ER=EPR but entanglement is not enough

What's makes quantum information so different?

Two things:

Entanglement: you can know everything about a system and know nothing about its parts.  

The capacity for exponentially large complexity.

Leonard Susskind, Stanford

KITP Entangled15
Conference, Jun 4, 2015
Ryu-Takayanagi construction

But what if the black holes are entangled?
How can entangled black holes be created?

Pair creation in an electric field

Collapse of entangled clouds
ER = EPR
ER = EPR
Einstein-Rosen bridges grow with time.

\[ \frac{dV}{dt} = A = S \cdot G \]
Einstein-Rosen bridges grow with time.

\[
\frac{dV}{dt} = \frac{A}{\ell_{ads}} = S \ T \ G
\]

A paradox:

Something is growing but the black holes are in perfect thermal equilibrium? What quantity in the holographic dual continues to grow long after equilibrium?
Computational Complexity

A classical example

Cellular Automaton

Simple state

Simple state
Run

Simple process

Complexity = minimum number of simple steps needed to get to configuration.

Not generally number taken by the CA.
\[ C_{max} = \frac{N}{2} \]

\[ S_{max} = N \log 2 \]
Classical

(001011100100101010......01) \quad N \text{ binary digits}

Quantum

Basis \quad |i\rangle = |001011100100101010......01\rangle

General state \quad |\Psi\rangle = \sum_{i=1}^{2^N} f(i) |i\rangle \quad 2^N \text{ complex numbers}
simple state $|000\ldots\ldots0\rangle$

simple process \[\square\] gate

Cellular automaton $\rightarrow$ quantum circuit

Simple state $|000\ldots\ldots0\rangle$

\[\text{gates}\]

Output $|\Psi\rangle$

Complexity = minimum number of gates to go from $|000\ldots\ldots0\rangle$ to $|\Psi\rangle$. 
\[ C_{\text{max}} = \exp N \]
\[ S_{\text{max}} = N \log 2 \]

Time to achieve \( S_{\text{max}} \sim \log N \)

Time to achieve \( C_{\text{max}} \sim \exp N \)
What goes on between $S_{max}$ and $C_{max}$?
Compare

\[
\frac{dC}{dt} = ST = \frac{A}{G \ell_{ads}}
\]
\[
dV/dt = A
\]

\[
C = \frac{V}{G \ell_{ads}}
\]

Coincidence?

Precursors geometries provide a strong test.

Shenker Stanford
Stanford  LS
Additional complexity due to $U(t) W U(-t)$

$$\Delta C = 2 S T t \quad \text{naively}$$

But what if $W = 1$? Then there is cancelation and $\Delta C = 0$.

$$\Delta C = 2 S T \left( t - \frac{l_{ads}}{2\pi} \log S \right)$$
\[ \Delta V = 2SG(t_w - t_*) \]

\[ t_* = \frac{l_{ads}}{2\pi} \log S \]

D. Stanford, LS
D. Roberts, D. Stanford, LS
LS, Y. Zhao
Implications

The holographic encoding of space behind the horizon is different. It is built out of the extremely subtle correlations that define quantum complexity, not the ordinary correlations that define entanglement and entropy.

Complexity and geometry are dual to one another: complexity of the gauge theory state, and geometry of the interior.

The classical evolution of geometry cannot persist for all time. Complexity and therefore volume must stop growing at times of order $\exp S$.

Questions

Does complexity actually grow for an exponential time or does it stall? What are the implications one way or another? (Scott Aaronson, LS to appear some day)

Do we need new non-linear rules for QM behind the horizon or is standard QM ok?

Is the growth of space always connected with the growth of complexity. Cosmological growth?
Different tools are needed to understand the emergent space behind horizons. Entanglement is not enough. Complexity theory will be one of these tools.

Questions:

Do $V$ and $C$ grow like $t$ for exponential time?
What are the implications for CS?
$PSPACE \not\subseteq BQP/poly$  Scott Aaronson, LS

Is the bulk-boundary dictionary behind the horizon non-linear (state dependent)?

Does the growth of ERB's and its relation to complexity tell us anything about the growth of cosmological space?
Entanglement is the hooks that hold space together.
From the outside black Holes are thermodynamic systems. They quickly come to almost perfect thermal equilibrium, maximizing their entropy. Their entropy is proportional to the area of their horizons. But you can fall through the horizon into an interior region.
Version 1

Alice and Bob independently create black holes.
V2

Alice and Bob create black holes out of entangled matter.
ER = EPR
EPR = ER

J. Maldacena, LS
Both Entanglement and ERBs are forms of non-local connectivity but they don't violate any physical laws or principles.

Entanglement cannot be used to send signals by local operations.

Signals cannot be sent through an ERB by local operations.

(ERBs are non-traversable)
Volume of an ERB grows linearly

\[ \frac{dV}{dt} = G_N \cdot S \]
Signals from outside to outside have to go the long way.
Einstein-Rosen bridges grow with time.

\[ \frac{dV}{dt} = G_N S \]
Paradox:
Black holes come to thermal equilibrium extremely rapidly (a millisecond for a solar mass black hole).

But the interior geometry grows for an extremely long time.

Q: What property of the quantum state continues to evolve long after thermal equilibrium?

Why is there an "arrow of time in the ERB?"
Might entanglement continue to spread among the degrees of freedom of the black holes?

