A New Perspective on Holographic Entanglement

Matthew Headrick
Brandeis University

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Closing the Entanglement Gap
KITP

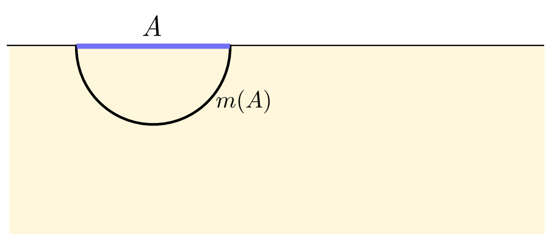
(work in progress with Michael Freedman)

Holographic mutual information: Review

Ryu-Takayanagi ['06] formula:

$$S(A) = \min_{m \sim A} \text{area}(m)$$

$m(A) :=$ minimizer
(in this talk $4G_N = \ln 2 = 1$)



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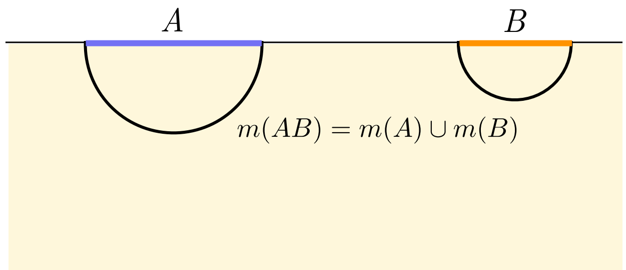
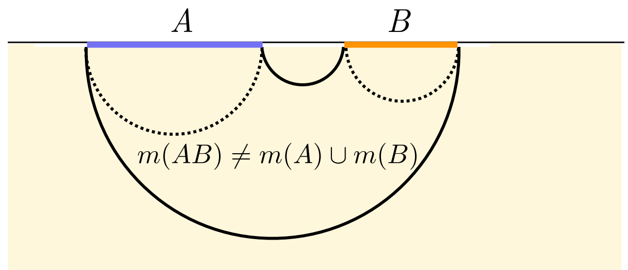
Mutual information: $I(A:B) := S(A) + S(B) - S(AB)$
Measures total amount of correlation between A & B





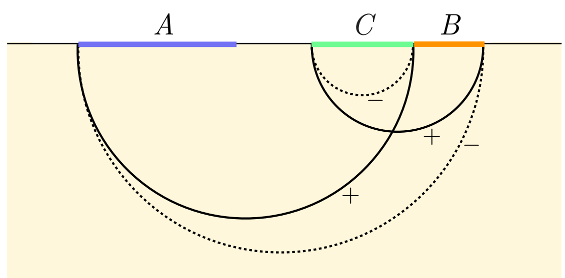
Properties:

1. Subadditivity: $I(A:B) \geq 0$

2. Strong subadditivity [Headrick-Takayanagi '07]:

$$I(A:B|C) := I(A:BC) - I(A:C) \geq 0$$

(conditional MI)

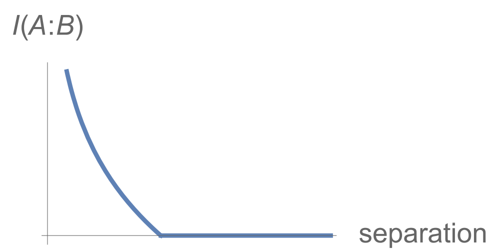


3. Monogamy [Hayden-Headrick-Maloney '11]:

$$I_3(A:B:C) := I(A:B) + I(A:C) - I(A:BC) \leq 0$$

(tripartite information)

4. "Phase transitions" [Headrick '10]

