Entanglement in Condensed Matter

(Overview)

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Talk Plan

- Introduction

- Overview of Overviews

Main Seminar Room

- 09:00AM Horacio Casini (Centro Atomico Bariloche) *C-theorems and entanglement entropy (overview)*

  09:00AM Guifre Vidal (Perimeter Institute) *Tensor Networks, renormalization and Holography (overview)*

- 01:30PM Dmitry Abanin (Perimeter Institute) *MBL: integrability, entanglement and dynamics (overview)*

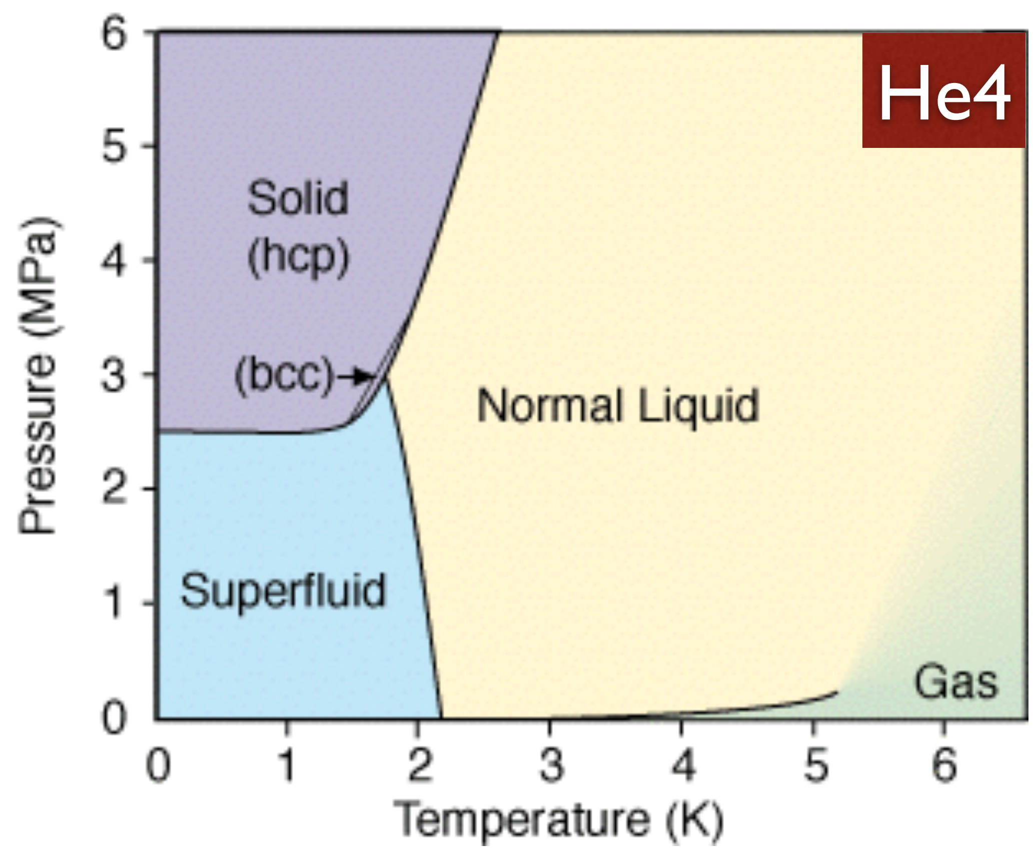
- Case Study of Entanglement as a tool - topological band structures. Filling enforced `quantum band insulator'.

Introduction

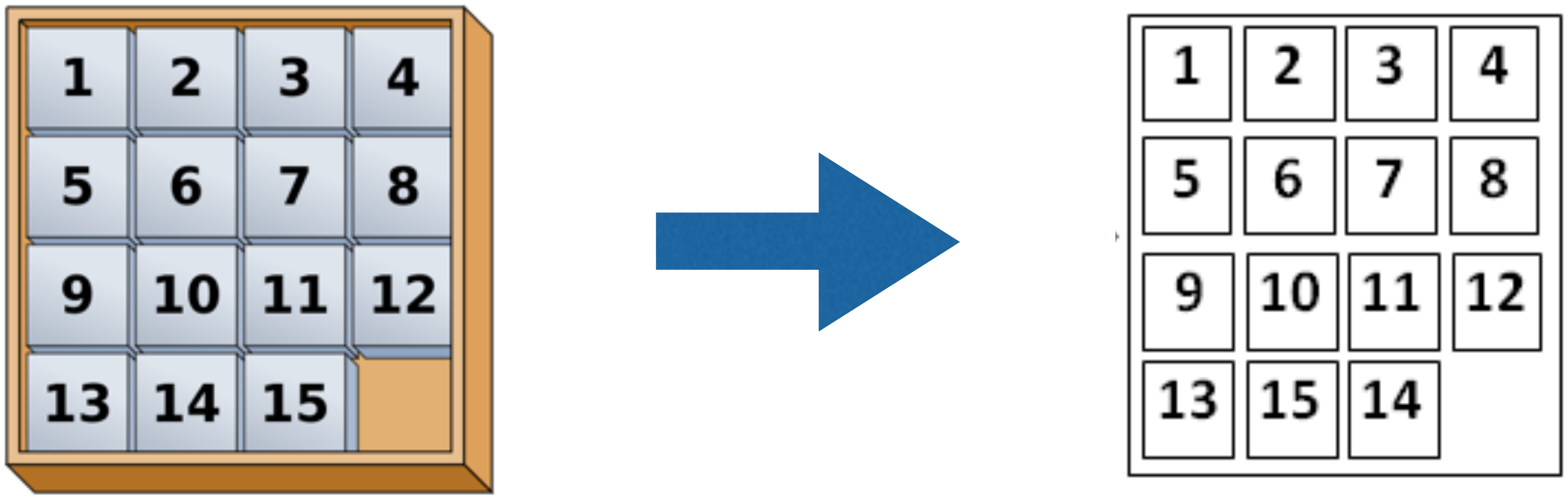
- **Qualitative** properties - of a quantum many body system.

$$N \to \infty$$

- Classifying ***phases*** - distinct phases are always separated by a phase transition (singular point).



Classifying Phases - An Analogy



Can they be connected?

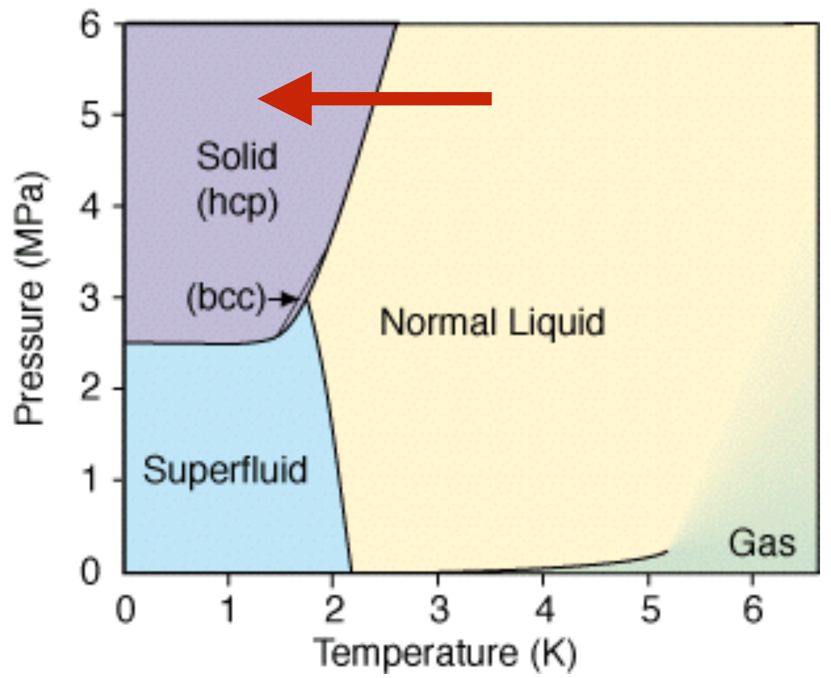
- NO
- Parity of permutation (+ row of blank) is conserved.

