Computational Methods for Entanglement in Lattice Models

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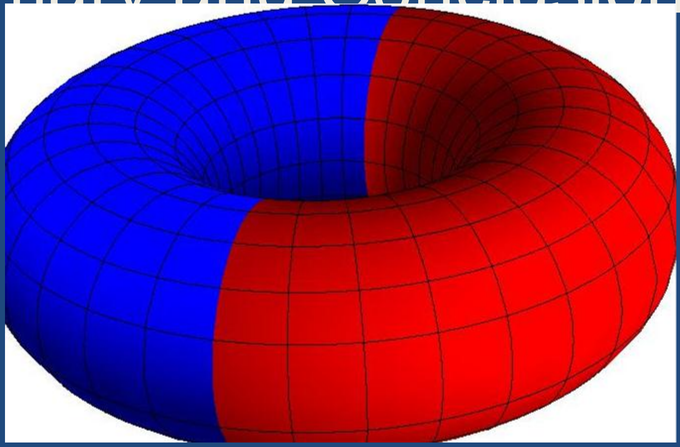
OUTLINE

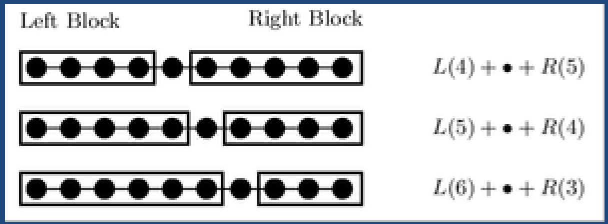
- Motivation
- Finite Size (QMC) and Linked Cluster Methods
- Spin, Boson and Fermion Models
- Entanglement at Quantum Phase Transitions
- Universal singularities
- Summary and Conclusions



The excitement about entanglement is relatively new in Condensed Matter

1. Fundamental understanding of the success of DMRG:

Starting with Haldane Gap in spin-one chains
DMRG can solve many 1D problems to machine precision

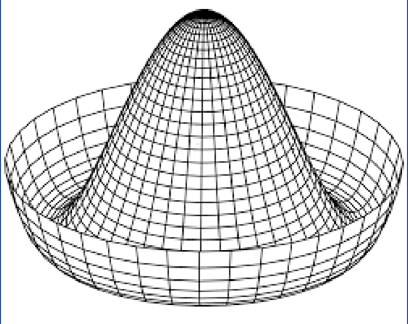


Basis set reduction works to machine precision!
It is related to low entanglement in the ground state

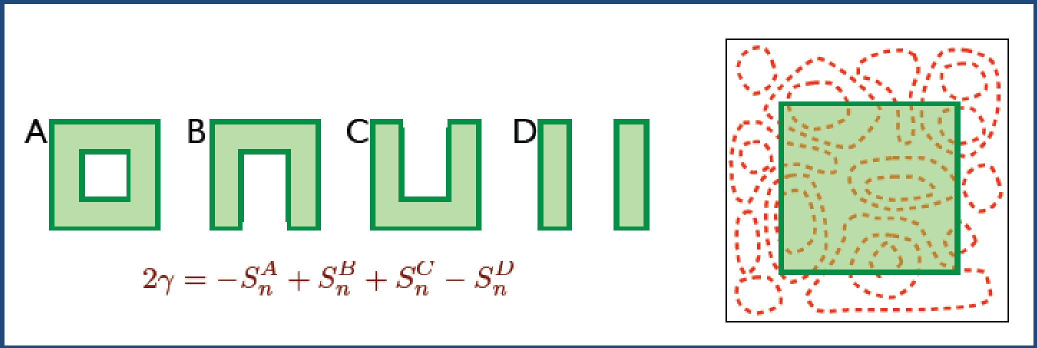
Powerful Variational Methods Matrix Product/Tensor Networks

Natural successor of Wilson's NRG (Kondo Problem)
Will it revolutionize our ability to connect Atoms to Materials (beyond DFT)?

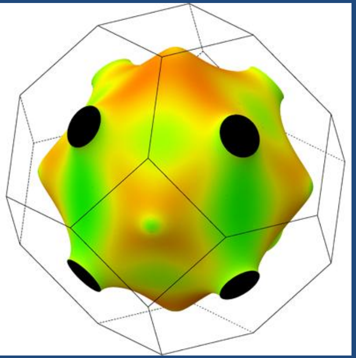
Entanglement contains information on all low lying physics in the system



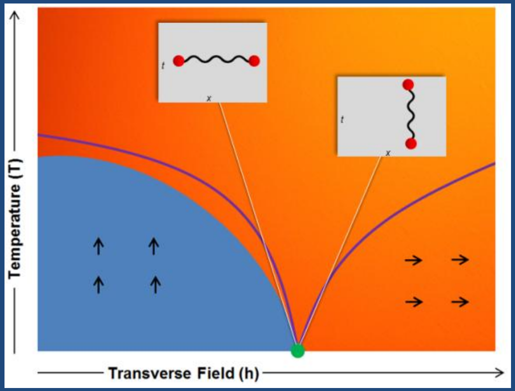
Broken Symmetry and Goldstone modes



Topological Order and QSL (KLHM)



Fermi surfaces

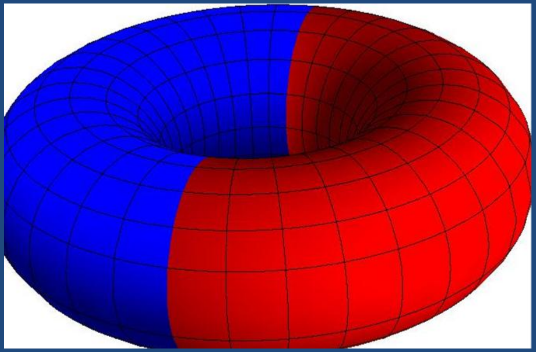


Quantum Critical Phenomena

Great expectation: Study of Entanglement will help in finding Novel Phases characterizing QC Points, understanding RG flows.

How do we compute entanglement entropies systematically for lattice models?

Non-interacting vs interacting systems: Correlation Matrix Method Peschel



$$C_{\vec{i}\vec{j}} = \langle \Psi_0 | c^\dagger_{\vec{i}} c_{\vec{j}} | \Psi_0 \rangle$$

L^d with PBC for L=1000

$$s_{VN} = \sum_i \lambda_i \ln(\lambda_i) + (1 - \lambda_i) \ln(1 - \lambda_i)$$

Fermions

Many-Body Hilbert space increases exponentially 2^N

Full Diagonalization 10^5 (N: 20s)

Low-lying states of sparse Hamiltonians 10^{12} (N: 48) Lauchli

DMRG in 1D

In this talk d>1

Stochastic Methods

Quantum Monte Carlo

Finite but relatively large systems

Linked Cluster Methods

Series expansions

Numerical Linked Cluster

Thermodynamic limit but only controlled when there is a small parameter

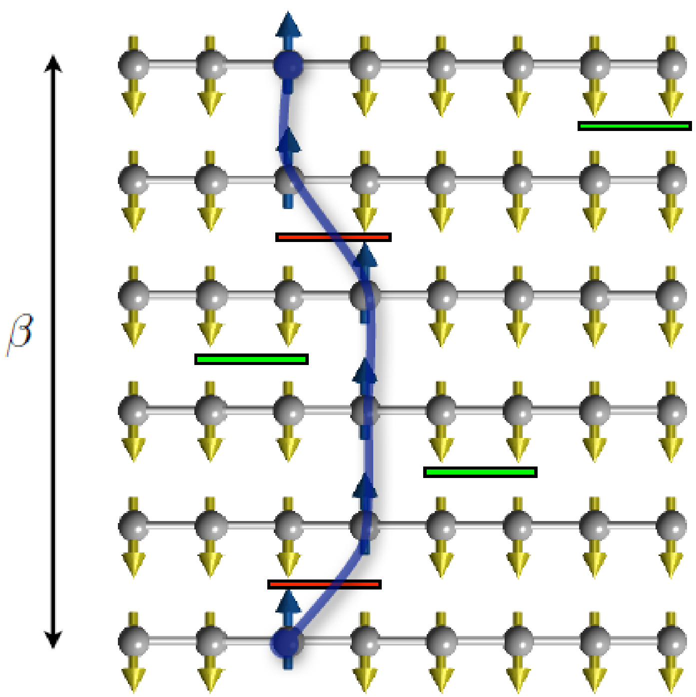
Measure of entanglement: Entanglement Entropy

Several posters at this conference

Stochastic Series Expansion

Sandvik

$$Z = \sum_{\alpha} \langle \alpha | e^{-\beta \hat{H}} | \alpha \rangle = \sum_{\alpha} \sum_{n} \frac{(-\beta)^n}{n!} \langle \alpha | H^n | \alpha \rangle = \sum_{\alpha} \sum_{n} \sum_{S_n} \frac{(-\beta)^n}{n!} \langle \alpha | \prod_{i=1}^{n} H_{b_i} | \alpha \rangle$$

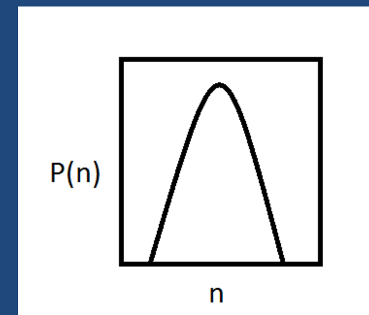


H_{b_i}

(green line) $= S_i^z S_j^z$

(red line) $= (S_i^+ S_j^- + S_i^- S_j^+)$

$$\sum_n P_n$$



Peaks at $n = N\beta$

An exact discrete-time representation for spin configurations in space and continuous imaginary-time

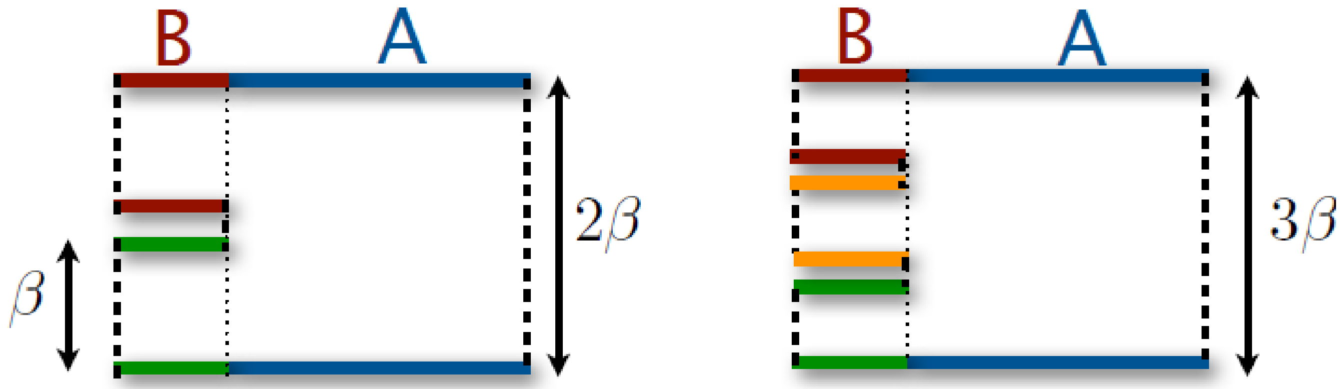
Entanglement entropy from QMC: Need to work with Renyi Entropies

Replica Trick

Calabrese and Cardy, J. Stat. Mech. 0406, P002 (2004).
Fradkin and Moore, Phys. Rev. Lett. 97, 050404 (2006)
Nakagawa, Nakamura, Motoki, and Zaharov, arXiv:0911.2596
Buividovich and Polikarpov, Nucl. Phys. B, 802, 458 (2008)
M. A. Metlitski, et.al, Phys.Rev. B 80, 115122 (2009).

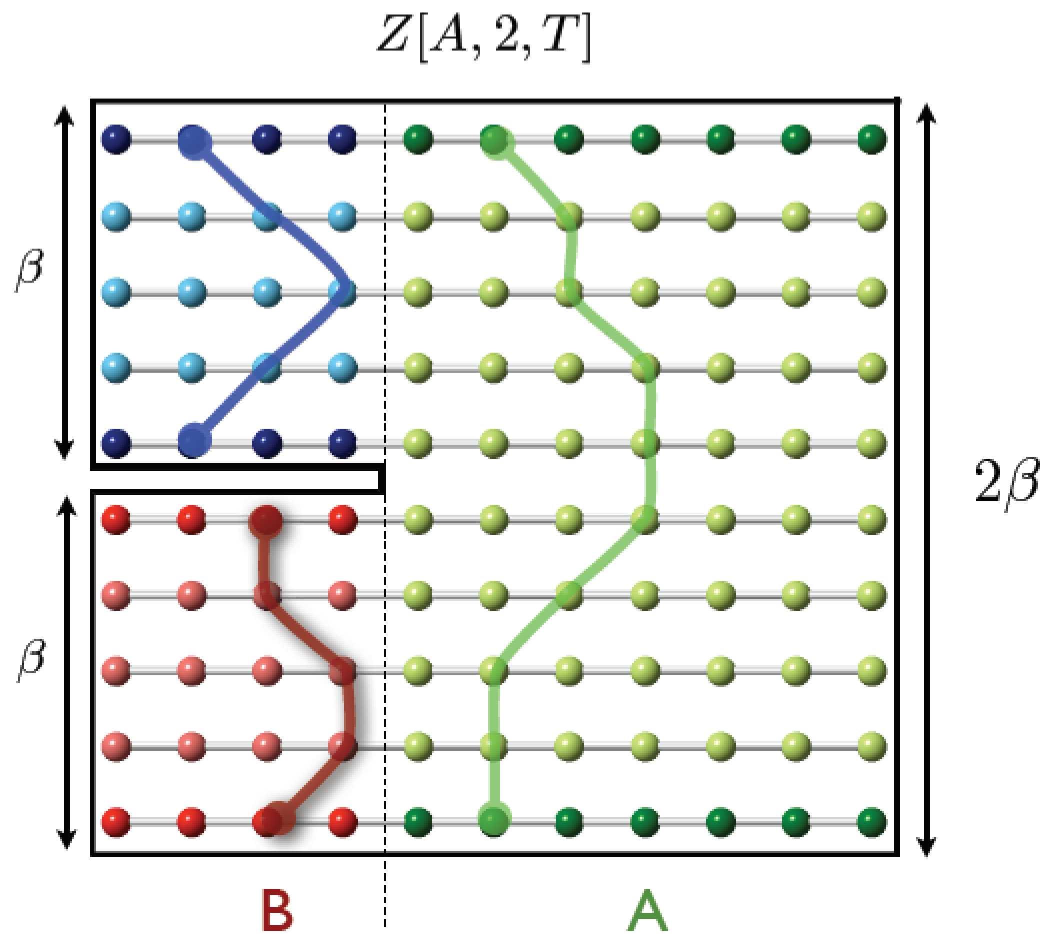
$$S_n(\rho_A) = \frac{1}{1-n} \ln \left[\mathrm{Tr} \left(\rho_A^n \right) \right] = \frac{1}{1-n} \ln \frac{Z[A,n,T]}{Z(T)^n}$$

where $Z[A,n,T]$ is the partition function of the systems having special topology – the n-sheeted Riemann surface.



