Closing the entanglement gap, June 1-5, 2015

On deformations of entanglement

(with V. Rosenhaus)

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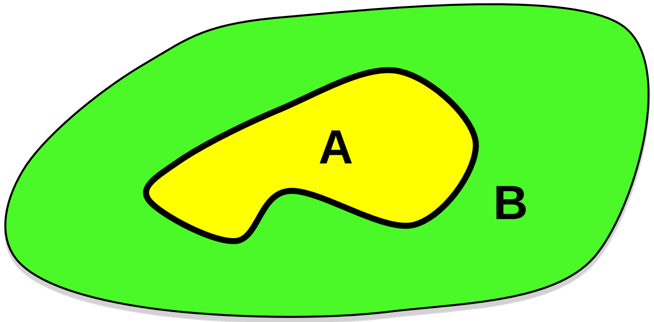
Motivation and main result

- *QFT state is vacuum* $|0\rangle$
- *trace out degrees of freedom associated with B:*

$$\rho_A \equiv Tr_B |0\rangle\langle 0|$$



- *calculate EE:*

$$S_{EE} = -Tr_A\left[\rho_A \log \rho_A\right] = Tr_A\left[\rho_A K_A\right], \; K_A \equiv -\log \rho_A$$

- *Two ingredients: geometry (entangling surface, background) and QFT*
- *Hard to compute. Existing tools: replica trick, holography, numerical*

Why deformations of entanglement are interesting?

- *Provides a new computational method*
- *Understanding the structure of the bulk using AdS/CFT and Ryu-Takayanagi*

Faulkner, Guica, Hartman, Myers, Van Raamsdonk '13

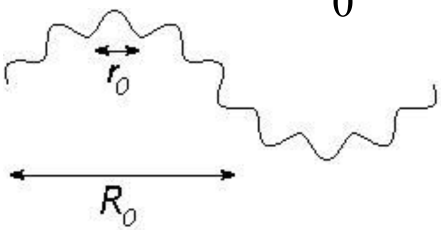
- *RG flows, C-theorems (in this context couplings and hence entanglement change)*

talk by H.Casini

- *Even if the geometry is flat at large (length) scale,* ℓ *, there are short wavelength geometric structures which contribute to EE at scales* $< \ell$

Jackson, Pourhasan, Verlinde '13

$\ell \sim r_0$



$\ell \sim R_0 \; ? \; r_0$

$\ell \gg R_0$

Main result:

$$I = I_0 + \lambda\, O - \frac{1}{2}\int \sqrt{g}\; T^{\mu\nu}\delta g_{\mu\nu} + \ldots$$

$$\frac{\partial S_{EE}}{\partial \lambda} = -\langle O\, K_A \rangle \;, \quad \frac{\delta S_{EE}}{\delta g_{\mu\nu}(x)} = \frac{\sqrt{g(x)}}{2}\langle T^{\mu\nu}(x)\, K_A \rangle$$

- *2 eqns. are in fact 1 eqn. if one thinks of the coupling constant as background field*
- *no replica trick!* The computation of entanglement entropy is reduced to the computation of correlation function

Explanation

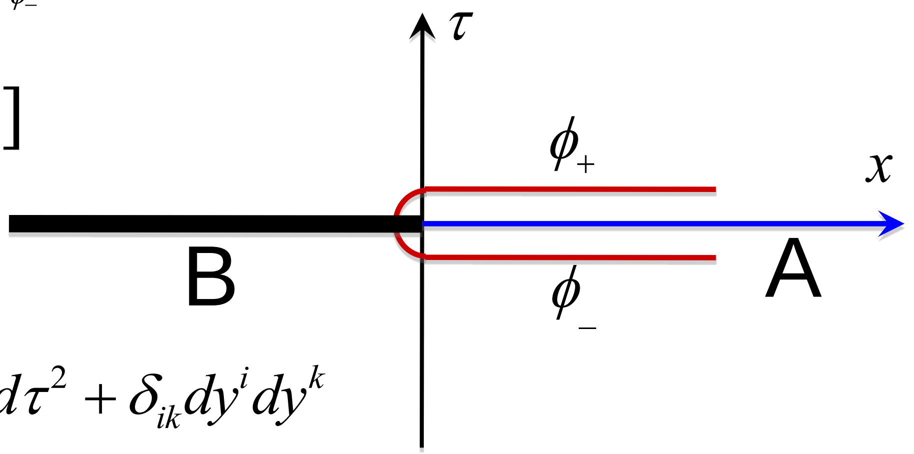
$$\frac{\partial S_{EE}}{\partial\lambda} = \frac{\partial Tr_A\left[\rho_A K_A\right]}{\partial\lambda} = Tr_A\left[\frac{\partial\rho_A}{\partial\lambda}K_A + \rho_A\frac{\partial K_A}{\partial\lambda}\right] = Tr_A\left[\frac{\partial\rho_A}{\partial\lambda}K_A\right] + \cancelto{0}{\frac{\partial}{\partial\lambda}Tr_A\rho_A}$$

