \int_{\text{intersect}} \frac{\partial^2 S(u,v)}{\partial u\,\partial v}\,du\,dv \;\; = \text{volume in kinematic space}$$

Reading a curve from a region

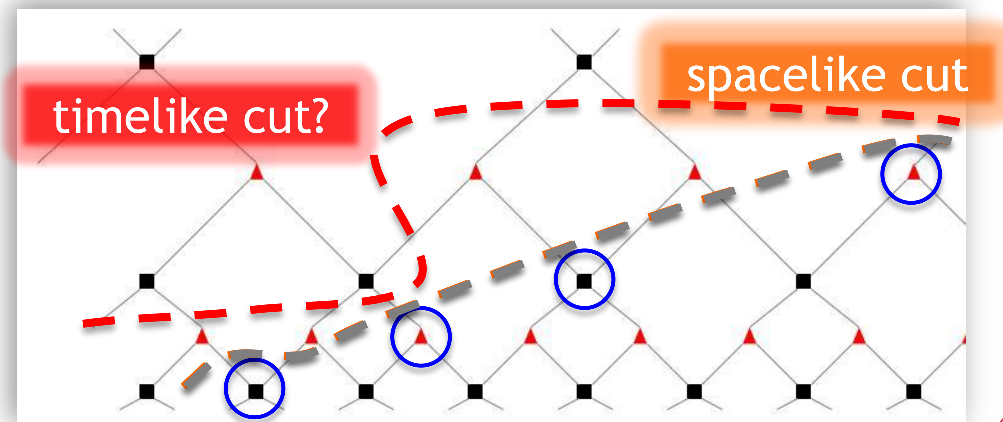


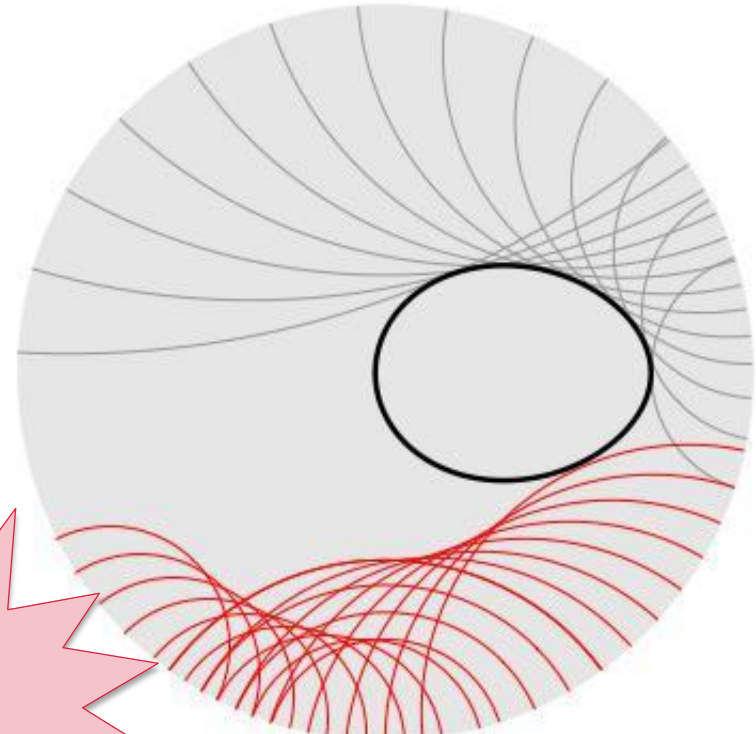


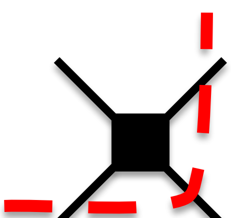
$$\frac{\text{circumference}}{4G} = \int_{\text{intersect}} \frac{\partial^2 S(u,v)}{\partial u\, \partial v}\, du\, dv \;=\; -\int du\, \frac{\partial S(u,v)}{\partial u}\bigg|_{v=v(u)}$$