$Z[A,2,T]$ $Z[A,3,T]$

SSE SIMULATION CELL

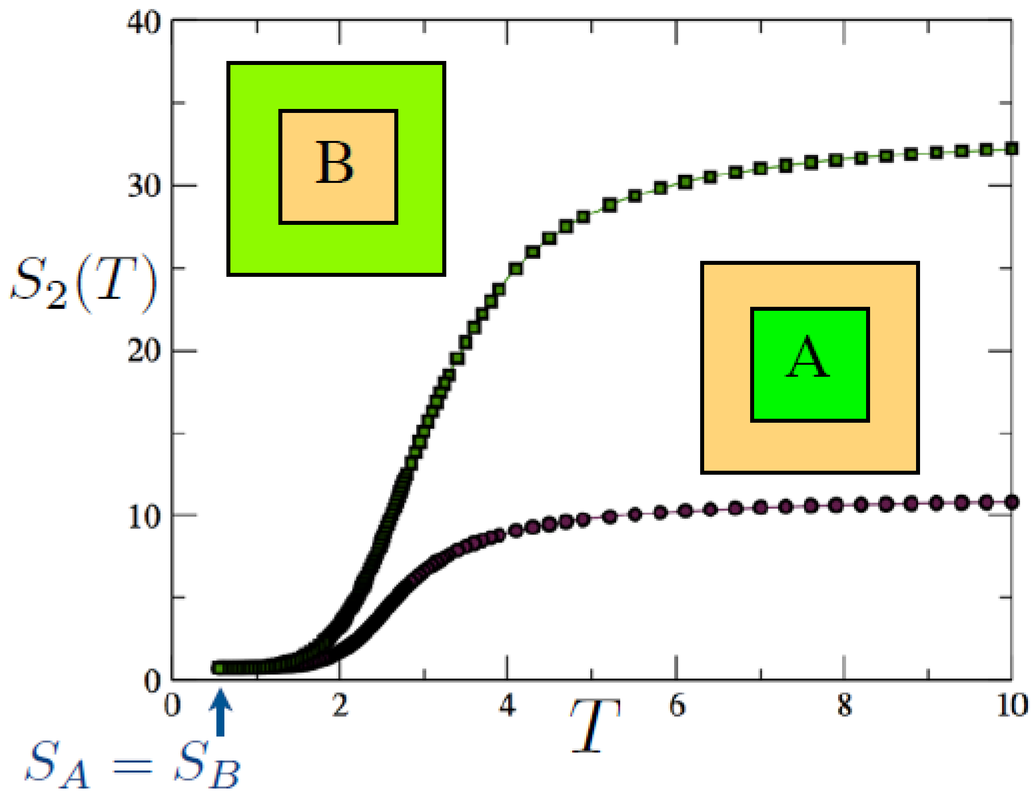


Hastings, Gonzalez, Kallin, Melko PRL 2010
Melko, Kallin, Hastings PRB 2010

With suitable thermodynamic integration S can be calculated

$$S_2 = -\ln \mathrm{Tr}(\rho_A^2) = -\ln \left\{ \frac{Z[A,2,\beta]}{Z(\beta)^2} \right\} = -\ln Z[A,2,\beta] + 2\ln Z(\beta)$$

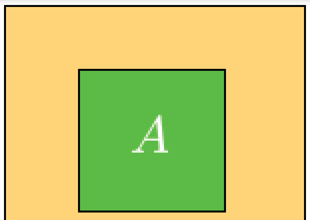
$$= -S_A(\beta=0) + \int_0^\beta \langle E\rangle_A d\beta + 2S_0(\beta=0) - 2\int_0^\beta \langle E\rangle_0 d\beta$$



Mutual Information for XXZ model $I_{AB} = S_A + S_B - S_{AB}$

Critical Scaling at a phase transition

Singh, Hastings, Kallin, RGM, Phys. Rev. Lett. 106, 135701 (2011)

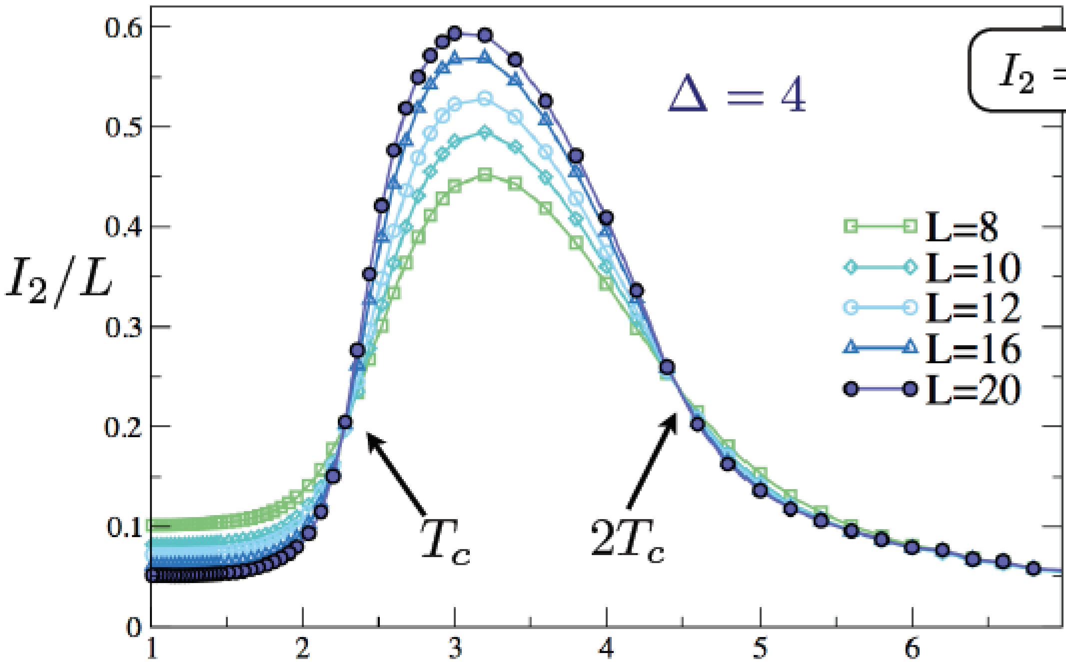


$$\ell = L/2$$

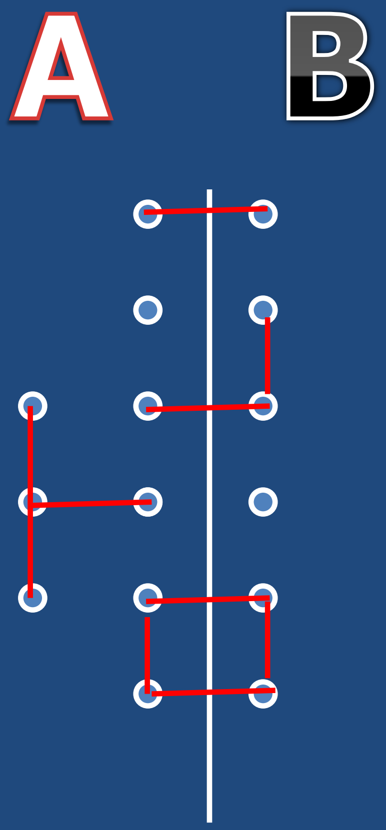
XXZ model

$$H = \sum_{\langle ij \rangle} \left(\Delta S_i^z S_j^z + S_i^x S_j^x + S_i^y S_j^y \right)$$

$$I_2 = c_1 L + c_2 \ln(L) + c_3$$



High Temperature Series Expansions in the Thermodynamic Limit



Any extensive/intensive quantity $S_A,\ S_B,\ S_{AB}$ can be expanded in powers of β using a Linked Cluster Method

$$S_n(\beta) = \sum_m s_m(n)\ \beta^m$$

$$S_n(\beta) = \sum_c L(c)W(c)$$

Divide the infinite lattice into two halves A and B.
Only Clusters that cross the dividing line contribute to I_{AB}
`Area Law' is built in to the series expansions

$$I_n(\beta)/l = \sum_m a_m(n)\ \beta^m$$

Coefficients $a_m(n)$ are ploynomials in n of order m-1.
They can be calculated for arbitrary Renyi index n.

$\exp(-\beta\ n\ H)$

Coefficient of β^m polynomial in n of order (m-1) β_c scales as 1/n

Consider the XXZ model on a bi-partite lattice

$$\mathcal{H} = \sum_{<i,j>} (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z)$$

Five traces are needed for full calculation to 4th order.

$$A_2 = \frac{Tr(\sigma_1 \cdot \sigma_2)^2}{Tr(1)} = 2 + \Delta^2$$

$$A_3 = \frac{Tr(\sigma_1 \cdot \sigma_2)^3}{Tr(1)} = -6\Delta$$

$$A_4 = \frac{Tr(\sigma_1 \cdot \sigma_2)^4}{Tr(1)} = (2 + \Delta^2)^2 + 4(1 + 2\Delta^2)$$

$$B_4 = \frac{Tr(\sigma_1 \cdot \sigma_2\ \sigma_2 \cdot \sigma_3\ \sigma_1 \cdot \sigma_2\ \sigma_2 \cdot \sigma_3))}{Tr(1)} = \Delta^4 - 4\Delta^2$$

$$C_4 = \frac{Tr(\sigma_1 \cdot \sigma_2\ \sigma_2 \cdot \sigma_3\ \sigma_3 \cdot \sigma_4\ \sigma_4 \cdot \sigma_1))}{Tr(1)} = 2 + \Delta^4$$

For the Square-Lattice, Mutual Information per unit length to 4th order is:

$$(\frac{\beta}{4})^2 \frac{nA_2}{2} - (\frac{\beta}{4})^3 \frac{n(n+1)A_3}{6} + (\frac{\beta}{4})^4 [n(n^2+n+1)(\frac{A_4}{24} - \frac{A_2^2}{8}) + n(n^2+n-1)(\frac{B_4 - A_2^2}{2} + C_4)]$$

SERIES EXTRAPOLATIONS

Domb, Baker, Fisher, …

$f(x) = \sum_m a_m \ x^m$

We know only a small number of terms (10-20)
How do we obtain singular behavior?

- Represent function by an Approximant F(x):

$F(x) = \frac{P^m(x)}{Q^n(x)}$ Simple pole (Pade' Approximants)

$\frac{1}{F}\frac{dF}{dx} = \frac{P^m(x)}{Q^n(x)}$ Power-law singularity

$\frac{dF}{dx} = \frac{P^m(x)}{Q^n(x)}$ log singularity

One can always tune approximants to type of singularity
Polynomials determined from series coefficients
They can be used to solve for critical parameters

XXZ Model Line Coefficient from QMC vs high-T expansions

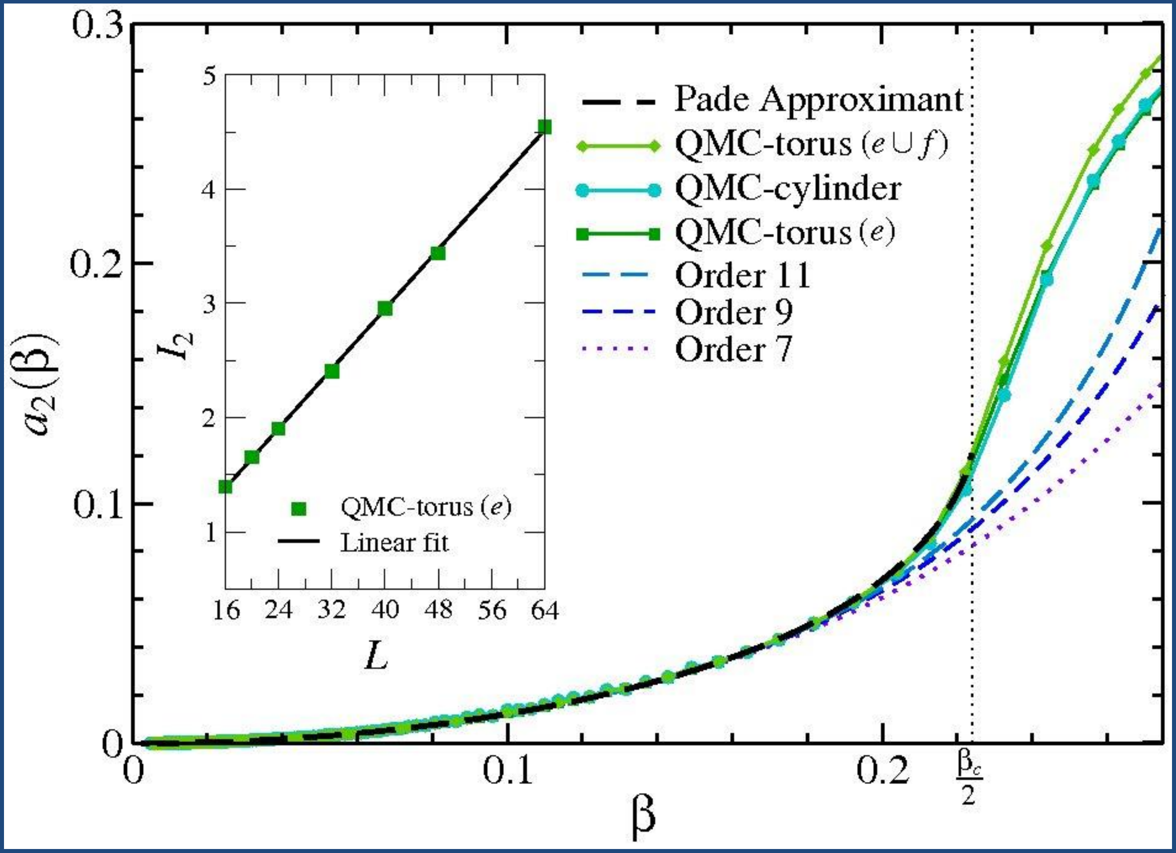
RRPS, A. Kallin, R. Melko, M. Hastings PRL 2011

High-T Series Diverges at $\beta_c/2$

Same universality class as at Tc

t ln(t) singularity

Represent 2nd derivative by Pade approximant



Corner log singularity related to central charge c

Cardy, Peschel NP 1988

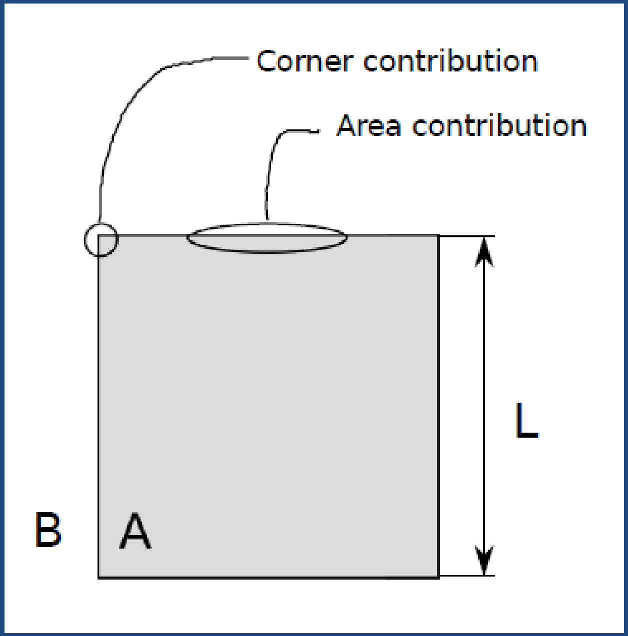
Quantum Entanglement in the Ground State

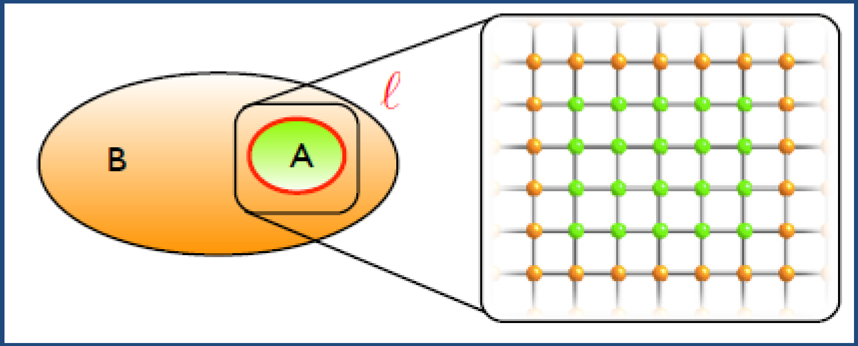
$$H|\psi> = E\ |\psi>$$

$$\varrho = |\psi><\psi|$$

$$\varrho_A = Tr_B \varrho$$

$$S_A = -Tr\,(\rho_A \ln \rho_A)$$





Bulk, corner, line singularities at QCP

Some representative models

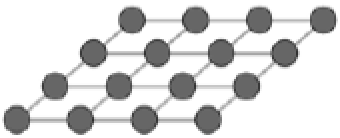
O(N)

E.M. Stoudenmire, Peter Gustainis, Ravi Johal, Stefan Wessel, Roger G. Melko

Phys. Rev. B 90, 235106 (2014)

– Transverse field Ising

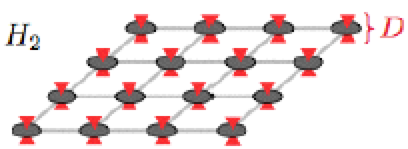
$$H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$



$(h/J)_c = 3.044$

– Anisotriopic S=1

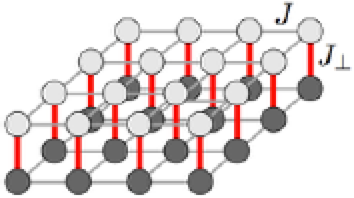
$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + D \sum_i (S_i^z)^2$$



$(D/J)_c = 5.625$

– Bilayer Heisenberg antiferromagnet

$$H = J \sum_{\langle i,j \rangle} (\mathbf{S}_{1i} \cdot \mathbf{S}_{1j} + \mathbf{S}_{2i} \cdot \mathbf{S}_{2j}) + J_\perp \sum_i \mathbf{S}_{1i} \cdot \mathbf{S}_{2i}$$



