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Surface/State Correspondence as a Generalized Holography

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Based on

[1] arXiv:1412.6226 (to be published in JHEP) -> QE in Boundary state

[2] arXiv:1503.08161 (to be published in PTEP)-> SS-duality proposal

[3] arXiv:1506.01353 (appeared last night) -> SS-duality in cMERA

Collaborators:

YITP, Kyoto: Masamichi Miyaji [1,2,3], Tokiro Numasawa [3],
Noburo Shiba [3], Kento Watanabe [3],
Illinois, Urbana–Champaign: Shinsei Ryu [1] and Xueda Wen [1].

Thanks to discussions with Horacio Casini and Xiao-liang Qi

① Introduction

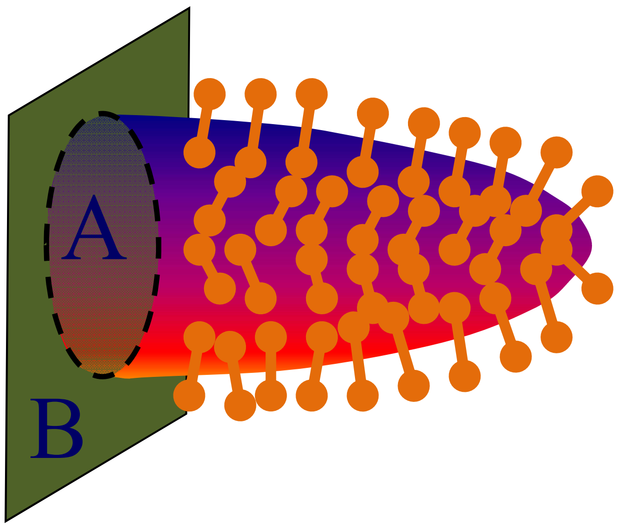
The main purpose of this talk:

developing a little forward the fascinating idea of

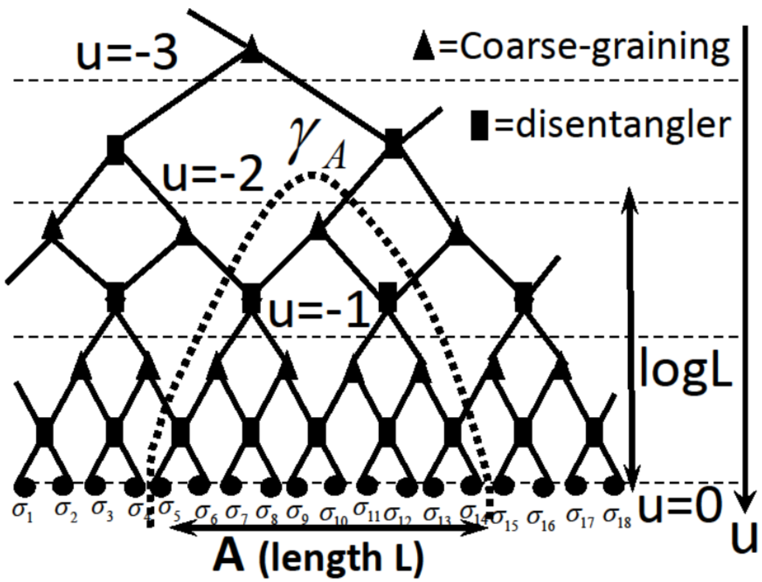
emergent spacetimes from tensor-networks

[Swingle 2009,… ; Vidal's overview, Czech's talk, Preskill's talk,..]

``quantum entanglement ~ a bit of spacetime''.



MERA [Vidal 2005,…]



Our strategy

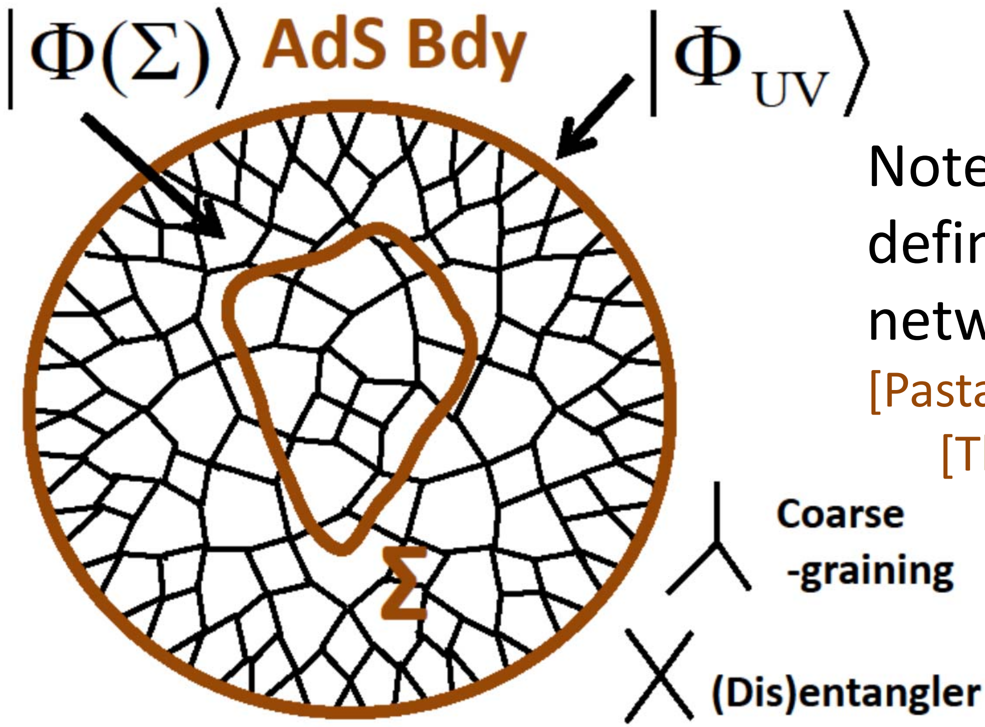
(1) Discrete lattice models of tensor networks seem to have lattice artifacts, which are absent in CFTs.