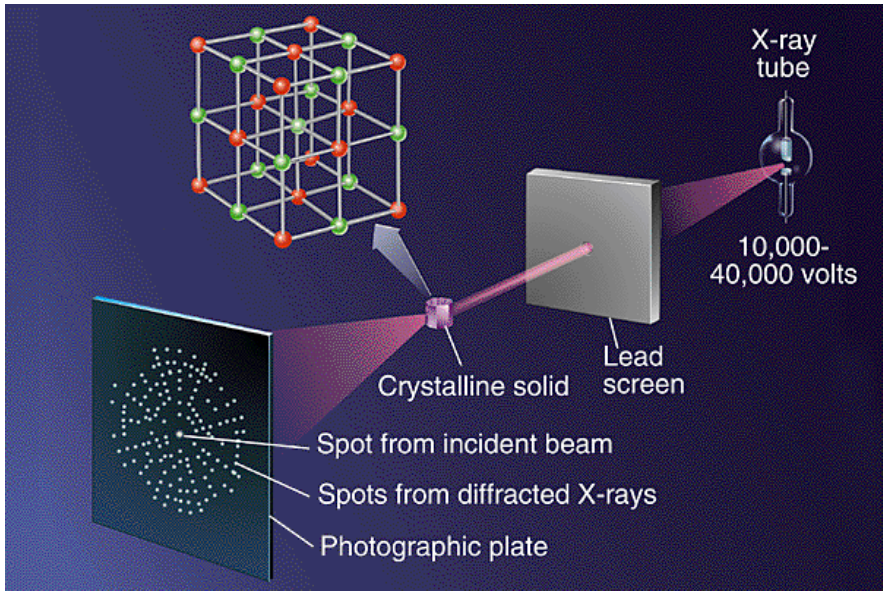
Entanglement and Many Body Hilbert Space

$$|0\rangle \otimes |1\rangle \qquad \textbf{vs} \qquad \frac{1}{\sqrt{2}}\left[|0\rangle|0\rangle + |1\rangle|1\rangle\right]$$

- Product states - no entanglement.
  - With N spins - need to specify just N bits for a product state.
- Generic state in Hilbert Space - need to specify 2^N complex numbers.
  - Arises due to Entanglement - makes quantum many body physics hard/rich.

Conventional Phases





- Phase distinction due to symmetry breaking. Captured by order parameter.
- Classify phases - different ways to break symmetry (230 types of crystals)
- Measure order parameter experimentally to diagnose a phase. *All* 230 realized in nature!
- Essentially weak entanglement/product state.

General Distinctions Between Quantum States?

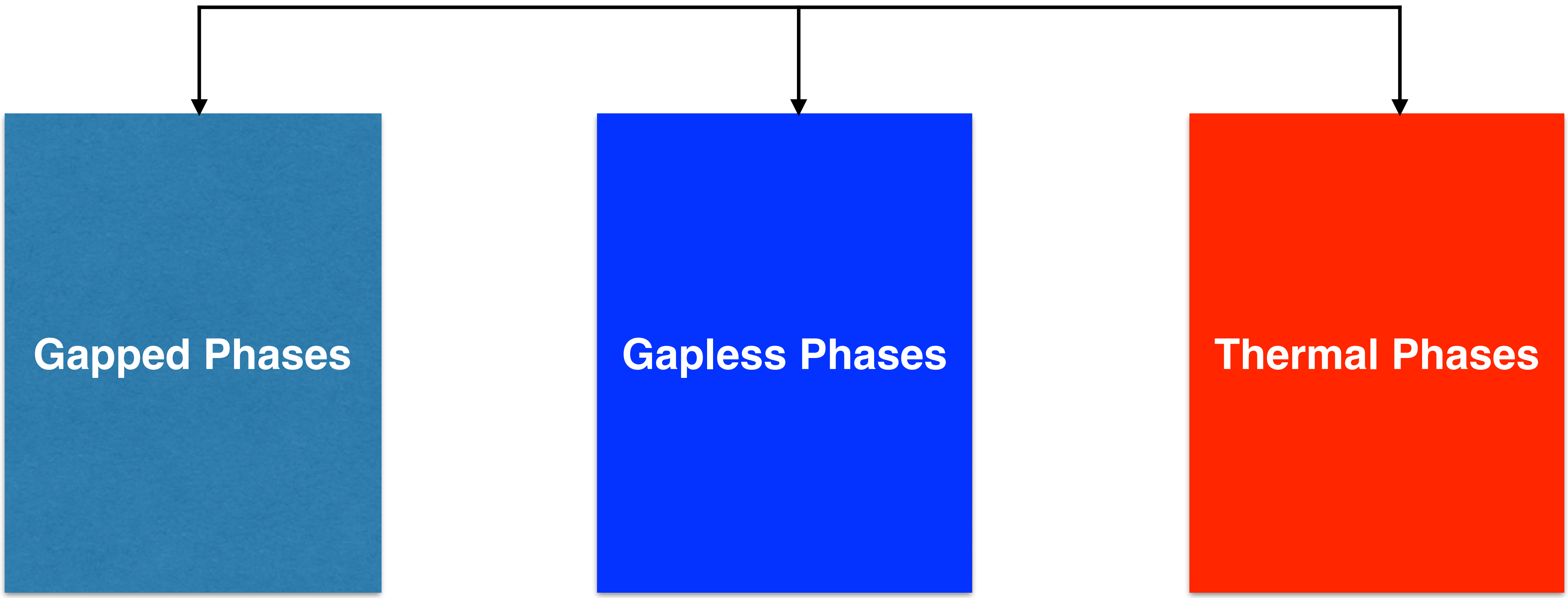
- Phase distinction due to symmetry breaking. Captured by order parameter.
- Classify phases -
- Measure order parameter experimentally

- Phase distinctions due to different patters of entanglement
- Helps classify and discover new phases (topological phases)
- Experimental measurement?

03:40PM Markus Greiner (Harvard University)
Entanglement detection through interference of quantum many-body twins in ultra cold atoms

2. Overview of Overviews

Entanglement in Condensed Matter



- Thermal Phases

Thermal Phases

- Volume law entanglement entropy of a highly excited state related to thermal entropy. (Eigenstate Thermalization hypothesis - Deutsch, Srednicki)

- Many body localization - one example of a non thermal phase with nonzero energy density.