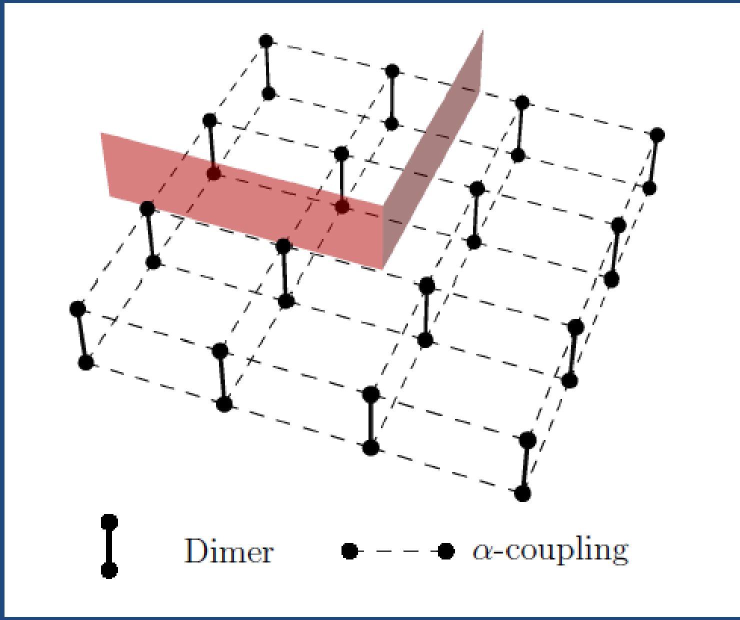
$(J_\perp/J)_c = 2.522$

O(N) UNIVERSALITY CLASSES

Devakul+RRPS PRB 2014

Anisotropic Bilayer Heisenberg Model

$$\mathcal{H} = \sum_{\langle i,j \rangle} (S_x S_x + S_y S_y + S_z S_z) + \alpha \sum_{\langle\langle i,k \rangle\rangle} (S_x S_x + S_y S_y + \lambda S_z S_z)$$



Small α : Dimerized Ground state

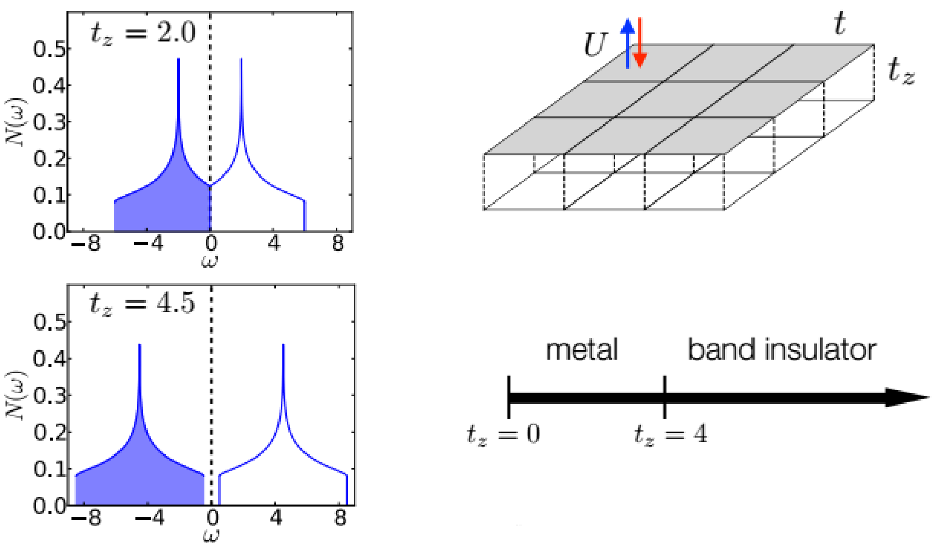
Large α

$\lambda < 1$ XY order
$\lambda = 1$ Heisenberg order
$\lambda > 1$ Ising order

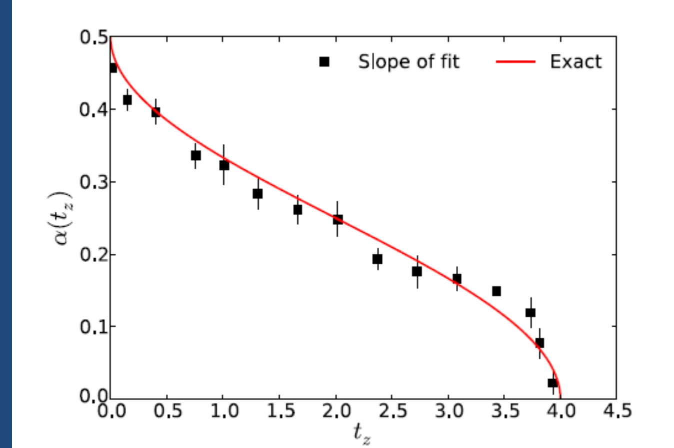
Tight-Binding and Hubbard Models

Half-filled bilayer square lattice Hubbard model

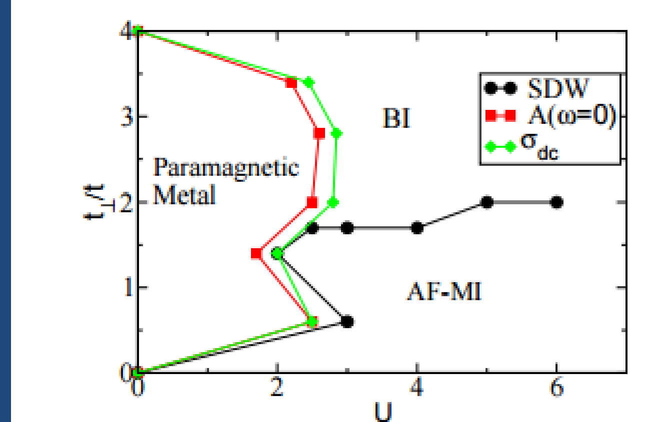
$$H = -t \sum_{\ell\sigma} \sum_{\langle ij \rangle} \left(c^\dagger_{i\ell\sigma} c_{j\ell\sigma} + h.c. \right) - t_z \sum_{i\sigma} \left(c^\dagger_{i1\sigma} c_{i2\sigma} + h.c. \right)$$



C.-C. Chang, RRPS, R. T. Scalettar



\`Widom conjecture' and logs



Phase diagram with U?

Ground State Entanglement: Quantum Monte Carlo Methods

Measurement of Swap Operator

Measure ratio of Partition Functions

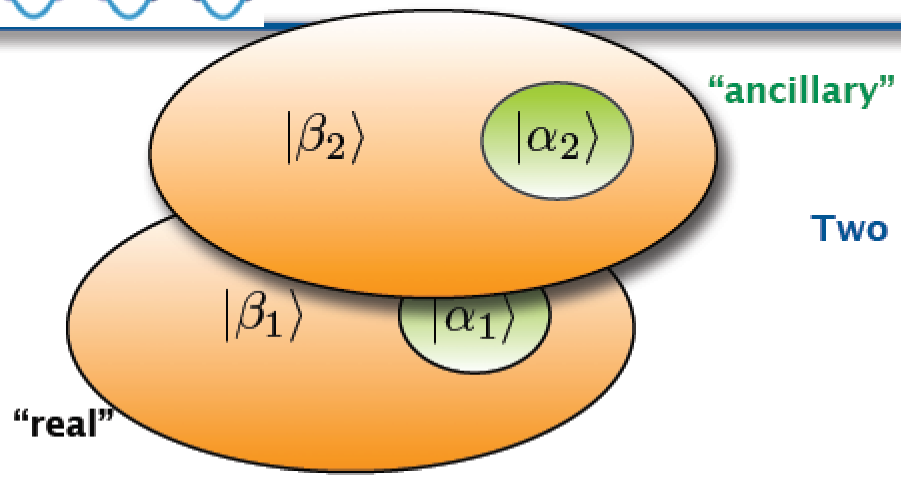
Generalize Correlation Matrix method (DQMC)

Projection Monte Carlo in Valence Bond Basis

Simulate two copies of the system and measure the SWAP operator

SWAP OPERATOR

Hastings, Gonzalez, Kallin, RGM
Phys. Rev. Lett. 104, 157201 (2010)



Two non-interacting copies

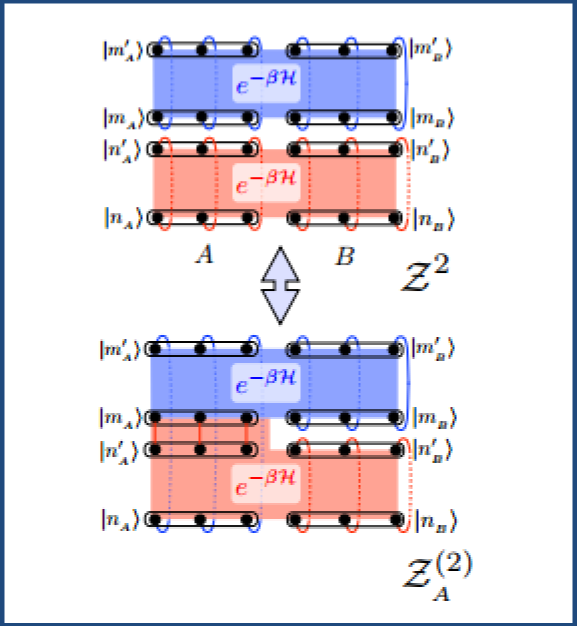
$$|\Psi_0 \otimes \Psi_0\rangle$$

$$\langle \Psi_0 \otimes \Psi_0 | Swap_A | \Psi_0 \otimes \Psi_0 \rangle = \sum_{\alpha_1,\alpha_2,\beta_1,\beta_2} C_{\alpha_1,\beta_1} \overline{C}_{\alpha_2,\beta_1} C_{\alpha_2,\beta_2} \overline{C}_{\alpha_1,\beta_2}$$

$$= \sum_{\alpha_1,\alpha_2} \langle \alpha_1 | \rho_A | \alpha_2 \rangle \langle \alpha_2 | \rho_A | \alpha_1 \rangle = \mathrm{Tr}(\rho_A^2)$$

$$S_2(\rho_A) = -\ln(\mathrm{Tr}(\rho_A^2)) = -\ln(\langle Swap_A \rangle)$$

Calculate ratio of partition functions in an extended ensemble



$$S_n = \frac{1}{1-n} \ln\left(\frac{\mathcal{Z}^A}{\mathcal{Z}^n}\right)$$

Humeniuk and Roscilde
Brocker and Trebst
Wang and Troyer
Helmes and Wessel

Find a QMC scheme that switches between partition functions, with a detailed balance condition leading directly to their ratio.

Determinantal Monte Carlo for Hubbard Models

Hubbard Stranotovich Transformation maps the interacting fermion problem into a bilinear one in presence of time varying Ising (HS) fields.

With replicas one can get Renyi entropies

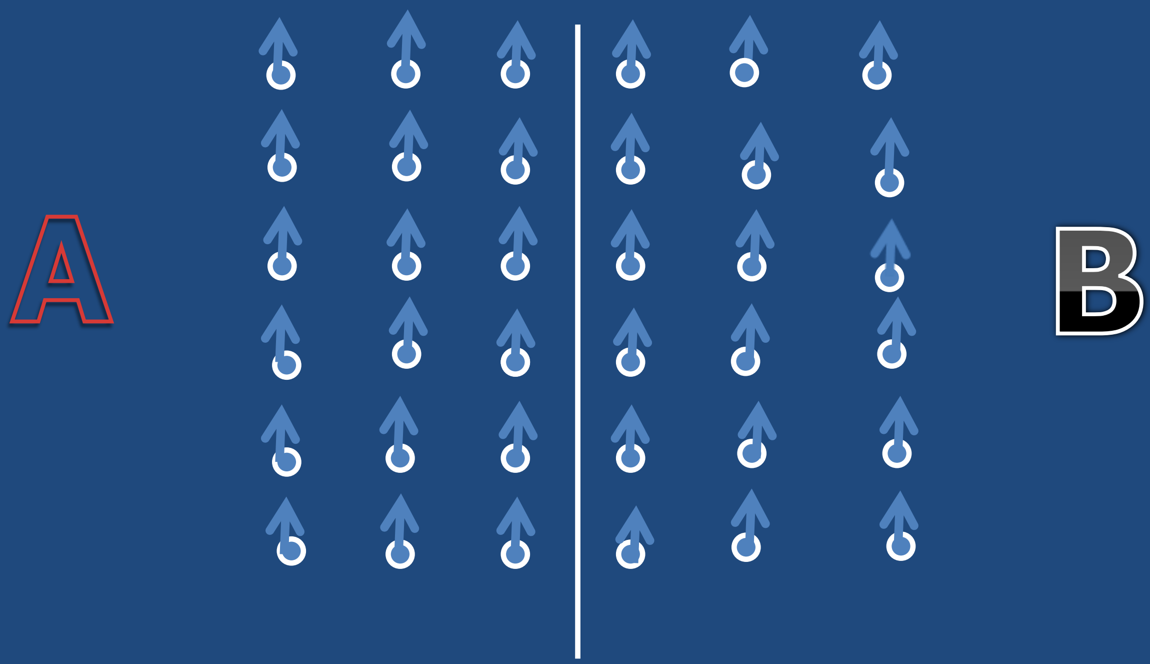
$$
\begin{aligned}
S_2 &= -\log\left[\sum_{\{s\},\{s'\}} P_s P_{s'} \operatorname{tr}(\rho_{A,s}\rho_{A,s'})\right] \\
&= -\log\Bigg[\sum_{\{s\},\{s'\}} P_s P_{s'} \{\mathrm{Det}(G_s G_{s'} \\
&\quad +(\mathbb{I}-G_s)(\mathbb{I}-G_{s'}))\}\Bigg]
\end{aligned}
$$

T. Grover
Assaad, Lang and Toldin

Mostly calculations have been done for small U

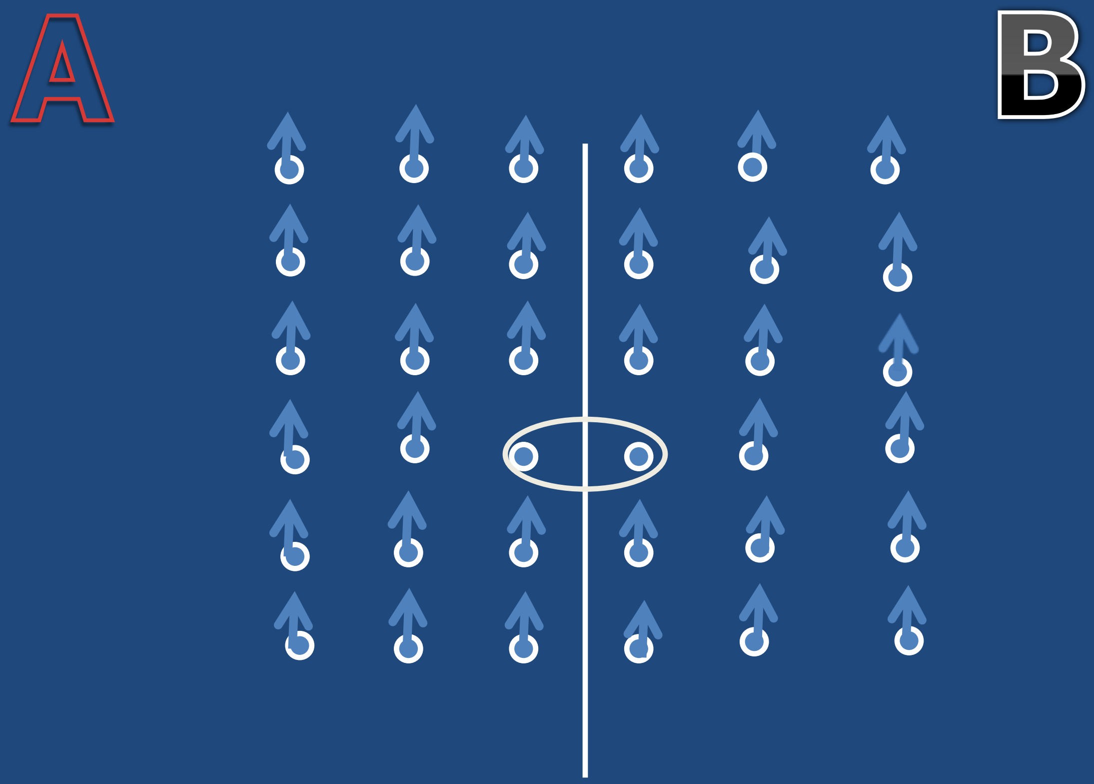
Ground State Series Expansions in J/h or h/J

$$H = -J \sum \sigma^z \sigma^z - h \sum \sigma^x$$



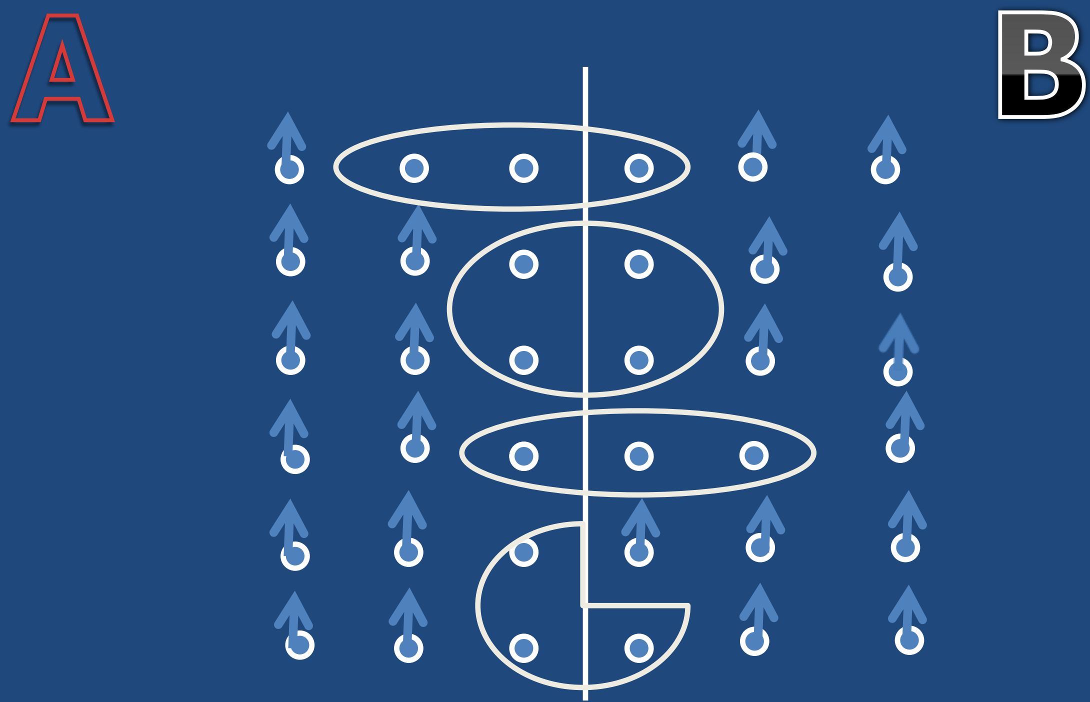
The unperturbed state is a product state and has no entanglement