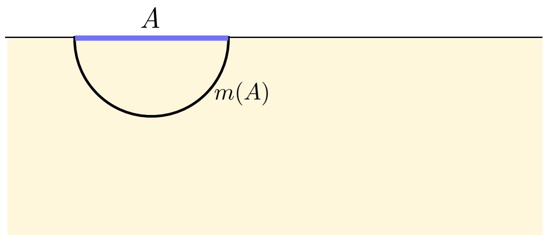


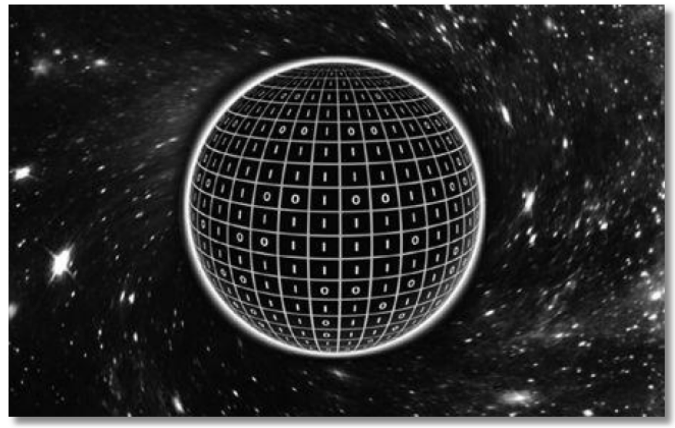
Note: 1 & 2 are general properties of MI, while 3 & 4 are special properties of RT

Interpretation?

Geometry of bulk encodes state of field theory
RT tells us something about that encoding

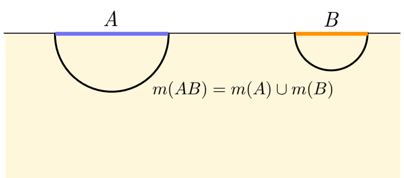
Do microstate bits of ρ_A "live" on $m(A)$?

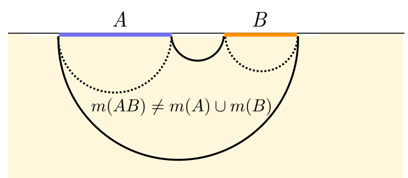


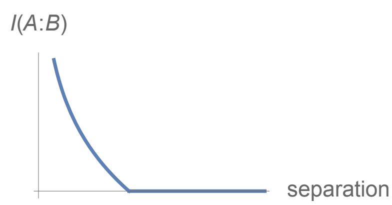


Questions:

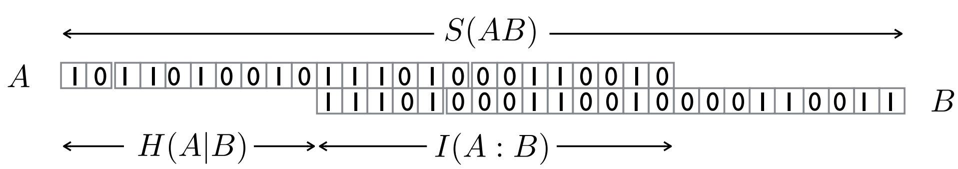
- Why does $m(AB)$ jump at phase transition, when ρ_{AB} presumably changes continuously?
  (Not a conventional exchange-of-dominant-macrostate phase transition [Headrick '13].)







- Recall information-theoretic meaning of MI
  Classical: $I(A:B)$ counts # of bits that are correlated (redundant) between A and B

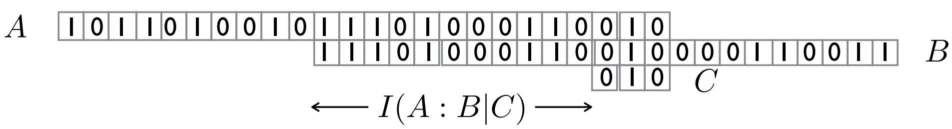


$H(A|B) := S(AB) - S(B)$ (conditional entropy)

Quantum: Each entangled bit counts like 2 correlated bits
Can lead to $H(A|B) < 0$

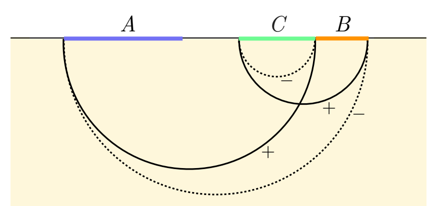
$$\frac{1}{\sqrt{2}}\left(\left|\begin{smallmatrix}1\\1\end{smallmatrix}\right\rangle + \left|\begin{smallmatrix}0\\0\end{smallmatrix}\right\rangle\right)$$

Conditional MI: $I(A:B|C) := I(A:BC) - I(A:C)$



Why do differences between areas of surfaces—in different parts of space—give MI, conditional entropy, and conditional MI?