⇒ Take the **continuum limit** directly: **cMERA** .

(2) Structures of tensor networks are described by **Surface/State correspondence**. This is useful in cMERA.

⇒ Employ SS-correspondence as a fundamental principle.

Surface/State Correspondence in Tensor Network



Note: this procedure is well-defined in selected tensor networks e.g.

[Pastawski-Yoshida-Harlow-Preskill 15]

[Thanks to Xiao-Liang Qi]

Σ Codim. two convex surface in Gravity ⟷ **Dual** $|\Phi(\Sigma)\rangle \in H_{dual}$

Contents

② Boundary States as Unentangled States and cMERA

(2-1) cMERA

[Haegeman-Osborne-Verschelde-Verstraete 11; Vidal's review; reformulation and AdS/CFT interpretation: Nozaki-Ryu-TT 12]

The cMERA formulation is defined by

$$\underbrace{|\Phi(u)\rangle}_{\text{State at scale u}} = P\cdot\exp\left(-i\int_{u_{IR}}^{u} ds\,\hat{K}(s)\right)\cdot\underbrace{|\Omega\rangle}_{\text{IR state}}.$$

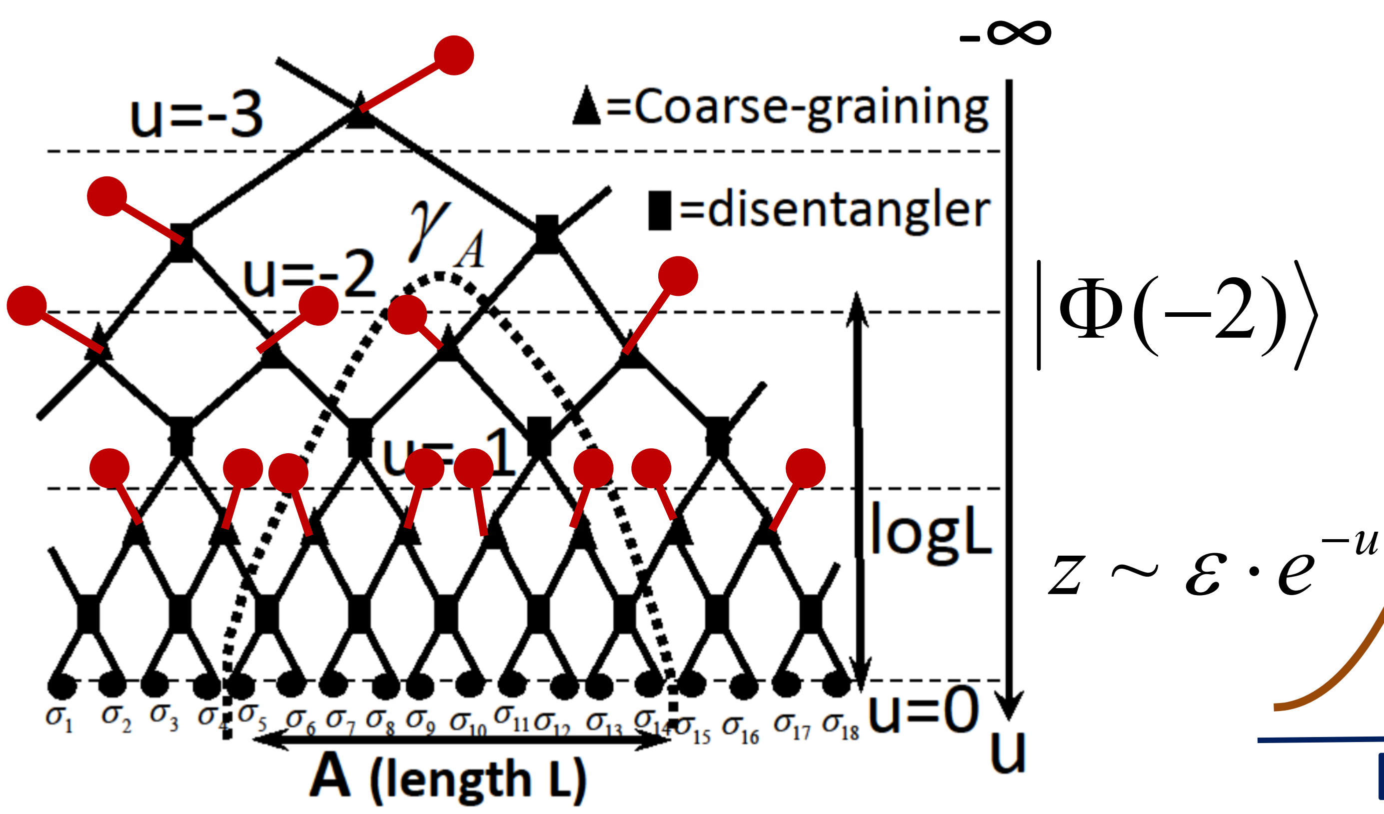
$$u_{IR} = -\infty$$

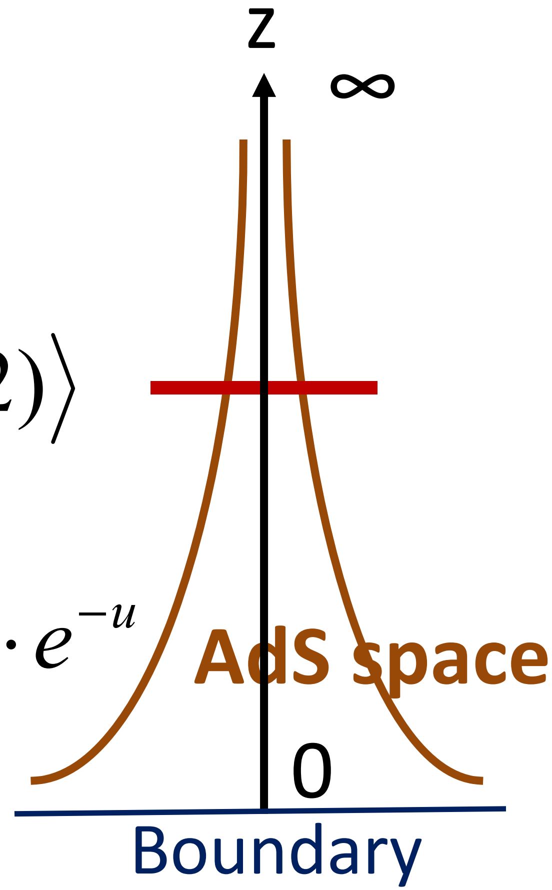
$\hat{K}(u)$: (dis)entangler at length scale $\sim \varepsilon\cdot e^{-u}$