Perturbation expansion in J/h



In leading order neighbors across the boundary are entangled.
Translated along the boundary gives an area-law contribution.

Series Expansions in J/h



In the following order larger clusters of spin get entangled across the boundary.

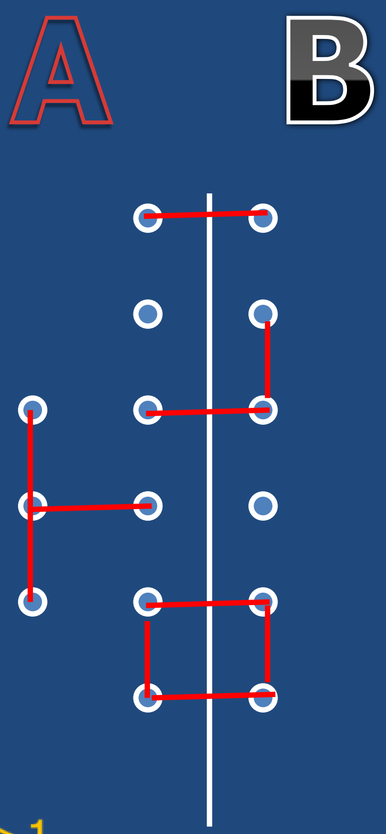
One can show existence of a `Linked-cluster expansion' for Renyi entropies, and area-law remains valid as long as perturbation theory converges

Perturbation expansion at T=0

Expand in powers of x=J/h or h/J using non-degenerate perturbation theory

$$S_n(x) = \sum_m a_m(n)\ x^m$$

$$S_n(x) = \sum_c L(c)W(c)$$



Only Clusters that cross the dividing line contribute to S_n
`Area Law' is built in to series expansion

Coefficients $a_m(n)$ can be calculated for any integer Renyi index n>1.

Non-integer Renyi entropies are singular at x=0

Non-integer Renyi entropies have no power series expansion in h/J or J/h

Writing the Renyi entropies in terms of eigenvalues of the reduced density matrix

$$S_\alpha = \frac{1}{1-\alpha} \ln\left(\lambda_0^\alpha + \lambda_1^\alpha + \lambda_2^\alpha + \ldots\right)$$

Separate into two terms $S_\alpha = E_\alpha + R_\alpha$

$$E_\alpha \ = \ \frac{\alpha}{1-\alpha} \ln \lambda_0$$

$$R_\alpha = \frac{1}{1-\alpha} \ln(1 + (\lambda_1/\lambda_0)^\alpha + (\lambda_2/\lambda_0)^\alpha + \ldots)$$

At x=0, density matrix is of the form→

1	0	0
0	0	0
0	0	0

$$\lambda_0 = 1 - O(x^2)$$

$$\lambda_i = O(x^{2m})$$

E_α has very simple dependence on Renyi index, R_α is analytic only for integer α

Von Neumann entropy has $x^m \ln x$ singularity at small x

Conformal field theory results: Large α may suffice to capture universality

$$A\;(1+\frac{1}{\alpha})\ln\xi$$

1+1

$$B\;(1+\frac{1}{\alpha}+\frac{1}{\alpha^2}+\frac{1}{\alpha^3})\ln\xi$$

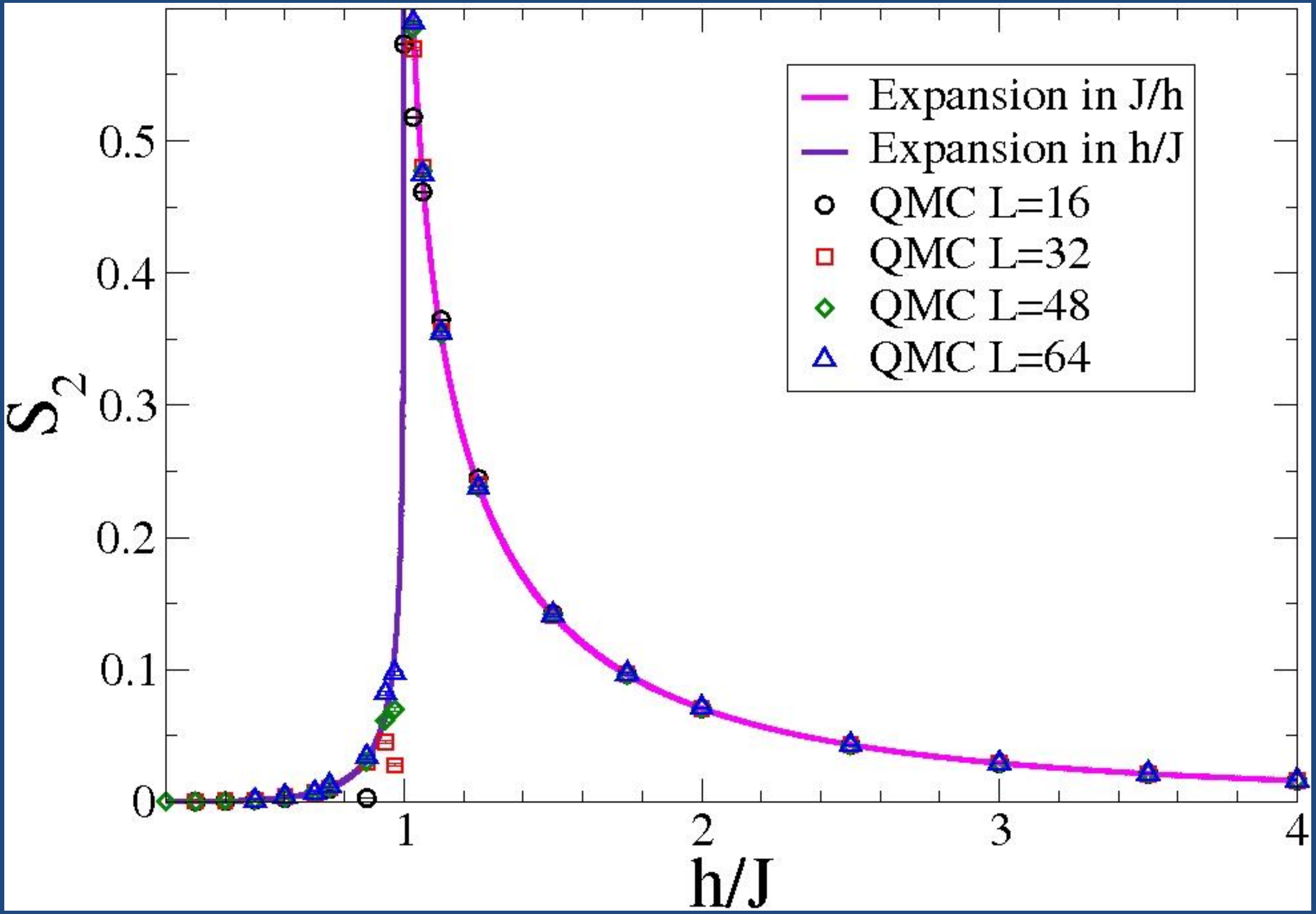
3+1

Suggest that large α or Ground state term may suffice to obtain the universal singularity

$$E_\alpha \;=\; \frac{\alpha}{1-\alpha}\ln\lambda_0$$

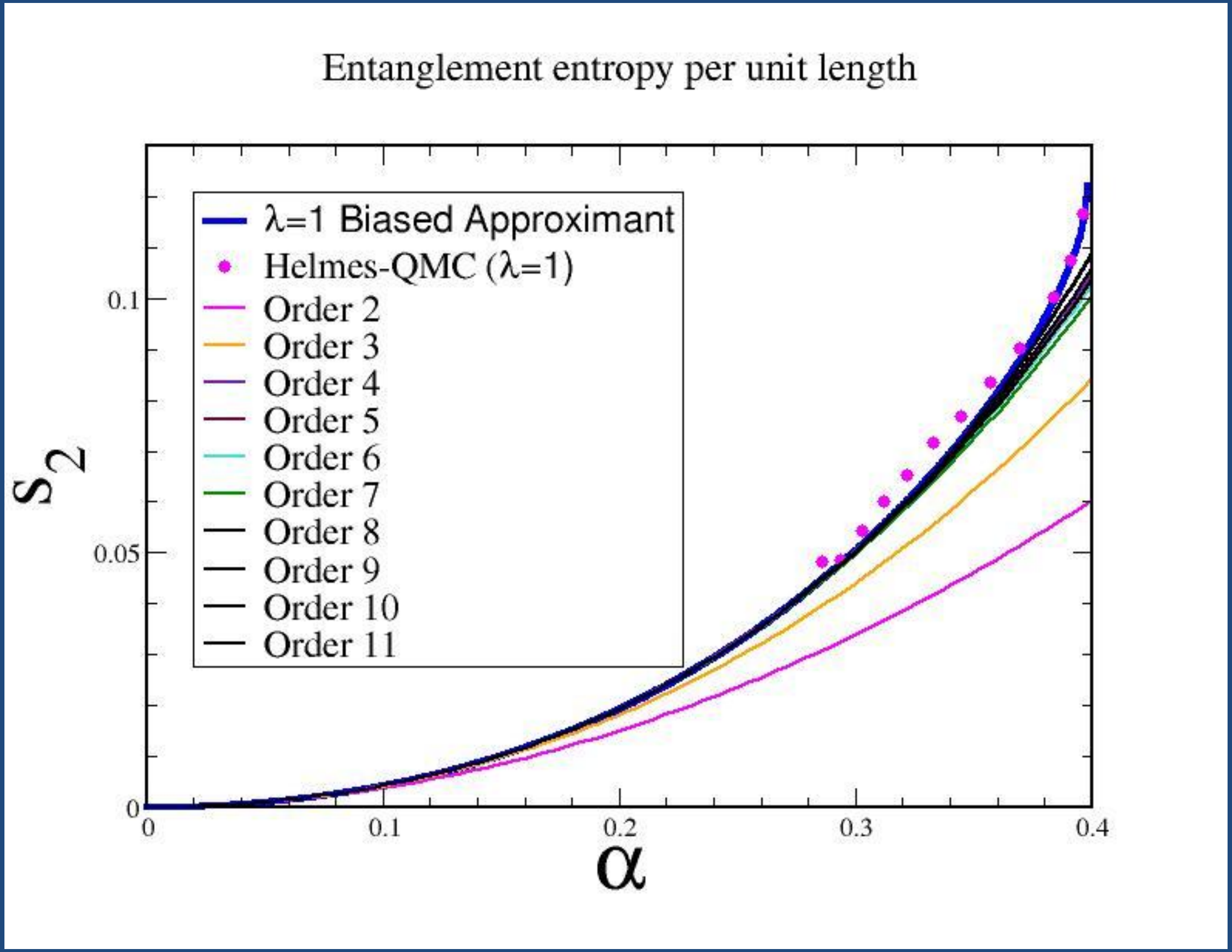
$$R_\alpha = \frac{1}{1-\alpha}\ln(1+(\lambda_1/\lambda_0)^\alpha+(\lambda_2/\lambda_0)^\alpha+\ldots)$$

SERIES EXPANSION VS QMC (1D TFIM)



Note: Renyi entropies only singular at true critical point!

Heisenberg Bilayer Model: Going up to critical point



Area-law behavior of second Renyi entropy
QMC data from Stefan Wessel group

Von Neumann Entropy

Combine Linked Cluster Method with Exact Diagonalization

Numerical Linked Cluster Methods

Ann B. Kallin, Katharine Hyatt, Rajiv R. P. Singh, Roger G. Melko — Phys. Rev. Lett. 110, 135702 (2013)

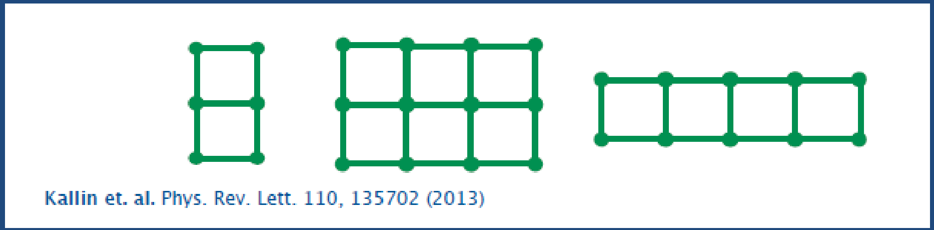
Ann B. Kallin, E. M. Stoudenmire, Paul Fendley, Rajiv R. P. Singh, Roger G. Melko — J. Stat. Mech. (2014) P06009

E.M. Stoudenmire, Peter Gustainis, Ravi Johal, Stefan Wessel, Roger G. Melko — Phys. Rev. B 90, 235106 (2014)

Numerical Linked Cluster Expansion (NLCE): Combine ED with Linked Cluster methods