Marolf,Minic,Ross '03 Bhattacharya,Nozaki,Takayanagi,Ugajin '12 Blanco,Casini,Hung,Myers '13
G.Wong, I.Klich, L.A.Pando Zayas, D.Vaman '13

Path integral representation of the density matrix:

$$\langle\phi_-|\rho_A|\phi_+\rangle = \langle\phi_-|Tr_B\left(|0\rangle\langle 0|\right)|\phi_+\rangle = \int_{\substack{\phi(0^+,x>0)=\phi_+\\ \phi(0^-,x>0)=\phi_-}} D\phi\exp\left[-I(\phi)\right]$$

$$\langle\phi_-|\frac{\partial\rho_A}{\partial\lambda}|\phi_+\rangle = -\int_{\substack{\phi(0^+,x>0)=\phi_+\\ \phi(0^-,x>0)=\phi_-}} D\phi\, O\exp\left[-I(\phi)\right]$$



For a plane in flat space: $ds^2 = dx^2 + d\tau^2 + \delta_{ik}dy^i dy^k$

O(2) symmetry $\Rightarrow K_A = -2\pi\int_A T_{\mu\nu}\,\xi^\mu\, n^\nu = -2\pi\int_\Sigma\int_0^\infty dx\, x\, T_{\tau\tau}$

Bisognano,Wichmann '75,
Kabat,Strassler `94

Example: relevant perturbations

$$\frac{\partial S_{EE}}{\partial \lambda} = -\langle O\, K_A \rangle \ , \quad \frac{\partial^2 S_{EE}}{\partial \lambda^2} = \langle O\, O\, K_A \rangle - \langle O\, O \rangle \, , \;\; K_A = -2\pi \int_\Sigma \int_0^\infty dx\; x\; T_{\tau\tau}$$

- *Perturbed CFT*

$$I = I_{CFT} + \lambda \int d^d x\; O(x) \quad , \quad \langle O(x)\, O(0) \rangle = \frac{N}{x^{2\Delta}}$$

$$\langle T_{\mu\nu}(x_1)\, O(x_2) O(x_3) \rangle$$ *see H.Osborn, A.Petkous '93*

$$\delta S_{univ} = 0 * \lambda \log(\delta / l) + N\lambda^2 \frac{d-2}{4(d-1)} \frac{\pi^{\frac{d+2}{2}}}{\Gamma\left((d+2)/2\right)} A_\Sigma \log(\delta / l) \;\; , \;\; \Delta = \frac{d+2}{2}$$

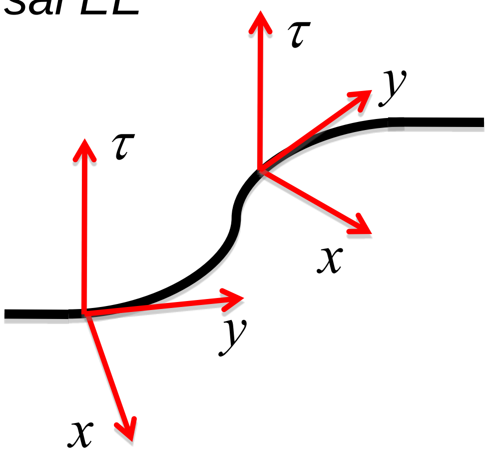
Comments:

- *same answer for strongly/weakly coupled CFT* — *A.Lewkowycz, R.C.Myers, MS '13*
- *agrees with holographic calculations* — *L-Y. Hung, R.C.Myers, MS '11*
- *will show up in any background and for any entangling surface* — *P. Jones, M.Taylor '15*
- *we ignored contact terms*
- *for spherical entangling surfaces see* — *T. Faulkner '14*