$|\Omega\rangle$: unentangled IR state

$\rightarrow S_A = 0$ for any A. ➡ **What is this state in general 2d CFTs ?**

Relation to (discrete) MERA





By adding dummy states |0>  , we keep the dimension of Hilbert space for any u to be the same.

⇒ We can formally describe the real space RG by a **unitary transformation**.

(2-2) Boundary State as Gravity Dual of Point-like Space

[Miyaji-Ryu-Wen-TT 14]

Q. A general construction of the IR states |Ω> in CFTs ?

Argument 1

We can realize disentangled states (IR states |Ω>)

⇔ Trivial (Point-like) spaces

by performing a (infinitely) massive deformation:

$$H_m = H_{CFT} + m^{d+1-\Delta_O} \int dx^d O(x),$$

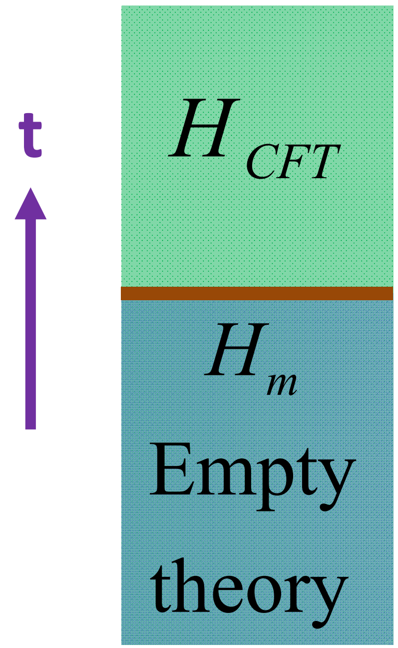
$$\underset{m\to\infty}{\Longrightarrow} |\Omega\rangle = \text{the ground state of } H_m.$$

Now we apply the idea of ***quantum quenches***.

⇒ For t<0, we assume the ground state of the massive Hamiltonian H_m. Then at t=0, we suddenly change the Hamiltonian into H_{CFT} as in [Calabrese-Cardy 05, Gravity dual: Hartman-Maldacena 10].