01:30PM Dmitry Abanin (Perimeter Institute) *MBL: integrability, entanglement and dynamics (overview)*

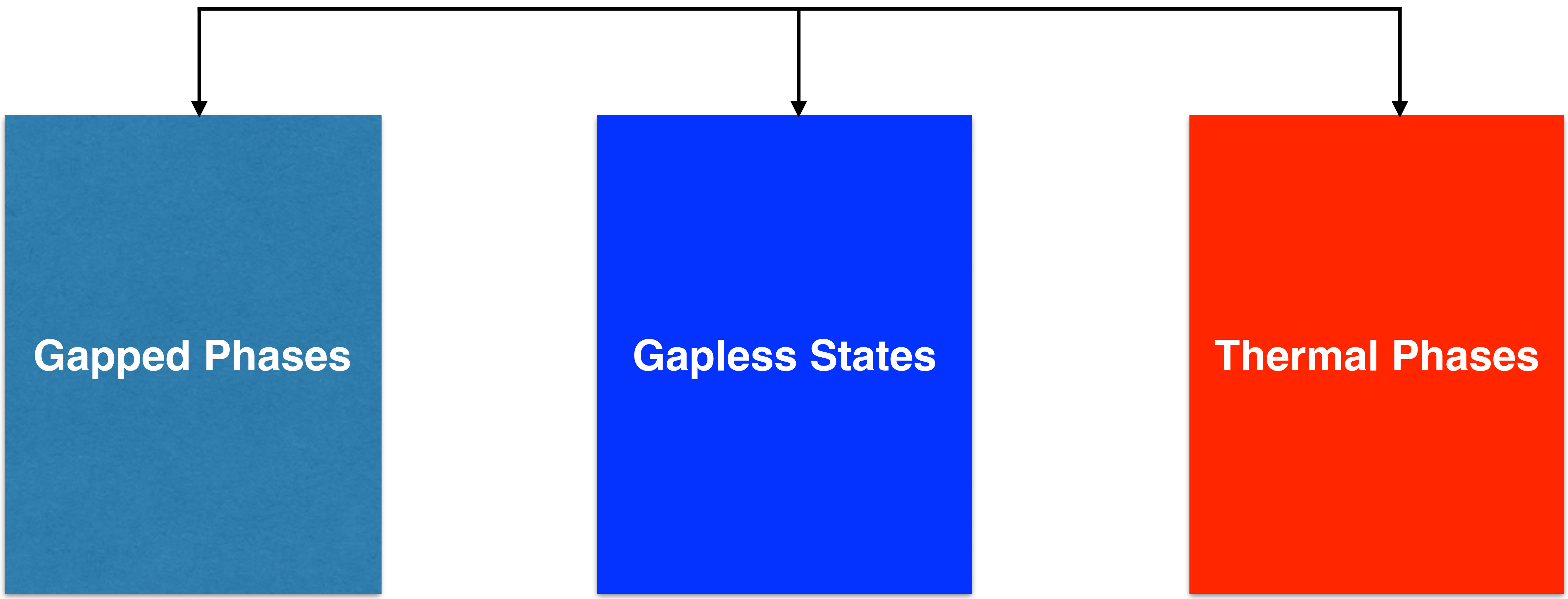
- Quantum phase transition from thermal to localized phase? (Classical to Quantum transition).

- 03:00PM Ehud Altman (Weizmann Institute) *Universal dynamics and entanglement patterns near a MBL transition*
- 03:40PM A. Chandran (Perimeter Institute) *Thermalization versus localization in a solvable circuit model*

  - General constraints from strong subadditivity (Grover)

- New phases with volume law entanglement? (Grover/Fisher)

Entanglement in Condensed Matter



- Gapless states

Entanglement of Gapless States

- Entanglement in Conformal Field theories
  - c & f & a from entanglement.

10:10AM John Cardy (University of Oxford) *The entanglement gap in CFTs*

09:00AM Horacio Casini (Centro Atomico Bariloche) *C-theorems and entanglement entropy (overview)*

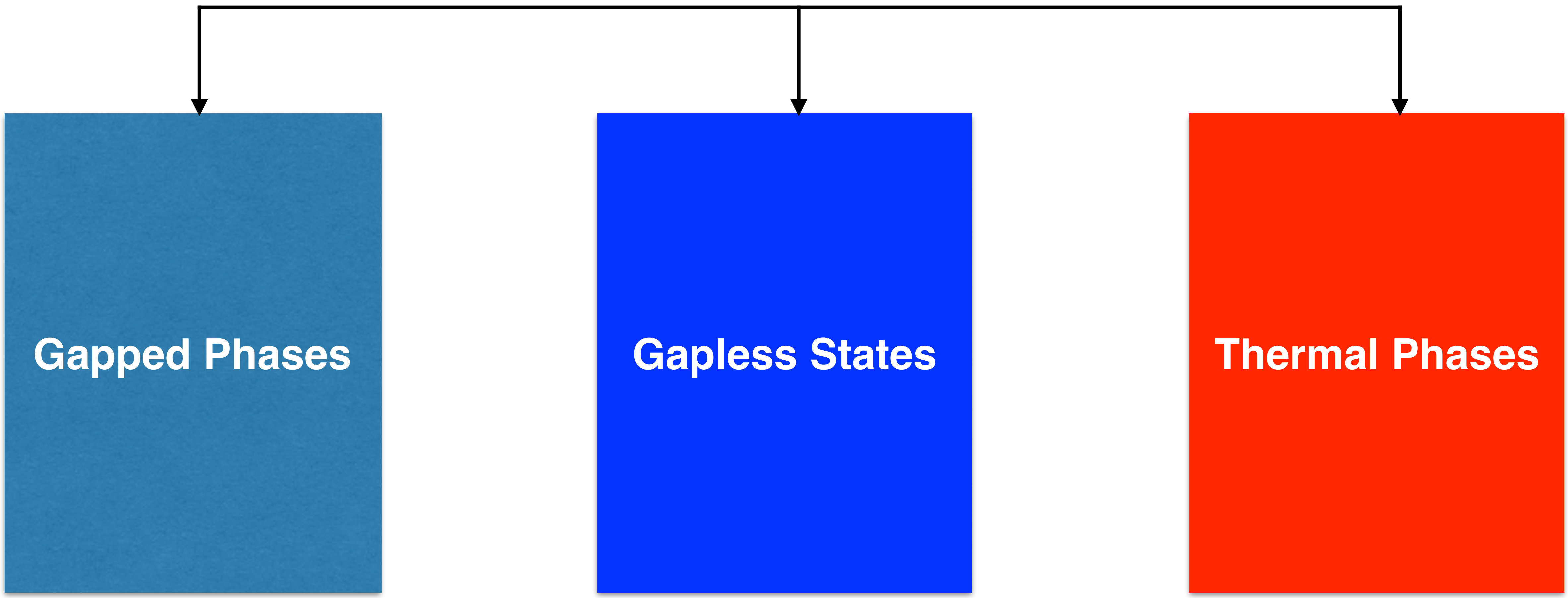
  - connections to condensed matter

03:00PM Tarun Grover (KITP) *Entanglement, RG flows and the stability of quantum matter*

- Entanglement of Phases with Fermi surfaces
  - Area law violation in fermi liquids and non-Fermi liquids

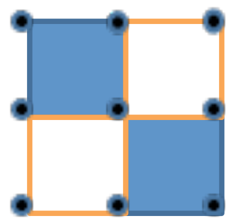
02:00PM Eun-Ah Kim (Cornell University) *Entanglement entropy of composite fermi liquids*