Geometric perturbations

$$\frac{\delta S_{EE}}{\delta g_{\mu\nu}(x)} = \frac{\sqrt{g(x)}}{2}\langle T^{\mu\nu}(x)\, K_A \rangle \;\Rightarrow\; \delta S_{EE} = \frac{1}{2}\int \sqrt{g(x)}\langle T^{\mu\nu}(x)\, K_A \rangle \delta g_{\mu\nu}(x) + \dots$$

- *universal EE in even space-time dimensions is local (log divergences). Therefore vicinity of the entangling surface is enough to compute universal EE*



- *For static space-time use Gauss normal coordinates*

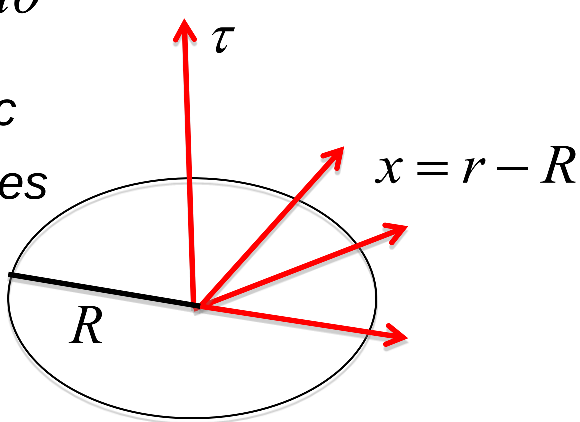
$$ds^2 = d\tau^2 + dx^2 + g_{ij}(x,y)dy^i dy^j$$

$$g_{ij}(x,y) = \gamma_{ij} + 2K_{ij}x + (R_{ixxj} + K_{il}K^l_j)x^2 + O(x^3)$$

Example (sphere of radius R)

$$ds^2 = d\tau^2 + dr^2 + r^2 d\theta^2 = d\tau^2 + dx^2 + \left(1 + 2\frac{x}{R} + \frac{x^2}{R^2}\right) R^2 d\theta^2$$

- *normal neighborhood can be also constructed for non-static space-times. Applications in holography and squashed cones*



A.Lewkowycz, J.Maldacena '13

D.Fursaev, S.N.Solodukhin,A. Patrushev '13

X.Dong '13 J.Camps '13

- *Final result for a plane in a weakly perturbed 4D space-time*

$$\delta S_{EE} = \frac{c}{2\pi}\int_{\Sigma} d^2x\, W^{ijkl}\delta_{ik}\delta_{jl}\,\log(l/\delta)$$

- *Universal EE for 4D CFT* S.N.Solodukhin ‘08

$$S_{EE} = \frac{1}{2\pi}\int_{\Sigma} d^2x\sqrt{\gamma}\left[c\left(W^{ijkl}\gamma_{ik}\gamma_{jl} - \tilde K_{ik}\tilde K^{ik}\right) - a\, R_{\Sigma}\right]\log(l/\delta)$$

where $\tilde K_{ik}$ *is the traceless extrinsic curvature and the constants are defined by*

$$\langle T^{\mu}_{\mu}\rangle = \frac{c}{16\pi^2}W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma} - \frac{a}{16\pi^2}E_4$$

passed many checks: numerical M.Huerta ’12 ,

holographic L-Y.Hung,Myers,MS ’11, X.Dong ’13

derivation using squashed cones D.Fursaev,A.Patrushev, S.N.Solodukhin ’13

- *Topological term can be derived without use of the replica trick*

  H.Casini,M.Huerta,R.C.Myers ’11

- *No direct derivation (without replica trick) of the shape-dependent term*

Summary and open questions

- *structure of the modular Hamiltonian* *A.Lewkowycz, E. Perlmutter '14*
- *perturbations of spherical entangling surface* *A.Allais,M.Mezei '14, M.Mezei '14*
- *relevant deformations of a CFT for spherical entangling surface* *T. Faulkner '14*
- *area terms in EE and renormalization of Newton's constant* *H.Casini, F.D.Mazzitelli,E.Teste'14*

- *C-theorems, C-functions in d>3?*
- *beyond linear Einstein-Hilbert equations of motion in the bulk?*
- *universal EE for non-smooth entangling surfaces?* *P. Bueno, R.C. Myers, W. Witczak-Krempa '15*
- *calculation of shape dependent terms of universal EE?*
- *field theory techniques for EE computation in the case of strongly coupled systems out-of-equilibrium?*

The End

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Thank You!