In this setup, the state at t=0 is identified with the boundary state(Cardy state):

$$\left|\Psi_m(t=0)\right\rangle = \left|\Omega\right\rangle = \left|B\right\rangle.$$



We may introduce the UV cut off like

$$\left|\Omega_m\right\rangle \propto e^{-H/m} \cdot \left|B\right\rangle \ .$$

Boundary states in CFTs (assume 2d CFT)

A **boundary state** (Ishibashi state) : |B>

= A state which gives a conformally invariant boundary condition:

$$\left[L_n - \tilde{L}_{-n}\right]|B\rangle = 0.$$

In terms of the Virasoro algebra: $|B\rangle = \sum_{\vec{k}} |\vec{k}\rangle_L |\vec{k}\rangle_R$

where $\vec{k} = (k_1, k_2, \ldots\ldots)$ represent

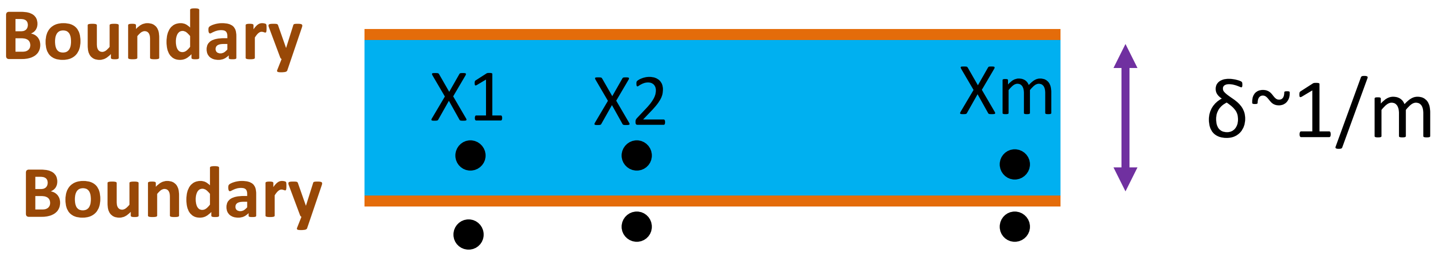
$$|\vec{k}\rangle = \sum (L_{-1})^{k_1} \cdot (L_{-2})^{k_2} \cdots |\Delta\rangle.$$

⇒ A maximally entangled state

between left and right moving sectors !

⇒ But, the real space entanglement is quite suppressed !

Argument 2: Correlation functions of local operators



$$\frac{\langle \Omega | O(x_1) O(x_2) \cdots O(x_n) | \Omega \rangle}{\langle \Omega | \Omega \rangle} \approx \prod_{i=1}^{n} \langle O(x_i) \rangle.$$

⇒ When (xi-xj)>>δ, there is no correlations !

⇒ Disentangled !

Argument 3: Direct calculation of EE

For the regularized IR state $\left|\Omega\right\rangle = e^{-H\delta}\left|B\right\rangle$,
we can compute the EE explicitly in free fermion CFTs:

[Ugajin-TT 10]

$$S_A \approx \frac{c}{3}\log\frac{\delta}{\varepsilon} + [\mathrm{Finite}], \qquad (\delta \to 0).$$

Thus we can set $S_A \approx 0 \quad \text{when} \quad \delta \approx \varepsilon.$

Note: Boundary states can still have non-zero finite *topological entanglement.*

③ SS-correspondence in AdS/CFT

[Miyaji-Numasawa-Shiba-Watanabe-TT, 2015]

Let us focus on a AdS3/CFT2 setup. It is useful to start with the symmetry of global AdS3 space:

$$ds^2 = R^2(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\phi^2),$$

whose isometry $SL(2,R)\times SL(2,R)$ is generated by

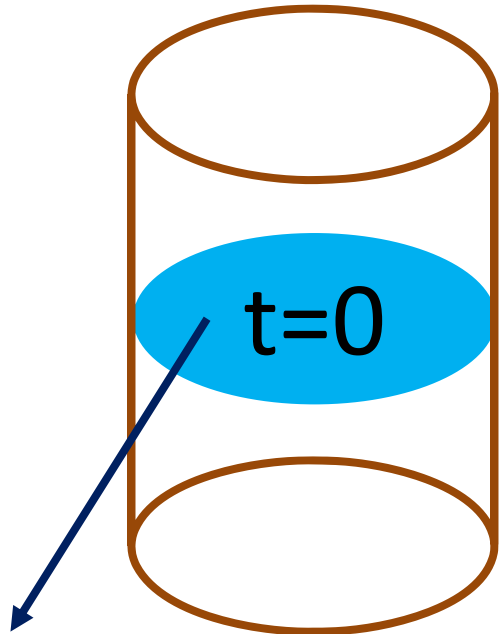
$$L_0 = i\partial_+, \quad \tilde{L}_0 = \partial_-,$$
$$L_{\pm1} = ie^{\pm ix^+}\left[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_+ - \frac{1}{\sinh 2\rho}\partial_- \mp \frac{i}{2}\partial_\rho\right],$$
$$\tilde{L}_{\pm1} = ie^{\pm ix^-}\left[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_- - \frac{1}{\sinh 2\rho}\partial_+ \mp \frac{i}{2}\partial_\rho\right].$$

In particular, we are interested in the SL(2,R) subgroup which preserves the time slice t=0 (i.e. H2) of the AdS3.