works in every geometry
that obeys the Ryu-Takayanagi proposal

Why only spacelike cuts are allowed



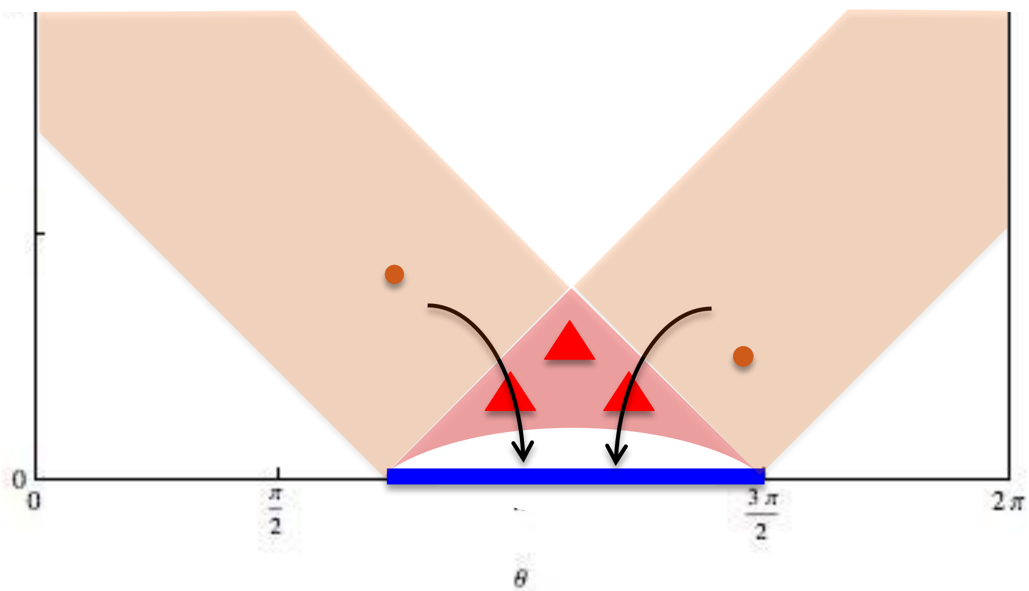


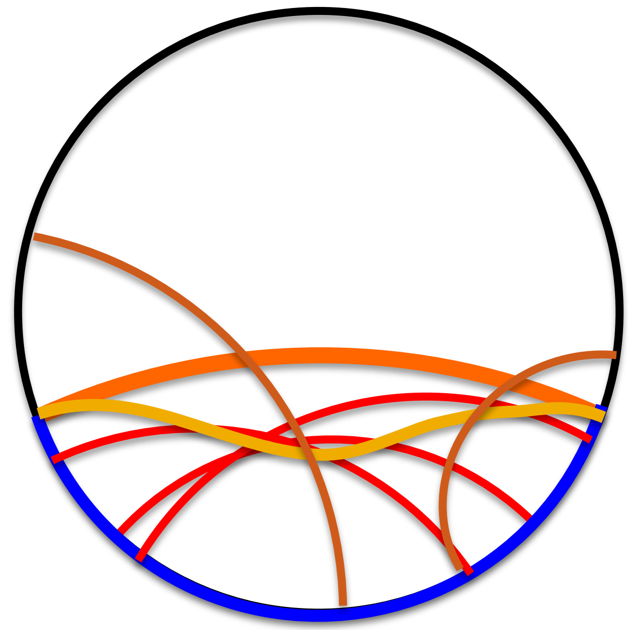


- Timelike cuts do not effect coarse-graining
- They introduce spurious degrees of freedom