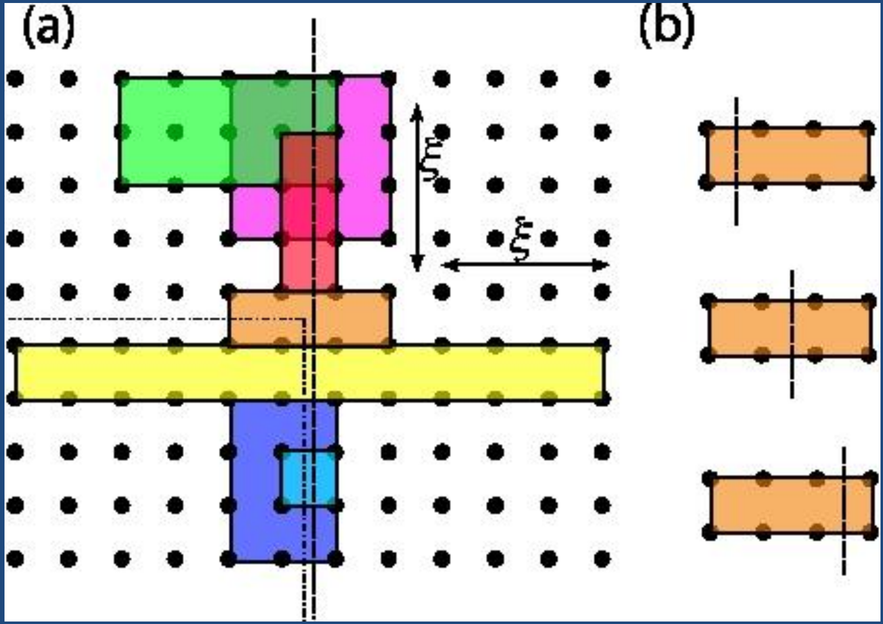
$$S_n(x) = \sum_c L(c) W(c)$$

Weights obtained numerically by ED and not as a series expansion



$$\frac{S}{L} = \sum_{m,n} W(m,n)$$

$$W(m,n) = P(m,n) - \sum_{m',n'} W(m',n')$$

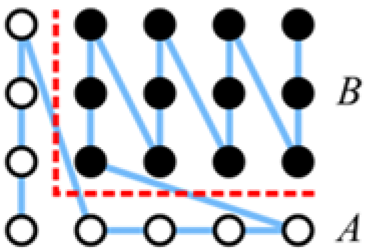


Works also for non-integer Renyi indices

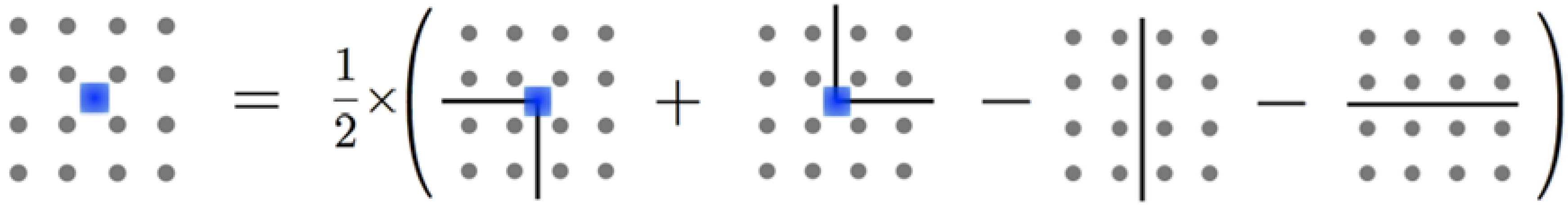
DMRG can help with bigger clusters making it much more accurate

Examining the corner with DMRG

Direct calculation of ρ_A

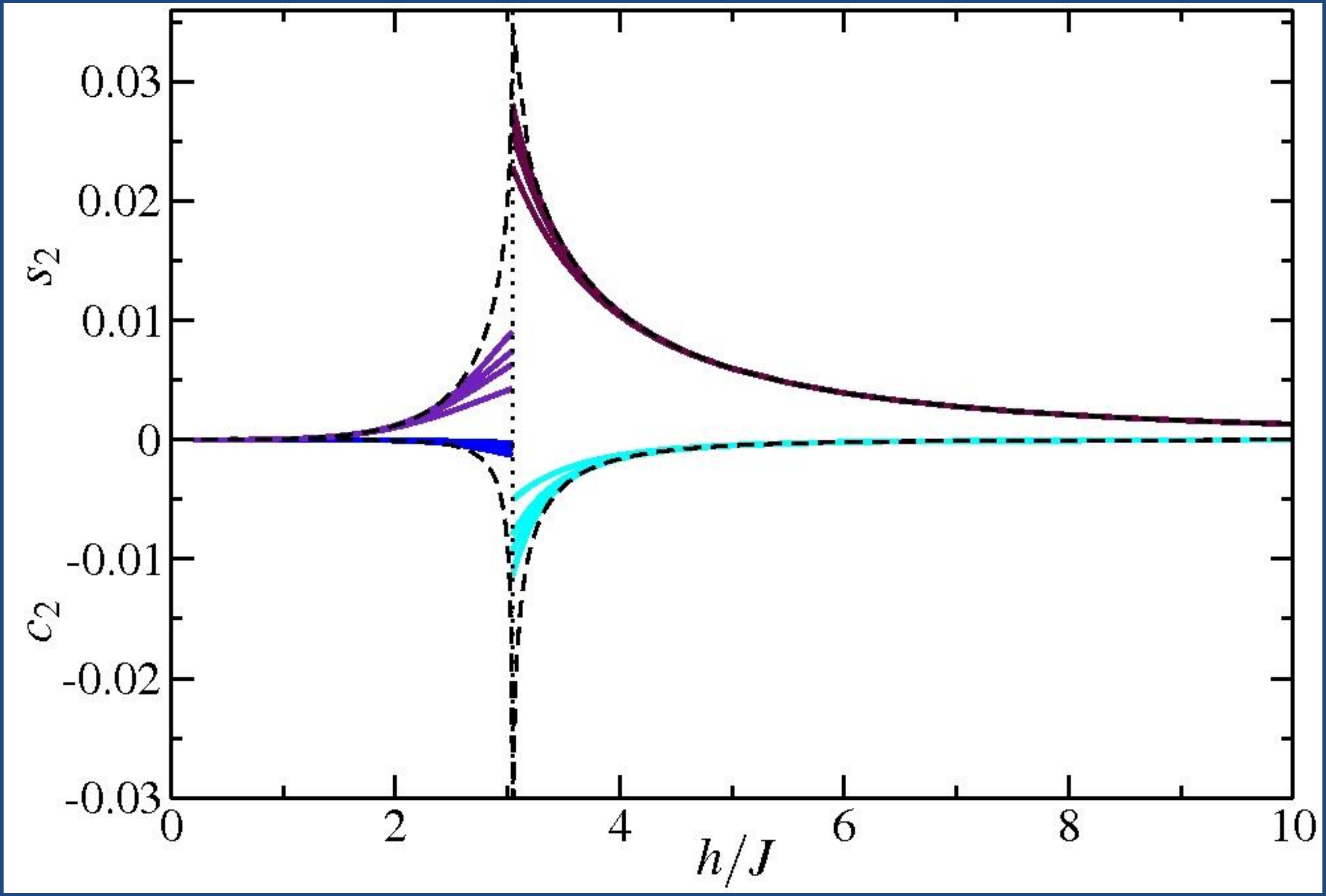


Isolate corner contribution cluster-by-cluster



NLC vs series expansions

Line and Corner entropies (2+1) TFIM



Near Critical Point NLC needs an extrapolation

Define a length scale to extrapolate in

Length scales

- Definition of a length scale is possible

$$\mathcal{O}_A = n_x + n_y \qquad \ell_A = \frac{1}{2}(n_x + n_y) = \frac{1}{2}\mathcal{O}_A$$

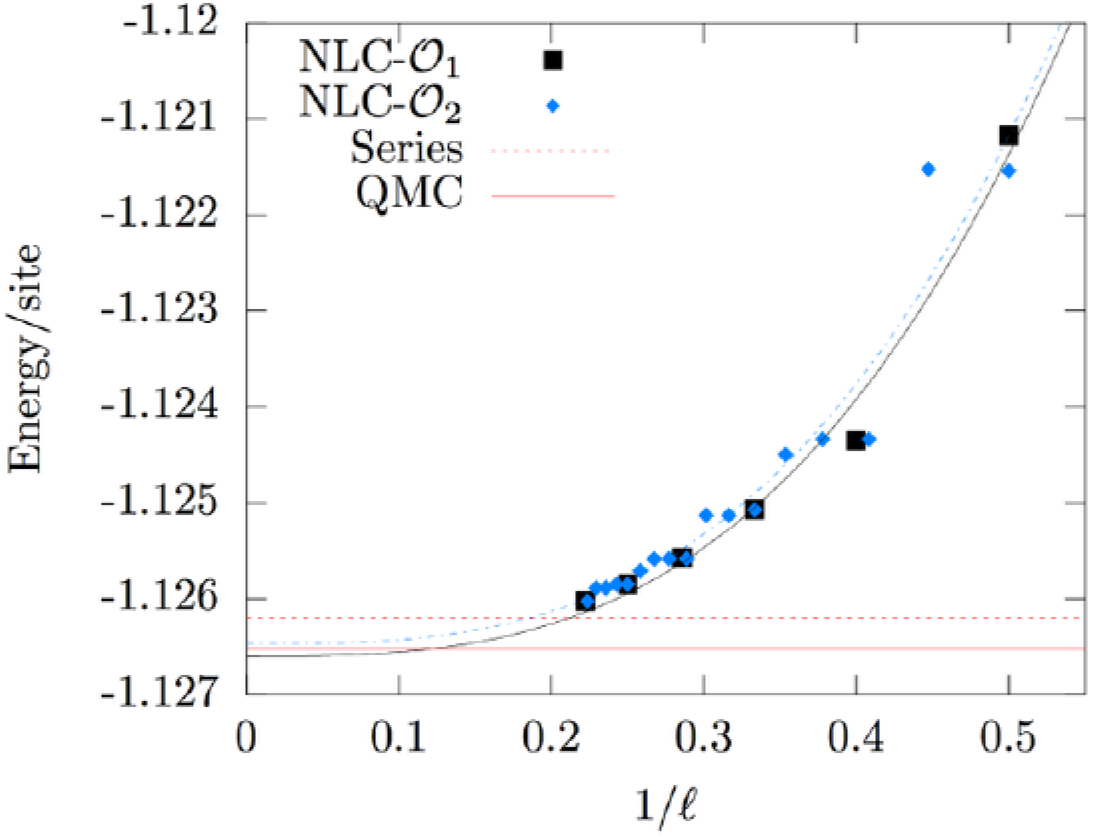
$$\mathcal{O}_G = n_x n_y \qquad \ell_G = (n_x n_y)^{\frac{1}{2}} = \sqrt{\mathcal{O}_G}$$

$$\mathcal{O}_Q = n_x^2 + n_y^2 \qquad \ell_Q = \sqrt{\frac{1}{2}(n_x^2 + n_y^2)}$$

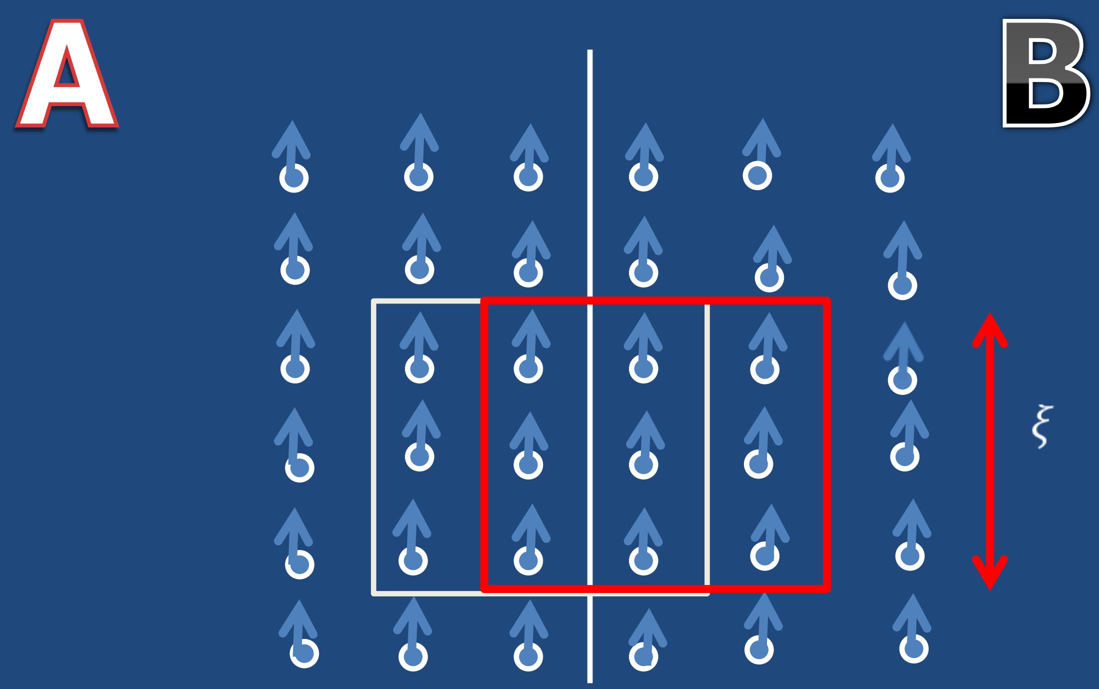
Clusters			Order			Length-scale		
id	$n_x \times n_y$	$L(c)$	$\mathcal{O}_G$	$\mathcal{O}_A$	$\mathcal{O}_Q$	ℓ_G	ℓ_A	ℓ_Q
(a)	1×1	1	1	2	2	1	1	1
(b)	1×2	2	2	3	5	1.414	1.5	1.581
(c)	1×3	2	3	4	10	1.732	2	2.236
(d)	1×4	2	4	5	17	2	2.5	2.915
(e)	2×2	1	4	4	8	2	2	2
(f)	2×3	2	6	5	13	2.445	2.5	2.550
(g)	2×4	2	8	6	20	2.828	3	3.162
(h)	3×3	1	9	6	18	3	3	3
(i)	3×4	2	12	7	25	3.464	3.5	3.536
(j)	4×4	1	16	8	32	4	4	4

Energy benchmark

$$H = J \sum_{\langle i,j \rangle} (\mathbf{S}_{1i} \cdot \mathbf{S}_{1j} + \mathbf{S}_{2i} \cdot \mathbf{S}_{2j}) + J_\perp \sum_i \mathbf{S}_{1i} \cdot \mathbf{S}_{2i}$$



Scaling arguments near the critical point



Near the critical point, blocks of size ξ get correlated

Critical fluctuations scales as $\frac{L}{\xi}$

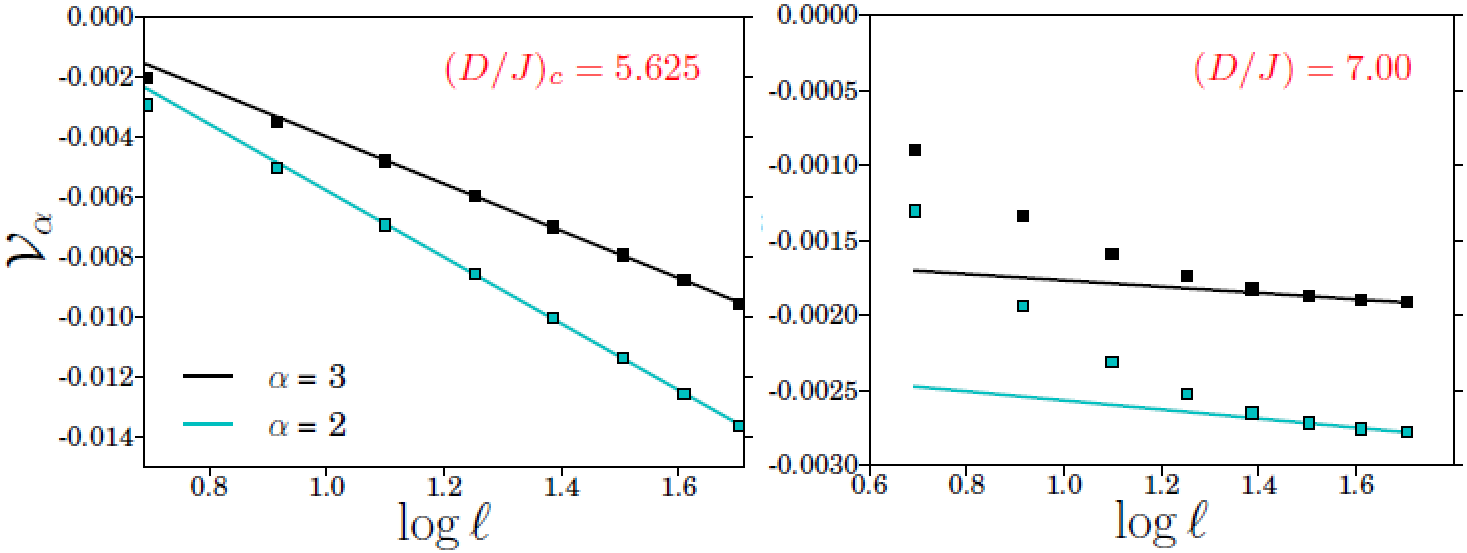
Area-law term has an $(x - x_c)^{\upsilon}$ singularity