They are generated by $l_n = \widetilde{L}_{-n} - L_n, \quad (n = 0, \pm 1),$
which annihilate the boundary states.

The SL(2,R) action which maps ρ=0 to the point (ρ, ϕ) is given by

$$g(\rho,\phi) = e^{i\phi l_0} e^{\frac{\rho}{2}(l_1 - l_{-1})}.$$



$$ds^2_{H_2} = R^2(d\rho^2 + \sinh^2 \rho d\phi^2)$$

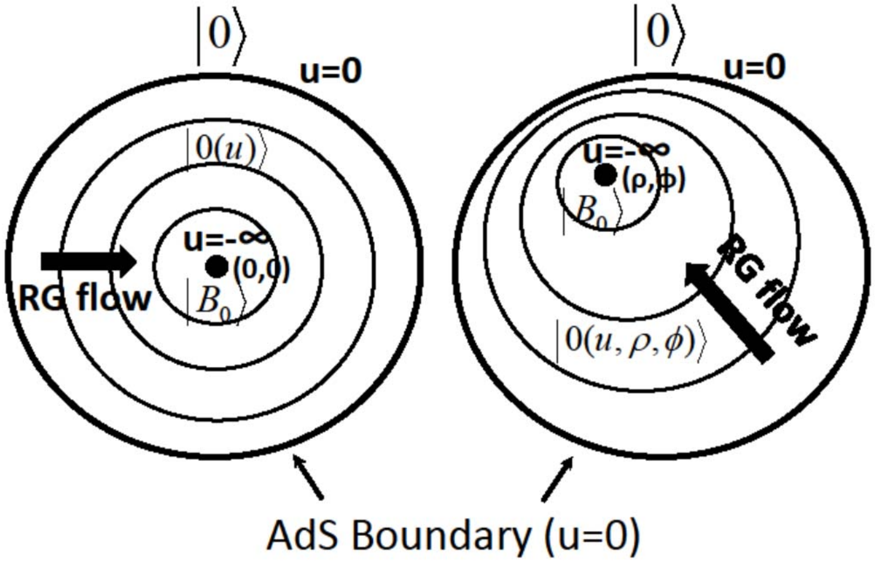
cMERA for the ground state of CFT2 is formulated as:

$$|0\rangle = P\exp\left(-i\int_{-\infty}^{0}\hat{K}(u)du\right)|B_0\rangle.$$ ← boundary (Ishibashi) state for the identity sector

If we act the SL(2,R) transformation $g(\rho,\phi)$ we find

$$|0\rangle = P\exp\left(-i\int_{-\infty}^{0}\hat{K}_{(\rho,\phi)}(u)du\right)|B_0\rangle,$$

where $\hat{K}_{(\rho,\phi)}(u) = g(\rho,\phi)\cdot\hat{K}(u)\cdot g(\rho,\phi)^{-1}.$



More generally, we can describe the **diffeomorphism** by taking into account $l_n = \tilde{L}_{-n} - L_n, \quad (|n| = 2,3,..)$:

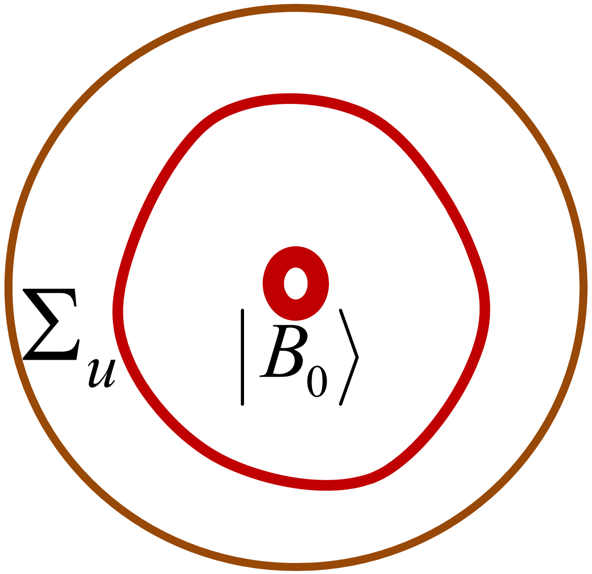
$$|0\rangle = P\exp\left(-i\int_{-\infty}^{0} \hat{K}_g(u)du\right)|B_0\rangle,$$

$$\hat{K}_g(u) = \hat{g}(u)\hat{K}(u)\hat{g}(u)^{-1} + \partial_u g(u)\cdot g(u)^{-1},$$

where $g(u) = \exp\left[\sum_n \xi_n(u)\, l_n\right]$ with $\xi_n(0) = 0.$

We can define a dual state $|\Phi(\Sigma_u)\rangle$ for any surface Σ_u as

$$|\Phi(\Sigma_u)\rangle = P\exp\left(-i\int_{-\infty}^{u} \hat{K}_g(s)ds\right)|B_0\rangle.$$