Entanglement in Condensed Matter



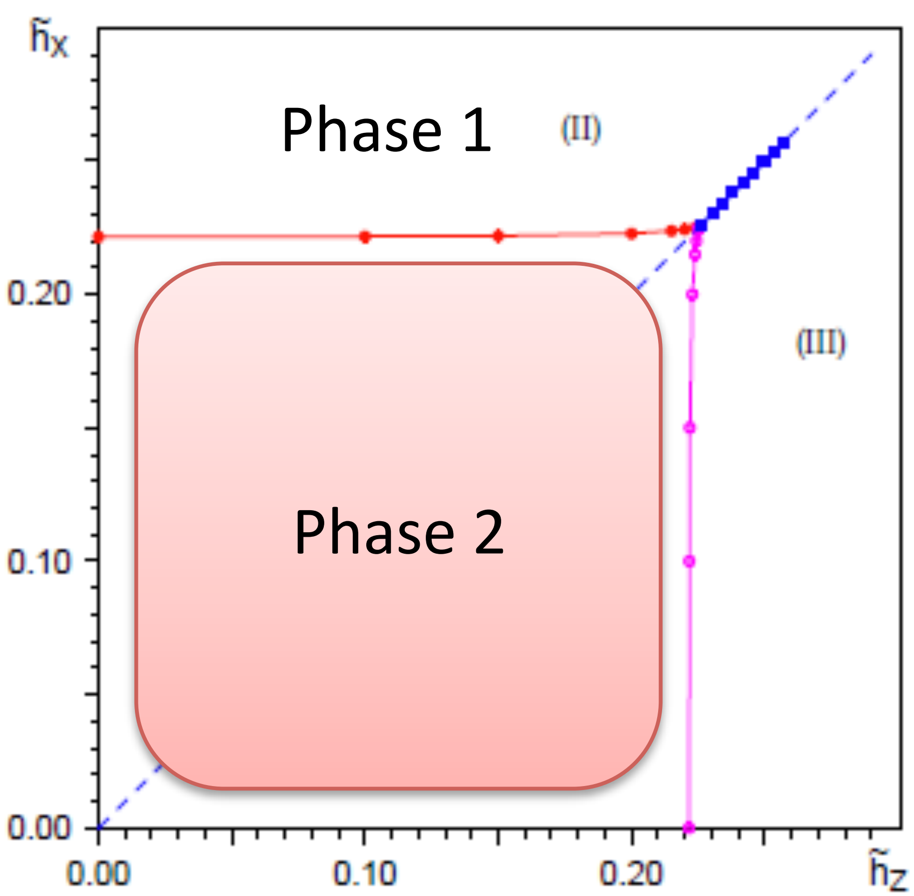
- Gapped states

Gapped Phases - Topological order



$$H = -\sum_{\blacksquare} \sigma^z\sigma^z\sigma^z\sigma^z - \sum_{\square} \sigma^x\sigma^x\sigma^x\sigma^x - h_x\sum\sigma^x - h_z\sum\sigma^z$$

A spin model with no spin symmetry



But two phases!
How to distinguish?

Kitaev; Tyupitsin et al.; Fradkin and Shenker.

Topological Entanglement Entropy

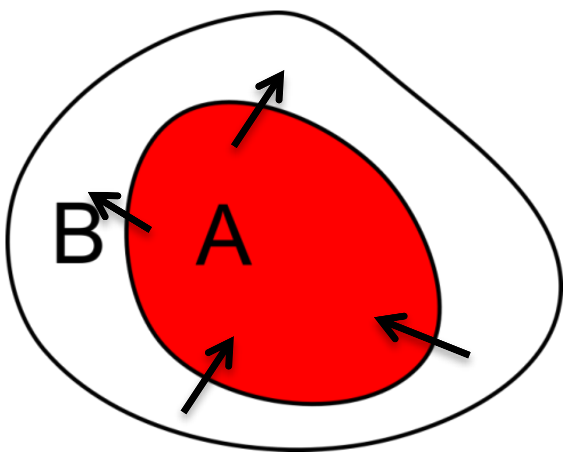
Gapped Phase with topological order.

eg. deconfined gauge theory. Smooth boundary, circumference L_A:

Topological Entanglement Entropy

(Levin-Wen;Kitaev-Preskill)

$$S_A = aL_A - \gamma$$



ϒ=Log D . (D : total quantum dimension).

Z_2 gauge theory: *ϒ=Log 2*

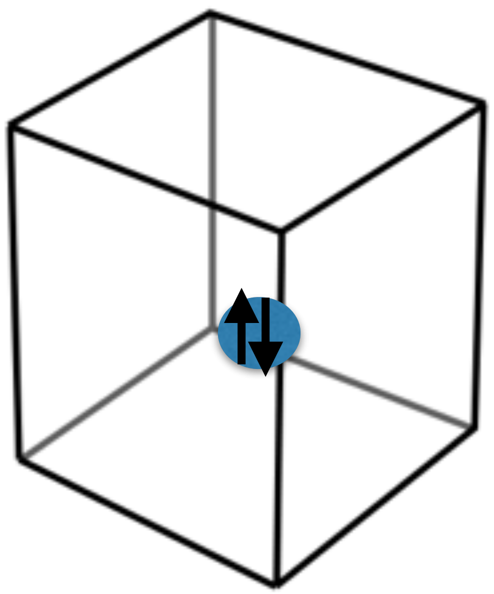
Gauss Law on boundary – no gauge charges inside.

Lowers Entropy by 1 bit of information.

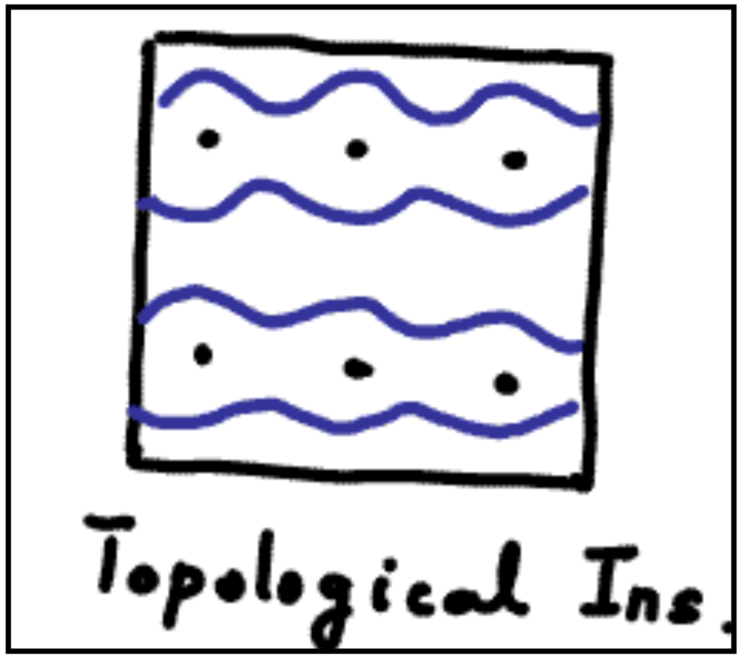
Applications to Kagome antiferromagnet (Yan-Huse-White/Jiang-Balents)

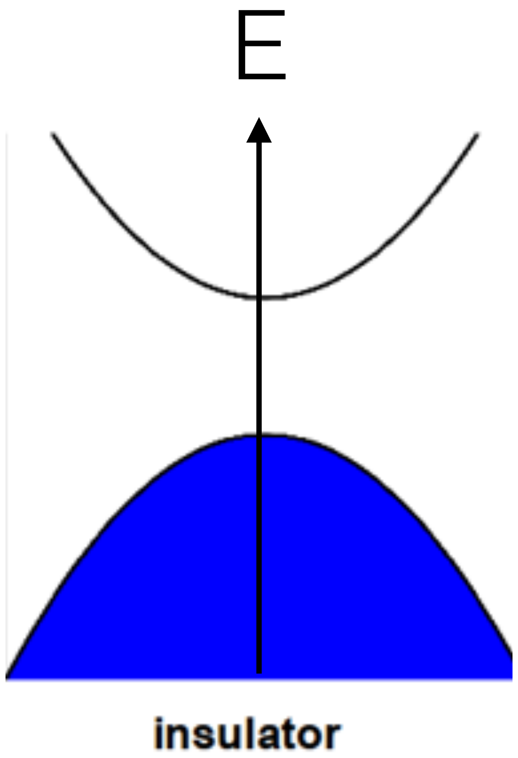
3. Case Study of Entanglement as a tool in Condensed Matter - Topological Phases of Free Fermion Insulators