What does holographic proof of SSA have to do with monotonicity of correlations?



To answer these questions, I will give a new formulation of RT

- Does not refer to minimal surfaces; they are demoted to a mere calculational device
- Suggests a new way to think about the connection between spacetime geometry and information

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Max-flow min-cut

(Originally on graphs, in context of network theory; continuous version [Federer '74, Strang '83, Nozawa '90])

Consider a Riemannian manifold with boundary

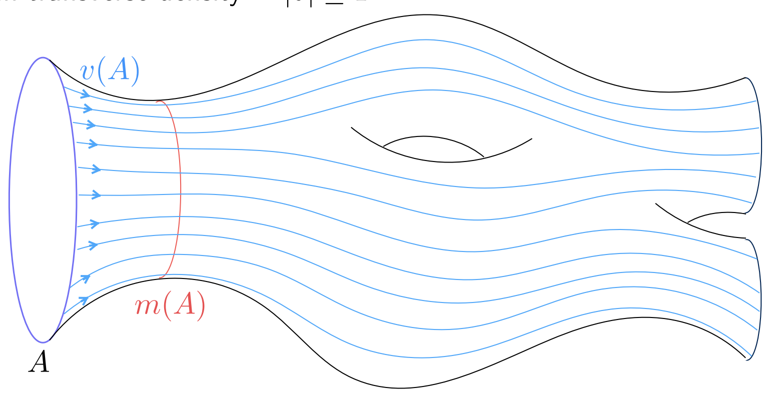
Flow $v :=$ vector field s.t. $\nabla \cdot v = 0$, $|v| \leq 1$
Equivalently, oriented threads (flow lines) with transverse density $= |v| \leq 1$

$A =$ subset of boundary

Max-flow min-cut theorem:

$$\max_v \int_A v = \min_{m \sim A} \text{area}(m)$$

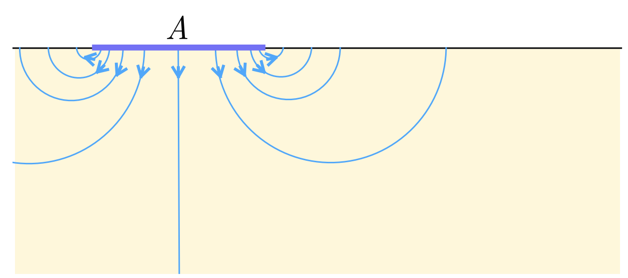
$v(A) :=$ maximizer



Ryu-Takayanagi 2.0:

$$\begin{aligned} S(A) &= \max_v \int_A v \\ &= \text{max \# of threads coming out of } A \end{aligned}$$

Threads have cross section of 4 Planck areas



Facts about max flows:

1. $v(A)$ is far from unique

   We will see that this "gauge freedom" is physically important

2. On $m(A)$, $v(A) =$ unit normal—threads are maximally crowded

3. $v(A)$ changes continuously under continuous changes in A

4. Different flows for different boundary regions:

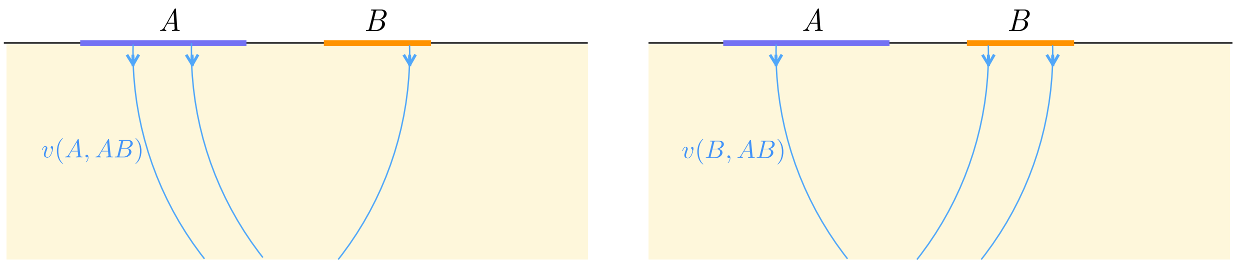
   one cannot necessarily simultaneously maximize flux on A & on B