⇒ An evidence for SS-correspondence

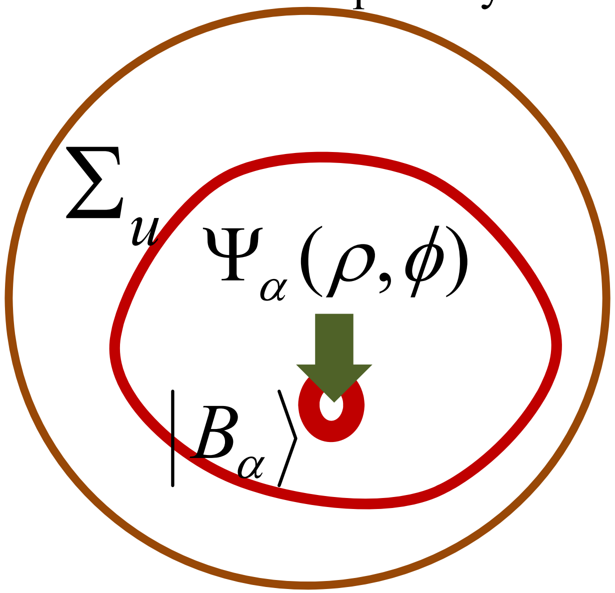
How to describe the bulk excitation ?

We argue the following identification:

$$\underbrace{\Psi_\alpha(\rho,\phi)}_{\text{Bulk local operator}} |0\rangle_{Bulk}$$

$$\Leftrightarrow \ |\Psi_\alpha(\rho,\phi)\rangle_{CFT} = P\exp\left(-i\int_{-\infty}^{0}\hat{K}_{(\rho,\phi)}(s)ds\right) \underbrace{|B_\alpha\rangle}_{\substack{\text{Ishibashi state} \\ \text{for primary } \alpha}} \ .$$

This is because the local operator insertion does not change the bulk metric (= entanglement).



We argue this state is evaluated as

$$\left|\Psi_\alpha(\rho,\phi)\right\rangle_{CFT} \approx g(\rho,\phi)\cdot e^{\frac{\pi}{2}i(L_0+\tilde{L}_0)} \cdot \underbrace{e^{-\varepsilon H}}_{\text{some UV cut off}} \cdot \underbrace{\left|J_\alpha\right\rangle}_{\substack{SL(2,R)\\ \text{Ishibashi State}}} .$$

This satisfies the correct EOM:

$$\Box_{\mathrm{AdS3}}\left|\Psi_\alpha(\rho,\phi,t)\right\rangle_{CFT} = 0.$$

We can compute the information metric:

$$\left|\left\langle\Psi_\alpha(\rho,\phi)\middle|\Psi_\alpha(\rho+\delta\rho,\phi+\delta\phi)\right\rangle\right| = 1 - G_{ab}dx^a dx^b ,$$

$$ds^2 = \frac{1}{\varepsilon^2}(d\rho^2 + \sinh^2\rho\, d\phi^2).$$

$\frac{1}{\varepsilon^2} \approx c^2$ (as in AdS/CFT) by choosing $\varepsilon \approx c^{-1}$.

④ Surface/State Correspondence [Miyaji-TT 15]

We propose SS-correspondence for general gravity theories.

(4-1) Basic Principle

Consider Einstein gravity on a d+2 dim. spacetime **M**.

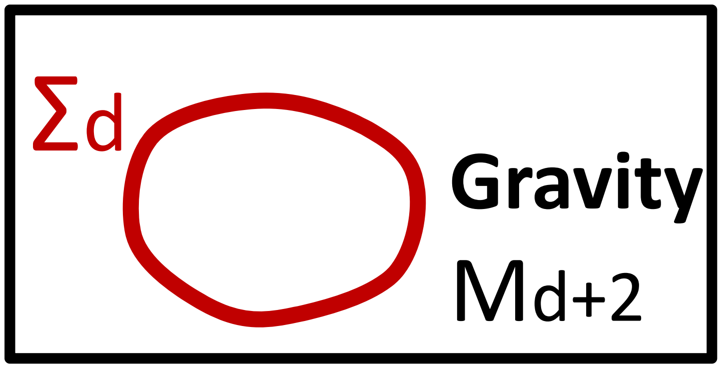
We argue the following correspondence:

Σ : an d dim. convex space-like surface in M which is closed and homologically trivial