Filling Enforced Quantum Band Insulators



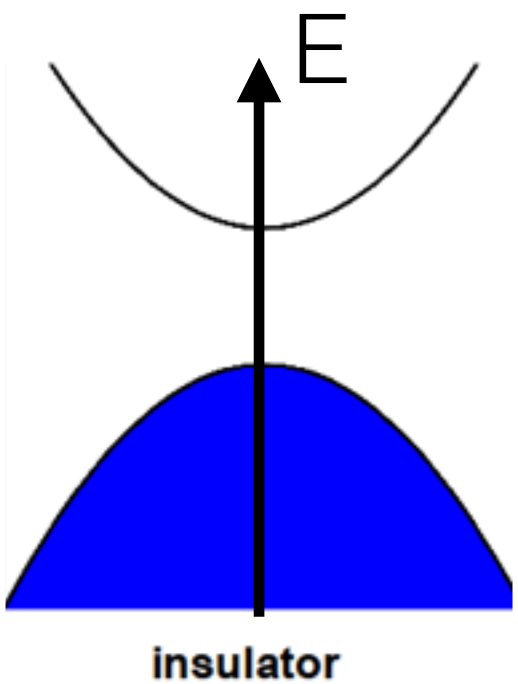
Atomic Picture



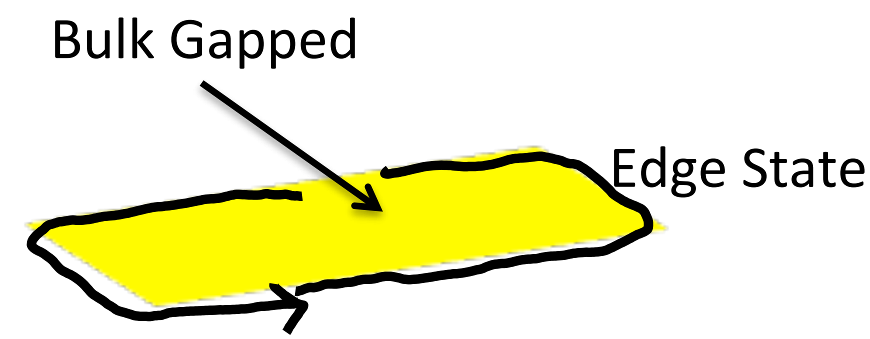


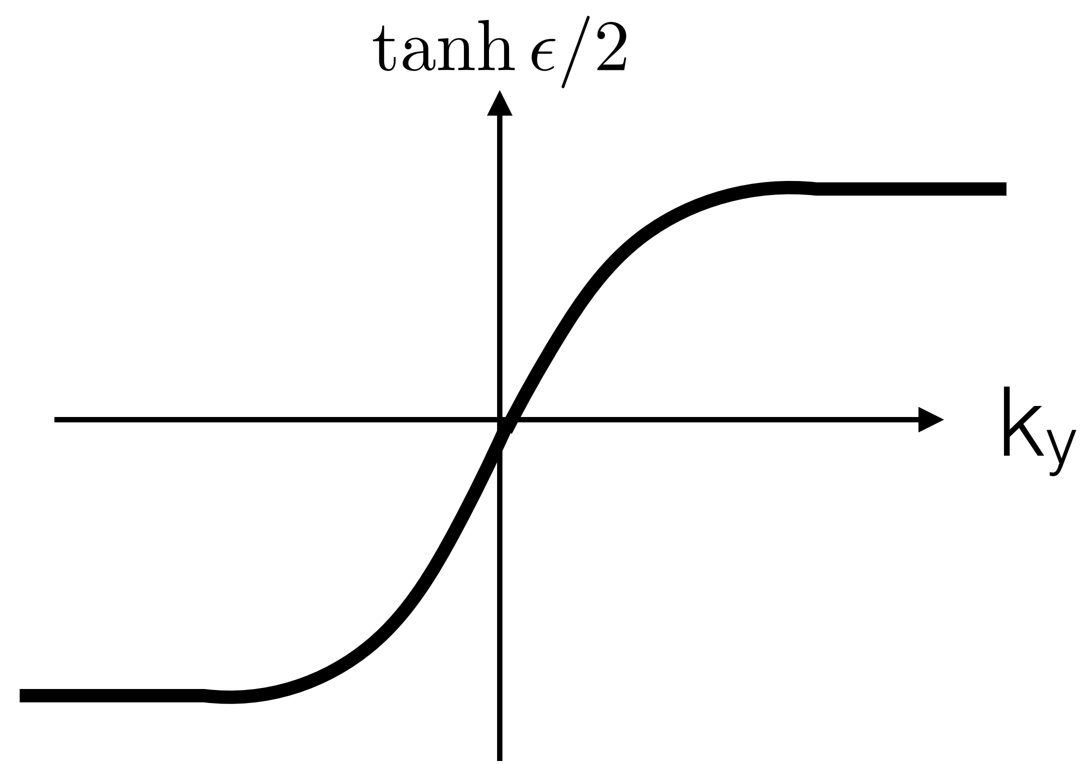
- Atomic picture of a band insulator
  - electrons localized on atomic site. Does this always apply?
  - not for topological phases - nontrivial entanglement in real space. No atomic picture