No, that's part of coming to thermal equilibrium and it's over very quickly.

Entanglement is not enough.
(Quantum) Computational Complexity
The program is the laws of motion.
The system can be initialized.
It updates itself.
One can measure the outcome.
a cellular automaton
What is complexity?

Simple state

Simple process

Complexity is the minimal number of simple processes required to produce a given state from a simple state.
Maximum complexity and maximum entropy

\[ C_{\text{max}} = \frac{N}{2} \quad \text{almost all} \]

\[ S_{\text{max}} = N \log 2 \]

When a classical system of bits reaches maximal entropy it also reaches maximal complexity.
Quantum States are far more complex.  

\[ |\psi\rangle = \sum_{1}^{2^N} C_i |i\rangle \]

Specify \(2^N\) complex coefficients.
Simple state: Same as before.

Simple process:

Unitary quantum gate

\[ u \]
Complexity is the minimum number of gates in any circuit that can simulate $|\Psi\rangle$.
$C_{\text{max}} \sim e^N$  

almost all

$S_{\text{max}} = N \log 2$
Chaotic Systems

Initially quantum complexity grows linearly with time, but it continues to grow long past the time that the entropy reaches its maximum.

It is much too subtle to affect the thermal properties of a system in an observable way.
Volume of an ERB grows linearly

\[ \frac{dV}{dt} = G_N \ S \]

Complexity of a chaotic quantum system grows linearly

\[ \frac{dC}{dt} = S \]

Volume – Complexity duality

\[ C = \frac{V}{G_N} \]
What evidence do we have for C/V duality? Lots. Precursors are good probes.

Consider Alice's lunch date problem
Added Complexity

\[ = 2S t_w \]
Added Complexity

\[ = 2S(t_w - t_*) \]

\[ t_* = \log S \]
Added Volume

\[ = 2S(t_w - t_*) \]

\[ t_* = \log S \]
The two essential differences that make quantum information quantum.

1. Entanglement: the hooks that hold space together.

2. The enormous capacity for complexity: It allows volume to grow almost indefinitely.
Potential for deep connections with theoretical computer science and complexity theory.

Black Holes and Complexity Classes

Scott Aaronson¹ and Leonard Susskind²

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Abstract

The purpose of this paper is to prove a theorem connecting the growth of complexity in certain types of quantum circuits, with plausible but unproved relations between complexity classes; for example that PSPACE is not contained in BQP/X, where X could stand either for polynomial or sub-exponential. The theorems are suggested by two conjectures about black holes. The first is that the volume of an Einstein-Rosen bridge is dual to the complexity of the quantum state of the black hole. The second is the the growth of an ERB follows the classical behavior as long as possible, namely for a time exponential in the entropy of the black hole. Showing that PSPACE is not contained in BQP/X would give evidence for the latter conjecture, and conversely.
a "computer"

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Simple state: Same as before.

Simple process:

Unitary quantum gate

Complexity is the minimum number of gates required....
$C_{\text{max}} \sim e^N$  

$S_{\text{max}} = N \log 2$
Quantum complexity may continue to grow long after entropy reaches its maximum.
(universal and unitary)
\[ U^t = \ldots \]

\[ |\Psi(t)\rangle = U^t |\Psi\rangle \]

The complexity of such circuits grows linearly in time.

\[
\frac{dC}{dt} = S T
\]

for some period.
Volume of ERB grows linearly
\[ \frac{dV}{dt} = S \, G \]

Complexity grows linearly
\[ \frac{dC}{dt} = S \]

Volume of Einstein-Rosen bridge \( \sim \) complexity of quantum state
\[ V = C \, G \]
What evidence do we have for C/V duality?
Lots. Precursors are good probes.

Consider Alice's lunch date problem
Precursor operators

\[ U(t) W U(-t) \]
$U_R(-t)$
$U_R(t)W U_R(-t)$
Additional Complexity due to precursor

\[ \dot{U}_R(t) W \dot{U}_R(-t) : \]

\[ \Delta C = 2 S (t - t^*) \]
All this was pure quantum circuitry and counting the minimum number of gates needed to make $U(t)WU(-t)$.

On the other hand the Einstein field equations of GR can be solved including the reaction of the geometry to the shock wave.

S. Shenker, D. Stanford
The effect of the shockwave is to add a segment of volume $\Delta V$ to the bridge.
$\Delta V$ is calculated by solving the Einstein GR equations with the shockwave as source.

$$\Delta V = G_N \Delta C$$
Assuming $V \sim C$ there is an upper bound on how long classical GR can describe the ERB.

\[ C_{\text{max}} \approx \exp\{S\} \approx t_{\text{max}} \]

\[
\frac{dC}{dt} = S \\
\frac{dV}{dt} = S \cdot G
\]
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J. Maldacena, LS
Complexity increases. But is it interesting?
It depends who asks.

No for a condensed matter physicist interested in thermodynamics, transport, correlation functions....

Computer scientist? The system could be a quantum computer solving harder and harder problems or even a theorem-proving machine. The growth of complexity is closely related to fundamental questions about how powerful QC are: questions such as: Is PSPACE contained in BQP/poly?
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Black hole theorist? The growth of complexity governs the emergence of the space behind the horizon.
$ER = EPR$
Einstein-Rosen bridges grow with time.

\[ \frac{dV}{dt} = \frac{A}{\ell_{ads}} = S \, T \, G \]