Corners always have log singularities

$$s \sim 1/\xi$$
$$c \sim \ln \xi$$

Distinguishing a QCP

– Anisotriopic S=1 model $H = J\sum_{\langle i,j\rangle} \mathbf{S}_i \cdot \mathbf{S}_j + D\sum_i (S_i^z)^2$



$$\ell_A = \frac{1}{2}(n_x + n_y) = \frac{1}{2}\mathcal{O}_A$$

$$\mathcal{V}_\alpha = a_\alpha \log \ell + b_\alpha$$

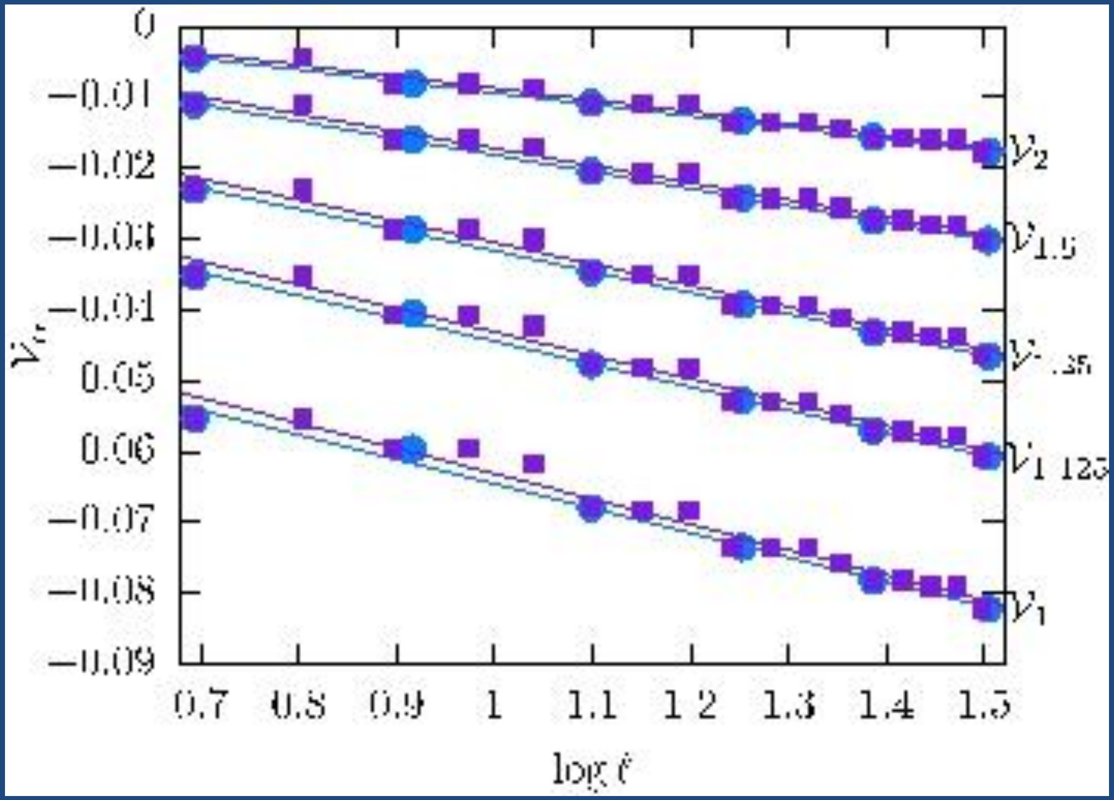
E.M. Stoudenmire, Peter Gustainis, Ravi Johal, Stefan Wessel, Roger G. Melko

Phys. Rev. B 90, 235106 (2014)

Log Divergence of Corner Entropy

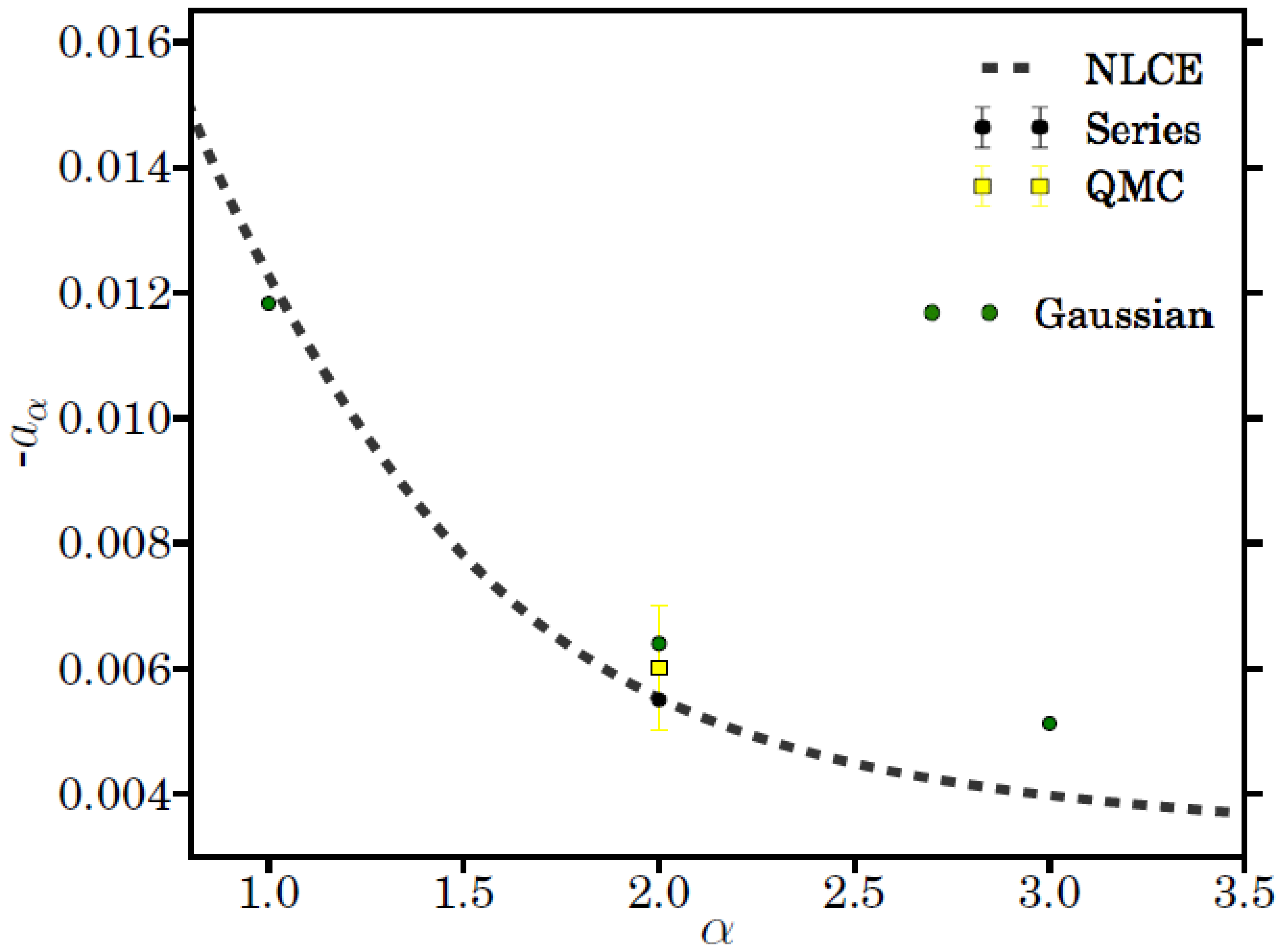
Kallin, Stoudenmire, Fendley, RRPS and Melko cond-mat 2014

Bilayer Heisenberg Model at criticality

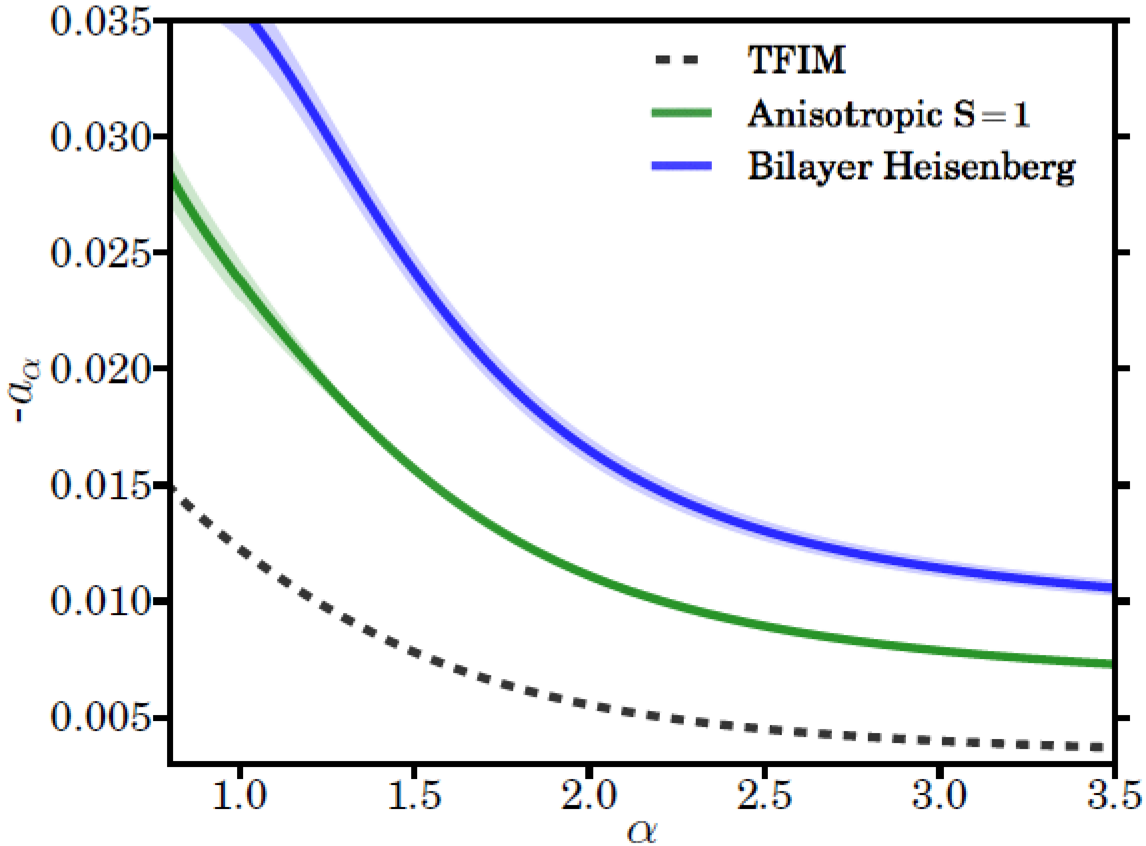


Different Renyi indices

– Transverse field Ising $H = -J \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$ $(h/J)_c = 3.044$

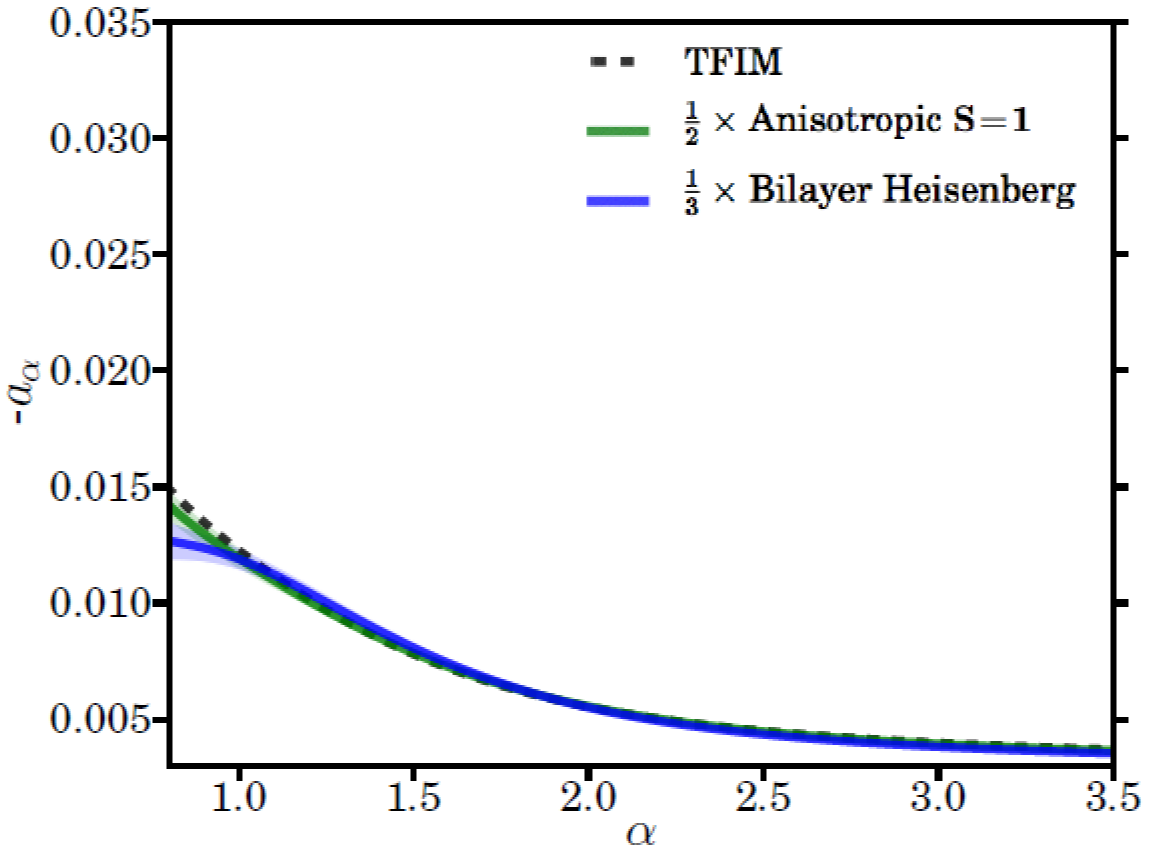


● Casini, Huerta, Nuclear Physics B 764, 183 (2007)



TFIM
Anisotropic S = 1
Bilayer Heisenberg
0.035
0.030
0.025
0.020
0.015
0.010
0.005
$-a_\alpha$
1.0
1.5
2.0
2.5
3.0
3.5
α

Key finding: Corner term scales with N in O(N) models



Bootstrapping the $O(N)$ Vector Models

Filip Kos[a], David Poland[a], David Simmons-Duffin[b]

N	Δ_ϕ	Δ_S	Δ_T	c/Nc_{free}
1	0.51813(5)	$1.4119^{+0.0005}_{-0.0015}$	–	$0.946600^{+0.000022}_{-0.000015}$
2	0.51905(10)	$1.5118^{+0.0012}_{-0.0022}$	$1.23613^{+0.00058}_{-0.00158}$	$0.94365^{+0.00013}_{-0.00010}$
3	0.51875(25)	$1.5942^{+0.0037}_{-0.0047}$	$1.2089^{+0.0013}_{-0.0023}$	$0.94418^{+0.00043}_{-0.00036}$
4	0.51825(50)	$1.6674^{+0.0077}_{-0.0087}$	$1.1864^{+0.0024}_{-0.0034}$	$0.94581^{+0.00071}_{-0.00039}$
5	0.5155(15)	$1.682^{+0.047}_{-0.048}$	$1.1568^{+0.009}_{-0.010}$	$0.9520^{+0.0040}_{-0.0030}$
6	0.5145(15)	$1.725^{+0.052}_{-0.053}$	$1.1401^{+0.0085}_{-0.0095}$	$0.9547^{+0.0041}_{-0.0027}$
10	0.51160	$1.8690^{+0.000}_{-0.001}$	$1.1003^{+0.000}_{-0.001}$	0.96394
20	0.50639	$1.9408^{+0.000}_{-0.001}$	$1.0687^{+0.000}_{-0.001}$	0.97936

Table 3: The values of the scalar and symmetric tensor operator dimensions and the values of the central charge saturating the obtained bound for the $O(N)$ vector model values of Δ_ϕ. For $N = 1, 2, 3, 4, 5, 6$, the value of Δ_ϕ is taken from Table 2; for $N = 10, 20$ the value of Δ_ϕ is the 3-loop large-N result. The errors reflect the uncertainty in the value of Δ_ϕ. In the determinations of $\Delta_{S,T}$ we have also included a contribution to the error due to our bisection precision of 0.001. This uncertainty is only in one direction, since the upper bound is rigorous.