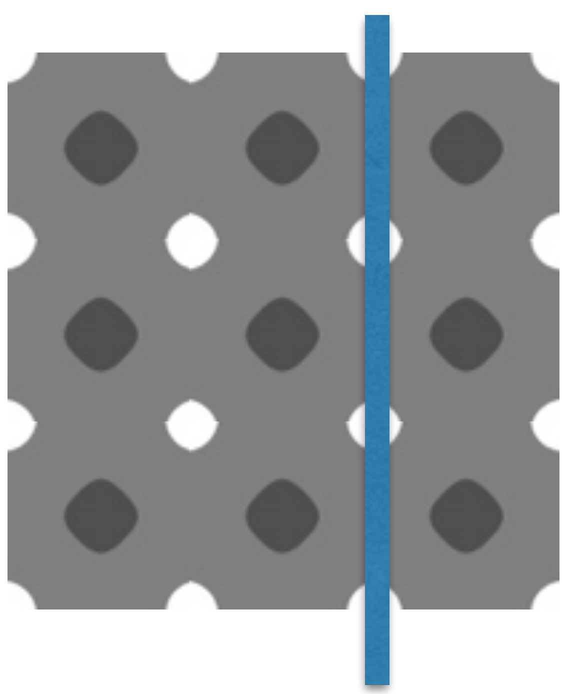
Chern insulator and Entanglement



$$C = \frac{1}{2\pi}\int F\, dk_x\, dk_y$$

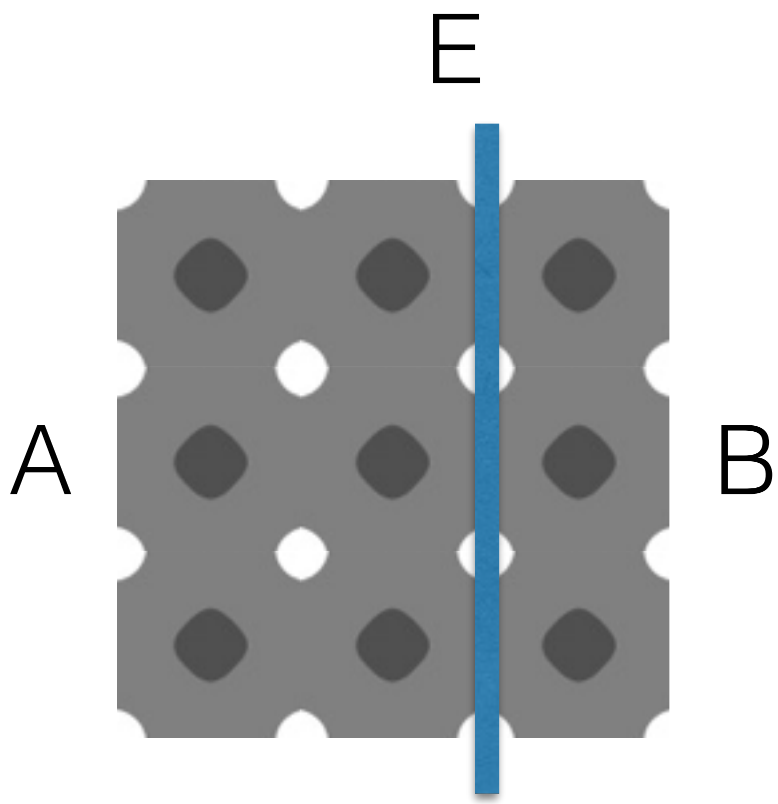


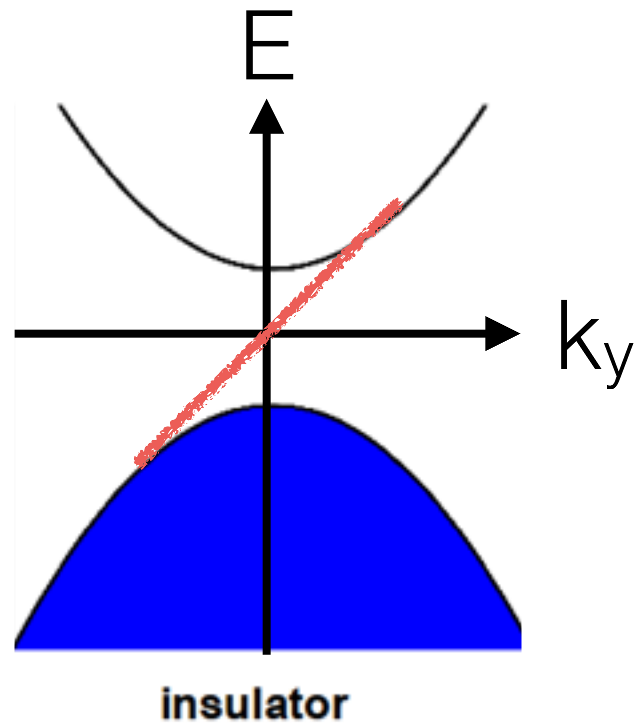


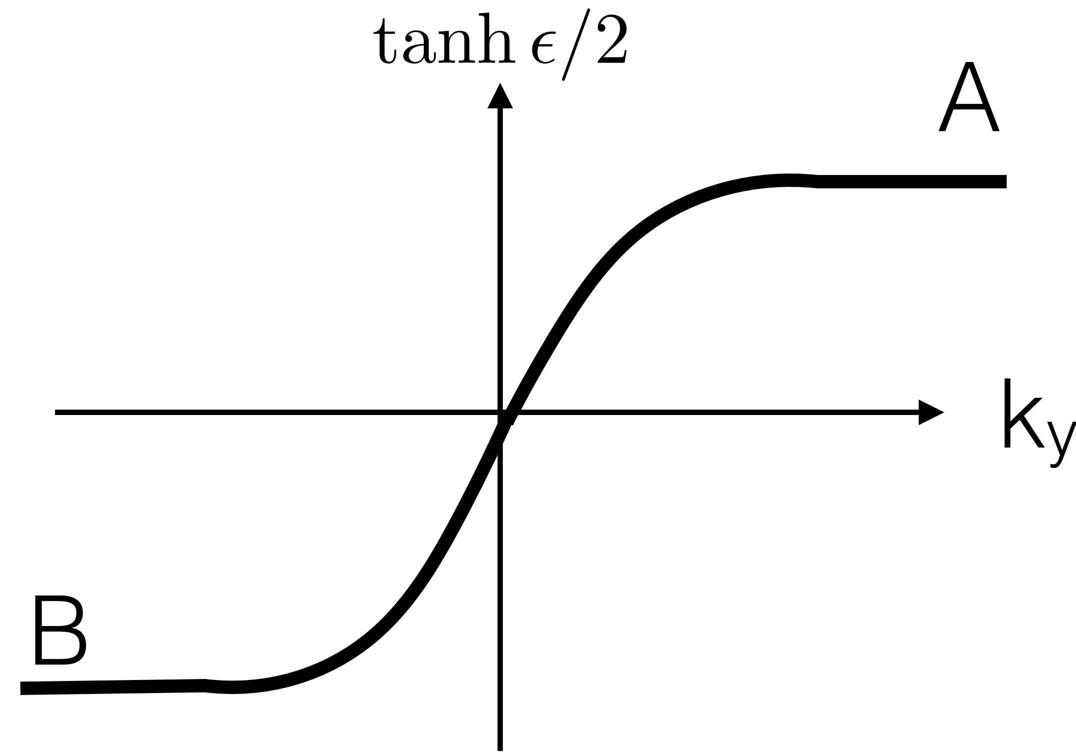


Signature in Entanglement?
Not in Entanglement entropy
BUT in Entanglement Spectrum

Chern insulator and Entanglement







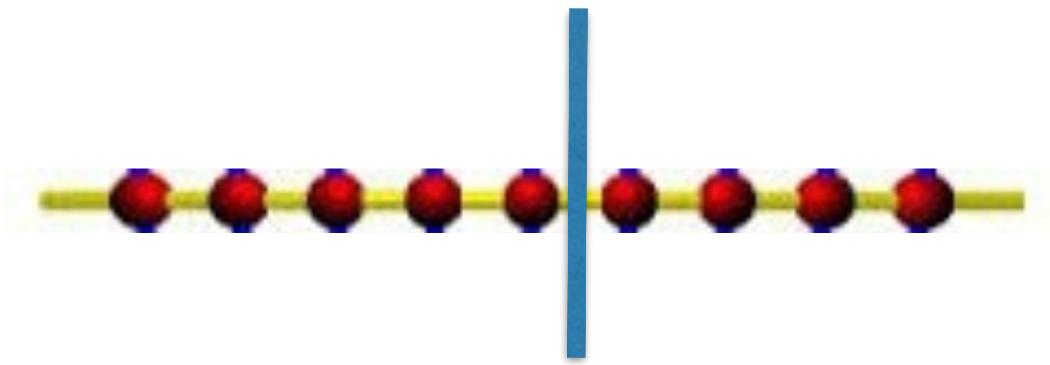
$$\rho_A = \mathrm{Tr}_B \left\{ |\Psi\rangle\langle\Psi| \right\}$$

$$\rho_A = \frac{1}{\mathcal{Z}} e^{-H_A}$$

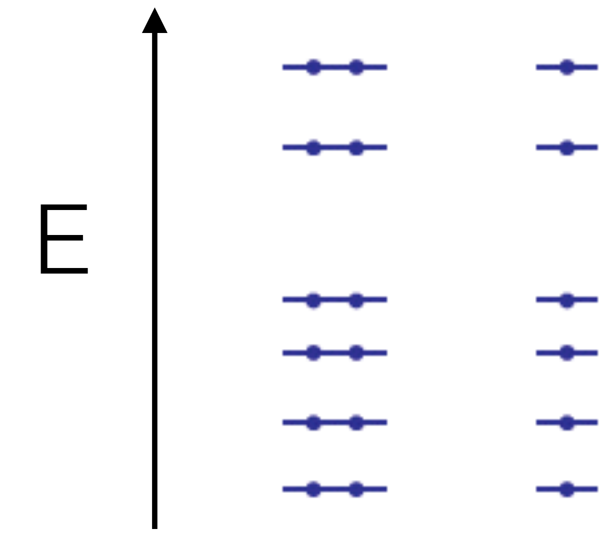
Obstruction to recovering atomic insulator