- Timelike cuts fail to define bulk curves

Curves with common endpoints





- length = volume of intersecting geodesics
- every geodesic intersecting orange also intersects yellow
- kinematic volume ≤ kinematic volume ⟷ orange curve is a geodesic
- length - length = **volume** of geodesics that intersect yellow but not orange

length - length = # isometries in this isometric embedding of states

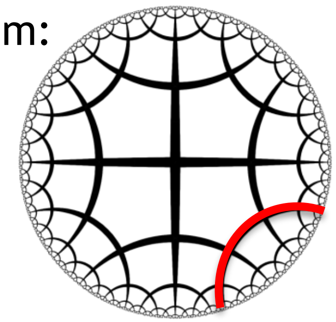
Kinematic space of vacuum is de Sitter

$$ds^2 = \frac{\partial^2 S_{\rm ent}}{\partial u\, \partial v}\, du dv$$

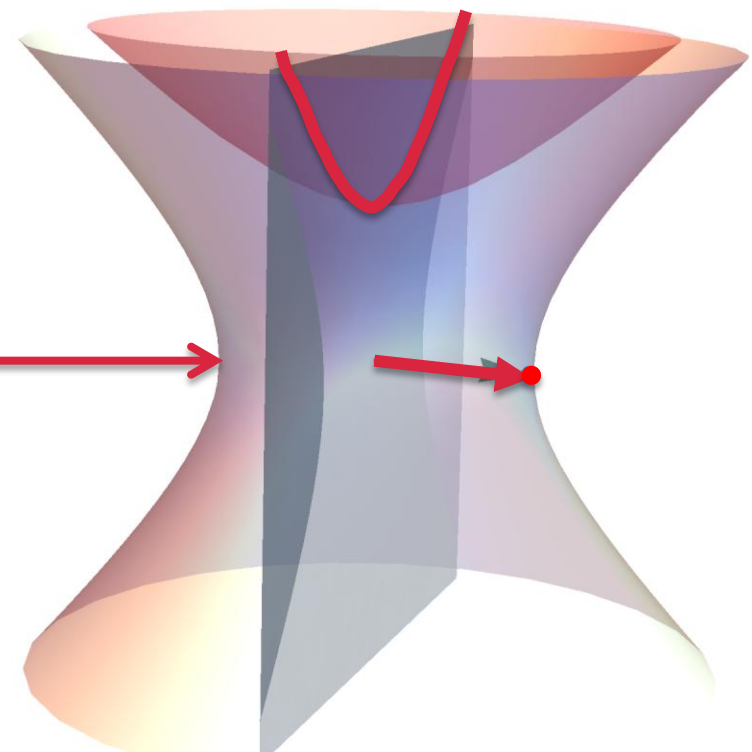
Plug in: $S_{\rm ent} = \frac{c}{3} \log \frac{\sin(v-u)/2}{\mu}$

$$ds^2 = \frac{c}{12} \frac{du\, dv}{\sin^2(v-u)/2} \quad \to \quad \frac{c}{12} \cdot \frac{-dt^2 + d\theta^2}{\sin^2 t}$$

dual to vacuum:



de Sitter geometry



We show that MERA corresponds to a discretization of de Sitter space.
Beny, 2011

Summary

- A quantitative connection between tensor networks (MERA) and a holographic geometry, mediated by integral geometry