Universality of corner entanglement in conformal field theories

Pablo Bueno,[1] Robert C. Myers,[2] and William Witczak-Krempa[2]

Both corner entanglement and central charge get contributions from all low energy physics

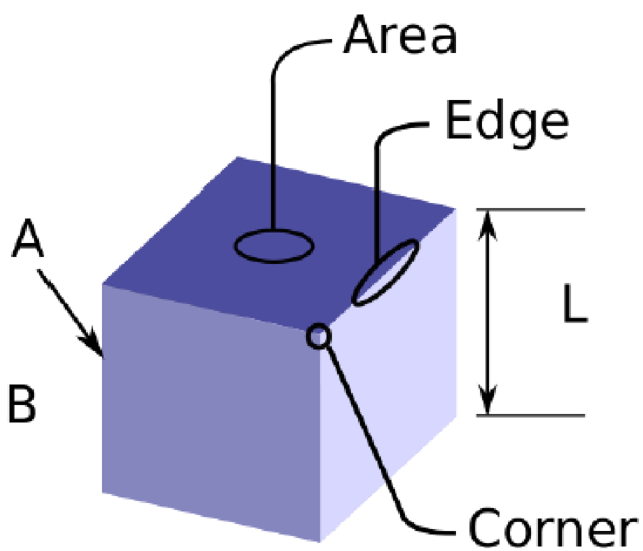
SIMPLE CUBIC LATTICE

Devakul+RRPS PRB 2014

In 3D, we can divide the system by a **cubic** interface, then the entanglement entropy the similar form

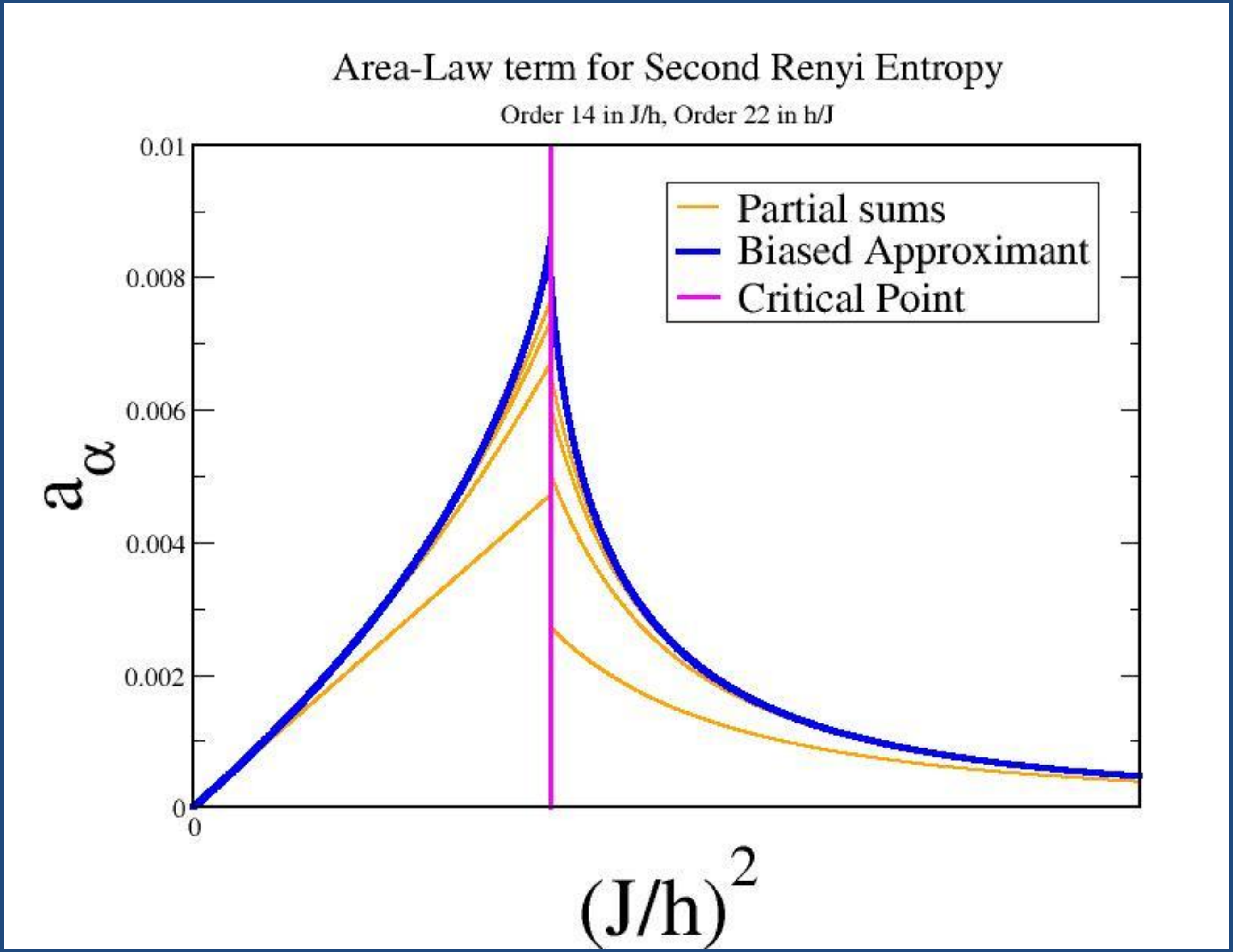
$$\mathcal{S} = a \cdot L^2 + s \cdot L + c + \text{Smaller terms that dissapear as } L \to \infty$$

a : Area contribution
s : Edge contribution
c : Corner contribution



Series expansions can be separately developed for area, edge and corner

TFIM: `Area-law' term from series expansions

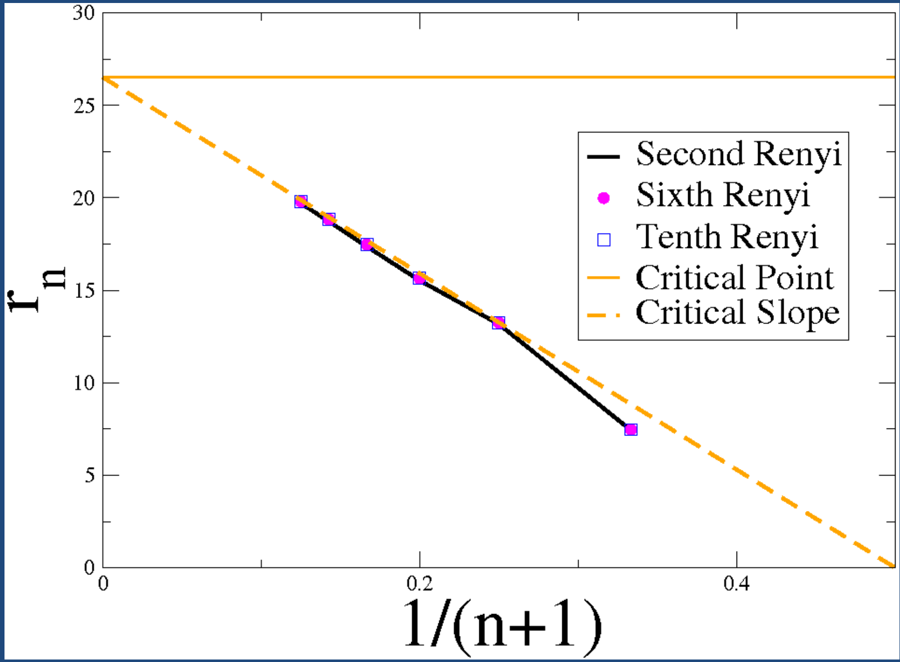


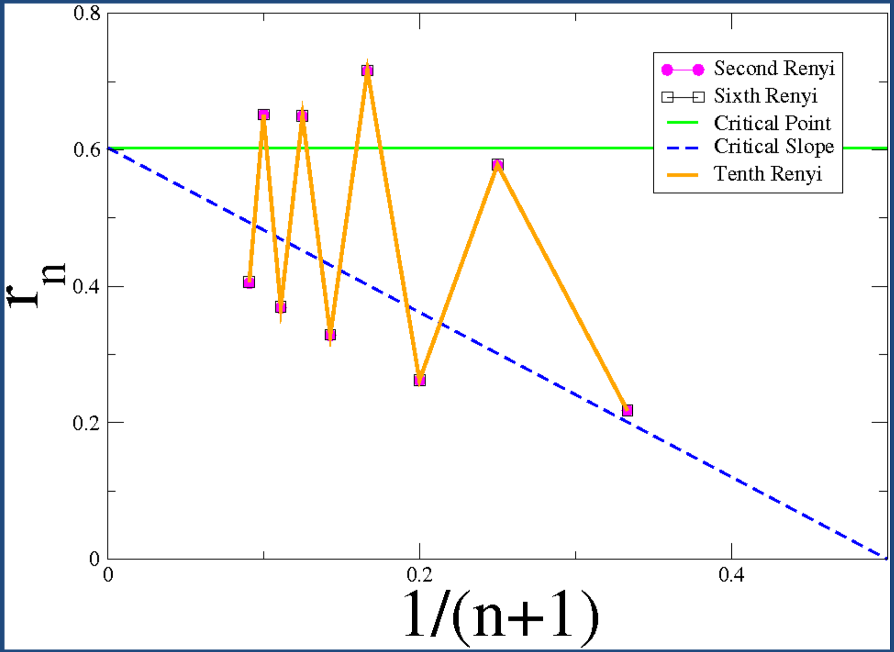
Sharp cusp: $1/\xi^2$ singularity, continuous from both sides

TFIM: Do critical points depend on Renyi Index?

Chandran, Khemani and Sondhi PRL 2014

Ratio of successive coefficients in J/h and h/J
Expected critical point and exponent from scaling





High-field expansion

Low-field expansion

RENYI CORRELATIONS

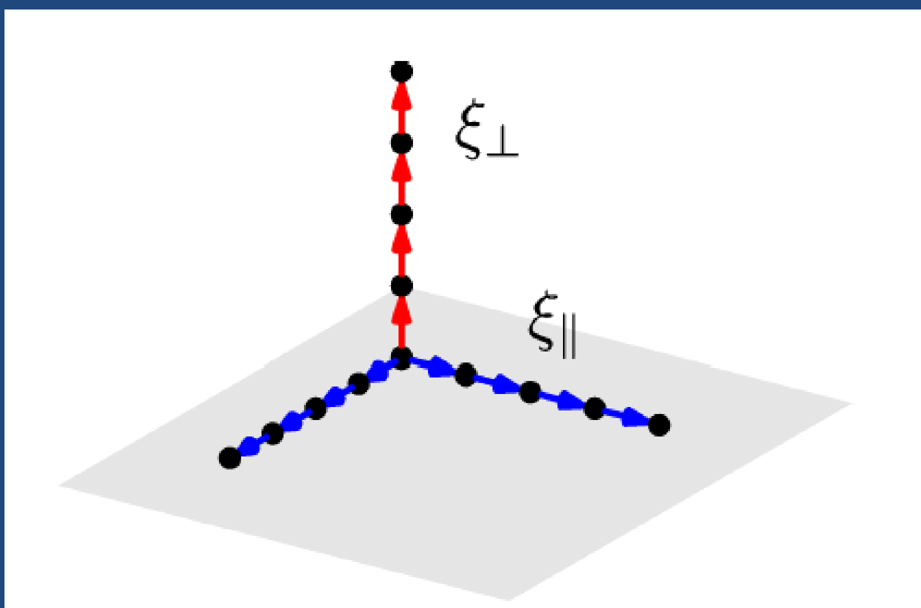
Devakul+RRPS

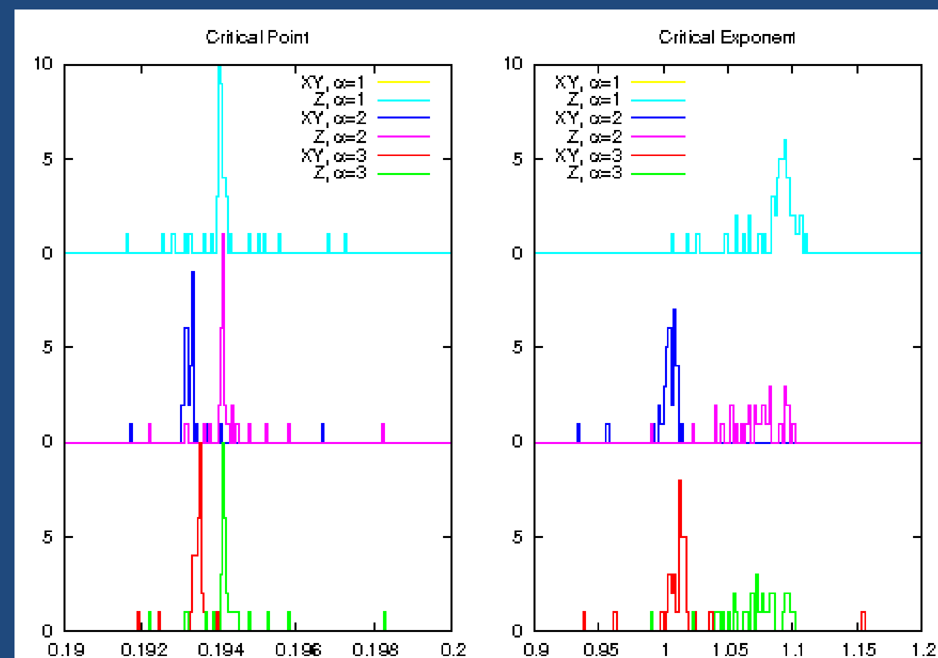
Metlitski

Is there a surface transition? $e^{-\alpha H_E}$

Correlation functions and correlation lengths calculated with Renyi weights $(\rho_A)^\alpha$

$2v$

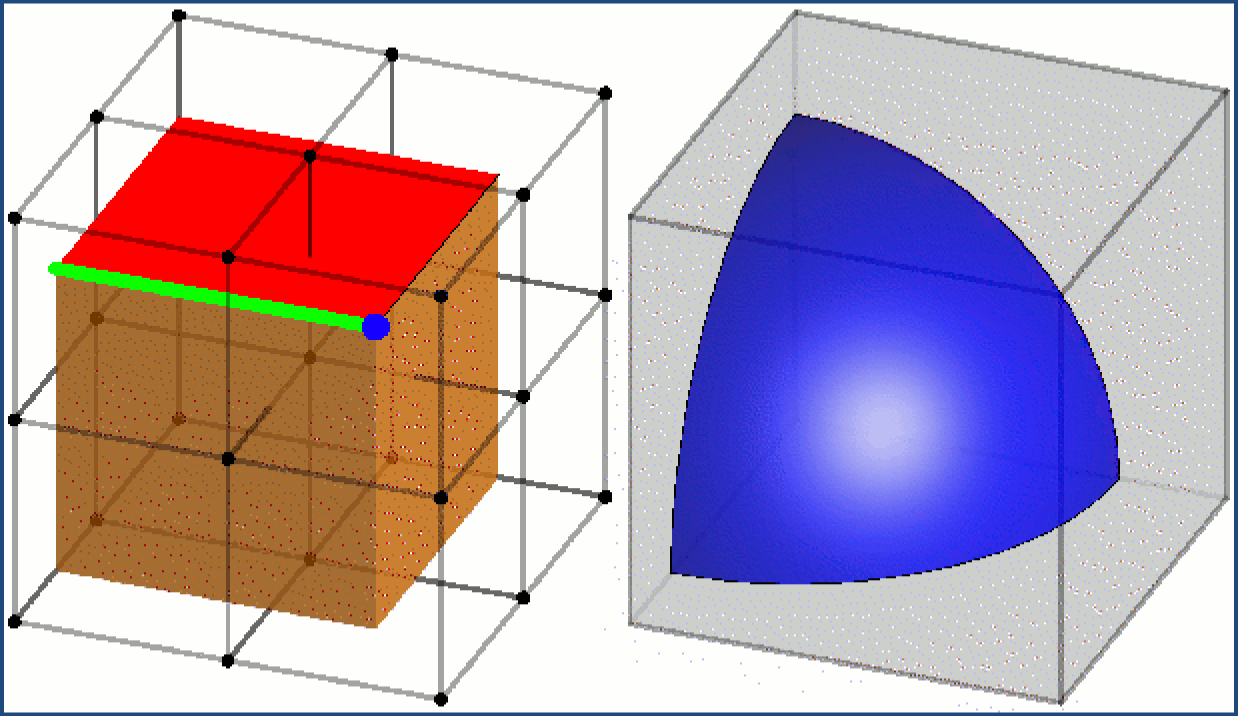




$\alpha = 1$ is usual bulk correlation

Exponents only consistent with d=4 not d=2

Corner singularity and continuum limit T. Grover

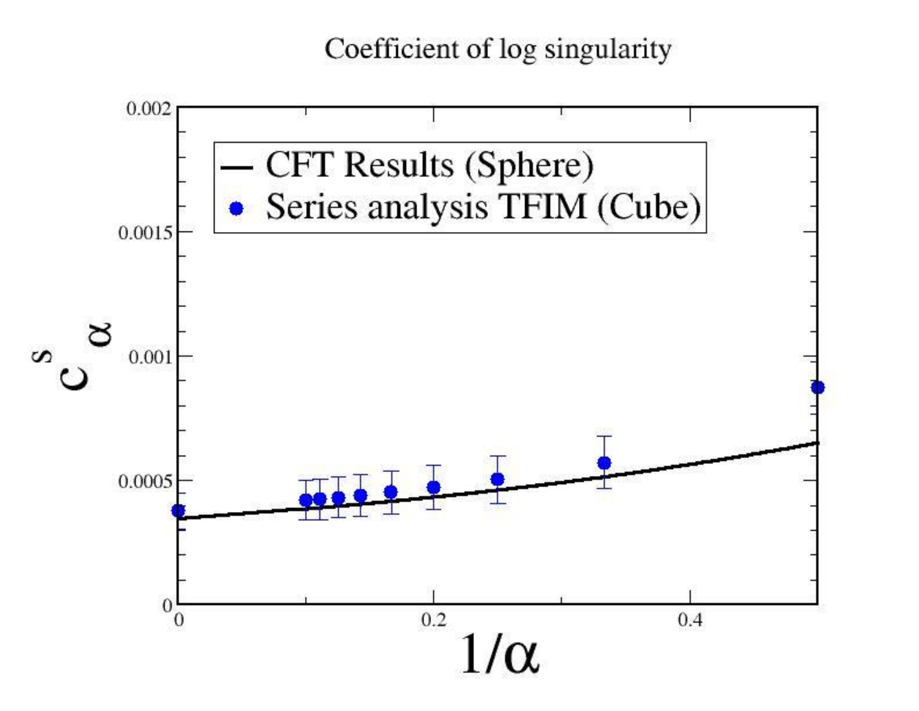


Do 8 corners on a lattice have same log as a sphere in continuum?