Physical Edge States $\nLeftarrow$ Entanglement \`edge' states

Diagnosing a topological phase with Entanglement Spectrum



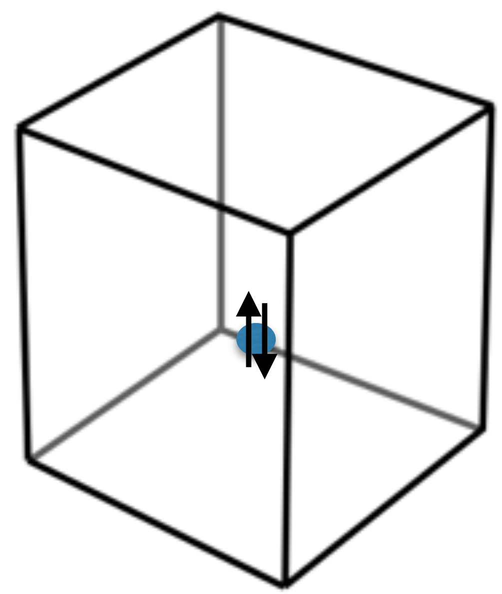
$$|\Psi\rangle = \sum_i \frac{e^{-E_i/2}}{Z} |R, i\rangle |L, i\rangle$$

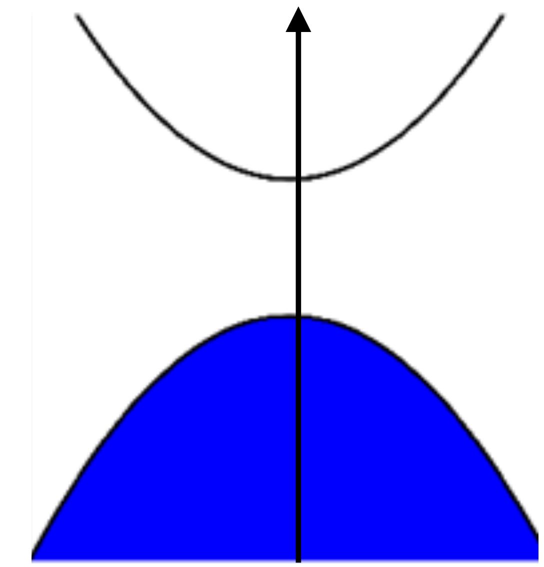


Pollman, Turner,Berg, Oshikawa
Turner, Zhang, Vishwanath
Fan, Gilbert, Bernevig

- Topological phase protected by inversion symmetry.
- Physical Edge breaks inversion - no edge state BUT entanglement spectrum secretly preserved inversion.
- Use Schmidt decomposition to come `back' to R. Antiunitary *Inv*

Filling Enforced Quantum Band Insulators

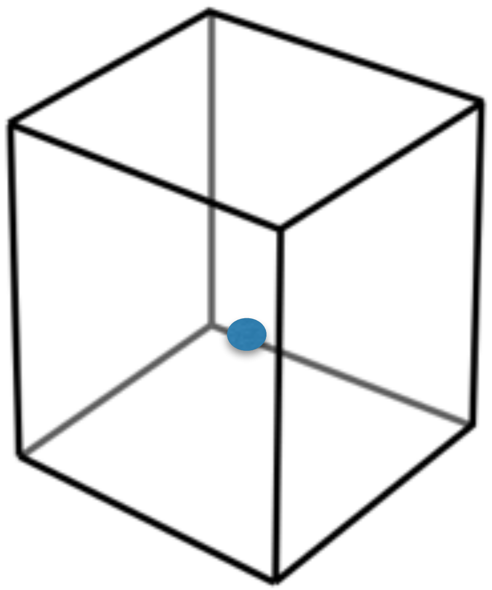

Atomic Picture



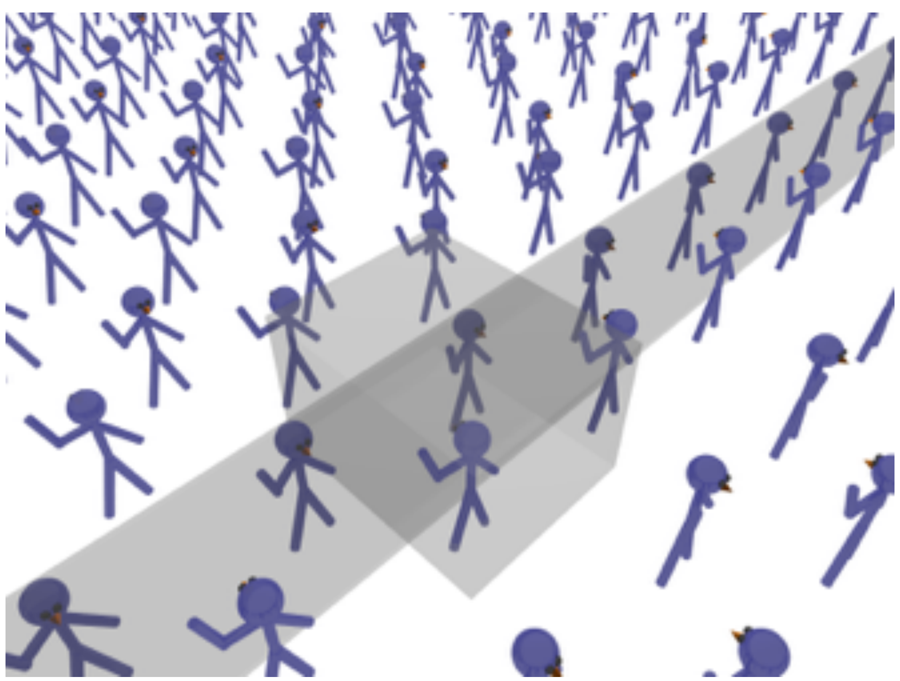
- Atomic limit of insulator - electrons localized on atomic site.
- An example where the atomic picture is forbidden by the band *filling* itself.

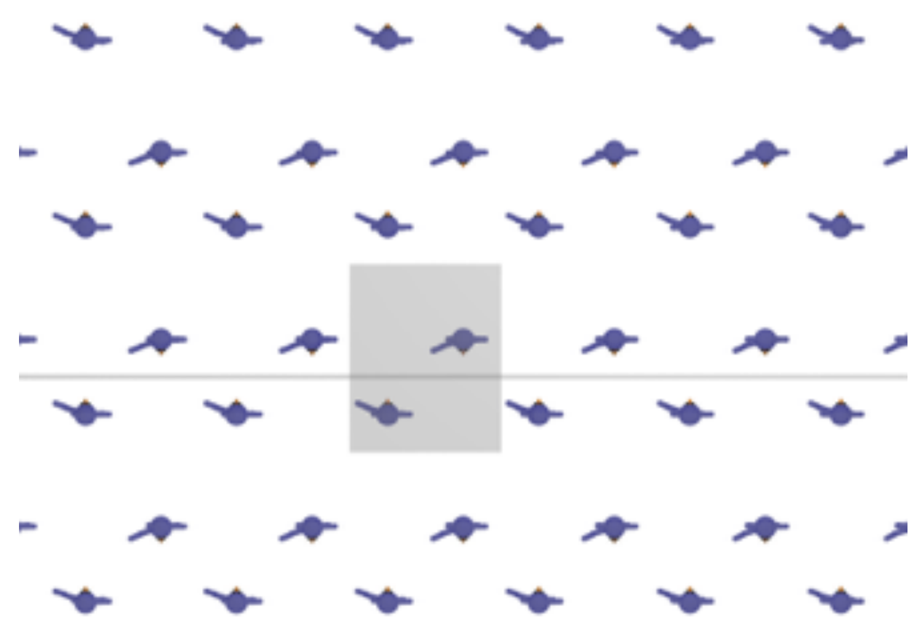
Adrian Po (Berkeley)
Haruki Watanabe (Berkeley-> MIT)
Mike Zaletel(Station Q)
(to appear)