↕

$$\left|\Phi(\Sigma)\right\rangle \in H_M$$

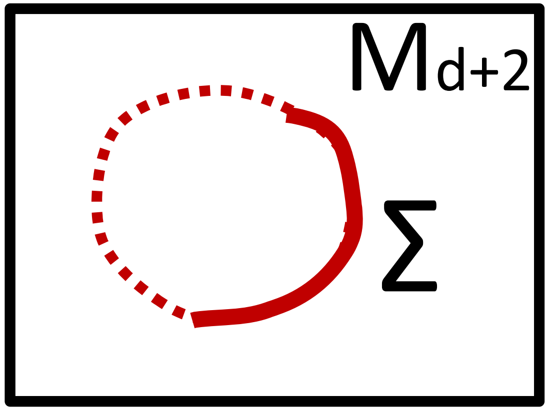
A pure state

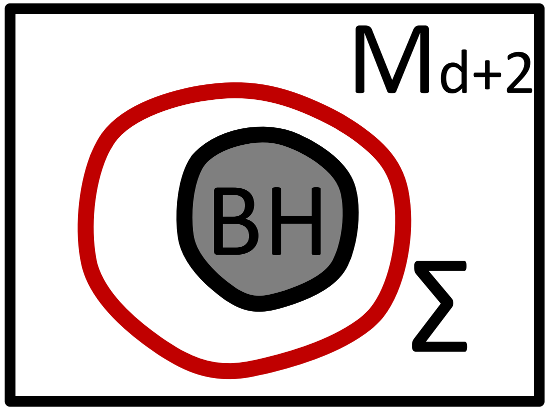


More generally,
the quantum state dual to a convex surface Σ is

a **mixed state** $\rho(\Sigma)$

if Σ is open or topologically non-trivial.





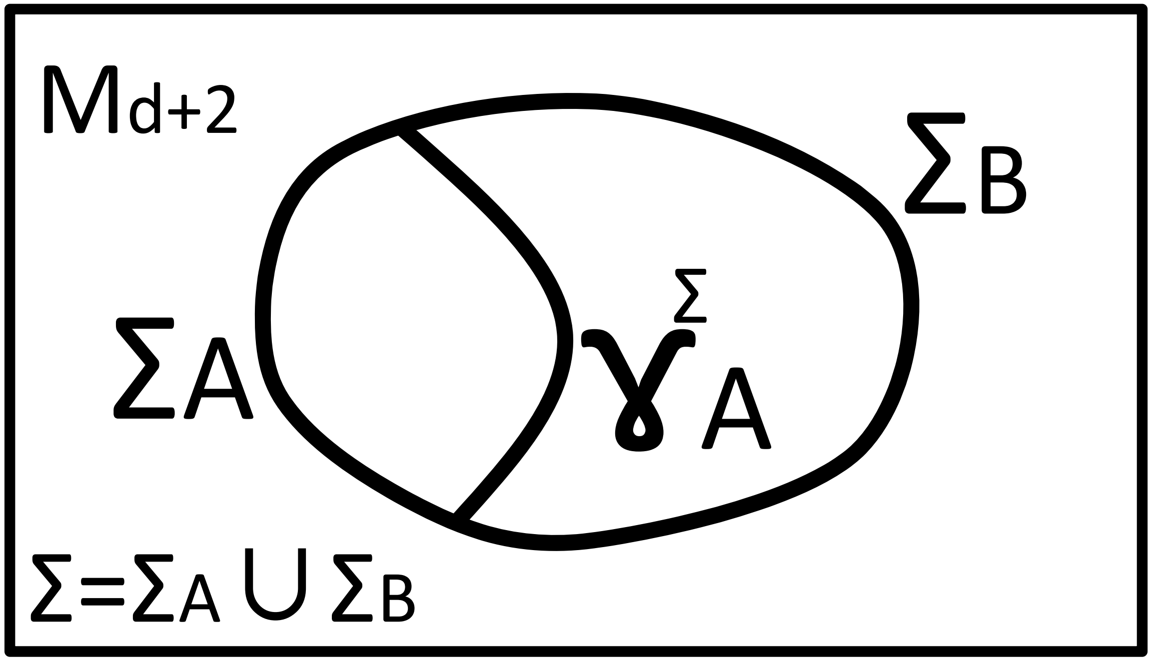
On the other hand, the **zero size limit of Σ** corresponds to the **trivial state |Ω>** with no real space entanglement.