Comparison to Field Theory

Casini and Huerta
Lee, McGough and Safdi

$$c_s^\alpha = \frac{1}{720}\frac{(1+\alpha)(1+\alpha^2)}{4\alpha^3}$$



Assume the log coefficients are same from high and low field sides

Summary and Conclusions

Several computational methods are being developed that allow calculation of ground state entanglement entropies of quantum critical lattice models (More results should be coming out soon)

Renyi entropies may be sufficient for studying universal properties

Log singularities at a corner are universal and scale with N in O(N) models

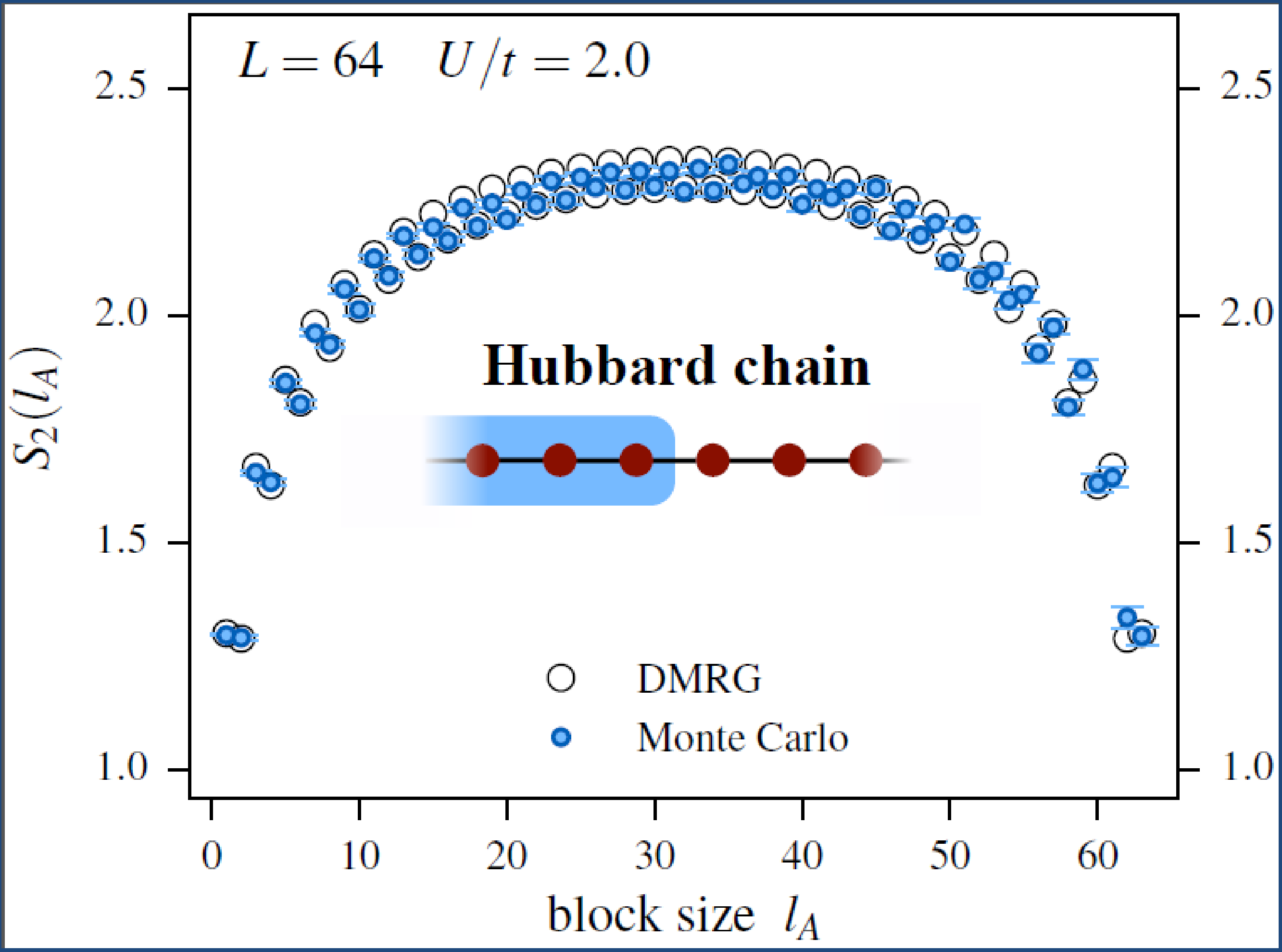
The log singularities for TFIM for a cube are close to free field theory results for a sphere

Can we calculate critical properties for interacting fermion models?

Can we use entanglement entropy to convincingly discover new phases and critical points in realistic lattice models?

THE END

Improved methods by Brocker and Trebst can study fairly large systems
(See also F. Assaad PRB 91, 125146 (2015)



Can they address singularities and phase transitions for d=2?

Projection Monte Carlo in Valence Bond Basis **Sandvik**

A singlet state can be expanded in the Valence Bond basis

$$|0\rangle = \sum_k f_k |(a_1^k, b_1^k)\cdots(a_{\frac{N}{2}}^k, b_{\frac{N}{2}}^k)\rangle = \sum_k f_k |S_k\rangle$$

Applying powers of the Hamiltonian, one can project out the ground state

$$[-(H-C)]^n|\Psi\rangle = \sum_k g_{n,k}|S_k\rangle \rightarrow c_0|E_0 - C|^n|0\rangle$$

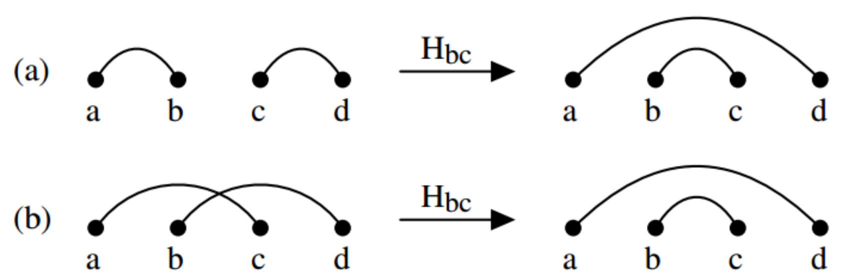
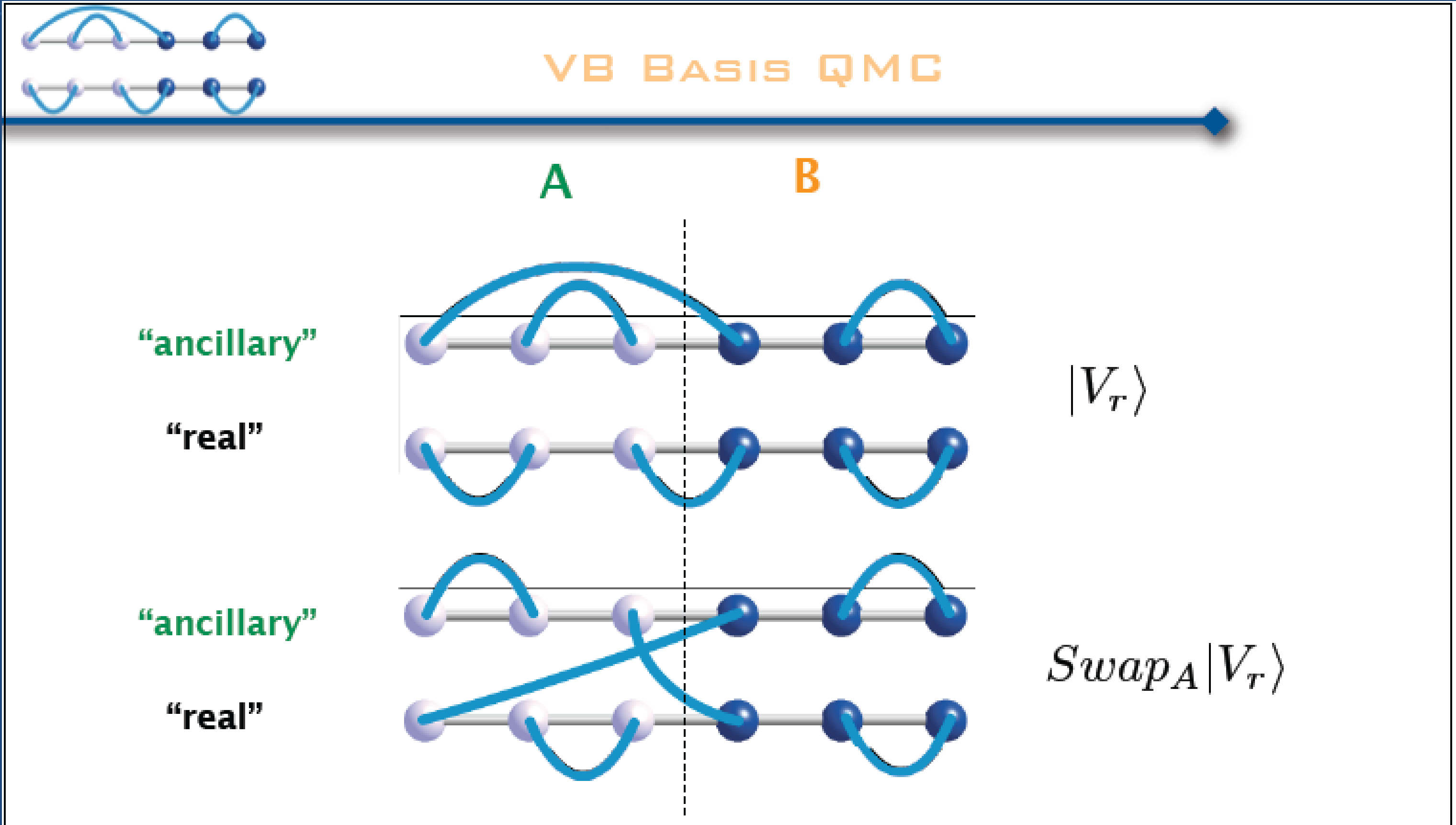


FIG. 1: Action of a bond operator on two VB states.

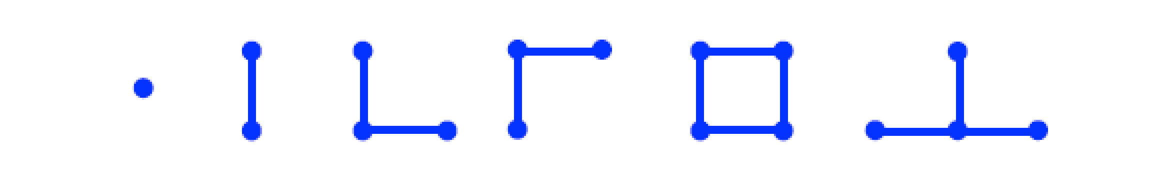
Free of minus signs on a bipartite lattice

Heisenberg Model: Valence Bond Monte Carlo



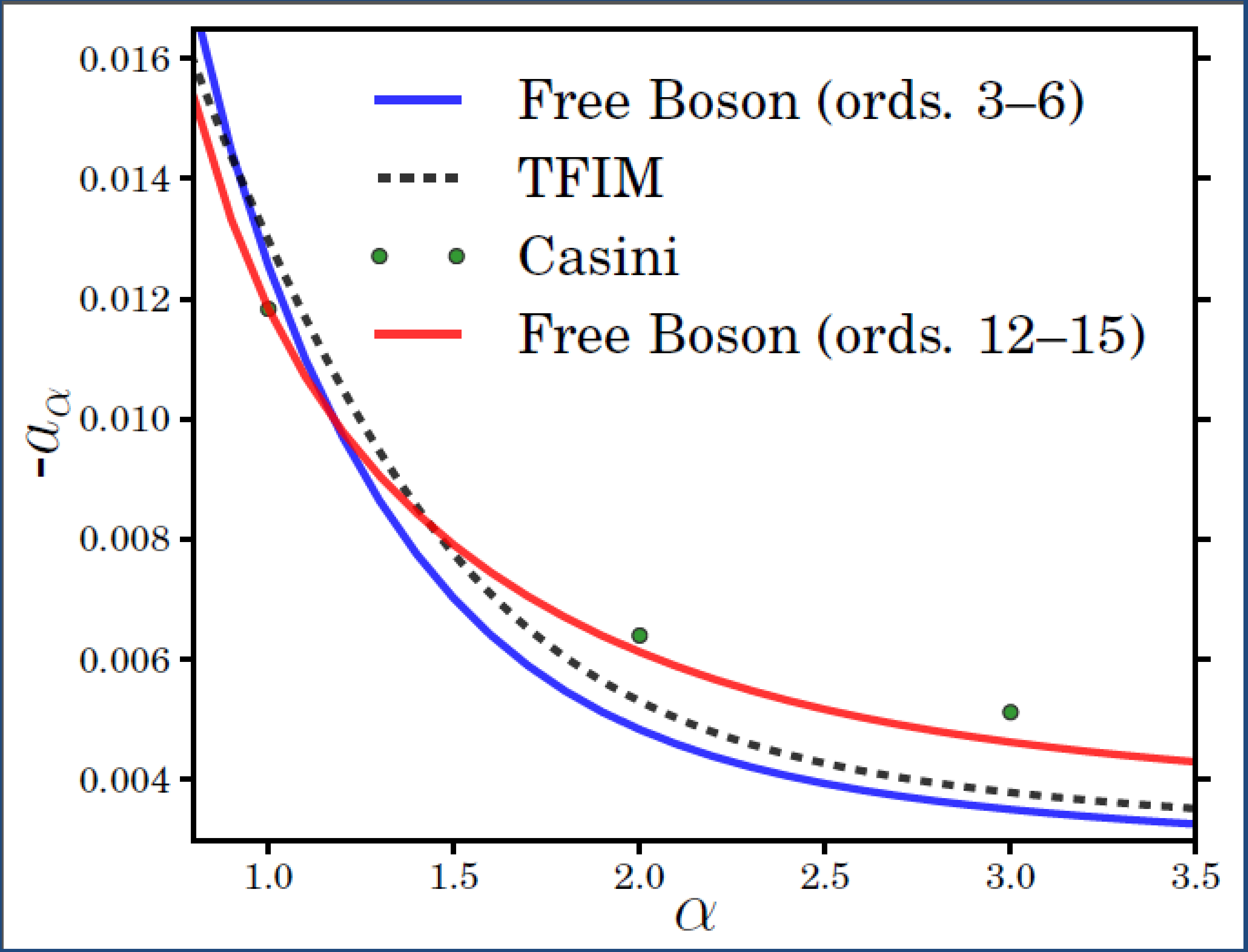
$$S_2(\rho_A) = -\ln(\mathrm{Tr}(\rho_A^2)) = -\ln(\langle Swap_A \rangle)$$

Connected clusters on the square-lattice



n	Connected	Sym. distinct	Topo.
1	1	1	1
2	2	1	1
3	6	2	1
4	19	5	3
5	63	12	4
6	216	35	10
7	760	108	19
8	2725	369	51
9	9910	1285	112
10	36446	4655	300
11	135268	17073	746
12	505861	63600	2042
13	1903890	238591	5450
14	7204874	901971	15197
15	27394666	3426576	42192
16	104592937	13079255	119561
17	400795844	50107909	339594

Tang, Khatami, Rigol
Computer Physics Communications 184, 557-564 (2013)


Free Boson (ords. 3–6)
TFIM
Casini
Free Boson (ords. 12–15)
$-a_\alpha$
0.016
0.014
0.012
0.010
0.008
0.006
0.004
1.0
1.5
2.0
2.5
3.0
3.5
α