Crystal Lattices



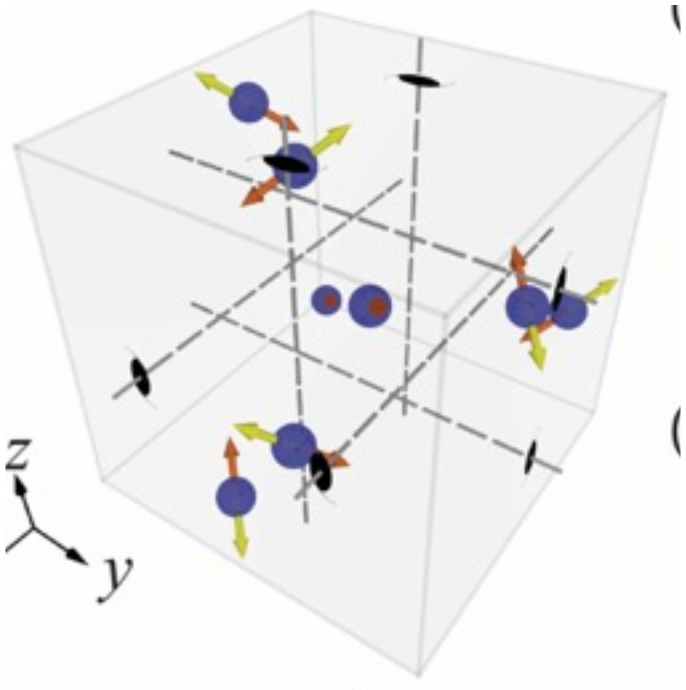
Symmorphic Lattices
Point Group x Translation

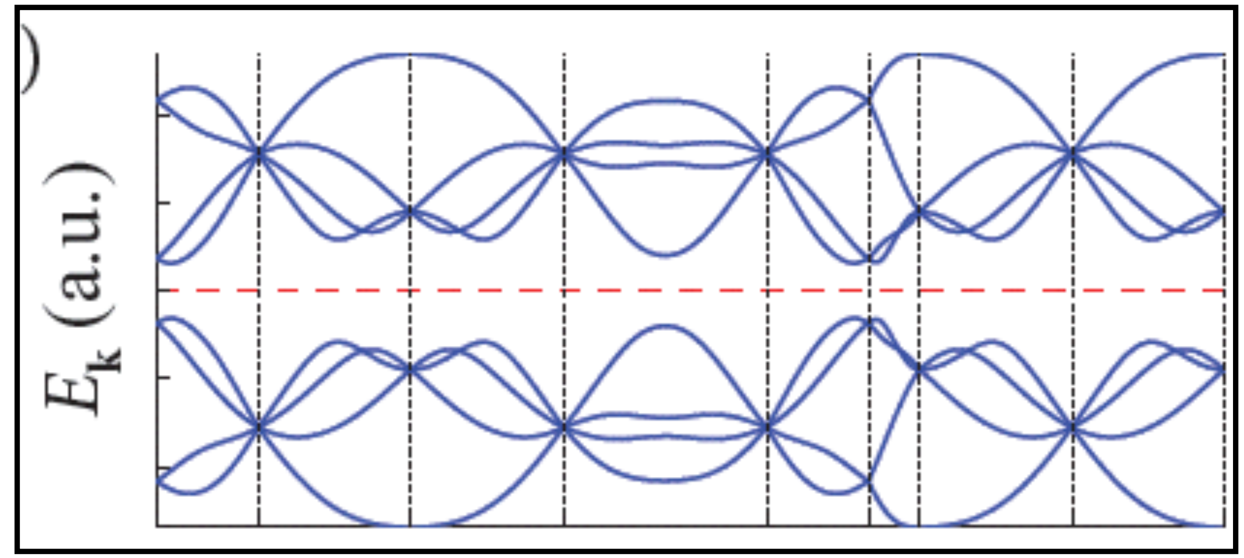




non-Symmorphic Lattice has eg. glides
(reflection+1/2 translation)
At least 2 atoms in the unit cell

Example of a Filling Enforced Quantum Band Insulator





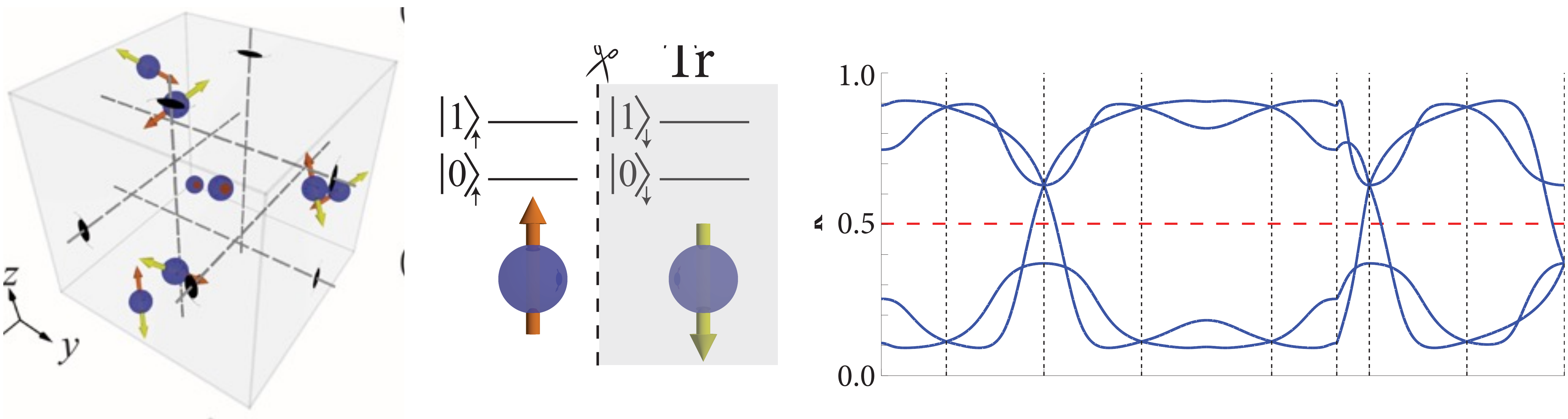
- Space Group -199 (non-symmorphic, *cubic* lattice)

- Minimum of 4 atomic sites in the unit cell - BUT - band structure with filling of 4 electrons. No atomic picture (incompatible with Time reversal and crystal symmetry - needs 2e per site).

- Requires spin-orbit coupling.

- Implies unremovable entanglement.

Entanglement Signature



- Choose entanglement cut that respects cubic symmetry - cut in spin up/down space.
- Time reversal is a unitary particle hole symmetry.
- Entanglement spectrum gapless - cannot be gapped without breaking symmetry. No `atomic' limit

Conclusions

- Entanglement provides a new way of thinking about quantum many body systems

    - How do we characterize short/long range entanglement?

    - What are the different patterns of entanglement allowed starting from local Hamiltonians?…