   However, for *nested* regions one can: there exists $v(A, AB)$ that maximizes $\int_A v$ & $\int_{AB} v$

Threads & information

Example 1: $S(A) = S(B) = 2$, $S(AB) = 3 \Rightarrow I(A:B) = 1$

Maximizing on AB, we can also maximize on *either* A *or* B



Lesson 1: Correlated bits are threads that can be moved (are redundant) between A & B .

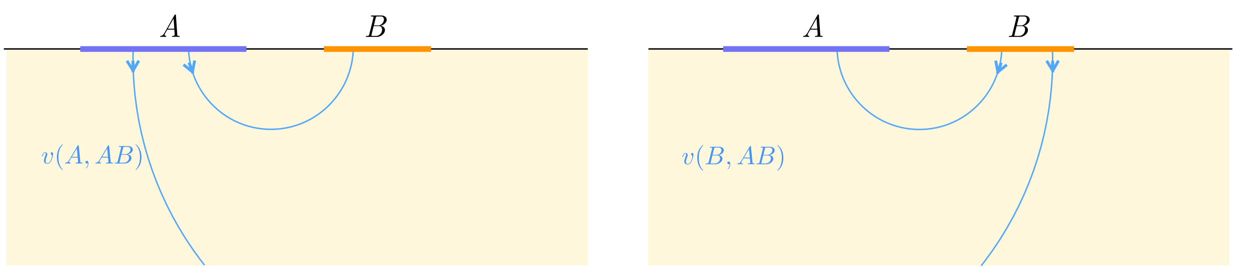
Conditional entropy:

$$\begin{aligned} H(A|B) &:= S(AB) - S(B) \\ &= \int_{AB} v(B, AB) - \int_B v(B, AB) \\ &= \int_A v(B, AB) \\ &= \text{number of threads remaining on } A \text{ when we "measure" } B \end{aligned}$$

.

Example 2: $S(A) = S(B) = 2$, $S(AB) = 1 \Rightarrow I(A:B) = 3$; $H(A|B) = -1 \Rightarrow$ entanglement!

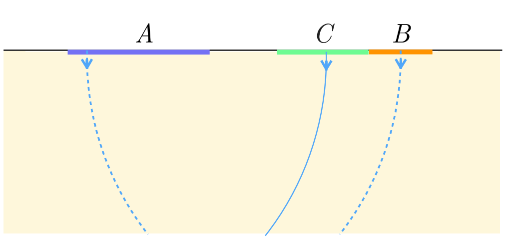
One thread leaving A *must* go to B, and vice versa!



Lesson 2: Entangled qubits are threads connecting A & B that switch direction; A measures B and vice versa.

Subadditivity is clear .

Conditional MI:



$$\begin{aligned} I(A:B|C) &= H(A|C) - H(A|BC) \\ &= \int_A v(C, AC) - \int_A v(BC, ABC) \\ &= (\text{max on } A) - (\text{min on } A), \text{ while maximizing on } C \text{ \& } ABC \\ &= \text{moveable between } A \text{ \& } B, \text{ while maximizing on } C \text{ \& } ABC \\ &= (\text{moveable between } A \text{ \& } BC) - (\text{moveable between } A \text{ \& } C) \\ &= I(A:BC) - I(A:C) \end{aligned}$$

Strong subadditivity is clear .

Exercise for reader: Find flow interpretation of other properties: Araki-Lieb, $S(A) = S(A^c)$ for pure states, ...

Open questions

- Flow-based proof/understanding of monogamy of MI? .
- Higher-derivative corrections (e.g. Gauss-Bonnet)? .
- Quantum corrections (perturbative & non-perturbative)? .
- Covariant holographic entanglement entropy [Hubeny-Rangamani-Takayanagi '07]:
  - Maximin [Wall '12] $\rightarrow$ maximax
  - Fully covariant flow version of HRT? .
- Can we understand the emergence of space from these threads? Is space a "string-net"?