<u>Assumptions:</u>

- Geometry: the Ryu-Takayanagi proposal
- Tensor networks: #cuts in MERA → EE

Not just the vacuum or thermal state

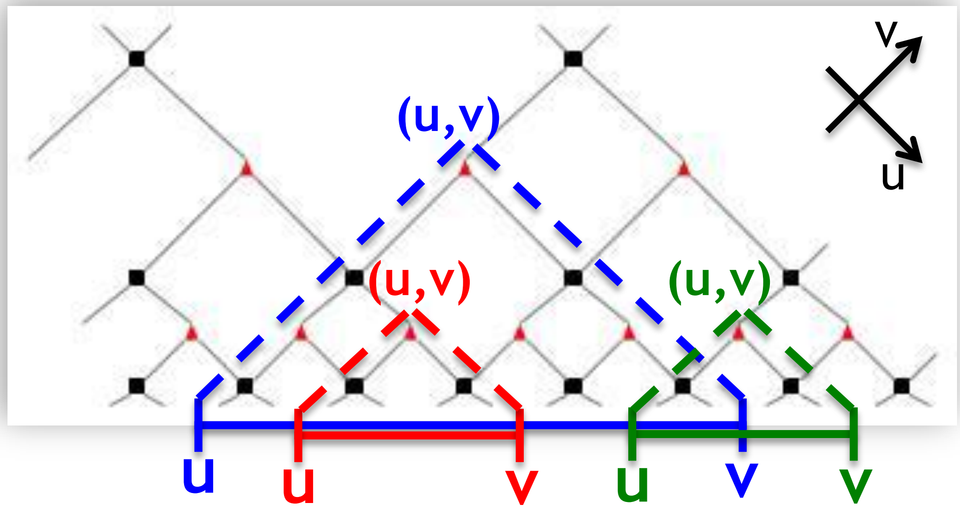
Does holography (large N) imply that we can construct a MERA-like tensor network with this property?

Questions

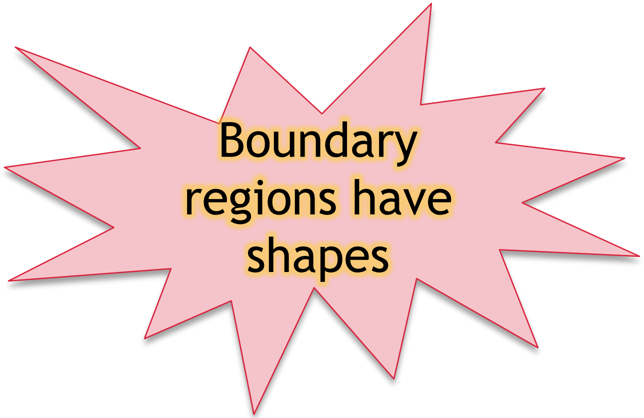
- Understand the kinematic space directly in the language of the path integral (cf. Tensor Network Renormalization)
- Understand the action of the conformal group on kinematic space
- Kinematic space versus cMERA
  (Haegeman, Osborne, Verschelde, Verstraete, 2011)

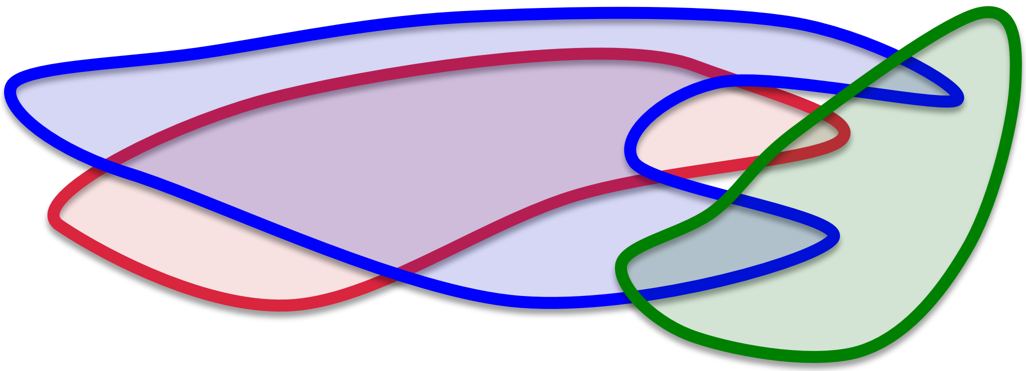
Kinematic Space in higher dimensions?

- 1+1-dimensional CFT:



- d+1-dimensional CFT:





My collaborators:

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arXiv:1505.05515
arXiv:1506.0xxxx

THANK YOU!