(4-2) Entanglement Entropy

We can naturally generalize HEE for our setup :

$$H_{\Sigma} = H_A \otimes H_B, \qquad \rho_A^{\Sigma} = \mathrm{Tr_B}[\rho(\Sigma)],$$

$$\Rightarrow \qquad S_A^{\Sigma} = \frac{\mathrm{Area}(\gamma_{\mathrm{A}}^{\Sigma})}{4G_N}.$$



(4-3) Effective Entropy

By dividing the surface Σ into infinitesimally small pieces $\Sigma = \bigcup A_i$, we easily find:

$$S_{eff}(\Sigma) \equiv \sum_i S_{A_i}^{\Sigma} = \frac{\mathrm{Area}(\Sigma)}{4G_N}.$$

↓

We interpret this as the log of effective dim. for Σ

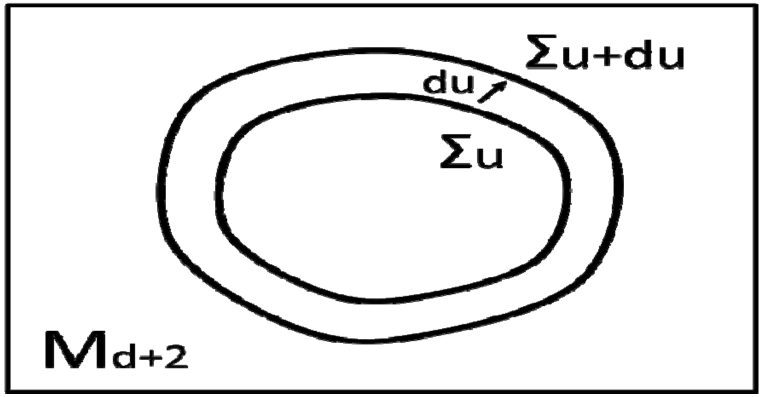
$$\log[\dim H_{\Sigma}^{eff}]$$

This is because $\rho_{A_i}^{\Sigma}$ is expected to be maximally entangled (except the dummy states).

[cf. Differential entropy: Balasubramanian-Chowdhury-Czech-deBoer-Heller 13]

(4-4) Inner Products and Information Metric

Another intriguing physical quantity is an inner product $\langle \Sigma | \Sigma' \rangle$ between two surfaces.



$$ds^2 = R^2 du^2 + g_{\mu\nu}(x,u) dx^\mu dx^\nu .$$

Here focus on the two surfaces separated infinitesimally.

⇒ Consider an information distance between them

The information metric is defined as

$$1 - \left| \langle \Phi(u) | \Phi(u + du) \rangle \right| = (du)^2 \cdot G_{uu}^{(B)}$$

If the metric is x-independent, we have

$$G^{(B)}_{uu} \sim \frac{1}{G_N}\int_{\Sigma u} dx^d \sqrt{g(x)}\left(K_u\right)^2 .$$ → Vanishes on extremal surfaces

Example 1: a flat spacetime ⇒ $G^{(B)}_{uu} = 0.$

[u-Translational inv. ⇒ $|\Phi(u+du)\rangle = |\Phi(u)\rangle$.]

Example 2: an AdS spacetime [Nozaki-Ryu-TT 12]:

$$G^{(B)}_{uu} = N_{\deg}\cdot\frac{V_d}{\varepsilon^d}\cdot e^{du} \Rightarrow \text{Agrees with cMERA for } \text{CFT}_{\text{d+1}}$$

⑤ Conclusions

- CFT states with no real space entanglement are given by boundary states. ⇒cMERA formulation
- cMERA can be generalized so that we have the surface/state correspondence. This SS-duality looks more general than AdS/CFT and even more general than holography.
- A bulk local operators is described by the cMERA network starting from the boundary state (Ishibashi state) for the corresponding primary.

  [cf. Recent paper by Verlinde 2015, maybe connected via
  the tensor network renormalization by Evenbly-Vidal 2015]

The SS-duality argues

Top. trivial convex surface	⇔	a pure state
Top. non-trivial surface	⇔	a mixed state
Zero size surface	⇔	boundary state
Area of surface	=	log[Eff. Dimension]
(Extrinsic curvature)^2	=	Information metric

Future problems

- Derivation of Einstein eq.
- AdS black holes
- Spacetimes without (T-like) boundary: de-Sitter spaces.
- Analysis of compact directions e.g. S5 in AdS5 × S5.

Quantum Estimation Theory

A quantum version of ***Cramer-Rao bound*** argues

$$\left\langle (\delta u)^2 \right\rangle \geq \frac{1}{G^{(B)}_{uu}}.$$

[Helstrom 76]

Mean square error

In the case of AdS/CFT, this leads to

$$\left\langle \frac{\delta z^2}{z^2} \right\rangle = \left\langle (\delta u)^2 \right\rangle \geq \frac{G_N}{\mathrm{Area}(\Sigma)} \sim \frac{1}{\log[\dim H^{eff}_{\Sigma}]} \propto N^{-2}.$$

In the large N limit, this error is highly suppressed.

⇒ Locality of the bulk in the large N limit ?

⇒ Some uncertainty principle of surfaces in QG ?

$$\left\langle \left(\delta \mathrm{Area}(\Sigma)\right)^2 \right\rangle \geq G_N \cdot \mathrm{Area}(\Sigma) \quad .$$